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Joint Power Allocation for Transmitter and Relay in a Full-Duplex Relay Covert Communication System

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ABSTRACT This paper considers a full-duplex relay-assisted system for covert communication, in which both the transmitter and the relay are the sender of covert messages. Different from current works that only consider the transmitter power optimization or relay power optimization individually, we propose a joint optimization approach for the transmitter and relay power, which is expected to enhance the system's covert performance. We establish the minimum error detection probability using the Kullback-Leibler (KL) divergence, which serves as the foundation for formulating our joint optimization problem. The objective is to achieve the highest covert transmission rate within the constraints of covertness requirement and total power limitation. Employing a graphical method, we effectively transform inequality constraints into equality constraints, leading to the derivation of an optimal closed-form solution. The simulation results confirm the accuracy of the theoretical derivation and demonstrate that the proposed power allocation method is effective in determining the optimal power for both the transmitter and the full-duplex relay within a two-hop covert communication system.

INDEX TERMS Wireless covert communication, full-duplex, relay, joint optimization.

I. INTRODUCTION

As WIRELESS communication technology rapidly advances, and individuals and institutions increasingly depend on wireless networks to share private information. Because of the inherent public nature, the safety and confidentiality of wireless communications have always been a concern. Available security technologies, such as traditional encryption-based approaches or physical layer security technology [1], [2], [3], focus on protecting information without regard to the presence of the transmission, which can be exploited for subsequent attacks. Therefore, covert communication, which aims to achieve wireless communication security by evading the detection of the transmission itself, has been considered as one of the advanced security technologies.

A. RELATED WORKS

Often referred to as low probability of detection (LPD) [3], covert communication is a method of communication designed to minimize the chances of being detected by a monitor or warden. The fundamental works can be found in [4], where the Square Root Law (SRL) was demonstrated, which posited that a secure transmission of $o(\sqrt{n})$ bits can be reliably achieved over channel use. Furthermore, plenty of research on positive covert rate was further conducted, mainly focusing on three categories [5]. For the first category, the works mainly focused on the theoretical performance limits (e.g., [6], [7], [8], [9], [10], [11], [12], [13]). Specifically, under a more credible premise, the study of [6] implemented the SRL, acknowledging that the warden lacked exact awareness of when the transmission occurs. The study conducted by [7] established the threshold for covert transmission in scenarios where the warden is oblivious to the background noise and the research presented in [8] proved that with the assistance of a cooperative jammer, the transmitter is capable of covertly and precisely conveying $o(n)$ bits in channel use. For the second category, the works mainly focused on the encoding schemes [14], [15], [16], [17]. Specifically, the work of [15] proved that, among multiple transmitter-receiver pairs, the most effective strategy is one that is straightforward to implement in coding: utilizing a point-to-point related approach. In [16], a coding scheme that utilized the systematic Reed-Solomon code to post uncertainty at the warden was proposed. For the third category, covert communication is applied to practical applications, often in conjunction with additional techniques [18], [19], [20], [21], [22]. Specifically, the work of [18] introduced unmanned aerial vehicles (UAVs) into covert communication to disrupt malicious detection by masking the UAV's interference signals as ambient noise. The work of [19] explored leveraging blockchain's decentralization and anonymity to facilitate covert communication. In [22], a multi-user scenario was considered where some users were opportunistically chosen to assist the covert user.

Different from the previous works with one-hop covert communication scenarios, relays are widely deployed to improve the transmission performance for long-distance covert communication in [23], [24], [25], [26], [27], [28]. Specifically, the work of [24] took advantage of the channel's unpredictable nature, affecting both the receiver's and the warden's capabilities to discern transmissions and it was demonstrated that $o(n)$ bits of secure information can be transmitted over n channel use. Subsequently, the work of [25] explored a relay selection scheme that leveraged the optimal relay effectively decreasing the probability of error detection and improving the covert transmission rate. To further improve the covert communication range, in [26], multiple relays were deployed to facilitate multi-hop transmission of the message and the work of [27] implemented not only cooperative jamming but also strategic relay selection.

B. MOTIVATIONS AND CONTRIBUTIONS

In two-hop wireless systems, the necessity for stringent control over both the transmitter's and the relay's transmit power cannot be overstated, as illustrated in [28]. Especially in covert communication contexts, the risk of detection by the overseeing warden in either hop is everpresent. Despite this, a significant gap exists in the existing literature regarding the joint power control of the transmitter and relay—a critical oversight given the profound impact of power allocation on system performance and security.

As far as we know, the studies on power allocation in FD-relay covert communication systems are far from sufficient. For instance, the work of [24] presupposes a dependency between transmitter and relay power without delving into its broader implications. Other studies, such as [25], [29] only controlled the power of the relay within a constraint to achieve the maximum covert rate in different relay selections, which depended on Alice's fixed transmit power. The work of [30] investigated covert communications over amplify-and-forward (AF) relay networks with an extra warden, who can combine two-hop received signals and introduce an optimization problem aimed at maximizing the transmission power to enhance the Average Effective Covert Throughput (AECT). It proposed a joint optimization approach for the transmission power applications of the transmitter and the relay. However, the work focused exclusively on the covertness constraint, neglecting the total power constraint, which may not be conducive to power-limited communication systems. The work of [31] focused on optimizing the power allocation for both the transmitter and the relay, aiming to maximize the covert transmission rate in both the UVA-relay and multi-antenna covert systems. However, this work was limited in its applicability to various scenarios. Additionally, the work did not address the sharing of the total system power between Alice and the relay, which can lead to suboptimal power utilization, especially in energy-constrained environments which suggests that there is room for improvement in power management strategies to ensure more efficient use of energy resources.

Recognizing these shortcomings, our work steps in to fill this void, offering a fresh perspective that prioritizes the efficient use of energy resources, especially in energy-constrained environments. We have undertaken a in-depth examination of total power control for transmitters and relays in two-hop wireless covert systems presenting a novel approach that fully leverages the system's total power without introducing artificial noise. Our primary contributions can be summarized as follows:

- **Practical System Model:** Our research makes a significant contribution by abandoning the idealized model of infinite channel use and instead employing finite blocklength codes. We have conducted an analysis of the covert rate, which effectively addresses the operational constraints present in covert communication. This approach not only provides a more accurate representation of real-world scenarios with limited channel resources but also enhances the applicability and practicality of our findings within such constrained environments.
- **An Effective Covert Transmission Strategy:** To ensure more efficient energy utilization for energy-constrained scenarios, we take a two-hop wireless covert communication system into account and formulate a joint optimization problem for the power allocation of the transmitter and relay, which aims to maximize the covert transmission rate while adhering to the covertness requirement and total power limitation. To decompose the complexity of the joint optimization problem, we adopt a systematic strategy by focusing on simplifying two critical factors: the covert transmission rate and the covertness requirement.
- **A Novel Methodology For Joint Optimization Problem:** We introduce a graphical method to break down the joint optimization problem into simpler cases: concealment-only, power-only, and a hybrid of both, strategically converting inequality constraints into equality constraints where possible, leading to the derivation of optimal closed-form solutions for power allocation.
- **Detailed Analysis Of Numerical Results:** We perform computational experiments to substantiate our

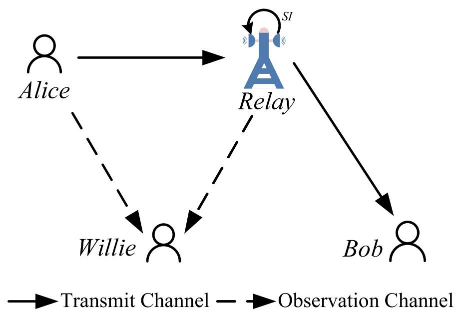


FIGURE 1. Two-hop covert system model, where relay operates in full-duplex mode.

theoretical results. We validate that our proposed algorithm distinguishes itself by adeptly ascertaining the optimal power allocation, with outcomes congruent to those derived from an exhaustive search method, yet with a more straightforward computational approach. Furthermore, our proposed algorithm facilitates an enhanced covert rate, through its dynamic power allocation strategy, thereby eclipsing the efficacy of both the single optimization method and the even power allocation method.

II. SYSTEM MODEL

A. SYSTEM SCENARIO

As shown in Fig. 1, we consider a two-hop wireless covert system consisting of a transmitter (Alice), a relay, a receiver (Bob), and a passive warden (Willie). Alice attempts to share private information covertly with Bob, utilizing the help of a relay. Meanwhile, Willie aims to detect the communication behavior from Alice to the relay or from the relay to Bob by silently observing the environment. Inspired by [28], the relay operates in full-duplex mode, which can simultaneously receive and forward information on the same channel, enhancing communication efficiency and spectrum utilization, but is negatively impacted by self-interference (SI).

We assume all channels to be additive white Gaussian noise (AWGN) channels, and h_{ij} denotes the channel coefficient, with subscripts indicating the nodes involved: ar for the Alice-to-relay channel, rb for the relay-to-Bob channel, aw for the Alice-to-Willie channel, rw for the relay-to-Willie channel, and rr for the relay-to-relay channel. Especially, h_{rr} is the relay's self-interference channel. The magnitude squared of the channel coefficient, denoted as $|h_{ij}|^2$, represents the corresponding channel gain, which can be ascertained by applying the channel estimation method outlined in [32].

Alice communicates private information to Bob in the energy-constrained system where the combined transmit power of Alice and relay is capped at the maximum allowable power P_{total}. Taking into account that channel gain and noise variance can be derived from channel estimation, our aim is to enhance the covert transmission rate with the aid of the relay. Willie carries out a hypothesis test to identify transmissions, rendering a binary verdict contingent on the gathered observations. We denote the NULL hypothesis H_0 as the scenario where Alice is not transmitting, and the alternative hypothesis H_1 as when Alice is transmitting.

B. SIGNAL MODEL

We assume that Alice utilizes the available channel uses, totaling L, to transmit private signals, and relay receives and transmits information with no delay. Thus, the received signal at the relay and Bob when Alice is transmitting can be expressed as

$$y_r[l] = \sqrt{P_a} h_{ar} x_a[l] + \sqrt{P_r} h_{rr} x_r[l] + n_r[l], \quad (1)$$

$$y_b[l] = \sqrt{P_r} h_{rb} x_r[l] + n_b[l], \quad (2)$$

where $i = 1, 2, \ldots, L$ index the channel uses, P_a denotes Alice's transmit power, P_r denotes relay's transmit power, $x_a[l]$ denotes Alice-to-relay signal, $x_r[l]$ denotes relay-to-Bob signal, $n_r[l]$ and $n_b[l]$ are the white Gaussian noise components affecting relay and Bob, and they satisfy $x_a[l] \sim \mathcal{CN}(0, 1)$, $x_r[l] \sim \mathcal{CN}(0, 1)$, $n_r[l] \sim \mathcal{CN}(0, \sigma_r^2)$ and $n_b[l] \sim \mathcal{CN}(0, \sigma_b^2)$, respectively. Hence, the signal-to-interference-plus-noise ratios (SINRs) at relay and Bob, which are denoted as Υ_R and Υ_B, respectively. Then we have

$$\Upsilon_R = \frac{P_a |h_{ar}|^2}{P_r |h_{rr}|^2 + \sigma_r^2} = \frac{P_a \rho_{ar}}{P_r \rho_{rr} + 1}, \quad (3)$$

$$\Upsilon_B = \frac{P_r |h_{rb}|^2}{\sigma_b^2} = P_r \rho_{rb}, \quad (4)$$

where $\rho_{ij} = \frac{|h_{ij}|^2}{\sigma_j^2}$. Hence, the signal-to-interference-plus-noise ratios from Alice to Bob in the two-hop covert system where the relay operates in FD mode is given by [33]

$$\Upsilon_{FD} = \frac{\Upsilon_R \Upsilon_B}{\Upsilon_R + \Upsilon_B + 1} = \frac{P_a \rho_{ar} P_r \rho_{rb}}{P_a \rho_{ar} + (P_r \rho_{rb} + 1)(P_r \rho_{rr} + 1)}. \quad (5)$$

Due to the finite number of channel uses, the covert rate of the system [34], [35], [36] is identified as

$$C_{FD} = log_2(1 + \Upsilon_{FD}) - \sqrt{\frac{V}{L}} Q^{-1}(\delta), \quad (6)$$

where $log_2(1 + \Upsilon_{FD})$ is the ideal Shannon capacity part of the channel without error, and $-\sqrt{\frac{V}{L}} Q^{-1}(\delta)$ represents the rate loss due to finite blocklength L and the packet error rate δ, where $V = \frac{\Upsilon_{FD}(\Upsilon_{FD}+2)}{(\Upsilon_{FD}+1)^2} \frac{1}{(ln 2)^2}$ is the channel dispersion and $Q^{-1}(\boldsymbol{.})$ is the inverse Q function.

C. COVERTNESS REQUIREMENT

Willie endeavors to discern between the two hypotheses H_0 and H_1, based on the signals he has received. When H_0 is true, both Alice and relay remain silent. Thus, Willie receives no signals but noise. When H_1 is true, Alice and relay are transmitting signal with power P_a and P_r. Thus, Willie can observe the combination signal from Alice and relay. The observation denoted as $y_w[l]$ is given by

$$y_w[l] = \begin{cases} n_w[l], & H_0, \\ \sqrt{P_a} h_{aw} x_a[l] + \sqrt{P_r} h_{rw} x_r[l] + n_w[l], & H_1, \end{cases} \tag{7}$$

where $n_w[l]$ is the white Gaussian noise at Willie and satisfy $n_w[l] \sim \mathcal{CN}(0, \sigma_w^2)$.

The transmission events occur with an equal probability of 0.5. Subsequently, contingent on Willie's selection between H_0 and H_1, the total detection error probability for Willie is

$$\mathbb{P}_{we} = \frac{1}{2}\mathbb{P}_{FA} + \frac{1}{2}\mathbb{P}_{MD}, \tag{8}$$

where the false alarm probability, denoted as $\mathbb{P}_{FA}$, signifies that Willie endorsing H_1 while H_0 actually prevails, and the missed detection probability, denoted as $\mathbb{P}_{MD}$, signifies that Willie endorsing H_0 endorsing H_1 is true. Based on [4], the total probability of detection error for Willie is

$$\mathbb{P}_{we} = \frac{1}{2}\left(1 - V\left(\mathbb{P}_0^L, \mathbb{P}_1^L\right)\right), \tag{9}$$

where $V(\mathbb{P}_0^L, \mathbb{P}_1^L)$ signifies the total variation distance, which measures the difference between $\mathbb{P}_0^L$ and $\mathbb{P}_1^L$. Following the Pinsker's inequality as referenced [4], [25], the relationship is given by the following equation

$$V\left(\mathbb{P}_0^L, \mathbb{P}_1^L\right) \leq \sqrt{\frac{1}{2} D\left(\mathbb{P}_0^L || \mathbb{P}_1^L\right)}, \tag{10}$$

where $D(\mathbb{P}_0^L || \mathbb{P}_1^L)$ denotes the relative entropy, which quantifies the difference between the two probability distributions $\mathbb{P}_0^L$ and $\mathbb{P}_1^L$. Utilizing the chain rule for relative entropy, as indicated in the references [4], [25], the relative entropy can be decomposed accordingly:

$$D\left(\mathbb{P}_0^L || \mathbb{P}_1^L\right) = LD(P_0 || P_1). \tag{11}$$

Thus, the overall probability of detection error for Willie is confined by

$$\mathbb{P}_{we} \geq \frac{1}{2} - \frac{1}{2}\sqrt{\frac{L}{2} D(P_0 || P_1)}. \tag{12}$$

Covert communication ensures that detection performance of Willie on Alice's transmission activities is statistically indistinguishable from mere chance, even under the most favorable conditions for detection. This means that Willie's ability to accurately identify when Alice is transmitting is no better than making a random guess, thereby maintaining the secrecy of the communication channel. Due to the transmissions of Alice occurring with an equal probability, covert communication is guaranteed by

$$\mathbb{P}_{we} \geq \frac{1}{2}(1 - \varepsilon), \tag{13}$$

where ε denotes a minor allowance, referred to as the tolerance value, which is established to define the acceptable performance metrics for the covert communication system.

Combination with (3), a lower bound on the covertness is given by

$$D(P_0 || P_1) \leq \frac{2\varepsilon^2}{L}, \tag{14}$$

where

$$D(P_0 || P_1) = \int_{-\infty}^{+\infty} f_0(x) ln \frac{f_0(x)}{f_1(x)} dx \tag{15}$$
$$\overset{(a)}{=} \frac{1}{2} ln(P_a \rho_{aw} + P_r \rho_{rw} + 1) - \frac{1}{2} \frac{P_a \rho_{aw} + P_r \rho_{rw}}{P_a \rho_{aw} + P_r \rho_{rw} + 1},$$

refers to the relative entropy between two probability distributions, $\mathbb{P}_0$ and $\mathbb{P}_1$, which is also known as the Kullback-Leibler divergence, and notation (a) refers to the specific case where $f_0(x) = \mathcal{CN}(0, \sigma_w^2)$ and $f_1(x) = \mathcal{CN}(0, P_a|h_{aw}|^2 + P_r|h_{rw}|^2 + \sigma_w^2)$.

Hence, the covertness requirement can be given by

$$ln(P_a \rho_{aw} + P_r \rho_{rw} + 1) - \frac{P_a \rho_{aw} + P_r \rho_{rw}}{P_a \rho_{aw} + P_r \rho_{rw} + 1} \leq \frac{4\varepsilon^2}{L}. \tag{16}$$

Given that $x \geq 0$, the inequality $ln(1+x) \leq x$ holds, hence a stricter covertness requirement can be

$$P_a \rho_{aw} + P_r \rho_{rw} - \frac{P_a \rho_{aw} + P_r \rho_{rw}}{P_a \rho_{aw} + P_r \rho_{rw} + 1} \leq \frac{4\varepsilon^2}{L}. \tag{17}$$

III. OPTIMAL POWER ALLOCATION FOR ALICE AND RELAY

In this section, we analyze the covert rate and the covert requirement within a two-hop FD relay-assisted communication system, based on which the optimization problem is formulated with the goal of maximizing the covert rate. This objective is realized through a simultaneous optimization of the transmit power P_a for Alice and P_r for relay while adhering to the system's covertness demands and the imposed total power constraint. Furthermore, by examining the interplay between the constraints, we manage to simplify the optimization problem to a point where we can derive a closed-form solution for the optimal power distribution profile.

A. PROBLEM FORMULATION

In this section, the focus is on constructing an optimization problem aimed at increasing the covert transmission rate. This is achieved by concurrently adjusting the transmission power of Alice, denoted as P_a, and the relay's transmission power, denoted as P_r, within the bounds set by the system's need for covertness and the total allowable power.

As elaborated in part II, the optimization problem is structured in the following manner:

$$\begin{aligned} (P1):\ & \max_{P_a, P_r} C_{FD}, \\ s.t.\ & C1: P_a \rho_{aw} + P_r \rho_{rw} - \frac{P_a \rho_{aw} + P_r \rho_{rw}}{P_a \rho_{aw} + P_r \rho_{rw} + 1} \leq \frac{4\varepsilon^2}{L}, \\ & C2: P_a + P_r \leq P_{total}, \\ & C3: P_a > 0, \\ & C4: P_r > 0, \end{aligned} \tag{18}$$

where C1 represents the system's need for covertness as defined by Equations (17), and C2, C3 and C4 correspond to the power limitations within a two-hop energy-constrained system. The complexity of C_{FD} in Equation (6), along with C1, renders the problem particularly challenging to resolve.

Lemma 1: For a certain packet error rate δ at Bob and a certain maximum channel uses L, the covert rate is monotonically increasing with the SINRs from Alice to Bob in the two-hop covert system where the relay operates in FD mode.

Proof: For the detailed proof, please refer to Appendix A. ■

Lemma 2: The covertness requirement in Equation (17) is equivalent to $P_a\rho_{aw} + P_r\rho_{rw} \leq \xi^*$, where $\xi^* = \frac{2\varepsilon^2}{L} + 2\sqrt{\frac{\varepsilon^2}{L}(\frac{\varepsilon^2}{L}+1)}$.

Proof: For the detailed proof, please refer to Appendix B. ■

Drawing on the conclusions of Lemma 1 and Lemma 2, the optimization problem $P1$ can be streamlined as follows:

$$\begin{aligned}(P2):\ &\max_{P_a,P_r} \frac{P_a\rho_{ar}P_r\rho_{rb}}{P_a\rho_{ar} + (P_r\rho_{rb}+1)(P_r\rho_{rr}+1)}, \quad (19)\\ s.t.\ \ &C1: P_a\rho_{aw} + P_r\rho_{rw} \leq \xi^*,\\ &C2: P_a + P_r \leq P_{total},\\ &C3: P_a > 0,\\ &C4: P_r > 0,\end{aligned}$$

where $\xi^* = \frac{2\varepsilon^2}{L} + 2\sqrt{\frac{\varepsilon^2}{L}(\frac{\varepsilon^2}{L}+1)}$. It's clear that the relationship between variable P_a and P_r is interdependent, complicating the solution to optimization problem $P2$. Despite the complexity, the constraints are all linear, which simplifies the analysis, as detailed in Lemma 3.

Lemma 3: For the optimization problem $P2$, it is established that an optimal power allocation exists for both Alice and the relay that maximizes the two-hop system's covert transmission rate assisted by an FD relay.

Proof: According to the optimization problem $P2$, we note that in addition to environmental factors, the objective function is determined by the transmit power P_a for Alice and P_r for relay. When P_r is fixed, the objective function progressively rises with an increment in P_a, indicating that the optimal power for Alice is specified by $P_a^* = min\{P_{total} - P_r, \frac{\xi^* - P_r\rho_{rw}}{\rho_{aw}}\}$. Therefore, the objective function can be regarded as a function of one variable P_r with an open interval $(0, min\{P_{total}, \frac{\xi^*}{\rho_{rw}}\})$. At the endpoints of the interval, the values of the objective function are both 0. By virtue of Rolle's Theorem, it is guaranteed that an optimal P_r^* exists within the open interval, allowing the objective function to reach its peak. This implies the existence of an optimal power allocation for both Alice and the relay, which serves to maximize the covert transmission rate, as stated in Lemma 3. ■

B. OPTIMAL POWER PROFILE FOR ALICE AND RELAY

In this section, we use graphical methods to classify feasible domains into three types of situations according to the relationship between the covertness requirement and the total power limitation. Each scenario is then individually addressed in the subsequent sections to resolve the optimization problem labeled as $P2$.

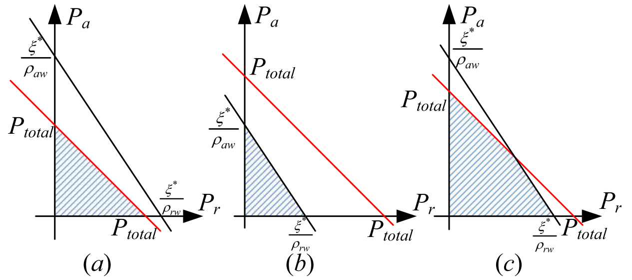


FIGURE 2. Three situations of the feasible domains for $P2$.

1) Maximizing covert rate with $P_{total} \leq min\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$

For the case $P_{total} \leq min\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$, the transmit power of Alice and relay is strictly limited by the total power constraint, as shown in Fig. 2(a). Therefore, the optimization problem $P2$ can be further rewritten as

$$\begin{aligned}(P3):\ &\max_{P_a,P_r} \frac{P_a\rho_{ar}P_r\rho_{rb}}{P_a\rho_{ar} + (P_r\rho_{rb}+1)(P_r\rho_{rr}+1)}, \quad (20)\\ s.t.\ \ &C2: P_a + P_r \leq P_{total},\\ &C3: P_a > 0,\\ &C4: P_r > 0,\end{aligned}$$

where the optimal transmit power $P_a^{*(1)}$ for Alice and $P_r^{*(1)}$ for relay can be derived by the subsequent theorem.

Theorem 1: For the case $P_{total} \leq min\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$, to achieve the highest covert transmission rate, the optimal power for relay is given by

$$P_r^{*(1)} = \frac{\sqrt{\mu\beta\tau} - \mu}{\mu\rho_{rb} + \rho_{rr} - \rho_{ar}},$$

where $\mu = P_{total}\rho_{ar} + 1$, $\beta = P_{total}\rho_{rb} + 1$, $\tau = P_{total}\rho_{rr} + 1$. Consequently, the corresponding optimal power for Alice to maximize the covert rate is given by $P_a^{*(1)} = P_{total} - P_r^{*(1)}$.

Proof: For the detailed proof, please refer to Appendix C. ■

2) Maximizing covert rate with $P_{total} \geq max\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$

For the case $P_{total} \geq max\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$, the transmit power of Alice and relay is strictly limited by the covertness requirement, as shown in Fig. 2(b). Therefore, the optimization problem $P2$ can be further rewritten as

$$\begin{aligned}(P4):\ &\max_{P_a,P_r} \frac{P_a\rho_{ar}P_r\rho_{rb}}{P_a\rho_{ar} + (P_r\rho_{rb}+1)(P_r\rho_{rr}+1)}, \quad (21)\\ s.t.\ \ &C1: P_a\rho_{aw} + P_r\rho_{rw} \leq \xi^*,\\ &C3: P_a > 0,\\ &C4: P_r > 0,\end{aligned}$$

where $\xi^* = \frac{2\varepsilon^2}{L} + 2\sqrt{\frac{\varepsilon^2}{L}(\frac{\varepsilon^2}{L}+1)}$, and the optimal transmit power of Alice $P_a^{*(1)}$ and the optimal transmit power of relay $P_r^{*(1)}$ can be obtained in the following theorem.

Theorem 2: For the case $P_{total} \geq max\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$, the optimal power for relay $P_r^{*(2)}$ in $P4$ can be derived as

$$P_r^{*(2)} = \frac{\sqrt{\gamma_{aw}\lambda\eta\theta} - \rho_{rw}\lambda}{\rho_{aw}\rho_{rr}\theta + \rho_{rw}(\rho_{rb}\rho_{aw} - \rho_{rw}\rho_{ar})},$$

where $\lambda = \xi^*\rho_{ar} + \rho_{aw}$, $\eta = \xi^*\rho_{rr} + \rho_{rw}$, $\theta = \xi^*\rho_{rb} + \rho_{rw}$. Consequently, the corresponding optimal power for Alice to maximize the covert rate is given by $P_a^{*(2)} = \frac{\xi^* - \rho_{rw}P_r^{*(2)}}{\rho_{aw}}$.

Proof: For the detailed proof, please refer to Appendix D. ■

3) Maximizing covert rate with $min\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\} < P_{total} < max\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$

In this case, we need to consider both the total power constraint and the covertness constraint, as shown in Fig. 2(c). It should be highlighted that the intersection of the constraints is within the feasible domain in this case, where the transmit power of Alice is $P_{ap} = \frac{\xi^* - \rho_{rw}P_{total}}{\rho_{aw} - \rho_{rw}}$ and the transmit power of relay is $P_{rp} = \frac{\rho_{aw}P_{total} - \xi^*}{\rho_{aw} - \rho_{rw}}$. Furthermore, the optimal power $P_a^{*(3)}$ for Alice and $P_r^{*(3)}$ for relay in this case can be derived from the subsequent theorem.

Theorem 3: For the case $min\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\} < P_{total} < max\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$, $P_a^{*(3)}$ and $P_r^{*(3)}$ in $P1$ can be obtained by

$$\left(P_a^{*(3)}, P_r^{*(3)}\right) = \underset{\{(P_a,P_r)\}}{argmax} \frac{P_a\rho_{ar}P_r\rho_{rb}}{P_a\rho_{ar} + (P_r\rho_{rb} + 1)(P_r\rho_{rr} + 1)},$$

where $\{(P_a, P_r)\}$ represents the set of $(P_a^{*(1)}, P_r^{*(1)})$, $(P_a^{*(2)}, P_r^{*(2)})$ and (P_{ap}, P_{rp}) that are in the feasible region. Specially, $(P_a^{*(1)}, P_r^{*(1)})$ is the optimal solution for $P1$ where the total power constraint is stricter, while $(P_a^{*(2)}, P_r^{*(2)})$ is the optimal solution for $P2$ where the covertness constraint is stricter.

Proof: For the case $min\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\} < P_{total} < max\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$, both the total power constraint and the covertness limitation must be taken into account. We observe that the maximum point may occur at $(P_a^{*(1)}, P_r^{*(1)})$, $(P_a^{*(2)}, P_r^{*(2)})$ or (P_{ap}, P_{rp}). However, $(P_a^{*(1)}, P_r^{*(1)})$ and $(P_a^{*(2)}, P_r^{*(2)})$ may not always in the feasible region for this case as shown in Fig. 2(c). Therefore, we must first judge whether $(P_a^{*(1)}, P_r^{*(1)})$ or $(P_a^{*(2)}, P_r^{*(2)})$ is within the feasible region. And then, find the one that maximum the objective function between the set of $(P_a^{*(1)}, P_r^{*(1)})$, $(P_a^{*(2)}, P_r^{*(2)})$ and (P_{ap}, P_{rp}) that are in the feasible region, as given in Theorem 3. ■

Algorithm 1 outlines the method for power allocation to Alice and Relay. According to the description of the algorithm, we can deduce that: if the total power Ptotal is relatively small, it can be calculated using Theorem 1; if Ptotal is relatively large, then Theorem 2 should be used for the calculation. In the worst-case scenario that lies between the two, a single substitution search is required to find the solution that maximizes the value of Υ_{FD}, which is the optimal power allocation strategy. From Algorithm 1, we can see that the method proposed in this paper is an expression for power allocation calculated directly by a closed-form solution, with a time complexity of constant time.

Algorithm 1 Algorithm to Compute P_r^* and P_a^*

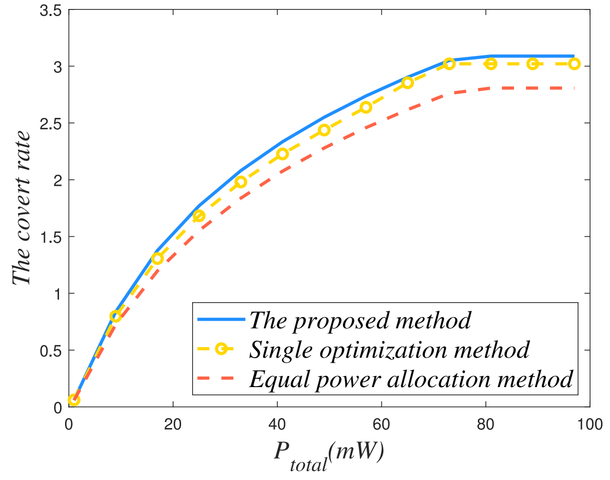
```
1: Input ε, L, P_total, ρ_ar, ρ_rb, ρ_rr, ρ_aw, ρ_rw;
2: Compute ξ* in Lemma 2;
3: Compute P_r^{*(1)} and P_a^{*(1)} in Theorem 1;
4: Compute P_r^{*(2)} and P_a^{*(2)} in Theorem 2;
5: Compute P_rp and P_ap in Theorem 3;
6: if P_total ≤ min{ξ*/ρ_aw, ξ*/ρ_rw} then
7:    P_r^* = P_r^{*(1)}, P_a^* = P_a^{*(1)};
8: else
9:    if P_total ≥ max{ξ*/ρ_aw, ξ*/ρ_rw} then
10:      P_r^* = P_r^{*(2)}, P_a^* = P_a^{*(2)};
11:   else
12:      find the one among (P_r^{*(1)}, P_a^{*(1)}), (P_r^{*(2)}, P_a^{*(2)}),
         and (P_rp, P_ap) that maximizes the calculated value
         of ϒ_FD, i.e., that one is the optimal solution (P_r^*,
         P_a^*).
13:   end if
14: end if
```

IV. NUMERICAL RESULTS

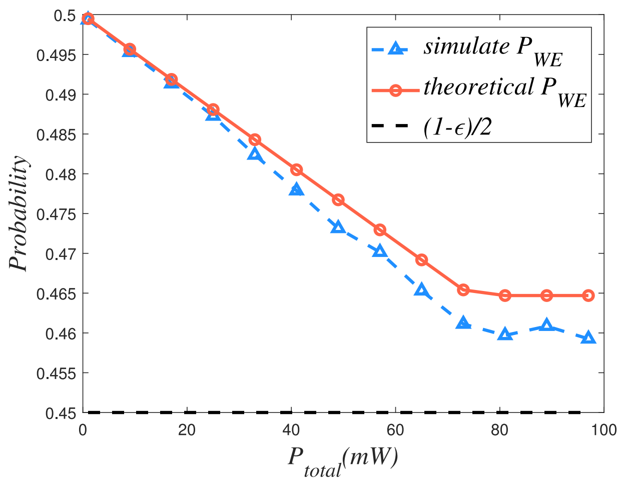
In this section, we present numerical findings that substantiate the theoretical analysis discussed in this article. The locations assigned to Alice, Bob, relay and Willie are at coordinates $(0, 0)$, $(0, 150m)$, $(-20m, 50m)$, $(600m, 0)$, respectively. The paper describes the large-scale fading of the communication channels using the formula $h_{ij} = h_0(d_{ij}/d_0)^{-\alpha_{ij}}$, where d_{ij} is the distance between i and j, d_0 is the reference distance set at 1 meter and $h_0 = -40dB$ is the channel gain at the reference distance, and α_{ij} are the path loss exponents with $\alpha_{ar} = 2.3$, $\alpha_{rb} = 2.5$,$\alpha_{aw} = \alpha_{rw} = 3.5$. The noise power spectral density is uniform across Bob, relay and Willie at $-90dBm$. A finite number of channels of use has been used and without special specification, the finite blocklength L is set to 500. The proposed power allocation method is compared with the exhaustive search technique and the even distribution method. The results presented are derived from a comprehensive set of 10^5 channel realizations, indicating a robust analysis of the system's performance under various channel conditions.

Fig. 3 plots the optimal covert rate C_{FD}^* and Willie's minimum detection error probability versus the total power P_{total} with the optimal transmit powers, with P_a^* for Alice, and P_r^* for relay, where tolerance level $\varepsilon = 0.1$ and the finite blocklength $L = 500$. In Sub-Figure a, our proposed algorithm demonstrates a consistently higher optimal covert rate C_{FD}^* compared to both the single optimization method and the equal power allocation method. The single optimization method's implementation is detailed in the following formula:

$$\begin{cases} P_a = 0.4 * min\left\{\frac{\xi^*}{\rho_{aw}}, P_{total}\right\} \\ P_r = min\left\{\left(\frac{\xi^*}{\rho_{aw}} - P_a\right) * \frac{\rho_{aw}}{\rho_{rw}}, P_{total} - P_a\right\}, \end{cases} \quad (22)$$



(a) The maximum covert rate versus P_{total}



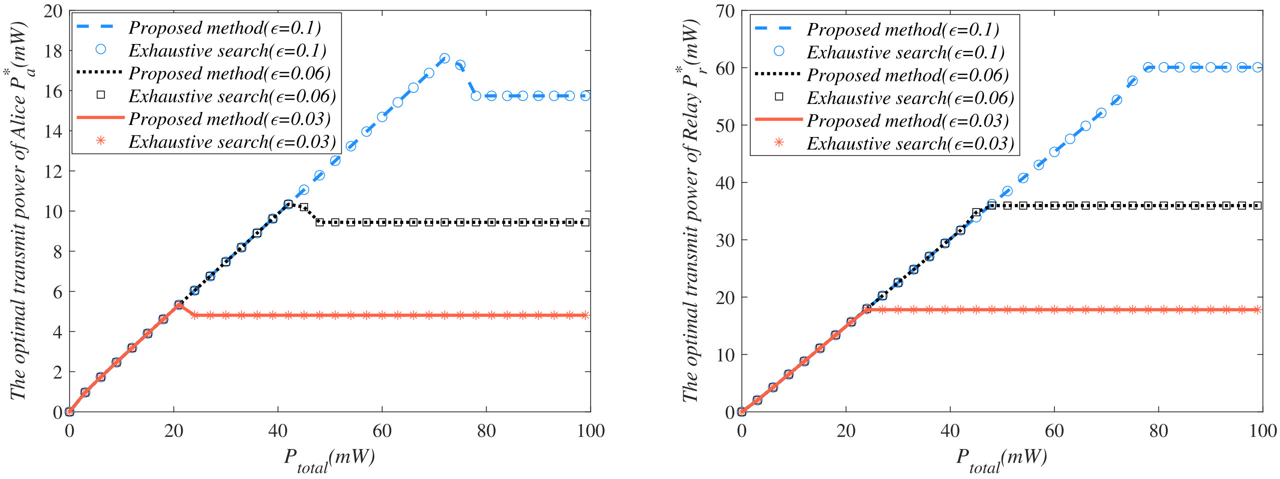
(b) The minimum detection error probability at Willie versus P_{total}

FIGURE 3. **The maximum covert rate and the minimum detection error probability at Willie versus the maximum total transmit power constraint P_{total}.**

where the transmit power of Alice is fixed at a 40% constraint while the power of the Relay is obtained by solving a single optimization problem similar to P1. The equal power allocation method simplifies to $P_a = P_r$ for optimization problem P1. This improvement in covert rate in Sub-Figure a is a direct result of the efficient power utilization strategy embedded in our algorithm, which dynamically allocates power between Alice and the relay based on the channel conditions and the total power constraint. The superior performance demonstrated by our algorithm highlights its optimality for the two-hop covert system in question. The Sub-Figure b plots Willie's minimum detection error probability demonstrating that the simulated values are between the theoretical values and the threshold of $\frac{1}{2}(1-\varepsilon)$. This threshold signifies the minimum detection error probability that must be achieved to ensure the communication is covert. In addition, we also observe that the simulated values of Willie's minimum detection error probability are smaller than the theoretical values. It is due to the fact that the theoretical minimum detection error probability is the upper bound based on Equation (17).

Fig. 4 illustrates a comparison between the power allocation algorithm we proposed for the full-duplex (FD) relay-assisted two-hop covert communication system and the exhaustive search method. We can observe that the data obtained from both methods are identical, validating the accuracy of our algorithm and confirming its capability to efficiently determine the optimal power allocation P_a^* and P_r^* within the constraints of limited total power. This consistency highlights the efficiency and reliability of our algorithm, proving that it offers a highly effective solution for optimal power allocation in covert communication scenarios without the need for the computationally expensive process of exhaustive searching. Fig. 4 also demonstrates how the constraint on total power, denoted as P_{total}, influences the optimal transmit powers, with P_a^* for Alice, and P_r^* for relay across various tolerance level ε, where the finite blocklength $L = 500$. Upon initial observation, it is noted that the optimal power P_a^* and P_r^* first increase and then keep unchanged as P_{total} increases. This is due to the fact that when P_{total} is small, P_a^* and P_r^* is restricted by total power constraint which can be obtained based on Theorem 1 thus P_a^* and P_r^* increase with P_{total}, and when P_{total} is large, $P_a^*$and P_r^* is restricted only by covertness constraint which can be obtained based on Theorem 2 thus P_a^* and P_r^* keep unchanged for a certain tolerance level ε. We can also observe that both P_a^* and P_r^* have transition periods which can be obtained based on Theorem 3. Intuitively, the larger tolerance level ε, leads to the larger P_a^* and P_r^*. This is due to that the larger tolerance level ε results in a later transition to the covertness constraint.

Fig. 5 plots the maximum covert rate C_{FD}^*, optimal transmit power for Alice P_a^*, and the relay P_r^* plotted against the tolerance level ε for various block lengths L. This visualization is intended to demonstrate how ε affects C_{FD}^*. In Sub-Figure a, it is initially observed that C_{FD}^* augments when ε is small. This is due to the fact that when ε is small, $P_a^*$and P_r^* is restricted by the covertness constraint which can be obtained based on Theorem 2 thus P_a^* and P_r^* increase with ε thus a larger C_{FD}^*. We can also observe that C_{FD}^* remains unchanged when ε is sufficiently large. This is due to the fact that when ε the is sufficient large, $P_a^*$and $P_r^*$is restricted only by the total power P_{total} and is independent of ε which can be obtained based on Theorem 1 thus an unchanged C_{FD}^*. In addition, in Sub-Figure b and c, we can observe that both P_a^* and P_r^* have transition periods, which further demonstrate that the covertness constraint and total power constraint can transition smoothly based on Theorem 3. Furthermore, we can also observe that the system achieves the same C_{FD}^*,P_a^* and P_r^* for different blocklength L when ε is sufficient large. This is due to the fact that when the ε is sufficient large, $P_a^*$and P_r^* are restricted only by the total power P_{total} and are independent of both ε and L which can be obtained based on Theorem 2.



(a) The optimal transmit power of Alice versus P_{total} (b) The optimal transmit power of relay versus P_{total}

FIGURE 4. The optimal transmit power of Alice and relay versus the maximum total transmit power constraint P_{total}.

In Fig. 6, we present a clearer investigation of how the relay's location affects the covert rate in a scenario where Alice, Bob, and Willie are situated at $(0, 0)$, $(150\text{m}, 0)$, $(0, 200\text{m})$, in which the total power constraint P_{total} is 0.1W, the covert tolerance level is 0.1 and the finite blocklength L is 500. In Sub-Figures a and b, we illustrate the variation in covert rate corresponding to different relay positions where the optimal transmit powers for Alice and the relay can be derived from Theorem 2. Firstly, we can observe that figure a is unimodal, indicating an optimal position for the deployment of the relay, which allows the entire two-hop covert system to achieve a higher covert rate. Moreover, we can observe that when the relay is deployed in the yellow area shown in Figure b, the system can attain a better covert rate which suggests that by using the method in this paper, we can improve our covert performance against potential warden by controlling the deployment position of the relay. Sub-Figure c intuitively compares the covert rates of two-hop and one-hop covert systems. The bright yellow areas in the graph highlight the advantage of the two-hop system in terms of covert rate. Upon observing these areas, it is evident that they are primarily distributed between Alice and Bob and in regions far from Willie, indicating that the covert rate of the two-hop system is significantly higher in these areas. This comparison reveals an important phenomenon: the strategic deployment of a full-duplex relay can significantly enhance the system's covert performance in this scenario. The reason behind this is that the relay's assistance effectively reduces the path loss during signal transmission. Although the optimal transmit power must be adjusted to meet the requirements for covertness, the two-hop system can still achieve a higher covert rate.

V. CONCLUSION

In this paper, we revealed an interaction between the Alice's power and relay's power by jointly considering the power constraints and the covertness requirements within a two-hop communication system that utilizes a full-duplex relay. In pursuit of an optimal transmission strategy, we have determined the optimal transmit power levels for both Alice and relay, which serve to maximize the covert transmission rate. This analysis reveals that employing the entire pool of available power and distributing it evenly is not necessarily the best approach.

APPENDIX A
PROOFS OF LEMMA 1

Taking derivative of C_{FD} in (6) with respect to Υ_{FD}, we have

$$\frac{\partial C_{FD}}{\partial \Upsilon_{FD}} = \frac{1}{(1+\Upsilon_{FD})ln2} \bullet \left(1 - \frac{Q^{-1}(\delta)}{\sqrt{L}}\frac{1}{\Delta}\right), \tag{23}$$

where $\Delta = (1+\Upsilon_{FD})\sqrt{\Upsilon_{FD}(\Upsilon_{FD}+2)}$.

There is only one solution that satisfies $\frac{\partial C_{FD}}{\partial \Upsilon_{FD}} = 0$ and is positive. We denote the solution as Υ_{FD}^*, which satisfies

$$(1+\Upsilon_{FD}^*)\sqrt{\Upsilon_{FD}^*(\Upsilon_{FD}^*+2)} = \frac{Q^{-1}(\delta)}{\sqrt{L}}, \tag{24}$$

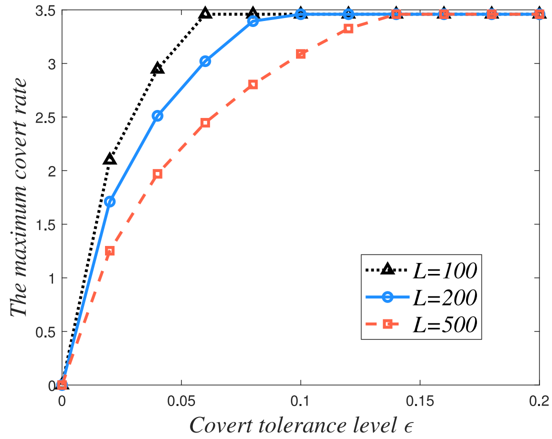
We observe that the value of Υ_{FD}^* is very small when the packet error rate δ and channel uses L take the proper value. In this regard, the small Υ_{FD}^* can be neglected, and thus $\frac{\partial C_{FD}}{\partial \Upsilon_{FD}} \geq 0$ is derived, i.e., the covert rate is monotonically increasing with the SINRs from Alice to Bob in the two-hop covert system where the relay operates in FD mode, as given in Lemma 1.

APPENDIX B
PROOFS OF LEMMA 2

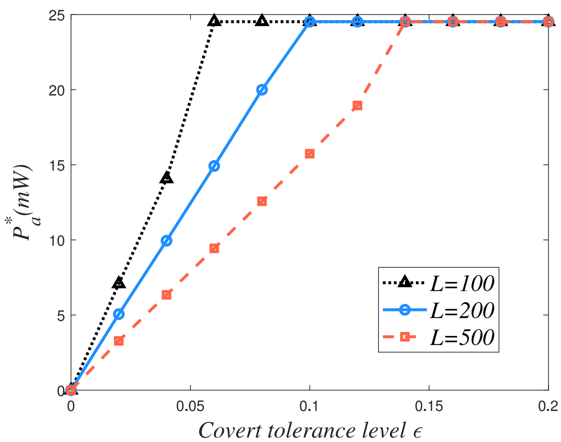
The covertness requirement in Equation (17) can be rewritten as

$$f(\xi) = \xi - \frac{\xi}{\xi+1} \leq \frac{4\varepsilon^2}{L}, \tag{25}$$

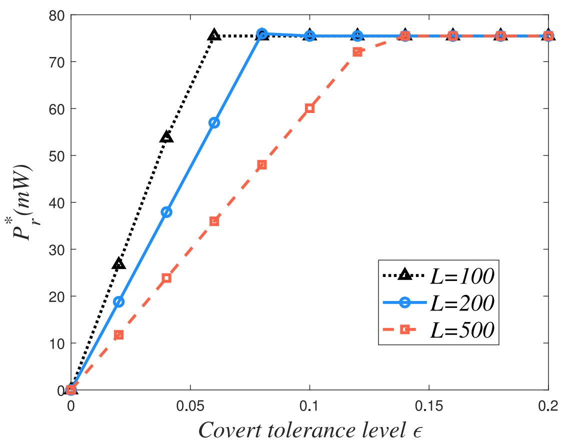
where $\xi = P_a\rho_{aw} + P_r\rho_{rw}$.



(a) The covert rate versus ε

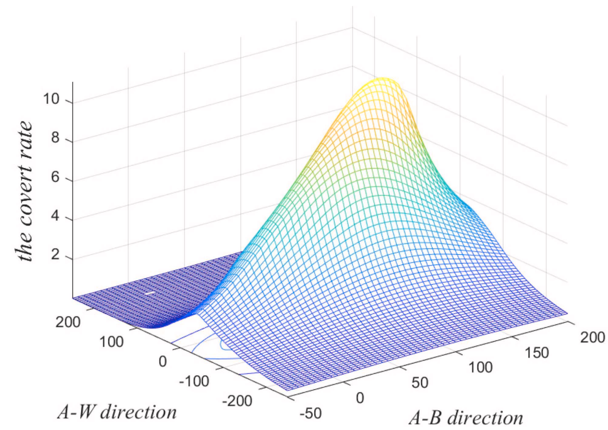


(b) The optimal transmit power of Alice versus ε

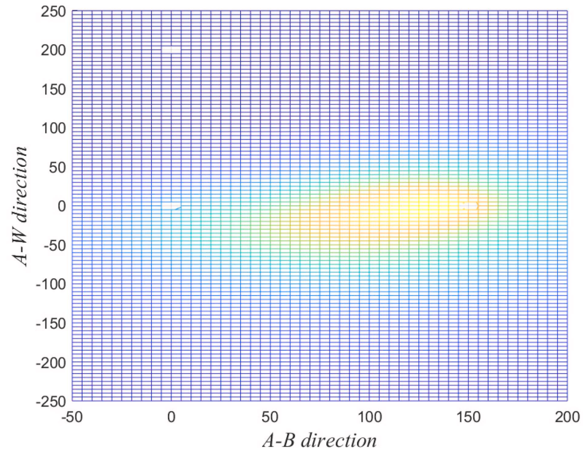


(c) The optimal transmit power relay versus ε

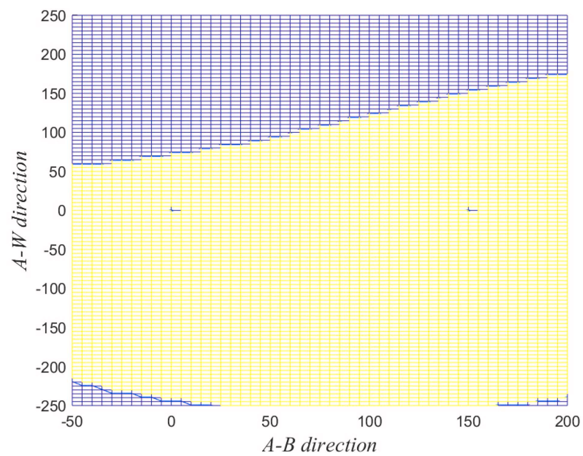
FIGURE 5. The optimal transmit power of Alice and relay versus the tolerance level ε.



(a) The maximum covert rate versus the position of relay



(b) Top view of Sub-Figure a



(c) Comparison between relay and direct transmission

FIGURE 6. The effect of the relay position in a specific scenario.

Taking derivative of $f(\xi)$ with respect to $\xi \geq 0$, we obtain

$$\frac{\mathrm{d}f(\xi)}{\mathrm{d}\xi} = 1 - \frac{1}{(\xi+1)^2} \geq 0, \tag{26}$$

which means that $f(\xi)$ is monotonically increasing as ξ increases, i.e., to obtain the maximum $f(\xi)$ of $\frac{4\varepsilon^2}{L}$ in Equation (25) is equivalent to obtain the maximum ξ of ξ^*.

For the case with $f(\xi^*) = \xi^* - \frac{\xi^*}{\xi^*+1} = \frac{4\varepsilon^2}{L}$, we obtain

$$\xi^* = \frac{2\varepsilon^2}{L} + 2\sqrt{\frac{\varepsilon^2}{L}\left(\frac{\varepsilon^2}{L}+1\right)}. \tag{27}$$

Hence, the covertness requirement in Equation (17) is equivalent to $\xi \le \xi^*$, i.e.,

$$P_a\rho_{aw} + P_r\rho_{rw} \le \xi^*, \tag{28}$$

where $\xi^* = \frac{2\varepsilon^2}{L} + 2\sqrt{\frac{\varepsilon^2}{L}(\frac{\varepsilon^2}{L}+1)}$, as given in Lemma2.

APPENDIX C
PROOFS OF THEOREM 1

Observing the optimization problem $P2$, the objective function is monotonically increasing as P_a increases when P_r is fixed, which implies that the ideal transmit power of Alice and relay must satisfy

$$P_a^{*(1)} + P_r^{*(1)} = P_{total}. \tag{29}$$

Thus, the inequality constraint problem $P3$ can be converted to an equality constraint problem, denoted by $P5$

$$\begin{aligned}(P5):\ &\max_{P_a,P_r} \frac{P_a\rho_{ar}P_r\rho_{rb}}{P_a\rho_{ar} + (P_r\rho_{rb}+1)(P_r\rho_{rr}+1)}, \\ s.t.\ &P_a + P_r = P_{total}, \\ &P_a > 0, \\ &P_r > 0.\end{aligned} \tag{30}$$

We note that P_a and P_r are deeply bundled by $P_a + P_r = P_{total}$, consequently, the optimization problem can be streamlined to

$$\begin{aligned}(P6):\ &\max_{P_r} \frac{(P_{total}-P_r)\rho_{ar}\rho_{rb}P_r}{(P_{total}-P_r)\rho_{ar} + (P_r\rho_{rb}+1)(P_r\rho_{rr}+1)}, \\ s.t.\ &0 < P_r < P_{total}.\end{aligned} \tag{31}$$

Take the derivative of the objective function, we can find two extreme points $\frac{\sqrt{\mu\beta\tau}-\mu}{\mu\rho_{rb}+\rho_{rr}-\rho_{ar}}$ and $\frac{-\sqrt{\mu\beta\tau}-\mu}{\mu\rho_{rb}+\rho_{rr}-\rho_{ar}}$, where $\mu = P_{total}\rho_{ar}+1$, $\beta = P_{total}\rho_{rb}+1$, $\tau = P_{total}\rho_{rr}+1$ and $\frac{\sqrt{\mu\beta\tau}-\mu}{\mu\rho_{rb}+\rho_{rr}-\rho_{ar}}$ is the only one that satisfies $0 < P_r < P_{total}$. So $\frac{\sqrt{\mu\beta\tau}-\mu}{\mu\rho_{rb}+\rho_{rr}-\rho_{ar}}$ is the solution to the optimization problem $P5$, which means that when $P_{total} \le min\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$, the optimal transmit power of relay to maximize the covert rate can be given by

$$P_r^{*(1)} = \frac{\sqrt{\mu\beta\tau}-\mu}{\mu\rho_{rb}+\rho_{rr}-\rho_{ar}}, \tag{32}$$

where $\mu = P_{total}\rho_{ar}+1$, $\beta = P_{total}\rho_{rb}+1$, $\tau = P_{total}\rho_{rr}+1$.

Finally, by substituting the optimal transmit power of relay $P_r^{*(1)}$ into (29), we can obtain

$$P_a^{*(1)} = P_{total} - P_r^{*(1)} = P_{total} - \frac{\sqrt{\mu\beta\tau}-\mu}{\mu\rho_{rb}+\rho_{rr}-\rho_{ar}}, \tag{33}$$

as given in Theorem 1.

APPENDIX D
PROOFS OF THEOREM 2

Observing the optimization problem $P4$, the objective function is monotonically increasing as P_a increases when P_r is fixed, which implies that the ideal transmit power of Alice and relay must satisfy

$$P_a^{*(2)}\rho_{aw} + P_r^{*(2)}\rho_{rw} = \xi^*, \tag{34}$$

where $\xi^* = \frac{2\varepsilon^2}{L} + 2\sqrt{\frac{\varepsilon^2}{L}(\frac{\varepsilon^2}{L}+1)}$.

Thus, the inequality constraint problem $P4$ can be transformed into an equality constraint problem, denoted by $P7$

$$\begin{aligned}(P7):\ &\max_{P_a,P_r} \frac{P_a\rho_{ar}P_r\rho_{rb}}{P_a\rho_{ar} + (P_r\rho_{rb}+1)(P_r\rho_{rr}+1)}, \\ s.t.\ &P_a\rho_{aw} + P_r\rho_{rw} = \xi^*, \\ &P_a > 0, \\ &P_r > 0.\end{aligned} \tag{35}$$

We note that P_a and P_r are deeply bundled by $P_a\rho_{aw} + P_r\rho_{rw} = \xi^*$, consequently, the optimization problem can be streamlined to

$$\begin{aligned}(P8):\ &\max_{P_r} \frac{(\xi^*-P_r\rho_{rw})\rho_{ar}\rho_{rb}P_r}{(\xi^*-P_r\rho_{rw})\rho_{ar} + \rho_{aw}(P_r\rho_{rb}+1)(P_r\rho_{rr}+1)}, \\ s.t.\ &0 < P_r < \frac{\xi^*}{\rho_{rw}}.\end{aligned} \tag{36}$$

Taking the derivative of the objective function, we can find two extreme points $\frac{\sqrt{\gamma_{aw}\lambda\eta\theta}-\rho_{rw}\lambda}{\rho_{aw}\rho_{rr}\theta+\rho_{rw}(\rho_{rb}\rho_{aw}-\rho_{rw}\rho_{ar})}$ and $\frac{-\sqrt{\rho_{aw}\lambda\eta\theta}-\rho_{rw}\lambda}{\rho_{aw}\rho_{rr}\theta+\rho_{rw}(\rho_{rb}\rho_{aw}-\rho_{rw}\rho_{ar})}$, where $\lambda = \xi^*\rho_{ar}+\rho_{aw}$, $\eta = \xi^*\rho_{rr}+\rho_{rw}$, $\theta = \xi^*\rho_{rb}+\rho_{rw}$ and $\frac{\sqrt{\rho_{aw}\lambda\eta\theta}-\rho_{rw}\lambda}{\rho_{aw}\rho_{rr}\theta+\rho_{rw}(\rho_{rb}\rho_{aw}-\rho_{rw}\rho_{ar})}$ is the only one that satisfies $0 < P_r < \frac{\xi^*}{\rho_{rw}}$. So $\frac{\sqrt{\rho_{aw}\lambda\eta\theta}-\rho_{rw}\lambda}{\rho_{aw}\rho_{rr}\theta+\rho_{rw}(\rho_{rb}\rho_{aw}-\rho_{rw}\rho_{ar})}$ is the solution to the optimization problem $P7$, which means that when $P_{total} \ge max\{\frac{\xi^*}{\rho_{aw}}, \frac{\xi^*}{\rho_{rw}}\}$, we have

$$P_r^{*(2)} = \frac{\sqrt{\rho_{aw}\lambda\eta\theta}-\rho_{rw}\lambda}{\rho_{aw}\rho_{rr}\theta+\rho_{rw}(\rho_{rb}\rho_{aw}-\rho_{rw}\rho_{ar})}, \tag{37}$$

where $\lambda = \xi^*\rho_{ar}+\rho_{aw}$, $\eta = \xi^*\rho_{rr}+\rho_{rw}$, $\theta = \xi^*\rho_{rb}+\rho_{rw}$.

Finally, by substituting (37) into (34), we can obtain $P_a^{*(2)}$ as given in Theorem 2.

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