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Parallelized Kalman Filters for Mitigation of the Excess Phase Noise of Fast Tunable Lasers in Coherent Optical Communication Systems

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Abstract: Numerical and experimental investigations are carried out on the performance of parallelized Kalman filters applied for mitigation of the excess phase noise of fast tunable lasers. Based on the characterization of the phase noise of a sampled grating distributed Bragg reflector (SG-DBR) laser, the proposed carrier phase recovery (CPR) scheme using Kalman filters is introduced. By performing simulations of data transmission with various advanced modulation formats in the presence of the excess phase noise, the Kalman filter based CPR scheme shows its ability to overcome the excess phase noise and this method is suitable for parallel processing. Then the results are further demonstrated by 12.5 Gbaud QPSK and 16-QAM transmission experiments employing the SG-DBR laser. We find that the Kalman filters have better performance than the 2nd-order phase-locked loop in parallel systems due to a better phase noise tolerance. The bit error rate performance is also examined in the whole tuning range (∼30 nm) of the tunable laser, which further proves the feasibility of the proposed scheme.

Index Terms: Tunable laser, Kalman filter, carrier phase recovery, coherent communication.

1. Introduction
Semiconductor tunable lasers can eliminate many of the limitations of fixed wavelength lasers, and enable new system architectures with increased functionality [1], [2]. This makes tunable lasers a key component in next-generation optical networks [3]. However, as a result of fast-growing internet traffic, high-speed optical fiber communication systems are adopting advanced modulation formats to achieve enhanced spectral efficiency (SE) [4]. When the data is modulated on the phase and amplitude of the laser output simultaneously, the phase noise of the transmitters becomes an important factor in determining the bit error rate (BER) performance of the coherent optical transmission systems [5], [6].
In order to track and compensate the phase noise of the laser source, carrier phase recovery (CPR) is one important part of the digital signal processing (DSP) based coherent receiver [7]. In general, the phase noise of a semiconductor laser can be modeled as a Wiener process and the frequency modulation (FM) noise spectrum is pure white noise [8]. This ideal case is usually assumed in the CPR algorithm design and it can work well for most lasers with fixed wavelength [9]–[11]. However, for the monolithic tunable lasers, the electronic tuning mechanism is commonly used and can achieve fast tuning speed (~ns) [2]. Due to the carrier noise in the passive sections, the electronic tuning normally results in significant additional phase noise, which has already been studied by the previous experiments [12], [13] and simulations [14], [15]. It has been found that the transmission performance is significantly degraded due to the excess phase noise of the fast tunable laser, especially for higher-order modulation formats (i.e., 16-QAM) [16]. Therefore, CPR improvements are necessary to cope with the excess phase noise of the fast tunable lasers, such as a scheme of 2nd-order phase-locked loop (PLL) [17]. However, massive parallelization and pipelining is usually used in real-time DSP processing to achieve a higher data throughput and hardware-efficiency, so the feedback loop will result in poor phase noise tolerance [9].

In this paper, a parallelized architecture implementing linear Kalman filters is proposed to mitigate the excess phase noise from a sampled grating distributed Bragg reflector (SG-DBR) laser. Previous research has investigated the application of Kalman filters for polarization state tracking and phase noise mitigation with fixed wavelength lasers [18]–[20], whereas in this current work we use Kalman filters with fast tunable lasers that have different phase noise properties. The CPR is first modified based on the phase noise characterization, and then simulations are performed to investigate the performance of the proposed scheme, which is further demonstrated by data transmission experiments.

2. Kalman Filter Based Carrier Phase Recovery

The phase noise of the SG-DBR laser is initially measured and analyzed, then the state parameter in the Kalman filters is adjusted to handle the phase noise problem according to the characterization. Fig. 1(a) shows the FM-noise spectrum measured by a coherent phase noise measurement technique [21]. Compared with the external cavity laser (ECL), the FM-noise spectrum of the SG-DBR laser is found to be enhanced considerably, especially in the low frequency range, which is known to be introduced by the carrier noise in the tuning sections [15]. In order to examine the
influence of this low frequency excess phase noise of the SG-DBR laser, the total measurement time of 40 $\mu$s is divided into several sections, the field spectrum in each section is then calculated and the center frequency is extracted, as presented in Fig. 1(b). For the ideal phase noise model, the center frequency is assumed unchanged. Then we find that the center frequency of the ECL is moving slowly and in the frequency range of $-2$ MHz to 2 MHz during the 40 $\mu$s time interval, which is similar to the ideal case. However, for the SG-DBR laser, due to the carrier noise and the low-pass property of the passive sections [15], the center frequency is fluctuating between $-20$ MHz and 20 MHz, making the phase noise cannot be approximated well by the ideal phase noise model. As most CPR algorithms are designed with only the white noise taken into account, this considerable frequency fluctuation makes it difficult to track the phase noise of the SG-DBR laser.

Kalman filter equations provide an optimal solution to the linear tracking problem [18]–[20], and the state parameter is adjusted based on the analysis instant. The block representation of the proposed Kalman filter based CPR is shown in Fig. 2. According to the phase noise characterization and analysis, both the frequency mismatch ($\omega_k$) and the phase mismatch ($\theta_k$) need to be estimated by the Kalman filters to overcome the excess phase noise of the fast tunable lasers. So the state parameter vector is set as $x_k = [\theta_k, \omega_k]^T$. For parallel processing, the received symbols are first separated into blocks. It is assumed that the frequency offset of the symbols in the same block is unchanged, then the carrier phase at the $n$th sample of the $k$th block can be obtained by

$$\varphi_{n,k} = \theta_k + \left(n - \frac{N + 1}{2}\right)\omega_k$$  \hspace{1cm} (1)

where $N$ is the block size.

At the prediction stage, the prior estimate of the parameters are obtained from that of the previous block based on the relationship between the phase and frequency, expressed as

$$\bar{\omega}_k^+ = \omega_{k-1}, \hspace{0.5cm} \bar{\theta}_k^+ = \theta_{k-1} + N\omega_{k-1}$$  \hspace{1cm} (2)

where $\bar{\theta}_k^+$ and $\bar{\omega}_k^+$ are the prior estimate of the phase and frequency, respectively. Therefore, the system evolution equation can be written as

$$\begin{bmatrix} \bar{\theta}_k^+ \\ \bar{\omega}_k^+ \end{bmatrix} = \begin{bmatrix} 1 & N \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{k-1} \\ \omega_{k-1} \end{bmatrix} + \begin{bmatrix} n_\theta \\ n_\omega \end{bmatrix}$$  \hspace{1cm} (3)

where $n_\theta$ and $n_\omega$ are the system noise for the phase and frequency, which are zero-mean normally distributed random variables having variance $\sigma_\theta^2$ and $\sigma_\omega^2$, respectively. And the expressions for $A$ and $B$ are

$$A = \begin{bmatrix} 1 & N \\ 0 & 1 \end{bmatrix}, \hspace{0.5cm} B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$  \hspace{1cm} (4)
Hence, the prior estimate of the state vector $x_{k-1}^-$ and its error covariance matrix $P_{k-1}^-$ are updated and obtained by

$$
\begin{align*}
    x_{k}^- &= Ax_{k-1}, \\
    P_{k}^- &= AP_{k-1}A^T + BH B^T
\end{align*}
$$

(5)

where $H = \begin{bmatrix} \sigma_0^2 & 0; 0, \sigma_0^2 \end{bmatrix}$ is the covariance matrix of the system noise. So the carrier phase is initially corrected based on the prior estimate of the state parameter and (1).

Then at the correction stage, the measurement noise still contributes to some estimation mismatch, which gives the description of the observation equation

$$
\begin{pmatrix}
    \tilde{\theta}_k \\
    \tilde{\omega}_k
\end{pmatrix} = \begin{pmatrix}
    \theta_k \\
    \omega_k
\end{pmatrix} + \begin{pmatrix}
    m_\theta \\
    m_\omega
\end{pmatrix}
$$

(6)

where $\tilde{\theta}_k$ and $\tilde{\omega}_k$ are defined as the observed phase and frequency offset, respectively. $m_\theta = \begin{bmatrix} m_\theta; m_\omega \end{bmatrix}^T$ accounts for the observation noise vector. Therefore, the posterior estimate of the state vector $x_k$ and its error covariance $P_k$ are corrected by

$$
\begin{align*}
    x_k &= x_{k}^- + G_k e_k, \\
    P_k &= P_{k}^- + (I - G_k) P_{k}^-
\end{align*}
$$

(7)

where $e_k$ represents the error between the predicted state and the measured state obtained by hard decisions, in which both the phase error and frequency error are estimated. $G_k$ is the Kalman gain, given by

$$
G_k = P_{k}^- (P_{k}^- + R)^{-1}
$$

(8)

where $R$ is the covariance of the observation noise $m_\theta$. After initialization, the state parameter for each block is obtained by (5) and (7), then applying the estimated phase and frequency to (1), the carrier phase for each symbol in the block is calculated and compensated.

### 3. Simulation and Analysis

The performance of the proposed scheme is investigated by simulations in this section. Fig. 3 shows the simulation setup. For simplicity, it is assumed that the optical output signal from the laser has negligible amplitude noise and the modulated signal is only degraded by additive white Gaussian noise and laser phase noise, expressed as

$$
r(k) = c(k) \exp(j(2\pi \omega(k) k T_ + \theta(k))) + n(k)
$$

(9)

where $c(k)$ is the ideal signal after modulation, $\omega(k)$ is the frequency offset between the signal and the local oscillator (LO), $T$ is the symbol interval, $\theta(k)$ is the sampled phase noise, $n(k)$ accounts for the white Gaussian noise, which is usually quantified by optical signal-noise-ratio (OSNR). As we are focused on the influence of the low frequency excess phase noise from the SG-DBR laser, the FM-noise spectrum without consideration of the resonance peak can be represented by

$$
S(f) = S_0 + S_p(f)
$$

(10)

where $S_0$ represents the white noise from the active section and $S_p(f)$ accounts for the phase noise introduced by the passive sections. Since the passive sections are sensitive to the $1/f$ noise [15] and their frequency response can be described as a low-pass filter, $S_p(f)$ is expressed as

$$
S_p(f) = K f_c \cdot \frac{1}{f + f_c}
$$

(11)

where $K$ represents the level of the low frequency excess noise and $f_c$ is the cut-off frequency of the passive sections. As shown in Fig. 3, the low frequency excess phase noise is generated as follows: first a white noise signal is generated in time domain and then passes through a digital filter that has a transfer function the same as (11), then another noise signal representing the white noise ($S_0$) is combined with the excess phase noise to construct the phase noise of the SG-DBR laser. The FM-noise spectrum of the generated phase noise is calculated and shown in the inset.
Fig. 3. Simulation setup to investigate the performance of the proposed Kalman filter based carrier phase recovery scheme.

of Fig. 3, which is also compared with the pure white noise and measured SG-DBR laser phase noise. By fitting the coefficients in (11), we can find a good agreement between the simulation and measurement. After that, the generated signal from (9) is applied to do DSP. After normalization, the frequency offset is corrected by a fast Fourier transform (FFT) method, and then the Kalman filter based CPR described in the previous section is applied to estimate and compensate the carrier phase. Finally, decoding is performed and the BER can be calculated and analyzed.

To start the CPR process shown in Fig. 2, some parameters need to be initialized. $R$ is dependent on the phase noise and the received OSNR, which can be updated by several samples of $\epsilon_k$, so it can automatically converge after several symbols. However, $H = \sigma^2_0, 0, 0$ is set as a constant in the demodulation process and should be optimized based on the variance of the phase difference between two consecutive blocks. In the simulations, $\sigma^2_0 = 10^{-7}$ is used and the optimal $\sigma^2_0$ is investigated. In Fig. 4, a 12.5 Gbaud 16-QAM transmission simulation is performed and the BER performance is analyzed at a fixed OSNR (16 dB). The BER as a function of $\sigma^2_0$ with different phase noise level is calculated and shown in Fig. 4(a). It indicates that the optimal $\sigma^2_0$ is both dependent on the white noise level ($S_0$) and excess phase noise level ($K$). Larger phase noise usually leads to an increase of the optimal $\sigma^2_0$. By changing the block size ($N$), the BER versus $\sigma^2_0$ is also obtained and presented in Fig. 4(b), we find that the optimal $\sigma^2_0$ increases with the increase of $N$, that is because the time interval is increasing with the block size, resulting in a larger phase difference between two consecutive blocks.

After applying the optimal parameters to the Kalman filters, the ability to mitigate the excess phase noise of the fast tunable lasers is examined at a fixed block size ($N = 4$). As we known, most standard CPR schemes designed based on the ideal phase noise model, in which the frequency is assumed to be zero, usually only estimate and compensate the phase change [10], [22]. For comparison, we employ Kalman filters whose state parameter is set as $x_k = [\theta_k]$ to represent a standard scheme. By keeping the white noise level ($S_0 = 10^5$ Hz$^2$/Hz) unchanged, Fig. 5(a) compared the BER performance of 16-QAM signals demodulated by the proposed scheme with that by the standard scheme. For the proposed scheme, the signals are demodulated successfully with
Fig. 4. BER vs. variable $\sigma_p^2$ with (a) different phase noise level and (b) different block size.

Fig. 5. Performance of the Kalman filters for mitigation of the excess phase noise including (a) BER vs. $K$ and (b) transmission Q factor vs. $S_0$.

$K$ increased from $1 \times 10^{20}$ Hz$^3$ to $5 \times 10^{21}$ Hz$^3$. However, the transmission fails when $K$ is larger than $1 \times 10^{21}$ Hz$^3$ for the standard scheme, which means it cannot track the phase noise of the SG-DBR laser, represented by the magenta dotted line with the fitting parameter in Fig. 3. Therefore, the analysis in the previous section is verified and the ability to overcome the low frequency excess phase noise is enhanced for the proposed Kalman filter based CPR. Then the BER performance simulation of 12.5 Gbaud QPSK, 16-QAM and 64-QAM transmission with different types of phase noise are carried out, as presented in Fig. 5(b). The graph of Q factor as a function of the white noise level with and without the low frequency excess phase noise ($K = 2 \times 10^{21}$ Hz$^3$) are shown in Fig. 5(b). The Q factor is calculated on the basis of BER:

$$ Q \ (dB) = 20 \ log_{10} \left( \sqrt{2}erfc^{-1} \ (2 \ BER) \right) $$

It is found that the Q factor is decreased with the increase of $S_0$, which is more obvious for higher-order modulation formats. We also observe that the transmission performance with the excess phase noise taken into account coincides with the case without the excess phase noise for all
the modulation formats. This further proves the excess phase noise in the low frequency range is mitigated by the proposed CPR scheme.

By taking the phase noise with the fitting parameters in Fig. 3 into the generated signal, the BER performance against the received OSNR with various block size are simulated. For the phase noise of the SG-DBR laser, the white noise is fitted with $S_0 = 10^{5} \text{Hz}^2/\text{Hz}$. From the simulation process in Fig. 5, we notice that when $S_0 = 10^5 \text{Hz}^2/\text{Hz}$, representing by the magenta dotted line, the phase noise level is too high to transmit 64-QAM signals. Therefore, only QPSK and 16-QAM signals are successfully demodulated at various OSNR for the given block size in Fig. 6. The signals are demodulated without degradation with $N$ varied from 2 to 8, which indicates that the Kalman filters can work well in parallel systems.

The $Q$ factor as a function of the block size is subsequently investigated and shown in Fig. 7. From the transmission performance of QPSK with excess phase noise ($K = 2.5 \times 10^{21}$ Hz$^3$) at OSNR = 6 and 9 dB in Fig. 7(a), there is an upper limit for $N$. When $N$ is below the limit, the $Q$ factor shows no degradation, while the CPR algorithm usually loses tracking with $N$ beyond the limit, resulting in decreasing $Q$ factor and degradation of system performance. Compared with the results with different OSNR, the limitation on $N$ is also changed. In general, high OSNR can get a larger upper limit for $N$. Because when OSNR is low, the serious additive white noise leads to more inaccurate estimations and the CPR process is more likely to lose track. Since 16-QAM signals are
more sensitive to the phase noise, the limitation on $N$ is reduced, as suggested in Fig. 7(b). We also find that the limitation on $N$ is extended considerably when the low-frequency excess phase noise is not included, which means the parallelized architectures need to be adjusted based on the phase noise level in practical applications.

As the 2nd-order PLL [17] has also been used to overcome the phase noise of the fast tunable lasers, the sensitivity penalty of 16-QAM signals at BER $= 1 \times 10^{-3}$ of the proposed Kalman filter based CPR against the linewidth times symbol duration product is compared with that of the 2nd-order PLL. The linewidth $\Delta v$ is calculated by the white noise level with a simple relation: $\Delta v = \pi \cdot S_0$ [23]. As shown in Fig. 8, for the Kalman filters, when the block length is increased from $N = 2$ to $N = 8$, the sensitivity is nearly unchanged, which coincides with the results in Figs. 6 and 7. For the 2nd-order PLL, the feedback delay $\Delta$ plays an important role in determining the phase noise tolerance of the receiver [9], [24]. When $\Delta = 1$, the sensitivity is similar to that of the Kalman filter, but when the feedback delay increases to $\Delta = 2$, the sensitivity suffers a substantial penalty. To achieve a sensitivity penalty of 1 dB, the linewidth times symbol duration product is about $1.2 \times 10^{-5}$ and $5 \times 10^{-5}$ for $\Delta = 1$ and 2, respectively. Since $\Delta = 1$ means the signals are processed symbol-by-symbol, it is impossible to achieve $\Delta = 1$ in a parallelized implementation, which means the Kalman filter based CPR scheme is a better choice for the parallelized systems and practical applications.

4. Experiment Setup and Results
The performance of the proposed CPR scheme is further examined by coherent optical communication experiments in this section. The setup is shown in Fig. 9. The SG-DBR laser is controlled by a four channel low noise current source. The laser output is modulated by an IQ modulator driven...
by an arbitrary waveform generator (AWG), which generates 12.5-Gbaud QPSK and 16-QAM signals. This optical signal is then coupled with a variable level of amplified spontaneous noise (ASE) generated by the EDFA to set the OSNR level. The resulting signal is subsequently split and fed into the optical spectrum analyzer (OSA) to measure the OSNR and the optical coherent receiver from which the electrical outputs are captured by a high-speed real-time scope at 50 Gs/s. Finally, the captured data is processed by offline DSP and the BER is calculated based on $1 \times 10^6$ symbols.

The BER performance against the received OSNR at a fixed wavelength (1555.747 nm) is first measured. By changing the block size $N$, the calculated BER is presented in Fig. 10(a). Both QPSK and 16-QAM signals are successfully transmitted. For $N = 2$, 4, and 8, the signals are demodulated without degradation, which is consistent with the simulation in Fig. 6. The required OSNR at the 7% FEC limit (BER = $3.8 \times 10^{-3}$) is nearly 8 dB and 17 dB for the QPSK and 16-QAM, respectively. Due to implementation penalty, this OSNR requirement is about 2 and 4.5 dB off the theoretically predicted performance for QPSK and 16-QAM, respectively. The performance of the Kalman filters is also compared with that of 2nd-order PLL and least mean square (LMS) [25], as shown in Fig. 10(b) and (c). For the LMS, which is designed based on the ideal phase noise model, the carrier phase cannot be tracked correctly and the tunable laser fails to transmit QPSK and 16-QAM signals. For the 2nd-order PLL, the feedback delay is set as $\Delta = 2$ for comparison. From the BER curves, the Kalman filter based CPR can obtain a better performance as the phase noise tolerance of the Kalman filters is larger than that of the 2nd-order PLL, as indicated in Fig. 8. This better tolerance can be observed more clearly by the constellation diagrams demodulated by these two algorithms at the same OSNR. As the 16-QAM signal is more sensitive to the phase noise, more improvement can be found.

The limitation of the block size on the system performance is then verified. Fig. 11 shows the measured Q factors and some constellation diagrams of QPSK and 16-QAM signals against $N$. As suggested in Fig. 7, the upper limit of operable block size for the signals is also observed in the experiments. From the constellation diagrams, the receiver signal distributions have more residual phase noise with the increase of $N$. That is because the assumption that the frequency offset is a constant in the same block results in more errors due to the random walk property of the phase noise with the increase of $N$, resulting in the limitation on $N$. From the results of QPSK transmission in Fig. 11(a), when OSNR = 12 dB, the upper limit is 30, but this limit reduces to 12 for OSNR = 7 dB. Comparing with the results for 16-QAM in Fig. 11(b), the largest operable $N$ is reduced, which is only $N = 7$ for OSNR = 27 dB and $N = 5$ for OSNR = 17 dB. In addition, these limitations are reduced by comparing with the simulation results in Fig. 7, which is caused by implementation penalty as well.

We also test the performance of the SG-DBR laser at 80 channels in the 30 nm tuning range (1529.163–1560.606 nm). As shown in Fig. 12, the transmission of QPSK ($N = 8$) and 16-QAM
Fig. 11. Q factor against block size and some constellation diagrams for 12.5-Gbaud (a) QPSK and (b) 16-QAM signals.

Fig. 12. Measured BER performance against the output wavelength of the SG-DBR laser.

\( (N = 4) \) are both performed. The signals can be demodulated correctly with the BER under the 7% FEC limit by the Kalman filter based CPR algorithm for all the wavelengths, further proving the feasibility of the proposed scheme

5. Conclusion

In this paper, we demonstrate the application of Kalman filters in the CPR part of the DSP based coherent receivers to mitigate the excess phase noise of the fast tunable lasers. Phase noise characterization is first performed, which suggests that the excess phase noise can be overcome by estimating both the frequency and phase in the CPR process. The state parameter of the Kalman filters is then modified according to the analysis. Afterwards, simulations are carried out to investigate the performance of the Kalman filter based CPR algorithm, which indicates the proposed scheme can track the excess phase noise from the SG-DBR laser and it is suitable for parallel processing. These results are further demonstrated by data transmission experiments, which shows good agreement with the simulations. Thus, the analysis about the excess phase noise is meaningful and the Kalman filter based CPR will be a good choice in the coherent optical networks employing fast tunable lasers or optical sources with excess low frequency phase noise.
References


