Series Impedance and Losses of Magnetic Field Mitigation Plates for Underground Power Cables

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Abstract—This paper deals with magnetic field mitigation techniques for underground power systems by using conductive or ferromagnetic shielding plates. Two new aspects are considered in this paper beside the shielding effectiveness analysis: the contribution to the series impedance and power losses due to the conducting set formed by the shielding plate/earth return path. A finite thickness ferromagnetic or conductive shielding is assumed buried parallel to the soil/air plane surface in order to model the induction magnetic field mitigation of a power underground cable system in flat configuration. An analytical method, appropriate to deal with possible optimization procedures, is used based on the magnetic vector potential for multilayered configurations where the shielding plate is treated as a different layer. Results for the magnetic field reduction factor at the soil surface are obtained for different shielding materials, different shielding thickness values, as well as different frequency values. Results are validated by employing the finite element analysis.

Index Terms—Magnetic field, mitigation shielding plates, power losses, series impedance, underground power cables.

I. INTRODUCTION

Remarkable efforts have been devoted to research activities aiming to attain suitable and accurate models to simulate the electromagnetic field created by underground power cables. Two facts justify the increment of these efforts: 1) spreading of urban centers and more demanding environment constraints require more usage of underground power transportation systems with increasing electric energy flux; and 2) danger to human health is still a current matter of concern and controversy regarding the exposure to electric and magnetic fields at extremely low frequency, namely, at power frequency. In particular, possible adverse health effects due to power frequency magnetic fields [1], [2] have led many countries to set limit reference values for exposure to magnetic fields.

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Different solutions have been analyzed to mitigate the magnetic field, in particular for high voltage transmission underground cables. These solutions, which may be included in the area of electromagnetic compatibility, are frequently classified into two groups: 1) “extrinsic methods” where external apparatuses are used such as shields made of ferromagnetic or conductive materials [3]–[8], including grids of insulated conductors [9] or even other technics like mitigation loops [10]; and 2) “intrinsic methods” where cable parameters themselves are under attention for mitigation purposes, for certain phase arrangements or compacting strategies [11].

This paper deals with extrinsic techniques by using a magnetic field mitigation shielding for underground power cables made of conductive or ferromagnetic materials.

Methods have been developed [3]–[7], [12]–[19] to evaluate accurately magnetic field quantities due to underground power transmission-line systems. These methods include analytical [13]–[18] or numerical methods, as the finite element analysis (FEA) [3]–[5], [19], hybrid methods [6], or integral methods combined with partial element equivalent circuits [7]. In particular, numerical approaches have grown in importance to obtain more realistic models taking into account the presence of inhomogeneity, nonlinearity, and complex configurations. On the other hand, when possible, analytical methods still continue to be up to date due to their high performance concerning computation efforts: time and memory consumptions.

An infinite ferromagnetic or conductive shielding with finite thickness is buried in the soil parallel to the soil/air plane surface covering a three-phase power underground system composed of three cables in flat configuration. Under these conditions, analytical methods can be used assuming a quasi-static approximation for the field inside the conducting media. In [12] and [13], Fourier developments centered in the cable axis are used showing that the zero-order term equivalent to the Pollaczek’s approximation [14] is accurate to describe the field in the earth for frequencies less than some 10 kHz. In these works, either Pollaczek’s integrals or the generalized ones [12], [13] are evaluated using power series developments in parallel with numerical methods. In [15], [16], and [18], Pollaczek’s integrals are evaluated using purely numerical techniques, and in [17], approximations using the first two terms of the power series developments are taken for frequencies less than 1 MHz. Finally, Pollaczek’s approximation has been also extended to treat power cables buried in multilayered soils [18]. Identical analytical rationale is adopted in [20] and [21] for overhead conductors. In
[21], perfect electric and magnetic conductor approximations were used for the shielding plates.

The studies performed in this paper consider a three-phase high-voltage underground cable in flat configuration, with the sheath bonded at one single point or cross-bonded and consequently the sheath current is taken to be equal to zero with good accuracy. In this case, where non-neglecting induction magnetic field values are observed on the earth surface, a plane conducting shield is the best way to mitigate the magnetic field due to currents in already installed cables. An analytical treatment is taken based on the vector potential for a layered media approach by using Pollaczek’s approximation. The novelty in the methodology lies mainly in two aspects: the consideration of the shielding plate as a different layer (1); and, the use of the fast Fourier transform (FFT) to evaluate Pollaczek’s integrals (2) avoiding numerical difficulties some of them referred in [18].

The validation of the presented method as well as the validation of the model assumptions was done by employing the FEA [22]. Results for the magnetic field reduction factor at the soil surface [4] are obtained for different shielding materials (conducting or ferromagnetic materials), different shielding thickness values, as well as different frequency values. Finally, two new, important but not common aspects are considered: the contribution to the series impedance and power losses due to the conducting set formed by the shielding plate/earth return path.

Section II presents the analytical formulation for the magnetic field, Section III describes the obtained numerical results, and Section IV deals with the conclusion.

II. ANALYTICAL FORMULATION FOR THE MAGNETIC FIELD

The field formulation is based in this paper on the magnetic vector potential, using the same rationale presented in [15]. The following assumptions are considered:

1) The earth is a linear and homogeneous medium inside each layer, separated from the air region by a plane surface;
2) The shielding plate is taken infinite with finite thickness located parallel to earth/air plane surface constituted by a linear and homogeneous conducting medium;
3) A three-phase power cable composed of three single core cables in flat configuration is buried below the shielding plate. The sheath currents are taken equal to zero;
4) The magnetic field sources are the axial currents of the three-phase power cable allowing a two-dimensional (2-D) treatment for the field; and
5) The quasi-static assumption is considered.

The cross section of the system under study is represented in Fig. 1. A single-circuit 400 kV power cable in flat formation is considered. The three cables are equal, with outer radius \( r_c \), separated by distance \( s \) (between two adjacent cables) and buried at a depth \( h \) from the earth/air plane interface. The shielding plate, considered with infinite width, is buried above the power cable system at a depth \( d \) from the earth surface and has thickness \( t \).

The system currents are assumed to flow along the \( z \)-axis, being \( I_k \) the total phasor current flowing through cable \( k \) (with \( k = 1, 2, 3 \)).

![Fig. 1. Cross section of the system under study: a 400 kV power cable in flat formation buried below an infinite mitigation shielding plate with finite thickness and the base case data used to characterize the system.](image)

A. Magnetic Vector Potential

To calculate the magnetic vector potential, four different regions are identified. Each region is characterized by a conductivity \( \sigma_i \) and a permeability \( \mu_i \) (with \( i = 1, 2, 3 \) and 4):

1) Region 1 is the earth region where the power cable is buried, characterized by \( \sigma_1 \) and \( \mu_1 \).
2) Region 2 is the shielding plate, characterized by \( \sigma_2 \) and \( \mu_2 \).
3) Region 3 is the earth region above the shielding plate, characterized by \( \sigma_3 \) and \( \mu_3 \).
4) Region 4 is the air above the ground, with permeability \( \mu_0 \).

The linearity of the problem allows to calculate the magnetic vector potential, for each region, by the superposition principle, adding the magnetic vector potential due to each cable current considering null currents in all other cables. For region 1, the magnetic vector potential \( A_1^{(k)} \), due to current \( I_k \), is given by

\[
A_1^{(k)}(x, y) = \int_{-\infty}^{+\infty} \left[ F_1(a) e^{j(y+e+t/2)\sqrt{a^2-q_x^2}} \right. \\
+ \frac{e^{i(y+h)\sqrt{a^2-q_x^2}}}{\pi} G_0(W_0(a) \bar{I}_k e^{ja(x-x_k)}) \right] da
\]

(1)

where \( x_k \) is the \( x \)-coordinate of conductor \( k \) and \( q_1, G_0 \) and \( W_0 \) are defined in [15]. Assuming that \( |q_1| r_c \ll 1 \), which is valid for low frequencies (less than 10 MHz), the Pollaczek solution [14] can be adopted [12], [13], allowing to express \( G_0 \) by an approximate solution, and where \( q_1 \) and \( W_0 \) of (1) are given by

\[
q_1 = \sqrt{\omega \mu_1 \sigma_1} e^{-j\pi/4}; \quad G_0 \approx \frac{\mu_1}{2\pi q_1 r_c} H_1^{(2)}(q_1 r_c); \\
W_0(a) = \frac{j}{\sqrt{a^2-q_1^2}}, \quad y > -h
\]

(2)

being \( H_1^{(2)} \) the Hankel function of the second kind of order one.

As much as Pollaczek’s approximation can be applied, meaning that each cable behaves like a filiform current source, (1) can be interpreted as the solution of the current of cable \( k \) in the absence of all other cables having null currents.
For region 2, \(-d + t/2 < y < -d - t/2\), the magnetic vector potential \(\vec{A}_2^{(k)}\) takes the form

\[
\vec{A}_2^{(k)}(x, y) = \int_{-\infty}^{+\infty} \left[ D_1(a) e^{i(y+d-t/2)\sqrt{a^2-q_1^2}} + D_2(a) e^{-i(y+d-t/2)\sqrt{a^2-q_2^2}} \right] e^{ja(x-x_3)} da
\]

(3)

where \(q_2\) has identical definition as \(q_1\) (2), but now calculated for the parameters of region 2, \(\mu_2, \sigma_2\), and related to the penetration depth \(\delta\) inside the plate

\[
q_2 = \frac{\sqrt{2}}{\delta} e^{-\pi/4}, \quad \delta = \frac{2}{\omega \mu_2 \sigma_2}.
\]

(4)

For region 3, the magnetic vector potential \(\vec{A}_3^{(k)}\) is given by

\[
\vec{A}_3^{(k)}(x, y) = \int_{-\infty}^{+\infty} \left[ R_1(a) e^{i\sqrt{a^2-q_1^2}} + R_2(a) e^{-i\sqrt{a^2-q_1^2}} \right] e^{ja(x-x_3)} da, \quad -(d - t/2) < y < 0
\]

(5)

where \(q_3\) has also the same meaning of \(q_1\) (2), but now calculated assigning the parameters of region 3, \(\mu_3, \sigma_3\).

For region 4, the magnetic vector potential \(\vec{A}_4^{(k)}\) is given by

\[
\vec{A}_4^{(k)}(x, y) = \int_{-\infty}^{+\infty} U(a) e^{-|a| y} e^{ja(x-x_3)} da, \quad y > 0.
\]

(6)

Functions \(F_1(a), j_1(a), D_2(a), R_1(a), R_2(a),\) and \(U(a)\) are calculated applying the boundary conditions on each interface [15], whose results are presented in Appendix A. The boundary conditions correspond to impose the continuity of the normal component of the induction magnetic field with the conjunction of imposing the continuity of the tangential component of the magnetic field strength on each interface between two different media. The first condition is satisfied if the continuity of the magnetic vector potential is imposed. As a consequence, boundary conditions can be expressed by

\[
\begin{align*}
\frac{\partial \vec{A}_i^{(k)}}{\partial y} \bigg|_{y=y_{i,j}} &= \frac{\partial \vec{A}_j^{(k)}}{\partial y} \bigg|_{y=y_{i,j}} \\
\frac{1}{\mu_i} \frac{\partial \vec{A}_i^{(k)}}{\partial x} \bigg|_{y=y_{i,j}^+} &= \frac{1}{\mu_j} \frac{\partial \vec{A}_j^{(k)}}{\partial x} \bigg|_{y=y_{i,j}^-}
\end{align*}
\]

(7)

where \(y_{i,j}\) represent the y-coordinate at the surface point between regions \(i\) and \(j\) (where \(j = i + 1\) with \(i = 1, 2, 3\)).

To solve the integrals, known as Pollaczek’s integrals, appearing in (1), (3), (5), and (6), an FFT algorithm is adopted, allowing the numerical difficulties referred in [18] to be overcome for the semi-infinite integrals.

B. Magnetic Induction Field Evaluation

The magnetic induction field may be obtained from the magnetic vector potential as

\[
\vec{B} = \nabla \times \vec{A} = \frac{\partial \vec{A}}{\partial y} u_x - \frac{\partial \vec{A}}{\partial x} u_y.
\]

(8)

Above ground, the \(x\) and \(y\) magnetic induction field components are obtained from (8), by summing the terms given in (6) for each power cable current

\[
\begin{align*}
\vec{B}_x &= \sum_{k=1}^{3} \left( f_{-\infty}^{+\infty} \frac{2jG_2 I_k}{\sqrt{a^2-q_1^2}} e^{-i(y-d-t/2)\sqrt{a^2-q_1^2}} \right) \frac{\sqrt{2}}{\pi} \sqrt{a^2-q_1^2} X(\alpha) \int_{-\infty}^{+\infty} e^{ja|x-x_3|} da \\
\vec{B}_y &= \sum_{k=1}^{3} \left( f_{-\infty}^{+\infty} \frac{2jG_2 I_k}{\sqrt{a^2-q_1^2}} e^{-i(y-d-t/2)\sqrt{a^2-q_1^2}} \right) \frac{\sqrt{2}}{\pi} \sqrt{a^2-q_1^2} X'(\alpha) \int_{-\infty}^{+\infty} e^{ja|x-x_3|} da
\end{align*}
\]

(9)

where functions \(X(\alpha)\) and \(X'(\alpha)\) are given in Appendix A by (A.5) and (A.6), respectively.

The influence of the shielding plate to mitigate the magnetic induction field above ground could be observed introducing the reduction factor \(F_r\). This factor is based on the root mean square (rms) value of the magnetic induction field \(B_{rms}\)

\[
B_{rms} = \sqrt{B_{rms}^2 + B_{rms}^2}
\]

(10)

and gives the ratio between the \(B_{rms}\) value evaluated at \(x = 0\) and \(y = 0\), without shielding plate \((B_0)\) and the one, at the same point, obtained with shielding plate \((B_{sh})\)

\[
F_r = B_0 / B_{sh}.
\]

(11)

C. Series-Impedance Evaluation

The set earth/shielding plate contribution to the series-impedance matrix is obtained assuming that the plate is in contact with the soil and the cable’s conductors are perfect. This contribution corresponds, for the self-impedance of each cable, to the term \(Z_e\) defined in an analogous way as done in [16]. Therefore, the series-impedance earth/plate contribution is evaluated by

\[
\begin{align*}
(\eta) &= [Z] (\overline{T}); \quad (\eta) &= \left( -\frac{dV}{dz} \right)
\end{align*}
\]

(12)

where \((\eta)\) and \(\overline{T}\) are column matrices \((3 \times 1)\), which represent the voltage drop per unit length (pul) and the current phasors, respectively, of the power cables, and \([Z]\) is a \((3 \times 3)\) matrix, which represents the earth/plate contribution to the series impedance. In fact, the complete series-impedance matrix should be of order \((6 \times 6)\) due to the composition of each cable: phase conductor and sheath. However, if the matrix is decomposed into \((2 \times 2)\) submatrices, one for each cable, the contribution of the conducting set formed by the earth/shielding plate return path to such submatrices is characterized to have all equal elements each of them given by the referred \((3 \times 3)\) matrix (12) [16], [17], [19].

As a consequence of the cable configuration used (see Fig. 1), the diagonal elements of the impedance matrix are equal and there are only two different off-diagonal elements. The diagonal elements contribute to the self-impedance of each cable and can be evaluated by

\[
Z_s = \left( \frac{\eta_k}{I_k} \right) I_{j=0 \forall \gamma \neq k} = \left( \frac{j\omega \vec{A}_1^{(k)}}{I_k} \right) I_{j=0 \forall \gamma \neq k}
\]

(13)

where \(\vec{A}_1^{(k)}\) is calculated using (1), for \(x = x_k\) and \(y = -h\).

Now, applying the integral definition of Hankel functions given
in [12], the following result is obtained
\[
Z_k = j\omega \int_{-\infty}^{+\infty} \frac{F_1(a)}{I_k} e^{-(h+d+\epsilon/2)\sqrt{a^2-q_i^2}} da
\]
\[+ j\omega \frac{\mu_1}{2\pi q_i r_e} \frac{H_0^{(2)}(q_i r_e)}{H_1^{(2)}(q_i r_e)}\] (14)

being \(H_0^{(2)}\) the Hankel function of the second kind of order zero.

The off-diagonal elements contribute to the mutual impedance between the cables. Thus, the mutual impedance between cables \(j\) and \(k\) can be evaluated by
\[
Z_{jk} = \left(\frac{\eta_j}{I_k}\right) \int_{x=0}^{\infty} \left(\frac{j\omega A_1^{(k)}}{I_k}\right) \int_{x=0}^{\infty} \eta_{j,k} \, dx \] (15)

where \(A_1^{(k)}\) is calculated using (1), for \(x = x_j\) and \(y = -h\), giving
\[
Z_{jk} = j\omega \int_{-\infty}^{+\infty} \left(\frac{F_1(a)}{I_k} e^{-(h+d+\epsilon/2)\sqrt{a^2-q_i^2}} + \frac{G_0 W_0(a)}{\pi}\right) e^{i\alpha(x_j-x_k)} da.\] (16)

D. Power Losses Evaluation

The contribution to the total power losses of the conducting set composed of the earth/shielding plate can be evaluated using two different approaches as follows
\[
P = \frac{1}{2} \text{Re} \left\{ \left(\mathbf{T}\right)^* \left(\mathbf{T}\right) \right\} = \frac{1}{2} \text{Re} \left\{ \left(\mathbf{T}\right)^* \begin{bmatrix} Z_1 \\ \vdots \\ Z_n \end{bmatrix} \right\}
\]
\[
P = \frac{1}{2} \sum_{i=1}^{3} \int |J_i|^2 dV_i
\] (17)

where the column matrix \(\mathbf{T}\) is for the three-phase currents of the cable, \(\mathbf{J}\) is the current density phasor given by
\[
\mathbf{J}_i = -j\omega\sigma \sum_{k=1}^{3} A_i^{(k)} \] (18)

and \(V_i\) is the volume, both of the conducting region \(i = 1, 2, 3\).

III. NUMERICAL RESULTS

The system analyzed in this paper and the parameters of the base case are shown in Fig. 1. All results are obtained using a cable current \(I_k = \sqrt{2} \times 1025 \times e^{-j(k-1)2\pi/3} [A]\) (with \(k = 1, 2, 3\)). Different materials are used to characterize the shielding plate: aluminum \((\sigma_2 = 3.5 \times 10^7 / \mu m, \mu_2 = \mu_0)\), steel 100 \((\sigma_2 = 10^7 / \mu m, \mu_2 = 100 \mu_0)\), and steel 500 \((\sigma_2 = 10^7 / \mu m, \mu_2 = 500 \mu_0)\).

The model proposed is validated by evaluating the rms value of the induction magnetic field, along \(y = 0\), at 50 Hz, using aluminum and steel 100 shielding plates for the base case data indicated in Fig. 1.

Fig. 2 shows a good agreement between the results achieved with the proposed model with those obtained by employing the finite element method magnetics (FEMM) [22] (with a maximum relative deviation of 1.4% at \(x = 0\)). The model constructed on FEMM uses a 2-D model for a planar shielding plate with finite thickness and width. The infinite space in the FEMM model has to be truncated. In the model, the truncation was performed using two semicircle truncations, with the radius equal to 40 m, which represent the air and the earth unlimited regions. The shielding plate is represented by a rectangle with variable width in order to validate and discuss the infinite width assumption for the shielding plate. Discretization inside the plate is taken dependent on the electromagnetic field penetration depth inside the conducting shielding plate. For the 50 Hz case, having the plate 3 mm thickness, 1 mm and 0.1 mm finite element transversal layers are taken, respectively, for the aluminum and steel 100 cases. Considering, as reference value, \(B_{\text{rms}}\) calculated at \((x = 0, y = 0)\) with the proposed model, the relative deviations obtained for aluminum are less than 1.4% for plate width equal to 60 m, which may be considered infinite so far as it corresponds to about 107 times the space occupied by the cable. Relative deviations obtained for steel 100 are less than 0.8% for the same plate width value. Decreasing the plate width, the obtained relative deviations increase, being greater than 4% for width values less than 3.75 m for both materials, as shown in Fig. 3. These results show that a shielding width about seven times greater than the space occupied by the cable may be treated with enough accuracy adopting the model proposed in the paper. These results are, therefore, decisive for the validation of the model where the analytical treatment has the assumption of an infinite plate width.

The magnetic field mitigation behavior on the earth surface is analyzed using the reduction factor \(F_r\) (11). Emphasis is given to the analysis of the influence of the shielding plate characteristics as well as of the frequency. Fig. 4(a) shows the dependence of \(F_r\) with the plate thickness, for the three materials and considering the frequency equal to 50 Hz. Fig. 4(b) and (c) shows the dependence of \(F_r\) with the frequency, for the three materials and considering the thickness equal to 1.5 and 3 mm, respectively. As expected, the results obtained show, for each material, that the mitigation of the magnetic field is better as the plate thickness or as the frequency increases.
When comparing aluminum with the ferromagnetic materials steel 100 or steel 500, it is interesting to realize that the aluminum is better to mitigate the magnetic field only for a range of plate thickness values, considering a constant frequency [see Fig. 4(a)], or for a range of frequency values, considering a constant plate thickness [see Fig. 4(b) and (c)]. Beyond that range of plate thickness values or frequency values, the ferromagnetic materials present better performances. This behavior is associated with two factors already referred in [21]. One is the magnetic permeability of the shielding plate, contributing to the “flux shunting” mechanism and the other one is due to the “eddy current cancellation” or the skin effect, which is dependent on the relation between the thickness and the plate penetration depth \( \delta \), defined in (4). The change of behavior between aluminum and steel 500, at 50 Hz, occurs for plate thickness equal to 2.4 mm [see Fig. 4(a)]. For the complete understanding of Fig. 4, it is important to have in mind the penetration depth \( \delta \), at 50 Hz, of each material: 12 mm for aluminum, 2.25 mm for steel 100, and 1 mm for steel 500.

Due to higher value of the magnetic permeability, the ferromagnetic material intensifies the magnetic field through the plate. For this reason, at 50 Hz frequency, using steel 500 with 1.5 mm thickness, the magnetic induction field above the shielding plate presents higher magnitude than using aluminum. Nevertheless, using steel 500 with 3 mm thickness, the skin effect is responsible for the concentration of the magnetic induction field in a layer on the bottom surface of the plate, close to the power cable.

It is important to notice that above the shielding plate, for a given \( y \) value, the results do not depend on the depth at which the plate is buried. This conclusion is valid for low frequency and for typical soils, presenting low conductivity and permeability equal to \( \mu_0 \), and can be confirmed theoretically, as shown in Appendix B. Numerically, this result can also be confirmed evaluating the relative differences between \( B_{\text{rms}} \) values calculated at a particular \((x, y)\) point, at 50 Hz, using a shielding plate with 3 mm thickness and buried at two different depth values: 1.352 m (taken as the reference value) and 1 m. With the aluminum shielding plate, the relative difference values obtained at \((x = 0, y = 0)\) and \((x = 0, y = -0.7 \text{ m})\) are \(0.28 \times 10^{-6}\%\) and \(0.16 \times 10^{-6}\%\), respectively. With the steel 500 shielding plate, the relative difference values obtained at \((x = 0, y = 0)\) and \((x = 0, y = -0.7 \text{ m})\) are \(0.76 \times 10^{-6}\%\) and \(0.56 \times 10^{-6}\%\), respectively. These results showing negligible deviations confirms that the mitigation effectiveness on and above the earth/air interface does not depend on the shielding plate burying depth since the plate is installed above the cables.

The contribution of the set earth/shielding plate to the series-impedance matrix and to the system power losses is analyzed.
Fig. 5. Shielding plate thickness and material (aluminum and steel 500) influence on the earth/shielding plate contribution to the (a) self-resistance and mutual resistance; (b) self-inductance and mutual inductance; and (c) total power losses.

for a frequency equal to 50 Hz, changing the plate thickness and using two different materials, aluminum and steel 500. Fig. 5(a) shows the plate thickness influence on the resistance, where \( R_s \) and \( R_m \) represent, respectively, the pull self and average value of the mutual resistance coefficients to obtain the equivalent symmetric three-phase configuration. Note that for both cases, with aluminum and steel, the resistance elements tend to finite values (\( R_s \) and \( R_m \) tend to about 0.0495 mΩ/m), when the plate thickness tends to zero, both having a maximum value for small values of the thickness. These maximum values are not shown in the figure due to the adopted axis scale. Fig. 5(b) is similar but related to the inductance, where \( L_s \) and \( L_m \) represent, respectively, the self and the average value of the mutual inductance coefficients both for unit length. Fig. 5(c) shows the plate thickness influence on the system power losses. Without a shielding plate, considering a null plate thickness, the contribution of the earth gives the following results (already referred): \( R_s \approx R_m = 0.0495 \text{ mΩ/m} \), \( L_s = 1.9 \text{ μH/m} \), \( L_m = 1.6 \text{ μH/m} \), and \( P = 20.5 \text{ μW/m} \).

The obtained results allow us to identify three different zones, as the plate thickness \( t \) increases.

Analyzing Fig. 5(a) the first zone, corresponding to the range where \( t \ll \delta \), the resistance tends to decrease monotonically. The existence of the shielding plate, presenting higher conductivity than the soil, alters the distribution of the current density in the soil, and as \( t \) increases, the shielding plate influence on the system increases, leading to reduced resistance values. In the zone where the resistance decreases, the first zone in Fig. 5(a), the distribution of the current density inside the plate is almost uniform. The zone corresponding to the range where \( t \approx \delta \), the resistance increases slightly due to the nonuniform distribution of the current density inside the plate. The last zone, corresponding to the range where \( t \gg \delta \), the resistance remains with a constant value, since the current density inside the plate is confined in a layer (with size related to \( \delta \)) in the bottom surface of the plate. For the aluminum shielding plate, the last two zones are not visible because the maximum value of the thickness considered is less than the aluminum penetration depth. As expected, when comparing both materials, lower resistance coefficients are obtained using the aluminum plate, since aluminum presents higher conductivity than steel.

The metallic shielding plate changes the configuration of the magnetic field lines, confining them in a region along the plate surface closest to the power cable. Therefore, the inductance values [see Fig. 5(b)] are always lower than values obtained without the shielding plate. The ferromagnetic shielding plate, due to the higher value of its magnetic permeability, flattens the magnetic field lines inside the plate and increases the inductance values. The shape of the curves presented is like the ones obtained for the resistance being associated with the confinement of the magnetic field in the region closest to the power cable, which is dependent on the ratio \( t/\delta \).

The contribution of the set earth/plate to the system power losses is evaluated using (17). Without the shielding plate, the earth only contributes with an increase of 20.5 μW/m for the system power losses, which is much lower than the values shown in Fig. 5(c). Thus, it can be concluded that the presence of the metallic shielding plate is responsible for the increased power losses in the system. The shape of the curves is related and could be analyzed through the three-phase modal resistance behavior with increasing plate thickness. The modal resistance is given by the difference between the self- and mutual resistances. In the range where the power losses increase with increasing thickness, the mutual resistance decreases faster than the self-resistance, see Fig. 5(a). By the contrary, in the range where the power losses
decrease, the mutual resistance decreases more slowly than the self-resistance. For small plate thickness, and despite having lower resistance values, the aluminum presents higher power losses than steel. This occurs because steel presents less modal resistance (less difference between self- and mutual-resistance values).

IV. Conclusion

In this paper, a specific technique to mitigate the magnetic field originated by an underground power cable, using a shielding plate, was analyzed by developing an analytical method adequate to treat a multilayered earth where the plate with infinite width occupies one more layer. The performance of the shielding plate on mitigation of the magnetic induction field is studied considering different: plate materials (aluminum, steel 100, and steel 500), plate thicknesses, plate burying depths, and frequency values. The contribution to the series impedance and power losses of the conducting set earth/shielding plate are also analyzed.

Validation of the proposed method is performed comparing the results obtained with those achieved by employing the FEMM. A good agreement is found since the relative deviations obtained are less than 4% for plate width values greater or equal to 3.75 m for the aluminum and steel 100 shielding plate cases.

Results show that the magnetic field mitigation is better as the plate thickness or the frequency increases. Comparing aluminum with steel, results also show that the ferromagnetic materials are better to mitigate the magnetic field when the plate thickness is greater than the penetration depth of the ferromagnetic material. Otherwise aluminum presents better performance. It is important to notice that the magnetic field mitigation is not influenced by the depth at which the plate is buried.

The existence of the shielding plate, presenting higher conductivity than the soil, alters the distribution of the current density in the soil, and as the plate thickness increases, the shielding plate influence increases, leading to reduced resistance values.

The metallic shielding plate changes the configuration of the magnetic field lines, confining them in a region along the plate surface closest to the power cable. As a consequence, the inductance values decrease relatively with those obtained without the shielding plate.

The presence of the metallic shielding plate is responsible for the increasing power losses in the system. For small plate thickness, and despite having lower resistance values, the aluminum presents higher power losses than steel.

APPENDIX A

Applying the boundary conditions in the plane $y = 0$ functions, $R_1$ and $R_2$ can be evaluated by

$$R_1(a) = \frac{1}{2} \left( 1 - \frac{|a|}{\mu_0 \sqrt{a^2 - q_3^2}} \right) U(a)$$

$$R_2(a) = \frac{1}{2} \left( 1 + \frac{|a|}{\mu_0 \sqrt{a^2 - q_3^2}} \right) U(a). \quad (A.1)$$

Applying the boundary conditions in the plane $y = -d + t/2$ and considering (A.1)

$$D_1(a) = \frac{U(a)}{2} \left[ \left( 1 - \frac{\mu_2|a|}{\mu_0 \sqrt{a^2 - q_3^2}} \right) ch(\xi) + \left( \frac{\mu_3|a|}{\mu_0 \sqrt{a^2 - q_3^2}} - \frac{\mu_2 \sqrt{a^2 - q_3^2}}{\mu_3 \sqrt{a^2 - q_3^2}} \right) sh(\xi) \right]$$

$$D_2(a) = \frac{U(a)}{2} \left[ \left( 1 + \frac{\mu_2|a|}{\mu_0 \sqrt{a^2 - q_3^2}} \right) ch(\xi) + \left( \frac{\mu_3|a|}{\mu_0 \sqrt{a^2 - q_3^2}} + \frac{\mu_2 \sqrt{a^2 - q_3^2}}{\mu_3 \sqrt{a^2 - q_3^2}} \right) sh(\xi) \right] \quad (A.2)$$

where $\xi = (d - t/2) \sqrt{a^2 - q_3^2}$. Applying the boundary conditions in the plane $y = -d - t/2$ and considering (A.1) and (A.2), functions $F_1$ and $U$ can be evaluated by

$$F_1(a) = \frac{\mu_2 \sqrt{a^2 - q_3^2} X(a) - \mu_1 \sqrt{a^2 - q_3^2} X'(a)}{\mu_2 \sqrt{a^2 - q_3^2} X(a) + \mu_1 \sqrt{a^2 - q_3^2} X'(a)} G(a)$$

$$U(a) = \frac{2\mu_2 \sqrt{a^2 - q_3^2} X(a) + \mu_1 \sqrt{a^2 - q_3^2} X'(a)}{\mu_2 \sqrt{a^2 - q_3^2} X(a) + \mu_1 \sqrt{a^2 - q_3^2} X'(a)} G(a) \quad (A.3)$$

where making

$$v = t \sqrt{a^2 - q_3^2}; G(a) = \frac{1}{\pi} e^{-(h - d - t/2) \sqrt{a^2 - q_3^2}} G_0 W_0(a) \tilde{I}_k \quad (A.4)$$

$$X(a) = \left( ch(v) + \frac{\mu_2|a|}{\mu_0 \sqrt{a^2 - q_3^2}} \right) ch(\xi)$$

$$+ \left( \frac{\mu_3|a|}{\mu_0 \sqrt{a^2 - q_3^2}} ch(v) + \frac{\mu_2 \sqrt{a^2 - q_3^2}}{\mu_3 \sqrt{a^2 - q_3^2}} sh(v) \right) sh(\xi) \quad (A.5)$$

$$X'(a) = \left( sh(v) + \frac{\mu_2|a|}{\mu_0 \sqrt{a^2 - q_3^2}} ch(v) \right) ch(\xi)$$

$$+ \left( \frac{\mu_3|a|}{\mu_0 \sqrt{a^2 - q_3^2}} sh(v) + \frac{\mu_2 \sqrt{a^2 - q_3^2}}{\mu_3 \sqrt{a^2 - q_3^2}} ch(v) \right) sh(\xi). \quad (A.6)$$

APPENDIX B

Assuming $q_1 = q_3 \approx 0$, which is valid for low frequency and for typical soils presenting low ground conductivity and permeability equal to $\mu_0$, then $\xi \approx (d - t/2) |a|$ and from (A.4)

$$X(a) \approx \left( ch(v) + \frac{\mu_2|a|}{\mu_0 \sqrt{a^2 - q_3^2}} sh(v) \right) e^\xi$$

$$X'(a) \approx \left( sh(v) + \frac{\mu_2|a|}{\mu_0 \sqrt{a^2 - q_3^2}} ch(v) \right) e^\xi. \quad (B.1)$$
Replacing (B.1) in (9), the result obtained does not depend on the position \( d \) of the shielding plate

\[
\bar{B}_x \approx \frac{3}{2\pi} \sum_{k=-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} \frac{-j2G_0 I_k |a|e^{-(h-t)|a|}e^{-|y|e^{j\alpha(x-x_k)}}}{2\pi |a|ch(v)} + \frac{\mu_0 |a|^2}{\mu_2} \frac{\sqrt{a^2-q_k^2}}{q_k} \right] d\alpha \\
\bar{B}_y \approx \frac{3}{2\pi} \sum_{k=-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} \frac{-2G_0 I_k |a|e^{-(h-t)|a|}e^{-|y|e^{j\alpha(x-x_k)}}}{2\pi |a|ch(v)} + \frac{\mu_0 |a|^2}{\mu_2} \frac{\sqrt{a^2-q_k^2}}{q_k} \right] d\alpha
\]

(B.3)

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REFERENCES


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