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Lane Changing Control of Autonomous Vehicle With Integrated Trajectory Planning Based on Stackelberg Game

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ABSTRACT Lane changing present a significant challenge for autonomous vehicles, as they must maintain safe driving and optimize time efficiency. This process is strongly affected by traffic environment and driver characteristics. This paper proposed a lane changing control method based on Stackelberg game theory, integrating lane changing decision and trajectory planning while comprehensively considering the driver's characteristics and the traffic environment. Firstly, considering the common characteristics of lane changing decision and trajectory planning, the two stages are integrated using the leader-follower game theory, enhancing the accuracy of lane changing decisions. Secondly, the cooperative game theory model is employed to design an adaptive weight adjustment strategy for the trajectory tracking controller. The weight coefficients for vehicle stability and path tracking accuracy are dynamically adjusted within the model predictive control method to adapt to the vehicle's stability state. Simulation results indicate a 24% improvement in decision-making accuracy with the proposed leader-follower game decision method over the rule-based lane changing model. The average relative error in lateral displacement, comparing the vehicle's actual trajectory to the planned one, is reduced by 6%. Additionally, the variable-weight trajectory tracking control enhances overall tracking performance by over 30% in scenarios involving high speeds and low adhesion. These findings verify the proposed vehicle lane changing method notably improves lane changing safety, stability, and precision.

INDEX TERMS Autonomous vehicle, lane changing control, Stackelberg game, lane changing decision, trajectory planning.

I. INTRODUCTION

SOME statistics indicate that lane changing are responsible for nearly one-third of all traffic accidents [1], [2], [3], [4]. This situation arises due to the high level of expertise and experience demanded from drivers during lane changing maneuvers. Not only does it call for a thorough comprehension of the traffic environment, but also necessitates the availability of competent decision-making abilities [5], [6]. Especially, driver errors in lane changing decisions are a key factor contributing to traffic accidents. Therefore, effective decision-making and control in lane changing are crucial for enhancing the road safety of autonomous vehicles [7], [8].

The lane changing process for autonomous vehicles typically comprises three stages: lane changing decision, trajectory planning, and trajectory tracking. Making informed decisions and planning precise trajectories for lane changes are vital in autonomous driving systems, ensuring the vehicle can change lanes safely and effectively. Lane changing decisions involve assessing the nearby traffic conditions

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and choosing the best moment for the maneuver, while trajectory planning encompasses designing an appropriate driving route and implementing trajectory tracking to ensure a smooth and safe lane changing process. Various driving styles affect the decision-making and planning strategies involved in lane changing. Several scholars have investigated the impact of different driving styles on lane changing behavior [9], [10]. Utilizing lane changing trajectory data from NGSIM, Pang et al. [11] classified drivers using pre-processed lane changing trajectory data and conducted a comparative analysis of the lane changing behaviors among different driver types. Du et al. [12] introduced an innovative method that integrates driving style-based lane changing contexts with trajectory-related data from both intelligent and connected vehicles (ICVs) and their surrounding vehicles to forecast lane changing behaviors of ICVs. Zhang et al. [13] proposed a tailored framework for predicting lane changing risks that incorporates driving style factors. They introduced several indices to assess driving volatility and developed a dynamic clustering technique to determine the most appropriate time windows and methods for categorizing driving styles.

Given that vehicle lane changing involves interactions similar to game dynamics, lane changing models grounded in game theory have increasingly garnered interest from researchers. Ji et al. [14] developed a lane-merging strategy for autonomous vehicles in high-density traffic by applying the Stackelberg game theory, allowing autonomous vehicles to make optimal decisions by anticipating the responses of adjacent lane vehicles. To address the potential for mutual interference in lane changing decisions among vehicles, a multi-vehicle cooperative lane changing model based on game theory was developed using a two-matrix approach, with solutions for lane changing decisions demonstrated [15]. Ding et al. [16] developed a strategy for coordinating lane changing among multiple vehicles, specifically designed for scenarios requiring mandatory lane changing. They employed Stackelberg game theory to represent the interactions between a lane changing vehicle and its surrounding vehicles, thereby identifying the best timing for executing the lane change. Liu et al. [17] proposed a novel nonlinear gamebased driver-automation cooperative steering control method to mitigate collision caused by the driver's limited experience on low adhesion road conditions. However, most of the aforementioned studies apply game theory to address only one aspect of lane changing.

Trajectory planning and path tracking control are closely related and are often designed in collaboration so that the feasibility of control is considered during the planning phase and the planned path is adapted during the control phase. In practical applications, trajectory planning often needs to adapt to the ability of path tracking control to ensure the stability and safety of the whole system. The primary goal of the trajectory tracking model is to regulate the vehicle's heading angle and speed in order to track the planned lane change path. Wang et al. [18] proposed a control strategy for trajectory planning and tracking in autonomous vehicles, utilizing a model predictive control-based approach for trajectory tracking. This strategy ensures that the model's outputs better match the vehicle's dynamic behavior and boosts the precision of trajectory tracking. Geng et al. [19] designed a dual-layer lateral trajectory tracking control system. By assessing the vehicle's real-time status, the system selects the optimal controller output to ensure the tracking accuracy and effectively improve the passenger comfort. Song et al. [20] developed a lateral controller based on a three-degree-of-freedom vehicle dynamic model, utilizing model predictive control. The resulting control algorithm exhibits excellent robustness and stability in tracking performance. Li et al. [21] introduced a combined planning layer for trajectory tracking control specifically for four-wheel-drive electric vehicles. This method involves planning an obstacle-avoidance trajectory within a dynamic obstacle environment.

Typically, the trajectory planning for a vehicle's lane change is conducted after the lane change decision has been made. In most contemporary research, the processes of lane changing decision and trajectory planning for vehicles are treated as separate entities. However, in practical applications, both the decision and planning modules must consider factors such as safety, driving efficiency, and the impact on the traffic environment [22], [23]. This indicates that the factors considered by both modules are relatively consistent. This segregation inevitably results in a decline in both the efficiency and safety of lane changing. Integrating these two modules allows for a more comprehensive assessment of safety risks and driving efficiency across different lane changing trajectories. Consequently, decisions regarding lane changes can be executed with higher precision, thereby enhancing the success rate and safety of lane changing.

Considering that there are numerous common factors between lane changing decision and trajectory planning, this article proposes a method for their integration. The vehicle lane changing decision game model employs a set of alternative lane changing trajectories as its decision set. This approach replaces the original binary decision set of changing lanes or not, resulting in the lane changing decision model outputting specific vehicle trajectories instead of abstract behavioral instructions. This streamlines the workflow of the vehicle lane changing model and enhances both the safety and efficiency.

The main vehicles involved in the interaction during lane changing are the main vehicle and the following vehicle in the target lane [24]. This article treats the main vehicle and the following vehicle in the target lane as game theory participants and their respective lane change decision sets as the game theory decision set. The benefits obtained by vehicles through lane changing are considered as the payoffs in game theory, thereby forming a game theory model for lane changing decisions. Currently, most lane changing decision models are non-cooperative static game theory models, where both players make decisions simultaneously without a defined order. However, in practical lane changing scenarios, the main vehicle observes the surrounding environment first, forms the intention to change lanes, and makes corresponding decisions. It signals its decisions to surrounding vehicles using traffic lights or other means, and then other vehicles make their own decisions based on the main vehicle's decision, which they feedback to it. This suggests that there is a sequential order in which both players make decisions during lane changing, and static game theory models cannot fully capture the interactions involved in vehicle lane changing. Considering the analysis provided, this paper proposed lane change methods using the stackelberg leader-follower game theory framework.

To address collision avoidance in emergency scenarios, Dai et al. [25] proposed a model predictive decision-making (MPDM) approach that incorporates the consideration of lane-changing time. Trajectory following model predictive control (MPC) is an effective control method for managing complex systems, including nonlinear and multivariable systems [26], [27]. The appropriate selection of weight coefficients is crucial when setting up MPC because they significantly impact the controller's performance [28], [29]. Specifically, the weight coefficient determines the extent to which the MPC controller emphasizes state deviation and control inputs during the control process. Optimizing the weight factor is a widespread and critical task that enhances controller performance, prevents over-control, and reduces energy consumption. However, the optimization of weight coefficients is an intricate problem, involving multiple interrelated variables that often conflict with each other. For example, increasing the weight of control inputs can enable the system to track a predetermined trajectory quickly but may lead to high energy consumption. In current practice, the control parameters of the MPC controller–such as the control horizon and weight coefficients are fixed values that may underperform under varying working conditions. The coordination between different weight coefficients is also a game process. This article introduces a weight optimization method grounded in leader-follower game theory.

To summarize, the key contributions of this paper are:

(1) Based on the shared characteristics, utilizing stackelberg leader-follower game theory, an integrated method for vehicle lane changing decision and trajectory planning is proposed. This method takes into account the impact of different driving styles by modifying the weights in the game payoff function, it enables the trajectory resulting from the decision to more closely align with actual situations.

(2) A adaptive weight adjustment strategy based on cooperative game theory is proposed for the trajectory tracking controller. During lane changing, this model dynamically adjusts the weight coefficients of the MPC controller for stability and accuracy, enhancing the overall tracking performance of the vehicle.

The structure of this paper is organized as follows: Section II introduces the framework of vehicle lane changing system proposed in this article. Section III describes the vehicle lane changing trajectory planning and decision-making method using the leader-follower game. Section IV provides the detailed process of the controller weight adaptive adjustment strategy based on cooperative game theory. Section V presents the simulation results., finally, Section VI concludes the paper and suggests directions for future research..

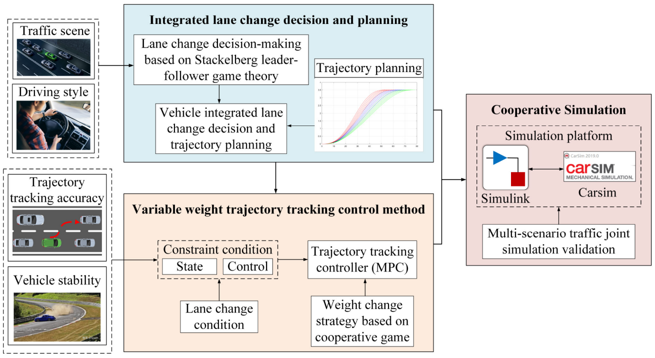


FIGURE 1. Vehicle lane changing system framework.

II. FRAMEWORK OF LANE CHANGING CONTROL

This paper introduces a lane-changing method that initially incorporates the driver's driving style and traffic conditions into a decision module based on Stackelberg leader-follower game theory, and then combines the decision module with the trajectory planning module as the basis for subsequent trajectory tracking. Within the trajectory tracking controller, the weight coefficient is set to be variable based on cooperative game, so as to achieve better control effect. Finally, the cooperative-simulation is carried out under various traffic scenarios.

The system scheme diagram of the whole article is illustrated in Fig. 1.

III. INTEGRATED LANE CHANGING DECISION AND TRAJECTORY PLANNING BASED ON GAME THEORY

A. ANALYSIS OF LANE CHANGING DRIVING STYLE

The way a vehicle is driven during lane changing is mainly reflected in lane changing time, lane changing speed, etc. Different driving styles significantly influence a vehicle's performance during lane changing. To effectively study the characteristics of the vehicle during lane changing, it is essential to classify and analyze the driving style of the vehicle. Studying the behavioral characteristics of vehicles requires a substantial amount of vehicle trajectory data. This paper used the data of NGSIM, which is a set of open public data with comprehensive vehicle traffic datasets [30]. Processed vehicle data can effectively reflect the characteristics of the lane changing process, making it suitable for analyzing behavior [31].

This article primarily concentrates on analyzing the driving styles of vehicles during the lane changing process. To classify vehicle driving styles during lane changing, the K-means clustering algorithm is utilized. Considering the

TABLE 1. Average vehicle speed under different driving styles.

Type	Minimum (m/s)	25% (m/s)	Median (m/s)	75% (m/s)	Maximum (m/s)
Aggressive	5.75	8.42	10.17	12.42	16.00
Common	3.50	6.67	7.67	8.92	11.92
Conservative	5.33	7.17	8.08	8.67	10.33

TABLE 2. Vehicle lane change time under different driving styles.

Type	Minimum (s)	25% (s)	Median (s)	75% (s)	Maximum (s)
Aggressive	4.29	5.95	6.70	7.13	8.40
Common	4.10	5.99	6.90	7.30	8.40
Conservative	4.60	6.59	7.30	8.11	8.40

typical characteristics of human drivers and the experimental verification of many scholars, this paper combines principal component analysis with K-means clustering to divide vehicle driving styles into three types: aggressive, conservative, and common.

According to the principle of K-means clustering algorithm, this paper sets the initial number of clustering centers for trajectory to 3, and uses SPSS data analysis software to conduct clustering analysis for the main components. The lane changing trajectory dataset is divided into three clusters, respectively representing three types of driving styles during lane changing. The dataset includes features such as speed, acceleration, and lane changing time.

This paper draws some of the feature data of the three clusters into tables. As illustrated in Table 1, Table 2 and Table 3.

(1) Average vehicle speed during lane changing From Table 1, it is evident that there is a significant gap in the range of the average speed for aggressive type vehicles when changing lanes, with the speed primarily centered around 10 m/s. The relatively high lane changing speed indicates an aggressive driving style. The average speeds of common type and conservative type vehicles during lane changing are lower, mainly around 7 m/s. Moreover, the speed distribution of conservative type is somewhat tighter than that of common type, indicating that the feature performance of vehicles in conservative type is more consistent. The range and distribution of the speed for common type are between aggressive type and conservative type, indicating that the driving style of common type is also between the two.

(2) Vehicle lane changing time The lane changing time for vehicles in different categories also has certain differences. The lane changing time required for aggressive type vehicles is shorter, mainly distributed between 6s to 7s, while the lane changing duration distribution for common type vehicles is slightly longer than aggressive type vehicles. Conservative type vehicles require a longer time for lane changing, mainly distributed between 7s to 8s. This indicates that both aggressive type and common type vehicles tend to consider lane changing efficiency during lane changing, hence their

TABLE 3. Speed ratio before and after lane changing for different driving styles.

Type	Minimum	25%	Median	75%	Maximum
Aggressive	1.09	1.39	1.54	1.63	1.90
Common	0.31	0.91	1.10	1.36	1.81
Conservative	0.43	0.72	0.87	0.98	1.35

lane changing time is shorter. On the other hand, conservative type vehicles value stability while changing lanes, resulting in a relatively longer lane changing time.

(3) Speed ratio before and after vehicle lane changing Under different driving styles, the speed changes of vehicles are different. For aggressive type vehicles, the increase rate of speed after lane changing compared to before is mainly distributed around 1.5, indicating that these vehicles increase a considerable speed through lane changing. The speed ratio before and after lane changing for common type vehicles is slightly greater than aggressive type, indicating that under this driving style, vehicles also get a certain speed gain from lane changing. For conservative type vehicles, the speed ratio before and after lane changing is slightly less than aggressive type, indicating that the speed gain from lane changing under this type is relatively small, suggesting a more conservative driving style.

Through the above analysis of various vehicle lane changing feature data, it can be seen that vehicles with an aggressive driving style tend to pursue efficiency when changing lanes, with short lane changing times and great speed increases after lane changing. Vehicles with a conservative driving style tend to have modest speed increases before and after lane changing, with long lane changing times. Vehicles of the common type perform between the two.

As indicated by the analysis results of average speed, time, and speed change rate before and after lane changing. It shows that the lane changing data after K-means clustering can reflect the features of different driving styles. Finally, the average vehicle speed, lane changing time and speed ratio before and after lane changing under different driving styles after classification are extracted as the basis for trajectory planning in the following sections.

B. VEHICLE LANE CHANGE TRAJECTORY PLANNING GROUNDED IN DRIVING STYLE

This section focuses on developing a set of potential trajectory curves for lane changes, tailored to various driving styles, to supply the essential decision-making framework to facilitate the subsequent vehicle lane change decision game model.

Here, a polynomial method for lane change trajectory planning is used. This planning method uses polynomial functions to plan a curved trajectory based on the vehicle's state of lane changes [32], [33], [34]. In this paper, polynomial functions f are used to plan the trajectory cluster of vehicle lane changes, and the degree of the polynomial

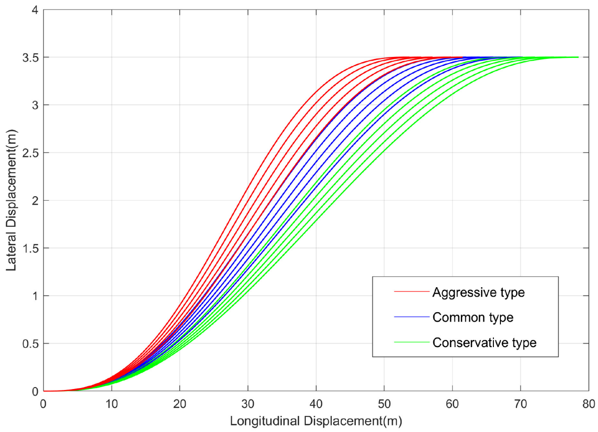


FIGURE 2. Lane changing trajectory curves under different driving styles.

is set to five considering the specifics of vehicle lane changes [35].

Solving the curve function requires the initial and end state data of the lane change. The initial state is mainly obtained from the data selected and extracted in the previous section, while the end state data needs to be calculated using the driving characteristic data extracted in the previous section. According to the selection and analysis data under various driving styles in the previous section, different driving styles have different lane change characteristics. When generating groups of lane changing trajectory, corresponding parameters need to be adjusted, and the formula for adjustment parameters is shown in (1)

$$\begin{cases} \dot{x}_{last} = \dot{x}_{first} \cdot F_i \\ d_i = T_i \cdot \left(\dot{x}_{first} + \dot{x}_{last}\right)/2 \\ x_{last} = d_i + x_{first} \\ y_{last} = y_{first} + Width \end{cases} \tag{1}$$

Among them, the starting points of x_{first}, $\dot{x}_{first}$ are lateral position and lateral speed respectively, x_{last}, $\dot{x}_{last}$ correspond to the end point lateral parameters, y_{first}, y_{last} are the longitudinal position at the starting point and the end point. T_i represents the vehicle lane changing time under various driving styles, F_i is the longitudinal speed ratio under various driving styles, d_i is the longitudinal lane changing distance of vehicles, and $Width$ is the road width, which is taken as 3.5m.

This article uses driving characteristics data under different styles to adjust the vehicle's final state, obtaining a set of trajectories for the vehicle under different driving styles, as illustrated in Fig. 2.

Due to the fact that the lane change feature data extracted from different driving styles is not a specific numerical value, but a range of values, there can be multiple possible lane changing trajectories corresponding to a driving style. When planning the trajectory curve corresponding to the style, it is also necessary to plan multiple trajectory curves based on the vehicle feature parameter range. Fig. 2 shows one of the alternative sets of trajectory curves for the main vehicle. It can be seen that the lane changing trajectory curves vary among vehicles with various driving styles. Among them, the aggressive vehicle has the shortest time and distance during lane changing when the initial vehicle speeds are close, with the common vehicles coming next and the conservative vehicles has the longest lane changing distance. However, as can be seen from the Fig. 2, there are also areas of overlap between trajectories under different driving styles, indicating that vehicles with various driving styles may also have similar behaviors.

In addition to planning a set of vehicles' trajectory curves under various driving styles, it is also necessary to plan a trajectory that does not change lanes, that is, a trajectory that continues to travel in the original lane, as the corresponding behavior for lane change decision.

C. INTEGRATED METHOD OF LANE CHANGING DECISION AND TRAJECTORY PLANNING

During the Stackelberg game [36], [37], the game participants choose appropriate strategies to maximize their own profit function. In combination with the actual situation during lane changing, further assumptions are needed for the Stackelberg game model, at this point, the vehicle lane changing model is shown in (2):

$$\begin{cases} \gamma_f = \underset{a_f \in A_f}{\arg\max} \left(\underset{a_l \in A_l}{\min} R_f\left(s, a_l, a_f\right)\right) \\ \gamma_l = \underset{a_l \in A_l}{\arg\max} \left(R_l\left(s, a_l, \gamma_f\right)\right) \end{cases} \tag{2}$$

wherein, (γ_l, γ_f) is the equilibrium solution of the game model, (A_l, A_f) is the rational decision-making set of leaders and followers, (a_l, a_f) is a specific decision in the rational decision-making set of leaders and followers, $(s \in S, S = (s_l, s_f))$ is the state space set of vehicles at the current time. R_l and R_f are the profit functions of leaders and followers, respectively. The subscripts l, f respectively represent parameters related to the lead and the following vehicle.

After obtaining a group of possible decisions for the players in the game decision making process, it is necessary to design an appropriate benefit function to characterize the benefits of different trajectories.

This article divides the benefit function into four parts: safety benefit $R_s(i, t)$, speed benefit $R_t(i, t)$, comfort benefit $R_c(i, t)$, and cooperation benefit $R_g(i, t)$, based on the characteristics of vehicles during lane changing. And determined the final vehicle decision model's benefit function as shown in (3) according to different weights.

$$\begin{cases} R_i(s_t, t + k\tau) = \omega_1 R_{si}(s_t, t + k\tau) \\ \qquad + \omega_2 R_{ti}(s_t, t + k\tau) + \omega_3 R_{ci}(s_t, t + k\tau) \\ \qquad + \omega_4 R_{gi}(s_t, t + k\tau) \\ R_i(t) = \sum\limits_{k=1}^{H} \xi^{k-1} R_i(s_t, t + k\tau), \xi \in [0, 1] \end{cases} \tag{3}$$

TABLE 4. Vehicle lane change decision model algorithm.

Steps	Description
1	Initialize the model and input the road environment information and the vehicle's driving style.
2	Considering various vehicle driving styles, generate candidate trajectory clusters corresponding to the front and rear vehicles using a fifth-order polynomial.
3	Using the benefit function to calculate the game decision-making revenue of the game participants.
4	Calculate the equilibrium solution of the Stackelberg game decision-making model to determine the current optimal trajectory curve for the main vehicle.
5	Transmit the obtained trajectory curve to the vehicle lane change execution module for lane change trajectory tracking.
6	Detect whether the vehicle has completed lane changing, otherwise repeat steps 2-4.

where ξ is the attenuation factor, $\omega_1, \omega_2, \omega_3, \omega_4$ is the weight of different return functions, k is the time frame for trajectory prediction, H is the total time window size of trajectory, and τ is the time period, which is taken as one tenth of the lane changing. The calculation formulas for each benefit function are as follows.

$$\begin{cases} R_s(i,t) = TH(i,t) = \frac{d}{v_i(t)} \\ R_t(i,t) = v_i(t) \\ R_c(i,t) = -|J_i(t)| = -\left|a_i'(t)\right| \\ R_g(i,t) = -\left|a_j(t')\right| \end{cases} \tag{4}$$

where d is the distance between the vehicle and the preceding vehicle in the target lane at the current time, and $v_i(t)$ is the speed at the current time. This section illustrates that a longer headway provides greater safety benefits. $v_i(t)$ is the current vehicle speed. It can be seen from this formula that the faster the speed, the higher the speed benefit. $a_i(t)$ is the acceleration of j at the current time. $a_j(t')$ is the acceleration of j at the t' time.

After obtaining the benefit function of the decision model, it is necessary to determine the weight of each benefit factor. This article uses the previously obtained vehicle feature data under different vehicle types to calibrate the weights of the four factors in the benefit function under different driving styles. Due to the faster speed and shorter lane change time of aggressive vehicles, with larger acceleration during lane changes, the weight of the speed benefit function should be higher, while the weights of other benefit functions should be lower. On the contrary, for conservative vehicles, the situation is the opposite. Therefore, the weight of the speed benefit function should be lower, and the weights of other benefit functions should be higher. For common vehicles, the weight of the benefit functions should fall between the other two. After computation and calibration, the factor weights for different vehicle types are obtained: aggressive: $\omega_1 = 0.1, \omega_2 = 2, \omega_3 = 5, \omega_4 = 8$; conservative: $\omega_1 = 2, \omega_2 = 1, \omega_3 = 10, \omega_4 = 15$; common: $\omega_1 = 1, \omega_2 = 1.5, \omega_3 = 5, \omega_4 = 10$.

The integrated lane change trajectory planning algorithm is shown in Table 4.

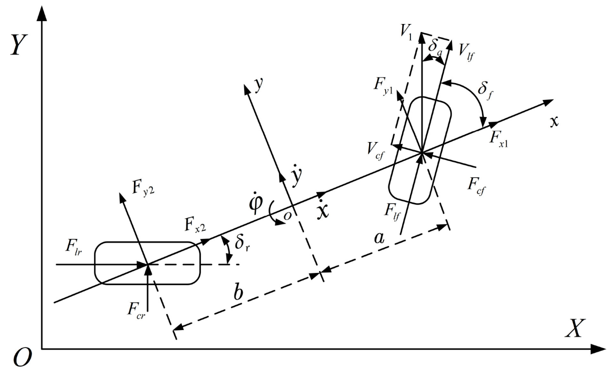


FIGURE 3. Two-degree-of-freedom vehicle model.

IV. VARIABLE WEIGHT TRAJECTORY TRACKING CONTROL GROUNDED IN COOPERATIVE GAME

A. DESIGN OF LANE CHANGING TRAJECTORY TRACKING CONTROLLER

In this context, the principles of game theory are applied to optimize the weight matrix, providing a systematic approach to improve the vehicle's stability and tracking accuracy during trajectory tracking. Consequently, this refined tuning enhances the MPC's capability to accurately follow the planned path, even in the presence of disturbances or uncertainties, thereby significantly improving the vehicle's overall stability and tracking accuracy during trajectory tracking [38]. The MPC-based vehicle trajectory tracking controller requires transferring the control output to the model to achieve the desired effect. Here, a two-degree-of-freedom model is used for the vehicle's dynamic modeling, as illustrated in Fig. 3.

The establishment of model is shown in (5):

$$\begin{cases} m\ddot{x} = m\dot{y}\dot{\varphi} + 2\left[C_{l1}s_1 + C_{c1}\left(\delta_f - \frac{\dot{y}+a\dot{\varphi}}{\dot{x}}\right)\delta_f + C_{l2}s_2\right] \\ m\ddot{y} = -m\dot{y}\dot{\varphi} + 2\left[C_{c1}\left(\delta_f - \frac{\dot{y}+a\dot{\varphi}}{\dot{x}}\right)\delta_f + C_{l2}\frac{b\dot{\varphi}-\dot{y}}{\dot{x}}\right] \\ I_z\ddot{\varphi} = 2\left[aC_{c1}\left(\delta_f - \frac{\dot{y}+a\dot{\varphi}}{\dot{x}}\right)\delta_f - bC_{l2}\frac{b\dot{\varphi}-\dot{y}}{\dot{x}}\right] \\ \dot{X} = \dot{x}\cos\varphi - \dot{y}\sin\varphi \\ \dot{Y} = \dot{x}\sin\varphi + \dot{y}\cos\varphi \end{cases} \tag{5}$$

where, the state variable of the model is $\xi_d = [\dot{y}, \dot{x}, \varphi, \dot{\varphi}, Y, X]^T$, where $u_d = \delta_f$ represents the vehicle's rotation angle. In this context, X represents the vehicle's longitudinal displacement, Y denotes the vehicle's lateral displacement, $\dot{x}, \dot{y}$ represent the vehicle's longitudinal and lateral speed, $\varphi, \dot{\varphi}$ denote the vehicle's yaw angle and yaw rate, m is the vehicle mass, $C_{l1,2}$ represents the longitudinal stiffness of front and rear tires, $C_{r1,2}$ denote the lateral stiffness of front and rear tires, a is the distance from the vehicle's center of gravity to the front axle, b is the distance to the rear axle, I_z is the moment of inertia of the vehicle about the z-axis, and $s_{1,2}$ represents the slip ratio.

This article selects linear time-varying model predictive control (LTVMPC), which has a small computational load and fast solution, and is suitable for vehicle lane changing scenarios. During a predictive horizon, the system input can be assumed constant, so its state trajectory can be obtained in this time domain. Then, the LTVMPC model is established using the differences between the actual system state and the predicted trajectory at this point.

Let the control system input control quantity $u_t(k) = u_t = \delta_f$ be set in the dimensional time domain from time t, resulting in a system state quantity trajectory $\xi_t(k)$:

$$\begin{cases} \dot{\xi}_t(k+1) = f(k, \xi_t(k+1), u_t(k)) \\ u_t(k) = u_t, k \geq 0, \xi_t(0) = \xi_t \end{cases} \tag{6}$$

Expanding the state (ξ_t, u_{t-1}) of the nonlinear system at time t, ignoring other high-order terms expected for the first-order term, we obtain the following linear time-varying system model:

$$\begin{cases} \dot{\tilde{\xi}}(t) = \frac{\partial f}{\partial \xi}\Big|_{\xi_t, u_{t-1}} \tilde{\xi}(t) + \frac{\partial f}{\partial \xi}\Big|_{\xi_t, u_{t-1}} \tilde{u}(t) \\ \tilde{\xi}(t) = \xi(t) - \xi_t, \tilde{u}(t) = u(t) - u_{t-1} \end{cases} \tag{7}$$

The linear system obtained after discretization is shown in (8):

$$\begin{cases} \tilde{\xi}(k+1|t) = A_t \tilde{\xi}(k|t) + B_t \tilde{u}(k|t) \\ \tilde{\xi}(k|t) = \xi(k|t) - \xi_t(k+1|t) \\ \tilde{u}(k|t) = u(k|t) - u_{t-1} \\ A_t = \left(I + T \frac{\partial f}{\partial \xi}\Big|_{\xi_t, u_{t-1}} \right), B_t = T \frac{\partial f}{\partial u}\Big|_{\xi_t, u_{t-1}} \end{cases} \tag{8}$$

Combining the state quantity error and the control quantity error, the state vector is defined as:

$$\Xi(k|t) = \begin{pmatrix} \tilde{\xi}(k|t) \\ \tilde{u}(k-1|t) \end{pmatrix} \tag{9}$$

Combining (8), the linear time-varying prediction model expression at time t is shown in (10):

$$\begin{cases} \Xi(k+1|t) = \begin{pmatrix} A_t & B_t \\ 0_{m \times n} & I_m \end{pmatrix} \Xi(k|t) \\ \qquad + \begin{pmatrix} B_t \\ I_m \end{pmatrix} \Delta \tilde{u}(k|t) \\ \Gamma(k|t) = \begin{pmatrix} C_t & 0 \end{pmatrix} \Xi(k|t) \end{cases} \tag{10}$$

where $C_t = [0\ 0\ 1\ 0\ 0\ 0;\ 0\ 0\ 0\ 0\ 1\ 0]$ represents the system output coefficient matrix, $\Xi(k|t) = [\dot{y}, \dot{x}, \varphi, \dot{\varphi}, Y, X, \delta_f]^T$ is the combination matrix, and $\Gamma(k|t) = [\varphi, Y]^T$ denotes the output matrix.

Constraints for the LTVMPC trajectory tracking controller include:

$$\begin{cases} \delta_{sw,\min} \leq u_{k,t} \leq \delta_{sw,\max}, k = 0, 1, \ldots, N_C - 1 \\ \Delta\delta_{sw,\min} \leq \Delta u_{k,t} \leq \Delta\delta_{sw,\max}, k = 0, 1, \ldots, N_C - 1 \\ P_{y,\min} \leq Y_{k,t} \leq P_{y,\max}, k = 0, 1, \ldots, N_C - 1 \\ a_{y,\min} \leq a_{y,k,t} \leq a_{y,\max}, k = 0, 1, \ldots, N_C - 1 \\ \alpha_{\min} \leq \alpha_{f,k,t} \leq \alpha_{\max}, k = 0, 1, \ldots, N_C - 1 \\ \dot{\varphi}_{\min} \leq \dot{\varphi}_{k,t} \leq \dot{\varphi}_{\max}, k = 0, 1, \ldots, N_C - 1 \end{cases} \tag{11}$$

Here $\delta_{sw,\min}, \delta_{sw,\max}$ represent the lower and upper bounds of the vehicle's front wheel angle, $\Delta\delta_{sw,\min}, \Delta\delta_{sw,\max}$ denote the lower and upper bounds of the vehicle's front wheel angle increment, $u_{k,t}, \Delta u_{k,t}$ represent the current front wheel angle and its increment, while N_C is the control time domain. $P_{y,\min}, P_{y,\max}$ represent the lower and upper bounds of the vehicle's lateral displacement, and $Y_{k,t}$ denotes the vehicle's lateral displacement. $a_{y,\min}, a_{y,\max}$ represent the lower and upper bounds of vehicle's lateral acceleration, and $a_{y,k,t}$ denotes the vehicle's lateral acceleration. $\alpha_{\min}, \alpha_{\max}$ represent the lower and upper bounds of the vehicle's tire slip angle, and $\alpha_{f,k,t}$ denotes the vehicle tire slip angle. $\dot{\varphi}_{\min}, \dot{\varphi}_{\max}$ represent the lower and upper bounds of the vehicle's yaw rate, and $\dot{\varphi}_{k,t}$ denotes the vehicle's yaw rate.

The objective function formulated according to the desired control target and is then converted into a quadratic programming problem to solve the control model. Finally, the predicted control quantity sequence at the current time is obtained. Below is the objective function formulated in this paper:

$$\begin{aligned} J(\xi_d(k), u_d(k-1), \Delta U_d(k)) \\ = \sum_{i=1}^{N_p} \|\eta_d(k+i|k) - \eta_r(k+i|k)\|_Q^2 \\ + \sum\nolimits_{i=1}^{N_{C-1}} \|\Delta u_d(k+i|k)\|_R^2 + \rho\varepsilon^2 \end{aligned} \tag{12}$$

where, ε represents the relaxation factor, ρ is the weight of the relaxation factor. Q, R denote the weight matrices of the control variable and control increment, respectively.

The weight of Q is divided into two parts, Q_Y and Q_φ,represents the tracking weights for the desired lateral displacement and desired yaw angle, respectively. By adjusting the weights, the controller can track the desired trajectory curve stably and rapidly, and its form is as follows:

$$Q = \begin{bmatrix} Q_\varphi & 0 \\ 0 & Q_Y \end{bmatrix} \tag{13}$$

By incorporating the above constraints and objective function, the optimization problem for solving the model predictive trajectory tracking controller, grounded in the vehicle dynamics model at each sampling period, is formulated as:

$$\begin{aligned} \min_{\Delta U, \varepsilon}\ & J(\xi_d(k), u_d(k-1), \Delta U_d(k)) \\ \text{s.t.}\ & \delta_{f,\min} \leq u_{k,t} \leq \delta_{f,\min}, k = t, \ldots, t + N_c - 1 \\ & \Delta\delta_{f,\min} \leq \Delta u_{k,t} \leq \Delta\delta_{f,\min}, k = t, \ldots, t + N_c - 1 \\ & \Gamma_{\min} - \varepsilon \leq \Gamma_{k,t} \leq \Gamma_{\max} - \varepsilon, k = t, \ldots, t + N_p \\ & \varepsilon \geq 0, k = t, \ldots, t + N_p \\ & \Delta u_{k,t} = 0, k = t + N_c, \ldots, t + N_p \end{aligned} \tag{14}$$

Here N_c represents the control time domain and N_p denotes the prediction time domain.

At time t, the model optimizes (12) using the vehicle's current state and the control input from the previous time step. Here, we use the quadprog solver function, and the planning result is the optimal control increment for the system output at the current time step.

$$\Delta U_t^* \triangleq \left[\Delta u_{t,t}^*, \ldots, \Delta u_{t+N_c-1,t}^*\right]^T \tag{15}$$

The first value in the control increment sequence is used as the optimal control output for the present moment, resulting in the subsequent state feedback control law:

$$u_t(\xi_t) = u_{t-1} + \Delta u^*_{t,t}(\xi_t) \tag{16}$$

When the control system executes control output to the next time node, the controlled object's system will update the vehicle system model by linearizing using the present state and prior input, and then use (12) to optimize the solution to obtain a new control increment sequence ΔU^*_{t+1}. This process is repeated until trajectory tracking is complete.

B. WEIGHT ADAPTATION OPTIMIZATION BASED ON COOPERATIVE GAME THEORY

Based on the above analysis, this article sets Q_{weight} as the controller tracking factor and R_{weight} as the controller stability factor, and changes the weights for the model predictive controller in the form shown below:

$$Q = Q_{weight}\begin{bmatrix} Q_{\varphi_0} & \\ & Q_{Y_0} \end{bmatrix} \tag{17}$$

$$R = R_{weight}R_0 \tag{18}$$

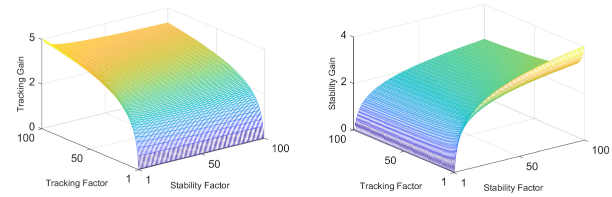
By establishing a weight adjustment strategy to improve Q_{weight} and R_{weight}, the controller weight coefficients are optimized and adjusted, resulting in good tracking performance of the controller.

In the trajectory tracking controller, the tracking weight coefficient and stability weight coefficient adjust their own values to affect the vehicle trajectory tracking performance. There is a certain competitive relationship between these two weights, and the stability weight and tracking weight jointly constitute the two parties of the game. These two weights can independently adjust to optimize the tracking capabilities of the controller to some extent, but they will also cause loss of performance in other aspects. For example, adjusting the stability weight alone will increase vehicle stability during trajectory tracking, but its tracking performance will decrease by coordinating and adjusting both weights simultaneously, the vehicle's stability and tracking accuracy can be maintained at a good level, resulting in good improvement of the controller's tracking performance. The stability weight is N_1, and the traceability weight is N_2, which constitutes the set of game members $N = \{N_1, N_2\}$. The stability weight factor is set to $u_1(u_1 = 1, 2 \cdots 100)$ and the traceability weight factor is set to $u_2(u_2 = 1, 2 \cdots 100)$. Design different revenue functions to reflect the revenue of each player and the cooperation between the two parties.

(1) Stability factor income function: Speed and surface adhesion will affect the vehicle's stability, and vehicle's yaw rate also reflects the stability to some degree. Therefore, with regard to the factors mentioned above, the income function of the stability weight is defined as the following form:

$$U_1(u_1, u_2) = \frac{\bar{\omega}\bar{v}}{\mu(1 + k_2\log(u_2))}\log(u_1) \tag{19}$$

$$\bar{\omega} = \frac{\omega - \omega_{\min}}{\omega_{\max} - \omega} \tag{20}$$



(a) 3D chart of tracking benefits **(b)** 3D chart of stability benefits

FIGURE 4. Returns under different tracking factors and stability factors.

$$\bar{v} = \frac{v - v_{miv}}{v_{\max} - v} \tag{21}$$

In order to facilitate calculation, data are normalized according to (20) and (21). $\bar{\omega}$ represents the normalized vehicle's yaw rate, $\bar{v}$ denotes the vehicle's speed, μ represents the road adhesion coefficient, and k_2 denotes the influence weight of tracking factors on vehicle stability. From (19), it is evident when the vehicle's speed and yaw rate are large and the surface adhesion is small, the improvement in stability factor can obtain greater benefits. However, when the tracking factor is increased, the return of the stability factor will decrease to some extent.

(2) Traceable factor return function: Trajectory tracking evaluation is generally conducted on two fronts: lateral displacement and yaw angle tracking. Therefore, when the vehicle's lateral displacement error and yaw angle error are large, increasing the value of the tracking factor will have higher returns, and the value of the stability factor will also affect the controller tracking to a certain extent. Consequently, the gain function for the tracking factor is designed as follows:

$$U_2(u_1, u_2) = \frac{\Delta\bar{Y}\Delta\bar{\varphi}\log(u_2)}{(1 + k_1\log(u_1))} \tag{22}$$

$$\Delta\bar{Y} = \frac{\Delta Y - \Delta Y_{\min}}{\Delta Y_{\max} - \Delta Y} \tag{23}$$

$$\Delta\bar{\varphi} = \frac{\Delta\varphi - \Delta\varphi_{\min}}{\Delta\varphi_{\max} - \Delta\varphi} \tag{24}$$

Here, $\Delta\bar{Y}$ and $\Delta\bar{\varphi}$ represent the lateral deviation of track and the deviation of vehicle's yaw angle after normalization according to (23) and (24), respectively, and k_1 is the influence weight of stability factor on controller traceability. According to the (22), when the vehicle's lateral error and yaw angle error trajectory are larger than that of the expected trajectory, increasing the value of the tracking factor can well improve the controller's tracking benefit, while increasing the stability factor will reduce the tracking benefit to some degree.

Taking into account the returns of traceability factors and stability factors, the corresponding 3D game returns are drawn as indicated in Fig. 4:

Obviously, As observed in Fig. 4, it is impossible to find a point to maximize the vehicle's stability and accuracy at the same time, and there are certain conflicts between the two sides. It is easy to know from the income function of both players that when two players play a non-cooperative game, the results in both players take the maximum value, which will lead to poor stability and traceability of the controller.

To enhance the controller's performance, the following work introduces a coalition N_3 consisting of stability and tracking factors. This coalition starts from the overall performance evaluation of vehicle trajectory tracking, and uses a forced external force to coordinate and adjust the values of both parties in the game, thus the controller can coordinate and obtain good tracking and stability. When both parties cooperate, a thorough evaluation of the vehicle's stability and tracking accuracy is necessary, hence the cooperative benefit function of coalition N_3 is given by form (25):

$$F(u_1, u_2) = K_1 U_1(u_1, u_2) + K_2 U_2(u_1, u_2)$$
$$K_1, K_2 > 1 \tag{25}$$

where K_1 is the influence weight of vehicle track tracking stability factor and K_2 is the influence weight of vehicle track tracking accuracy factor. From (25), it is evident that the benefits resulting from the alliance formed by the cooperation between the two parties are greater than the comprehensive benefits of their own.

The evaluation index of vehicle trajectory tracking performance includes stability and tracking, representing a problem involving multiple optimization targets. Given the conflicting nature between these two aspects, achieving a strict Pareto optimal solution is not feasible. Therefore, based on the principle of cooperative game theory, this paper adjusts the two weight coefficients. Compared with the Nash equilibrium solution of non-cooperative game theory, this adjustment will cause a certain loss in one aspect of trajectory tracking. This loss will not cause the performance of the controller to decline too much, which is within an acceptable range. However, the performance improvement of the other aspect of the controller will be great. Therefore, the comprehensive performance of the controller is improved.

After adding the cooperative game weight adjustment strategy, the solution process for weight coefficient adjustment in vehicle trajectory tracking control is as shown in Fig. 5. From Fig. 5, firstly, the controller stability factor and tracking factor are calculated based on the reference trajectory, vehicle state, and road environment parameters, and the cooperation benefit of both parties is calculated on this basis. Secondly, the equilibrium solution of the cooperative game is solved to obtain the values of the stability factor and tracking factor, thus obtaining the most suitable weight coefficient for the vehicle at the current time. The additional benefit obtained through cooperation is fully reflected in the improvement of vehicle trajectory tracking control performance.

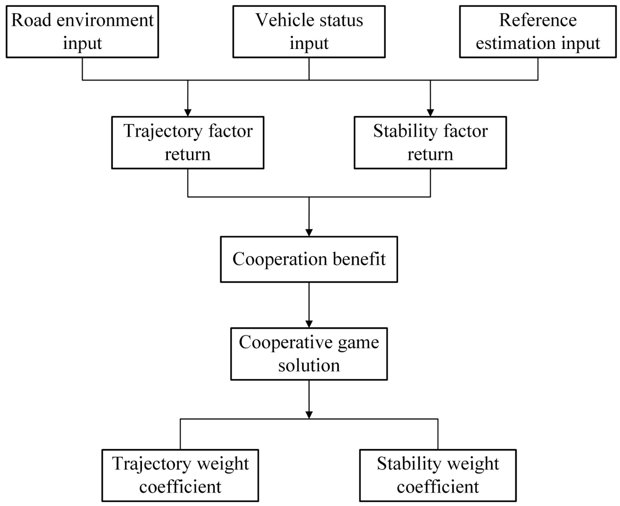


FIGURE 5. Weight coefficient solving process based on cooperative game theory.

From Fig. 5, firstly, the controller stability factor and tracking factor are calculated based on the reference trajectory, vehicle state, and road environment parameters, and the cooperation benefit of both parties is calculated on this basis. Secondly, the equilibrium solution of the cooperative game is solved to obtain the values of the stability factor and tracking factor, thus obtaining the most suitable weight coefficient for the vehicle at the current time. The additional benefit obtained through cooperation is fully reflected in the improvement of vehicle trajectory tracking control performance.

V. SIMULATION VERIFICATION

A. VEHICLE LANE CHANGE DECISION MODEL VALIDATION

According to lane changing trajectory set screened out in Section III, data of the trajectory curves for 20 vehicles that remained in their lanes are also added to the trajectory set as the comparison data. In this paper, the first 2/3 of the selected trajectory data set are used for computation and calibration. The weights of four factors are determined according to the driving style. The computation and calibration results are shown in Section III-C.

After obtaining the factor weights of the objective function, the model's validity can be demonstrated. To ensure the simulation's credibility, the remaining 1/3 of the data set is used for simulation verification. By comparing the actual vehicle driving trajectory with the trajectory data obtained from the game decision-making model, the accuracy of the model planning trajectory is verified.

For comparison, this article also developed a leader-follower decision model for vehicle lane changing without considering the vehicle driving style, as well as a lane changing decision-making and planning model utilizing traditional static game theory. By comparing the accuracy in decision-making behavior and the match between the planned and real trajectories, this article successfully demonstrated the effectiveness of the model.

Firstly, the accuracy of vehicle's behavior should be verified by comparing the lane changing decisions obtained

TABLE 5. Accuracy in behavioral decision-making under various lane changing models.

Model		Sample number	Accurate number	Accuracy rate (%)
Leader-follower game model considering driving style	Conservative	30	30	100
	Common	60	55	91.6
	Aggressive	30	28	93.3
	Overall	120	113	94.1
Leader-follower game model without considering driving style		120	101	84.1
Traditional decision-making and planning model grounded in static game theory		120	84	70

from different lane changing models with actual behaviors. Table 5 presents the simulation results:

From Table 5, it can be seen that the traditional game decision-making model has a behavior prediction accuracy of only 70%, meanwhile, the precision of the leader-follower game model, which does not account for vehicle driving style, is 84.1%, which has a certain improvement in accuracy. After considering the influence of vehicle driving style, the precision of the leader-follower game model reaches 94.1%, and its accuracy increases by 24.1%. Based on the analysis of the behavior prediction accuracy under different styles in Table 5, it can be found that the behavior prediction accuracy of common vehicles in the three driving styles is low. This is because common vehicles exhibit a performance between aggressive and conservative during lane changes, and their characteristic parameters are less distinguishable compared to other two styles of vehicles.

Further analysis of the vehicle's trajectory during lane changes. This article uses the mean relative error (MRE) of the lateral displacement to characterize the overall precision, as shown in (26):

$$MRE = \frac{1}{K}\sum_{i=1}^{K}\frac{1}{J}\sum_{m=1}^{J}\frac{\left|y(i,m)-y'(i,m)\right|}{y'(i,m)} \times 100\% \quad (26)$$

Here, $y(i, m)$ is the predicted lateral displacement for vehicle i at time m, $y'(i, m)$ is the real lateral displacement for vehicle i at time m, J represents the overall number of time steps within the decision-making and planning model, and K represents the quantity of simulated vehicle instances. Table 6 illustrates the results.

According to Table 6, the MRE for the lane changing model's planning curve based on static game theory reaches 12.0428%, while the MRE for the trajectory decision model without considering driving style is 9.9925%. The trajectory error is reduced to a certain extent. After considering driving style, the MRE of the trajectory curve obtained by the decision model further decreases to 5.9297%, which is significantly improved. The average relative error in lateral displacement across the vehicle's actual and planned trajectories is reduced by 6%. The table also shows that the common lateral displacement error for common and

TABLE 6. Average relative errors under different driving styles.

Driving style		Mean relative error(%)
Leader-follower game model considering driving style	Conservative	5.0229
	Common	7.4706
	Aggressive	7.1093
	Overall	5.9297
Leader-follower game model without considering driving style		9.9925
Traditional decision-making and planning model grounded in static game theory		12.0428

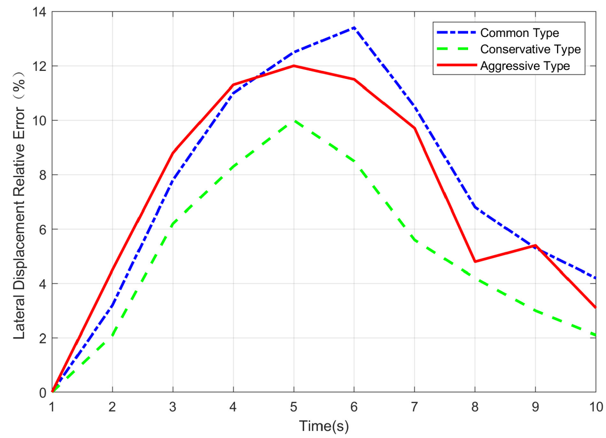


FIGURE 6. Lateral displacement error throughout lane changing under different driving styles.

aggressive vehicles is larger than that of conservative vehicles, because the characteristic parameter distribution range of these two types of vehicles is larger, and the possible lane change trajectory changes are also more, resulting in lower predictive performance for the model.

Further examination of the simulation results indicates that during the lane changing process, the vehicle's decision-making model cycles through every time node to update the lane change trajectory when solving the decision-making model. Therefore, for the trajectory data obtained from the decision-making process, the data at each time node is relatively important. Due to differences in lane changing times among vehicles, to analyze the changes in the planned trajectory during each stage of lane changing, this article segments the entire lane change maneuver across ten time nodes, and calculates the vehicle's lateral displacement error trajectory at each node using (27).

$$RE = \frac{1}{K}\sum_{i=1}^{K}\frac{\left|y(i,m)-y'(i,m)\right|}{y'(i,m)} \times 100\% \quad (27)$$

Plot the mean relative error of the lateral displacement for the predicted trajectory at each time node to analyze the situation of different driving style planned trajectories.

As demonstrated in Fig. 6, the trajectory lateral displacement deviation trends under different driving styles are similar, the vehicle's lateral deviation reaches its maximum midway through the lane changing, while the error is the smallest at the beginning and end. This is because the state changes in the initial and final stages of the lane change are relatively small, so the prediction accuracy is high. However, throughout the lane transition, due to the vehicle's horizontal and vertical speed influence, the unpredictability in the actual lane-changing path increases, and the deviation in the trajectory planned by the vehicle lane changing model and the real trajectory increases to some extent. Fig. 6 also shows that the relative error in the trajectory of aggressive vehicles increases to some extent at the final stage, indicating that the behavior of aggressive vehicles is relatively unstable at this stage, so the difficulty of trajectory prediction increases and the lateral displacement deviation increases.

The above results indicates that the vehicle lane changing decision planning method derived from Stackelberg game theory designed in this paper has higher accuracy and efficiency in practical operation than traditional static game theory based lane changing and planning methods. After considering the influence of different vehicle driving styles, the precision of its decision-making planning method has been further enhanced, providing a clearer depiction of how vehicles perform during lane changes.

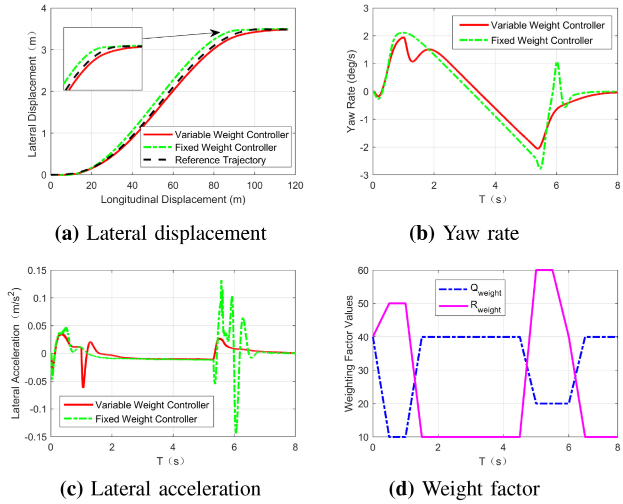
B. VALIDATION OF THE EFFECTIVENESS OF WEIGHTED ADAPTIVE TRAJECTORY TRACKING CONTROLLERS

This paper develops a vehicle lane changing trajectory tracking controller model in Simulink using the algorithm, and the C-Class vehicle model in Carsim is employed as the control object to conduct simulations under different conditions, comparing and analyzing the effects of both the fixed weight controller and the variable weight controller. In this setup, the lane change trajectory is specified as a single-shift maneuver with a width of 3.5 meters.

This paper takes into account the effects of vehicle speed and road adhesion coefficient on control effectiveness, and designs two operating conditions: $V = 60km/h, \mu = 0.85$, and $V = 90km/h, \mu = 0.4$.

1) CASE ONE: $V = 60KM/H, \mu = 0.85$

The results are presented in Fig. 7. As illustrated in Fig. 7(a), both controllers track the desired trajectory curve relatively well. During the final stage of the lane changing, the vehicle with fixed weight shows some overshoot, which reduces both tracking performance and stability. In the whole process of lane changing, different stages focus on stability and accuracy is different, variable weight controller can adjust the weight in time, so that the vehicle reach better vehicle stability and tacking accuracy than the fixed weight controller. Fig. 7(b) shows that during the initial and final stages of the lane change, the stability of the vehicle with fixed weight fluctuates significantly, while the vehicle with variable weight effectively reduces the yaw fluctuation amplitude in the final phase of the lane change.



(a) Lateral displacement **(b)** Yaw rate **(c)** Lateral acceleration **(d)** Weight factor

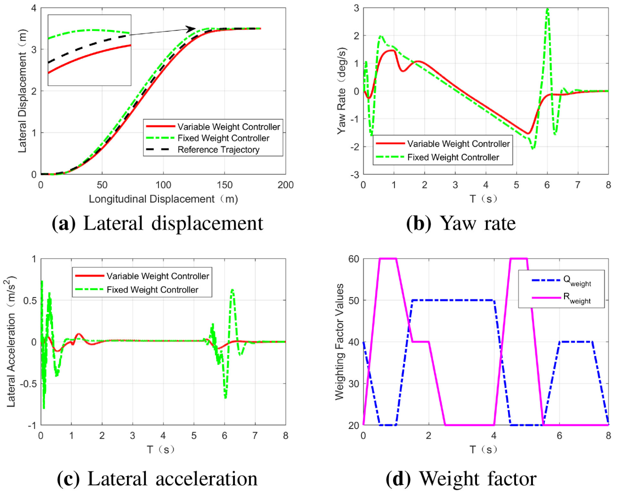
FIGURE 7. **Relevant results for lane changing of case one.**

Fig. 7(c) shows that both types of vehicles exhibit some fluctuations in lateral acceleration during the initial phase of the lane change, while the vehicle with fixed weight exhibits significant fluctuations in the final phase of the lane change. Fig. 7(d) illustrates that the stability weight of the vehicle increases throughout the initial and final phases of the lane change, thereby improving vehicle's stability. During the middle, due to the vehicle's good stability itself, the control tracking weight increases, allowing the vehicle to accurately follow the intended path.

2) CASE TWO: $V = 90KM/H, \mu = 0.4$

The results of the simulation are illustrated in Fig. 8. From Fig. 8(a), it is evident that the variable weight vehicle closely follows the desired trajectory curve throughout the lane changing process, while the fixed weight vehicle has a trajectory deviation during the final stage of the lane changing, resulting in a decrease in tracking accuracy. From Fig. 8(b) and Fig. 8(c), noticeable fluctuations in yaw rate and lateral acceleration occur in the fixed weight vehicle towards the end of the lane change, indicating that the controller increases the control increment output to reduce the deviation after the trajectory deviation, leading to a reduction in vehicle stability. From Fig. 8(d), it is apparent that the variable weight controller adjusts the stability weight factor to a higher value in the initial and final phases of the lane change to improve the stability of the vehicle during these stages, which also ensures that the vehicle does not lose stability and can maintain high tracking accuracy to follow the path.

The data presented reveal that the fixed weight controller performs relatively stable in medium and low speed environments on high adhesion coefficient road, but performs poorly in situations where the vehicle speed is high or the road adhesion coefficient is low. The variable weight controller performs better than the fixed weight controller under different vehicle states and road environments.



(a) Lateral displacement **(b)** Yaw rate

(c) Lateral acceleration **(d)** Weight factor

FIGURE 8. Relevant results for lane changing of case two.

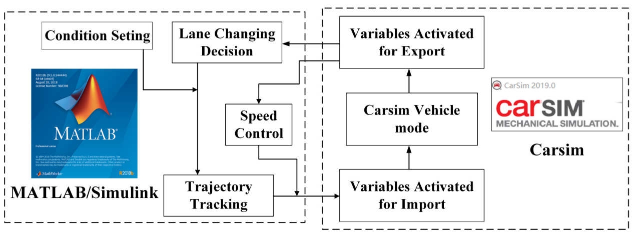


FIGURE 9. Joint simulation algorithm flowchart.

C. JOINT SIMULATION OF VEHICLE LANE CHANGING DECISION-MAKING PLANNING AND TRACKING CONTROL IN MULTIPLE SCENARIOS

Path tracking control is affected by driving style, which is mainly reflected in the fact that path tracking control is carried out on the basis of trajectory planning. This section combines the previously designed vehicle lane changing decision-making planning model and tracking control model to formulate a lane change model for vehicles, and designs different traffic scenarios to examine the vehicle performance traits with various driving styles in the scenarios, so as to assess the validity of the lane-changing theory presented in this article.

The construction of the vehicle lane changing model algorithm is entirely completed in MATLAB/Simulink. Based on the algorithm theory set above, this article compiles and designs relevant function models, builds and connects each module in Simulink, and finally connects the interface with the Carsim vehicle model to establish a virtual simulation platform required for the simulation. The joint simulation block diagram is depicted in Fig. 9.

According to Fig. 9, the Condition Setting module is a vehicle lane changing environmental parameter setting module, responsible for setting relevant parameters for vehicles. The Lane Changing Decision module is a vehicle lane changing decision and planning module, responsible for analyzing the vehicle's own state and environmental parameters to derive the vehicle's lane change decision and anticipated trajectory. The Trajectory Tracking module is a vehicle trajectory tracking control model, responsible for following the trajectory curve set by the vehicle decision and planning module, controlling the vehicle's active steering, and transmitting expected control data to the vehicle dynamics model. Since the trajectory tracking module only controls the vehicle's active steering, a vehicle longitudinal speed control module Speed Control is developed using PID control principles.

After establishing the vehicle lane changing simulation model, it is essential to configure the settings for each model and the environment. Because of the disparities between the simulation's traffic conditions and those in the NGSIM dataset, the weight parameters selected here are adjusted based on the model parameters in Section IV to ensure that the module can generate trajectory function curves that are more suitable for the control module. The adjusted weight parameters are: aggressive: $\omega_1 = 0.5, \omega_2 = 0.5, \omega_3 = 5, \omega_4 = 2$; conservative: $\omega_1 = 2, \omega_2 = 1.5, \omega_3 = 10, \omega_4 = 8$; common: $\omega_1 = 1, \omega_2 = 2, \omega_3 = 5, \omega_4 = 4$.

The study focuses on a decision and planning model for vehicles. The vehicles mainly include the front vehicle in the current lane (FV), the rear vehicle in the current lane (RV), the front vehicle in the target lane (TFV), and the rear vehicle in the target lane (TRV). Thus, when configuring the parameters, it is important to set the relevant state parameters for these vehicles. By altering the state data of these vehicles, the traffic environment of the main vehicle(MV) can be changed. This article refers to the lane changing condition data in NGSIM and designs the following three simulation condition scenarios to reflect the lane changing decision-making and planning in various traffic scenarios.

TABLE 7. Main simulation parameters of vehicles (Scenario 1).

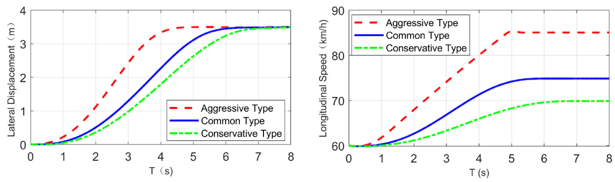
Vehicle	Longitudinal Position(m)	Lateral Position(m)	Speed(km/h)	Driving style
MV	0	0	60	—
FV	60	0	60	Common
RV	-30	0	60	Common
TRV	-30	3.5	60	Conservative
TFV	80	3.5	80	Common

1) SCENARIO 1

In Scenario 1, Table 7 provides the essential parameters for each vehicle.

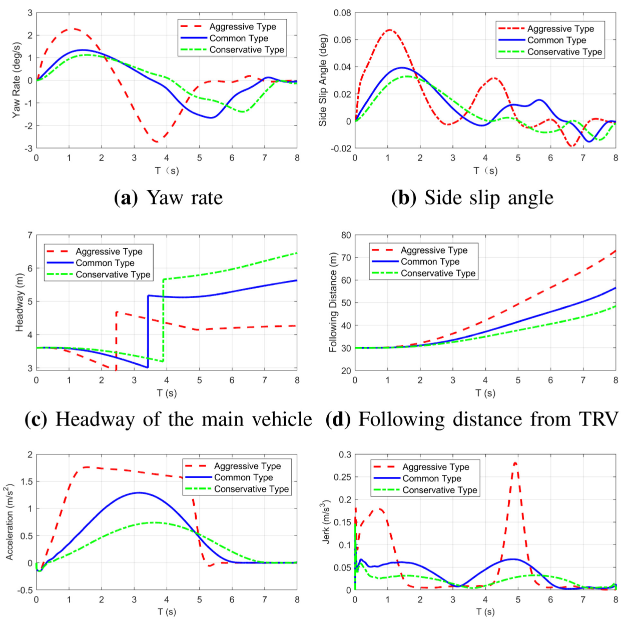
For this scenario, the spatial distance between the main vehicle and the target lane is relatively large, and the driving behaviors of adjacent vehicles are basically conservative and common. The TFV has a fast speed, and the lane change benefits are high, suggesting that the traffic conditions for vehicle lane changing is relatively suitable.

The simulation results of the lane change process are illustrated in Fig. 10 and Fig. 11.



(a) Lateral displacement **(b)** Longitudinal speed

FIGURE 10. Position and speed of the main vehicle in scenario 1.



(a) Yaw rate **(b)** Side slip angle

(c) Headway of the main vehicle **(d)** Following distance from TRV

(e) Longitudinal acceleration **(f)** Jerk

FIGURE 11. Results of the main vehicle in scenario 1.

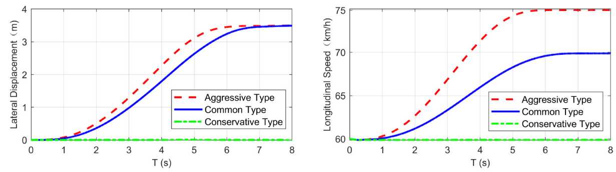
From Fig. 10(a), it is evident that all three types of vehicles in this scenario elect to change lanes, and the time of lane changing varies with varying driving styles. Fig. 10(b) shows that the longitudinal speed of vehicles elevates slightly after lane changing. The speed of aggressive vehicles after lane changing is the highest, the speed of conservative vehicles is the lowest, and the speed of common vehicles is between them.

Fig. 11 illustrates the variations in relevant outcomes of the main vehicle under different driving styles. As observed in Fig. 11(a) and Fig. 11(b), the aggressive vehicles have large fluctuations in yaw rate and side slip angle, indicating generally poor stability in these vehicles. In contrast, the fluctuations in these parameters are smaller for the common and conservative vehicles, indicating better stability. Fig. 11(c) and Fig. 11(d) mainly reflect the lane changing benefits. The data indicate that all three types of vehicles obtain certain benefits after lane changing, but the benefits vary under different driving styles. Aggressive vehicles obtain higher speeds and driving spaces after changing lane, leading to the highest lane changing benefits. The common and conservative vehicles also obtain higher speeds and driving spaces, resulting in certain lane changing benefits. Fig. 11(e) and Fig. 11(f) reflect the vehicle's comfort level during driving. As shown in them, the longitudinal acceleration for the conservative and common vehicles remains low, and the jerk is also below $0.1m/s^3$ throughout the process, indicating that the impact during lane changing is small and the driving comfort is good.

In summary, all three types of vehicles in this scenario choose to change lanes and obtain certain lane changing benefits. The aggressive vehicles have the highest benefits, followed by common vehicles, and the conservative vehicles have the lowest benefits. In contrast, the safety and comfort levels of conservative and common vehicles during lane changing are higher than those of aggressive vehicles.

TABLE 8. Main simulation parameters of vehicles (Scenario 2).

Vehicle	Longitudinal Position(m)	Lateral Position(m)	Speed(km/h)	Driving style
MV	0	0	60	—
FV	40	0	60	Conservative
RV	-30	0	60	Common
TRV	-20	3.5	65	Common
TFV	60	3.5	75	Common



(a) Lateral displacement **(b)** Longitudinal speed

FIGURE 12. Position and speed of the main vehicle in scenario 2.

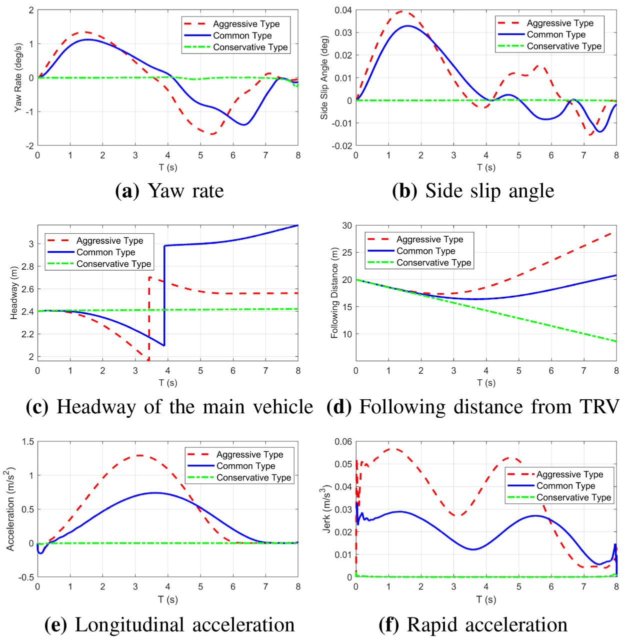
2) SCENARIO 2

In Scenario 2, Table 8 details the fundamental parameters for each vehicle.

For this scenario, the lane changing space for the main vehicle is relatively small, but the driving styles of the vehicles around the main vehicle tend to be conservative. Therefore, there is still a good benefit from lane changing, and it can be carried out.

Scenario 2 reduces the target lane space distance based on scenario 1, leading to reduced potential benefits for vehicles and a decrease in lane changing safety. The findings are illustrated in Fig. 12 and Fig. 13.

In scenario 2, due to the change in the lane changing environment, the responses of vehicles with diverse driving styles has also changed. As shown in Fig. 12(a), conservative vehicles choose not to change lanes and continue driving in the initial position, while common and aggressive vehicles continue to choose to change lanes. According to Fig. 12(b), after the lane change is completed, the speeds of aggressive and common types both increase, with the speed of aggressive vehicles being the highest.



(a) Yaw rate **(b)** Side slip angle **(c)** Headway of the main vehicle **(d)** Following distance from TRV **(e)** Longitudinal acceleration **(f)** Rapid acceleration

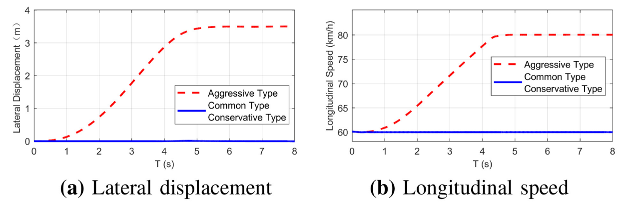
FIGURE 13. Results of the main vehicle in scenario 2.

Fig. 13 shows some results curves during vehicle lane changing, which allows for further study of the characteristics of vehicle lane changing. As represented in Fig. 13(a) and Fig. 13(b), the fluctuation amplitude and maximum yaw rate and side slip angle of common vehicles are lower than those of aggressive vehicles, indicating that common vehicles have better stability. As demonstrated in Fig. 13(c), the headway distance of common and aggressive vehicles continues to decrease before lane changing, indicating poorer safety. As shown in Fig. 13(d), the distance separating the main vehicle and the rear vehicle in the target lane decreases first and then increases during the lane change. Common and aggressive vehicles obtain greater driving space after successful lane changing and obtain certain lane changing benefits. Finally, as presented in Fig. 13(e) and Fig. 13(f), the acceleration and jerk of common vehicles during lane changing are smaller than those of aggressive vehicles, indicating better vehicle comfort. Conservative vehicles choose to drive at a fixed speed in the starting lane throughout the procedure, ensuring the highest level of safety and stability but without obtaining any lane changing benefits.

Given the analysis above, the conclusion can be drawn that when the lane changing environment becomes worse, conservative vehicles will make the decision not to change lanes due to lower potential benefits from lane changing, prioritizing the safety and stability of their own vehicle's driving. Common and aggressive vehicles continue to choose to change lanes and obtain potential benefits from lane changing, gaining greater driving space and higher speeds. However, their vehicle safety and stability are lower than those of conservative vehicles during lane changing.

TABLE 9. Main simulation parameters of vehicles (Scenario 3).

Vehicle	Longitudinal Position(m)	Lateral Position(m)	Speed(km/h)	Driving style
MV	0	0	60	—
FV	40	0	60	Common
RV	-20	0	60	Aggressive
TRV	-10	3.5	65	Aggressive
TFV	50	3.5	75	Common



(a) Lateral displacement **(b)** Longitudinal speed

FIGURE 14. Position and speed of the main vehicle in scenario 3.

3) SCENARIO 3

In Scenario 3, Table 9 outlines the key specifications for each vehicle.

For this scenario, the lane changing space is further compressed, and the driving styles of vehicles around the main vehicle tend to be aggressive. The main vehicle operates in a relatively harsh lane changing environment, and the lane changing benefits are relatively small, making it unsuitable for lane changing maneuvers.

In this scenario, the distance between the MV and the TFV is short, the velocity of the TFV is low, and the benefit of lane changing is small. However, gap separating between the MV and the TRV is close, and the driving style of the TRV is aggressive, resulting in low safety during lane changing. Scenario 3 describes a driving environment that is unsuitable for lane changes. The results are presented in Fig. 14 and Fig. 15.

As illustrated in Fig. 14(a), only aggressive vehicles choose to change lanes in this scenario, while common and conservative vehicles choose not to perform lane changing behavior. As shown in Fig. 14(b), common and conservative vehicles maintain constant speeds in the original lane, while aggressive vehicles increase their speed compared to the other two styles of vehicles after lane changing.

Since only aggressive vehicles performed lane changing, the analysis here focuses on the situation of aggressive vehicles using the relevant parameter curves in Fig. 15. As demonstrated in Fig. 15(a) and Fig. 15(b), the yaw rate and side slip angle of aggressive vehicles fluctuate significantly, indicating poor vehicle stability. According to Fig. 15(c), although the headway distance of aggressive vehicles increases after lane changing, the minimum headway distance is less than 2m, indicating low security. Fig. 15(d) shows that the gap separating the aggressive vehicle and the rear vehicle continues to increase following lane changing, providing more driving space, but the minimum distance between them is close to 5m, posing a significant safety

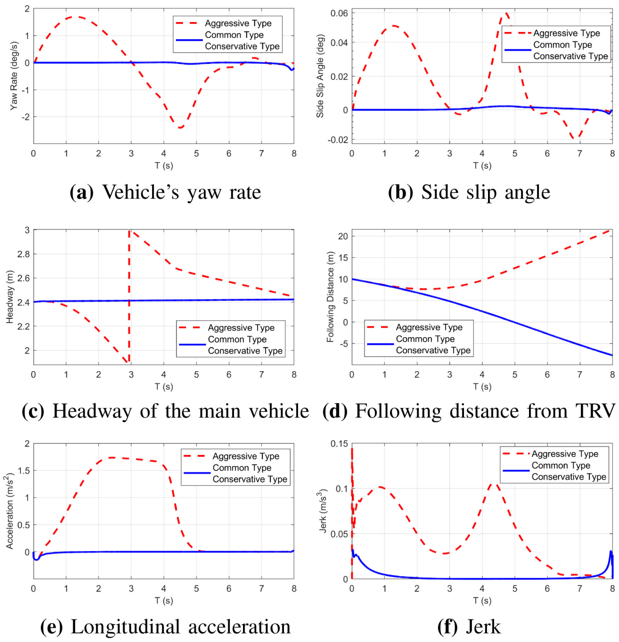


FIGURE 15. Results of the main vehicle in scenario 3.

risk. As illustrated in Fig. 15(e) and Fig. 15(f), the impact of aggressive vehicles during lane changing is relatively high, resulting in poor vehicle comfort. Common and conservative vehicles do not perform lane changing, maintaining a stable state throughout the simulation, and the gap between them and the rear vehicle keeps expanding.

The traffic environment in scenario 3 is not suitable for lane changing, with small potential benefits from lane changing. Therefore, conservative and common vehicles choose not to change lanes and continue driving in the original lane. Aggressive vehicles still choose to change lanes in this environment and increase their vehicle's longitudinal speed after lane changing. The distance between them and the rear vehicle also increases, providing more driving space and obtaining lane changing benefits. However, the security and steadiness of aggressive vehicles while changing lanes are relatively low, posing a risk of accidents.

Through the simulation and examination of the three scenarios discussed above, the analysis provides information on lane changing decisions and the behavioral traits of vehicles during lane changes across different driving styles. The above results reflect the characteristics of vehicles with different driving styles as lanes are being changed, and validate the theoretical model of vehicle lane changing presented in this article.

VI. CONCLUSION

The research presented in this paper addresses the vehicle lane changing decision planning method and trajectory tracking control method according to the basic principles of game theory, formulates a model for vehicle lane changing control, and verifies its performance through simulation. The major conclusions of this study are outlined below:

(1) Driving style is analyzed based on the NGSIM data set. Three different vehicle driving styles are defined based on the parameters:$\omega_1\omega_2\omega_3\omega_4$. Driving style recognition method can be used in the lane changing decision-making and path planning.

(2) In light of the shared traits in vehicle lane changing decision-making and planning, this paper integrates both aspects, and establishes the integrated lane changing decision planning model following the basic tenets of Stackelberg's leader-follower game theory. The proposed method can improve the lane changing accuracy and driving safety.

(3) Weight adaptive change model of vehicle trajectory tracking controller is established using cooperative game theory. According to the simulation results, the control and tracking capabilities of the variable weight controller can be enhanced under different working conditions.

REFERENCES

[1] Y. He, Z. Liu, X. Zhou, and B. Zhong, "Analysis of urban traffic accidents features and correlation with traffic congestion in large-scale construction district," in *Proc. Int. Conf. Smart Grid Electr. Autom. (ICSGEA)*, May 2017, pp. 641–644.

[2] Y. Qu, Z. Lin, H. Li, and X. Zhang, "Feature recognition of urban road traffic accidents based on GA-XGBoost in the context of big data," *IEEE Access*, vol. 7, pp. 170106–170115, 2019.

[3] Q. Jiang and S. Li, "Traffic accident data analysis based on Internet of Things and cloud computing technology," in *Proc. 3rd Asia-Pacific Conf. Image Process., Electron. Comput.*, 2022, pp. 1045–1048.

[4] K. R. Reddy and A. Muralidhar, "Machine learning-based road safety prediction strategies for Internet of Vehicles (IoV) enabled vehicles: A systematic literature review," *IEEE Access*, vol. 11, pp. 112108–112122, 2023.

[5] S. Wang, Y. Zhou, F. Han, and N. Li, "Study on traffic accident characteristics of provincial trunk highway based on data mining," in *Proc. SPIE Int. Soc. Opt. Eng.*, 2022, Art. no. 123400F.

[6] T. T. Wang, "Research on road traffic safety accident data mining based on multi-objective decision-making," *Adv. Transp. Stud.*, vol. 2, pp. 105–112, Mar. 2022.

[7] L. Hu, J. Ou, J. Huang, Y. Chen, and D. Cao, "A review of research on traffic conflicts based on intelligent vehicles," *IEEE Access*, vol. 8, pp. 24471–24483, 2020.

[8] M. Manzoor et al., "RFCNN: Traffic accident severity prediction based on decision level fusion of machine and deep learning model," *IEEE Access*, vol. 9, pp. 128359–128371, 2021.

[9] J. Ugan, M. Abdel-Aty, and Z. Islam, "Using connected vehicle trajectory data to evaluate the effects of speeding," *IEEE Open J. Intell. Transp. Syst.*, vol. 5, pp. 16–28, 2024.

[10] L. Vasile, N. Dinkha, B. Seitz, C. Däsch, and D. Schramm, "Comfort and safety in conditional automated driving in dependence on personal driving behavior," *IEEE Open J. Intell. Transp. Syst.*, vol. 4, pp. 772–784, 2023.

[11] M. Y. Pang, "Trajectory data based clustering and feature analysis of vehicle lane-changing behavior," in *Proc. 4th Int. Conf. Electromech. Control Technol. Transp. (ICECTT)*, Apr. 2019, pp. 229–233.

[12] L. Y. Du, W. Chen, J. Ji, Z. H. Pei, B. M. Tong, and H. J. Zheng, "A novel intelligent approach to lane-change behavior prediction for intelligent and connected vehicles," *Comput. Intell. Neurosci.*, vol. 2022, Jan. 2022, Art. no. 9516218.

[13] Y. Zhang, Y. Chen, X. Gu, N. N. Sze, and J. Huang, "A proactive crash risk prediction framework for lane-changing behavior incorporating individual driving styles," *Accid. Anal. Prevent.*, vol. 188, Aug. 2023, Art. no. 107072.

[14] K. Ji, M. Orsag, and K. Han, "Lane-merging strategy for a self-driving car in dense traffic using the Stackelberg game approach," *Electronics*, vol. 10, no. 8, p. 894, Apr, 2021.

[15] M. Sun, Z. Chen, H. Li, and B. Fu, "Cooperative lane-changing strategy for intelligent vehicles," in *Proc. 40th Chin. Control Conf. (CCC)*, Jul. 2021, pp. 6022–6027.

[16] N. Ding, X. Meng, W. Xia, D. Wu, L. Xu, and B. Chen, "Multivehicle coordinated lane change strategy in the roundabout under Internet of Vehicles based on game theory and cognitive computing," *IEEE Trans. Ind. Informat.*, vol. 16, no. 8, pp. 5435–5443, Aug. 2020.

[17] J. Liu, H. Guo, Q. Meng, W. Shi, Z. Gao, and H. Chen, "Game-theoretic driver-automation cooperative steering control on low-adhesion roads with driver neuromuscular delay," *IEEE Trans. Intell. Transp. Syst.*, vol. 25, no. 8, pp. 10115–10130, Au. 2024.

[18] W. Wang, F. F. Qu, C. Guo, and W. B. Li, "Research on the trajectory tracking of a curved road in an active lane change scenario based on model predictive control algorithm," *Adv. Mech. Eng.*, vol. 15, no. 9, Sep. 2023, Art. no. 16878132231200389.

[19] G. Q. Geng, P. C. Ding, C. Duan, H. B. Jiang, and C. Li, "Research on human-like lane-changing control of intelligent vehicles based on extension goodness evaluation," *Proc. Inst. Mech. Eng. Part D, J. Automob. Eng.*, vol. 237, no. 7, pp. 1543–1554, Jun. 2023.

[20] H. Song, D. Y. Qu, H. B. Guo, K. K. Zhang, and T. Wang, "Lane-changing trajectory tracking and simulation of autonomous vehicles based on model predictive control," *Sustainability*, vol. 14, no. 20, Oct, 2022, Art. no. 13272.

[21] A. Li, W. Han, G. Liu, X. Huang, X. Wang, and Q. Zhang, "Four-wheel-drive vehicle trajectory tracking control at joint planning layer," *Front. Energy Res.*, vol. 10, Aug. 2022, Art. no. 960879.

[22] Z. Wang, J. Guo, Z. Hu, H. Zhang, J. Zhang, and J. Pu, "Lane transformer: A high-efficiency trajectory prediction model," *IEEE Open J. Intell. Transp. Syst.*, vol. 4, pp. 2–13, 2023.

[23] M. Geisslinger, R. Trauth, G. Kaljavesi, and M. Lienkamp, "Maximum acceptable risk as criterion for decision-making in autonomous vehicle trajectory planning," *IEEE Open J. Intell. Transp. Syst.*, vol. 4, pp. 570–579, 2023.

[24] M. Zhao, S. H. Wang, D. Sun, and X. J. Wang, "A car-following model considering preceding vehicle's lane-changing process," *IEEE Access*, vol. 7, pp. 89913–89923, 2019.

[25] Q. Dai, J. Liu, H. Guo, H. Chen, and D. Cao, "Model predictive decision-making considering lane-changing time under emergency collision avoidance for intelligent vehicles," *IEEE Trans. Ind. Electron.*, vol. 71, no. 9, pp. 11250–11261, Sep. 2024.

[26] S. Peicheng, L. Li, X. Ni, and A. Yang, "Intelligent vehicle path tracking control based on improved MPC and hybrid PID," *IEEE Access*, vol. 10, pp. 94133–94144, 2022.

[27] X. Song, Y. Shao, and Z. Qu, "A vehicle trajectory tracking method with a time-varying model based on the model predictive control," *IEEE Access*, vol. 8, pp. 16573–16583, 2020.

[28] S. Mata, A. Zubizarreta, and C. Pinto, "Robust tube-based model predictive control for lateral path tracking," *IEEE Trans. Intell. Veh.*, vol. 4, no. 4, pp. 569–577, Dec. 2019.

[29] E. Siampis, E. Velenis, S. Gariuolo, and S. Longo, "A real-time nonlinear model predictive control strategy for stabilization of an electric vehicle at the limits of handling," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 6, pp. 1982–1994, Nov. 2018.

[30] (Dept. Transp., Federal Highw. Admin., Washington, DC, USA). *NGSIM: Next Generation Simulation*. Accessed: Apr. 24, 2024. [Online]. Available: https://catalog.data.gov/dataset/next-generation-simulation-ngsim-vehicle-trajectories-and-supporting-data

[31] K. Gao et al., "Dual transformer based prediction for lane change intentions and trajectories in mixed traffic environment," *IEEE Trans. Intell. Transp. Syst.*, vol. 24, no. 6, pp. 6203–6216, Jun. 2023.

[32] Y. Liu et al., "Dynamic lane-changing trajectory planning for autonomous vehicles based on discrete global trajectory," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 7, pp. 8513–8527, Jul. 2022.

[33] S. Xin, L. Liang, S. Wei, and X. Shuyuan, "A trajectory planning method based on feedforward compensation and quintic polynomial interpolation," in *Proc. 14th IEEE Int. Conf. Electron. Meas. Instrum. (ICEMI)*, Nov. 2019, pp. 434–441.

[34] Z. Zhang, C. Wei, B. Ma, L. Hu, B. Zhao, and M. Lv, "An optimization-based trajectory planning method with polynomial curves," in *Proc. IEEE Int. Conf. Unmanned Syst. (ICUS)*, Oct. 2023, pp. 1–6.

[35] M. Piccinini, S. Gottschalk, M. Gerdts, and F. Biral, "Computationally efficient minimum-time motion primitives for vehicle trajectory planning," *IEEE Open J. Intell. Transp. Syst.*, vol. 5, pp. 642–655, 2024.

[36] D. Nan, A. Maria, M. Xianghua, Nasibullah, and D. Wu, "Mandatory lane change strategy in VANET based on coordinated Stackelberg game model," in *Proc. Chin. Control Decision Conf. (CCDC)*, Aug. 2020, pp. 1895–1900.

[37] Y. Yan et al., "A multi-vehicle game-theoretic framework for decision making and planning of autonomous vehicles in mixed traffic," *IEEE Trans. Intell. Veh.*, vol. 8, no. 11, pp. 4572–4587, Nov. 2023.

[38] J. Xu, Y. Wu, and Y. Lin, "Path curvature based variable weight multi-constraint MPC for smart vehicle trajectory tracking control," in *Proc. China Autom. Congr. (CAC)*, Nov. 2023, pp. 7554–7559.