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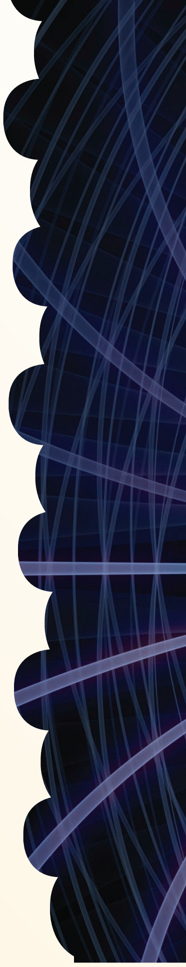
Computational Electromagnetics and the IEEE Antennas and Propagation Society



Seventy-five years of shared history, part 1.

Computational electromagnetics (CEM) is heavily intertwined with the IEEE Antennas and Propagation Society (AP-S). Effective designs for antennas and electromagnetic systems have motivated accurate simulation tools that have continuously exhausted available computer resources. This two-part article traces the development of computational tools and techniques and ties them to milestones in computer hardware, the growth and development of the AP-S, and some historical events throughout the world.

INTRODUCTION

The development of CEM over the past 75 years parallels the creation and growth of the AP-S (1949) and advancements in computers. With these two articles, we trace the development of the CEM discipline by following the development of computer technology. We include the eventual transfer of CEM technology from applied academic research to commercial field solvers.

CEM has deep roots in applied mathematics and science. The theoretical foundations of classical variational calculus were laid in the 18th century by Leonhard Euler and Giuseppe Luigi Lagrange [1], [2], among others, with scientific contributions continuing throughout the 19th and 20th centuries [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16]. These developments were contemporary to those of James Clerk Maxwell, Heinrich Hertz, and Oliver Heaviside, the founding fathers of modern electromagnetic theory, and to those of Alexander S. Popov, Jagadish C. Bose, Guglielmo Marconi, Nikola Tesla, Roberto L. De Moura, and Jozef Murga, who are considered the pioneers of radio communications [17].

Science and engineering advanced rapidly during World War II and the Cold War years, when Western countries experienced very strong demographic growth, industrial development, and extensive international trade. During the 1950s and 1960s, the bloc of Western countries, led by the United States, and the Eastern bloc, led by the Soviet Union, competed over the development of science and culture rather than large-scale wars. In particular, space exploration programs (the "Space Race") had broad support from public opinion in both blocs. Important advances in computer science, aerospace and materials engineering, and telecommunications originated from these programs and other lesser-known military programs, such as the development of stealth technology.

The rapid evolution of computers beginning in the early 1960s was enabled by huge funding allocated for the development of these innovative R&D programs, which in turn needed more robust methods to enable the engineering design of aircraft, spacecraft, and many supporting technologies. Growing computer power enabled the development of numerical codes for structural analysis based on the finite-element method (FEM) during the 1960s. Similar developments in CEM were motivated by the very rapid progress of electronics,

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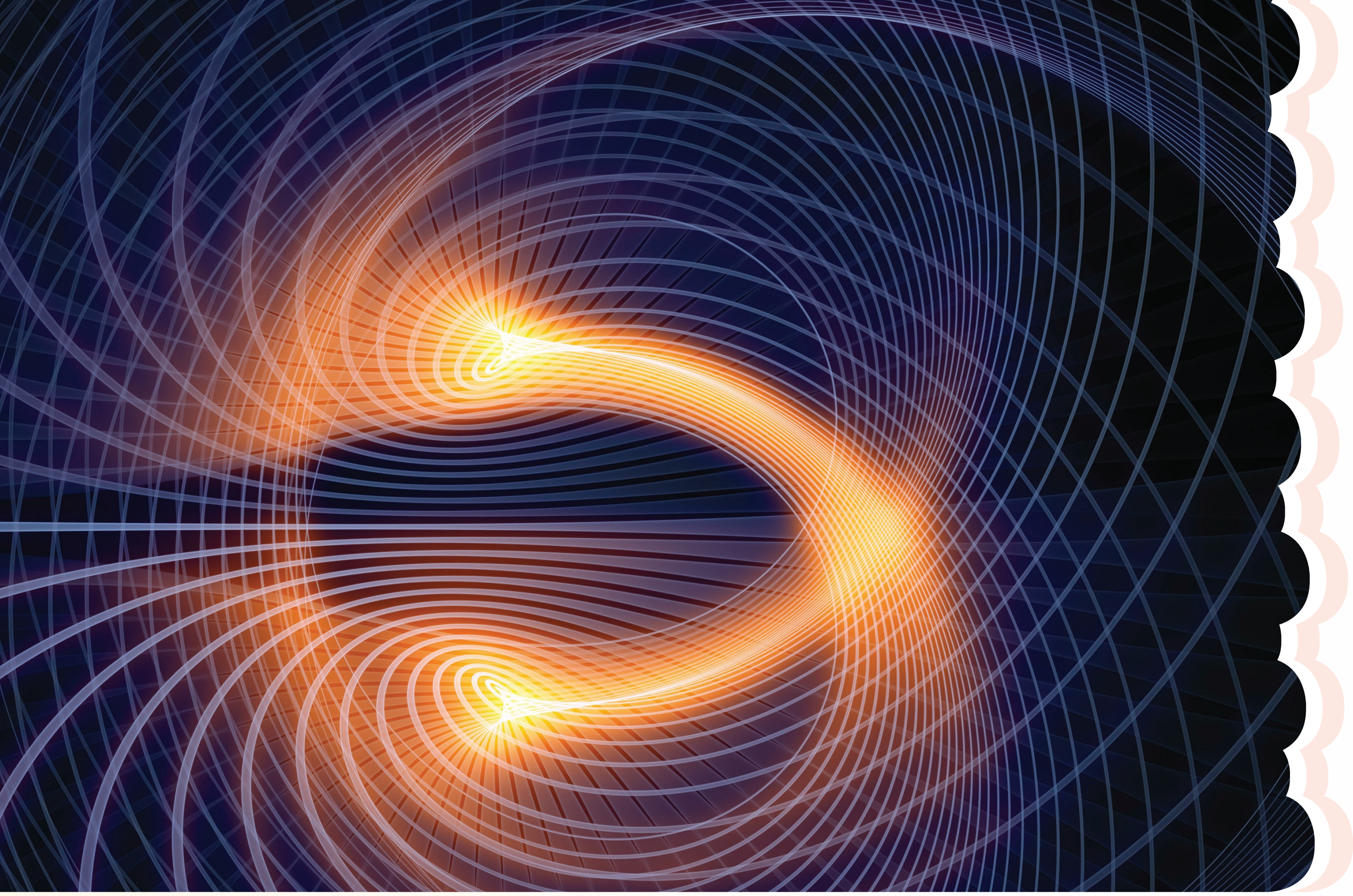

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telecommunications, and information technology, and the growing need for antennas and microwave devices [18]. The evolution of computers and numerical analysis since the 1960s is synergistic with the development of CEM:

- Computers have become increasingly powerful, faster, cheaper, widely available, and finally, portable.
- The availability of high-level programming languages (e.g., ALGOL, Ada, BASIC, C, Fortran, Pascal, and so on) has made computers easily programmable.
- Robust software libraries, commercially available or free, have enabled engineers to more easily carry out advanced computations.
- The development of telecommunications networks and protocols has made it possible for computers to communicate, transfer data, and execute software programs in parallel.
- Computer graphics and GPUs have made exceptional advances, driven by the development of the video game market and, more recently, the need to efficiently train artificial intelligence (AI) algorithms.
- The formation of interuniversity research groups or entrepreneurial consortia supported by public bodies has enabled the development of advanced analysis codes.
- The creation of software tools for symbolic calculation and algebraic manipulation has simplified design work, much as the evolution of AI systems now enable program debugging and generation.

The tables that follow divide some of the achievements of CEM into different "eras" related to the evolution of computers, which we consider a driving factor in the history of CEM. These tables are the authors' attempts to list in chronological order the significant milestones that we observed while working in North America and Western Europe, and are by no means a complete historical record. The tables include milestones in computers, in CEM, and within the AP-S, and other significant historical events that are capable of identifying the era.

We have loosely associated the eras of evolution with

- mainframes
- minicomputers and microprocessors
- personal computers and workstations
- IT revolution
- widespread use of commercial electromagnetic field solvers
- GPU computing.

In this regard, it is rather interesting to note in Figure 1 how the trend in the number of pages published each year in *IEEE Transactions on Antennas and Propagation* (*TAP*) grew regularly until around 2002 and then exponentially in the fifth era, up to our days. Since 2000, there has been widespread availability of powerful computers and commercial solvers at moderate costs, as well as easy access to powerful new databases updated in real time, such as the IEEE *Xplore* digital library, which opened in May 2000. (The distribution of revenue from IEEE *Xplore* allowed the AP-S to increase its journal page budget annually since 2003, which, in an example of "positive feedback,"

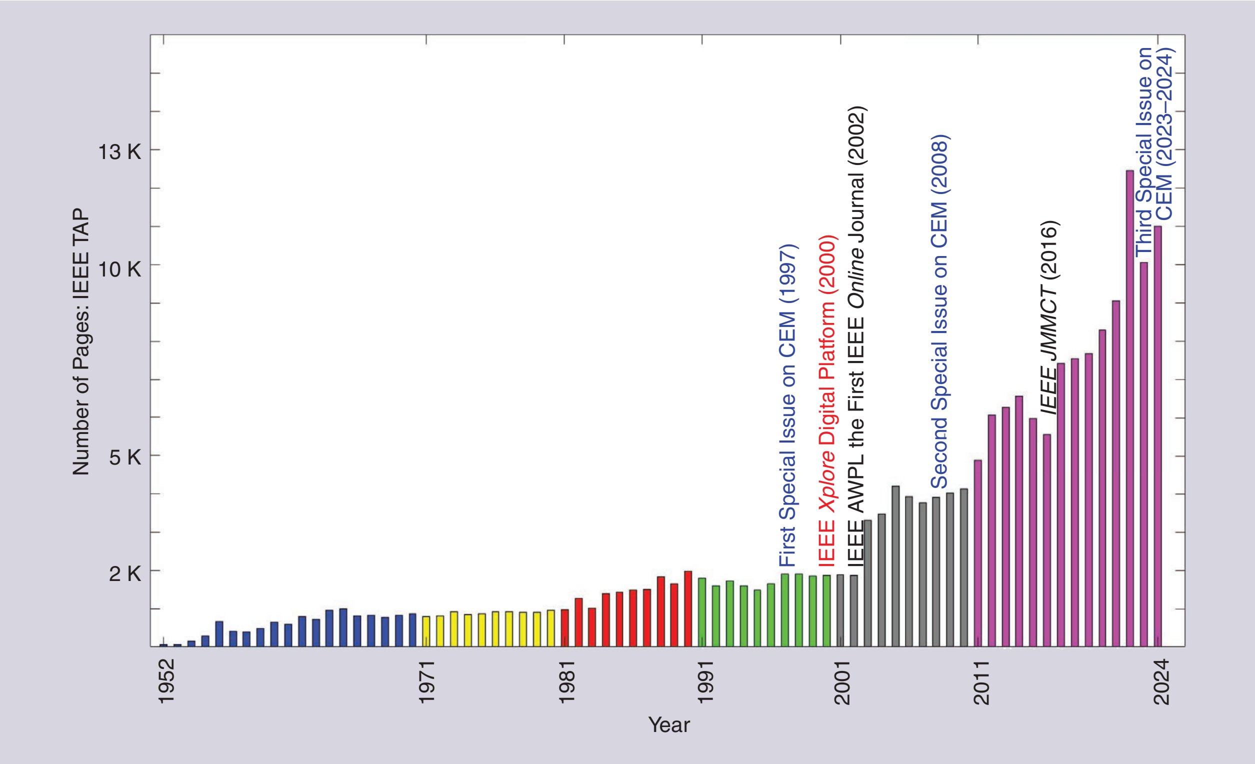

FIGURE 1. The number of pages per year published by *Transactions of the IRE Professional Group on Antennas and Propagation* (1952–1962) and its successor, *IEEE Transactions on Antennas and Propagation* (*TAP*) (1963–present). The journal grew from 63 pages in 1952 to a stable level of ~1,900 pages per year by 2000. Beginning in 2003, the journal exhibited rapid growth and currently exceeds 10,000 pages per year, despite the Society's creation of new journals (e.g., *IEEE Antennas and Wireless Propagation Letters* in 2002 and *IEEE Journal on Multiscale and Multiphysics Computational Techniques* in 2016).

increased the revenue stream from IEEE *Xplore*.) As shown in Figure 1, *TAP* has published three special issues on CEM. The first (1997) was edited by Graglia et al. [19], the second (2008) was edited by Michielssen and Jin [20], and the third (2023) was edited in two parts by Notaros et al. [21].

THE ERA OF MAINFRAMES (1949–1970)

Table 1 spans the years associated with second-generation mainframe computers; that is, those made with transistors instead of vacuum tubes. We highlight the IBM 7000 and IBM System/360 series (see Figure 2) but note that many other manufacturers offered mainframe computers during this time. (These manufacturers were sometimes known as *IBM and the Seven Dwarfs*, where the latter referred to Burroughs, Control Data, General Electric, Honeywell, NCR, RCA, and UNIVAC.) It was towards the end of this era that the Olivetti P101, the first programmable desktop calculator was produced on a large scale and marketed (see Figure 3). The appearance of the first software libraries can be traced back to this first phase, such as the International Mathematics and Statistics Library (IMSL) of 1970, and the Numerical Algorithm Group (NAG) Library of 1971.

It is noteworthy that aerospace engineering and, more generally, all applied mathematics, underwent a substantial revolution after the NASA STRuctural ANalysis code (NASTRAN) became available [46]. This code is primarily a finite-element problem solver that was created for NASA through a cooperative project initially funded (in the 1960s) with U.S. government aerospace funds. Commercial versions of NASTRAN have been available since 1971, while the source code was released to the public for a fee with the "NASA Classics" package in 2001. The popularity of NASTRAN spurred the emergence of many powerful pre- and postprocessing programs (e.g., PATRAN [47], marketed since 1971) because all of NASTRAN's inputs and outputs are in the form of text files, and the code had no functionality to graphically create a model or a mesh. The commercial pre- and postprocessing codes used for NASTRAN can also be used by many other finite-element codes as well as by many electromagnetic codes involving the solution of integral or differential equations. Typically, it took hundreds of man-years of work to create each of these codes, and in fact, many of them now have extraordinary rendering capabilities, although it often takes great experience to fully exploit them.

The appearance in the early 1960s of fast computing systems and robust computer libraries spread quickly to a wide variety of engineering disciplines, e.g., mechanics, thermodynamics, hydrodynamics, and electromagnetism. Few problems of interest could be solved by traditional analytical techniques (see [48] for a collection of problems that could be solved adequately), and there was tremendous interest in systematic numerical solutions, such as those offered by the FEM, for which the first text had appeared in 1967 [49]. The first known use of computers to solve integral equations

TABLE 1. THE ERA OF MAINFRAMES (1949–1970).

1949	—	Founding of the AP-S, formerly known as the *IRE Professional Group on Antennas and Propagation* (1949–1963); IEEE Professional Technical Group on Antennas and Propagation (1963–1965); Antennas and Propagation Group (1965–1975); and then, the AP-S since 1975.
1949	—	The People's Republic of China, with Beijing as its capital, was founded on 1 October 1949.
1949	—	Maue's seminal article [22] on integral equations (IEs) for conducting bodies was published.
1950s	—	The early numerical solutions of IEs [23], [24], [25].
1951	UNIVAC I	UNIVersal Automatic Computer I: the first computer used for data processing and automatic storage of letters and numbers. Forty-six units were sold. It was made of 5,200 vacuum tubes, cost ~US$1.5 million, weighed 13 tons, consumed 125 kW, and operated at a speed of 2.25 MHz.
1952	—	*Transactions of the IRE Professional Group on Antennas and Propagation*, now named *IEEE TAP*, was founded.
1956	FORTRAN	The first FORTRAN manual was released. FORTRAN was the first widely used computer language to use a compiler.
1957	*Sputnik 1*	The first artificial satellite to orbit the Earth was built by the Soviet Union and launched on 4 October 1957 from the Baikonur Cosmodrome. It burned up in the Earth's atmosphere on 4 January 1958.
1958	—	NASA was established on 1 October 1958.
1960s	IBM 7000	The IBM 7000 series mainframes reached the market.
1960	Ruby laser	Theodore H. Maiman created the first working ruby laser at Hughes Research Laboratories on 16 May 1960.
1961	Man in space	12 April: Vostok 1 mission with Jurij A. Gagarin.
1963–1965	—	The first IE method of moments (MoM) articles were written by Mei and Van Bladel [26], Andreasen [27], Andreasen [28], Richmond [29], and Richmond [30].
1965	IBM System 360	The IBM System 360 series mainframes entered the market.
1965	Olivetti P101	The first programmable desktop calculator produced on a large scale, was made in Italy by Olivetti. Approximately 44,000 units were sold, 90% of which were on the North American market. NASA used it to carry out the orbital calculations that brought the Apollo 11 mission to the Moon in 1969.
1966	—	Yee's seminal article [31] on finite difference time domain (FDTD), which is used to solve Maxwell's equations in the time domain, was published.
1966–1968	—	Additional early IE/MoM articles were written by Oshiro and Cross [32], Richmond [33], Oshiro et al. [34], Harrington [35], Tanner and Andreasen [36], Oshiro and Mitzner [37], Harrington [38], and Wallenberg and Harrington [39]. Harrington's *Field Computation by Moment Methods* [38] was published.
1968	NASTRAN	NASTRAN, a finite-element structural analysis program written primarily in FORTRAN, was released to NASA.
1967–1970	—	The first time-domain IE/MoM articles were written by Mitzner [40], Sayre and Harrington [41], and Bennett and Weeks [42], [43].
1969	—	Silvester's seminal article [44] on the FEM applied to the Helmholtz equation for homogeneous waveguides.
1969	Man on the Moon	… and live on television. Neil Armstrong, Buzz Aldrin, and Michael Collins piloted the Apollo 11 mission on 21 July 1969.
1969	ARPAnet	The first wide-area packet-switched network with distributed control, and one of the first computer networks to implement the TCP/IP protocol suite (see year 1982 in Table 2).
1970	—	The International Mathematics and Statistics Library.
1971	—	The Numerical Algorithm Group Library.

for electromagnetics dates back to ~1960 and involve the Fourier series solution of Hallen's integral equation for the study of wire antennas [24] and the use of the scalar potential for the calculation of the capacitance of a cube [25]. In the early 1960s, a handful of research groups began to solve integral equations numerically for problems involving structures of perfect electrically conductive materials; these were restricted mostly to 1D and 2D problems, such as scattering from cylindrical structures, bodies with revolution (BoR) symmetry, bodies with impedance surfaces, and wire-grid problems [26], [27], [28], [29], [30]. These efforts expanded in the mid-1960s with many more publications of this type reported, of which we cite only a few [32], [33], [34], [35], [36], [37], [38], [39]. We note that the article that introduced the name *method of moments* (*MoM*) appeared in 1967 [35] and was soon followed by the first article on the numerical solution of integral equations to appear in *IEEE Spectrum* [36]. Roger F. Harrington's monograph [38], which included the solution of differential and integral equations, was published in 1968. Aperture problems were also investigated using integral equations [39]. Figure 4 shows a photo of Roger Harrington and other illustrious names in the CEM community of those years.

Beginning in 1966, a series of reports was produced at the Northrop Corporation [32], [34], [37], [40] (A picture of K. Mitzner is shown in Figure 4). Because of their restricted access, few were (and are) aware of these works. These reports proposed the use of E-field, H-field, and combined field-integral

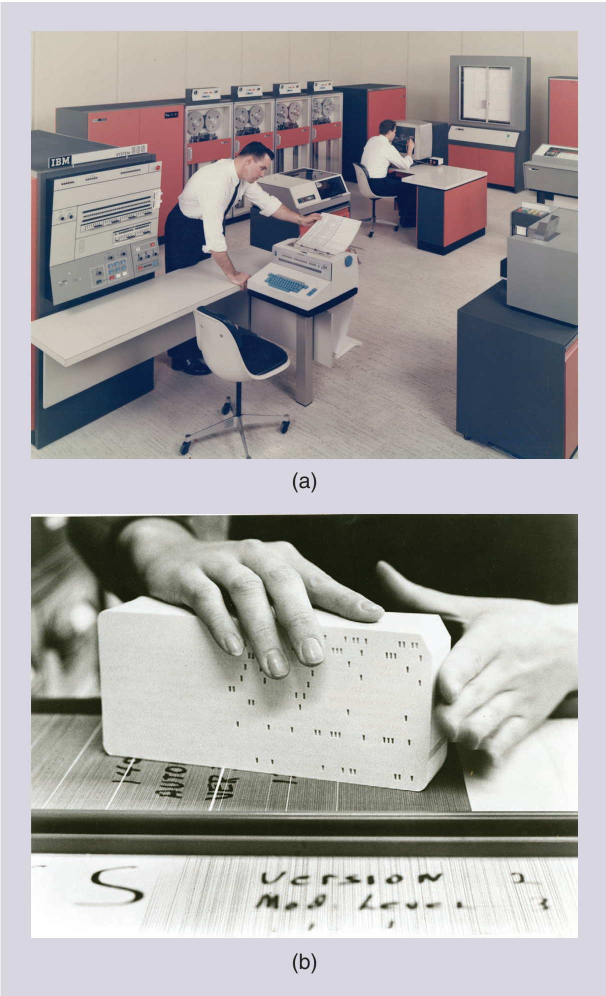

FIGURE 2. The mainframe era. (a) IBM System 360 Model 44 was a specialized system introduced in 1966, with drives to read data tapes and terminals to enter and record information. In that era, programs and data were written on punched cards and fed into computers using optical readers. (b) A set of computer punch cards from the 1970s, before the availability of terminals. Punched cards and paper tape served as information storage devices prior to hard drives, floppy disks, and solid-state memory. (Reprint courtesy of IBM Corporation©.)

FIGURE 3. A NASA scientist works with the Olivetti P101 (1969). The P101 was used as a calculation tool for the Apollo 11 mission in preparation for the various phases of the journey, from the compilation of lunar maps to the choice of moon landing location and the trajectory of the journey. (Courtesy of Associazione Archivio Storico Olivetti, Ivrea—Italy. Used with permission.)

equations to address 2D and 3D scattering problems, and the use of impedance boundary conditions as well as numerical codes that were capable of exploiting the symmetries of the structure to reduce the number of unknowns. They also mention Calderon identities, which play an important role in recent research on preconditioned iterative methods.

Growing computer power enabled the development of numerical codes for structural analysis based on the finite-element method (FEM) during the 1960s.

Although the previous endeavors dealt primarily with frequency-domain situations, some numerical treatments of integral equations in the time domain were investigated in the late 1960s, with application to wires and 2D scattering problems [41], [42], [43].

Formulations based on differential equations were also being developed for treating electromagnetics problems, but they were not widely adopted for many years. The finite-difference time-domain (FDTD) method for solving first-order Maxwell equations was proposed by Kane S. Yee in 1966 [31] (Figure 4). This technique was used throughout the 1970s and 1980s, as mentioned later, but did not explode in popularity until the 1990s. The first use of the FEM for high-frequency electromagnetics problems was its application in 1968 by Peter P. Silvester to solve the scalar Helmholtz equation for homogeneous waveguide problems [44]. Many extensions that attempted to treat inhomogeneous waveguides throughout the 1970s and early 1980s encountered difficulties with spurious modes, and were not fully successful until the introduction of vector-finite elements. Another factor that hindered the use of differential equations was the lack of techniques for truncating the computational domain in open-region problems.

To better understand this historical period, a 1967 article by Harrington was previously rejected by *IEEE Transactions on Antennas and Propagation* for reasons including that "it is not possible to represent continuous physical quantities, such as current, using discontinuous quantities," and "it is not possible for a computer to invert even a 100 × 100 matrix because the magnetic tape will wear out going back and forth." Harrington is sometimes considered the "father of MoM," not only for having first named the method but, above all, because he was instrumental in consolidating its foundations, understanding its generality and power, and developing and applying appropriate techniques to a wide variety of problems [45].

In a general sense, the MoM and the FEM are equivalent procedures. The emphasis placed in the 1960s, and later on scattering and antenna problems, for which integral equations were natural because they did not require a means of truncating the domain as did differential equations, led to the label "MoM" being associated almost exclusively with integral equations. The FEM was developed almost independently within the electromagnetics community, driven initially by the modeling of microwave circuits and guided-wave devices (closed-region problems). Within the electromagnetics community, the label "FEM" became associated almost exclusively with partial differential equations. Computers evolved rapidly and the first IBM computer with all the major memory elements fabricated on a single silicon chip appeared on the market in 1970 (see Figure 5).

THE ERA OF MINICOMPUTERS AND MICROPROCESSORS (1970–1980)

The second era began ~1970 and is characterized by the widespread adoption of minicomputers, such as the PDP-11 series from Digital Equipment Corporation (DEC), which were used until the mid-1990s because they were perfect laboratory machines and were interfaceable with various instruments (see Figure 6). Prior to the 1970s, the average

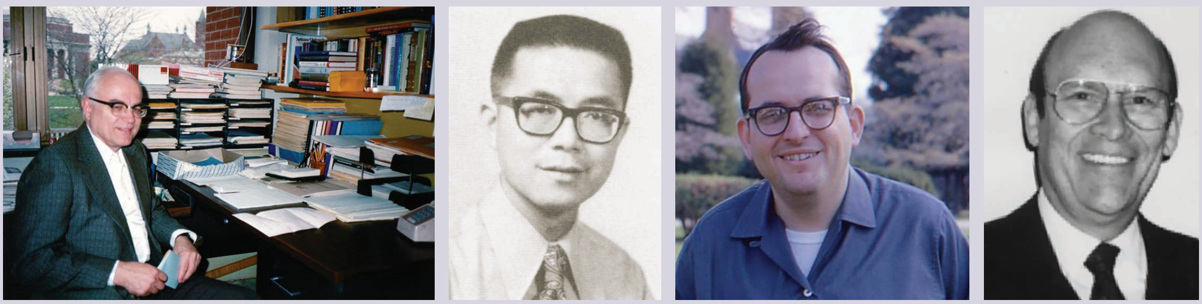

FIGURE 4. From left, Roger Harrington in his office at Syracuse University, note the absence of computers on his desk (reprint courtesy of the Applied Computational Electromagnetics Society); Kane S. Yee; Kenneth Mitzner; and Peter Silvester. The photo of Harrington was taken by James C. Rautio in spring 1985 and published in [135].

FIGURE 5. The 1970 System 370 Model 145 was the first IBM computer with all the major memory elements fabricated on a single silicon chip. It marked IBM's first move away from the magnetic-ferrite-core technology that had been the mainstay of computer memory since the mid-1950s. Six months after the announcement of the System 370, the nickname *Silicon Valley* appeared in print for the first time in an issue of *Electronic News*. The era of silicon had begun. (Reprint courtesy of IBM Corporation ©.)

FIGURE 6. The *Digital PDP11/70 Processor Handbook*. (Courtesy of the Politecnico di Torino DET Library.)

person had no access to a computer due to the prohibitive costs of purchasing and operating it. Because of this perception, DEC avoided giving the "computer" label to the PDP, but instead used its name, *Programmed Data Processor*, as a generic term. The PDP-11, the only 16-bit computer DEC ever made, was clocked at roughly 15 MHz and cost ~US$10,000. These machines, unlike the large IBM systems, covered the market segment represented by small work groups or university departments, which could not afford a mainframe.

During the 1970s, there was an explosion of CEM numerical techniques and applications. The techniques introduced for 2D and BoR formulations were extended to true 3D approaches, although integral equation formulations for 3D problems remained limited to electrically small targets due to computer hardware constraints. The formulations for homogeneous and heterogeneous dielectric targets were introduced. The singularity expansion method was introduced by Carl Baum in 1971 [50], and asymptotic approaches such as the Uniform Geometric Theory of Diffraction [51], [52] reached a certain level of maturity, enabling their use for problems that were not within the range of purely numerical methods. Figure 7 shows a photo of Carl Baum and Allen Taflove.

In 1972, Ruehli [53], [54] introduced the partial-element equivalent circuit (PEEC) framework for calculating multiloop inductances formed by complicated interconnection conductors to transfer problems from the electromagnetic domain to the circuit domain, where conventional SPICE-like circuit solvers can be employed to analyze the equivalent circuit. PEEC is used today to tackle combined electromagnetic and circuit problems in various areas, such as power electronics, electromagnetic compatibility, antenna design, and signal-integrity analysis.

Short courses on CEM were developed at several universities and led to the publication of texts on the subject. One such text, "Computer Techniques for Electromagnetics," [55] is focused primarily on integral equation approaches and includes the well-known chapter by Poggio and Miller, "Solutions of Integral Equations of 3D Scattering Problems" [56], which reported general formulations and included many results. Their formulation for homogeneous dielectric targets was extended by [57] and [58] and subsequently named the *PMCHWT* approach, after the authors' initials [56], [57], [58].

In the early 1970s, a few articles appeared on the topic of using integral equations to truncate the domain associated with the FEM solution of differential equations so that they could be used to treat open problems [59], [60]. These were applied to capacitance problems for illustration. In 1974, a technique, the unimoment method, that used an exterior eigenfunction expansion for this purpose and was applied to 2D scattering problems was introduced [61], [62]. During the 1970s, local absorbing boundary conditions (ABCs) were under investigation by the applied mathematics community [63], [64], but these were not widely adopted for frequency-domain applications for another decade. The FEM and finite-difference approaches for treating closed heterogeneous problems, such as dielectric loaded waveguide, were limited by the "spurious mode" problem [65], [66], [67].

Applications in the biomedical area helped drive the development of techniques that could be used to model electromagnetic fields in animal and human tissue. One such approach was developed in 1974 based on the 3D volume electric-field integral equation discretized with pulse basis functions on cubical-cell "block" models [68]. The technique was studied by several research groups with mixed results; it was concluded that the accuracy was often poor for the range of permittivity associated with tissue [69].

Applications in the biomedical area helped drive the development of techniques that could be used to model electromagnetic fields in animal and human tissue.

Biomedical applications were more amenable to treatment using the FDTD approach, which is a fully explicit ("marching-in-time") technique that does not require a matrix solution. Although it is a time-domain approach, the scheme is easily used in the frequency domain, as was first reported by Allen Taflove (Figure 7) in 1975 [70], [71]. The FDTD approach was used throughout the late 1970s to model a variety of transient and frequency-domain problems [72], [73], [74]. The development of the Mur ABC facilitated the treatment of open problems [75].

A method that competed with the FDTD algorithm was the transmission-line matrix (TLM) algorithm, which was introduced in 1971 by Johns and Beurle [76], [77]. The TLM approach treated regions of space as interconnected meshes of transmission lines and was used for scattering, waveguide, and other applications in the time and frequency domains [78].

Volume-integral equations had the drawback of requiring a number of unknowns that grew with the volume of the target. One way around that limitation was to employ iterative solvers. The first use of an iterative solver in electromagnetics dates back to 1971, when Bojarski combined a simple linear iterative algorithm with a fast Fourier transform (FFT)-based discretization that permitted the matrix to be compressed to save on storage and computation time [79]. Similar ideas were proposed under the name *spectral iterative technique* later in the decade [80], [81], [82]. These suffered from simple iterative algorithms that often failed to converge for problems of interest, which was later remedied by the use of conjugate gradient methods in the mid-1980s.

Another notable advancement during the 1970s was the development of the Numerical Electromagnetics Code (NEC) for the modeling of wire antennas and surfaces and was based on integral equations and the three-term sinusoid representation of the current. The NEC evolved from a series of codes written in the late 1960s into the 1970s and resulted in the release of NEC-2 in 1980 [83]. NEC-2 incorporated lossy grounds and was extended in subsequent years to NEC-3 and NEC-4. The Electromagnetic Surface Patch (ESP) code was developed at The Ohio State University and released at roughly the same time [84]. The ESP used piecewise sinusoidal basis functions on quadrilateral patch models. A competing approach, also employing rectangular patches but using piecewise linear functions, was proposed by Glisson and Wilton [85]. These articles marked the start of using vector-basis functions in electromagnetics, a topic of importance during the third era that is discussed later.

We note that the development of computers also enabled the practical application of asymptotic methods used for applied electromagnetics to realistic structures. The NEC–Basic Scattering Code (1979) employs geometric optics and the geometric theory of diffraction and is intended for applications

FIGURE 7. Dr. Carl Baum (left) with permission by SUMMA Foundation, and Dr. Allen Taflove, courtesy of Northwestern University.

such as the determination of antenna patterns on large vehicles or spacecraft [86].

It had long been recognized that common integral equations suffered from the *internal resonance* problem, a phenomenon that can be observed (but was not recognized) in a 1963 article [26] and arises when the surface of the target in question can also represent the exterior of a resonant cavity. Some remedies to this problem were discussed in

TABLE 2. THE ERA OF MINICOMPUTERS AND MICROPROCESSORS (1970–1980).

1970s	PDP-11	The DEC minicomputer series was sold from 1970 through the late 1990s.
1970s	—	IE applications were extended from perfect electrically conductive to dielectric, aperture problems, periodic structures, and so on [55], [56], [68], [88], [89], [91].
1971	—	The early use of fast, iterative solvers [79].
1971	First eBook	On 4 July in Chicago, M.S. Hart launched Project Gutenberg and digitized the U.S. Declaration of Independence, becoming the first eBook in the world.
1971	UNIX	The *UNIX Programmer's Manual* was published on 3 November 1971.
1971	Intel 4004	The first microprocessor in history was produced on 15 November 1971. In 1974, the Intel 8080 was the first microprocessor to be inserted into a home computer. The Intel 8088, introduced in 1979, would later be installed on millions of IBM PCs.
1971	—	Carl Baum introduced the singularity expansion method [50].
1972	HP-35	Hewlett-Packard's (HP's) first pocket calculator and the world's first scientific pocket calculator was introduced.
1972	—	The PEEC method was applied to low-frequency applications [53], [54].
1974	—	The first use of 3D volume IE applications for biological and other applications [68], [69].
1975		The European Space Agency was established 30 May 1975.
1975		Taflove and Brodwin's article [70] on FDTD was published.
1975	Olivetti P6060	The first PC was sold as a preassembled working system and incorporated a floppy-disk (FD) reader. It was made in Italy by Olivetti and featured 32–64 kB of read-only memory; 8 kB of random-access memory, with a maximum of 16 kB; a built-in alphanumeric keyboard; 32-alphanumeric character display; and a single or double FD drive (FDD), for 8-in FDs, with a capacity of 256 kB. It was replaced in 1981 by the P6066.
1975–1977	—	Articles by Lindman [63] and Enquist and Majda [64] on ABCs led to their use with FDTD [75].
1976	TI-30	Texas Instruments' pocket calculator performed nearly all of the logarithmic and trigonometric functions and marked the end of slide rules, which became museum pieces.
1976	Cray-1	The first Cray-1, designed, manufactured, and marketed by Cray Research, was installed at Los Alamos National Laboratory. Cray-1 was one of the most successful supercomputers in history, selling 80 units.
1976	The electric pencil: the first commercial word processor	Released in 1976 by Michael Shrayer Software Inc., it was the first microcomputer word processor to implement word wrapping, in which lines are adjusted as words are inserted and deleted. It was sold on the market until the mid-1980s.
1977	APPLE II	Released in 1977, it was the first fully assembled computer with a color display.
1978	VAX-11/780	Released by DEC in 1978, it was the first superminicomputer to implement the Virtual Address eXtension (VAX) architecture, which addresses virtual memory pages, and was the first 32-bit calculator marketed in the history of computing. The CPU, built from Schottky transistor–transistor logic devices, had a cycle time of 200 ns (5 MHz) and a 2-KB cache.
1978	TeX	TeX, the typesetting system capable of typesetting complex mathematical formulas, was first released in 1978. The most popular software system for typesetting documents that uses TeX is LaTeX, which was written in the early 1980s.
1980	NEC2	NEC2 code for wires was released [83].
1980–1981	—	Rectangular patches were used to discretize surface IEs [84], [85].
1981	IBM 5150	The first commercially successful PC was produced and exceeded 200 sales in its first year of sales.

earlier publications, such as [34] and [87]. During 1978–1979, Mautz and Harrington provided a systematic discussion of several remedies that ensured the uniqueness of numerical solutions [88], [89].

Table 2 provides a summary of the milestones of this time period. It was also during this second phase, from 1971 to 1979, that Intel created and marketed the first microprocessors, with a contribution from the Italian designer Federico

TABLE 3. THE ERA OF PERSONAL COMPUTERS AND WORKSTATIONS (1980–1990).

1980s	Computer environments for symbolic calculation	MACSYMA (1982), MAPLE (1984), and Mathematica (1988).
1980s	Video games	The golden age of video games (1988).
1980	—	Nedelec's article [103] on mixed finite elements is published.
1981	Xerox Star: the first computer with a mouse became available on the market	The first commercially available computer with a mouse (two buttons), bitmap display, GUI, and Ethernet networking was the Xerox Star workstation and was officially named the *Xerox 8010 Information System*. Its precursor, which was actually never marketed, was the Xerox Alto and was distributed in ~2,000 units, starting in March 1973.
1982	Compact Disc	The first CD was produced on 17 August. In a few years, vinyl records and cassettes were replaced by CDs.
1982	TCP/IP	The word *Internet* was defined.
1982	—	Rao et al.'s seminal article [100] on EFIE on flat-patch models was published.
1983	—	Silvester and Ferrari's FEM book [116] (the first applied to electromagnetics) was published.
1983	—	The Internet was born on 1 January with the official transition of ARPAnet to Internet protocol
1983	Motorola Dynatac X8000X	The "brick phone" was the first mobile phone marketed since 6 March 1983.
1983	Sun-2	Sun Microsystems launched the Sun-2 series of UNIX workstations and servers with virtual memory management, high-resolution graphics, and Ethernet networking.
1984	HP LaserJet	HP LaserJet was the first mass-market laser printer to use the Canon CX engine, controlled by HP software. It could print at 300 dots per inch and could print eight pages per minute on cut sheets. The HP LaserJet was quickly followed by printers from Brother Industries, IBM, and others.
1984	HP 9000	HP 9000 is a line of workstations and servers produced by HP. The HP 9000 brand was introduced in 1984.
1984	Matlab	Matlab was first released as a commercial product in 1984. It featured a numerical solver for ordinary differential equations and could run on IBM PCs.
1984	Macintosh 128 K	Marketed by Apple Computer it was among the first personal computers to have a GUI, thanks to Macintosh's Mac OS (operating system).
1984	—	Schaubert et al.'s article [102] on volume integral equations (VIE) with tetrahedral cells is published.
1985	Windows 1.0	Microsoft Corporation's Windows 1.0 was released in 1985. Windows 2.0, released in 1987, was the first version of the Windows OS in which icons appeared.
1985–1988	—	CG and improved iterative algorithms were used for IE problems [108], [109], [110], [111], [112], [113].
1987	VGA monitor	Video Graphics Array (VGA) is a video display controller and graphics standard. It was first introduced with the IBM PS/2 line of computers in 1987. IBM released the first VGA monitor in 1987.
1987–1989	—	Simple radiation/ABCs were first used with the scalar Helmholtz equation [120], [121].
1989	—	The shooting and bouncing rays method was introduced [133].
1989	—	The fall of the Berlin Wall on 9 November 1989.
1989–1990	C	Standardization of the C language.

Faggin, who was the project leader and designer of the Intel 4004, as well as being responsible for the development of the 8008, 4040, and 8080 microprocessors and their architectures. Faggin was also the developer of MOS silicon-gate technology, which allowed the creation of the first microprocessors, EPROM, and random-access memory dynamic memories and charge-coupled device sensors, all essential elements for the digitization of information. Later, in 1974, Faggin founded and directed the ZiLOG company, the first company dedicated exclusively to microprocessors, where he created the Z80 microprocessor. (In 1986, Faggin cofounded and directed Synaptics, a company that developed the first touchpads and touchscreens, but this is part of the third era.)

Roger F. Harrington's monograph, which included the solution to differential and integral equations, was published in 1968.

THE ERA OF PERSONAL COMPUTERS AND WORKSTATIONS (1980–1990)

Although we listed the IBM 5150 PC at the end of Table 2, we can consider that the third phase began in the early 1980s with the true and widespread diffusion of personal and domestic computers and, subsequently, workstations and computer clusters [also called *high-performance computing* (*HPC*) clusters] (see Table 3). The IBM 5150 (see Figure 8) included a central processing unit and qwerty keyboard with 83 keys (mouse not yet invented), with the monitor sold separately to keep the price low (consumers were free to connect any display or television to the PC). Those who wanted to purchase the complete IBM kit could choose from between two monitor models: the 5151 (monochrome green) and the 5153 (color). The IBM 5150 could accommodate a memory of just 64 kB on the motherboard (extendable to 256 kB), but a 16-kB version was also marketed. The first computer equipped with a mouse and desktop with a graphical interface was the Xerox Star, which became available on the market in April 1981.

FIGURE 8. Introduced in 1981, the IBM 5150 was IBM's first PC to use the Intel 8088 microprocessor. (Reprint courtesy of IBM Corporation ©.)

We associate software instruments with symbolic calculation and algebraic manipulation, such as Macsyma, Maxima, Maple, and Mathematica, with the third era. In fact, Macsyma, one of the oldest computer algebra systems still in use, was developed from 1968 to 1982 at the Massachusetts Institute of Technology in Boston with funding from the U.S. Department of Energy but only became a commercial product in 1982. Maxima is a free software based on a 1982 version of Macsyma. Maple is a computer environment for symbolic calculation that was developed in 1980 and then commercialized starting in 1984. Finally, Mathematica, launched in 1988, is quite a popular and fairly easy-to-use computing environment that uses an interpreted programming language, called the *Wolfram* language. Practically all software environments for symbolic calculation and algebraic manipulation were originally developed by university research groups before becoming commercial products, which happened, for example, at the end of the 1960s for Macsyma, one of the oldest general-purpose computer algebra systems still in use, or for Maple and Mathematica in the 1980s.

For CEM, the 1980s saw more widespread use of the FDTD method ([90] provides a comprehensive survey of FDTD articles from this time frame), and continued widespread use of integral equation formulations, for applications including the analysis of periodic structures [91], [92], [93], [94] and the incorporation of substrates into the treatment of microstrip patch antennas (later used in the 1990s for wireless communications) by means of Sommerfeld integrals [95], [96], [97], [98], [99].

A work of considerable significance at the start of this era was the publication by Rao et al. [100]. This article described the use of triangular patch models to treat the electric-field integral equation (EFIE), using vector-basis functions now known as *RWG* functions. These functions were previously used by Raviart and Thomas [101] and were independently proposed by Allen Glisson in his 1978 Ph.D. dissertation. This publication was followed a few years later by an analogous approach for the volume EFIE [102], which introduced the Schaubert–Wilton–Glisson bases for tetrahedral cells. Both types of basis function constitute the lowest-order members of the family of mixed-order "divergence-conforming" functions described by Nedelec in his 1980 article [103]. The treatment of the volume-integral equations using tetrahedral-cell models in [102] was a significant improvement over the "block models" used a decade earlier [68], and the treatment of surfaces using triangles proved to be so useful that the approach of [100] is still in widespread use 40 years later and has been adopted in several commercial codes. The RWG basis functions were also used to discretize the PMCHWT integral equations for homogeneous dielectric bodies in 1986 [104]. Articles addressing the efficient computation of the matrix entries arising within these formulations began to appear during the mid-1980s [105], [106], and continue to the present day.

Another significant step in the 1980s was the use of conjugate gradient and related iterative algorithms to improve the convergence of the simple iterative methods that had been considered previously [107], [108], [109], [110], [111], [112], [113], [114]. These techniques typically employed discrete-convolutional discretizations to compress the matrix operator that arose from an integral equation formulation, and became known as *FFT conjugate gradient* (*CG-FFT*) methods. These techniques evolved into the "fast multipole" iterative solvers of the 1990s, based on the work of Rokhlin and others in the late 1980s [115] (era 4 will be covered in part 2).

In 1983, the first textbook dealing with the FEM applied specifically to electromagnetic field problems was published by Silvester and Ferrari [116]. This book presented scalar FEM analysis, as had been developed to treat Laplace's equation or the scalar Helmholtz equation, and was applicable to electrostatic or 2D field problems in electromagnetics. Open-region problems also require that a radiation condition be imposed on the outer boundary of the domain to ensure a unique solution. This relationship between the field and its normal derivative at the boundary could be obtained from an eigenfunction expansion, such as the one used in [62], or from an integral equation [61], [117], [118]. Those approaches involve a dense matrix block, motivating the use of local ABCs, such as those proposed in [63], [64], or [119]. Those ABCs were suitable for use in 2D problems, and these began to be demonstrated by the end of the 1980s [120], [121].

To treat 3D problems with the FEM, one needs to discretize the full set of Maxwell's equations, or, more typically, the curl–curl form of the vector Helmholtz equation. Attempts to treat that equation for cavity and waveguide analysis with scalar FEM usually encountered spurious modes, apparently arising from distorted solutions from the null space of the vector Helmholtz operator. Vector-basis functions that belonged to the Nedelec curl-conforming spaces [103], [122] were being used in eddy current calculations in the early 1980s [123], [124], and it was soon discovered that when they were used to discretize the vector Helmholtz equation, the eigenvalues associated with the null space remained well separated from those of interest in the numerical results [125], [126]. Local ABCs were proposed for use with the vector Helmholtz equation [127], [128], and the FEM based on vector bases became widely used for treating open-region 3D problems.

Although most of the computations in this era were carried out using flat-cell models, the first articles that discussed using curved cells/elements for the MoM solution of volume and surface integral equations were published between the end of this decade and the beginning of the 1990s [129], [130], [131]. Similar extensions were available for vector FEM approaches [132].

In the years from 1986 to 1990, software based on ray methods, such as the shooting and bouncing rays (SBR) method, were developed for radar cross-section analysis and antenna design [133]. These methods can be implemented for GPUs, which makes the computation very efficient. In recent years, the SBR method has been implemented in commercial codes such as Altair Feko, CST Microwave Studio, and Ansys high-frequency structure simulator SBR+, previously Delcross Savant XGTD.

Available computational power in this era was growing but was insufficient to routinely enable full electromagnetic codes to be implemented within optimization loops. Despite this, attempts to numerically optimize the performance of antennas and arrays date to the 1960s, and early attempts to optimize the shape of wire antennas were reported in [134].

CONCLUSION

Part 1 of this history traced some of the major milestones in CEM from the founding of the AP-S to approximately the year 1990, with an emphasis on developments that were important to antenna and electromagnetic system design. Part 2 will continue the synopsis to the present day.

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