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Abstract: An excessively tilted LPFG-assisted surface plasmon polaritons sensor (Ex-TLPFG assisted SPP sensor) with ultrahigh sensitivity is proposed and numerically investigated using the finite-element-method-based full-vector complex coupled mode theory. We show that the SPP mode is transited (or excited) gradually from both the p-polarized TM\textsubscript{0,j} and EH\textsubscript{v,j} (v ≥ 1) modes, and hence, the proposed SPP sensor can be tuned to achieve strong resonance of either the degenerate TM\textsubscript{0,j} and EH\textsubscript{2,j} modes or EH\textsubscript{1,j} mode to optimize the sensitivity for analyte refractive index sensing. The results confirm that a transition point corresponding to the phase matching curve of the SPP mode is obtained, which can be used to predict the optimized grating period. By this approach, ultrasensitive SPP refractometric sensor can be obtained and the sensitivity can be further improved through a simple method: reducing the fiber cladding combined with an optimized grating period. We demonstrate that a giant sensitivity as high as 10\textsuperscript{100} nm/RIU is achieved for the degenerate TM\textsubscript{0,32} and EH\textsubscript{2,32} modes (or 7400 nm/RIU for the EH\textsubscript{1,32} mode). These appealing characteristics make the proposed Ex-TLPFG-assisted SPP sensor ideal for biochemical analyte sensing applications.

Index Terms: Tilted fiber gratings, surface plasmon polaritons, refractive index sensing, sensitivity optimization.

1. Introduction
Surface plasmon polariton (SPP) based optical fiber sensors, due to its high sensitivity, good portability and strong robustness, have emerged as a highly-efficient tool in the areas of label-free biochemical and environmental measurements in recent years [1], [2]. The SPPs, in its simplest form, are resulted from the oscillation of free electrons on the metal surface, which is closely related to the frequency of the light propagating in the fiber [3]. The SPP mode is an electromagnetic
resonance that propagates in a wave along the metal surface that is often immersed into the surrounding analytes (such as gas and solution), and whose amplitude decays exponentially with increasing distance into surrounding medium from the interface between the metal and the surrounding medium. Thus, a much higher interaction of the electromagnetic field with the surrounding analyte can be obtained for the SPP based fiber sensors than that for the conventional fiber sensors [4], [5]. This enhancement of interaction leads to an extraordinary sensitivity to the changes of the surrounding conditions.

Apparently, the sensing performance of the SPP based fiber devices is dependent significantly on the excitation of the SPP mode in the fiber. The most intuitive approach to excite the SPP mode in metal coated fiber is to use the cladding removal or cladding decrease architectures (i.e., structure modified fiber). As the fiber structure is modified, the guided mode that is confined within the core of standard fiber is brought into contact directly with the metal layer and hence to match the oscillation frequency. On this basis, numerous configurations are proposed in recent years, such as side-polished fiber or D-shaped fiber [6], [7], U-shaped fiber [8], and tapered fiber [9], etc. The SPP sensor based on the structure modified fibers can be easily fabricated and constitute a simple and portable sensing configuration, which promotes greatly the development of the SPP devices, as compared to the Kretschmann-Raether prism scheme [10]. However, they are in general characterized by lower mechanical strength and a broadband resonance in the transmission spectrum, which limits the sensing resolution and hence prevents their practical applications.

To improve the excitation of the SPP mode in the fiber with structural integrity, fiber grating assisted surface plasmon resonance sensors have been gaining greatly interest, due to numerous advantages over the structure modified fiber based SPP sensors, such as narrow band resonance, high resolution and self-referencing capability. Fiber gratings are passive fiber devices that have periodic refractive index perturbation in fiber core, cladding or both of them [11]. This periodic perturbation enables a strong coupling between the guided core modes and cladding modes. Thus the SPP mode can be always efficiently excited by the cladding modes of which the field penetrates into the surrounding medium in a form of evanescent wave when the grating region is coated with a metal layer. Recently, two major kinds of fiber grating assisted SPP sensors, i.e., the TFBG assisted SPP sensors and the LPFG assisted SPP sensors, have found applications in label-free biochemical detection [5].

TFBGs are short period fiber Bragg gratings with the index perturbation angled with respect to the fiber axis (with a typical tilted angle of $8^\circ \sim 10^\circ$). The tilted index perturbation enables the TFBGs coupling the guided forward-going core mode to both the backward-propagating core mode and a large number of cladding modes that are utilized to excite the SPP modes. As a result, the transmission spectrum is most generally composed of tens of resonance bands corresponding to backward-going cladding modes, the SPP modes and the guided core mode respectively [12]. In addition, since the tilted index perturbation breaks the symmetry of the single mode fiber, the resonance is highly polarization dependent. Hence, only the radially polarized mode with the electric field perpendicular to the metal interface can excite the SPP modes [13]. The resonance of the TFBG assisted SPP sensors displays very much narrower band (with a typical FWHM of 100 $\sim$ 200 pm) than that for the structure modified fiber based SPP resonance [14]. Therefore, TFBG assisted SPP sensors exhibit a much higher figure of merit (FOM) than that for other fiber based SPP sensors, which facilitates greatly the exact measurement of the resonance location [5]. On this basis various TFBG assisted SPP sensors have been proposed in label-free biological and biochemical applications [15].

LPFGs have the periodic index perturbation of a few hundreds of micrometers. This periodic perturbation gives rise to the co-directional coupling between the guided core mode and the higher order cladding modes. As the LPFGs are combined with a thin metal film, such as the silver and gold, the grating can couple the guided core mode to the SPP modes that can be observed in the metal interface [16], [17]. Since the changes in surrounding medium have a greater influence on higher order cladding modes than the guided core mode, much higher sensitivity can be achieved for the LPFG assisted SPP sensors compared to the TFBG assisted SPP sensors [18]–[20]. The sensing performance of the LPFG assisted SPP sensors is significantly dependent upon the influence of...
surrounding medium on the mode coupling between the guided core mode and HE/EH$_{1,j}$ cladding modes. However, LPFGs can also couple the guided core mode to the higher order degenerate TE/TM$_{0,j}$ and HE/EH$_{v,j}$ ($v \geq 2$) cladding modes, especially for the excessively tilted Ex-LPFGs (Ex-TLPFGs) as well as the non-uniform perturbation LPFGs [11], [21]. The Ex-TLPFGs are a kind of LPFG of which the grating is tilted, typically with the tilted angle as much as $\sim 81^\circ$ or greater, and hence strong coupling between the guided core mode and the degenerate modes TE/TM$_{0,j}$ and HE/EH$_{v,j}$ ($v \geq 2$) can be observed [22]. Zhang et al. reported a series of researches on the Ex-TLPFG involving the unique property of polarization-dependent mode coupling and the applications in bio-analyte sensing [23]–[29]. They focused on the mode coupling between the guided core mode and polarized TE/TM$_{0,j}$ cladding modes of the Ex-TLPFGs. As compared to HE/EH$_{1,j}$ cladding modes, the field of degenerate modes TE/TM$_{0,j}$ and HE/EH$_{v,j}$ ($v \geq 2$) distributes further into the adjacent medium. As a consequence, a much higher sensitivity could be expected by realizing the coupling between the guided core mode and the degenerate modes TE/TM$_{0,j}$ and HE/EH$_{v,j}$ ($v \geq 2$) that are used to excite SPP modes for the Ex-TLPFG assisted SPP sensors.

In this work a novel Ex-TLPFG assisted SPP sensing platform with enhanced sensitivity is proposed and theoretically investigated for the analyte refractive index (RI) sensing. The Ex-TLPFG assisted SPP sensor consists of the conventional metal-coated Ex-TLPFG but the transmission can be tuned flexibly to match the resonance of either the lower order or the higher order cladding modes, i.e., EH$_{1,j}$ mode or the degenerate TM$_{0,j}$ and EH$_{2,j}$ modes, respectively. The polarization-dependent characteristics of the SPP mode and hence the corresponding sensing performance are investigated by using the full-vector complex mode solver and complex coupled mode theory (CCMT) [30], [31]. The optimum design rules for the achievement of ultimate refractometric sensor are provided and analyzed. It is found that for the dispersion property, i.e., the phase matching condition (PMC), of the SPP modes, there is a transition point at which the optimized grating period can be achieved for the Ex-TLPFG assisted SPP sensor. It is demonstrated that as the designed SPP sensor works at the optimized grating period, giant sensitivity and high resolution corresponding to the resonances of both EH$_{1,j}$ mode and the degenerate TM$_{0,j}$ and EH$_{2,j}$ modes can be achieved at the analyte RI in the range from 1.33 to 1.337 which is of great interest in the field of biochemical and biological sensing.

2. Degenerate and Polarized Properties of the SPP Mode
The schematic of the Ex-TLPFG assisted SPP sensor is shown in Fig. 1. A four-layer waveguide is terminated by a perfectly matched layer (PML) enclosed by a perfectly reflecting boundary conditions (PRB) to form a closed single mode waveguide model. The considered parameters of the single mode fiber are: core radius of $r_{co} = 4.15 \, \mu m$, a cladding radius of $r_{cl} = 62.5 \, \mu m$, fiber core refractive index of 0.36% times larger than that of cladding that is silica. The metal layer is considered as silver with the thickness of $d_{Ag} = 40 \, nm$. The surrounding medium is considered as the watery environment with initial refractive index of $n_s = 1.33$. The wavelength-dependent dispersion is taken into account in the calculation of effective refractive index (ERI) by considering the Sellmeier coefficients which are available in [32]. In the case of silver layer, the wavelength dependence of the dielectric function is obtained based on the experimental value in [33], and
the wavelength-dependent dispersion at a wide wavelength window is depicted in Fig. S1 in the Supplementary materials. To obtain a clear presentation, the notation for the modes of the metal coated single mode fiber are defined as: HE\_11\_j for guided core mode, HE\_v\_j (v ≥ 1) cladding mode, EH\_11\_j for the first EH\_v\_j (v ≥ 1) cladding mode, TE\_01\_j for the first TE\_0\_j cladding mode, TM\_01\_j for the first TM\_0\_j cladding mode, and so on. Here, v represents the azimuthal order and j is the radial order of the fiber modes.

Due to the complex dielectric of the silver layer, the fiber modes display complex ERI, especially for the radially polarized modes, of which the imaginary part corresponds to the energy loss. In general, the metal coated single mode fiber supports four kinds of modes, i.e., guided core mode, cladding modes, leaky modes, and SPP modes [19]. Leaky modes with the highest imaginary part of ERI loss almost all of the energy and cannot propagate in the fiber. SPP modes have larger imaginary ERI than that of guided modes and propagate in the metal surface. This is illustrated in Fig. 2, from which the degenerate property of the higher order modes and the evolution of the SPP excitation are clearly observed. To improve the readability, only the mode spectra corresponding to TE/TM\_0\_j and HE/EH\_2\_j modes are considered. In fact, the influence of fiber modes with the azimuthal order of v ≥ 3 on the transmission spectrum of the Ex-TLPFG assisted SPP sensor can be ignored since they contribute very little to the mode coupling with the guided core mode, which will be analyzed in Section 3.

As shown in Fig. 2(a), the dispersion curve corresponding to the TM\_0\_j (or TE\_0\_j) mode is almost exactly the same with that of EH\_j (or HE\_j) mode, which confirms the degeneracy of TM\_0\_j and EH\_j modes (or TE\_0\_j and HE\_j modes). The wavelength-dependent dispersion curve of the EH\_j mode gradually intersects with that of HE\_j mode at the corresponding circle position for each HE/EH\_1\_j mode group, and the same is true for the degenerate TE/TM\_0\_j and HE/EH\_2\_j mode group. After the intersection point at longer wavelength, the TM\_0\_j (or EH\_j (v ≥ 1)) mode displays a larger real part of ERI than that of TE\_0\_j (or HE\_j (v ≥ 1)) mode. It clearly indicates that the SPP mode is transited from the TM\_0\_j and EH\_j (v ≥ 1) modes. This is further verified by the evolution of the imaginary ERI as depicted in Fig. 2(b), which is characterized by a single wide peak for the TM\_0\_j and EH\_j (v ≥ 1) modes. It is obvious that the maximum value (in absolute, the same below) of imaginary ERI is obtained at longer wavelength after the intersection point, which is different from that in [19]. This difference stems from the fact that the wavelength-dependent dispersion is considered both for the optical fiber and the metal layer. The imaginary ERI corresponds to mode power in the metal layer and hence the resonance of a SPP mode. As shown in Fig. 2(b), the imaginary ERI firstly increases to the maximum value and then decreases, which demonstrates the evolution of the SPP mode excitation. It is therefore obvious that the largest imaginary ERI corresponds to the strongest resonance of a SPP mode. Here it must be pointed that all TM\_0\_j and EH\_j (v ≥ 1) cladding modes exhibit a similar property at the interest wavelength window. This indicates that the SPP mode can
be selectively excited through either the degenerate TM$_{0,j}$ and EH$_{2,j}$ modes or the EH$_{1,j}$ mode, which facilitates greatly the design of the Ex-TLPFG assisted SPP sensor.

The polarization-dependent resonance of the SPP mode can be identified by the field profile in the metal layer that is proportional to the imaginary ERI. In general, the field of a fiber mode consists of radial and azimuthal field components that can be expressed as:

$$
E_r = e_{r,v}(r) \cos (v\phi + \psi), \quad E_\phi = e_{\phi,v}(r) \sin (v\phi + \psi) \quad (1)$$
$$
H_r = h_{r,v}(r) \sin (v\phi + \psi), \quad H_\phi = h_{\phi,v}(r) \cos (v\phi + \psi). \quad (2)
$$

The mode field described by (1)~(2) can be solved numerically using the finite-element-method based full-vector complex mode solver [30], and the exact expressions are presented in [34] for reference. For the TE$_{0,j}$ and TM$_{0,j}$ modes, there are no azimuthal dependence, indicating that the TE$_{0,j}$ mode is azimuthally polarized while the TM$_{0,j}$ mode is radially polarized. For the hybrid mode it is polarization degenerate, depending on the value of radial order $j$ and the angle $\psi$. Thus different polarization properties can be obtained for the guided core mode and cladding modes. When $\psi = 0$, the guided core mode HE$_{1,1}^{co}$ is p-polarized mode of which the electric field is in parallel with $x$ axis in the $x-z$ plane where the grating is tilted. In the cases of $\psi = \pi/2$, HE$_{1,1}^{co}$ is s-polarized mode with the electric field perpendicular to the $x-z$ plane, as shown in Fig. 1. The hybrid cladding mode displays much more complicated polarization-dependent degeneracy, which is depicted in Fig. 3 by calculating the radial fraction of the total power [34]. It is obvious that the higher order hybrid mode have a similar polarization property at different wavelengths. The higher order EH$_{v,j}$ cladding mode shows a radially polarized state and corresponds to the p-polarized mode, while higher order HE$_{v,j}$ mode exhibits a azimuthal polarization and is the s-polarized mode. Here in particular, there is a region (denoted by a ellipse in Fig. 3) for the higher order EH$_{v,j}$ mode where a large radial fraction is clearly observed. Apparently, the mode at this region has a larger radial component $E_r$ of the electric field which is decomposed into radial and azimuthal components as shown in Fig. 4. The electric field of the TM$_{0,42}$ and EH$_{2,42}$ ($v = 1, 2$) modes have the maximum imaginary ERI at wavelength close to 1.54 $\mu$m, at which the maximum radial fraction is also obtained for these modes. It is further confirmed utilizing the electric field of the EH$_{1,42}$ mode which decomposed into radial and azimuthal components shown in Fig. 4. The electric field of the TM$_{0,42}$ and EH$_{2,42}$ modes are plotted in Fig. S2 and Fig. S3 in the Supplementary materials. Compared Fig. 4 with Fig. 2, it reveals that the strongest resonance of the SPP mode occurs at the wavelength $\lambda = 1.549 \mu$m where the maximum imaginary ERI is obtained, and the same is true for the TM$_{0,42}$ and EH$_{2,42}$ modes. Therefore, it is demonstrated that the SPP mode can be transited from (or excited by) the degenerate and radially polarized (or p-polarized) TM$_{0,j}$ and EH$_{v,j}$ ($v \geq 1$) modes.
Fig. 4. Radial $E_r$ (top) and azimuthal $E_\phi$ (bottom) components of the electric field of the EH$_{1,42}$ mode at different wavelengths.

3. Transmission Spectrum of the Ex-TLPFG Assisted SPP Sensor

The grating period $\Lambda_g$ is related to the period $\Lambda$ along the fiber axis through $\Lambda_g = \Lambda \cos \theta$ as shown in Fig. 1. The tilted structure of the grating leads to the coupling between the guided core mode HE$_{11}$ and the higher order degenerate modes TE/TM$_{0,j}$ and HE/EH$_{v,j}$ ($v \geq 2$) which are polarized and hence, the transmission spectrum is degenerate and polarized dependence. In this work we follow the full-vector CCMT which is described in detail in [31] to simulate the mode coupling of the Ex-TLPFG assisted SPP sensor. When analyzing the resonance property of the sensor, the phase matching condition (PMC) is a powerful tool to predict the resonance wavelength, which is defined as [19]:

$$\delta = \text{Re} \left( \Delta \beta_{v,j} + \frac{1}{2} \left( \delta n_{eff}^c - \delta n_{eff}^cl \right) \right)$$  \hspace{1cm} (3)

in which $\Delta \beta_{v,j}$ is phase detuning factor determined by (4), $\delta n_{eff}^c$ and $\delta n_{eff}^cl$ are the induced ERI changes of the fiber core and cladding respectively, which are given as (5)

$$\Delta \beta_{v,j} = \frac{1}{2} \left( \beta^c - \beta^cl - \frac{2\pi}{\Lambda} \right)$$  \hspace{1cm} (4)

$$\delta n_{eff}^c = \delta n \cdot \kappa^c, \quad \delta n_{eff}^cl = \delta n \cdot \kappa^cl$$  \hspace{1cm} (5)

where $\delta n$ is the maximum index modulation, $\kappa^c$ and $\kappa^cl$ are the self-coupling constant of the guided core mode and the $v,j$ cladding mode respectively. The PMC is depicted in Fig. S4 in the Supplementary materials, in which three periods intersect with the corresponding PMC curves. As shown in Fig. S4, two period lines, $\Lambda_1$ and $\Lambda_3$, pass through the intersection points of the PMC curves corresponding to HE/EH$_{1,j}$ and the degenerate TE/TM$_{0,j}$ and HE/EH$_{2,j}$ modes, respectively. Here the grating period of $\Lambda_2 = 15.6 \mu m$ is selected in this section without loss of generality.

Fig. 5 gives the convergence property of the transmission spectra of the Ex-TLPFG assisted SPP sensor when different fiber modes are considered. The Ex-TLPFG has the tilted angle of $\theta = 81^\circ$, the maximum index perturbation of $\delta n = 2.0 \times 10^{-3}$ and the grating length of $L = 50$ mm. As shown in Fig. 5, the resonance wavelength is generally in agreement with that predicted by the PMC (3) as depicted in Fig. S4. It is apparent from the figure that the p-polarized core mode only couples with the p-polarized cladding modes (i.e., p-p coupling), such as TM$_{0,j}$ and EH$_{v,j}$ ($v \geq 1$) modes, while the same property goes for the s-polarized modes (i.e., s-s coupling), such as TE$_{0,j}$ and HE$_{v,j}$ ($v \geq 1$) modes. For the resonance of p-polarized modes, the band at shorter wavelength corresponds to the resonance of the degenerate TM$_{0,j}$ and HE$_{v,j}$ modes, while the band at longer wavelength arises from the resonance of EH$_{1,j}$ mode. In addition, the higher order modes with the azimuthal order $v \geq 3$ (both the p-polarized and s-polarized modes) contribute very little to the mode coupling with the guided core mode. This is understandable due to the small overlap of the mode field between the guided core mode and the higher azimuthal order cladding modes. Hence, the transmission spectrum is obviously convergent when the TE/TM$_{0,j}$ and HE/EH$_{v,j}$ ($v = 1, 2$) modes are considered for the mode coupling of the Ex-TLPFG assisted SPP sensor. This means that though numerous
Fig. 5. Transmission spectra of the Ex-TLPFG assisted SPP sensor with the tilted angle of $\theta = 81^\circ$ considering different fiber modes. Here, $v = 0$ represents the TE$_{0,j}$ and TM$_{0,j}$ modes, and $v \geq 1$ represents the HE$_{v,j}$ and EH$_{v,j}$ modes. The radial order of $j = 1 \sim 50$ is considered for all fiber modes.

Fig. 6. Transmission spectra of the Ex-TLPFG assisted SPP sensor with different tilted angle. Here, the TE/TM$_{0,j}$ and HE/EH$_{v,j}$ ($v = 1, 2$) modes are considered for the mode coupling. The radial order of $j = 1 \sim 50$ is considered for all fiber modes.

cladning modes that match the PMC could couple with the guided core mode of the Ex-TLPFG, only the lower azimuthal order cladding modes need to be considered, which enhances greatly the device-design flexibility in practical applications.

Fig. 6 shows the transmission spectra of the Ex-TLPFG assisted SPP sensor with different tilted angle. For the grating without tilted index perturbation ($\theta = 0^\circ$), only the HE/EH$_{1,j}$ modes can couple with the guided core mode. The resonance is polarization-independent since both the p-polarized and s-polarized modes have the exact same transmission spectrum with each other. As the tilted angle changes, a very different transmission is observed for both polarized modes. It clearly reveals from Fig. 6 that the increase in the tilted angle gradually gives rise to the separation of the transmission spectra corresponding to the resonances of p-polarized and s-polarized modes. When the tilted angle is equal to $\theta = 81^\circ$, the mode coupling occurs only between the modes with the same polarization state (i.e., p-p coupling and s-s coupling, respectively), and the strongest resonance of the p-polarized mode that excites the SPP mode is achieved. As the tilted angle continuously increases to $\theta = 89^\circ$, the resonances of both polarized modes drop rapidly to almost zero. This is because of the fact that the coupling coefficient between the guided core mode and cladding modes (including the SPP modes) becomes zero as the tilted angle approaches to 90° [35]. Therefore, the tilted angle of $\theta = 81^\circ$ is considered to achieve the maximum coupling between the guided core mode and the SPP mode of the Ex-TLPFG assisted SPP sensor.

4. Sensing Characteristics of the Ex-TLPFG Assisted SPP Sensor

For the fiber grating based SPP sensors, the method of wavelength interrogation is in general utilized to investigate the sensing performance. When the analyte RI of $n_s$ varies, the PMC changes
Accordingly, this leads to the shift in resonance wavelength corresponding to the p-polarized mode whereas the resonance wavelength of the s-polarized mode keeps consistent. Hence the variation of the analyte RI can be identified by the resonance wavelength shift. To investigate the sensing characteristics, the PMC in (3) by setting $\delta = 0$ can be expressed as:

$$\lambda_{res} = R\theta \left( \frac{2\pi \cdot \Delta n_{eff}}{2\pi - \delta n \cdot \Delta \kappa \cdot \Lambda} \right) \cdot \Lambda \quad (6)$$

where $\Delta n_{eff} = n_{eff, cl}^0 - n_{eff, cl}^v$ and $\Delta \kappa = \kappa^0 - \kappa^v$ represent the difference of the real ERI and self-coupling coefficient between the guided core mode and cladding modes, respectively. In general, $\Delta n_{eff}$ has the magnitude of $10^{-1}$ for the SPP mode and depends on the parameters of SPP sensor, such as fiber radius, wavelength, surrounding medium, and etc. $\Delta \kappa$ is determined by the self-coupling coefficient which can be approximated to the magnitude of $10^{-4}$. In this work, the grating period $\Lambda$ is in the magnitude of $10^1$. Therefore compared to $2\pi$, $\delta n \cdot \Delta \kappa \cdot \Lambda$ is negligible since it is in the magnitude of $10^{-6} \sim 10^{-5}$. Thus for the Ex-TLPFG assisted SPP sensor with a given grating period $\Lambda$, when the analyte RI changes, the resonance wavelength $\lambda_{res}$ shifts approximately by:

$$\frac{\partial \lambda_{res}}{\partial n_s} \approx \frac{\partial(\Delta n_{eff})}{\partial n_s} \cdot \Lambda. \quad (7)$$

Equation (7) can be further approximated as follow to obtain the theoretical maximum sensitivity. Apparently, the sensitivity can be optimized through two approaches, i.e., by increasing either (both) the $\Delta n_{eff}$ or (and) the grating period $\Lambda$, when the analyte RI changes. $\Delta n_{eff}$ can be in general increased by reducing the fiber diameter and optimizing the metal-layer thickness. Here a efficient alternate method is to match the resonance of higher order modes by designing the SPP sensor with appropriate parameters which is investigated below. In this work, the analyte RI is considered to be from 1.33 to 1.337 with a step variation equal to $\partial n_s = 10^{-3}$. In this case, the variation of $\Delta n_{eff}$, i.e., $\partial \Delta n_{eff}$, is depicted as a function of wavelength in Fig. 5 in the Supplementary materials. It can be obtained from the figure that as the analyte RI increases by $\partial n_s = 10^{-3}$, the $\partial \Delta n_{eff}$ is in the magnitude of $10^{-4} \sim 10^{-3}$. Thus the theoretical maximum sensitivity is given by:

$$\frac{\partial \lambda_{res}}{\partial n_s} \leq \Lambda. \quad (8)$$

Equation (8) confirms that for the Ex-TLPFG assisted SPP sensor with given parameters, the sensitivity can be optimized to the value close to the grating period $\Lambda$.

The resonance wavelength depends on the grating period $\Lambda$ as highlighted by the PMC in (3) or (6), which is depicted in Fig. 7. Compared Fig. 7(a) to Fig. 2, it confirms that there is a region corresponding to the transition (or excitation) of the SPP mode from the p-polarized TM$_{0,42}$ and EH$_{1,42}$ modes. Apparently, a transition point can be expected, at which the PMC curve varies sharply with the wavelength. This behavior is illustrated in Fig. 7(b) where the derivative of $\Lambda$ versus wavelength, i.e., $\partial \Lambda / \partial \lambda$, is plotted according to (6) for the p-polarized modes. It is clearly
verified that a transition point exist at the position where the maximum derivative for the PMC of p-
 polarized modes is obtained, indicating that the sharpest variation of the PMC with the wavelength,
 and accordingly the highest sensitivity, can be achieved. At the transition point, the optimized grating
 period is obtained corresponding to $\lambda_1$ and $\lambda_2$ for the SPP modes, i.e., p-polarized $EH_{1,42}$ mode
 and the degenerate $TM_{0,42}$ and $EH_{2,42}$ modes, respectively.

To get a deeper insight, we plot in Fig. S6 the PMC curve corresponding to the $HE/EH_{1,42}$ modes
 and in Fig. S7 the evolution of the transmission spectrum in the Supplementary materials, when
 different grating periods close to the transition region are considered for the Ex-TLPFG assisted
 SPP sensor. The resonance wavelength predicted by the PMC in (3) or (6) as well as the wavelength
 obtained in practical situation are included in Table 1. The result reveals that there is a difference
 between the wavelength predicted by the PMC and that obtained in practice. The same trend goes
 for the TE/TM$_{0,42}$ and HE/EH$_{2,42}$ modes though the transmission spectrum is not depicted here. It
 is evident that for the grating period $\lambda > 15.65 \mu m$ corresponding to the transition point of the PMC
 curve as shown in Fig. 7, the wavelength predicted is larger than that obtained, while for the grating
 period $\lambda < 15.65 \mu m$, the wavelength predicted becomes smaller than that obtained in practice.
 It turns out that when the grating period shifts far away from the transition point, the wavelength
 predicted by the PMC gradually matches the wavelength obtained in practice. This is because of the
 fact that the imaginary part of the PMC is in general ignored when predict the resonance wavelength
 of fiber grating based devices coated with the materials having complex refractive index [36]. The
 PMC in (3) or (6) can be used to predict the resonance wavelength with very high accuracy for the
 fiber modes without imaginary ERI, whereas the error occurs when analyze the SPP modes having
 large imaginary ERI, especially at the transition region. On this basis, the resonance wavelength
 obtained in practice is considered to investigate the sensing performance of the Ex-TLPFG assisted
 SPP sensor.

Fig. 8 depicts the transmission spectra when the optimized grating period of $\lambda_1 = 15.65 \mu m$ is
 selected to match the resonance of $HE/EH_{1,42}$ modes. The other parameters are the same as that
 in section 3. By this approach, the sensing characteristic of the Ex-TLPFG assisted SPP sensor
 is identified by tracking the resonance wavelength as a function of analyte RI. The result clearly
 demonstrates that the resonance wavelength of the p-polarized mode shifts to longer wavelength
 as the analyte RI increases, whereas the resonance wavelength of the s-polarized mode keeps
 consistent and hence can be used for reference. It can be seen that a very narrower band (smaller
 FWHM) with strong resonance close to $-100 \text{ dB}$ can be achieved, which facilitates greatly the
 improvement of the FOM and hence the sensing resolution. Interestingly, a strong resonance of the
degenerate TE/TM$_{0,42}$ and HE/EH$_{2,42}$ modes can also be observed as the analyte RI increases.
This indicates the sensing performance can also be identified by the resonance of TE/TM$_{0,42}$ and
 HE/EH$_{2,42}$ modes which are investigated below

$$S_{\text{SPP}} = \frac{\partial (\Delta \lambda_{\text{res}})}{\partial (\Delta n_s)}.$$ (9)
Fig. 8. Transmission spectra of the Ex-TLPFG assisted SPP sensor with grating period of $\Lambda_1 = 15.65 \mu m$ as the analyte RI changes from $n_s = 1.33$ to $n_s = 1.337$.

Fig. 9. Resonance wavelength of the EH$_{1,42}$ mode and local sensitivity as a function of analyte RI.

The sensing characteristics are depicted in Fig. 9 where the local sensitivity is defined as (9). As the analyte RI increases, the resonance wavelength shifts to the longer wavelength that is gradually away from the transition point. As a consequence, the local sensitivity decreases. The results confirm that for the Ex-TLPFG assisted SPP sensor with optimized parameters, the maximum sensitivity reaches as high as 6900 nm/RIU when the analyte RI is close to 1.33, and the minimum sensitivity of 800 nm/RIU is obtained for the analyte RI near 1.337. It is found that the enhanced sensitivity (at $n_s = 1.33$) obtained here increases by a factor of 4.16 and 5.75 compared to that for the conventional LPFG based SPP sensor [16] and cladding-modulated LPFG based SPP sensor [19], respectively.

The evaluation of the sensing performance can also be carried out by tracking the wavelength shift in correspondence of p-polarized degenerate TM$_{0,42}$ and EH$_{2,42}$ modes as a function of analyte RI. This is depicted in Fig. 10 where the optimized grating period of $\Lambda_2 = 15.48 \mu m$ is selected to match the resonance of TM$_{0,42}$ and EH$_{2,42}$ modes. The variation in the transmission spectrum as a function of analyte RI exhibits the same trend with that shown in Fig. 8. The similar sensing characteristics but with a larger sensitivity are obtained as shown in Fig. 11. It is evident that the maximum and minimum sensitivities increase up to 9300 nm/RIU (by a factor of 1.35) and 1000 nm/RIU (by a factor of 1.25), respectively, compared to that for the EH$_{1,42}$ mode. The sensitivity enhancement stems from the greater influence of the analyte on higher-azimuthal order modes whose field distributes further into the surrounding medium as previously aforementioned in Section 2. As a consequence, the $\Delta n_{eff}$ and accordingly the sensitivity increase as expected by (7).
Fig. 10. Transmission spectra of the Ex-TLPFG assisted SPP sensor with grating period of $\Lambda_2 = 15.48 \, \mu m$ as the analyte RI changes from $n_s = 1.33$ to $n_s = 1.337$.

Fig. 11. Resonance wavelength of the TM$_{0,42}$ and EH$_{2,42}$ modes and local sensitivity as a function of analyte RI.

Fig. 12. The PMC curve as a function of wavelength for the cladding-reduced Ex-TLPFG assisted SPP sensor. The optimized grating period $\Lambda_1$ and $\Lambda_2$ are equal to 15.52 $\mu m$ and 15.31 $\mu m$ for the p-polarized EH$_{1,32}$ mode and degenerate TM$_{0,32}$ and EH$_{2,32}$ modes, respectively.

5. Sensitivity Optimization of the Ex-TLPFG Assisted SPP Sensor

Reducing the fiber diameter has been proved to be one of the simple and effective way to greatly enhance the interaction between the mode field and the surrounding medium, and hence to optimize the sensitivity for the fiber grating based sensor. Fig. 12 gives the PMC curve versus the resonance wavelength for the Ex-TLPFG assisted SPP sensor with the cladding radius of $r_{cl} = 48.5 \, \mu m$. The grating length is $L = 40 \, \text{mm}$ in this section and other parameters are the same with that analyzed.
above. These two optimized grating periods of $\Lambda_1$ and $\Lambda_2$ in Fig. 12 are obtained to match the resonance of the EH$_{1,32}$ mode and the degenerate TM$_{0,32}$ and EH$_{2,32}$ modes respectively through the same method presented in Section 4. The same trend with that in Fig. 7 is observed, but the most important property is that the cladding-reduced Ex-TLPFG assisted SPP sensor experiences larger variation in the PMC curve versus the wavelength, indicating that higher sensitivity can be achieved.

The evolution of the transmission spectra corresponding to the resonances of HE/EH$_{1,32}$ modes and degenerate TE/TM$_{0,32}$ and HE/EH$_{2,32}$ modes as a function of the analyte RI are plotted in Fig. S8 and Fig. S9 in the Supplementary materials for the cladding-reduced Ex-TLPFG assisted SPP sensor, respectively. The figures clearly reveal that the transmission spectrum exhibits strong resonance and narrow band for all p-polarized modes as the analyte RI changes. The sensing performance of the cladding-reduced Ex-TLPFG assisted SPP sensor is depicted in Fig. 13. The results demonstrate that much higher sensitivities are obtained for both p-polarized modes. As shown in Fig. 13(a), the maximum sensitivity as high as 7400 nm/RIU is obtained at the analyte RI around 1.33, while the minimum sensitivity reaches 1200 nm/RIU, when the Ex-TLPFG assisted SPP sensor is tuned to match the resonance of EH$_{1,32}$ mode. The sensitivities are enhanced by a factor of 1.07 and 1.5 respectively, as compared to that for the standard-diameter SPP sensor as shown in Fig. 9. Much higher sensitivity is achieved for the degenerate TM$_{0,32}$ and EH$_{2,32}$ modes as plotted in Fig. 13(b). The maximum and minimum sensitivities of up to 10100 nm/RIU and 1100 nm/RIU are obtained respectively. The sensitivities are increased by a factor of 1.08 and 1.1 respectively, compared to that as shown in Fig. 11. Apparently, the sensitivity could be further improved by reducing the fiber diameter and optimizing the grating period of the Ex-TLPFG assisted SPP sensor.

6. Conclusion
In this work we have numerically investigated the Ex-TLPFG assisted SPP sensor for the realization of ultimate refractometric sensitivity. The transmission spectrum and the sensing performance are exploited based on the finite element full-vector complex coupled mode theory. We have shown that both the higher order p-polarized TM$_{0,j}$ and EH$_{v,j}$ ($v \geq 1$) cladding modes can excite the SPP mode and hence the Ex-TLPFG assisted SPP sensor can be tuned to match the resonance of either TM$_{0,j}$ or EH$_{v,j}$ ($v \geq 1$) mode. The results further confirm that there is a transition region where the SPP mode is transited gradually from the p-polarized TM$_{0,j}$ and EH$_{v,j}$ ($v \geq 1$) modes. Due to the tilted refractive index perturbation, the Ex-TLPFG primarily couples the guided core mode to the higher order cladding modes and SPP modes with the azimuthal order $v \leq 2$, i.e., TE/TM$_{0,j}$ and HE/EH$_{v,j}$ ($v = 1, 2$) modes, while the modes having $v \geq 3$ contribute very little to the excitation of the SPP mode. At the transition region, there is a transition point of the PMC curve in correspondence of the SPP mode that can be used to predict the optimized grating period. It is demonstrated that for the
Ex-TLPFG assisted SPP sensor operating at the optimized grating period, ultrahigh sensitivity up to 9300 nm/RIU (or 6900 nm/RIU) can be obtained for the resonance of degenerate TM_{0,42} and EH_{2,42} (or EH_{1,42}) modes when the analyte RI is close to 1.33. The sensitivity can be further enhanced just by a simple way: reducing the fiber cladding combined with an optimized grating period. By this approach, it is found that the sensitivity can be increased to as high as 10100 nm/RIU when the cladding-reduced Ex-TLPFG assisted SPP sensor is tuned to the resonance of TM_{0,32} and EH_{2,32} modes (or 7400 nm/RIU for the EH_{1,32} mode), which is much higher than that for the conventional LPFG devices and LPFG based SPP sensors. We believe that this Ex-TLPFG assisted SPP sensor can find applications in the important fields with low refractive index, such as biochemical and biological parameters' sensing.

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References


