Theoretically analyze the tunable wide band-stop filtering in plasmonic waveguide coupled with fixed height to length stubs

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Abstract A metal-dielectric-metal (MDM) waveguide coupled with fixed height to length \( \frac{b_i}{a_i}=0.8 \) stubs is proposed in our work. Then, an obvious band-stop filtering phenomenon can be observed in MDM waveguide coupled with single fixed height to length stub. Interestingly, the MDM waveguide coupled with two fixed height to length stubs exhibits more excellent band-stop filtering performance than that in single fixed height to length stub coupled waveguide structures. The phase difference analysis and transmission line theory are established to discuss the transmission spectrum and the wide band-stop filtering phenomenon of our proposed structure. Inconceivably, the bandwidth of band-stop filtering can reach to 1750nm through tuning structure parameters and the number of the fixed height to length stub. These results may provide great potential applications for plasmonic filtering devices.

Index terms: Plasmonics, Tunable filters, Waveguides.

1. Introduction

Surface plasmons (SPs) are excited at the surface of the metal and dielectric [1]. The surface plasmon polaritons (SPPs) and localized surface plasmon resonance (LSPR) are two branches of SPs, which have been widely investigated in recent years. The SPPs can propagate along the surface of metals and dielectric [2]. However, the LSPR can’t spread, but it’s local resonance effect has important application potential in the field of nanotechnology [3]. Many abnormal optical phenomena have been proved in recent research, such as plasmon induced transparency [4-7], breaking through the diffraction limit of light [8-9] and extraordinary optical transmission [10-11]. Numbers of nanoscale structures based on SPs have been theoretically and experimentally investigated in recent years. In the large number of nanoscale structures, MDM waveguides can not only support the propagating of SPPs on the surface of metal and dielectric, but also have the advantages of long propagation distances, small size, lossless and simple fabrication technique. Thus, the MDM
Waveguides have been widely investigated in nano devices, such as all-optical switches [12-13], all-optical logic gates [14, 15], plasmonic filters [16-20] and plasmonic sensors [21-24].

As a promising application, the plasmonic filter based on MDM waveguide systems has been attracted many people’s attention in recent years [16,19, 25-29]. For example, Bragg-reflector filters [25], disk-shaped resonator filters [26] and ring-rectangular filters [27]. All mentioned filters above can only select the wavelength in a narrow bandwidth. The wide band-stop filtering has been reported in recent years. Many people aimed at raising the bandwidth of the band-stop filtering. Elbialy et al. proposed a MDM plasmonic slot waveguide filtering and the bandwidth of band-stop can reach to 75nm [28]. Chen et al. proposed a single-stub coupled waveguide filtering, and the bandwidth of band-stop can reach up to 150nm [29]. Wang et al. introduced a double ring-resonators coupled waveguide filtering and got the 700nm bandwidth of band-stop filtering [19]. In our recent work, we proposed a multiple-mode stub coupled MDM waveguide structure, and it showed long transmission forbidden band when the MDM waveguide is coupled with the fixed height to length stub [23]. Here, we suspect that whether the fixed height to length stub coupled waveguide can show good band-stop filtering performance?

In this work, we first investigate the transmission spectrum and filtering performance when the MDM waveguide coupling with single fixed height to length stub and two fixed height to length stubs through finite-difference time-domain (FDTD) simulation method in detail. Then, we introduce the transmission line theory and phase difference analysis to discuss the transmission spectrum and the ultra-wide band-stop filtering performance. At last, the structure parameters and the number of fixed height to length stub effecting on the transmission spectrum and the filtering performance are investigated.

2. Structure model and Analytical Expression

Fig. 1 schematically shows the plasmonic filter and its equivalent model. Each stub with length $a_i$ and height $b_i$ is coupled to the bus waveguide. The parameter $d$ is the coupling distance between adjoined two stubs. $h=50$nm is the width of the bus waveguide. In this work, the height $b_i$ to length $a_i$ is set to be a constant as 0.8. And the metal is silver, with permittivity $\varepsilon_m$ defined by the Drude model:

$$\varepsilon_m(\omega) = \varepsilon_{\infty} - \omega_p^2 / (\omega^2 + i\omega\gamma_p),$$

where $\omega$ stands for the angle frequency of the incident wave,
ε∞=3.7, ωp=1.38×10^{16} \text{rad/s} is the bulk plasmon frequency, and γp=2.73×10^{13} \text{rad/s} stands for the damping rate. And the part in white is chosen to be the air with εa=1 for simplicity. The transmission and reflection spectra responses of our proposed structure are found by using the FDTD simulations. The effective area is divided into uniform Yee cells with ∆x=∆y=2 \text{nm} and ∆t=∆x/2c (c is the velocity of light in vacuum), and the perfectly matched layer (PML) is chosen in our simulation. The p-polarized input plane wave is considered in our simulation.

Fig. 2. Transmission line model of the MDM waveguide side-coupled with fixed height to length stubs.

For discussing the transmission spectrum and filtering performance of our proposed waveguide structure, we establish a complex transmission line theory in our paper. In Fig. 2, the bus waveguide can be regarded as an infinite transmission line with the characteristic impedance $Z_{MDM}$. According to the recent report [30], the $i$th rectangle stub can be treated as two coupled finite transmission lines. The one possesses the characteristic impedance $Z_{Sxi}$ with the propagation constant $\beta(b)$, terminated at the end with a loaded impedance $Z_{Lxi}$, the other one possesses the characteristic impedance $Z_{Syi}$ with the propagation constant $\beta(a)$, terminated at the end with a loaded impedance $Z_{Lyi}$. Thus, the characteristic impedance of the infinite transmission line and the finite transmission line can be written as follows

\begin{equation}
Z_{MDM}(h) = \frac{E_h}{H_c} = \frac{\beta(h)h}{\omega_c \varepsilon_a}
\end{equation}

\begin{equation}
Z_{Sxi}(b) = \frac{\beta(b)h}{\omega_c \varepsilon_a}, \quad Z_{Syi}(a) = \frac{\beta(a)a}{\omega_c \varepsilon_a}
\end{equation}

where the propagation constant $\beta(h)$, $\beta(a)$, $\beta(b)$ can be calculated referring to the recent articles [31]. According to the circuit analysis [30-31], the terminate impedance can be derived as

\begin{equation}
Z_{Lxi}(b) = \frac{\varepsilon_a Z_{Sxi}}{\varepsilon_a} - \tan (\beta(b)h) \quad Z_{Lyi}(a) = \frac{\varepsilon_a Z_{Syi}}{\varepsilon_a} - \tan (\beta(a)a)
\end{equation}

The characteristic impedances of the two finite transmission lines for each stub are replaced by effective impedances $Z_{ul}$ and $Z_{ul}$ so as to simplify the transmission line model. Thus, the value of the effective impedance can also be obtained from the transmission line theory [31]

\begin{equation}
Z_{ul} = Z_{Sxi} \cdot Z_{Lxi} \cdot \tan (\beta(b)h)
\end{equation}

\begin{equation}
Z_{ul} = Z_{Syi} \cdot Z_{Lyi} \cdot \tan (\beta(a)a)
\end{equation}
where \(i\) respects the \(i\)th stub, and the \(j\) is symbol for imaginary. According to the transfer matrix method [31], the transmittance efficiency of MDM waveguide coupled with single stub and double stubs can be derived as follows

\[
T_i = \left| \frac{V_{i+}}{V_{i-}} \right|^2 = |C_i|^2 \exp\left( \frac{-L}{L_{spp}} \right)
\]

\[
T_s = \left| \frac{V_{s-}}{V_{s+}} \right|^2 = \left| C_i C_s - (B_{s+} A_i \exp(-j\beta_1 L_{spp}) + A_i B_{s+} \exp(j\beta_1 L_{spp})) \right|^2 \exp\left( - \frac{L}{L_{spp}} \right)
\]

\[
A_i = \frac{Z_{MEM}}{2Z_{e}} \quad A_s = \frac{Z_{MEM}}{2Z_{e}}
\]

\[
B_{s+} = 1 \pm A_s \quad B_{s-} = 1 \pm A_s
\]

\[
C_i = B_{s+} B_{s+} \exp(-j\beta_1 L_{spp}) - A_s A_s \exp(j\beta_1 L_{spp})
\]

where \(\beta_1\) and \(\beta_2\) are effective propagation constants in the first and second stub, respectively. The effect coupling distance \(L_{spp}\) stands for the coupling effect between the two finite transmission lines in \(i\)th stub. \(L\) is the length of the plasmonic waveguide system. \(L_{spp}\) is the propagation length of SPPs which can be obtained from the relation \(L_{spp} = (2Im(\beta))^{-1}\) [30-31]. The exponential factor describes the attenuation of the SPPs while it propagates along the bus waveguide.

3. Simulation and analysis

A long transmission forbidden band has been found in our recent work [23], but we didn’t discuss the formation of the forbidden band in detail. Here, we aim at illuminating the phenomenon through transmission line theory and the FDTD simulation method. Fig. 3 shows the transmission and reflection spectrum of MDM waveguide coupling with only one fixed height to length stub with \(a=300\)nm and \(b=0.8a\). The electric field distribution at the transmission peak and dip is discussed, respectively. The blue solid line is the transmission spectrum, and the green solid line is the reflection spectrum. Observing from the Fig. 3, the transmission ratio is very small at infrared wavelength bands, however, the reflection ratio is very large. It is caused by the destructive interference between horizontal reflected SPPs and vertical reflected SPPs.

![Fig. 3](image_url)

(a) The transmission and reflection spectrum of MDM waveguide coupling with only one fixed height to length stub with \(a=300\)nm and
\(b=0.8*a.\)

For analyzing the mechanism of the transmission spectrum in Fig. 3, the phase delay of the fixed height to length stub coupled waveguide structures is discussed. When the incident light spreads from the input side, the SPPs can divide into two parts, which are the horizontal reflected SPPs along the input direction in the stub and the vertical reflected wave is perpendicular to the bus waveguide in the stub. This will generate the phase difference between the horizontal reflected SPPs and the vertical reflected SPPs. These phase delays of the horizontal reflected SPPs and vertical reflected SPPs can be expressed by the following equations [19]  
\[
\phi_1 = \frac{4\pi}{\lambda} n_{\text{eff}} a + \theta_1 \quad \phi_2 = \frac{4\pi}{\lambda} n_{\text{eff}} b + \theta_2
\]  
(11)

where \(n_{\text{eff}}\) is the effective refractive index of the SPPs. \(\theta_1\) and \(\theta_2\) are the phase shift caused by the reflections on the metal-air interface at the horizontal and the vertical direction, respectively. For the vertical reflected SPPs wave, there is once half-wave loss, however, there are twice half-wave loss for the horizontal reflected SPPs. Thus, \(\theta_1 = 2\pi\) and \(\theta_2 = \pi\). Combining Eq. (11), the total phase difference of this structure is  
\[
\phi = \phi_1 + \phi_2 = \frac{4\pi}{\lambda} n_{\text{eff}} (a+b) + 3\pi
\]  
(12)

We can see the total phase difference caused by the horizontal reflected SPPs and the vertical reflected SPPs, which is dependent on length \(a\), height \(b\) and the effective refractive index. In our proposed structure, the height \(b\) to length \(a\) is set to be a constant as 0.8 for satisfying the destructive interference condition. The SPPs waves can’t transmit through the bus waveguide due to the destructive interference of the horizontal reflected SPPs and the vertical reflected SPPs, which results in a flat transmission stop band [19].

Then we will introduce Eqs. (8) and (9) fitting with FDTD simulation results for further illuminating the wide forbidden band and explore its application of filtering. Fig. 4 (a) shows the transmission spectrum of MDM waveguide side-coupled with only one fixed height to length stub. The red line is the FDTD result and red triangular symbol is the transmission line theory result with \(a=300nm\) and \(b=0.8*a\). The blue line is the FDTD result and blue triangular symbol is the transmission line theory result with \(a=450nm\) and \(b=0.8*a\). We can see both the two spectra show low transmission ratio. In addition, the transmission line theory result can well agreement with the FDTD simulation result. For the case of \(a=300nm\), the fitting parameters \(\beta_f = \beta(270nm)\) and \(L_{\text{eff}}=6nm\). The other case of \(a=450nm\), the fitting parameters \(\beta_f = \beta(405nm)\) and \(L_{\text{eff}}=8.5nm\). Imagining what happens when two fixed height to length stubs are side-coupled to the bus waveguide with \(d=50nm\), \(a_1=300nm\), \(b_1=0.8*a_1\), \(a_2=450nm\) and \(b_2=0.8*a_2\) ? Transmission spectra of the MDM waveguide coupled with two fixed height to length stubs are shown in Fig. 4 (b). We can see the transmission forbidden band is very long, and the transmission ratio is also very small. We simulate electric field distributions at 644.1nm, 816nm, 973.8nm and 1500nm in Fig. 4 (c-f) for illustrating the physical mechanism of transmission forbidden phenomenon. The electric field distributions in Figs. 4(c-f) are agreement with transmission spectra at wavelengths I, II, III and IV in Fig. 4 (b). Figs. 4 (d-f)
show strong interference in the reflection area. Thus, the transmission spectra show wide forbidden bands. The blue triangular symbol in Fig. 4 (b) is the transmission line theory result, which can demonstrate the transmission spectrum well with $\beta_1=\beta(278\text{nm})$, $L_{\text{eff}1}=6.4\text{nm}$, $\beta_2=\beta(400\text{nm})$, $L_{\text{eff}2}=9\text{nm}$. Then, we also introduce the definition of bandwidth of the band-stop filtering as the difference between the two wavelengths smaller than 1% transmittance [19]. Under this definition, the bandwidth of the band-stop filtering in two fixed height to length stubs coupled waveguide system can reach to 860nm, this result is much longer than that in recent articles [19, 28-29]. Therefore, I think our proposed structure may have important applications in plasmonic filtering devices.

Fig. 4. (a) The transmission spectrum of MDM waveguide coupled with only one fixed height to length stub, red line (FDTD result) and red triangular symbol (transmission line theory result) with $a=300\text{nm}$ and $b=0.8*a$, blue line (FDTD results) and blue triangular symbol (transmission line theory result) with $a=450\text{nm}$ and $b=0.8*a$. (b) The transmission spectra of MDM waveguide coupled with two fixed height to length stubs with $d=50\text{nm}$, $a_1=300\text{nm}$, $b_1=0.8*a_1$, $a_2=450\text{nm}$ and $b_2=0.8*a_2$, blue line is FDTD result and blue triangular symbol is transmission line theory result results. (c-f) The electric field distribution at 644.1nm, 816nm, 973.8nm and 1500nm corresponding to I, II, III and IV in Fig. 4 (b).

Here, we investigate the transmission spectrum as a function of the coupling distance $d$ between the two stubs with $a_1=300\text{nm}$, $b_1=0.8*a_1$, $a_2=450\text{nm}$ and $b_2=0.8*a_2$. We can see the stop band shows red shift and a sharp peak occur around
1000nm when \( d=125\text{nm} \). For discussing the bandwidth of band-stop filtering as a function of the coupling distance \( d \) between the two fixed height to length stubs, we plot the bandwidth as a function of the coupling distance in the inset of the Fig. 5 (a). we can see the bandwidth of our proposed filtering structure first decreases and then increases with the increasing of the coupling distance \( d \), and the maximum of the bandwidth of band-stop can reach to 1200nm. In Fig. 5 (b), we investigate the transmission spectrum as a function of the parameter \( a_2 \) with \( a_1=300\text{nm} \), \( b_1=0.8*a_1 \), \( d=50\text{nm} \). We can see the transmission ratio decreases much soon as \( a_2 \) increases from 250nm to 450nm. Thus, the bandwidth will increase with the increasing of the \( a_2 \) as shown in inset of Fig. 5 (b). These results will provide the tuning method for the plasmonic filters.

**Fig. 5.** (a) The transmission spectra as a function of the coupling distance with \( a_1=300\text{nm} \), \( b_1=0.8*a_1 \), \( a_2=450\text{nm} \) and \( b_2=0.8*a_2 \). Inset figure shows the bandwidth of band-stop as a function of \( d \). (b) The transmission spectra as a function of length of the second stub with \( a_1=300\text{nm} \), \( b_1=0.8*a_1 \), \( d=50\text{nm} \). Inset figure shows the bandwidth of band-stop as a function of \( a_2 \).

At last, we will discuss the transmission spectrum and the bandwidth of band-stop filtering as a function of the number of the stub. Firstly, we investigate the transmission spectrum and the band-stop filtering when the MDM waveguide is coupled with the multiple same size stubs. Fig. 6(a) shows the transmission spectrum as the number of the same stub increases from 1 to 4 with \( a_1=300\text{nm} \) and \( b_1=0.8*a_1 \). And the inset of Fig. 6 (a) shows the bandwidth of band-stop filtering increases as the number of the same stub increases. We can see that the transmission ratio will decrease as the number of the stub increases from 1 to 4. Thus, the bandwidth of our proposed plasmonic filtering increases from 0 to 1250nm with the \( n \) ranging from 1 to 4. Then, the transmission spectrum and the bandwidth of band-stop filtering with the number of the different size stub increases from 1 to 4 as shown in Fig. 6 (a). Red line corresponds to \( n=1 \) with \( a_1=300\text{nm} \), black line corresponds to \( n=2 \) with \( a_1=300\text{nm} \) and \( a_2=350\text{nm} \), blue line corresponds to \( n=3 \) with \( a_1=300\text{nm} \), \( a_2=350\text{nm} \) and \( a_3=400\text{nm} \), green line corresponds to \( n=4 \) with \( a_1=300\text{nm} \), \( a_2=350\text{nm} \), \( a_3=400\text{nm} \) and \( a_4=450\text{nm} \). Comparing with the Fig. 6(b), we can see that the transmission ratio will go down fast and the bandwidth of the filtering increases with the increasing of \( n \). The maximum of the bandwidth of band-stop filtering can reach to 1750nm, which will provide important application in band-stop filtering devices.
Fig. 6. The transmission spectrum and the bandwidth of band-stop filters as a function of the number of the stub. (a) Same size stubs are coupled to the bus waveguide with $a_1=300$nm, red line corresponds to $n=1$, black line corresponds to $n=2$, blue line corresponds to $n=3$, green line corresponds to $n=4$. (b) Different size stubs are coupled to the bus waveguide, red line corresponds to $n=1$ with $a_1=300$nm, black line corresponds to $n=2$ with $a_1=300$nm and $a_2=350$nm, blue line corresponds to $n=3$ with $a_1=300$nm, $a_2=350$nm and $a_3=400$nm, green line corresponds to $n=4$ with $a_1=300$nm, $a_2=350$nm, $a_3=400$nm and $a_4=450$nm.

4. Conclusion

In summary, the ultra-wide band-stop filtering is observed in one fixed height to length stub, two fixed height to length stubs and multiple fixed height to length stubs coupled waveguide systems. And we establish the phase difference analysis and transmission line theory to discuss the transmission spectrum and the filtering performance of our proposed structure. The transmittance ratio decreases with the increasing of the number of the fixed height to length stub. At the same time, the bandwidth of band-stop filtering increases as the number of the fixed height to length stub increases. Then we investigate transmission spectrum and the filtering performance as functions of structure parameters. The maximum of the bandwidth of band-stop filtering can reach to 1750nm. These results may provide great potential applications for plasmonic filtering in integrated optical circuits.

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References


