

Letter

Data-Driven Adaptive Predictive Control Method With Autotuned Weighting Factor for Nonlinear Systems Using Triangular Dynamic Linearization

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Dear Editor,

In this letter, a novel data-driven adaptive predictive control method is proposed using the triangular dynamic linearization technique. The proposed method only contains one time-varying parameter with explicit physical meaning, which can prevent severe deviation in parameter estimation. Specifically, a triangular dynamic linearization (TDL) data model is employed to predict future system outputs, and then to correct inaccurate predictive outputs, a feedback regulator is designed. An autotuned weighting factor is introduced to alleviate the computational burden in practical applications and further improve output tracking performance. Closed-loop stability conditions are derived by rigorous analysis. Simulation results are provided to demonstrate the efficacy of the proposed method.

Model predictive control (MPC) has received a surge of interests in various applications, such as autonomous vehicles [1], power electronic systems [2], and networked control systems [3], [4]. In practice, however, it is generally difficult to develop an accurate model for those systems with nonlinearity, uncertainty, or time-varying characteristics. To address this, data-driven predictive control methods were proposed [5]–[9]. For example, a data-driven model predictive control method was presented in [8], where the unknown system was estimated directly from measured data by the subspace technique. A model-free adaptive predictive control (APC) method based on an equivalent data model with partial form dynamic linearization (PFDL), called PFDL-APC, was proposed in [9]. However, there exist two restrictions stated as follows. Firstly, most of the existing data models lack explicit physical significance pertaining to their model parameters. Secondly, massive parameters in these data models require online identification. In practical applications, the two problems would lead to the deviation of parameter estimates in the online identification process and also impose an additional computational burden.

To solve those problems, based on the TDL technique in [10], a novel data-driven adaptive predictive control method called TDL-APC is proposed in this letter for a class of nonlinear systems. Our contributions are summarized as follows. 1) To solve the parameter estimate deviation problem in the PFDL-APC method, a data-driven adaptive predictive control method utilizing triangular dynamic linearization is designed. 2) An autotuned weighting factor is introduced to alleviate the computational burden and also provide a better tracking performance compared to the corresponding method with a

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fixed weighting factor. 3) The rigorous analysis is given to obtain closed-loop stability conditions.

Notations: Throughout this paper, $\|\cdot\|$ denotes the Euclidean norm. It is defined that $\Delta x(k) = x(k) - x(k-1)$. $\rho(\cdot)$ means the spectral radius of a matrix. I_m and $0_{m \times n}$ denote an $m \times m$ -dimensional identity matrix and an $m \times n$ -dimensional zero matrix, respectively.

TDL-APC method: Consider a general nonlinear system as follows:

$$z(k+1) = f(z(k), \dots, z(k-n_z), v(k), \dots, v(k-n_v)) \quad (1)$$

where $z(k)$ and $v(k)$ denote the system output and the control input at time k , respectively; $f(\cdot)$ represents an unknown nonlinear function; and n_z and n_v are the unknown system output and input orders.

According to [10], system (1) can be expressed as the following TDL data model:

$$z(k+1) = z(k) + p(k)\Delta\omega(k) \quad (2)$$

where $p(k) \in [p^l, p^u]$ is a scalar with positive constants p^l and p^u , and $\omega(k)$ is defined as

$$\Delta\omega(k) = \frac{1}{n} \sum_{i=1}^n i \Delta v(k-i+1) + \frac{1}{M-n} \sum_{i=n+1}^M (M-i) \Delta v(k-i+1) \quad (3)$$

where $M \in [2, +\infty)$ and $n \in [1, M)$ are both integer constant parameters of the TDL data model. For details of data model (2), please refer to [10].

The proposed TDL-APC scheme consists of four parts, i.e., a parameter estimator, a TDL predictor, a feedback regulator, and an optimal controller. Each part is presented below in detail.

1) Parameter estimator: The estimate of $p(k)$ is given as

$$\hat{p}(k) = \hat{p}(k-1) + \frac{\Delta\omega(k-1)}{\eta + \Delta\omega^2(k-1)} (\Delta z(k) - \hat{p}(k-1)\Delta\omega(k-1)) \quad (4)$$

where η is a constant weighting parameter.

2) TDL predictor: Define $\Delta\hat{V}(k) = [\Delta\hat{v}(k|k-1), \dots, \Delta\hat{v}(k+N_p-1|k-1)]^T$, $\Delta V_p(k-1) = [\Delta v(k-1), \dots, \Delta v(k-M+2)]^T$, and N_p is the predictive horizon of the system output. Based on the historical control input sequence $\Delta V_p(k-1)$ and the future control input sequence $\Delta\hat{V}(k)$, future system outputs are predicted by using TDL data model (2), which are given as

$$\begin{aligned} \hat{z}(k+i|k) &= \hat{z}(k+i-1|k) + \hat{a}_i(k)\Delta\hat{V}(k) + \hat{b}_i(k)\Delta V_p(k-1) \\ &= z(k) + \sum_{j=1}^i \hat{a}_j(k)\Delta\hat{V}(k) + \sum_{j=1}^i \hat{b}_j(k)\Delta V_p(k-1) \end{aligned} \quad (5)$$

where $i = 1, 2, \dots, N_p$, and

$$\hat{a}_j(k) = \begin{cases} \hat{p}(k) \left[\frac{j}{n}, \frac{j-1}{n}, \dots, \frac{1}{n}, 0_{1 \times (N_p-j)} \right], & 1 \leq j < n \\ \hat{p}(k) \left[\frac{M-j}{M-n}, \dots, 1, \frac{n-1}{n}, \dots, \frac{1}{n}, 0_{1 \times (N_p-j)} \right], & n \leq j < M \\ \hat{p}(k) \left[0_{1 \times (j-M+1)}, \frac{1}{M-n}, \dots, 1, \frac{n-1}{n}, \dots, \frac{1}{n}, 0_{1 \times (N_p-j)} \right], & M \leq j \leq N_p \end{cases} \quad (6)$$

$$\hat{b}_j(k) = \begin{cases} \hat{p}(k) \left[\frac{j+1}{n}, \dots, 1, \frac{M-n-1}{M-n}, \dots, \frac{1}{M-n}, 0_{1 \times (j-1)} \right], & 1 \leq j < n \\ \hat{p}(k) \left[\frac{M-j-1}{M-n}, \dots, \frac{1}{M-n}, 0_{1 \times (j-1)} \right], & n \leq j < M \\ 0_{1 \times (M-2)}, & M \leq j \leq N_p. \end{cases} \quad (7)$$

The compact form of (5) can be rewritten as

$$\hat{Z}(k+1) = Qz(k) + \hat{A}(k)\Delta\hat{V}(k) + \hat{B}(k)\Delta V_p(k-1) \quad (8)$$

where $\hat{Z}(k+1) = [\hat{z}(k+1|k), \dots, \hat{z}(k+N_p|k)]^T$, $Q = [1, 1, \dots, 1]^T$, $\hat{A}(k) = [\hat{a}_1^T(k), \hat{a}_2^T(k), \dots, \sum_{i=1}^{N_p} \hat{a}_i^T(k)]^T$, and $\hat{B}(k) = [\hat{b}_1^T(k), \hat{b}_1^T(k) + \hat{b}_2^T(k), \dots, \sum_{i=1}^{N_p} \hat{b}_i^T(k)]^T$.

If $\Delta v(k+j-1) = 0$ for $j > N_v$, (8) is simplified as

$$\hat{Z}(k+1) = Qz(k) + \hat{A}_1(k)\Delta\hat{V}_{N_v}(k) + \hat{B}(k)\Delta V_p(k-1) \quad (9)$$

where $N_v \leq N_p$ is the predictive horizon of the control input, $\Delta\hat{V}_{N_v}(k) = [\Delta\hat{v}(k|k-1), \dots, \Delta\hat{v}(k+N_v-1|k-1)]^T$, and $\hat{A}_1(k) \in \mathbb{R}^{N_p \times N_v}$ consists of the first N_v columns of $\hat{A}(k)$.

3) Feedback regulator: In order to further correct inaccurate predictive outputs, a feedback regulator is designed. Define the feedback regulation term as

$$\hat{\beta}(k) = F(z(k) - \hat{z}(k|k-1)) \quad (10)$$

where $F = [f_1, f_2, \dots, f_{N_p}]^T$ is the weighting coefficient vector. As a result, the modified predictive output vector is expressed as

$$\hat{Z}_p(k+1) = \hat{Z}(k+1) + \hat{\beta}(k) \quad (11)$$

which is used for the design of the subsequent optimal controller.

4) Optimal controller: By considering the future system behavior, the following receding horizon optimization performance criterion with an autotuned weighting factor is constructed

$$J = \|Z^*(k+1) - \hat{Z}_p(k+1)\|^2 + \lambda \hat{p}^2(k) \|\Delta\hat{V}_{N_v}(k)\|^2 \quad (12)$$

where $Z^*(k+1) = [z^*(k+1), \dots, z^*(k+N_p)]^T$ is the reference input, and λ is a positive control weighting factor. Minimizing (12) yields the following control prediction sequence:

$$\begin{aligned} \Delta\hat{V}_{N_v}(k) &= (\hat{A}_1^T(k)\hat{A}_1(k) + \lambda\hat{p}^2(k)I)^{-1} \hat{A}_1^T(k)(Z^*(k+1) \\ &\quad - Qz(k) - \hat{B}(k)\Delta V_p(k-1) - \hat{\beta}(k)) \\ &= \hat{p}^{-1}(k)H_0^{-1}A_0^T(Z^*(k+1) - Qz(k) \\ &\quad - \hat{B}(k)\Delta V_p(k-1) - \hat{\beta}(k)) \end{aligned} \quad (13)$$

where $H_0 = A_0^T A_0 + \lambda I_{N_v}$, and $A_0 = \hat{A}_1(k)/\hat{p}(k)$ are constant matrices. According to the receding horizon optimization mechanism of MPC [6], the first element of the control prediction sequence is applied to the controlled plant at the current time instant, i.e.,

$$v(k) = v(k-1) + g\Delta\hat{V}_{N_v}(k) \quad (14)$$

where $g = [1, 0_{1 \times (N_v-1)}]$.

Remark 1: It is noted that by introducing the autotuned weighting factor $\lambda\hat{p}^2(k)$ in (12), the computational burden can be considerably reduced by avoiding the calculation of matrix inversion, since $(\hat{A}_1^T(k)\hat{A}_1(k) + \lambda\hat{p}^2(k)I)^{-1} \hat{A}_1^T(k)$ in (13) is turned into $\hat{p}^{-1}(k)H_0^{-1}A_0^T$, where $H_0^{-1}A_0^T$ can be calculated offline. Furthermore, it is easy to see from (2) and (13) that the autotuned weighting factor can preserve smooth output response for different operating points of a nonlinear system.

Stability analysis:

Theorem 1: If $p(k)$ is bounded, the parameter estimate $\hat{p}(k)$ is also bounded, i.e., $\hat{p}(k) \in [\hat{p}^l, \hat{p}^u]$, where positive constant scalars \hat{p}^l and \hat{p}^u denote the lower and upper bound, respectively.

Proof: For a detailed proof, please refer to Theorem 17 of [9]. ■

To analyze the convergence of the system output tracking error, without loss of generality, the reference input $z^*(k)$ is set as a constant r^* . Thus, the predictive control law (14) becomes

$$\Delta v(k) = \hat{\psi}_1(k)e(k) + \hat{\psi}_2(k)\Delta V_p'(k-1) \quad (15)$$

where the system output tracking error $e(k) = r^* - z(k)$, $D = [\frac{1}{n}, \frac{2}{n}, \dots, 1, \frac{M-n-1}{M-n}, \dots, \frac{1}{M-n}]$, $\Delta V_p'(k-1) = [\Delta v(k-1), \dots, \Delta v(k-M+1)]^T$, $\hat{\psi}_1(k) = \hat{p}^{-1}(k)gH_0^{-1}A_0^T Q$, $\hat{\psi}_2(k) = -gH_0^{-1}A_0^T B_0 - \frac{g}{\hat{p}(k)}(p(k-1) - \hat{p}(k-1))H_0^{-1}A_0^T F D$, $\hat{B}_1(k) = [\hat{B}(k), 0_{N_p \times 1}]$, and $\hat{B}_1(k) = \hat{p}(k)B_0$.

As shown in Theorem 1, $\hat{p}^l \leq \hat{p}(k) \leq \hat{p}^u$. Thus, the scalar $\hat{\psi}_1(k)$ is also bounded, i.e., $\hat{\psi}_1(k) \in [\hat{\psi}_1^l, \hat{\psi}_1^u]$, where $\hat{\psi}_1^l$ and $\hat{\psi}_1^u$ are constant scalars. Then, with (15), $\Delta V_p'(k)$ can be written as

$$\Delta V_p'(k) = \hat{\Psi}_1(k)e(k) + \hat{\Psi}_2(k)\Delta V_p'(k-1) \quad (16)$$

where

$$\hat{\Psi}_1(k) = [\hat{\psi}_1(k), 0_{1 \times (M-2)}]^T, \quad \hat{\Psi}_2(k) = \begin{bmatrix} \hat{\psi}_2(k) \\ I_{M-2} 0_{(M-2) \times 1} \end{bmatrix}. \quad (17)$$

Since $p(k)$ is bounded, using Theorem 1, $p(k) - \hat{p}(k)$ is bounded for any time k , which further leads to $\hat{\psi}_2(k)$ being bounded. Then, it can be obtained from (17) that $\|\hat{\Psi}_2(k)\|$ is also bounded, i.e., $\|\hat{\Psi}_2(k)\| \leq \mu_1$, where μ_1 is a scalar.

Theorem 2: If the weighting factor λ satisfies

$$0 < \mu_1 < 1 \quad (18)$$

$$0 < \frac{p^l \hat{\psi}_1^l}{n} < 1, \quad 0 < \frac{p^u \hat{\psi}_1^u}{n} < 1 \quad (19)$$

$$0 < \frac{p^l \hat{\psi}_1^l}{n} + p^u \|D\| \hat{\psi}_1^u \leq 1 \quad (20)$$

where μ_1 , $\hat{\psi}_1^l$, and $\hat{\psi}_1^u$ are all only related to the constant λ , then $\lim_{k \rightarrow \infty} |e(k)| = 0$.

Proof: Let $\|\Delta V_p'(0)\| = 0$, and by taking norm of both sides of (16), $\|\Delta V_p'(k)\|$ satisfies

$$\begin{aligned} \|\Delta V_p'(k)\| &\leq \mu_1 \|\Delta V_p'(k-1)\| + \hat{\psi}_1^u |e(k)| \\ &\leq \mu_1^k \|\Delta V_p'(0)\| + \hat{\psi}_1^u \sum_{g=0}^{k-1} \mu_1^g |e(k-g)| \\ &= \hat{\psi}_1^u \sum_{g=0}^{k-1} \mu_1^g |e(k-g)|. \end{aligned} \quad (21)$$

Therefore, with (16), it can be obtained that

$$e(k+1) = (1 - \frac{p(k)\hat{\psi}_1(k)}{n})e(k) - p(k)D\hat{\Psi}_2(k)\Delta V_p'(k-1). \quad (22)$$

Let $\mu_2 = 1 - \frac{p^l \hat{\psi}_1^l}{n}$ and $\mu_3 = p^u \|D\|$. With (19), $0 < \frac{p^l \hat{\psi}_1^l}{n} < \frac{p^u \hat{\psi}_1^u}{n} < 1$ holds, and it can be derived that

$$0 < 1 - \frac{p^u \hat{\psi}_1^u}{n} \leq 1 - \frac{p(k)\hat{\psi}_1(k)}{n} \leq \mu_2 < 1.$$

With (21), the norm of (22) is

$$\begin{aligned} |e(k+1)| &\leq \mu_2 |e(k)| + \mu_1 \mu_3 \|\Delta V_p'(k-1)\| \\ &\leq \mu_2^k |e(1)| + \mu_1 \mu_4 \sum_{g=0}^{k-2} \mu_2^g \sum_{h=0}^{k-2-g} \mu_1^h |e(k-1-g-h)| \\ &= \kappa(k+1) \end{aligned} \quad (23)$$

where $\mu_4 = \mu_3 \hat{\psi}_1^u$, $\kappa(2) = \mu_2 |e(1)|$, and

$$\begin{aligned} \kappa(k+1) &= \mu_2 \kappa(k) + \mu_4 \sum_{h=1}^{k-2} \mu_1^{h+1} |e(k-1-h)| + \mu_1 \mu_4 |e(k-1)| \\ &= \mu_2 \kappa(k) + \ell(k) \end{aligned} \quad (24)$$

with $\ell(k) = \mu_4 \sum_{h=1}^{k-2} \mu_1^{h+1} |e(k-1-h)| + \mu_1 \mu_4 \kappa(k-1)$. According to (20), $0 < \frac{p^l \hat{\psi}_1^l}{n} + p^u \|D\| \hat{\psi}_1^u \leq 1$, i.e., $\mu_4 \leq \mu_2$. Thus, one has

$$\begin{aligned} \ell(k) &\leq \mu_1 \left[\mu_2^{k-1} |e(1)| + \mu_1 \mu_4 \sum_{g=0}^{k-3} \mu_2^g \right. \\ &\quad \left. \times \sum_{h=0}^{k-3-g} \mu_1^h |e(k-2-g-h)| \right] = \mu_1 \kappa(k). \end{aligned} \quad (25)$$

Substituting (25) into (24) gives

$$\kappa(k+1) \leq (\mu_1 + \mu_2) \kappa(k) \leq (\mu_1 + \mu_2)^{k-1} \kappa(2). \quad (26)$$

Since $\mu_1 < \frac{p^l \hat{\psi}_1^l}{n}$, i.e., $\mu_1 + \mu_2 < 1$, it is obtained that

$$\lim_{k \rightarrow +\infty} |e(k+1)| \leq \lim_{k \rightarrow +\infty} (\mu_1 + \mu_2)^{k-1} \kappa(2) = 0. \quad (27)$$

■

Numerical simulation: To verify the effectiveness of the proposed method, the following time-varying nonlinear system is considered:

$$z(k+1) = \frac{\alpha(k)z(k)z(k-1)}{1+z^2(k)+z^2(k-1)} + v^3(k) + 2v(k-1) \quad (28)$$

where $\alpha(k) = 1 + 0.9 \sin(\frac{k\pi}{40})$. The parameters of the proposed method are chosen as $\eta = 1$, $N_p = 4$, $N_v = 2$, $F = [0.95, 0.95, 0.95, 0.95]^T$, $M = 5$, $n = 2$, $\lambda = 5$, and $\hat{p}(0) = 1$. The parameters of the PFDL-APC method are set as $\eta = 1$, $\mu = 0.1$, $N_p = 4$, $N_u = 2$, $L = 4$, $\lambda = 10$, and $\hat{\varphi}_L(1) = [1, 0, 0, 0]^T$.

Fig. 1 shows the comparative simulation results of the PFDL-APC and TDL-APC methods for a reference signal $z^*(k) = \sin(\frac{k\pi}{40})$. As shown in Fig. 1(a), both methods possess similar tracking performance. Moreover, the TDL-APC method can always maintain good tracking performance while that of the PFDL-APC method deteriorates significantly as time increases (see Figs. 1(b) and 1(c)). This is because the estimates of the four model parameters of the PFDL-APC method gradually deviate, as displayed in Fig. 1(d), while the estimate $\hat{p}(k)$ of the TDL-APC method is relatively stable.

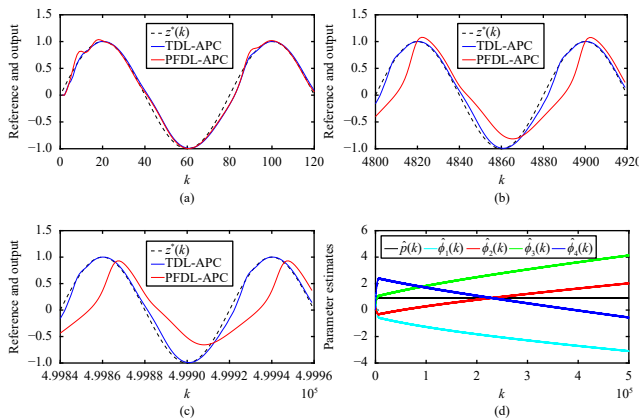


Fig. 1. Tracking performance and estimation performance of the proposed TDL-APC method and PFDL-APC method. (a) Tracking performance ($k \in [0, 120]$); (b) Tracking performance ($k \in [4800, 4920]$); (c) Tracking performance ($k \in [499840, 499960]$); (d) Estimation performance.

To test the superiority of the autotuned weighting factor in (12), simulation is carried out for the TDL-APC method with a fixed weighting factor and the following the performance criterion:

$$J = \|Z^*(k+1) - \hat{Z}_p(k+1)\|^2 + \lambda \|\Delta \hat{V}_{N_v}(k)\|^2. \quad (29)$$

The simulation time of the three control schemes is given in Table 1. It can be seen from Table 1 that the TDL-APC scheme with the autotuned weighting factor (i.e., aTDL-APC) is 3.38 times faster than the PFDL-APC control scheme and 2.33 times faster than the TDL-APC scheme with the fixed weighting factor (i.e., fTDL-APC), which verifies that the autotuned weighting factor can strongly alleviate computational burden. At the same time, Fig. 2 shows that the aTDL-APC method can further improve the tracking performance compared to the fTDL-APC method.

Conclusion: A novel TDL-APC method has been presented based on the triangular dynamic linearization technique for nonlinear systems, where a TDL data model is used to predict future outputs, a feedback regulator is designed to correct inaccurate predictive outputs, and an autotuned weighting factor is introduced to alleviate the computational burden. The closed-loop stability analysis and comparative simulation results have been provided to illustrate the effectiveness and superiority of the proposed TDL-APC method.

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Table 1. Simulation Time of Three Control Schemes

Scheme	Set 1 (ms)	Set 2 (ms)	Set 3 (ms)	Set 4 (ms)	Set 5 (ms)	Average (ms)
PFDL-APC	11.83	11.05	12.10	11.33	12.55	11.77
fTDL-APC	7.82	8.10	8.12	8.67	7.87	8.12
aTDL-APC	3.85	3.62	3.69	3.20	3.06	3.48

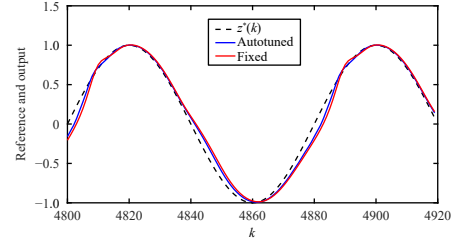


Fig. 2. Tracking performance of TDL-APC method under the fixed and autotuned weighing factors.

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