Independently Tunable Ultrasharp Double Fano Resonances in Coupled Plasmonic Resonator System

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A coupled plasmonic resonator system is investigated by both the finite element method (FEM) and the multimode interference coupled-mode theory (MICMT). The resonator system is made up of a cross rectangular cavity on two metal–insulator–metal (MIM) waveguides separated by a metal baffle. The asymmetric cross rectangular in the structure induces an additional Fano resonance in the transmission spectrum, which only single Fano resonance arises in the structure consisted of the symmetric cross-rectangular cavities. The positions of double Fano resonances in the transmission spectrum can be manipulated independently by changing the size of the horizontal rectangular cavity. It is of great help for designing photonic component at fixed wavelengths. The proposed structure based on our independently tunable double Fano resonances has a wide application in the sensors, splitters, switches, and nano-photonic integrated circuits devices.

Index Terms: Surface plasmons, coupled resonators, optical sensing and sensors.

1. Introduction
The Fano resonance was first described in the theory of quantum system [1]. Unlike conventional Lorentzian resonance with symmetric line shape, the shape of the Fano resonance is asymmetric and sharp, which is caused by the interference between the discrete state and the continuous state [2]–[4]. Recently, the Fano resonance in nanoplasmonic structures have attracted more attentions due to their sharp spectral responses and potential applications in the enhanced bio-chemical sensing [4]–[6], lasing, switching [7], nonlinear and slow light devices [3], [8]. Until now, varieties of nanoplasmonic structures have been proposed to generate the Fano resonance, such as ring-disk-like structures [9], [10], nanoparticle dimmers [11], [12], plasmonic nanoclusters [13], [14], stun pair in the metal-insulator-metal (MIM) waveguides [15]. Among them, the MIM waveguide structures arouse great concern because the surface-plasmon-polariton (SPP) modes in the MIM waveguide [15] can significantly overcoming the classical diffraction limit and having acceptable propagation...
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Fig. 1. Sheme diagram of the plasmonic resonator system composed of a MIM waveguide with metal nanowall and a cross resonator.

lengths, low bend losses [16], and convenience for sample fabrications. Therefore, various kinds of structures have been designed based on the MIM waveguides, which include broken symmetry in rectangular cavities [16], [17], stub coupled with resonators of different shapes [18], [19], rectangular cavity with slot resonators [20]–[22] and circle cavity with circle resonators [23]. These structures can generate single or multiple Fano resonances. Compared with single Fano resonance system, multiple Fano resonances system will have more dominance in highly integrated photonic circuits due to their parallel processing capability. Nevertheless, it is difficult to manipulate each Fano resonance independently in the transmission spectrum [17].

In this paper, an independently tunable ultra-sharp double Fano resonances system is numerically proposed with the FEM and MICMT. This structure is composed of a cross rectangular cavity and two MIM waveguides separated by a metal baffle. Both symmetric and asymmetric horizontal rectangular cavities are considered to investigate the changes of the transmission spectra. The positions of double Fano resonances in the transmission spectrum can be manipulated independently by changing the horizontal rectangular cavity. The sensitivity and figure of merit of each Fano resonance could reach to 1040 nm/RIU, 980 nm/RIU and 198.9, 197.6 in a certain condition. The proposed structure has a wide application in the highly integrated refractive index sensing area, switches, and slow-light devices.

2. Models and Theories

Fig. 1 shows the two-dimensional geometry of the proposed plasmonic structure. A cross rectangular cavity is on two MIM waveguides separated by a metal baffle. The MIM waveguide consists of an input port ($S_1$) and an output port ($S_2$). $D$ denotes the width of the cross resonator. The vertical length of the cross resonator is $H$ and the horizontal length of the cross resonator is divided to $L_1 + L_2$, respectively. $w$ stands is the width of MIM waveguides, and $t$ is the thickness of the Ag baffle. The coupling distance between the MIM waveguide and the cross resonator is $g$. The gray and white parts represent silver ($\varepsilon_{Ag}$) and air ($\varepsilon_d = 1.0$). The dielectric constant of the silver is characterized by the Drude model [24]:

$$\varepsilon_m(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{(\omega^2 + j\gamma\omega)}$$

(1)

where, $\varepsilon_\infty$ is the dielectric constant at the infinite angular frequency, $\gamma$ is the electron collision frequency, $\omega_p$ is bulk plasma frequency, and $\omega$ is the angular frequency of incident light. In our work, these parameters are set as $\varepsilon_\infty = 3.7$, $\omega_p = 9.1$ eV, $\gamma = 0.018$ eV. These parameters provide well-described empirical dielectric constant data for Ag [25], over the wavelength range of 800 ~ 1800 nm of interest. In order to excite the SPPs, the incident light is set to be transverse magnetic (TM) plane wave.

In this paper, we utilize FEM to investigate the optical responses of the proposed structure. And the transmittance is calculated by using commercial software (COMSOL Multiphysics). The
transmittance is defined as $T = P_{\text{output}}/P_{\text{input}}$ [16], where $P_{\text{output}}$ and $P_{\text{input}}$ are the SPP power flows of the output and input, respectively.

In the coupled resonator system, we usually use coupled mode theory (CMT) [26] to analyze the transmission spectrum. This theory expresses the total optical fields by a superposition of various resonant modes. For the single mode coupling, we can ignore the effect of the coupled phase by choosing the appropriate reference plane. Nevertheless, when we research multiple modes coupling, each resonant mode has different coupled phase and modulus, which can influence the transmittance. So, based on the CMT, the multiple interference coupled mode theory (MICMT) including coupled phase and modulus are given as follows: [23]

$$\frac{da_n}{dt} = -j\omega_n a_n - \left( \frac{1}{\tau_{n0}} + \frac{1}{\tau_{n1}} + \frac{1}{\tau_{n2}} \right) a_n + \kappa_n s_{n,1+} + \kappa_{n2} s_{n,2+}$$

(2)

where, $a_n$ and $\omega_n$ are the field amplitude and resonant frequency of the nth resonant mode. $\tau_{n0}$ is the decay time of internal loss of the nth resonant mode. $\tau_{n1}$, $\tau_{n2}$ are the decay time of the coupling between the resonator and waveguide ($S_1$ and $S_2$). $\kappa_n$ and $\kappa_{n2}$ are $\kappa_n = \sqrt{\frac{2}{\tau_{n1}}} e^{j\phi_n}$, and $\kappa_{n2} = \sqrt{\frac{2}{\tau_{n2}}} e^{j(\phi_n-\phi_{n1})}$, respectively. $s_{i,z} (i = 1, 2)$ stands for the normalized amplitudes of SPPs in the output and input MIM waveguides:

$$s_{1-} = -s_{1+} + \sum_n \kappa_{n1}^* a_n, \quad s_{2-} = -s_{2+} + \sum_n \kappa_{n2}^* a_n, \quad s_{n,1+} = \gamma_n e^{j\phi_n} s_{1+} \quad \text{and} \quad s_{n,2+} = \gamma_n e^{j\phi_n} s_{2+}$$

(3)

$\gamma_n e^{j\phi_n}$, $\gamma_n e^{j\phi_{n2}}$ are the normalized coefficients ($\gamma_n = \gamma_{n2} \approx 1$) $\theta_n$ and $\theta_{n2}$ are the coupling phases of the nth resonant mode. $\phi_n$ is the phase difference between output port and input port of the nth resonant mode. When SPPs are only launched into the MIM waveguide from the left side, the $s_{2+} = 0$. In this case, the complex amplitude transmission coefficient from input to output is derived as

$$t = \frac{s_{2-}}{s_{1+}} = \sum_n \frac{\gamma_n |\kappa_{n1}| |\kappa_{n2}| e^{j\phi_n}}{-j(\omega - \omega_n) + \frac{1}{\tau_{n1}} + \frac{1}{\tau_{n2}}} \quad \text{with} \quad \psi_n = \psi_{n1} + \phi_n + \theta_n - \theta_{n2}$$

(4)

Then, the transmittance $T$ can be expressed as $T = |t|^2$. $\psi_n$ is the total coupling phase difference of the nth resonant mode. In our plasmonic structure, because of the equal width of our waveguide ($S_1$ and $S_2$), $\tau_{n1} = \tau_{n2} = \tau_{ne}$ and $\theta_{n1} = \theta_{n2}$, then the transmittance formula of the plasmonic system is simplified as

$$T = |t|^2 = \left| \sum_n \frac{2e^{j\phi_n}}{-j(\omega - \omega_n) \tau_{ne} + 2 + \frac{\tau_{ne}}{\tau_n}} \right|^2, \quad \psi_n = \psi_{n1} + \phi_n$$

(5)

3. Simulation Results and Discussions

Firstly, the symmetric coupled plasmonic system is investigated. The geometrical parameters of the plasmonic system are set as $w = 50$ nm, $D = 100$ nm, $L_1 = L_2 = 200$ nm, $H = 500$ nm, $g = 10$ nm and $t = 20$ nm. The transmission spectrum is shown in Fig. 2(a) donated by blue solid line. Obviously, there are two resonant peaks in the spectrum. The right resonant peak is called as Lorentzian resonance (LR) at 1270 nm with a transmittance of 0.63, and the left resonant peak is called as Fano Resonance (FR1) at 1026 nm with a transmittance of 0.68. In order to analyze the spectral response, we use MICMT to plot the transmission spectrum, which is plotted in Fig. 2(a) with red-circle line. In order to simplify the calculation process, the complex amplitude transmission coefficient of each resonant mode is set as $t_n = 2e^{j\phi_n}/[-j(\omega - \omega_n)\tau_{ne} + 2 + \tau_{ne}/\tau_n]$. We can only obtain two strong resonant modes ($t_1$ and $t_2$) in the simulation result. So, the complex amplitude transmission coefficient of the other resonant modes uses a constant $t_0$ to represent, then $t$ can be assumed as $t = t_0 + t_1 + t_2$. We can use the curve fitting to determine the parameters in the formula, which are $t_0 = 0.24$, $\tau_{1e} = 126$ fs, $\tau_{2e} = 155$ fs, $\tau_1 = 142$ fs, $\tau_2 = 250$ fs and $\psi_1 = 0.40\pi$. 

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Fig. 2. (a) Transmission spectra based on FEM (blue line) and MICMT (red circle line) methods, The distribution of the normalized magnetic field of (b) the FR at 1026 nm and (c) the LR at 1270 nm.

Fig. 3. The transmission spectra for different lengths of \( H \) (a) and \( L \) (c). (b) The transmittance variations of LR and FR1. (d) The peak-wavelength and dip-wavelength variations with the lengths (L) of horizontal cavity.

\[ \varphi_2 = 0.52 \pi, \] respectively. As can be seen from the figure, the computation result (MICMT) is in good agreement with simulation result (FEM) at the incident wavelength of 900 nm \( \sim \) 1400 nm and it has a little deviation at the incident wavelength of 800 nm \( \sim \) 900 nm and 1400 nm \( \sim \) 1600 nm, which is caused by using \( t_0 \) to represent the sum of the transmission coefficient of other resonant modes. The distribution of the normalized magnetic field of the resonant mode (FR1 and LR) is plotted in the Fig. 2(b) and (c). We can find that the magnetic field energy of the LR is distributed in the vertical rectangular cavity. However, the magnetic field of FR1 is symmetrically distributed in the horizontal rectangular cavity of the cross cavity. Therefore, we can control the FR1 by changing geometrical parameters of the horizontal rectangular cavity.

Successively, we research on the influence of the length of the vertical rectangular cavity \( (H) \) and the symmetric horizontal rectangular cavity \( (L = 2L_1 = 2L_2) \). The transmission spectra for different lengths of \( H \) are plotted in Fig. 3(a). When \( H \) increases from 450 nm to 550 nm, LR and FR1 all have a red shift. Fig. 3(b) shows that they keep a linear relationship and the growth step of LR is bigger than FR1. And \( L \) is changed from 300 nm to 500 nm with the step of 50 nm, and the transmission
Fig. 4. (a) Transmission spectra for different lengths of $\Delta L$. The distribution of the normalized magnetic field at 898 nm, when (b) $\Delta L = 0$ nm, (c) $\Delta L = 1$ nm.

spectra are shown in Fig. 3(c), respectively. As Fig. 3(c) shown, it can be seen that the wavelength of FR1 has a red shift when the length of the horizontal rectangular cavity increases. However, the transmittance and the wavelength of the LR are substantially unchanged. Fig. 3(d) reveals the wavelength shifts of the peak and the dip with the length of the horizontal cavity. We can find that they approximately keep a linear relationship. In addition, the wavelength shifts from peak to dip decreases from 30 nm to 12 nm with the increase of the length of the horizontal cavity. The growth step of the peak wavelength and the dip wavelength are about 58.5 nm and 54.0 nm. The large tuning range of the FR1 is benefitted for designing the broad-spectrum photonic components.

In the following, we discuss the influence of the symmetry-breaking parameter of the horizontal rectangular cavity. Here, $L_1 = 180$ nm, $\Delta L = L_2 - L_1$ is the length difference between the left and right parts of the horizontal cavity, and other parameters are unchanged. $\Delta L$ is adopted as 1 nm, 10 nm, 20 nm, 30 nm, 40 nm and 50 nm, and the transmission spectra are shown in Fig. 4(a). As Fig. 4(a) shown, a new sharp Fano resonance (FR2) forms in the spectra with the increase of $\Delta L$. When $\Delta L$ changes from 1 nm to 50 nm, the transmission of FR2 becomes from 0.13 to 0.53, and the transmission of the dip is close to 0. It is obvious that the position of the FR2 changes little, especially unchanged (the shift<1 nm) with the increase of $\Delta L$ from 40 nm to 50 nm. In the illustration of Fig. 4(a), a weak Fano resonance appear at 898 nm when $\Delta L = 1$ nm. The distribution of the normalized magnetic field at 898 nm when $\Delta L = 0$ nm, $\Delta L = 1$ nm are shown in Fig. 4(b) and (c). We can see that a new anti-symmetric waveguide mode is immediately induced with slightly symmetry breaking. And with the spatial asymmetry increasing, the position of FR2 moves slightly and tends towards stability, which implies that we could obtain a stable FR2 with $\Delta L$ more than 40 nm.

Next, we make a detail analysis on the symmetry-breaking structure. We set $L_1 = 180$ nm, $L_2 = 220$ nm and other parameters are unchanged. The simulation curve and the theoretical curve of the transmittance are plotted in Fig. 5(a). As shown in Fig. 5(a), the wavelength shifts of FR1 and FR2 from peak to dip are only 14 nm and 10 nm, which is an important factor to influence the sensing property. In the theoretical analysis, we only need to decide $\tau_2$, $\tau_3$, $\tau_{2e}$, $\tau_{3e}$, $\varphi_2$, $\varphi_3$, because the breaking of the symmetry structure do not influence the wavelength and transmittance of LR. This can be verified by the distribution of the normalized magnetic field in Fig. 5(d). By the curve fitting, we can obtain that $\tau_2 = 430$ fs, $\tau_3 = 280$ fs, $\tau_{2e} = 280$ fs, $\tau_{3e} = 260$ fs, $\varphi_2 = 0.62 \pi$, $\varphi_3 = 0.63 \pi$. From the Fig. 5(a) we can see that theoretical result and simulation result are in good agreement. Fig. 5(b)–(d) show that the distribution of the normalized magnetic field of FR2, FR1 and LR. We can find that the distribution of the normalized magnetic field of LR is still unrelated to the length of the horizontal cavity. The magnetic field energy of FR1 is distributed in the vertical rectangular cavity and the longer one of the horizontal rectangular cavity ($L_2$) rather than the whole
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Fig. 5. (a) Transmission spectra based on FEM (blue line) and MICMT (red circle line) methods. The distribution of the normalized magnetic field of (b) the FR2 at 940 nm, (c) the FR1 at 1078 nm and (d) the LR at 1270 nm.

Fig. 6. Transmission spectra for different lengths of $L_1$ (a) and $L_2$ (b). The peak-wavelength and dip-wavelength variations with the lengths of $L_1$ (c) and $L_2$ (d).

structure. Moreover, the magnetic field energy of FR2 is mainly distributed in the shorter one of the horizontal rectangular cavity ($L_1$). According to the field distributions, we can independently control the resonant wavelengths of FR1 and FR2 by adjusting the length of the longer one and the shorter one of the horizontal rectangular cavity.

Next, we simulate the variation of the wavelengths of the FR1 and FR2 in the different lengths of $L_1$ and $L_2$, respectively. Firstly, we set $L_2 = 220$ nm and observe the movement of the resonant peak when the $L_1$ increases from 160 nm to 180 nm. The transmission spectra are plotted in Fig. 6(a). Only FR2 shows a red shift from 870 nm to 940 nm. The positions of FR1 and LR have no change. Next, we set $L_1 = 180$ nm and the lengths of $L_2$ are increased from 220 to 240 nm with a step of 5 nm to observe the influence on the resonant peak. The transmission spectra are shown in Fig. 6(b). In this case, only FR1 shows a red shift from 1076 nm to 1140 nm. The change of the
Fig. 7. Transmission spectra of the plasmonic resonator system for different lengths of \( g \) (a) and \( t \) (b).

Fig. 8. (a) Transmission spectra of the plasmonic resonator system with \( w = 50 \text{ nm}, D = 100 \text{ nm}, L_1 = 180 \text{ nm}, L_2 = 220 \text{ nm}, t = 20 \text{ nm} \) and different refractive indices of the dielectric in the waveguides and cavity, \( n = 1.00 \) (green line), \( n = 1.01 \) (red line). (b) The FOM of the plasmonic resonator system.

FR2 position is too small to be observed (<3 nm). Fig. 6(c) and (d) give more details analysis on the wavelength variations. Fig. 6(c) shows that the wavelength shifts of the FR2 and FR1 from the peak to the dip is around 10 nm and 14 nm with the increase of \( L_1 \), which can provide a stable and high sensitivity in the spectrum. In Fig. 6(d), the wavelength shifts of the FR1 from the peak to the dip decrease from 14 nm to 8 nm with the increase of \( L_2 \). In addition, the wavelength shifts of the FR2 from the peak to the dip increase slightly (<3nm). The adjustment of the two Fano resonances almost does not interfere with each other. Therefore, this kind of tuning is called 'independent tuning'. We can use the independently-tunable ability to achieve the precise control of spectrum. It is of great help for designing highly integrated photonic circuits component at fixed wavelengths.

In consideration of the device fabrication, the effect of coupling distance \( g \) and width of the metal baffle \( t \) on double Fano resonances are analyzed. The other parameters are the same as Fig. 5(a). The transmission spectra of the structure with different \( g \) are shown in Fig. 7(a). We can see that the resonant wavelength of LR shows an obvious blue-shift and the resonant wavelength of FR1 and FR2 have a little change. But the coupling distance \( g \) had great influence on the transmittance. In comparison with coupling distance \( g \), the width of the metal baffle \( t \) has insignificant effect on the transmittance and the resonant wavelength, as shown in Fig. 7(b). This shows our structure has high expectations for the coupling distance \( g \).

Our plasmonic double Fano resonances system has great potential as a high refractive index sensitivity sensor due to its sharp asymmetric spectra. Therefore, we investigate the transmission spectra with different refractive index \( n \). The result is shown in Fig. 8(a). It can be seen that the three resonant peaks have a red shift when \( n \) increases from 1.00 to 1.01. The sensitivity (nm/RIU) is defined as the shifts in the resonance wavelength per unit variation of the refractive index. The
sensitivity of LR, FR1 and FR2 are 1200 nm/RIU, 1040 nm/RIU, 980 nm/RIU, which are superior to [23] (840 nm/RIU), [27] (900 nm/RIU) and [30] (900 nm/RIU). Moreover, to better evaluate the sensor performance of our structure, the figure of merit \( (FOM) \) is investigated in the Fig. 8(b). \( FOM \) is defined as: [28]

\[
FOM = \frac{\Delta T/\Delta n}{T}
\]

where \( T \) can be assumed as \( T = 1/2 \times (T_{1.00} + T_{1.01}) \). Fig. 8(b) shows the calculated \( FOM \) at different wave-lengths when the refractive index changes from 1.00 to 1.01. The \( FOM^* \) of FR1 and FR2 are as high as 198.9 and 197.6, which is much higher than previous reports (100 in [23], 87 in [29]). The great \( FOM^* \) is due to the narrow wavelength shifts between the peak and dip and large sensitivity.

Next, we calculate the variation of the sensitivity and \( FOM^* \) of different parameters. As shown in Fig. 9(a) and (b), with the increase of the \( L_1 \) and \( L_2 \), the sensitivity of FR1 and FR2 increase slightly. And the \( FOM^* \) of FR1 and FR2 change slightly, no more than 5\%. This means that the sensitivity and \( FOM^* \) of the double Fano resonances keep high value in the tuning process. Fig. 9(c) and (d) shows that coupling distance \( g \) had great influence on the \( FOM^* \) due to drastic changes of transmittance. But when \( g = 25 \) nm, the \( FOM^* \) of FR1 is still large than 159.

4. Conclusion
In summary, we design an asymmetric structure consisted of a cross rectangular cavity and two MIM waveguides separated by a metal baffle, which can achieve independently tunable and sharp double Fano resonances. Firstly, we use MICMT to analyze the Fano resonance in the symmetric and asymmetric structure and get consistent result with simulation. The transmittance decreases fast from peak to dip in FR1 and FR2 with wavelength shifts of only 14 nm and 10 nm. In addition, we find that the position of the new Fano peak moves slightly and tends towards stability when the spatial asymmetry increases. Moreover, compared with other multiple Fano resonances system, we
can manipulate each Fano resonance on the transmission spectra independently by changing the horizontal rectangular cavity. Based on the sharp Fano resonance, the sensitivity and \( FOM' \) of FR1 and FR2 can be as high as 1040 nm/RIU, 980 nm/RIU and 198.9, 197.6, respectively. Therefore, this structure is very promising for the applications in sensors, splitters, switches and nano-photonic integrated circuits devices.

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