

# De-Correlation and De-Bias Post-Processing Circuits for True Random Number Generator

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**Abstract**—True random number generators (TRNGs) are commonly used in hardware security for secure authentication, data encryption, etc. The raw random numbers often exhibit defects. The most commonly observed defects are bias and correlations. Post processing techniques have been developed to address them. The von Neumann method addresses bias, but it requires input that is uncorrelated and has an identical distribution. On the other hand, the Markov chain can address correlation but introduce bias. In this work, we research the lightweight combination of two techniques. We verified that MKV2(QL4)/VN2 performs well for both Markov and non-Markov model bitstreams. MKV1(QL8)/VN8W is effective for the Markov model. The randomness is verified by NIST SP 800-22 and 800-90B, and ENT, respectively. Both of these circuits require only 16 bits of memory, which is 12 times smaller than in previous work. MKV1(QL8)/VN8W is implemented using 65-nm CMOS. A prototype chip demonstrates a minimum energy consumption of 0.149 pJ/bit at 0.45V. When applied to a latch-based TRNG, it can double the operation frequency thanks to the enhanced decorrelation. The total energy consumption is reduced by 21%.

**Index Terms**—De-autocorrelation, de-bias, Markov chain, von Neumann post-processing, true random number generator, hardware security.

## I. INTRODUCTION

A TRUE random number generator (TRNG) that harvests physical noise to generate true random and unpredictable numbers is a fundamental component of hardware security.

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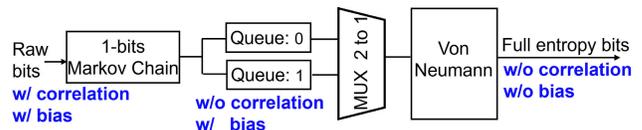


Fig. 1. Diagram of de-correlation and de-bias by Markov chain and von Neumann post-processing.

These random numbers are utilized as keys or nonces in secure operations, such as communication protocols and authentication in Internet of Things (IoT) devices, smart cards, etc. The raw data from TRNGs often exhibit statistical defects due to processes, voltage variations, temperature fluctuations, clock period variations, or intentional attacks [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14]. The most commonly observed defects are bias and auto-correlation (AC). Bias indicates the deviation from 0.5, while AC indicates the correlations between bits. Both of these issues can be addressed by post-processing techniques, which can be divided into two groups: cryptographic algorithm-based methods and arithmetic-based methods.

Cryptographic algorithm-based post-processing techniques, such as hash functions, AES, CBC-MAC, CMAC, and HMAC, etc., are typically used in a cryptographic secure random number generator [5]. These techniques require a minimum entropy guarantee of the entropy source and can extract full randomness per bit. However, these techniques come with energy and hardware costs, making them unsuitable for resource-limited IoT devices. To provide a low-cost alternative, mathematical constructs such as finite field addition, matrices, substitution, and permutation networks have been employed. Examples include BIW [3], strong blenders [15], and the PRESENT cipher [16], etc. However, these methods still require a guarantee of minimum entropy in the raw data, necessitating additional circuits for entropy monitoring.

On the other hand, arithmetic methods provide a more lightweight solution. According to the extraction efficiency (ExE, which is defined as the output bitstream length over the input bitstream length), these methods can be further divided into three groups: 1. ExE less than one, such as the XOR and von Neumann methods; 2. ExE equal to one, such as Linear Feedback Shift Registers (LFSR) and Markov chains; 3. ExE greater than one, such as the middle square method. Among the above-mentioned arithmetic methods, XOR with 50% ExE is the simplest technique. However, bias can never be fully eliminated. LFSRs are often used for whitening the bitstream.

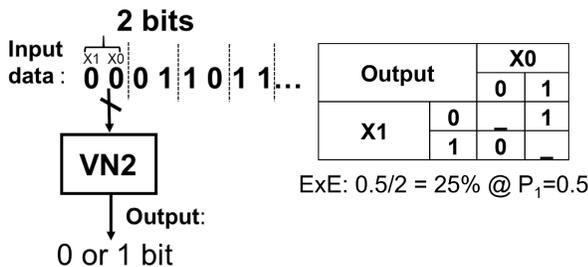


Fig. 2. Diagram of von Neumann (VN2) post-processing.

Even though the apparent entropy is increased, the true entropy per bit cannot be improved since no bits are discarded during processing. The middle square method, once a pseudo-random number generator, is used as a post-processing circuit by combining a 16-bit shift register to feed the raw TRNG's bitstream [14]. However, in that work, 8 bits are generated with only one raw bit. The raw bits with bias and correlations are XORed with the bits in the last square stage to obtain the final output. This presents a risk for attacks.

The von Neumann (VN) [17] can achieve zero bias without a minimum entropy guarantee. However, its ExE decreases with input bias. While this property can serve as an entropy detector to indicate the current bias condition in the raw bits. It leads to increased energy consumption due to the reduced ExE. To improve ExE, iterative von Neumann (IVN) [18] methods, and N-bit von Neumann [19] methods have been proposed. However, all these methods cannot solve the correlation problem. 8-bit von Neumann with waiting (VN8W) [19] can tolerate less than 0.03 lag 1 correlation (AC lag1) at input. However, the application is limited for low correlations. Based on our measurement results on latch-based TRNG [11], if the equalization time, which is the setting time to initialize the latch internal node to metastable voltage, is insufficient, the correlation can exceed 0.03, reaching values as high as 0.3.

The N-bits Markov chain works for decorrelation by separating the input bits into  $2^N$  memory queues based on the current N-bits. The bits within the same queue are assumed to be independent of each other. However, bias is added to each queue's bits. Combining the Markov chain for decorrelation and von Neumann methods for debiasing presents a promising solution. Research in [4] proposes a combination of a 4-bit Markov chain and IVN structure. However, it requires significant memory of 192 bits (12 bits in each queue by 16 states) to store the decorrelated bitstreams before sending them to IVN. Additionally, a 16-bit LFSR is still necessary for removing residual correlation after IVN processing.

In this study, we explore a lightweight approach to combining a Markov chain and von Neumann. We extend the theoretical analysis of the Markov model for generating correlated bitstreams presented in [20]. We verify that even with 1-bit MKV (MKV1) and 2-bit (MKV2), they work efficiently: MKV2(QL4)/VN2 performs well for both Markov and non-Markov model bitstreams. MKV1(QL8)/VN8W is effective for the Markov model. The randomness is verified by NIST SP 800-22 [21], 800-90B [22], and ENT [23], respectively. Both of these approaches require only 16 bits of memory, which is 12 times smaller than in previous work.

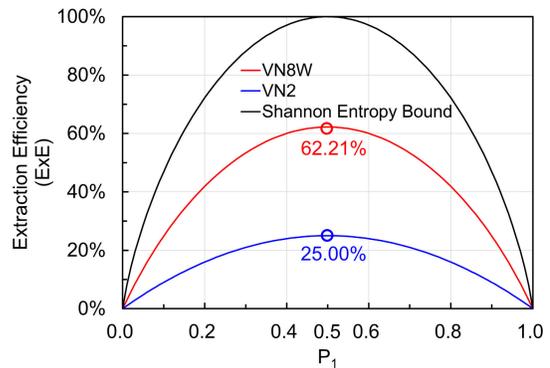


Fig. 3. Extraction efficiency for VN2 and VN8W.

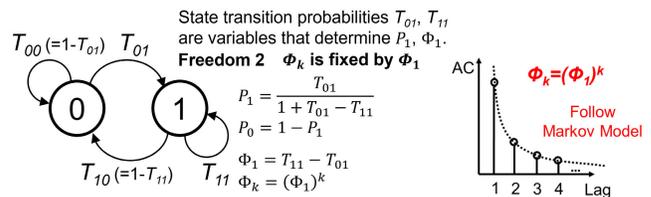


Fig. 4. Diagram of MKV1 random bitstream generation and autocorrelation of generated random bitstream. (Source: [20] modified).

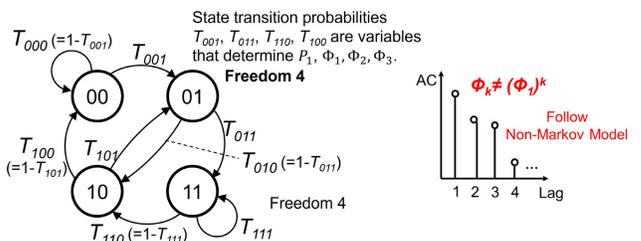


Fig. 5. Diagram of MKV2 and autocorrelation based on MKV2 model. (Source: [20] modified).

MKV1(QL8)/VN8W, as illustrated in Fig. 1, is implemented using 65-nm CMOS. When applied to a latch-based TRNG, it can double the operation frequency thanks to the enhanced decorrelation. The total energy consumption is reduced by 21%.

The following sections are organized as follows: Section II reviews the basic concepts of autocorrelation, the Markov model, and von Neumann post-processing. Section III presents the theoretical analysis of MKV1 and MKV2 for generating both Markov and non-Markov model-based bitstreams. Section IV introduces the usage of the MKV model for decorrelation. Section V explores the combination of the MKV model and the von Neumann method. Chip measurement results are presented in Section VI, followed by the conclusion in Section VII. The details of relationship between transition probabilities and generated bitstream in MKV2 are provided in the Appendix.

## II. REVIEW OF AUTOCORRELATION, MARKOV MODEL AND VON NEUMANN

This section reviews the fundamental knowledge of autocorrelation and Markov model. The information presented in

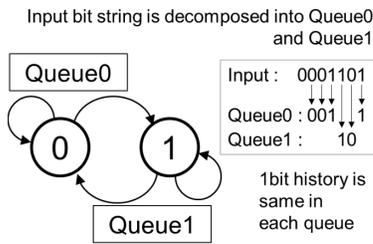


Fig. 6. Decorrelation by MKV1. (Source: [20] modified).

this section will support the analysis conducted in subsequent sections.

### A. Autocorrelation

Given a random  $n$ -bit bitstream  $X$ , the autocorrelation at lag  $k$ , denoted by  $\phi_k$ , characterizes the degree of correlation between bits at positions  $t$  and  $t+k$ , where  $t$  ranges from 1 to  $n-k$ .  $\phi_k$  is calculated as:

$$\phi_k = \frac{\text{Cov}(X_t, X_{t+k})}{\sigma_X^2} = \frac{\sum_{t=1}^{n-k} (X_t - \mu)(X_{t+k} - \mu)}{\sum_{t=1}^{n-k} (X_t - \mu)^2}, \quad (1)$$

where  $\mu$  represents the mean value of  $X$ . In particular, for  $k = 1$ , the autocorrelation  $\phi_1$  can be computed utilizing Pearson's autocorrelation formula:

$$\phi_1 = \frac{n_{11}n_{00} - n_{10}n_{01}}{n_1n_0}, \quad (2)$$

where  $n_{11}$ ,  $n_{00}$ ,  $n_{10}$ , and  $n_{01}$  denote the frequencies of 1-bit overlap patterns of 11, 00, 10, and 01, respectively.  $n_1$  and  $n_0$  represent the total counts of 1s and 0s within the bitstream, respectively. For an  $n$ -bit bitstream, the acceptable correlation level falls within the 95% confidence interval (CI), which is approximately  $\pm 1.96/\sqrt{n}$ .

### B. Markov Model

A Markov model is a memoryless model where the future state of a system is determined solely by its current state. The most common type of Markov model is the Discrete-Time Markov chain, where the state space and time are discrete. A Markov chain consists of the following components: state space, transition probability matrix, and initial state distribution. The state space is a finite set of possible states that the system can be in. Each state is represented by binary bits, resulting in  $2^N$  possible states, where  $N$  denotes the number of binary bits. For example, in a 1-bit Markov chain, there are 2 states. The transition probability matrix specifies the probabilities of transitioning from one state to another. The initial state distribution is a probability distribution that specifies the initial state of the system. Markov chains provide a flexible framework for generating bit sequences with specific correlation. Further details will be presented in the next Section.

### C. Von Neumann

The diagram of von Neumann (VN) [17] is shown in Fig. 2. It processes each pair of input bits and retains only the first

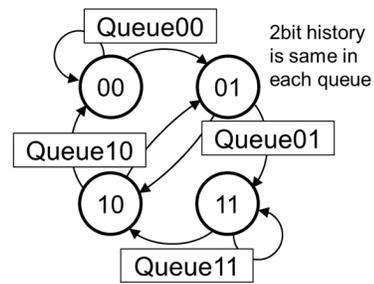


Fig. 7. Decorrelation by MKV2.

bit when the two input bits are different. It can achieve zero bias when input follows independent, identically, distributed. The ExE is defined as the output bit length over the input bit length. As shown in Fig. 3, when the probability of ones  $P_1$  is 0.5, ExE = 25%, and 75% of bits are discarded. To improve ExE, iterative von Neumann (IVN) [18] methods, which reuse discarded bits, and N-bit von Neumann [19] methods, which process more than two bits simultaneously, have been proposed. For example, in the work [19], the author proposed 8-bit von Neumann with waiting strategy (VN8W) achieves 62.21% ExE when  $P_1 = 0.5$ , as shown in Fig. 3. By separating the 8-bit into odd and even numbers and processing separately, VN8W can tolerate less than 0.03 lag1 correlation.

## III. BITSTREAM GENERATION BY MARKOV CHAIN

### A. Bitstream Generation by MKV1

The diagram of 1-bits Markov chain (MKV1) is illustrated in Fig. 4. Its transition probabilities from the current state  $i$  to the next state  $j$  are denoted as  $T_{ij}$ , where both  $i$  and  $j$  are values of either 0 or 1. These transitions adhere to the constraints  $T_{00} + T_{01} = 1$  and  $T_{10} + T_{11} = 1$ . The probabilities associated with states 0 and 1 are represented by  $P_0$  and  $P_1$ , respectively. Upon achieving stability after time  $t$ , the following relationship holds:

$$[P_0, P_1] \begin{bmatrix} T_{00} & T_{01} \\ T_{10} & T_{11} \end{bmatrix} = [P_0, P_1], \quad (3)$$

which leads to the expressions:

$$P_1 = \frac{T_{01}}{1 + T_{01} - T_{11}}, \quad P_0 = 1 - P_1. \quad (4)$$

In the scenario where  $T_{01} + T_{11} = 1$ , equilibrium is reached with  $P_0 = P_1 = 0.5$ .

Now, let's reconsider  $\phi_1$  as illustrated in Equation (2). For the  $n$  bits, then we have:  $n_{11} = P_1 * T_{11} * n$ ,  $n_{00} = P_0 * T_{00} * n$ ,  $n_{10} = P_1 * T_{10} * n$ ,  $n_{01} = P_0 * T_{01} * n$ ,  $n_1 = P_1 * n$  and  $n_0 = P_0 * n$ . Thus,  $\phi_1$  can be expressed as:

$$\phi_1 = T_{11} - T_{01}, \quad (5)$$

If  $T_{11}$  equals  $T_{01}$ ,  $\phi_1$  equals zero, indicating the absence of autocorrelation. For a general  $\phi_k$ , it can be obtained by substituting  $T_{11}$  and  $T_{01}$  with  $T_{1-k}$  and  $T_{0-k}$  in Equation (5), where  $-$  denotes all intermediate states. For instance, when  $k = 2$ ,  $\phi_2$  can be calculated as,

$$\phi_2 = T_{1-1} - T_{0-1}$$

$$\begin{aligned}
&= (T_{111} + T_{101}) - (T_{011} + T_{001}) \\
&= (T_{11} \cdot T_{11} + T_{10} \cdot T_{01}) - (T_{01} \cdot T_{11} + T_{00} \cdot T_{01}), \quad (6)
\end{aligned}$$

substituting  $T_{10} = 1 - T_{11}$ ,  $T_{00} = 1 - T_{01}$ , we get

$$\phi_2 = (T_{11} - T_{01})^2 = \phi_1^2. \quad (7)$$

Hence,

$$\phi_k = (T_{11} - T_{01})^k = \phi_1^k. \quad (8)$$

Thus, MKV1 offers two degrees of freedom through  $T_{01}$  and  $T_{11}$ . As MKV1 represents a 1-bit state machine, it can be employed to generate Markov model bitstreams with target  $P_1$  and  $\phi_1$  by calculation:

$$T_{01} = P_1(1 - \phi_1), \quad (9)$$

$$T_{11} = \phi_1 + T_{01}, \quad (10)$$

therefore, new bits are generated based on the transition matrix.

### B. Bitstream Generation by MKV2

The diagram of 2-bits Markov chain (MKV2) is illustrated in Fig. 5. MKV2 has four states. And the transition probabilities from the current state  $ij$  to the subsequent state  $jk$ , with a one-bit of  $j$  overlap, are represented by  $T_{ijk}$ , where  $i, j$  and  $k$  are either 0 or 1. For instance,  $T_{101}$  represents the probability of transition from state 10 to state 01.

The characteristics of  $P_1$  and autocorrelations at lag 1, 2, 3 ( $\phi_1, \phi_2, \phi_3$ ) in random bitstream generated by MKV2 can be extracted from its four independent transition probabilities  $T_{001}, T_{011}, T_{110}, T_{100}$ . For simplicity, they are denoted as  $a, b, c, d$ , respectively. The equations are show in following:

$$\begin{cases}
P_1 = \frac{1+s}{2+q+s} \\
\phi_1 = \frac{qs-1}{(1+q)(1+s)} \\
\phi_2 = 1 - \frac{2+q+s}{(1+q)(1+s)}(b+d) \\
\phi_3 = \frac{qs-1+(2+q+s)(b^2/s+d^2/q-2bd)}{(1+q)(1+s)},
\end{cases} \quad (11)$$

where  $q = d/a$ ,  $s = b/c$ .

Random bit streams with arbitral characteristics  $P_1, \phi_1, \phi_2, \phi_3$  can be generated by applying calculated  $T_{001}, T_{011}, T_{110}, T_{100}$  to MKV2 state transition model. Inverse functions that calculate  $T_{001}, T_{011}, T_{110}, T_{100}$  from  $P_1, \phi_1, \phi_2, \phi_3$  are also clarified. The details are exhibited in Appendix.

## IV. MARKOV DE-AUTOCORRELATION

In this section MKV1 and MKV2 are used inversely for decorrelation. Here, random bitstream with correlation is input of them, while it is output in Section III.

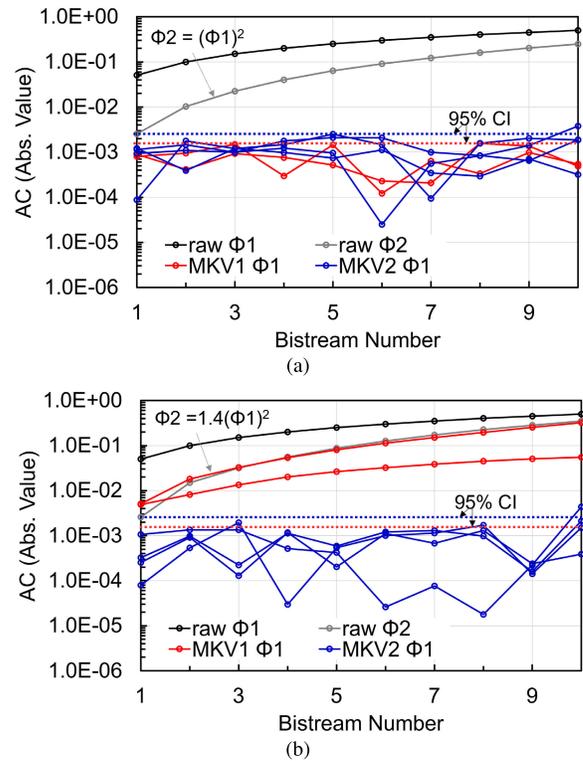


Fig. 8. Decorrelation analysis with (a) Markov model bitstreams. (b) Non-Markov model bitstream.

TABLE I  
PARAMETERS:  $\phi_1$  AND  $r$  IN 41 PSEUDORANDOM BITSTREAMS

$\phi_1$	$r$	Count
$\pm 0.05, \pm 0.1, \pm 0.15, \pm 0.2$	1, 1.4, 2, 3	32
0.25, 0.3, 0.35	1, 1.4, 2	9

### A. De-Correlation by MKV1 and MKV2

The decorrelation diagram using MKV1 is depicted in Fig. 6. The input bitstream is routed into two queues, Queue0 and Queue1, depending on the preceding bit: if the previous bit is 0, the current bit is routed into Queue0; if it's 1, it is routed into Queue1. As a result, correlation due to one bit history in the raw bitstream is eliminated.

Similarly, the diagram for decorrelation using MKV2 is shown in Fig. 7. The input bitstream is routed into four queues, Queue00 to Queue11, based on the previous 2-bit states. MKV2 is expected to handle both Markov model and non-Markov model bitstreams effectively.

### B. Evaluation of Decorrelation by MKV1 and MKV2

To evaluate the decorrelation achieved by MKV1 and MKV2, pseudorandom bitstreams, each with a length of 3 million bits, are generated based on MKV2 equations shown in Appendix.  $P_1 = 0.5$ , the autocorrelation (AC) lag 1 ranges from 0.05 to 0.5 with increments of 0.05. The first ten bitstreams are generated with  $\phi_2 = (\phi_1)^2$ , representing Markov model bitstreams. The other ten bitstreams are generated with  $\phi_2 = 1.4(\phi_1)^2$ , where the ratio of 1.4 is multiplied, indicating non-Markov model bitstreams. The absolute values of ACs for raw data and Markov post-processed data are shown in Fig. 8.

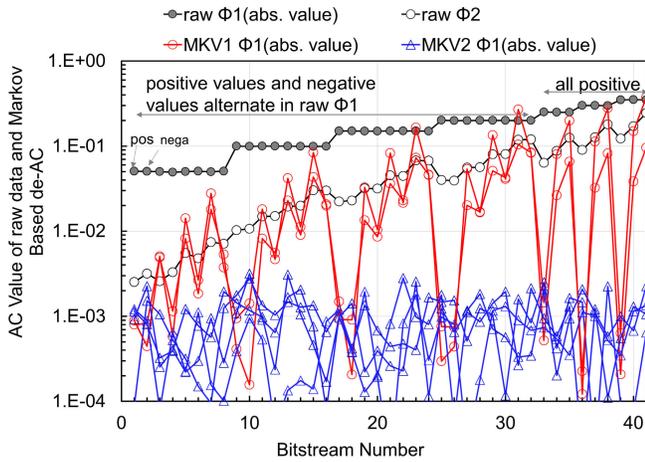


Fig. 9. Autocorrelation values for raw data and post-processed data by MKV for each queue. (Source: [20] modified).

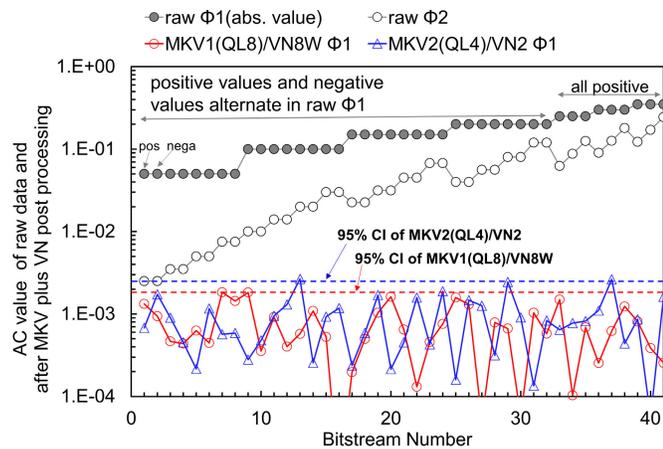


Fig. 10. Autocorrelation values for raw data and post-processed data by MKV+VN. (Source: [20] modified).

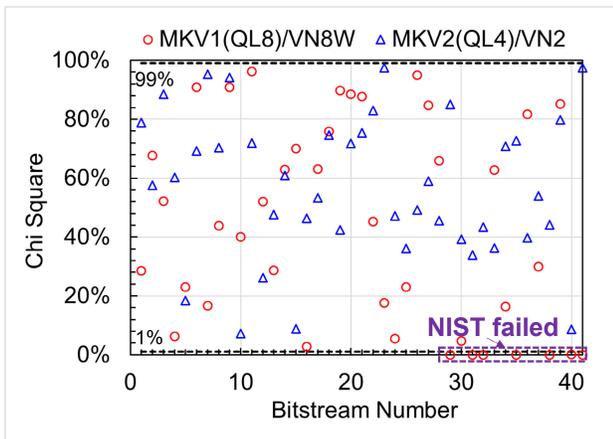


Fig. 11. Chi-square analysis using ENT.

There are two lines for MKV1, representing the bitstreams in Queue0 and Queue1, respectively. Similarly, there are four lines for MKV2, representing the bitstreams in Queue00, Queue01, Queue10, and Queue11, respectively. As observed, by MKV2, AC are removed to below 95% CI boundary (within stochastic error) in all bitstreams including non-MKV

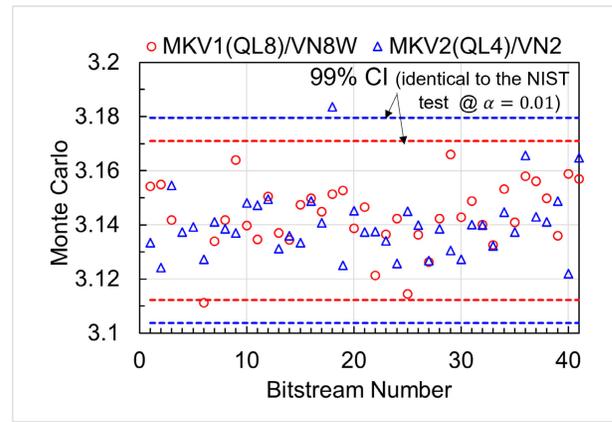


Fig. 12. Monte Carlo analysis using ENT.

TABLE II  
NIST SP 800-22 RESULTS FOR RANDOM DATA FROM MARKOV CHAIN

	MKV1(QL8)/VN8W Input: 1M*41		MKV2(QL4)/VN2 Input: 600k*41	
	Pass Rate	Ave. P-Val	Pass Rate	Ave. P-Val
Frequency	34/41 <sup>a</sup>	0.44	41/41	0.57
Block Frequency	41/41	0.42	41/41	0.44
Cumulative Sums	34/41 <sup>a</sup>	0.43	41/41	0.57
Runs	36/41 <sup>a</sup>	0.39	40/41	0.50
Longest Run	40/41	0.47	40/41	0.49
Rank	41/41	0.48	41/41	0.54
FFT	41/41	0.48	41/41	0.49
Non-overlapping Template	34/41 <sup>a</sup>	0.48	39/41	0.50
Overlapping Template	40/41	0.37	41/41	0.48
Universal	39/41	0.39	40/41	0.44
Approximate Entropy	41/41	0.35	41/41	0.49
Random Excursions	22/22	0.50	-	-
Random Excursions Variant	22/22	0.49	-	-
Serial	39/41	0.43	41/41	0.50
Linear Complexity	39/41	0.49	41/41	0.59

<sup>a</sup> Acceptable pass rate is greater than 38/41 [21]. Failure happened in non-Markov model with high  $\phi_1$  bitstreams ( $r = 1.4$   $\phi_1 = 0.35$ ,  $r \geq 2$   $|\phi_1| \geq 0.2$ ).

model. MKV1, can remove AC by MKV model. For non-MKV model, while it can not remove perfectly, it still reduces AC several times smaller. For example in case of bistream #6, AC is reduced from 0.30 to 0.11 or 0.03, bistream #4 AC is reduced from 0.20 to 0.05 or 0.02.

## V. COMBINATION OF MARKOV AND VON NEUMANN

This section presents Markov chain and von Neumann combined circuits.

### A. Discussion on Decorrelation Ability Combined With Markov Chain and Von Neumann

There can be many combinations of N1-bit Markov chains and N2-bit VN or IVN. We focus on 2-bit MKV and VN2, denoted as MKV2/VN2; and 1-bit MKV and VN8W, denoted as MKV1/VN8W. MKV2/VN2 has stronger decorrelation ability, with a maximum ExE of 25.0%. MKV1/VN8W is applied for removing Markov model-based correlation with an ExE of 62.2%. As mentioned, based on measurement

TABLE III  
ENT TEST RESULTS FOR PSEUDORANDOM DATA AFTER POST-PROCESSING

	MKV1(QL8)/VN8W, Input: 1M*41			MKV2(QL4)/VN2, Input: 600k*41		
	Average	Pass Rate	Failed Bitstream #	Average	Pass Rate	Failed Bitstream #
Mean	0.4994	34/41	29,31,32,35,38,40,41	0.4999	41/41	-
Entropy	1.0000	34/41	29,31,32,35,38,40,41	1.0000	41/41	-
Chi square	48.53%	34/41	29,31,32,35,38,40,41	57.12%	41/41	-
Monte Carlo	3.1439	40/41	6	3.1401	40/41	18
Serial correlation	-0.0001	38/41	9,25,41	0.0001	40/41	21

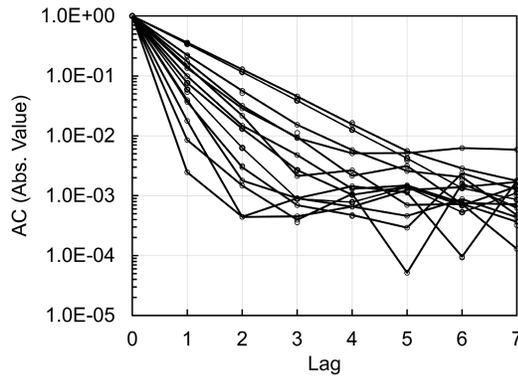


Fig. 13. Autocorrelation analysis of raw bitstreams of latch-based TRNG.

results, we found that under insufficient equalization time, the AC lag 1 in latch-based TRNG can approach 0.3, and the correlations are mostly positive. Therefore, to test most correlation cases, 41 bitstreams, each with a length of 3 million bits, are generated based on the MKV2 model.  $P_1$  is set to 0.5,  $\phi_1$  is set to  $\{\pm 0.05, \pm 0.1, \pm 0.15, \pm 0.2, 0.25, 0.3, 0.35\}$ .  $\phi_2 = \phi_1^2 \times r$ ,  $\phi_3 = \phi_1^3 \times r^2$ , where  $r$  quantifies the deviation from the MKV1 model. When  $r$  equals 1, the bitstream is a Markov model bitstream; otherwise, the bitstream is a non-Markov bitstream. The 41 combinations of  $\phi_1$  and  $r$  are summarized in Table I. When  $|\phi_1|$  is less than or equal to 0.2,  $r$  is set to 1, 1.4, 2, or 3. When  $\phi_1$  exceeds 0.2, since it is already large,  $r$  is set to 1, 1.4, and 2 for each  $\phi_1$  condition.

First, the AC value and post-processed data using MKV1 and MKV2 are shown in Fig. 9. As observed, MKV1 cannot handle highly correlated raw data. Conversely, MKV2 effectively processes nearly all cases and achieves an AC lag 1 less than 0.004. When combined VN, the queue length should also be considered. For a lightweight implementation, MKV2 with a 4-bit queue length with VN2 [MKV2(QL4)/VN2] and MKV1 with an 8-bit queue length with VN8W [MKV1(QL8)/VN8W] are considered. The results are summarized in Fig. 10. As observed, under both circuits, almost all correlations at lag 1 are reduced within the 95% CI. For a full randomness check, NIST SP 800-22 was tested, and the results are summarized in Table II. Note that the NIST results in Table I in our previous work [20] were incorrect due to a bitstream misconnection. The corrected results are presented in this work. It is observed that MKV2(QL4)/VN2 performs well for both Markov and non-Markov model bitstreams. MKV1(QL8)/VN8W works for Markov model ( $r = 1$ ) and non-Markov model when  $r = 1.4$  with  $|\phi_1| \leq 0.3$ ; and  $r = 2$  with  $|\phi_1| \leq 0.15$ .

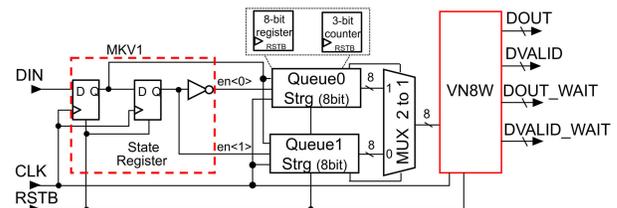


Fig. 14. Block diagram of MKV1(QL8)/VN8W. (Source: [20] modified).

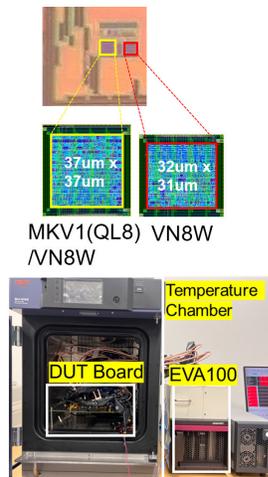


Fig. 15. Die-photo, layout, and measurement setup.

To visually display the random characteristics, we applied the ENT [23] test battery, which includes tests for mean, entropy, serial correlation, chi-square, and Monte Carlo. The results are summarized in Table III. The mean test calculates the  $P_1$  in the bitstream. Entropy is the Shannon entropy. Serial correlation is calculated at the byte level. The chi-square test detects deviations from a uniform distribution by comparing observed frequencies to expected frequencies within 8 bits. The results are depicted in Fig. 11. As can be seen, 7 bitstreams fall below 1%, indicating they are almost certainly not random. These 7 bitstreams also failed the NIST test. The Monte Carlo test estimates the value of  $\pi$  using 48 bits by calculating the ratio of points inside a circle to those in a square. Its results are summarized in Fig. 12. As observed, in almost all cases, the values are close to the ideal value of  $\pi$  within the 99% confidence interval (identical to the NIST test at  $\alpha = 0.01$ ).

### B. Circuit Design

Many combinations of MKV and VN are possible depending on the correlation conditions and throughput requirements

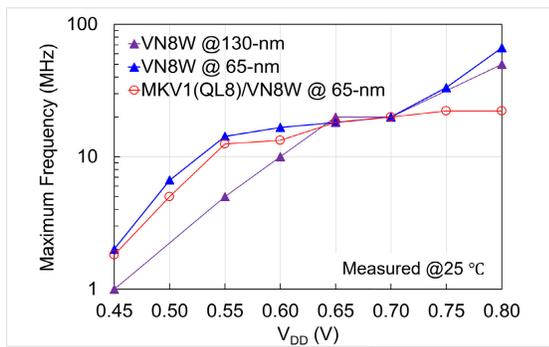


Fig. 16. Maximum frequency versus supply voltage.

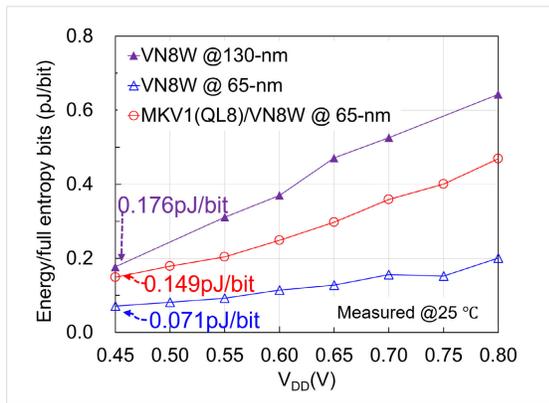


Fig. 17. Energy per full entropy bit versus supply voltage.

of the entropy source. In this work, a latch-based TRNG [11] is used. It has two operation phases: the equalization phase and the evaluation phase. During the evaluation phase, the internal node of the latch is forced to the metastable voltage. The random data is obtained during the evaluation phase. We observed that, under insufficient equalization time, the correlation of raw data exhibits properties similar to MKV1, as shown in Fig. 13. Although increasing the equalization time can reduce the correlation, this approach consumes a lot of energy. Therefore, MKV1(QL8)/VN8W is chosen in this work for hardware implementation to reduce the equalization time and achieve lower energy consumption.

Fig. 14 displays the block diagram of MKV1(QL8)/VN8W. It consists of the MKV1 part, queue storage registers, MUX part for sharing VN8W, and VN8W part. The MKV1 part only includes two registers and one inverter. The input ports are DIN, CLK, and RSTB. The initial bit is first registered in the state register. Then, the subsequent bits are routed to either Queue0 Storage or Queue1 Storage based on the previous bit's state. Each Queue Storage comprises an 8-bit shift register for storing the bits and a 3-bit counter. Once one of the 3-bit counters reaches '111', the stored bits in the Queue Storage are sent to the VN8W block for debiasing. The VN8W processes 8-bits simultaneously and generates output in two parts: the direct output denoted by DOUT and DVALID, and the waiting output denoted by DOUT\_WAIT and DVALID\_WAIT. The details of VN8W are provided in [19].

TABLE IV  
NIST SP 800-22 TEST RESULTS FOR RANDOM DATA FROM LTRNG

	MKV1(QL8)/VN8W Input: 1M*5	
	Pass Rate	Ave. P-Val
Frequency	5/5	0.25
Block Frequency	5/5	0.36
Cumulative Sums	5/5	0.26
Runs	5/5	0.62
Longest Run	5/5	0.44
Rank	5/5	0.58
FFT	5/5	0.54
Non-overlapping Template	5/5	0.49
Overlapping Template	4/5 <sup>a</sup>	0.50
Universal	5/5	0.66
Approximate Entropy	5/5	0.32
Random Excursions	2/2	0.46
Random Excursions Variant	2/2	0.42
Serial	5/5	0.38
Linear Complexity	5/5	0.56

<sup>a</sup> One failure is reasonable due to the limited bitstream numbers [21].

TABLE V  
ENT TEST RESULTS FOR RANDOM DATA FROM LTRNG

	MKV1(QL8)/VN8W Input: 1M*5	
	Average	Pass Rate
Mean	0.4995	5/5
Entropy	1.0000	5/5
Chi square	25.14%	5/5
Monte Carlo	3.1480	5/5
Serial correlation	-0.0002	5/5

TABLE VI  
NIST SP 800-90B IID TEST RESULTS FOR RANDOM DATA FROM LTRNG

	MKV1(QL8)/VN8W Input: 1M*5
	Passed
Chi square independence	5/5
Chi square goodness of fit	5/5
Length of longest repeated substring test	5/5
IID permutation tests	5/5
Min-Entropy(max/min/ave.)	0.995/0.994/0.993

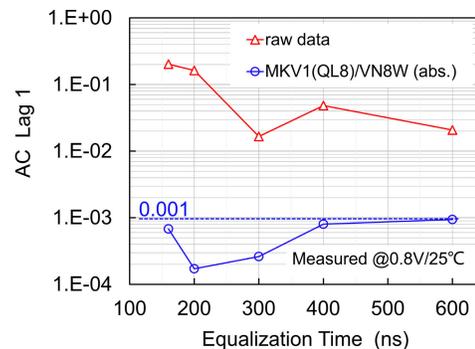


Fig. 18. Autocorrelation under equalization time variations. (Source: [20] modified).

## VI. CHIP MEASUREMENT RESULTS

MKV1(QL8)/VN8W was designed using an automatic Place and Route tool (IC Compiler) and fabricated using

TABLE VII  
COMPARISON WITH PRIOR POST-PROCESSING TECHNIQUES

	This work		IEICE'2022 [19]	SSCL'2018 [4]	JSSC'2016 [3]	TCAS-IF'2019 [15]	TCAS-I'2015 [16]
	MKV1(QL8)/VN8W	VN8W	VN8W	MKV4(QL12)/IVN16/LFSR	Decorrelators/BIW	Strong Blenders	PRESENT
Process Technology	65-nm CMOS	65-nm CMOS	130-nm CMOS	65-nm CMOS	14-nm CMOS	45-nm NanGate	32-nm CMOS PTM
Gate Equivalent(GE)	950	689	381	NA	586	166.3 ~13K	1171
Queue Memory (bits)	16	-	-	192	-	-	-
Max. Extraction Efficiency	62.21%	62.21%	62.21%	≈ 78%	12.5%	4%~20%	50%/80%
De-Biasing	Yes	Yes	Yes	Yes	Yes	Yes	Yes
De-Correlation	AC lag 1 ≤0.35 <sup>b</sup>	AC lag 1 ≤0.03	AC lag 1 ≤0.03	Yes <sup>a</sup>	Mutual Correlation <0.003	Yes <sup>a</sup>	Yes <sup>a</sup>
Energy per full-entropy bit (pJ/bit)	0.149 @0.45 V	0.071 @0.45 V	0.176 @0.45 V	2.58 <sup>c</sup> @0.53 V	9 @0.75 V	NA	1.0-2.5 <sup>c</sup> @0.9V

<sup>a</sup> No specific data has been provided in the paper. MKV4(QL12) in [4] is designed for removing AC up to lag 4. The works in [15], [16] are described for removing any correlation.

<sup>b</sup> MKV1(QL8)/VN8W works for Markov model ( $r = 1$ ) with  $|\phi_1| \leq 0.35$  and non-Markov model when  $r = 1.4$  with  $|\phi_1| \leq 0.3$ ; and  $r = 2$  with  $|\phi_1| \leq 0.15$ .

<sup>c</sup> Including TRNG cores.

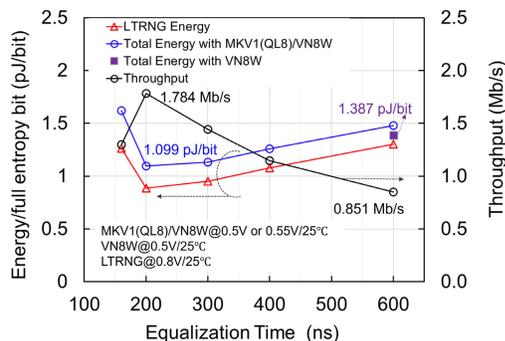


Fig. 19. Energy and throughput under equalization time variations.

TSMC 65-nm CMOS technology. To ensure the comparison, VN8W, as previously implemented in 130-nm CMOS according to [19], was also redesigned using the 65-nm CMOS process. MKV1(QL8)/VN8W occupies an area of  $1369 \mu\text{m}^2$ , equivalent to 950 gate equivalents (GEs). VN8W occupies an area of  $992 \mu\text{m}^2$ , equivalent to 689 GEs. The area of MKV1(QL8)/VN8W is 1.38 times larger than that of VN8W, primarily due to the memory register storing the queue data, leading to area overhead. The die photo, layout image and measurement setup are depicted in Fig. 15. One test chip is utilized to verify the performance of the proposed circuit. Further details are provided in the following.

#### A. Frequency and Energy Results of Post-Processing Circuits

MKV1(QL8)/VN8W and VN8W were tested with a supply voltage ranging from 0.45 V to 0.8 V, incremented by 0.05 V per step. Fig. 16 displays the maximum frequency results. The tested input data consists of raw output data from latch-based TRNG [11] with 1.2% bias and 0.0006 AC at lag 1. For comparison purposes, the measurement results of VN8W in 130-nm technology are also depicted in the

graph. As observed, the maximum frequency increases with the rise in supply voltage, a trend consistent across all three circuits. At 0.45 V, MKV1(QL8)/VN8W and VN8W @ 65-nm operate at 1.82 MHz and 2.00 MHz, respectively, while VN8W @ 130-nm operates at 1 MHz. The maximum frequency of MKV1(QL8)/VN8W increases slowly after 0.65 V and achieves 22.22 MHz at 0.8 V. The reduced highest frequency in MKV1(QL8)/VN8W is attributed to the delay in the memory shift registers for storing queue data. The maximum frequency of VN8W @ 65-nm is, on average, 1.63 times larger than VN8W @ 130-nm, demonstrating the merit of technology scaling.

Fig. 17 shows the energy results, wherein the energy per full entropy bit [19] is calculated as follows:

$$E_{\text{post-processing}} = \frac{\text{Power}}{\text{Frequency} \times \text{ExE}}, \quad (12)$$

where the ExE of input test data is 62%, a value identical across all three circuits. As can be seen, the minimum energy at 0.45 V of MKV1(QL8)/VN8W, VN8W @ 65-nm, and VN8W @ 130-nm are 0.149 pJ/bit, 0.071 pJ/bit, and 0.176 pJ/bit, respectively. Compared with VN8W @ 65-nm, MKV1(QL8)/VN8W costs 2.3 times, on average, larger energy. This is due to the fact that MKV1(QL8)/VN8W has 2 times larger register count than VN8W @ 65-nm, which consumes a significant amount of energy. Additionally, VN8W @ 130-nm consumes 3.2 times larger energy than VN8W @ 65-nm.

#### B. Application in Latch-Based TRNG

MKV1(QL8)/VN8W is utilized in the latch-based TRNG (LTRNG) [11], which requires a prolonged equalization time to reduce correlation. The autocorrelation lag 1 value versus equalization time, ranging from 160 ns to 600 ns, is shown in Fig. 18. As observed, the raw data consistently exhibits values larger than 0.02 across all equalization times. After

post-processed by MKV1(QL8)/VN8W, all AC lag 1 values remain consistently below 0.001. The randomness of post-processed data is verified by NIST SP 800-22, ENT, and NIST SP 800-90B, as summarized in Table IV, Table V, and Table VI, respectively.

The energy and throughput results are depicted in Fig. 19. As observed, by utilizing MKV1(QL8)/VN8W, the total energy, calculated as the energy of LTRNG (divided by ExE) + the energy of post-processing [19]—can be reduced to 1.099 pJ/bit (= 0.522/0.588+0.212). This represents a 21% improvement compared to the total energy with VN8W, which is 1.387 pJ/bit. Additionally, the throughput is also improved by 2.09 times (= 1.784/0.851), owing to the enhanced decorrelation ability by MKV1(QL8)/VN8W.

### C. Comparisons

Table VII shows the comparisons with previous works. For higher correlation post-processing circuit, MKV1(QL8)/VN8W with 62.21% ExE costs considerably fewer GEs when compared with [15] and [16]. Compared with work in [4], the queue memory is reduced 12 times. Although MKV1(QL8)/VN8W is 1.38 times larger than the VN8W implemented in 65-nm, it can handle Markov model data with AC lag 1 up to 0.35. VN8W implemented in 65-nm achieves the minimum energy among others, which is more than two times smaller than the energy of the previous work [19], demonstrating the merit of using advanced technology.

## VII. CONCLUSION

In this study, we introduce a lightweight combination of Markov chains and von Neumann. We verify the decorrelation capability of the combined circuits. MKV1(QL8)/VN8W was implemented in a 65-nm CMOS. Compared to previous works, the queue memory was reduced by a factor of 12. It achieved a low energy consumption of 0.149 pJ/bit at 0.45 V. When applied in a latch-based TRNG, the throughput improved by 2.09 times, and the total energy decreased by 21% due to the decorrelation enhancement provided by MKV1(QL8)/VN8W.

As for the practical use of MKV and VN combined post-processing circuits, the designer should first characterize their TRNGs' correlation properties. If the correlation follows a Markov model, i.e.,  $\phi_k = \phi_1^k$ , then MKV1 is appropriate. If the correlation follows a non-Markov model, first judge if it has less than AC lag 3 correlations. If yes, MKV2 is appropriate. MKV2 has four degrees of freedom:  $P_1$ ,  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$ . In theory, it can solve up to AC lag 3 correlations. If the raw bitstream has long-term correlations such as AC lag 4 or more complex statistical defects, a higher-order MKV model is needed, such as a 4-bit 16-state Markov model. Then, based on the trade-off between logic complexity and throughput requirements, choose a VN circuit with an appropriate queue memory bit length.

The correlation test cases in this work are still not enough due to the limited variety and number of tested scenarios. In future work, we would like to survey more practical TRNG types, including those used in different applications and environments, to better understand their correlation properties.

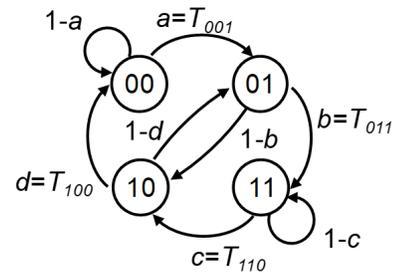


Fig. 20. The diagram of MKV2.

Additionally, we plan to extend our MKV model and VN circuits to provide appropriate solutions.

## APPENDIX

In this appendix, characteristics of frequency  $P_1$  and auto-correlations at lag 1, 2, 3 ( $\phi_1$ ,  $\phi_2$ ,  $\phi_3$ ) in random bitstream generated by two-bit (four state) Markov chain MKV2 are extracted from its four independent transition probabilities  $T_{001}$ ,  $T_{011}$ ,  $T_{110}$ ,  $T_{100}$ . Inverse functions that calculate  $T_{001}$ ,  $T_{011}$ ,  $T_{110}$ ,  $T_{100}$  from  $P_1$ ,  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  are also clarified. Random bit streams with arbitral characteristics  $P_1$ ,  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$  can be generated by applying calculated  $T_{001}$ ,  $T_{011}$ ,  $T_{110}$ ,  $T_{100}$  to MKV2 state transition model.

### A. Definitions

$T_{ijk}$ : Transition probability from state  $[i, j]$  to  $[j, k]$   
( $i, j, k \in \{0, 1\}$ )

$P_{ij}$ : Probability to be in a state  $[i, j]$ .

This is same as the probability that two adjacent (lag 1)

bit pair in generated bit stream is  $(i, j)$ .

$P_{ijk}$ : Probability that three consecutive bits in bit stream is  $(i, j, k)$

$P_{ijkl}$ : Probability that four consecutive bits in bit stream is  $(i, j, k, l)$

$P_{i-j}$ : Probability that pair of bits with a distance 2 (lag 2) in bit stream is  $(i, j)$

$P_{i--j}$ : Probability that pair of bits with a distance 3 (lag 3) in bit stream is  $(i, j)$

For simplicity, variables  $a$ ,  $b$ ,  $c$ ,  $d$  are used as  $a = T_{001}$ ,  $b = T_{011}$ ,  $c = T_{110}$ ,  $d = T_{100}$ , as shown in Fig. 20.

### B. Relation Between $P_{ij}$ and Four Independent Transition Probabilities

In the equilibrium, probabilities of the four states  $P_{00}$ ,  $P_{01}$ ,  $P_{10}$ ,  $P_{11}$  don't change after transition. Thus,

$$P_{00} = (1 - a)P_{00} + dP_{10} \quad (13)$$

$$P_{01} = aP_{00} + (1 - d)P_{10} \quad (14)$$

$$P_{10} = (1 - b)P_{01} + cP_{11} \quad (15)$$

$$P_{01} = bP_{00} + (1 - c)P_{10} \quad (16)$$

By adding equations (13) and (14),

$$\begin{aligned} P_{00} + P_{01} &= (1 - a)P_{00} + dP_{10} + aP_{00} + (1 - d)P_{10} \\ &= P_{00} + P_{11} \end{aligned}$$

Thus,

$$P_{01} = P_{10}. \quad (17)$$

Same result can be obtained by adding equations (15) and (16). From (13),  $aP_{00} = dP_{10}$ ,  $P_{00} = qP_{10} = qP_{01}$ , where  $q = d/a$ . From (15),  $bP_{01} = cP_{11}$ ,  $P_{11} = sP_{01} = sP_{10}$ , where  $s = b/c$ .

### C. Extract $P_1$ , $P_0$ , $\phi_1$ From $q$ , $s$

Since  $P_{00} + P_{01} + P_{10} + P_{11} = 1$ ,  $(q + s + 2)P_{01} = 1$ . Thus,

$$\begin{cases} P_{01} = P_{10} = \frac{1}{2 + q + s} \\ P_{00} = \frac{q}{2 + q + s} \\ P_{11} = \frac{s}{2 + q + s} \end{cases} \quad (18)$$

$$\begin{cases} P_1 = \frac{P_{01} + P_{10} + 2P_{11}}{2} = \frac{1 + s}{2 + q + s} \\ P_0 = 1 - P_1 = \frac{1 + q}{2 + q + s} \end{cases} \quad (19)$$

Applying (18) (19) in Pearson's autocorrelation formula,

$$\phi_1 = \frac{n_{11}n_{00} - n_{10}n_{01}}{n_1n_0} = \frac{P_{11}P_{00} - P_{10}P_{01}}{P_1P_0} = \frac{qs - 1}{(1 + q)(1 + s)} \quad (20)$$

### D. Extract $\phi_2$ From $q$ , $s$ , $b$ , $d$

$P_{i-j}$  are decomposed as

$$\begin{cases} P_{0-0} = P_{000} + P_{010} \\ P_{0-1} = P_{001} + P_{011} \\ P_{1-0} = P_{100} + P_{110} \\ P_{1-1} = P_{101} + P_{111} \end{cases} \quad (21)$$

Bit string [000] happens when 0 appears next to state [00]. Thus,

$$P_{000} = (1 - a)P_{00} \quad (22)$$

In the same way,

$$\begin{cases} P_{001} = aP_{00}P_{010} = (1 - b)P_{01}P_{011} = bP_{01} \\ P_{100} = dP_{10}P_{101} = (1 - d)P_{10} \\ P_{110} = cP_{11}P_{111} = (1 - c)P_{11} \end{cases} \quad (23)$$

Substituting (22), (23) and (20) into (21),

$$\begin{cases} P_{0-0} = (1 - a)P_{00} + (1 - b)P_{10} \\ = \frac{1 + q - (aq + b)}{2 + q + s} = \frac{1 + q - (b + d)}{2 + q + s} \\ P_{0-1} = aP_{00} + bP_{01} = \frac{aq + b}{2 + q + s} = \frac{b + d}{2 + q + s} \\ P_{1-0} = dP_{10} + cP_{11} = \frac{d + cs}{2 + q + s} = \frac{b + d}{2 + q + s} \\ P_{1-1} = (1 - d)P_{10} + (1 - c)P_{11} \\ = \frac{1 + s - (cs + d)}{2 + q + s} = \frac{1 + s - (b + d)}{2 + q + s} \end{cases} \quad (24)$$

Applying (24) (19) in Pearson's autocorrelation formula,

$$\begin{aligned} \phi_2 &= \frac{P_{1-1}P_{0-0} - P_{1-0}P_{0-1}}{P_1P_0} \\ &= \frac{(1 + q)(1 + s) - (1 + q)(b + d) - (1 + s)(b + d)}{(1 + q)(1 + s)} \\ &= 1 - \frac{2 + q + s}{(1 + q)(1 + s)}(b + d) \end{aligned} \quad (25)$$

### E. Extract $\phi_3$ From $q$ , $s$ , $b$ , $d$

$P_{i--j}$  are decomposed as

$$\begin{cases} P_{0--0} = P_{0000} + P_{0010} + P_{0100} + P_{0110} \\ P_{0--1} = P_{0001} + P_{0011} + P_{0101} + P_{0111} \\ P_{1--0} = P_{1000} + P_{1010} + P_{1100} + P_{1110} \\ P_{1--1} = P_{1001} + P_{1011} + P_{1101} + P_{1111} \end{cases} \quad (26)$$

Bit string [0000] happens when 0 and 0 appear next to state [00]. Thus,

$$P_{0000} = (1 - a)^2P_{00} \quad (27)$$

In the same way,

$$\begin{cases} P_{0001} = (1 - a)aP_{00} \\ P_{0010} = a(1 - b)P_{00} \\ P_{0011} = abP_{00} \\ P_{0100} = (1 - b)dP_{01} \\ P_{0101} = (1 - b)(1 - d)P_{01} \\ P_{0110} = bcP_{01} \\ P_{0111} = b(1 - c)P_{01} \\ P_{1000} = d(1 - a)P_{10} \\ P_{1001} = daP_{10} \\ P_{1010} = (1 - d)(1 - b)P_{10} \\ P_{1011} = (1 - d)bP_{10} \\ P_{1100} = cdP_{11} \\ P_{1101} = c(1 - d)P_{11} \\ P_{1110} = (1 - c)cP_{11} \\ P_{1111} = (1 - c)^2P_{11} \end{cases} \quad (28)$$

Substituting (27), (28) and (20) into (26),

$$\begin{cases} P_{0-0} = (1-a)^2 P_{00} + a(1-b)P_{00} + \\ (1-b)dP_{01} + bcP_{01} \\ = (1-A)P_{00} + DP_{01} \\ P_{0-1} = (1-a)aP_{00} + abP_{00} + \\ (1-b)(1-d)P_{01} + b(1-c)P_{01} \\ = AP_{00} + (1-D)P_{01} \\ P_{1-0} = d(1-a)P_{10} + (1-d)(1-b)P_{10} + \\ cdP_{11} + (1-c)cP_{11} \\ = (1-B)P_{10} + CP_{11} \\ P_{1-1} = daP_{10} + (1-d)bP_{10} + \\ c(1-d)P_{11} + (1-c)^2 P_{11} \\ = BP_{10} + (1-C)P_{11} \end{cases} \quad (29)$$

where  $A = a+ab-a^2$ ,  $B = b+ad-bd$ ,  $C = c+cd-c^2$ ,  $D = d+bc-bd$ . Applying (29) (19) in Pearson's autocorrelation formula,

$$\begin{aligned} \phi_3 &= \frac{P_{1-1}P_{0-0} - P_{1-0}P_{0-1}}{P_1 P_0} \\ &= \frac{qs - 1 + (D - Aq)(s + 1) + (B - Cs)(q + 1)}{(1 + q)(1 + s)} \end{aligned} \quad (30)$$

Here,

$$\begin{aligned} D - Aq &= d + bc - bd - d(1 + b - a) \\ &= ad + bc - 2bd \end{aligned} \quad (31)$$

$$\begin{aligned} B - Cs &= b + ad - bd - b(1 + d - c) \\ &= ad + bc - 2bd \end{aligned} \quad (32)$$

Substituting (31) (32) into (30)

$$\begin{aligned} \phi_3 &= \frac{qs - 1 + (2 + q + s)(ad + bc - 2bd)}{(1 + q)(1 + s)} \\ &= \frac{qs - 1 + (2 + q + s)(b^2/s + d^2/q - 2bd)}{(1 + q)(1 + s)} \end{aligned} \quad (33)$$

F. Extract  $q, s$  From  $P_1, \phi_1$

Set  $t = 1 + q$ ,  $u = 1 + s$ , From (19)

$$P_1 = \frac{u}{t + u} \quad (34)$$

$$P_0 = \frac{t}{t + u} \quad (35)$$

From (20)

$$\begin{cases} \phi_1 = \frac{tu - t - u}{tu} = 1 - \frac{t + u}{tu} \\ \frac{1}{1 - \phi_1} = \frac{tu}{t + u} \end{cases} \quad (36)$$

Divide (36) with (34)

$$t = \frac{1}{P_1(1 - \phi_1)}$$

Divide (36) with (35)

$$u = \frac{1}{P_0(1 - \phi_1)}$$

Thus,

$$\begin{aligned} q &= \frac{1}{P_1(1 - \phi_1)} - 1 \\ s &= \frac{1}{P_0(1 - \phi_1)} - 1 = \frac{1}{(1 - P_1)(1 - \phi_1)} - 1 \end{aligned}$$

Extracted  $q$  and  $s$  are used in the followings.

G. Extract  $b$  From  $q, s, \phi_1, \phi_2, \phi_3$

Substituting (36) into (25)

$$\begin{cases} \phi_2 = 1 - \frac{2 + q + s}{(1 + q)(1 + s)}(b + d) \\ = 1 - \frac{t + u}{(1 + q)(1 + s)}(b + d) = 1 - (1 - \phi_1)(b + d) \\ b + d = \frac{1 - \phi_2}{1 - \phi_1} = E \\ d = E - b \end{cases} \quad (37)$$

where  $E = \frac{1 - \phi_2}{1 - \phi_1}$ . Substituting (20) and (36) into (33)

$$\begin{cases} \phi_3 = \frac{qs - 1 + (2 + q + s)(\frac{b^2}{s} + \frac{d^2}{q} - 2bd)}{(1 + q)(1 + s)} \\ = \phi_1 + (1 - \phi_1)\left(\frac{b^2}{s} + \frac{d^2}{q} - 2bd\right) \\ \frac{b^2}{s} + \frac{d^2}{q} - 2bd = \frac{\phi_3 - \phi_1}{1 - \phi_1} = F \end{cases} \quad (38)$$

where  $F = \frac{\phi_3 - \phi_1}{1 - \phi_1}$ . Substituting (37) into (38),

$$(1/s + 1/q + 2)b^2 - 2E(1/q + 1)b + \frac{E^2}{q} - F = 0.$$

Solving this quadratic equation,

$$\begin{aligned} b &= \frac{E(1/q + 1) \pm \sqrt{E^2(1/q + 1)^2 - (1/s + 1/q + 2)(E^2/q - F)}}{1/s + 1/q + 2}. \end{aligned}$$

Generally,  $b$  has two solutions. But in the case of the Markov model where bit stream is generated by MKV1, the square root term becomes zero, and  $b$  has only one solution.

H. Find  $T_{001}, T_{011}, T_{110}, T_{100}$  From  $b$  and Other Extracted Parameters

$$T_{011} = b$$

$$T_{100} = d = E - b$$

$$T_{001} = a = \frac{d}{q}$$

$$T_{110} = c = \frac{b}{s}$$

End.

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