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RESEARCH ARTICLE

An Integrated External Archive Local Disturbance Mechanism for Multi-Objective Snake Optimizer

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Abstract — It is an interesting research direction to develop new multi-objective optimization algorithms based on meta-heuristics. Both the convergence accuracy and population diversity of existing methods are not satisfactory. This paper proposes an integrated external archive local disturbance mechanism for multi-objective snake optimizer (IMOSO) to overcome the above shortcomings. There are two improved strategies. The adaptive mating between subpopulations strategy introduces the special mating behavior of snakes with multiple husbands and wives into the original snake optimizer. Some positions are updated according to the dominated relationships between the newly created individuals and the original individuals. The external archive local disturbance mechanism is used to re-search partial non-inferior solutions with poor diversities. The perturbed solutions are non-dominated sorting with the generated solutions by the next iteration to update the next external archive. The main purpose of this mechanism is to make full use of the non-inferior solution information to better guide the population evolution. The comparison results of the IMOSO and 7 state-of-the-art algorithms on WFG benchmark functions show that IMOSO has better convergence and population diversity.
Keywords — Multi-objective optimization, Snake optimizer, Adaptive mating between subpopulations strategy, External archive local disturbance mechanism.
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I. Introduction

A real-world problem often requires to tackle multiple conflicting objectives at once, called a multi-objective optimization problem (MOP) [1]. Different from a single-objective problem, solving MOPs needs to coordinate each objective to make it as optimal as possible. For an MOP, Pareto optimal set is very important, which is a set of optimal solutions weighing various objectives [2]. MOPs are widely existed in various fields, e.g., feature selection [3] and power system distribution [4]. As prac tical problems tend to more and more large-scale and diversified, MOPs are confronted with severe challenges from nonlinearity, high dimension, and multimodality. To solve these problems, evolutionary multi-objective optimization algorithms (EMOAs) have become the major methods for MOPs [5], since they have the strong randomness [6] and can find multiple Pareto optimal solutions in one run.

Non-dominated sorting is a strategy commonly used in EMOAs. Benefitting from its simple mechanism and few parameters, it has become an important research direction for MOPs. For example, Coello *et al.* [7] developed the multi-objective particle swarm optimization (MOPSO) algorithm by using the Pareto dominance to determine particles’ flight directions and saving the discovered non-dominated solutions into a global knowledge base. Deb *et al.* [8] improved the non-dominated sorting genetic algorithm (NSGA) and presented a fast non-dominated sorting genetic algorithm (NSGA-II) based on elites’ guidance, which employs the crowding distance to maintain population uniformity. In multi-objective artificial bee colony optimization (ε-MOABC) [9], an external

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archive is used to save inferior solutions for each iteration. Besides, many new EMOAs are constantly being developed, e.g., multi-objective cuckoo search (MOCS) [10], multi-objective grey wolf optimizer (MOGWO) [11], multi-objective bonobo optimizer (MOBO) [12], and multi-objective slime mould algorithm (MOSMA) [13].

However, although many EMOAs have been proposed, the convergence accuracy and population diversity still need to be enhanced. Most directly employ primitive optimization strategies, such as MOPSO, MOCS, and MOGWO. And some use the canonical processing tricks. For instance, MOSMA uses the non-dominated sorting and crowding distance mechanism in NSGA-II to update non-dominated solutions in each iteration. MOBO3, based on decomposition, divides objective functions into sub-problems with the same population number, where these subpopulations optimize each sub-problem independently. As a result, the performance of EMOAs can not be effectively enhanced.

Snake optimizer (SO) [14] is a novel swarm-based algorithm with the good iterative optimization capacity. To realize SO for solving MOPs effectively, a multi-objective version snake optimizer is developed. The main contributions of this paper are as follows. An adaptive mating between subpopulations strategy is presented to adequately cross information between two subpopulations. Then, to better guide population evolution, an external archive local disturbance mechanism is developed. Based on the above, an integrated external archive local disturbance mechanism for multi-objective snake optimizer is proposed. Finally, experimental results verify the effectiveness of the proposed algorithm.

II. Multi-Objective Snake Optimizer (MOSO)

SO mimics the mating behaviors of snakes and its specific mathematical descriptions are referred to [14]. However, SO cannot be directly used to solve MOPs. Hence, a multi-objective snake optimizer (MOSO) is constructed based on the non-dominated sorting and the crowding distance.

For each iteration, the updated population is equipped with an external archive of maximum capacity N to save the non-inferior solutions obtained so far. If the number of non-inferior solutions exceeds the maximum capacity, truncation is adopted. Meanwhile, each subpopulation also needs to set an external archive with a maximum capacity of $N/2$.

SO divides the entire population into the male and female subpopulations, and the male (female) subpopulation moves towards the best female (male). Furthermore, the current population also evolves with reference to the position of the elite (X_{food}). By analysis, an external archive maintenance should be carried out for each subpopulation in the updating process. After each subpopulation update, the male (female) subpopulation and the previous generation are combined for non-dominated sorting [8]. Based on crowding distance [8], the first $N/2$ individuals formed the initial population of the next iteration and saved in their corresponding sub-archives. It needs to be mentioned that since there are many Pareto optimal solutions, the best female (male) for their subpopulation is randomly selected from the non-dominated solutions in the current sub-archive. Similarly, the global optimal solution (X_{food}) based on the entire external archive is determined. For other individuals, parameters containing fitness values are used to guide evolution. However, corresponding to the MOP, the fitness value of the objective with a tendency replaces the fitness value of the single objective problem.

III. Integrated External Archive Local Disturbance Mechanism for MOSO

The proposed integrated external archive local disturbance mechanism for multi-objective snake optimizer (IMOSO) is introduced in detail. Firstly, the mathematical models of the two improved strategies are described. Then, the implementation framework of the proposed IMOSO algorithm is given.

1. Adaptive mating between subpopulations strategy

In the mating stage of SO, each male (female) only has a mating relationship with a current female (male). For the fighting stage, each male (female) is also updated only by the best female (male). Except for the above two stages, for the other stages, males or females only search for renewal within their independent subpopulation. This may lead to the underutilization of the solution information and the poorer evolutionary directions of individuals. To solve these problems, an adaptive mating between subpopulations strategy is developed to strengthen the exploration and exploitation.

In nature, the mating system of snakes is polygamy, which can effectively inherit good information and improve the survival rate of offspring. The new behavior of finding for a spouse is introduced. This model is constructed to achieve the re-searching evolution of snakes. Different from the mating behavior in SO, snakes do not produce new offspring during the process of finding mates, but rather update new locations where they may meet. However, not all males (females) will update their current positions. They also have to make a choice based on the degree of compatibility between their partners.

To guarantee that male (female) snakes select more beneficial spouses, the dominated relationship is used to determine whether the found mates are suitable for them. If a new male (female) dominates the original male (female), the current position is updated. Otherwise, keep the original position unchanged. It can be calculated as follows:

$$X_{i,m}^{\text{new}}(t+1) = \begin{cases} X_{i,m}^{\text{current}}, & f\left(X_{i,m}^{\text{current}}\right) \prec f\left(X_{i,m}\right) \\ X_{i,m}, & \text{otherwise} \end{cases}$$
$$X_{i,f}^{\text{new}}(t+1) = \begin{cases} X_{i,f}^{\text{current}}, & f(X_{i,f}^{\text{current}}) \prec f\left(X_{i,f}\right) \\ X_{i,f}, & \text{otherwise} \end{cases} \tag{1}$$

where $X_{i,m}^{\text{current}}$ and $X_{i,f}^{\text{current}}$ are male and female to be renewed based on adaptive mating between subpopulations strategy, respectively. Their specific expressions are

$$\begin{aligned} X_{i,f}^{\text{current}} &= r \times X_{i,f}(t) + (1-r) \times X_{\text{rand},m}(t) \\ X_{i,m}^{\text{current}} &= r \times X_{i,m}(t) + (1-r) \times X_{\text{rand},f}(t) \end{aligned} \tag{2}$$

where r is defined as the random number between $[0, 1]$. $X_{\text{rand},m}$ and $X_{\text{rand},f}$ are the male and female randomly selected from their subpopulation in the t iteration.

Figure 1 shows an illustration of the adaptive mating between subpopulations strategy. The population size is set as 10. Males and females are distinguished by different colors (only female positions updated are shown). It can be seen that newly generated female snakes are conducive to explore more new locations in the search space. These positions are mapped from the decision space to the objective space and updated to better positions by the domination relation. With the iterative evolution, the new dominated solution is closer to the Pareto front of MOP.

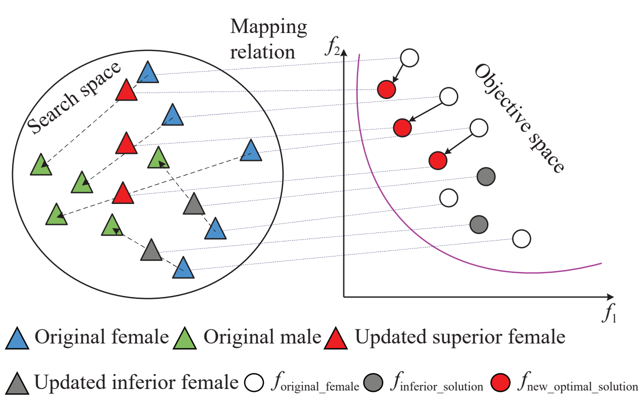

Figure 1 Adaptive mating between subpopulations strategy.

2. External archive local disturbance mechanism

Traditional non-dominated sorting is performed on the hybrid of the current population and original population. The traditional external archive maintenance mechanism directly deletes individuals with small crowding distances to avoid an infinite expansion of the population. Factly, although these solutions show poor diversity for the current population, they still have some certain potential abilities to guide population evolution than other solutions. If these solutions are deleted, some positive solution information is easily lost. Moreover, in the late iterations, the elite set traps into local optimal. In view of the fact that SO divides the snake population into two sub-populations, female and male. It is necessary to carry out archives' maintenance for two subpopulations. Therefore, an external archive local disturbance mechanism is proposed, which is expected to enhance the ability of the algorithm to jump out of the local extremum and advance towards the Pareto front. It should be emphasized that the external archive local disturbance mechanism is used to maintain external archive for subpopulations.

The external archive local disturbance mechanism does not need to perturb all individuals. When each subpopulation is updated, only the 50% individuals with the lowest level of dominance and the smaller crowding distances are selected for local re-search. For early iterations, there are a few non-dominated solutions in the external archive. These individuals with low domination levels should perform less disturbance in their local search areas because they have a lower guiding ability for the next generation. In the late iterations, the number of non-dominated solutions in the external archive is large, and non-inferior solutions are disturbed more. The new solutions and the next generation solutions are non-dominated to ensure the dominated relationship between the solutions. The updated external archive is used as the initial population for the next iteration. The mathematical expression for the local disturbance of a solution is

$$X^{\text{new}} = X^* + X^* \times \alpha \tag{3}$$

where X^* represents a non-inferior solution with the small crowding distance in the external archive. α is the factor of random disturbance, and its mathematical expression is

$$\alpha = r \times \frac{t}{T} \tag{4}$$

With the increase of iteration number, the disturbance range gradually expands, and the disturbance rate is regulated by $\dfrac{t}{T}$.

Figure 2 depicts the external-archive local disturbance mechanism in the late iterations. The maximum capacity of the external archive is 10. In the current iteration process, there are 16 non-dominated solutions used to update the external archive. First, the top 10 non-dominated solutions with large crowding distances are directly stored in the current external archive. In the remaining 6 individuals with low crowding distances, 3 individuals are randomly selected for local disturbance. These solutions are then performed non-dominated sorting to update the external archive for the next iteration.

3. Execution process of the IMOSO algorithm

An IMOSO is proposed based on the above two strategies. Algorithm 1 gives the pseudocode of the proposed IMOSO algorithm.

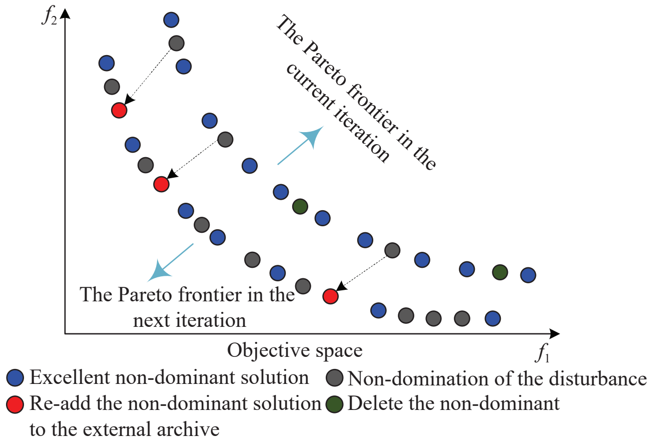

Figure 2 External-archive local disturbance mechanism.

Algorithm 1 Integrated external archive local disturbance mechanism for multi-objective snake optimizer

Require: Population size N, dimension D, variable upper bound Ub and lower bound Lb, maximum iterations T, set females (K_f) and males (K_m) to be disturbed to empty.

1: Initialize the population;
2: Calculate the fitness value for each individual;
3: Randomly divide the population into the male subpopulation (P_m) and the female subpopulation (P_f);
4: Perform non-dominated sorting for P_m and P_f;
5: Update the male sub-archive (Arch_m), the female sub-archive (Arch_f), and the whole archive (Arch);
6: **while** ($t < T$) **do**
7: Update P_m^{new} and P_f^{new} by the SO algorithm;
8: Cross the male and female subpopulations using (2);
9: Update the positions of males and females using (1);
10: **if** $K_f = [\ \ ]$ **then**
11: Merge the female population $P_f^1 = P_f \cup P_f^{\text{new}}$;
12: Select the top $N/2$ individuals and store in P_f;
13: **else**
14: Merge the female population $P_f^1 = P_f \cup P_f^{\text{new}} \cup K_f$;
15: Select the top $N/2$ individuals and store in P_f;
16: Update the females that need to be disturbed using (3);
17: **end if**
18: Update the Arch_f;
19: Update the Arch_m, as in steps 12–18 of Algorithm 1;
20: Merge the whole population $P = P_0 \cup P_m \cup P_f$;
21: Select the top N individuals and store in P;
22: Update the whole archive (Arch);
23: **end while**
24: **return** Arch.

IV. Experimental Results and Analysis

To verify the convergence and diversity of the proposed IMOSO algorithm, the ZDT, DTLZ, and WFG test suites are used in this section. It should be noted that the performance of IMOSO on ZDT and DTLZ functions is similar to WFG functions. Limited by space, this paper only shows the test results of all algorithms on WFG functions. All experiments were run on the 11th Gen Intel(R) Core(TM) i7-11700@2.50 GHz operating system, MATLAB R(2019a) version.

1. Benchmark functions and comparison algorithms

For the dual-objective functions, WFG1 function's Pareto front is nonlinear and WFG2 function's Pareto front turns on discontinuous concave surface. The Pareto front of WFG3 function is linear. The Pareto fronts of WFG4, WFG5, WFG6, WFG7, and WFG8 functions show convex. On the three-objective functions, the Pareto fronts of WFG1 and WFG2 functions are presented as a waterfall in 3D space. The Pareto fronts of WFG8 and WFG4 functions are a hypersurface in 3D space, and the Pareto front of WFG3 function is showed as a straight line in 3D space. These functions are the international common test standards and are challenging.

IMOSO is compared with MOPSO, NSGA-II, MOBO3, MOGWO, MOSMA, and MOCS. To better evaluate two proposed strategies, MOSO based on the elite non-dominated sorting is also compared. The maximum iterations (T), the population size (N), and the external archive capacity (Arch) of all algorithms are set to 200, 100, and 100, respectively. Meanwhile, in the IMOSO algorithm, the capacity of each sub-archive is set to 50. The remaining parameters of comparison algorithms are set according to the original literature. To ensure fair competition among algorithms, each algorithm is run independently for 30 times on each function. The mean (mean) and standard deviation (std) of the experimental results run for 30 times are taken as the statistical indicators.

Two crucial aspects of an EMOA are the convergence and diversity of Pareto optimal solutions. Inverted generational distance (IGD) [15] and hypervolume (HV) [16] are used as comprehensive metrics.

2. Performance analysis on the dual-objective test functions

Tables 1 and 2 show the IGD and HV values of the IMOSO algorithm and 7 comparison algorithms on dual-objective WFG suite. To avoid accidental results, IMOSO is treated as a control algorithm and paired with each competitor for the Wilcoxon rank sum test. This paper sets the significance level at 5%. If the generated p value is lower than 5%, it indicates that there is a significant difference between the two algorithms in statistical significance, represented by "+". Otherwise, it is considered that the difference between the two algorithms is not obvious, represented by "–".

It is observed that for IGD and HV metrics, IMOSO has 14 best and 13 best results out of 16 instances, respectively. The IGD results of IMOSO on WFG1 and WFG4 functions are more significant. For WFG8 func-

Table 1 Mean and standard deviation values of IGD metric obtained by different algorithms on dual-objective functions (WFG)

Fun.	Metric Val.	IMOSO	MOSO	MOPSO	NSGA-II	MOBO3	MOGWO	MOSMA	MOCS
WFG1	mean	**6.29E−2**	4.58E−1 (+)	6.02E−1 (+)	4.58E−1 (+)	4.10E−1 (+)	6.24E−1 (+)	6.19E−1 (+)	4.41E−1 (+)
	std	4.05E−2	1.36E−2	4.61E−2	6.95E−2	6.56E−2	1.81E−2	**1.33E−2**	2.17E−2
WFG2	mean	**5.20E−3**	6.59E−3 (+)	1.59E−1 (+)	3.13E−1 (+)	5.61E−2 (+)	2.08E−2 (+)	1.23E−1 (+)	5.61E−3 (+)
	std	**5.26E−4**	9.63E−4	8.31E−2	1.27E−1	3.20E−2	4.42E−3	3.01E−2	5.41E−4
WFG3	mean	**5.35E−3**	7.91E−3 (+)	8.97E−2 (+)	1.97E−1 (+)	5.16E−2 (+)	2.40E−2 (+)	1.14E−1 (+)	7.28E−3 (+)
	std	**3.49E−4**	8.43E−4	4.70E−2	3.79E−2	3.11E−2	3.24E−3	2.38E−2	4.49E−4
WFG4	mean	**5.57E−3**	2.07E−2 (+)	7.67E−2 (+)	1.07E−1 (+)	3.25E−2 (+)	2.79E−2 (+)	1.01E−1 (+)	2.19E−2 (+)
	std	**2.96E−4**	1.90E−3	5.03E−2	8.44E−2	9.21E−3	2.13E−3	2.03E−2	1.54E−3
WFG5	mean	**2.43E−2**	2.60E−2 (+)	6.53E−2 (+)	1.59E−1 (+)	3.24E−2 (+)	3.48E−2 (+)	2.86E−2 (+)	2.67E−2 (+)
	std	**1.09E−4**	1.51E−3	5.28E−2	6.29E−2	1.81E−3	5.05E−3	1.11E−3	1.64E−3
WFG6	mean	**1.87E−2**	3.80E−2 (+)	5.21E−2 (+)	2.48E−1 (+)	7.06E−2 (+)	2.50E−2 (+)	4.93E−2 (+)	4.51E−2 (−)
	std	**3.99E−4**	5.01E−3	3.74E−2	3.89E−2	2.32E−2	6.31E−3	3.20E−3	3.62E−2
WFG7	mean	**5.40E−3**	6.22E−3 (+)	4.96E−2 (+)	2.14E−1 (+)	1.20E−2 (+)	2.44E−2 (+)	1.38E−1 (+)	6.65E−3 (+)
	std	**3.16E−4**	3.64E−4	3.38E−2	4.74E−2	2.52E−3	5.10E−3	1.16E−2	3.46E−4
WFG8	mean	4.64E−2	4.84E−2 (+)	1.49E−1 (+)	1.59E−1 (+)	6.19E−2 (+)	6.74E−2 (+)	1.36E−1 (+)	**4.63E−2** (−)
	std	**5.62E−4**	1.54E−3	6.36E−2	5.09E−2	1.13E−2	8.20E−3	1.53E−2	7.95E−4
Num. of +/−			8/0	8/0	8/0	8/0	8/0	8/0	6/2

Note: Bold indicates that the results obtained by the algorithm rank the first; "+" and "–" denote the Wilcoxon rank sum test $P < 0.05$ and $P > 0.05$ between the comparison and IMOSO algorithms.

Table 2 Mean and standard deviation values of HV metric obtained by different algorithms on dual-objective functions (WFG)

Fun.	Metric Val.	IMOSO	MOSO	MOPSO	NSGA-II	MOBO3	MOGWO	MOSMA	MOCS
WFG1	mean	**6.16E−1**	1.72E−1(+)	5.95E−3(+)	1.10E−1 (+)	2.04E−1 (+)	2.20E−3 (+)	4.66E−3 (+)	9.47E−2 (+)
	std	4.97E−2	1.88E−2	1.45E−2	5.42E−2	5.81E−2	**9.30E−3**	9.98E−3	2.42E−2
WFG2	mean	**6.31E−1**	6.28E−1 (+)	5.18E−1 (+)	4.62E−1 (+)	5.65E−1 (+)	6.10E−1 (+)	5.11E−1 (+)	6.29E−1 (+)
	std	**5.49eE−4**	1.34E−3	5.98E−2	7.44E−2	4.25E−2	4.17E−3	2.88E−2	7.42E−4
WFG3	mean	**5.79E−1**	5.74E−1 (+)	4.82E−1 (+)	3.98E−1 (+)	5.14E−1 (+)	5.46E−1 (+)	4.27E−1 (+)	5.75E−1 (+)
	std	**4.60E−4**	1.34E−3	3.51E−2	2.83E−2	4.06E−2	4.93E−3	2.71E−2	6.69E−4
WFG4	mean	**3.44E−1**	3.21E−1 (+)	2.66E−1 (+)	2.79E−1 (+)	3.06E−1 (+)	3.10E−1 (+)	2.26E−1 (+)	3.21E−1 (+)
	std	**4.41E−4**	2.57E−3	2.20E−2	2.39E−1	1.32E−2	2.20E−3	2.35E−2	1.64E−3
WFG5	mean	**3.13E−1**	3.10E−1 (+)	2.68E−1 (+)	2.39E−1 (+)	3.01E−1 (+)	3.00E−1 (+)	3.07E−1 (+)	3.09E−1 (+)
	std	**2.37E−4**	2.07E−3	4.47E−2	2.64E−2	1.82E−3	4.24E−3	2.52E−3	2.10E−3
WFG6	mean	**3.21E−1**	2.93E−1 (+)	3.00E−1 (+)	2.05E−1 (+)	2.46E−1 (+)	3.15E−1 (+)	2.84E−1 (+)	2.85E−1 (−)
	std	**5.59E−4**	7.44E−3	2.04E−2	1.68E−2	3.37E−2	8.93E−3	4.64E−3	5.27E−2
WFG7	mean	**3.45E−1**	3.44E−1 (+)	3.05E−1 (+)	2.28E−1 (+)	3.38E−1 (+)	3.18E−1 (+)	1.78E−1 (+)	3.43E−1 (+)
	std	4.98E−4	4.83E−4	2.03E−2	2.23E−2	3.72E−3	7.20E−3	8.49E−3	**4.80E−4**
WFG8	mean	**2.83E−1**	2.80E−1 (+)	2.08E−1 (+)	2.23E−1 (+)	2.61E−1 (+)	2.59E−1 (+)	2.18E−1 (+)	2.83E−1 (+)
	std	9.17E−4	1.86E−3	2.87E−2	2.14E−2	1.64E−2	8.09E−3	1.14E−2	**8.70E−4**
Num. of +/−			8/0	8/0	8/0	8/0	8/0	8/0	7/1

Note: Bold indicates that the results obtained by the algorithm rank the first; "+" and "–" denote the Wilcoxon rank sum test $P < 0.05$ and $P >$ 0.05 between the comparison and IMOSO algorithms.

tion, MOCS's IGD result has the best mean optimization performance, while IMOSO gets a sub-optimal value. In terms of HV metric, the mean results of IMOSO on WFG functions are superior to other algorithms. The Wilcoxon rank sum test results show that IMOSO has the highest score, which prove the effectiveness of IMOSO more convincingly in the statistical significance.

Figure 3 provides the boxplots of the HV metric obtained by 8 algorithms running 30 experiments. Larger HV results mean better performance. Clearly, the results obtained by IMOSO can be located in the higher Y-axis positions among these functions. The differences between the upper and lower limits are small, which further verify the strong robustness of IMOSO. Even

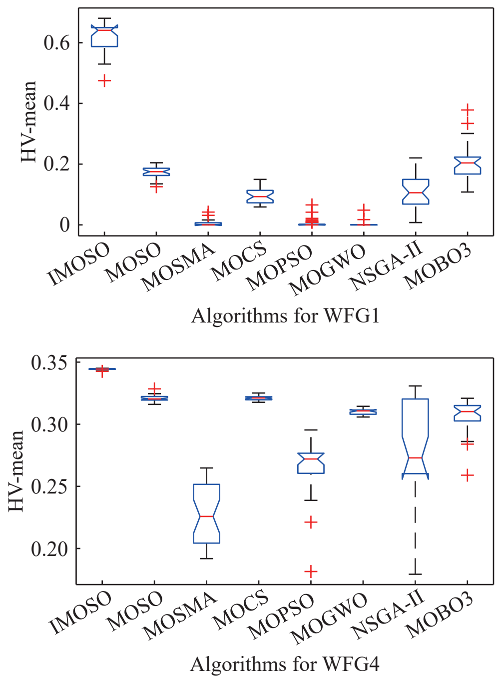

Figure 3 HV boxplot comparisons between IMOSO and compared algorithms on dual-objective functions.

though IMOSO does not appear the same "+" on the top as the other algorithms, its mean results are superior to the other algorithms "+" on the top. On WFG1 function, the extreme value of IMOSO is better than other comparison algorithms. The results of boxplot further show the strong convergence and diversity of the IMOSO algorithm.

Overall, the IGD and HV metrics of IMOSO achieve better results on almost all dual-objective functions. It is stated that the proposed adaptive mating between subpopulations strategy can generate better non-dominated solutions, and the external archive local disturbance mechanism can guide the population to update to the real Pareto front better.

3. Performance analysis on three-objective functions

With the increase of the number of objectives, it becomes more and more difficult to optimize for EMOAs. In this subsection, the different algorithms are tested on the three-objective WFG functions and their IGD and HV results are shown in Tables 3 and 4.

In terms of IGD and HV metrics, IMOSO achieves 12 and 14 better results. Specifically, the IGD mean and std results of IMOSO are more obvious and strong robustness on WFG5 function. For HV metric, the mean values by IMOSO rank the first in all functions. On WFG2 function, the HV metric of IMOSO can reach 9.02. The results can be inferred that the adaptive mating between subpopulations strategy can create better dominated solutions. Meanwhile, the external archive local disturbance mechanism can make full use of the information of the non-inferior solutions and guide the evolution of the population towards the real Pareto front in each evolution process. On WFG6 function, IMOSO is obviously superior to the MOSO, MOPSO, NSGAII, MOBO3, MOGWO, and MOSMA algorithms. Compared with the second-ranked MOCS, it also has a certain improvement. Based on the Wilcoxon rank sum test results, the IGD and HV metrics of IMOSO have obvious advantages over

Table 3 Mean and standard deviation values of IGD metric obtained by different algorithms on three-objective functions (WFG)

Fun.	Metric Val.	IMOSO	MOSO	MOPSO	NSGA-II	MOBO3	MOGWO	MOSMA	MOCS
WFG1	mean	5.48E−1	**5.47E−1** (−)	7.93E−1 (+)	7.85E−1 (+)	5.54E−1 (−)	6.44E−1 (+)	6.85E−1 (+)	5.61E−1 (+)
	std	9.69E−3	**8.15E−3**	5.95E−2	6.69E−2	6.05E−2	4.23E−2	2.48E−2	2.33E−2
WFG2	mean	**6.28E−2**	6.91E−2 (+)	1.72E−1 (+)	2.71E−1 (+)	1.33E−1 (+)	1.08E−1 (+)	2.04E−1 (+)	6.37E−2 (+)
	std	**3.65E−3**	4.71E−3	2.19E−2	4.90E−2	3.35E−2	1.93E−2	3.65E−2	3.77E−3
WFG3	mean	**6.71E−2**	7.61E−2 (+)	5.51E−1 (+)	7.48E−1 (+)	3.16E−1 (+)	3.26E−1 (+)	1.95E−1 (+)	9.22E−2 (+)
	std	**8.15E−3**	1.04E−2	9.34E−2	1.19E−1	8.21E−2	1.35E−1	1.57E−2	9.73E−3
WFG4	mean	**9.43E−2**	9.76E−2 (+)	1.92E−1 (+)	3.62E−1 (+)	1.14E−1 (+)	3.15E−1 (+)	1.36E−1 (+)	9.78E−2 (+)
	std	**4.04E−3**	5.20E−3	2.54E−2	4.68E−2	1.30E−2	2.67E−2	9.97E−3	4.32E−3
WFG5	mean	**7.85E−2**	1.04E−1 (+)	4.06E−1 (+)	4.18E−1 (+)	1.03E−1 (+)	3.61E−1 (+)	9.42E−2 (+)	8.19E−2 (+)
	std	3.16E−3	8.69E−3	2.93E−2	4.91E−2	1.46E−2	2.77E−2	7.18E−3	**3.03E−3**
WFG6	mean	**1.08E−1**	1.29E−1 (+)	4.53E−1 (+)	4.47E−1 (+)	1.60E−1 (+)	4.29E−1 (+)	1.45E−1 (+)	1.16E−1 (+)
	std	8.95E−3	**8.09E−3**	3.83E−2	3.54E−2	3.11E−2	3.24E−2	9.97E−3	1.30E−2
WFG7	mean	**8.72E−2**	8.89E−2 (+)	4.34E−1 (+)	4.33E−1 (+)	1.13E−1 (+)	3.53E−1 (+)	2.21E−1 (+)	9.92E−2 (+)
	std	**4.66E−3**	4.75E−3	2.95E−2	2.79E−2	2.66E−2	2.59E−2	7.08E−3	4.74E−3
WFG8	mean	**1.34E−1**	1.45E−1 (+)	4.65E−1 (+)	4.65E−1 (+)	1.54E−1 (+)	5.13E−1 (+)	2.36E−1 (+)	1.42E−1 (+)
	std	**3.99E−3**	5.33E−3	3.39E−2	3.39E−2	2.06E−2	4.36E−2	1.21E−2	4.42E−3
Num. of +/−			7/1	8/0	8/0	7/1	8/0	8/0	8/0

Note: Bold indicates that the results obtained by the algorithm rank the first; "+" and "−" denote the Wilcoxon rank sum test $P < 0.05$ and $P > 0.05$ between the comparison and IMOSO algorithms.

Table 4 Mean and standard deviation values of HV metric obtained by different algorithms on three-objective functions (WFG)

Fun.	Metric Val.	IMOSO	MOSO	MOPSO	NSGA-II	MOBO3	MOGWO	MOSMA	MOCS
WFG1	mean	**2.98E−1**	2.91E−1 (+)	2.68E−2 (+)	2.93E−2 (+)	2.61E−1 (+)	1.57E−1 (+)	1.12E−1 (+)	2.56E−1 (+)
	std	7.65E−3	**7.25E−3**	3.88E−2	3.93E−2	6.41E−2	4.74E−2	2.79E−2	2.08E−2
WFG2	mean	**9.02E−1**	8.87E−1 (+)	7.29E−1 (+)	6.04E−1 (+)	7.73E−1 (+)	8.40E−1 (+)	6.95E−1 (+)	8.90E−1 (+)
	std	**4.23E−3**	9.09E−3	3.08E−2	5.17E−2	5.71E−2	2.38E−2	3.98E−2	5.22E−3
WFG3	mean	**3.72E−1**	3.52E−1 (+)	5.52E−2 (+)	1.27E−2 (+)	1.41E−1 (+)	1.65E−1 (+)	2.49E−1 (+)	3.42E−1 (+)
	std	**5.41E−3**	1.01E−2	2.94E−2	1.69E−2	4.91E−2	7.71E−2	9.27E−3	8.85E−3
WFG4	mean	**4.64E−1**	4.55E−1 (+)	3.75E−1 (+)	2.67E−1 (+)	4.52E−1 (+)	2.82E−1 (+)	3.91E−1 (+)	4.52E−1 (+)
	std	**5.98E−3**	8.30E−3	2.23E−2	2.23E−2	1.25E−2	1.80E−2	1.64E−2	7.85E−3
WFG5	mean	**4.88E−1**	4.39E−1 (+)	1.83E−1 (+)	1.77E−1 (+)	4.52E−1 (+)	2.26E−1 (+)	4.53E−1 (+)	4.75E−1 (+)
	std	**4.75E−3**	1.58E−2	1.63E−2	2.42E−2	1.65E−2	1.68E−2	1.23E−2	8.13E−3
WFG6	mean	**4.46E−1**	3.89E−1 (+)	1.81E−1 (+)	1.86E−1 (+)	3.66E−1 (+)	2.11E−1 (+)	3.75E−1 (+)	4.08E−1 (+)
	std	2.02E−2	1.58E−2	2.12E−2	2.18E−2	3.06E−2	1.94E−2	**1.54E−2**	3.33E−2
WFG7	mean	**4.92E−1**	4.86E−1 (+)	1.93E−1 (+)	1.96E−1 (−)	4.92E−1 (+)	2.12E−1 (+)	2.48E−1 (+)	4.56E−1 (+)
	std	**7.30E−3**	1.09E−2	1.90E−2	1.77E−2	1.74E−2	1.12E−2	9.48E−3	9.00E−3
WFG8	mean	**3.98E−1**	3.76E−1 (+)	1.64E−1 (+)	1.64E−1 (+)	3.55E−1 (+)	1.39E−1 (+)	2.59E−1 (+)	3.76E−1 (+)
	std	**5.96E−3**	8.19E−3	1.64E−2	1.64E−2	3.58E−2	2.17E−2	1.63E−2	1.01E−2
Num. of +/−			8/0	8/0	7/1	8/0	8/0	8/0	8/0

Note: Bold indicates that the results obtained by the algorithm rank the first; "+" and "−" denote the Wilcoxon rank sum test $P < 0.05$ and $P > 0.05$ between the comparison and IMOSO algorithms.

other algorithms.

Figure 4 describes the comparisons between the Pareto fronts obtained by different algorithms and the real Pareto front on three-objective WFG2 function. According to the figure analysis, the Pareto fronts obtained by IMOSO, the MOSO and MOCS are more evenly distributed on the real Pareto front. However, MOSMA, NSGA-II, and MOBO3 obtain poor distribution uniformity of Pareto fronts.

In summary, the above results prove the better convergence and population diversity of the proposed IMOSO algorithm for the three-objective problems.

V. Conclusion and Future Works

In this paper, we propose an integrated external archive local disturbance mechanism for multi-objective snake optimizer. First, the adaptive mating between subpopulations strategy, inspired by the polyamorous mating system of the snake swarm, is constructed to make the information exchange between the two subpopulations effective and decide to mate according to the dominated relationship between the newly created individual

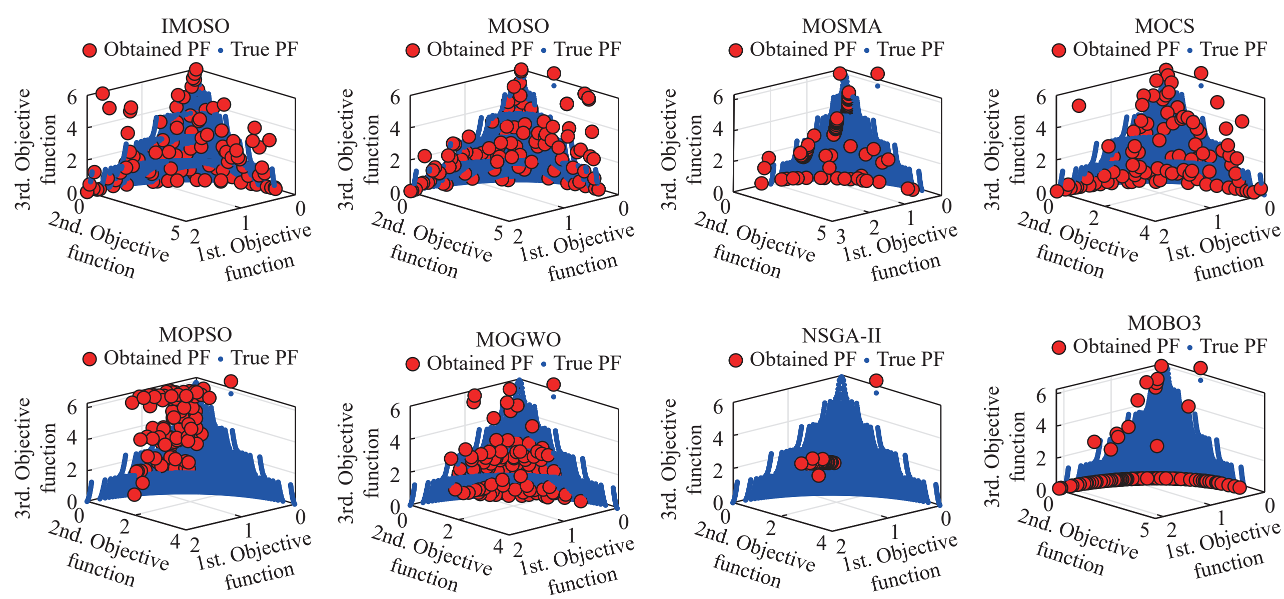

Figure 4 Contrast diagram of Pareto fronts obtained by different algorithms on WFG2 function.

and the original individual. The external archive local disturbance mechanism randomly selects 50% individuals with small crowding distances in each iteration to conduct local disturbance and non-dominated sorting with the solutions generated for the next iteration. Compared with state-of-the-art EMOAs, IMOSO shows better convergence performance and population diversity.

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