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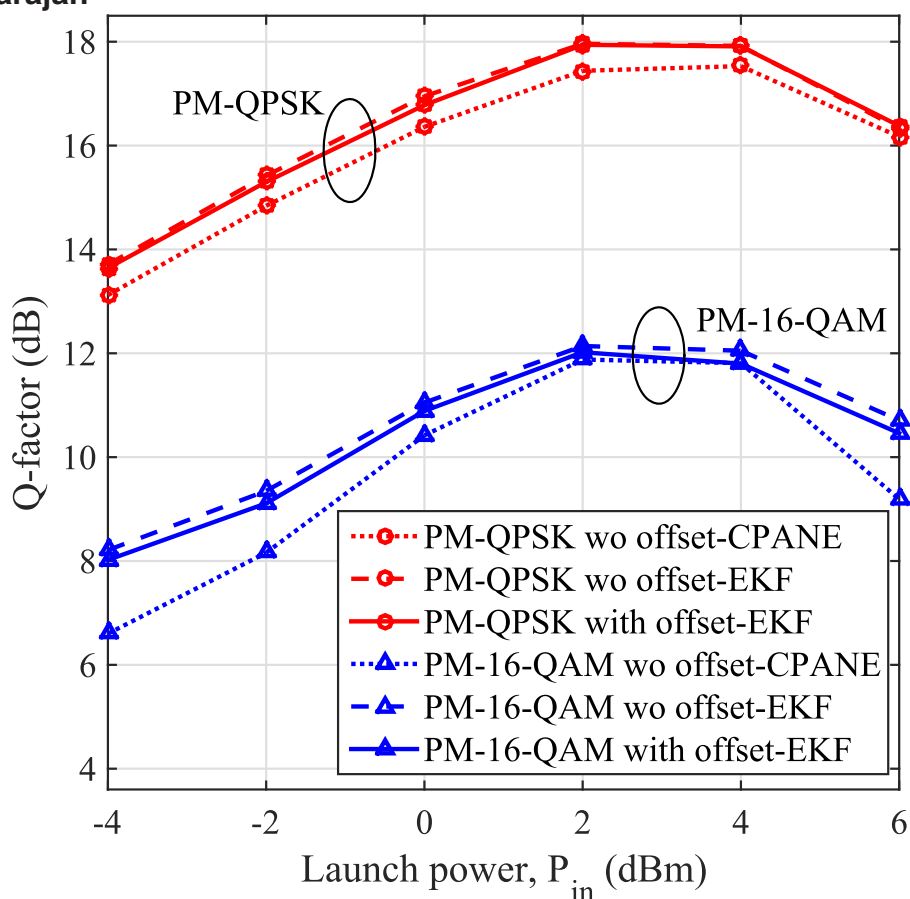**Pradeep Kumar Krishnamurthy**
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# EKF for Joint Mitigation of Phase Noise, Frequency Offset and Nonlinearity in 400 Gb/s PM-16-QAM and 200 Gb/s PM-QPSK Systems

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**Abstract:** The performance of higher order modulation formats such as 16-quadrature amplitude modulation (QAM) coherent optical communication systems are limited due to several linear and nonlinear impairments such as phase noise, frequency offset, chromatic dispersion, and fiber nonlinearities. For a satisfactory system performance, these impairments need to be eliminated either by all-optical and/or electronic means. In this paper, we use the extended Kalman filter (EKF) algorithm to jointly mitigate laser phase noise, frequency offset, and nonlinear channel impairments, namely self phase modulation and nonlinear phase noise in polarization multiplexed 400 Gb/s 16-QAM and 200 Gb/s quadrature phase shift keying (QPSK) systems. We consider a two-state EKF with phase as one state and frequency offset as another. Simulation results for transmission over 1000 km (10 × 100 km) of standard single mode fiber show a *Q*-factor of 12 dB for PM-16-QAM at frequency offset of 1 GHz and laser linewidth of 100 kHz and a *Q*-factor of 18 dB for PM-QPSK at 1 MHz linewidth. Further, phase trajectories corresponding to tracked laser phase noise and frequency offset show that EKF can suppress laser phase noise up to 10 MHz and frequency offset up to 5 GHz, thereby eliminating the requirement of using separate algorithms for phase noise and frequency offset estimation.

**Index Terms:** Coherent communication, digital signal processing, phase noise, adaptive Kalman filtering, phase estimation.

## 1. Introduction

With the growing need for networking, the demand for bandwidth and capacity of data transmission is increasing day-by-day. Looking beyond 100 Gbps, work is already underway towards 400 Gbps data rate as the next ethernet (IEEE 802.3) standard [1]–[3]. For a fixed bandwidth, data rate can be increased by using higher order modulation formats such as 16 and 64 quadrature amplitude

modulation (QAM). While these modulation formats lead to efficient spectrum utilization, they are highly sensitive to frequency offsets, accumulated phase noise and fiber nonlinearities.

Several DSP algorithms have been developed to mitigate these impairments. Viterbi-Viterbi carrier phase estimation (VV-CPE) and its variants are widely used to mitigate frequency offset (FO) and laser phase noise (PN) effects whereas digital backpropagation (DBP) is used to alleviate fiber nonlinearities [4]–[7]. However, DBP can eliminate only deterministic nonlinearities viz. self phase modulation (SPM) and fails to converge to the exact transmitted signal in the presence of nonlinear phase noise (NLPN) induced due to amplified spontaneous emission (ASE) noise interacting with fiber kerr nonlinearity [8]–[10]. Further, VV-CPE algorithm which mainly employs $M^{th}$-power method is prone to cycle-slipping [11].

The Kalman filtering algorithm is a well-known recursive algorithm for signal estimation and tracking in time-varying systems. At the discrete time step k, the system is specified by a state vector $\mathbf{x_k}$, input vector $\mathbf{u_k}$, and measurement vector $\mathbf{z_k}$. In the Kalman filter (KF) framework, these states are updated according to following equations:

$$\text{Process equation :} \qquad \mathbf{x_{k+1}} = \mathbf{f(x_k)} + \mathbf{g(u_k)} + \mathbf{w_k}$$

$$\text{Measurement equation :} \quad \mathbf{z_k} = \mathbf{h(x_k)} + \mathbf{v_k}$$

$$\text{State-update equation :} \quad \mathbf{\hat{x}_{k|k}} = \mathbf{\hat{x}_{k|k-1}} + \mathbf{K_k(z_k} - \mathbf{h(\hat{x}_{k|k-1})}). \qquad (1)$$

KF is optimal in the minimum mean square sense for linear systems, i.e., $\mathbf{f(x_k)} = \mathbf{Ax_k}$ with process noise $\mathbf{w_k}$ and measurement noise $\mathbf{v_k}$ as Gaussian and uncorrelated with each other [12], [13]. In optical communication systems, KF has been proposed for estimation and tracking of polarization, phase and frequency offset by treating them as state variables and received samples forming the measurement variables [14], [15]. In [16], KF was used to estimate FO. PN and nonlinearity were not considered. We previously proposed the use of KF in [17] to mitigate PN and NLPN in 100 Gbps single polarization quadrature phase shift keying (QPSK) systems and showed that KF provides 6 dB improvement in Q-factor in comparison with DBP algorithm (1 step/span).

When $\mathbf{f(x_k)}$ is nonlinear, a modified version of KF, suitable for nonlinear dynamic systems, known as extended Kalman filter (EKF) is used [18]. This is desirable in coherent optical communication systems since the received sample $r_k$ is a nonlinear function of phase noise $\theta_k$, i.e. $r_k = s_k e^{j\theta_k} + n_k$. Thus, in EKF $r_k$ can be processed directly without the extraction of phase from $r_k$ as required in KF. In [19], EKF-based carrier phase and amplitude noise estimation (CPANE) algorithm was suggested to jointly mitigate phase and amplitude noise in 224 Gbps coherent optical systems. However, FO was not considered. In real time systems, the frequency offset between transmitter and local oscillator lasers ranges up to few GHz and it is not possible to estimate the phase correctly until FO is compensated [20]. The algorithm in [19] cannot be extended in a straightforward way to include the effects of FO. Thus, a separate algorithm must be used to estimate FO or their algorithm must be reformulated by considering FO as additional state variable. Hence, the computational complexity of EKF is inevitable.

In this paper, we demonstrate the use of extended Kalman filter to simultaneously track and compensate laser PN, FO, and nonlinearity in 400 Gbps polarization multiplexed 16-QAM (PM-16-QAM) and 200 Gbps polarization multiplexed QPSK (PM-QPSK) system. We also compare our results with CPANE. We show that even at high FO of 1 GHz, EKF performs better than CPANE for both PM-16-QAM and PM-QPSK eliminating the requirement of a separate algorithm for mitigating FO.

The remainder of the paper is organised as follows: in Section II, we discuss the system model and DSP techniques used at the receiver end. The extended Kalman filter used for phase and offset estimation is described in Section III. In Section IV, we present simulation results showing the performance of EKF for back-to-back as well as transmission over 10-span link for both 400 Gbps PM-16-QAM and 200 Gbps PM-QPSK. We conclude by summarizing our results in Section V.

## 2. System Model

In order to access the performance of EKF for the joint mitigation of laser phase noise, nonlinearity, and FO, we consider PM-16-QAM and PM-QPSK systems. The two systems operate at 50 Gbaud
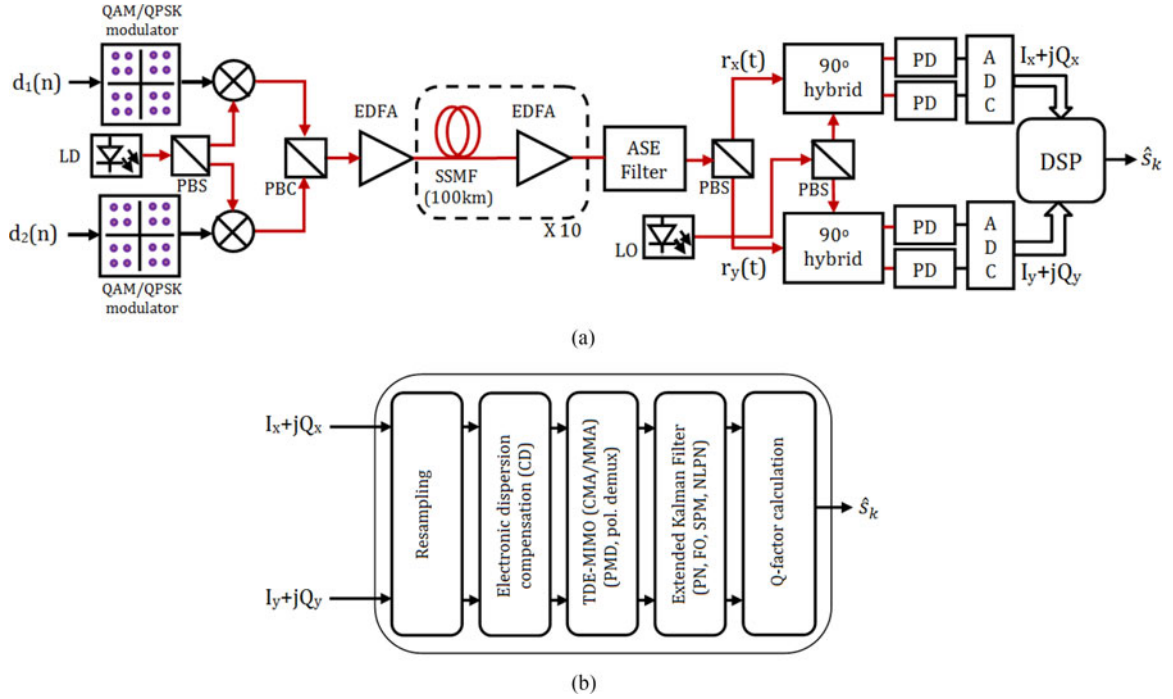
Fig. 1. (a) Simulation model for 50 Gbaud PM-16-QAM/PM-QPSK transmission system. (b) DSP algorithms used at the receiver end. LD: laser diode, PBS: polarization beam splitter, PBC: polarization beam combiner, EDFA: erbium doped fiber amplifier, SSMF: standard single mode fiber, ASE: amplified spontaneous emission noise, LO: local oscillator, PD: photodetector, ADC: analog-to-digital converter, DSP: digital signal processing. Red line: optical fiber, black line: electric wire.

with an overall data rate of 400 and 200 Gbps, respectively. Fig. 1(a) shows a schematic of our transmission model for the 50 Gbaud PM-16-QAM/PM-QPSK system. At the transmitter, the symbols $d_1(n)$ and $d_2(n)$ are QAM or QPSK modulated onto the $x$ and $y$ polarizations of the laser. The baseband modulated signals for 16-QAM and QPSK are given by

$$16\text{-QAM}: \quad s_{x,y}(t) = \sqrt{P_{in}} \cos\left(\frac{u_I(t)\pi}{2V_\pi}\right) \exp\left(\frac{ju_P(t)\pi}{V_\pi}\right)$$

$$u_I(t) = -V_\pi + \frac{2V_\pi}{\pi} \sum_k \left( \arcsin\left(\frac{\sqrt{i_k^2 + q_k^2}}{\sqrt{2}}\right) \cdot p(t - kT_s) \right)$$

$$u_P(t) = \frac{V_\pi}{\pi} \sum_k (\arg[i_k, q_k] \cdot p(t - kT_s)) \tag{2}$$

$$\text{QPSK}: \quad s_{x,y}(t) = \sqrt{P_{in}} \exp\left(\frac{ju_1(t)\pi}{V_\pi}\right) \exp\left(\frac{ju_2(t)\pi}{V_\pi}\right)$$

$$u_n(t) = \frac{V_\pi}{2^{n-1}} \sum_k [b_{n_k} \cdot p(t - kT_s)] \tag{3}$$

where $i_k$ and $q_k$ represent the normalized coordinates of $k^{th}$ 16-QAM symbol; $b_{n_k}$ represents the $n^{th}$ bit of a QPSK symbol in the $k^{th}$ symbol interval [21]. The pulse shaping function $p(t - kT_s)$ = 1 for the NRZ case used in the system. The symbols are then combined using a polarization beam combiner (PBC), amplified using power-controlled erbium doped fiber amplifier (EDFA) and transmitted through the channel consisting 10-spans (100 km each) of standard single mode fiber

(SSMF) and in-line amplifiers. The amplifiers used within a span are gain-controlled and assumed to fully compensate the span losses. The received signal vector can be represented as $\mathbf{r_r}(t) = [r_x(t), r_y(t)]^T$ and is given by

$$\mathbf{r_r}(t) = (\mathbf{r_t}(t) \star \mathbf{h_l}(t))h_{nl}(t) + \mathbf{n}(t) \qquad (4)$$

$\mathbf{r_t}(t)$ is the transmitted signal given by $\mathbf{r_t}(t) = \mathbf{s}(t)\exp(j\theta(t))$, where $\mathbf{s}(t) = [s_x(t), s_y(t)]^T$ and $\theta(t)$ is the PN of the source laser. $\mathbf{n}(t) = [n_x(t), n_y(t)]^T$ is the noise vector. $\mathbf{h_l}(t)$ and $h_{nl}(t)$ are the linear and nonlinear impulse responses of the channel respectively and are given by

$$\mathbf{h_l}(t) = \mathcal{F}^{-1}\left\{ \begin{pmatrix} \cos\xi & -\sin\xi \\ \sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} e^{j\omega\tau/2} & 0 \\ 0 & e^{-j\omega\tau/2} \end{pmatrix} \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix} e^{-j\omega^2\frac{\beta_2}{2}L_{(fiber)}} \right\} \qquad (5)$$

$$h_{nl} = \exp\left[j(\theta^{NL}(t) + \Phi(t))\right] \qquad (6)$$

where, $\xi$ is the angle between the reference polarization and principal state of polarization (PSP) of the fiber; $\tau$ is the differential group delay between the PSPs; $\beta_2$ is dispersion coefficient; and $\theta^{NL}(t)$ is the phase shift due to SPM and $\Phi(t)$ is the NLPN due to interaction of SPM with ASE noise [22].

At the receiver, $\mathbf{r_r}(t)$ is filtered using a second-order Gaussian bandpass filter to reduce the out-of-band ASE noise. The two polarizations of the received signal, $r_x(t)$ and $r_y(t)$ are separated using a polarization beam splitter (PBS) and demodulated using a polarization diverse coherent receiver with a local oscillator (LO). The LO laser is considered free of phase noise, with a fixed frequency offset (FO). FO is the fixed frequency difference between transmitter laser and LO. The $r_x(t)$ and $r_y(t)$ signals are downconverted using photodetectors and sampled by the ADCs. The in-phase and quadrature components of each polarization thus extracted, are combined to form complex discrete signal samples $r_x$ and $r_y$, where $r_x = I_x + jQ_x$ and $r_y = I_y + jQ_y$. $r_x$ and $r_y$ are then processed using a DSP module expanded in Fig. 1(b).

The DSP module consists of successive application of the following four algorithms: (a) Resampling to resample the incoming ADC samples at twice the symbol rate, (b) Electronic dispersion compensation (EDC) using overlap FDE method to mitigate the acclumulated chromatic dispersion (CD) [23], (c) Adaptive TDE-MIMO algorithm to compensate polarization mode dispersion (PMD) and polarization crosstalk in the linearly equalized samples and separate out the two polarizations [5]. It is adapted by constant modulus algorithm (CMA) for QPSK and multi-modulus algorithm (MMA) for 16-QAM with optimized number of taps. The data of $x$ and $y$ polarizations are then sampled to one sample/s and fed to the EKF for further processing. (d) EKF to mitigate laser PN, FO and nonlinearity and this process is described in detail in the next section. We have used EKF for each polarization separately. However, the two polarizations can be jointly considered along with other impairments [15].

# 3. Extended Kalman Filtering

After compensation of the linear impairments CD and PMD, (4) is reduced to the form $\mathbf{r_k} = \mathbf{s_k}\exp(j\Psi_k) + \mathbf{n_k}$, where $\mathbf{r_k} = [r_{k_x} r_{k_y}]^T$. For simplicity, we drop the vector notation and represent $r_{k_x}$ and $r_{k_y}$ individually by $r_k$ as

$$r_k = s_k\exp(j\Psi_k) + n_k \qquad (7)$$

where $k$ denotes the $k^{th}$ sample over the observed 16-QAM/QPSK symbol. $s_k$ are the samples of modulated signal, given by $s_k = \sqrt{P_{in}}d_k$, where $P_{in}$ is the input launch power and $d_k$ are the mapped data symbols represented by $[\pm 1/3\pm 1/3j, \pm 1/3\pm 1j, \pm 1\pm 1/3j, \pm 1\pm 1j]$ for 16-QAM and $[\pm 1, \pm 1j]$ for QPSK. $\Psi_k = \omega_k T + \theta_k + \theta_k^{NL} + \Phi_k$ is the uncompensated phase due to frequency offset, inherent phase noise of the laser diode at the transmitter and fiber nonlinearity; where, $\omega_k$ is the fixed frequency offset at LO, $T$ is the symbol duration, $\theta_k$ is the laser PN, $\theta_k^{NL}$ is the average fixed SPM phase and $\Phi_k$ is the NLPN. $n_k$ is the complex ASE noise sample modeled as zero mean AWGN process.
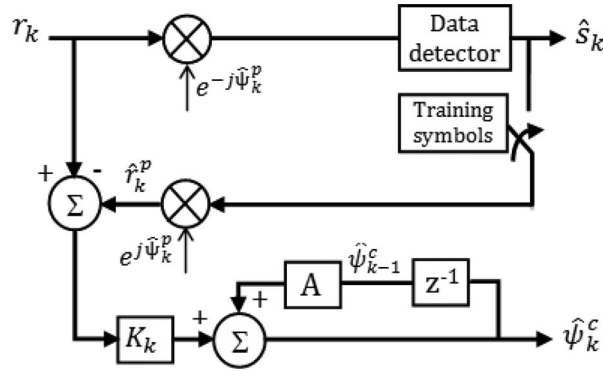
Fig. 2. Schematic diagram of EKF for FO, PN, and nonlinearity estimation.

We model the laser phase noise at the transmitter as a discrete Weiner process: $\theta_k = \theta_{k-1} + \delta\theta_k$. $\delta\theta_k$ is a zero-mean Gaussian random variable with a variance of $\sigma_\theta^2 = 2\pi\delta\nu/R$, where $\delta\nu$ denotes full-width half-maximum (FWHM) linewidth of the laser and $R$ is the data rate of the transmission system. The fixed aevarge SPM phase shift for $N$ spans is given by $\theta_k^{NL} = N\gamma L_{eff}[P_{in} + (N+1)\sigma_n^2] \approx N\gamma L_{eff}P_{in}$, where $\gamma$ is the nonlinear coefficient, $L_{eff}$ is the effective length of SSMF within one span, $N$ is the total no. of spans and $\sigma_n^2$ is the variance of ASE noise [10]. $\Phi_k$ is the resultant phase due to NLPN with variance $\sigma_\Phi^2 = (4/3)N^3(\gamma L_{eff})^2 P_{in}\sigma_n^2$.

**Two-state EKF model for frequency and phase estimation:**

The complex discrete samples, $r_k$ of each polarization are processed using a two-state EKF for the estimation and compensation of FO, PN and nonlinearity. The schematic diagram of EKF is shown in Fig. 2. We take $[\Psi_k, \omega_k]^T$ as the two-component state vector. The state model is given by

$$\begin{pmatrix} \Psi_{k+1} \\ \omega_{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Psi_k \\ \omega_k \end{pmatrix} + \begin{pmatrix} w_{1k} \\ w_{2k} \end{pmatrix} \tag{8}$$

where $\Psi_k$ is the uncompensated phase due to offsets and phase noises at the $k^{th}$ sample, $\omega_k$ is the LO frequency offset, and $w_{1k}$ and $w_{2k}$ are the process noises assumed to have Gaussian probability density function (PDF). $r_k$ is taken as measurement variable, represented as $r_k = s_k\exp(j\Psi_k) + n_k$, where $n_k$ is the measurement noise modelled as AWGN process. The estimates of $\Psi_k$ and $\omega_k$ are obtained using the following recursive EKF equations:

Prediction stage :
$$\hat{\Psi}_k^p = \hat{\Psi}_{k-1}^c + \hat{\omega}_{k-1}^c$$
$$\hat{\omega}_k^p = \hat{\omega}_{k-1}^c$$
$$P_k^p = A P_{k-1}^c A^T + Q$$
$$\hat{r}_k^p = \hat{s}_k \exp(j\hat{\Psi}_k^p) \tag{9}$$

Correction stage :
$$H_k = \begin{pmatrix} -\Im(\hat{r}_k^p) & \Re(\hat{r}_k^p) \\ 0 & 0 \end{pmatrix}$$
$$K_k = P_k^p (H_k^*)^T (H_k P_k^p (H_k^*)^T + R)^{-1}$$
$$P_k^c = P_k^p - K_k H_k P_k^p$$
$$\begin{pmatrix} \hat{\Psi}_k^c \\ \hat{\omega}_k^c \end{pmatrix} = A \begin{pmatrix} \hat{\Psi}_{k-1}^c \\ \hat{\omega}_{k-1}^c \end{pmatrix} + K_k \begin{pmatrix} \Re(r_k - \hat{r}_k^p) \\ \Im(r_k - \hat{r}_k^p) \end{pmatrix} \tag{10}$$
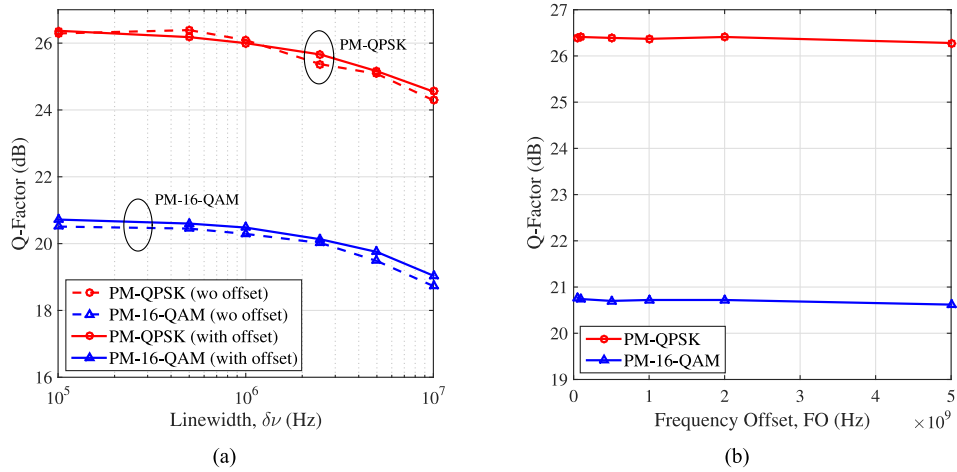
Fig. 3. Performance for b2b propagation. (a) Q-factor vs. linewidth $\delta\nu$ with and without FO of 1 GHz. (b) Q-factor vs. frequency offset, FO at $\delta\nu = 100$ kHz.

where $A$ is the state transition matrix given by $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $K_k$ is the Kalman gain, and $Q$ and $R$ are the covariance matrices of process and measurement noise, respectively. The performance of EKF depends on $Q$ and $R$. As $Q$ increases, the filter bandwidth increases giving rise to faster transient response. But with an increase in $Q$, the error covariance $P$ also increases indicating a rise in uncertainty [24]. The same effect is also observed by decreasing $R$. In our simulations, $Q$ and $R$ are calculated as $Q = Q_o I_2$ and $R = R_o I_2$ where, $I_2$ is a 2x2 identity matrix. $Q_o$ and $R_o$ are initiated with $\sigma_\theta^2$ and $N\sigma_n^2$ respectively and tuned further to achieve a better tradeoff between fast acquisition and small phase tracking error. The estimation is initiated with $\hat{\Psi}_0^c$ and $\hat{\omega}_0^c$ set as 0 and $P_0^c$ taken to be $I_2$ [25]. The discrete symbols $r_k$ are first derotated by $\hat{\Psi}_k^p$ to produce soft decisions on the QAM/QPSK symbols $\hat{s}_k$ which are used in estimating the state in the next iteration as shown in Fig. 2. The estimates $\hat{\Psi}_k^c$ are used to derotate $r_k \Rightarrow r_k \exp(-j\hat{\Psi}_k^c)$, thereby giving the correct estimates of $s_k$.

## 4. Results and Discussion

In this section, we discuss the performance of EKF for mitigating PN, FO and nonlinearity in 400 Gbps PM-16-QAM and 200 Gbps PM-QPSK systems. The system model used in simulation is shown in Fig. 1(a). We evaluate the performance in terms of Q-factor for both back-to-back (b2b) and fiber transmission. Q-factor is calculated on the basis of error vector magnitude (EVM) given in [26], [27], with the following simplified equations:

$$\text{BER}_{(QPSK)} = \frac{1}{2}erfc\left(\frac{1}{\sqrt{2}EVM}\right); \qquad \text{BER}_{(QAM)} = \frac{3}{8}erfc\sqrt{\frac{\sqrt{2}}{10EVM^2}} \tag{11}$$

$$Q(dB) = 20\log_{10}(\sqrt{2}erfc^{-1}(2\text{BER})) \tag{12}$$

For b2b transmission, the channel is bypassed in the simulation, while for fiber transmission, a total of 10 spans each of 100 km leading to a total link length of 1000 km is used. Further, for b2b transmission, the filter is operated in decision-directed mode without using any training symbols. However, for span transmission the filter is operated in the data-aided mode for the first 5 time samples and then switched to the decision-directed mode. In both the cases, a total number of $2^{14}$ symbols are transmitted. We used VPI transmission maker 9.5 to simulate the system shown in Fig. 1(a) and processed the data using MATLAB.
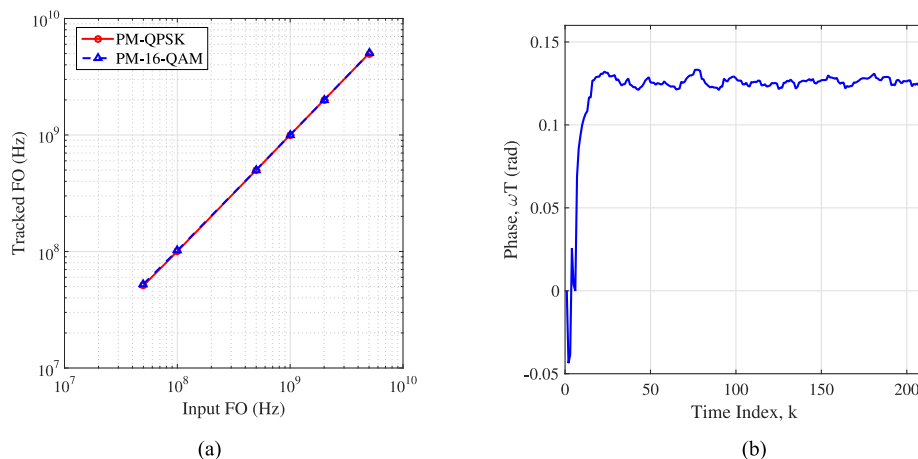
Fig. 4. FO estimation. (a) Tracked FO vs. input FO. (b) Tracked phase offset corresponding to FO of 1 GHz.

### 4.1. B2B Transmission

For b2b propagation, we set the launch power $P_{in}$ to 0 dBm. Since fiber is not included in this case, the EKF tracks and compensates laser PN and FO only. Fig. 3(a) shows the graph of Q-factor as a function of laser linewidth, $\delta\nu$ for PM-16-QAM and PM-QPSK for two cases: without considering LO-frequency offset, i.e., FO = 0 GHz, and with FO of 1 GHz. We observe that the Q-factors, with FO taken into account, coincide with the case without FO for both modulated formats. Moreover, a Q-factor of ≈19 dB and 24.5 dB are achieved for PM-16-QAM and PM-QPSK respectively even at a linewidth as high as 10 MHz.

Fig. 3(b) shows Q-factor as a function of FO while linewidth of the laser is kept fixed at 100 kHz. We observe that the Q-factor remains nearly constant for all the values of FO from 50 MHz to 5 GHz. Thus, we conclude that EKF can eliminate offsets as large as 5 GHz. This is further justified by Fig. 4(a) where the FO tracked by EKF is plotted against the FO applied to the system. We see that estimated FO coincides with the applied FO for both PM-16-QAM and PM-QPSK. The tracked phase offset corresponding to FO of 1 GHz is shown in Fig. 4(b). The tracked FO trajectory shows that the filter converges within <20 symbols.

### 4.2. Span Transmission

We simulate the PM-16-QAM/QPSK modulated system transmitted over 10 spans (10 × 100 km) as shown in Fig. 1(a). Each span of the channel consists of SSMF simulated using the split-step fourier method (SSFM) for attenuation, $\alpha$ = 0.2 dB/km; dispersion, $D$ = 17 ps/nm-km; dispersion slope, $S = 0.08e^3$ s/m$^3$; nonlinear coefficient, $\gamma$ = 1.3/W/km and PMD, $D_P$ = 0.01 ps/$\sqrt{km}$. The SSMF within a span is followed by an EDFA with gain, G = 20 dB to compensate for span losses. The EDFA noise figure is taken to be 3.5 dB. At the receiver, a second order Gaussian band pass filter is used to reduce out-of-band ASE noise. After coherent detection, the signal is processed with DSP module. Electronic dispersion compensation using overlap FDE method is used to compensate for CD and TDE-MIMO (using adaptive CMA/MMA) is used for mitigating PMD. For the mitigation of FO, PN and nonlinearity, the EKF algorithm is used as explained in Section III. In this section we also compare the Q-factor performance of EKF with that of CPANE assuming FO = 0 for the latter.

Fig. 5(a) shows the plot of Q-factor vs. input launch power $P_{in}$ for PM-16-QAM and PM-QPSK under three scenarios: a) without FO-EKF, where FO = 0 GHz and EKF is used to mitigate PN and nonlinearity; b) with FO-EKF where EKF tracks PN and nonlinearity along with FO of 1 GHz; c) without FO-CPANE, where FO = 0 GHz and CPANE is used to mitigate PN and nonlinearity. Laser linewidth is fixed at 100 kHz for PM-16-QAM and 1 MHz for PM-QPSK systems.
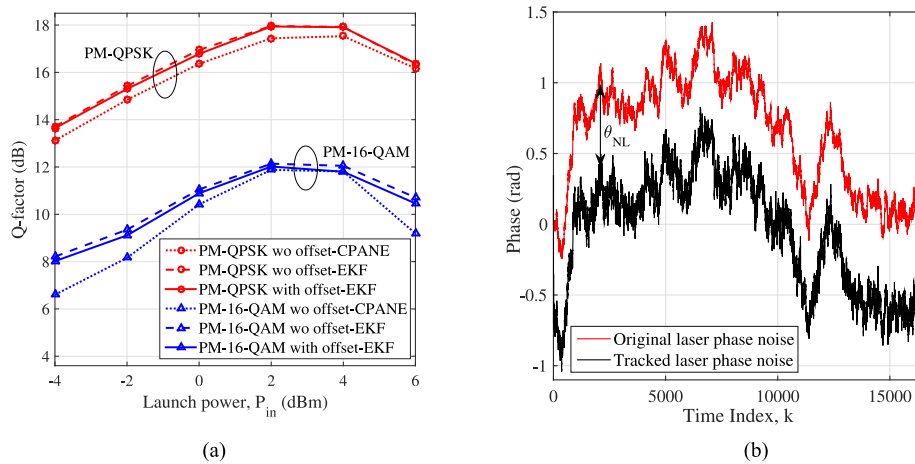
(a)                                                       (b)

Fig. 5. Performance for span transmission. (a) Q-factor vs. launch power. N = 10 spans, FO = 1 GHz, $\delta\nu_{(QPSK)}$ = 1 MHz, and $\delta\nu_{(QAM)}$ = 100 kHz. (b) Tracked phase noise at EKF output. $P_{in}$ = 2 dBm, N = 10 spans, $\delta\nu$ = 1 MHz, and FO = 0 GHz.



(a)

(b)
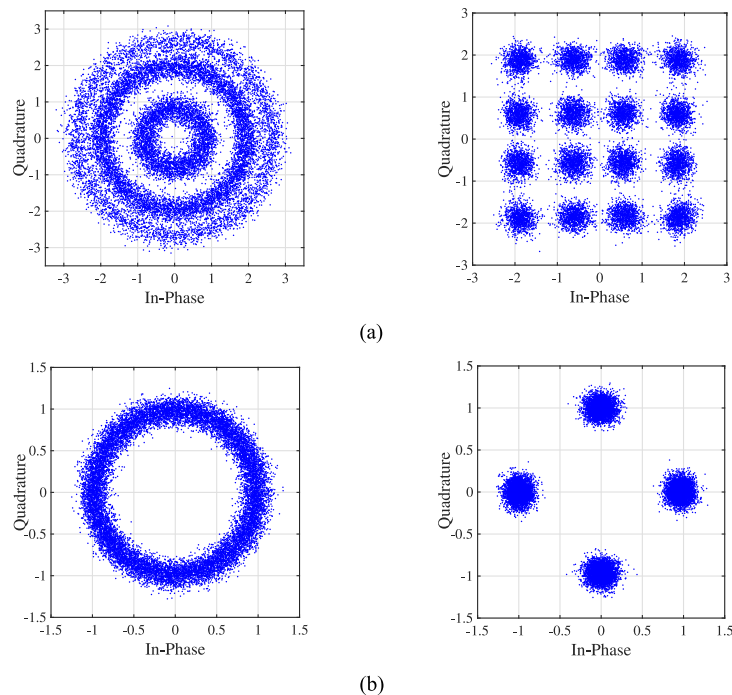
Fig. 6. Constellation diagrams before and after EKF at $P_{in}$ = 2 dBm, FO = 1 GHz and N = 10 spans. (a) PM-16-QAM system at $\delta\nu$ = 100 kHz. (b) PM-QPSK system at $\delta\nu$ = 1 MHz.

We see that Q-factor performance for cases a) and b) coincide with each other for both systems and a maximum Q-factor of ≈12 and 18 dB are achieved for PM-16-QAM and PM-QPSK respectively for $P_{in}$ = 2 dBm. This shows that EKF is capable of compensating PN and nonlinearity along with high FO (of 1 GHz) without severe performance degradation. Comparing cases a) and b) with case c), we find that EKF performs better than CPANE by ≈0.3 dB for PM-16-QAM and ≈0.5 dB for PM-QPSK at 2 dBm. Further, for PM-16-QAM, the performance difference rises to ≈1.5 dB at 6 dBm. While due to scalar EKF, CPANE has reduced complexity as compared to two-state EKF in

this paper, but in the presence of FO the former does not converge. To compensate FO, a separate algorithm should be used or CPANE should be reformulated by adding FO as another state variable. In both the cases, the complexity of the overall parameter estimation rises which cannot be avoided. The advantage of using a single EKF is not only in terms of Q-factor but also the fact that EKF eliminates PN, nonlinearity and a high FO of 1 GHz simultaneously and does not require separate algorithms.

Fig. 5(b) shows the original and tracked PN trajectories at $P_{in}$ = 2 dBm, N = 10 spans, and $\delta\nu$ = 1 MHz. We observe that the tracked PN is similar to the original PN. The phase difference between the two trajectories corresponds to the fixed average phase shift due to SPM, i.e., $\theta^{NL}$ as explained in Section III.

Fig. 6(a) and (b) shows the constellation diagrams for PM-16-QAM and PM-QPSK, respectively, before and after using EKF. They are captured at $P_{in}$ = 2 dBm, after transmission over 10 spans. $\delta\nu$ is set at 100 kHz for PM-16-QAM and 1 MHz for PM-QPSK while maintaining FO of 1 GHz. While the presence of nonlinearity gives a fixed phase shift to the constellation points along with the helical-shaped distribution due to NLPN, high PN and FO in the system also tend to rotate the constellation diagram exhibiting the arcs about the ideal points. Due to the combined effect of these impairments, the constellation diagram tends to close [28], [29]. We observe in Fig. 6(a) and (b) that for both the cases, constellation diagram is initially closed due to the presence of PN, FO and nonlinearity in the system. After applying EKF, the constellation points return to their original positions indicating the elimination of impairments. The uniform spread around the points is attributed to ASE noise present in the system.

## 5. Conclusion

In this paper, we demonstrated the use of the extended Kalman filter to track and mitigate laser phase noise, frequency offset and nonlinearity for 50 Gbaud coherent PM-16-QAM (400 Gbps) and PM-QPSK (200 Gbps) systems. Simulations are carried out for back-to-back transmission and channel transmission over 10 × 100 km with frequency offset as high as 1 GHz. For back-to-back transmission, we obtained a Q-factor of 19 dB for PM-16-QAM and 24.5 dB for PM-QPSK at high linewidth of 10 MHz. For span transmission, maximum Q-factor of 12 dB is achieved for PM-16-QAM at a linewidth of 100 kHz and 18 dB is obtained for PM-QPSK at 1 MHz linewidth. We also provide phase trajectories corresponding to laser phase noise and frequency offsets which shows that EKF can suppress laser phase noise up to 10 MHz and frequency offset up to 5 GHz. Constellations diagrams obtained after EKF seem to be consistent with the original signal constellations proving that EKF can jointly mitigate multiple impairments thereby eliminating the requirement of separate algorithms. Thus, EKF can be an interesting and optimized choice for joint mitigation of laser PN, FO, and nonlinearities.

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