Single LED Based Indoor Positioning System
Using Multiple Photodetectors

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Abstract: This paper presents a visible light positioning (VLP) system using a single light emitting diode (LED) and a novel receiver. In the proposed system, the receiver is composed of a horizontal photodetector (PD) and two tilted PDs, and a known calibration point is deployed to improve the accuracy of the positioning. Based upon the multi-PDs' relative positions and the received signal strength (RSS), the location of the receiver can be estimated regardless of the LED's and PDs' characteristics. The influences of tilted PDs on the positioning errors are discussed, i.e. the polar angle α and the difference θ between two azimuth angles. The experimental results show that high accuracy can be achieved when α=20° and θ ranges from 30° to 90°. The average and maximum errors are 2.15cm and 4.01cm respectively in the case of α=20° and θ=45°. The proposed system avoids the inter-cell interference (ICI) caused by different transmitters and can be applied in scenarios with limited LEDs.

Index Terms: Visible light communication (VLC), indoor positioning, light-emitting diodes (LEDs), received signal strength (RSS).

1. Introduction

Recently, the indoor positioning system (IPS) has attracted much attention due to the increasing demands of location-based services (LBSs). In outdoor areas, Global Positioning System (GPS) works well and has been widely used. However, it does not fit for indoor scenarios due to the attenuation and multipath fading [1]. The traditional indoor positioning techniques, such as infrared, ultrasound, Bluetooth and Wi-Fi, are limited by extra infrastructures, low security, high complexity and electromagnetic interference [2-4]. In recent decades, as the traditional lightings are replaced by light emitting diodes (LEDs) increasingly, the visible light positioning (VLP) becomes a promising technique [5,6] because of its advantages of high accuracy, low cost, energy saving, anti-electromagnetic and applications in special environments, e.g., hospitals, industrial sites, air crafts, etc. [7-9].

In the common VLP systems using no less than three LEDs, the inter-cell interference (ICI) has to be eliminated by adopting some techniques, for instance, wavelength division multiplexing (WDM) [10], time division multiplexing (TDM) [11-13], carrier allocation (CA) [14,15] and orthogonal frequency division multiplexing (OFDM) [16]. However, these technologies will bring other problems. For example, the RF filters and other compensations are necessary if CA is used. Ref. [17] proposed a single LED-based VLP system using multiple horizontal photodetectors (PDs), in which the diameter of the receiver is set to 40cm to distinguish the signals received by different PDs. By tilting multi-PDs, the diameter is reduced to 3cm and based on the angle gain profile measured in advance, the system achieves the error of less than 4cm [18,19]. In Ref. [20], a feasible VLP system using multi-PDs is proposed and the total number of LEDs and PDs are no less than four. In particular, with a single LED and three horizontal PDs, the average positioning error and the maximum positioning error are 6cm and 14cm, respectively. Ref. [21] presented a light intensity based positioning system (LIPS) with a single transmitter and multiple light sensors in which the location can be estimated when three linearly independent sensing planes on the receiver are available.

In this paper, a novel VLP system using a single LED and three PDs is proposed. Different from other systems with multi-PDs, a horizontal PD and two tilted PDs are coconformant in the receiver. According to the physical position of the LED, the relative positions of PDs and the received signal strength (RSS), the location can be estimated regardless of the LED’s and PDs’ characteristics. In the established experiments, a calibration point is deployed to improve the positioning accuracy. Besides, the influences of parameters of tilted PDs on the positioning errors are discussed, and the optimum angles can be determined.

The rest of this paper is arranged as follows: Section 2 introduces the model of the proposed novel receiver and the positioning algorithm. Section 3 evaluates the performance of the proposed system and analyzes the parameters related to positioning errors. Finally, the conclusions are given in section 4.

2. Principle of Proposed VLP System

The diagram of the indoor VLP system is shown in Fig. 1 (a). According to the Lambertian radiation pattern, the channel gain H of line-of-sight (LoS) is expressed as [5]...
where \( m \) is the order of Lambertian emission related to the semi-angle at half power \( \Phi_{1/2} \). \( A \) is the physical area of an optical detector and \( d \) represents the transmission distance between the LED and detector. \( \Phi \) and \( \psi \) are the radiation angle and incident angle respectively. \( T_s(\psi) \) and \( G(\psi) \) are the gain of the optical filter and optical concentrator, which are both absent in our system. \( \psi_c \) is the field of view (FOV) of the detector, and it is usually large enough to make \( 0 \leq \psi \leq \psi_c \) always hold.

Assuming \( P_t \) is the average transmitted power of the LED, the received optical power \( P_r \) can be calculated by \( P_r = P_t \cdot H \).

Fig. 1 (b) shows the model of the proposed receiver, which is composed of a horizontal PD (denoted as PD\(_0\)) and two tilted PDs (denoted as PD\(_i\), \( i=1, 2 \)). The coordinates of PD\(_0\) are desired and associate with the coordinates of PD\(_i\).

\[
\begin{align*}
x_{r,i} &= x_r + r \cos(\alpha) \cos(\omega_i) \\
y_{r,i} &= y_r + r \cos(\alpha) \sin(\omega_i) \\
z_{r,i} &= z_r + r \sin(\alpha)
\end{align*}
\]

where \((x_r, y_r, z_r)\) and \((x_{r,i}, y_{r,i}, z_{r,i})\) are the coordinates of PD\(_0\) and tilted PDs, respectively. \( r \) is the distance from the center of PD\(_i\) to that of PD\(_0\). \( \alpha \) is the polar angle of the detector, which is defined as the angle between the z-axis and the normal vector of the detector. In our experiments, \( r \) and \( \alpha \) are the same for tilted PDs. \( \omega_i \) is the angle between the x-axis and the line through PD\(_0\) as well PD\(_i\).

![Diagram](image-url)
The received optical power $P_{r,0}$ and $P_{r,i}$ of PD0 and PDi can be calculated by

$$P_{r,0} = \frac{k h^m}{d^{m+3}}$$

$$P_{r,i} = \frac{k h^m}{d^{m+3}} \left[ (x_i - x_{r,i}) \sin(\alpha) \cos(\beta_i) + (y_i - y_{r,i}) \sin(\alpha) \sin(\beta_i) + h \cos(\alpha) \right]$$

where $k=P_{t}(m+1)A/2\pi$ is constant, because there is only a single LED and the same optical detectors are used.

Formula (5) is divided by formula (4), we have

$$T_i - h \cdot RSSR_i = x_i \cdot A_i + y_i \cdot B_i \quad (i=1,2)$$

where

$$A_i = \sin(\alpha) \cos(\beta_i)$$
$$B_i = \sin(\alpha) \sin(\beta_i)$$
$$T_i = [x_i - r \cos(\alpha) \cos(\omega)] A_i + [y_i - r \cos(\alpha) \sin(\omega)] B_i + h \cos(\alpha)$$

$$RSSR_i = \frac{P_{r,i}}{P_{r,0}}$$

Using two tilted PDs (denoted as PD1 and PD2), the measured coordinates of the receiver $(x_{r,m}, y_{r,m})$ can be calculated by

$$\begin{align*}
x_{r,m} &= \frac{B_2 (T_1 - h \cdot RSSR_1) - B_1 (T_2 - h \cdot RSSR_2)}{A_1 B_2 - A_2 B_1} \\
y_{r,m} &= \frac{-A_2 (T_1 - h \cdot RSSR_1) + A_1 (T_2 - h \cdot RSSR_2)}{A_1 B_2 - A_2 B_1}
\end{align*}$$

(7)

Generally, the relative positions of multi-PDs and the coordinates of LED $(x_t, y_t, z_t)$ are known, which means $A_i$, $B_i$ and $T_i$ $(i=1,2)$ are constant and known in advance. Therefore, RSSR is the only parameter needed to be measured. Assuming $\Delta_1$ ($\Delta_2$) is the measurement error of RSSR1 (RSSR2), the measured coordinates can be also expressed as

$$\begin{align*}
x_{r,m} &= x_{r,\text{real}} + \frac{-\sin(\beta_2) h \Delta_1 + \sin(\beta_1) h \Delta_2}{\sin(\alpha) \sin(\frac{\theta}{2})} \\
y_{r,m} &= y_{r,\text{real}} + \frac{\cos(\beta_2) h \Delta_1 - \cos(\beta_1) h \Delta_2}{\sin(\alpha) \sin(\frac{\theta}{2})}
\end{align*}$$

(8)

where $(x_{r,\text{real}}, y_{r,\text{real}})$ is the real location of the receiver. The positioning error (denoted as pe) is calculated by

$$pe = \sqrt{e_x^2 + e_y^2} = \frac{h}{\sin(\alpha)} \sqrt{\Delta_1^2 + \Delta_2^2 - 2 \Delta_2 \cos(\theta)}$$

(9)

where $\theta$ is the difference between two azimuth angles and it is defined as $\theta=\beta_2-\beta_1$. In the two-dimensional localization experiments, h is constant and fixed at 1.5m. $\alpha$ and $\theta$ are determined by the tilted PDs in the receiver. These angles influence the system’s performance significantly. After appropriate choosing of the angles, the positioning error is only influenced by the measurement errors of RSSR. Compared with other VLP algorithms, only the RSSR is needed for the proposed system which exhibits simple arithmetical operations and low complexity regardless of LED’s and PD’s characteristics.
3. Performance of Proposed VLP System

To assess the performance of the proposed scheme, we build an experimental system as illustrated in Fig. 1 (a). A single LED is located on the ceiling with the coordinates of (0.5m, 0.5m, 1.5m). There are 25 measured points arranged in an area of 1m×1m. The model of the proposed novel receiver is shown as Fig. 1 (b). We are going to estimate the location of PD0 which is 1.2cm away from two tilted PDs (r=1.2cm). The other parameters of the receiver are shown below: α=15°, ω1=0°, ω2=90°, β1=180° and β2=270° (θ=90°). These angles are chosen arbitrarily in order to test the measurement errors of RSSR. In particular, several frameworks with three faces are 3D printed, on which the PDs can be placed and the angles (α and θ) are determined. When the receiver is moving, ω1 is fixed at 0° according to the reference lines on the received plane and the other angles (ω2 and βi) are also known. Besides, if more flexibility is required for users to obtain these angles, the electrical machinery can be utilized. In addition, to reduce the random errors, each point has been measured three times and the average results are given.

Fig. 2 shows the measurement errors of RSSR, ranging from 0.01 to 0.025 and their average values are 0.0178 and 0.0183, respectively. The results show that the measured RSSR is always larger than the theoretical value, which is because there are reflections from four walls and the tilted PDs can receive the more reflections compared with the horizontal PD. As expressed in formula (9), there will be an average error of about 16.39cm if Δi=Δ2=0.02 in the case of α=15° and β=90°. In our experiments, a calibration point with the known coordinates (xc, yc) is used to increase the accuracy of the actual VLP system with reflections, which can avoid the increment of extra PDs. It is assumed that the estimated errors of x-coordinate and y-coordinate at this point are denoted as xc,xc and yc,yc, respectively. The final coordinates of the receiver can be corrected by xc′=xc+Δxc and yc′=yc+Δyc. Generally, the central point with the location of (0.5, 0.5) is regarded as this calibration point. As shown in Fig. 2, Δ1 and Δ2 at the central point are 0.0186 and 0.0192, which approach the average values of 0.0178 and 0.0183, respectively.

![Fig. 2. (a) The measurement error of RSSR: Δ1. (b) The measurement error of RSSR: Δ2.](image)

The error distribution without the calibration point is shown in Fig. 3 (a). The positioning error ranges from 11.40cm to 19.36cm and it is 15.24cm on average. By contrast, Fig. 3 (b) is the error distribution when using the calibration point. It can be observed that the minimum error is about 0cm at the center while the maximum error is 8.82cm near the corner. Compared with Fig. 3 (a), the average error is decreased from 15.24cm to 3.90cm. The results show that the calibration point can largely reduce the positioning errors caused by the measurement errors of RSSR.
On the other hand, the accuracy can be increased further by optimizing $\alpha$ and $\theta$. Firstly, we test the performance of different angles as shown in Fig. 4 and Fig. 5. When $\theta$ is fixed at 90°, the error distribution with $\alpha=10°$ and $\alpha=20°$ are given in Fig. 4 ($\alpha=15°$ is shown in Fig. 3 (b)). It is obvious that best performance is achieved in the case of $\alpha=20°$ compared with $\alpha=15°$ and $\alpha=10°$, which is in conformity with the formula (9). The average errors of them are 2.49cm, 3.90cm and 5.06cm, respectively. Fig. 5 is the error distribution with different $\theta$ when $\alpha$ is equal to 20°. As shown in Fig. 5 (a), the maximum error of 4.01cm and the average error of 2.15cm are achieved in the case of $\theta=45°$. However, when $\theta$ increases to 135°, the maximum and average values deteriorate to 8.14cm and 4.40cm, respectively, as shown in Fig. 5 (b). In addition, the minimum errors of them are all about 0cm at the central point, which is regarded as the calibration point.

Fig. 3. The error distribution: (a) Without the calibration point. (b) With the calibration point.

Fig. 4. The error distribution of $\theta=90°$ with the polar angle $\alpha$: (a) $\alpha=10°$. (b) $\alpha=20°$. 
Fig. 5. The error distribution of $\alpha=20^\circ$ with the difference between two azimuth angles $\theta$: (a) $\theta=45^\circ$. (b) $\theta=135^\circ$.

Fig. 6 gives the relationships between positioning errors and various angles. The red, green, blue and yellow lines correspond to the polar angle of 10°, 15°, 20° and 25°, respectively. Obviously, the positioning error decreases as $\alpha$ increases. However, there is little improvements when $\alpha=25^\circ$ compared with $\alpha=20^\circ$ especially as $\theta$ changes from 15° to 105°. Moreover, the maximum error of $\alpha=25^\circ$ is 4.15cm while it is 3.91cm in the case of $\alpha=20^\circ$. These results can be explained that, with $\alpha=25^\circ$, it is not better for PDs to receive the direct light as much as possible especially when the receiver moves to the corner. Besides, the difference caused by $\alpha$ is unapparent since it agrees with $1/\sin(\alpha)$, which can be hardly identified by the receiver. Consequently, the polar angle fixed at 20° is the optimum value for the proposed system.

Because of the symmetry, the positioning errors with $\theta$ ranging from 0° to 180° are given in Fig. 6, but except 0° and 180°. It is clear that with the increasing $\theta$, the positioning error increases too, especially when the angle is more than 90°. However, little differences can be observed as the angle changes from 15° to 90°. Furthermore, all curves have the same phenomenon that the positioning error will be higher when $\theta=15^\circ$ compared with $\theta=30^\circ$, which is not in conformity with the theoretical results given by formula (9). It is possibly because that the difference between two azimuth angles is too small to distinguish the RSS received by tilted PDs since they have the same polar angle and they are arranged closely. In conclusion, higher accuracy can be achieved when $\theta$ ranges from 30° to 90° which can be chosen as 45° in this work.

Fig. 6. The curves of average positioning errors and $\theta$ with different $\alpha$.

The performance of the proposed system with $\alpha=20^\circ$ and $\theta=45^\circ$ is further evaluated. Fig. 7 (a) is the distribution of actual points and measured points of three times. It can be seen that the estimated points match the actual points (black points) well. Moreover, the cumulative distribution function (CDF) of the average positioning error is given in Fig. 7 (b). It shows that 80% of estimated errors are less than 2.9cm and 95% are less than 3.5cm. As mentioned above, the average and maximum errors are 2.15cm and 4.01cm, respectively, which means high accuracy is achieved in the case of $\alpha=20^\circ$ and $\theta=45^\circ$. 
Fig.7. (a) The distribution of actual points and measured points of three times. (b) The CDF of average positioning error in the case of $\alpha=20^\circ$ and $\theta=45^\circ$.

4. Conclusions

In this paper, a novel VLP system is proposed. By using a single LED as the transmitter, the ICI can be avoided. The receiver is composed of a horizontal PD and two tilted PDs. By considering their relative positions and RSS, the location is estimated regardless of LED’s and PD’s characteristics. In the experiments, a calibration point is used to increase the accuracy for the RSS-based VLP system. Furthermore, we analyze the performance of the proposed system with different parameters of tilted PDs. The experimental results show that higher accuracy can be achieved when $\alpha$ is 20° and $\theta$ ranges from 30° to 90°. In the case of $\alpha=20^\circ$ and $\theta=45^\circ$, the average and maximum errors are 2.15cm and 4.01cm, respectively. The proposed VLP system can be simply realized and applied in some specific scenarios with limited LEDs.

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References


