Letter

Distributed Predefined-Time Control for Cooperative Tracking of Multiple Quadrotor UAVs

Kewei Xia, Xinyi Li, Kaidan Li, and Yao Zou

Dear Editor,

This letter addresses the predefined-time control for cooperative tracking of multiple quadrotor unmanned aerial vehicles (UAVs) under a directed communication network. A predefined-time distributed estimator is first introduced to accurately get the reference velocity and acceleration for each UAV. Then, a cascade predefined-time control strategy is proposed to guarantee that all the UAVs track the reference trajectory while maintaining a preassigned configuration, where an attitude constraint algorithm is developed to avoid the flipping over of each UAV. Stability analysis demonstrates that the tracking errors of the closed-loop systems converge to zero within a predefined time. Finally, experiment results validate the proposed control strategy.

Recently, the cooperative mission of multiple quadrotor UAVs has received considerable attentions by researchers and scientists [1], [2]. Different from the application of a single UAV, the UAV team requires all the members achieving a common goal while maintaining a desired configuration cooperatively [3]. As for the cooperative formation of quadrotor UAVs, geometric control [4] and adaptive control [5] approaches have been reported, where the communication topology among the UAVs was undirected. In addition, these results are obtained by assuming that the desired acceleration [4] (as well as velocity [5]) is available to all the UAVs. To improve the control accuracy, a distributed estimator was introduced to get the accurate estimate of the desired trajectory [6]. However, the upper bound of the desired trajectory is supposed to be available for the UAV who has no access to it. Besides, the control strategies in [4]–[6] fail to ensure the achievement of the formation within a finite time or predefined time. The attitude of each UAV should be maintained within a given set to avoid the flipping over [1], [2], which has not been considered by previous results [4]–[6].

To solve the cooperative control problem of UAVs with finite-time convergence property, a wealth of distributed control algorithms were studied [7], [8]. Although these results can achieve the cooperative tracking of UAVs within a finite time, the settling time is coupled with both initial value of the system and the communication topology. Any changes of this would result in a redesign of the control parameters. For the sake of avoiding this issue, a distributed fixed-time control strategy was studied for the formation-containment of quadrotors [9]. Despite of the fact that the settling time in [9] is no longer related to the initial state of the UAV system, it still could not provide an intuitive expression by a predefined single parameter. In addition, the calculated settling time in the fixed-time control would be conservative since the estimate of its upper bound may be quite larger than the real settling time [10]. The predefined-time control that provides a preassigned settling time which is independent of both initial system state and control parameters is studied [11]. To provide the consensus estimate of the leader's state, a distributed prescribed-time estimator was developed for the multi-agent systems [12], where the upper bound of the leader's state is available for all the agents. A distributed prescribed-time optimization was investigated in [13], and the experimental validation was conducted on the quadrotors, where only the consensus was achieved rather than the trajectory tracking.

Motivated by aforementioned discussions, this paper focuses on developing a distributed control strategy that achieves the cooperative tracking of multiple quadrotors over a directed communication topology within a predefined time. The main contributions are presented as follows: 1) Compared with [6], [9], [12], an adaptive predefined-time distributed estimator is proposed where the upper bound of the desired trajectory is merely available for the UAV who can access it. 2) In contrast to [4]–[9], a predefined-time attitude constraint torque is developed. Since the roll and pitch of the UAV are constrained within a safe range, the reliability of the control system can be enhanced.

Preliminaries: Let $\bar{\lambda}(\cdot)$ and $\underline{\lambda}(\cdot)$ denote the maximum and minimum eigenvalues of a square matrix. Define $\mathrm{s}\cdot \triangleq \sin(\cdot)$ and $\mathrm{c}\cdot \triangleq \cos(\cdot)$. For a positive constant $t_t > 0$ and $t_0 \geq 0$, the time function is defined as $\mathcal{T}_{t_t,t_0}(t) = 1/(t_t + t_0 - t)$, $\forall t \in [t_0, t_t + t_0)$, and $\mathcal{T}_{t_t,t_0}(t) = 1$, $\forall t \in [0, t_0) \cup [t_t + t_0, \infty)$. For $x \in \mathbb{R}^n$, the following function is defined $\mathfrak{s}(x) = x/\|x\|$, $\forall \|x\| \neq 0$, and $\mathfrak{s}(x) = 0$, $\forall \|x\| = 0$.

Communication topology: To formulate the communication networks among the UAVs, a graph $\mathcal{G}_n \triangleq (\mathcal{V}, \mathcal{E})$ consisting of a node set $\mathcal{V} \triangleq \{1, 2, \ldots, n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ are introduced. For a directed graph, $(i, j) \in \mathcal{E}$ indicates that the information of node j is available to node i, but not conversely. Define $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ as the adjacent matrix and $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ as the Laplacian matrix of a UAV graph $\mathcal{G}_n$, where $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise, $l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$ for $j \neq i$. Denote $\mathcal{G}_{n+1} \triangleq \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$ as a directed graph (the reference trajectory is labeled as 0), where $\bar{\mathcal{V}} = \{0, 1, \ldots, n\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. The adjacent matrix and Laplacian matrix are denoted by $\bar{\mathcal{A}} \in \mathbb{R}^{(n+1)\times(n+1)}$ and $\bar{\mathcal{L}} \in \mathbb{R}^{(n+1)\times(n+1)}$. Let $\mathcal{B} = \mathrm{diag}(b_1, b_2, \ldots, b_n)$, where $b_i = 1$ if node i is accessible to the reference trajectory and $b_i = 0$ otherwise.

Suppose that the UAV graph $\mathcal{G}$ is directed and the graph $\mathcal{G}_{n+1}$ has a directed spanning tree with the reference trajectory being the root. Therefore, $\mathcal{W} = [w_{ij}] \triangleq \mathcal{L} + \mathcal{B}$ is positive definite [6]. Let $[1/h_1, 1/h_2, \ldots, 1/h_n]^T = \mathcal{W}^{-1} 1_n$. Then, $\Psi = \mathcal{W}^T \mathcal{H} + \mathcal{H}\mathcal{W}$ is positive definite, where $\mathcal{H} = \mathrm{diag}(h_1, h_2, \ldots, h_n)$.

Problem description: Based on the Euler-Newton formula [1], for $i \in \mathcal{V}$, the kinematics and dynamics of the under-actuated quadrotor UAV are given by

$$\begin{cases} \dot{p}_i = v_i \\ \dot{v}_i = -g\hat{e}_3 + \dfrac{T_i}{m_i} R_i \hat{e}_3 \\ \Gamma_i \dot{\gamma}_i = \omega_i, \\ J_i \dot{\omega}_i = -\omega_i^{\times} J_i \omega_i + \tau_i \end{cases} \tag{1}$$

where $p_i \in \mathbb{R}^3$ and $v_i \in \mathbb{R}^3$ are the position and velocity of the center of gravity of the UAV in the earth-fixed inertial frame, m_i is the mass, g is the local gravitational acceleration, $R_i = R(\gamma_i) \in \mathrm{SO}(3)$ is the rotation matrix, $\hat{e}_3 \triangleq [0, 0, 1]^T$, $T_i \in \mathbb{R}$ is the thrust command, $\gamma_i = [\phi_i, \theta_i, \psi_i]^T$ is the Euler angle, $\omega_i \in \mathbb{R}^3$ is the angular velocity, $J_i \in \mathbb{R}^{3\times 3}$ is the inertial matrix, $\tau_i \in \mathbb{R}^3$ is the torque command, and

Corresponding author: Yao Zou.

Citation: K. Xia, X. Li, K. Li, and Y. Zou, "Distributed predefined-time control for cooperative tracking of multiple quadrotor UAVs," *IEEE/CAA J. Autom. Sinica*, vol. 11, no. 10, pp. 2179–2181, Oct. 2024.

K. Xia is with the Advanced Research Institute of Multidisciplinary Sciences, Beijing Institute of Technology, Beijing 100081, China (e-mail: kwxia@bit.edu.cn).

X. Li is with the School of Aerospace Engineering, and also with the Advanced Research Institute of Multidisciplinary Sciences, Beijing Institute of Technology, Beijing 100081, China (e-mail: 3120222037@bit.edu.cn).

K. Li is with the School of Automation, Beijing Institute of Technology, Beijing 100081, China (e-mail: kaidan@bit.edu.cn).

Y. Zou is with the School of Intelligence Science and Technology, and also with the Key Laboratory of Intelligent Bionic Unmanned Systems, Ministry of Education, University of Science and Technology Beijing, Beijing 100083, China (e-mail: zouyao@ustb.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JAS.2023.123861

$\Gamma_i = [1, 0, -\mathrm{s}\theta_i; 0, \mathrm{c}\phi_i, \mathrm{s}\phi_i\mathrm{c}\theta_i; 0, -\mathrm{s}\phi_i, \mathrm{c}\phi_i\mathrm{c}\theta_i]$.

Control objective: Consider the reference trajectory labeled by 0. Suppose that the reference position p_0, velocity v_0 and acceleration $\dot{v}_0$ are all bounded. In particular, $\|\ddot{v}_0\| \le \gamma$, where γ is a positive constant. Given a desired position offset σ_i between the i-th UAV and the reference trajectory, the cooperative predefined-time tracking objective can be achieved if all the UAV track the reference trajectory while maintaining the desired configuration within a predefined time. More specifically, for $i \in \mathcal{V}$, define the tracking errors $\tilde{p}_i = p_i - p_0 - \sigma_i$ and $\tilde{v}_i = v_i - v_0$, the control objective is to design the thrust command T_i and torque command τ_i for each UAV described by (1) such that $\lim_{t\to t_o} \tilde{p}_i(t) = 0$ and $\lim_{t\to t_o} \tilde{v}_i(t) = 0$ with a predefined time t_o.

Main results: In this section, the main design procedures of the distributed estimator, thrust command and torque command are provided.

Predefined-time distributed estimator: For $i \in \mathcal{V}$, define $\hat{v}_i$, $\hat{a}_i$ and $\hat{\gamma}_i$ as the estimates of v_0, $\dot{v}_0$ and γ, respectively. Design the following adaptive distributed estimator:

$$\begin{cases} \dot{\hat{v}}_i = \hat{a}_i - k_v \mathcal{T}_{t_v, t_a+t_\gamma}(t)\bar{v}_i^e \\ \dot{\hat{a}}_i = -k_a \mathcal{T}_{t_a, t_\gamma}(t)\bar{a}_i^e - \hat{\gamma}_i \mathrm{s}(\bar{a}_i^e) \\ \dot{\hat{\gamma}}_i = -k_\gamma \mathcal{T}_{t_\gamma, 0}(t)\bar{\gamma}_i^e \end{cases} \tag{2}$$

where k_v, k_a and k_γ are constant parameters, t_v, t_a and t_γ are positive time constants, $\bar{v}_i^e = \sum_{j\in N_i} a_{ij}(\hat{v}_i - \hat{v}_j) + b_i(\hat{v}_i - v_0)$, $\bar{a}_i^e = \sum_{j\in N_i} a_{ij}(\hat{a}_i - \hat{a}_j) + b_i(\hat{a}_i - \dot{v}_0)$, and $\bar{\gamma}_i^e = \sum_{j\in N_i} a_{ij}(\hat{\gamma}_i - \hat{\gamma}_j) + b_i(\hat{\gamma}_i - \gamma)$.

Theorem 1: If the estimator parameters are chosen such that $k_v \ge 2\bar{\lambda}(\mathcal{H})/\underline{\lambda}(\Psi)$, $k_a \ge 2\bar{\lambda}(\mathcal{H})/\underline{\lambda}(\Psi)$, and $k_\gamma \ge 2\bar{\lambda}(\mathcal{H})/\underline{\lambda}(\Psi)$, then the proposed adaptive distributed estimator (2) guarantees that all the estimate errors converge to zero with a predefined time $t_e = t_v + t_a + t_\gamma$.

Proof: Define $\tilde{v}_i^e = \hat{v}_i - v_0$, $\tilde{a}_i^e = \hat{a}_i - \dot{v}_0$ and $\tilde{\gamma}_i^e = \hat{\gamma}_i - \gamma$ as the estimate errors. Note that $\bar{v}_i^e = \sum_{j=1}^n w_{ij}\tilde{v}_j^e$, $\bar{a}_i^e = \sum_{j=1}^n w_{ij}\tilde{a}_j^e$ and $\bar{\gamma}_i^e = \sum_{j=1}^n w_{ij}\tilde{\gamma}_j^e$. Let $\bar{v}^e$, $\bar{a}^e$, $\bar{\gamma}^e$, $\tilde{v}^e$, $\tilde{a}^e$ and $\tilde{\gamma}^e$ be the column stack vectors of $\bar{v}_i^e$, $\bar{a}_i^e$, $\bar{\gamma}_i^e$, $\tilde{v}_i^e$, $\tilde{a}_i^e$ and $\tilde{\gamma}_i^e$, respectively. Choose a Lyapunov function candidate $\mathrm{L}_\gamma = \bar{\gamma}^{eT}(\mathcal{H}\otimes I_3)\bar{\gamma}^e$. Its derivative satisfies $\dot{\mathrm{L}}_\gamma = -k_\gamma \mathcal{T}_{t_\gamma,0}(t)\bar{\gamma}^{eT}(\mathcal{W}^T\mathcal{H} + \mathcal{H}\mathcal{W})\bar{\gamma}^e \le -k_\gamma^* \mathcal{T}_{t_\gamma,0}(t)\mathrm{L}_\gamma$, where $k_\gamma^* = k_\gamma\underline{\lambda}(\Psi)/\bar{\lambda}(\mathcal{H}) \ge 2$. According to the Comparison principle [14], we have $\mathrm{L}_\gamma(t) \le \mathrm{L}_\gamma(0)((t_\gamma - t)/t_\gamma)^{k_\gamma^*}$, $\forall t \in [0, t_\gamma)$. This implies that $\lim_{t\to t_\gamma^-} \mathrm{L}_\gamma(t) = 0$. Based on the continuity of L_γ, it follows that $\mathrm{L}_\gamma(t_\gamma) = 0$. This further implies that $\dot{\mathrm{L}}_\gamma(t) \le -k_\gamma^*\mathrm{L}_\gamma \le 0$, $\forall t \ge t_\gamma$. It can be concluded that $\mathrm{L}_\gamma(t) = 0$ is invariant $\forall t \ge t_\gamma$. Therefore, we have $\bar{\gamma}^e(t)$ converges to zero at $t = t_\gamma$ while maintaining $\bar{\gamma}^e(t) = 0$, $\forall t \ge t_\gamma$. By recalling $\bar{\gamma}^e = (\mathcal{W}\otimes I)\tilde{\gamma}^e$, we finally have $\tilde{\gamma}^e(t) = 0$, $\forall t \ge t_\gamma$. Following similar analysis, we can conclude that $\tilde{a}^e(t) = 0$, $\forall t \ge t_\gamma + t_a$, and $\tilde{v}^e(t) = 0$, $\forall t \ge t_\gamma + t_a + t_v$. Therefore, it can be concluded that $\tilde{v}^e$, $\tilde{a}^e$ and $\tilde{\gamma}^e$ converge to zero with a predefined-time $t_e = t_\gamma + t_a + t_v$. ■

Remark 1: Based on (2), for $t \in [0, t_\gamma)$, we have $\|\mathcal{T}_{t_\gamma,0}(t)\bar{\gamma}_i^e\| \le \sqrt{\mathrm{L}_\gamma(0)/\underline{\lambda}(\mathcal{H})}/t_\gamma$. This implies that $\dot{\hat{\gamma}}_i^e \in \mathcal{L}_\infty$. Then, we have that $\hat{\gamma}_i^e$ is uniformly continuous and bounded. Similarly, the boundedness and uniform continuity of $\hat{v}_i^e$ and $\hat{a}_i^e$ can be also proved.

Force command development: Introduce an auxiliary manifold as follows:

$$s_i = v_i - \hat{v}_i + \kappa_1 \mathcal{T}_{t_p,t_s}(t)\bar{p}_i \tag{3}$$

where $\kappa_1 > 0$, t_p and t_s are positive time constants, and $\bar{p}_i = \sum_{j\in N_i} a_{ij}(p_i - p_j - \sigma_{ij}) + b_i(p_i - p_0 - \sigma_i) = \sum_{j=1}^n w_{ij}\tilde{p}_j$. Taking the derivative of s_i gives $\dot{s}_i = -g\hat{e}_3 + u_i - (\hat{a}_i - \mathcal{T}_{t_p,t_s}(t)\bar{v}_i^e) + \kappa_1\dot{\mathcal{T}}_{t_p,t_s}(t)\bar{p}_i + \kappa_1\mathcal{T}_{t_p,t_s}(t)\bar{v}_i + T_i(R_i - R_{ci})\hat{e}_3/m_i$, where $u_i = T_iR_{ci}\hat{e}_3/m_i$, $R_{ci} = R(\gamma_{ci})$, $\gamma_{ci} = [\phi_{ci}, \theta_{ci}, \psi_{ci}]^T$ is the attitude command (the detailed expression of γ_{ci} can be found in [1]), and $\bar{v}_i = \sum_{j\in N_i} a_{ij}(v_i - v_j) + b_i(v_i - v_0)$. Therefore, the thrust command $T_i = m_i\|u_i\|$ is obtained since $\|R_{ci}\hat{e}_3\| = 1$. Develop the following force command:

$$u_i = -\kappa_2\mathcal{T}_{t_s,0}(t)s_i + g\hat{e}_3 + \hat{a}_i - \kappa_1(\dot{\mathcal{T}}_{t_p,t_s}(t)\bar{p}_i + \mathcal{T}_{t_p,t_s}(t)\bar{v}_i) \tag{4}$$

where κ_2 is a positive constant.

Torque command development: To ensure the safety during the operation and avoid the singularity caused by the representation of the Euler angle, inspired by [1], a nonlinear transformation function is introduced as $\eta(\nu_i) \triangleq \nu_i/(\pi^2/4 - \nu_i^2)$, $\nu_i = \phi_i, \theta_i$. According to [1], $\eta_i \to \eta_{ci}$ is sufficient to ensure $\nu_i \to \nu_{ci}$ with $\eta(\nu_i) \in \mathcal{L}_\infty$, for ν_i, $\nu_{ci} \in (-\pi/2, \pi/2)$. Define a new attitude error $\tilde{\rho}_i \triangleq [\tilde{\eta}_{\phi i}, \tilde{\eta}_{\theta i}, \tilde{\psi}_i]^T = \rho_i - \rho_{ci}$, where $\rho_i = [\eta(\phi_i), \eta(\theta_i), \psi_i]^T$ and $\rho_{ci} = [\eta(\phi_{ci}), \eta(\theta_{ci}), \psi_{ci}]^T$. Then, the derivative of ρ_i satisfies $\dot{\rho}_i = Q_i\dot{\gamma}_i$, where $Q_i = Q(\gamma_i) \triangleq \mathrm{diag}((\pi^2/4 + \phi_i^2)/(\pi^2/4 - \phi_i^2)^2, (\pi^2/4 + \theta_i^2)/(\pi^2/4 - \theta_i^2)^2, 1)$. Define an auxiliary manifold

$$z_i = \dot{\gamma}_i + Q_i^{-1}(\kappa_3\mathcal{T}_{t_\rho,t_z}(t)\tilde{\rho}_i - \dot{\rho}_{ci}) \tag{5}$$

where $\kappa_3 > 0$, t_z is a positive time constant, and $\dot{\rho}_{ci} = Q(\gamma_{ci})\dot{\gamma}_{ci}$. The derivative of Γ_iz_i satisfies $J_i\mathrm{d}(\Gamma_iz_i)/dt = \varrho_i + \tau_i$, where $\varrho_i = J_i\dot{\Gamma}_iQ_i^{-1}(\kappa_3\mathcal{T}_{t_\rho,t_z}(t)\tilde{\rho}_i - \dot{\rho}_{ci}) - J_i\Gamma_i(Q_i^{-1}\dot{Q}_iQ_i^{-1}(\kappa_3\mathcal{T}_{t_\rho,t_z}(t)\tilde{\rho}_i - \dot{\rho}_{ci}) - Q_i^{-1}(\kappa_3\dot{\mathcal{T}}_{t_\rho,t_z}(t)\tilde{\rho}_i + \kappa_3\mathcal{T}_{t_\rho,t_z}(t)\dot{\tilde{\rho}}_i - \ddot{\rho}_{ci})) - \omega_i^\times J_i\omega_i$. Design the following torque command:

$$\tau_i = -\varrho_i - \kappa_4\mathcal{T}_{t_z,0}(t)\Gamma_iz_i \tag{6}$$

where κ_4 is a positive constant.

Stability analysis: Two propositions are first stated for the predefined time stabilities of the attitude and position error dynamics.

Proposition 1: If the initial roll and pitch satisfy $\phi_i(0)$, $\theta_i(0) \in (-\pi/2, \pi/2)$, and the control parameters are chosen such that $\kappa_3 \ge 2$ and $\kappa_4 \ge \bar{\lambda}(J_i)$, the proposed torque command (6) ensures that the closed-loop attitude error dynamics converges to zero with a predefined time $t_r = t_z + t_\rho$ and $\phi_i(t)$, $\theta_i(t) \in (-\pi/2, \pi/2)$, $\forall t \ge 0$.

Proof: For $i \in \mathcal{V}$, choose a Lyapunov function $\mathrm{V}_{zi} = (\Gamma_iz_i)^TJ_i\times\Gamma_iz_i/2$. Taking its derivative along the closed-loop trajectory satisfies $\dot{\mathrm{V}}_{zi} \le -\lambda_{zi}\mathcal{T}_{t_z,0}(t)\mathrm{V}_{zi}$, where $\lambda_{zi} = 2\kappa_4/\bar{\lambda}(J_i)$. It can be easily shown that $z_i(t)$ converges to zero at $t = t_z$ while maintaining $z_i(t) = 0$, $\forall t \ge t_z$. Next, choose a Lyapunov function $\mathrm{V}_{\rho i} = \tilde{\rho}_i^T\tilde{\rho}_i/2$. The derivative of $\mathrm{V}_{\rho i}$ is derived as $\dot{\mathrm{V}}_{\rho i} \le -\lambda_\rho\mathcal{T}_{t_\rho,t_z}(t)\mathrm{V}_{\rho i} + \tilde{\rho}_i^TQ_iz_i$, where $\lambda_\rho = 2\kappa_3$. Based on previous analysis, Q_iz_i converges to zero at $t = t_z$. According to the Comparison principle, we can conclude that $\mathrm{V}_{\rho i}(t) = 0$ is invariant, $\forall t \ge t_\rho + t_z$. Therefore, we finally have $\tilde{\rho}_i(t) = 0$, $\forall t \ge t_r$. ■

Proposition 2: If the control parameters are chosen such that $\kappa_1 \ge 2\bar{\lambda}(\mathcal{H})/\underline{\lambda}(\Psi)$ and $\kappa_2 \ge 1$, then the proposed force command (4) with adaptive distributed estimator (2) guarantees that closed-loop position error system converges to zero with a predefined time $t_o \triangleq t_p + t_s$.

Proof: By substituting the force command (4), we have the nominal closed-loop system $\dot{s}_i = -\kappa_2\mathcal{T}_{t_s,0}(t)s_i$ perturbed by $\mathcal{T}_{t_v,t_a+t_\gamma}(t)\bar{v}_i^e + T_i(R_i - R_{ci})\hat{e}_3/m_i$. For $i \in \mathcal{V}$, choose a Lyapunov function candidate $\mathrm{V}_{si} = s_i^Ts_i/2$. Its derivative can be derived as $\dot{\mathrm{V}}_{si} \le -\lambda_s\mathcal{T}_{t_s,0}(t)\mathrm{V}_{si}$, where $\lambda_s = 2\kappa_2$. Thus, all the closed-loop signals are bounded and $T_i \in \mathcal{L}_\infty$. According to Theorem 1 and Proposition 1, we know that the perturbations $\mathcal{T}(t)_{t_v,t_a+t_\gamma}\bar{v}_i^e$ and $T_i(R_i - R_{ci})\hat{e}_3/m_i$ converge to zero at t_e and t_r, respectively. Then, we have $\mathrm{V}_{si}(t) \le \mathrm{V}_{si}(\max(t_e, t_r))((t_s - t)/(t_s - \max(t_e, t_r)))^{\lambda_s}$, $\forall t \in [\max(t_e, t_r), t_s)$. Therefore, it can be concluded that $s_i(t)$ converges to zero at $t = t_s$ while maintaining $s_i(t) = 0$, $\forall t \ge t_s$. Define $\tilde{p}$ and $\bar{s}$ as the column stack vectors of $\tilde{p}_i$ and s_i, respectively. Choose a Lyapunov function $\mathrm{V}_p = \tilde{p}^T(\mathcal{H}\otimes I_3)\tilde{p}$. Taking its derivative gives that $\dot{\mathrm{V}}_p \le -\lambda_p\mathcal{T}_{t_p,t_s}(t)\mathrm{V}_p + 2\tilde{p}^T(\mathcal{H}\otimes I_3)(\tilde{v}^e + \bar{s})$, where $\lambda_p = \kappa_1\underline{\lambda}(\Psi)/\bar{\lambda}(\mathcal{H}) \ge 2$. Since $\tilde{v}^e + \bar{s}$ are bounded and converge to zero at t_s, it can be concluded that $\mathrm{V}_p(t) \le \mathrm{V}_p(t_s)((t_s + t_p - t)/t_p)^{\lambda_p}$, $\forall t \in [t_s, t_s + t_p)$. Then, we can conclude that $\mathrm{V}_p(t) = 0$ is invariant, $\forall t \ge t_p + t_s$. Therefore, the closed-loop position error system is predefined-time stable with $t_o = t_p + t_s$.

Based on Propositions 1 and 2 and the hierarchical system stability theory, the main result can be summarized as following theorem.

Theorem 2: Consider the n quadrotor UAVs described by (1). The proposed force command (4) and torque command (6) with the adaptive distributed estimator (2) ensure that the cooperative predefined-time tracking of multiple quadrotor UAVs is achieved with a predefined time t_o.

Remark 2: It is worthy noting that although there exist seven time parameters, they can be easily selected based on the following criteria: $t_r < t_s$ and $t_e < t_s$, since the convergence of attitude-loop tracking and estimator is required to be faster than the position-loop tracking.

Experiment results: To evaluate the proposed strategy, a flight experiment that describes a group of 4 crazyflie quadrotors cooperatively tracking a reference trajectory is provided, where the position and velocity are obtained by the motion capture system, and the information exchange is operated by a control center under a directed communication topology in Fig. 1. The proposed adaptive distributed estimator and predefined-time force command are implemented in the position loop of the crazyflie UAV. The mass of the UAV is 0.032 kg. The reference trajectory is designed as: $p_0 = [0.5\times\cos(\pi t/10); 0.5\sin(\pi t/10); 1-0.5e^{-0.1t}]^T$ m. The desired position offsets of the UAVs are assigned as: $\sigma_1 = [0,0,0]^T$ m, $\sigma_2 = [0.5,0,0]^T$ m, $\sigma_3 = [0,0.5,0]^T$ m and $\sigma_4 = [0.5,0.5,0]^T$ m. The parameters of the distributed estimator are chosen as: $t_v = 1$, $t_a = 1$, $t_\gamma = 3$, $k_v = 8$, $k_a = 10$ and $k_\gamma = 5$. The control parameters are chosen as: $t_p = 0.5$, $t_s = 5.5$, $\kappa_1 = 5.4$ and $\kappa_2 = 3$. The experiment results are provided in Figs. 2 and 3. (The experiment video can be found in https://b23.tv/crJUxX5.)

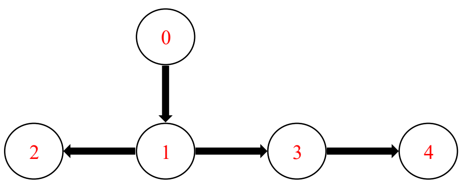

Fig. 1. Communication topology.

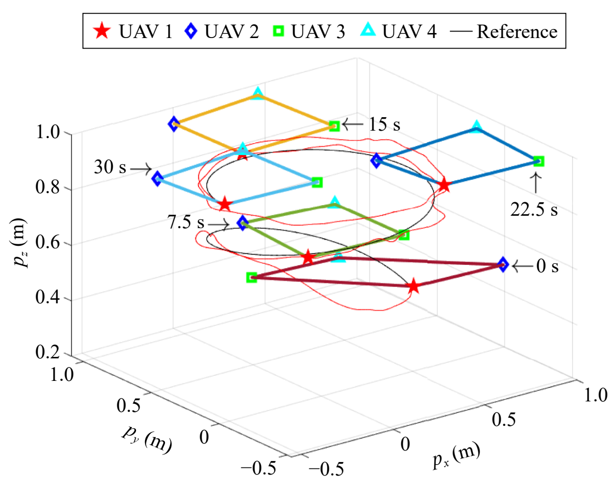

Fig. 2. Experiment result: Trajectories of the quadrotors and the target.

Fig. 2 shows the formation evolution of the quadrotors with respect to the reference trajectory, where the formation is depicted at regular 7.5 s intervals. Fig. 3 exhibits the position tracking errors of each quadrotor on three axes. It can be seen from these two figures that the formation tracking is constructed in 6 s and then it is maintained afterwards. Note that although there are errors in the final tracking arising from measurement noise, the convergence results are within reasonable ranges. Therefore, the flight experiment has verified and accessed the effectiveness of the proposed control strategy.

Conclusion: In this letter, a predefined-time control strategy consisting of an adaptive distributed estimator and a hierarchical control algorithm has been developed for the cooperative tracking of multiple quadrotor UAVs under a directed communication topology. Experimental validations have been provided to assess the proposed theoretical results.

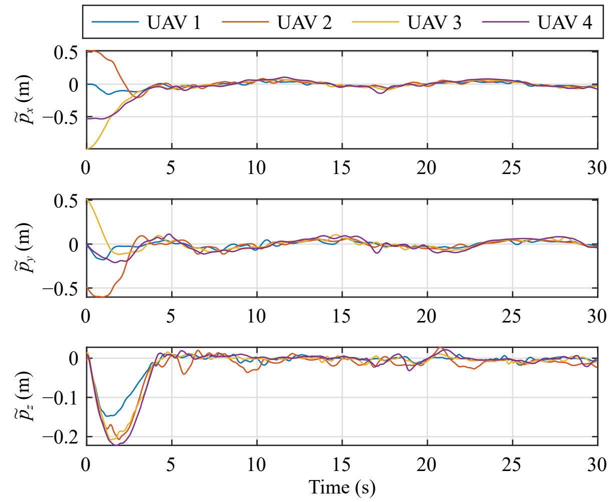

Fig. 3. Experiment result: Position tracking error.

Acknowledgments: This work was supported in part by the National Natural Science Foundation of China (62203054) and the Beijing Institute of Technology Research Fund Program for Young Scholars.

References

[1] K. Xia, M. Shin, W. Chung, M. Kim, S. Lee, and H. Son, "Landing a quadrotor UAV on a moving platform with sway motion using robust control," *Control Engineering Practice*, vol. 128, p. 105288, 2022.

[2] K. Xia, S.-M. Lee, W. Chung, Y. Zou, and H. Son, "Moving target landing of a quadrotor using robust optimal guaranteed cost control," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 3, pp. 819–821, 2023.

[3] Z. Zuo, C. Liu, Q.-L. Han, and J. Song, "Unmanned aerial vehicles: Control methods and future challenges," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 4, pp. 601–614, 2022.

[4] Z. Miao, H. Zhong, J. Lin, Y. Wang, and R. Fierro, "Geometric formation tracking of quadrotor UAVs using pose-only measurements," *IEEE Trans. Circuits and Systems-II: Express Briefs*, vol. 69, no. 3, pp. 1159–1163, 2022.

[5] Y. Zou, K. Xia, and W. He, "Adaptive fault-tolerant distributed formation control of clustered vertical takeoff and landing UAVs," *IEEE Trans. Aerospace and Electronic Systems*, vol. 58, no. 2, pp. 1069–1082, 2022.

[6] Y. Zou and Z. Meng, "Coordinated trajectory tracking of multiple vertical take-off and landing UAVs," *Automatica*, vol. 99, pp. 33–40, 2019.

[7] D. Wang, Q. Zong, B. Tian, F. Wang, and L. Dou, "Finite-time fully distributed formation reconfiguration control for UAV helicopters," *Int. J. Robust and Nonlinear Control*, vol. 28, no. 18, pp. 5943–5961, 2018.

[8] J. Wang, C. Bi, D. Wang, Q. Kuang, and C. Wang, "Finite-time distributed event-triggered formation control for quadrotor UAVs with experimentation," *ISA Trans.*, vol. 126, pp. 585–596, 2022.

[9] Y. Xu, Y. Qu, D. Luo, and H. Duan, "Distributed fixed-time time-varying formation-containment control for networked underactuated quadrotor UAVs with unknown disturbances," *Aerospace Science and Technology*, vol. 130, p. 107909, 2022.

[10] A. Garza-Alonso, M. Basin, and P. C. Rodriguez-Ramirez, "Predefined-time backstepping stabilization of autonomous nonlinear systems," *IEEE/CAA J. Autom. Sinica*, vol. 9, no. 11, pp. 2020–2022, 2022.

[11] Y. Wang, Y. Song, D. J. Hill, and M. Krstic, "Prescribed-time consensus and containment control of networked multiagent systems," *IEEE Trans. Cybern.*, vol. 49, no. 4, pp. 1138–1147, 2019.

[12] X. Gong, Y. Cui, T. Wang, J. Shen, and T. Huang, "Distributed prescribed-time consensus observer for high-order integrator multi-agent systems on directed graphs," *IEEE Trans. Circuits and Systems-II: Express Briefs*, vol. 69, no. 4, pp. 2216–2220, 2022.

[13] X. Gong, Y. Cui, J. Shen, J. Xiong, and T. Huang, "Distributed optimization in prescribed-time: Theory and experiment," *IEEE Trans. Network Science and Engineering*, vol. 9, no. 2, pp. 564–576, 2022.

[14] H. Khalil, *Nonlinear Systems*, 3rd Eds. Englewood, USA: Prentice-Hall, 2002.