Letter

Fuzzy-Inverse-Model-Based Networked Tracking Control Frameworks of Time-Varying Signals

Shiwen Tong, Dianwei Qian, Keya Yuan, Dexin Liu, Yuan Li, and Jiancheng Zhang

Dear Editor,

Tracking control in networked environment is a very challenging problem due to the contradiction of rapid response to the time-varying signal and the inevitable delay introduced by networks. This letter has proposed several fuzzy-inverse-model-based network tracking control frameworks which are helpful in handling the system with nonlinear dynamics and uncertainties. The control frameworks have adopted different strategies such as feedback correction, internal model structure and adaptive technology. Simulations have proved the validity of the strategies. Moreover, the combination of two or more technologies can greatly improve the control performance.

As we all know, set-point control is applicable to most processes in industry, in which the controlled variable is required to keep the vicinity of a given value. The control object is easily to realize and many control strategies such as PID control [1], predictive control [2], fuzzy control [3], neural network control [4], etc., can all be used. However, for some systems, the set-point is no longer a fixed value, but a time-varying signal, which is named as a tracking control system [5]. This requires the output of the controlled process timely and accurately follow the changes of the time-varying signal which increases the difficulty of control.

In network environment, tracking control has become more difficult because on one hand there is time-delay in networks and on the other hand timely and accurate are basic requirements for tracking control [6]. This contradiction puts higher demands on the design of the controller. The controller not only overcomes delay, but also needs to be fast and accurate. Predictive control [7] and sliding mode control [8] have been successfully implemented in the networked tracking control system.

The main contribution of the letter is to extend the fuzzy inverse model control theory of professor Babuska [9] from local control to the networked control environment. Moreover, three fuzzy-inverse-model-based networked control frameworks are proposed to realize the control algorithm, especially the input-oriented control framework which can greatly improve the control performance in the networked control of time-varying signals. The letter is organized as follows: The core idea and strategy to solve the networked tracking control problem by using fuzzy inverse model technology is first given. Then three networked tracking control frameworks implementing the fuzzy inverse models are presented and the simulations of the proposed methods are illustrated. Finally, the conclusions are drawn.

Model and inverse model: Suppose a controlled process which can be depicted by a T-S fuzzy model

Corresponding author: Keya Yuan.

Citation: S. Tong, D. Qian, K. Yuan, D. Liu, Y. Li, and J. Zhang, "Fuzzy-inverse-model-based networked tracking control frameworks of time-varying signals," *IEEE/CAA J. Autom. Sinica*, vol. 11, no. 7, pp. 1708–1710, Jul. 2024.

S. Tong, K. Yuan, D. Liu, Y. Li, and J. Zhang are with the College of Robotics, Beijing Union University, Beijing 100101, China (e-mail: shiwen.tong@buu.edu.cn; jqrkeya@buu.edu.cn; liudexin@buu.edu.cn; zdhtliyuan@buu.edu.cn; jdtjiancheng@buu.edu.cn).

D. Qian is with the School of Control and Computer Engineering, North China Electric Power University, Beijing 102206, China (e-mail: dianwei.qian@ncepu.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/JAS.2024.124293

$$
\begin{aligned}
R_i:\ & \textbf{If } y(k) \text{ is } A_{i1} \textbf{ and } y(k-1) \text{ is } A_{i2} \textbf{ and } \ldots \\
& \textbf{and } y(k-n_y+1) \text{ is } A_{in_y} \textbf{ and } u(k) \text{ is } B_{i1} \textbf{ and} \\
& u(k-1) \text{ is } B_{i2} \textbf{ and } \ldots \textbf{ and } u(k-n_u+1) \text{ is } B_{in_u} \\
& \textbf{then } y(k+1) = \sum_{j=1}^{n_y} a_{ij}\, y(k-j+1) + \sum_{j=1}^{n_u} b_{ij}\, u(k-j+1)
\end{aligned} \tag{1}
$$

where $A_{i1},\ldots,A_{in_y}$, $B_{i1},\ldots,B_{in_u}$, $i=1,\ldots,c$ are fuzzy sets of the antecedent parts, and a_{ij}, b_{ij} are parameters of the consequent parts.

The model of such a system can be obtained from the fuzzy clustering technology by analysing the input-output data. The grades of memberships for the antecedent variables can be estimated by the G-K algorithm.

$$
\begin{aligned}
&\min_{(V,U,\mathcal{A})} \left\{ J(W;V,U,\mathcal{A}) = \sum_{i=1}^{c} \sum_{k=1}^{N} (\mu_{ik})^m D^2_{ik\mathcal{A}_i} \right\} \\
&D^2_{ik\mathcal{A}_i} = (z_k - v_i)^T \mathcal{A}_i (z_k - v_i)
\end{aligned} \tag{2}
$$

where $W=\{w_k \mid k=1,\ldots,N\}$ is observations, $\mathcal{A}=[\mathcal{A}_1,\ldots,\mathcal{A}_c]$ is a matrix of c induced norm, $U=\mu_{ik}$ is a $c\times N$ fuzzy partition matrix, m is a parameter relating fuzziness of the clusters, $D^2_{ik\mathcal{A}_i}$ is a squared distance norm, $V=[v_1,\ldots,v_c]$ is cluster prototypes,

$$
\mu_{ik} = \frac{1}{\sum_{j=1}^{c} (D_{ik\mathcal{A}_i}/D_{jk\mathcal{A}_i})^{2/(m-1)}} \tag{3}
$$

and the subsequent parameters can be identified by the L-S estimation method.

To obtain the inverse model, one method is to use fuzzy cluster modeling after swapping input and output data. Another option is to transform the process model into an identical fuzzy singleton model which make the inversion of the model be possible.

$$
\begin{aligned}
R_i:\ & \textbf{If } y(k) \text{ is } A'_{i1} \textbf{ and } y(k-1) \text{ is } A'_{i2} \textbf{ and } \ldots \\
& \textbf{and } y(k-p+1) \text{ is } A'_{ip} \textbf{ and } u(k) \text{ is } B'_{i1} \textbf{ and} \\
& u(k-1) \text{ is } B'_{i2} \textbf{ and } \ldots \textbf{ and } u(k-q+1) \text{ is } B'_{iq} \\
& \textbf{then } y(k+1) = C_i.
\end{aligned} \tag{4}
$$

Simplified equation (4) with the following form:

$$
\textbf{If } x(k) \text{ is } X \textbf{ and } u(k) \text{ is } B \textbf{ then } y(k+1) \text{ is } C. \tag{5}
$$

Then, $y(k+1)$, a model output, can be derived by (6)

$$
y(k+1) = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} \beta_{ij}(k) c_{ij}}{\sum_{i=1}^{M}\sum_{j=1}^{N} \beta_{ij}(k)} = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} \mu_{X_i(x(k))}\mu_{B_j(u(k))} c_{ij}}{\sum_{i=1}^{M}\sum_{j=1}^{N} \mu_{X_i(x(k))}\mu_{B_j(u(k))}}. \tag{6}
$$

Let $\otimes$ be a minimum operator, and $\oplus$ be a maximum operator, the invertibility conditions and the approach to get the control actions are directly given which can be found in the works of Professor Babuška Robert.

Invertibility conditions [9]: Define $b_j = \mathrm{core}(B_j)$. The fuzzy singleton model, described by the rule base (5) and the defuzzification method (6), is invertible if and only if:

1) The core b_j for each B_j is a single point, that is, $|B_j| = 1$, $j=1,\ldots,N$, and

2) $b_1 < \cdots < b_N \to c_{i1} < \cdots < c_{iN}$ or $c_{i1} > \cdots > c_{iN}$, $i=1,\ldots,M$.

Inverse of singleton fuzzy model [9]: Suppose the process described by an invertible fuzzy singleton model (5) following the defuzzification method (6). Moreover, define the partition of the antecedent membership functions. i.e., $\sum_{i=1}^{M}\mu_{X_i}(x)=1,\ \forall x$, and $\sum_{j=1}^{N}\mu_{B_j}(u)=1,\ \forall u$. The control action will be derived, for a given state $x(k)$, by the following rules on the basis of an inverse of the fuzzy singleton model:

$$
\textbf{If } r(k+1) \text{ is } C_j(k) \textbf{ then } u(k) \text{ is } B_j,\quad j=1,\ldots,N \tag{7}
$$

where $C_j: Y \to [0\ \ 1]$ are defined by the triangular form of membership functions

$$\begin{aligned}
&\mu_{C_1}(y) = \oplus(0, \otimes(1, \frac{c_2 - y}{c_2 - c_1})) \\
&\mu_{C_j}(y) = \oplus(0, \otimes(\frac{y - c_{j-1}}{c_j - c_{j-1}}, \frac{c_{j+1} - y}{c_{j+1} - c_j})), \quad 1 < j < N \\
&\mu_{C_N}(y) = \oplus(0, \otimes(\frac{y - c_{N-1}}{c_N - c_{N-1}}), 1)
\end{aligned} \tag{8}$$

where the cores c_j are calculated by

$$c_j = \sum_{i=1}^{M} \mu_{X_i}(x(k)) c_{ij}, \quad j = 1, \ldots, N \tag{9}$$

and are sorted by $c_1 \le \cdots \le c_N$. Fuzzy sets B_j are sorted consequently. Then, the inference of the rules (7) can be realized by

$$u(k) = \sum_{j=1}^{N} \mu_{C_j}(r(k+1)) b_j \tag{10}$$

where b_j are the cores of B_j.

Things to be considered in networks and their solutions: The most worth considering question in networks is the time-delay. Some other issues can be ultimately transformed into the time-delay problem to be concerned. For example, data packet dropout can be regarded as a special case of time-delay that the duration of it is infinite. Prediction is a feasible solution to the time-delay problem by generating a sequence of future control actions on the networked controller side and selecting the suitable control input from these control sequence received on the plant side.

The second worth concerning issue in networked control is how to get the future control actions. This can be done by iteration of a fuzzy inverse model which is from the fuzzy cluster modeling or other similar techniques [10]. Professor Babuska Robert proposed a basic method as mentioned earlier to set up the local fuzzy inverse model and gave a strict proof [9]. In fact, due to the disturbances, un-modeling dynamics and other uncertainties in the controlled process, generating a strict fuzzy inverse model of the process is unreliable and the difference between the model and the system is inevitable. Fortunately, feedback correction, internal model and adaptive strategies can be utilized to form the networked control frameworks based on fuzzy inverse models to narrow the differences.

Fuzzy-inverse-model-based networked tracking control frameworks: Fuzzy inverse model is a useful tool to cope with networked tracking control problems. Three basic control frameworks based on the fuzzy inverse model are proposed in this letter.

Internal model output-oriented networked tracking control framework: The first fuzzy-inverse-model based networked tracking control framework (see Fig. 1), is a basic structure that is composed by a fuzzy model predictor, a fuzzy inverse model networked controller and a delay compensator. The function of the fuzzy model predictor is to generate a sequence of calculated future process outputs by iterating an identical fuzzy model of the controlled system. The fuzzy inverse model networked controller calculates future control actions according to the error between the references and the results of predicted outputs subtracted by process output. Then, the candidate control actions are packed with the time stamp and sent from the controller side to the plant side through the network. On the plant side, the time delay, which can be estimated by comparing the time stamps of sending and receiving, can be compensated by selecting appropriate control action from the received candidate control actions. For example, if one-step delay is estimated, the next time control action of the controller side will be selected. By the same token, if three-step delay is calculated, the third future control action of the controller side will be chosen. As time goes on, the basic built fuzzy model will not be able to represent the process completely because of the disturbance, un-modeling dynamic and uncertainty etc. Internal model control strategy is introduced to compensate for the output difference between the controlled system and the built fuzzy model.

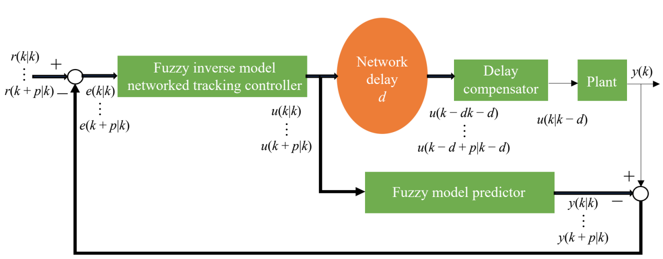

Fig. 1. Fuzzy-inverse-model-based networked tracking control framework 1.

Output-oriented networked tracking control framework with adaptation: Although the basic internal model strategy can improve the control performance to some extent, the parameters in the fuzzy model are kept constant all the time no matter how the process changes. This is clearly unreasonable. In this part, an adaptive fuzzy-inverse-model-based networked tracking control framework (see Fig. 2) has been proposed. Different from the basic networked tracking control framework, the subsequent parameters in the fuzzy model and the fuzzy inverse model are dynamically updated to make the fuzzy and fuzzy inverse model adapt to the process. Thus, the fuzzy and fuzzy inverse model are kept tuned from the beginning to the end. This scheme can further improve the control performance to some extent. It should be pointed out that the adaptive strategy can be utilized in the fuzzy model individually or both the fuzzy model and the fuzzy inverse model.

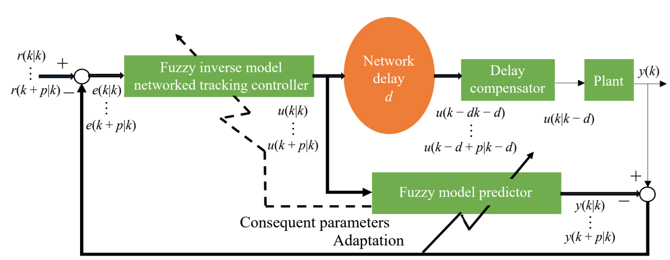

Fig. 2. Fuzzy-inverse-model-based networked tracking control framework 2.

Input-oriented networked tracking control framework with adaptation: The above two networked tracking control frameworks are output-oriented. In this part, an input-oriented networked tracking control framework with adaptation (see Fig. 3) is proposed. The characteristic of the framework only makes use of fuzzy inverse models as the internal model and the networked tracking controller. The error between the output of the fuzzy inverse model in the feedback channel and the output of the fuzzy inverse model networked tracking controller in the forward channel is utilized to adjust the subsequent parameters in the fuzzy inverse models. Since the error is from the difference of the control inputs, it is called as input-oriented fuzzy-inverse-model-based networked tracking control framework.

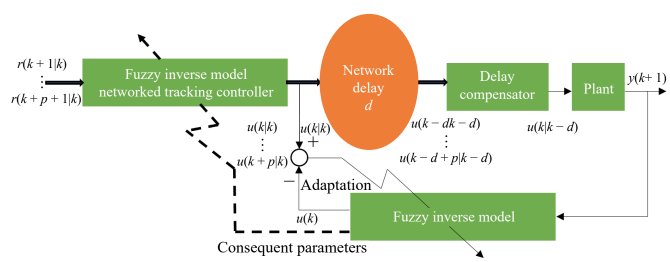

Fig. 3. Fuzzy-inverse-model-based networked tracking control framework 3.

Comparing the three control frameworks, it can be seen that the first two frameworks are calibrated at the output end of the system, while the third framework is calibrated at the input end of the process. In terms of timeliness, the input-oriented framework is more timely than the output-oriented structure, which can quickly improve control performance. From the perspective of model matching, the input correction uses the same model for both the forward and feedback channels, while the output correction uses different models. Therefore, the input correction can better achieve ideal control performance.

Simulations: To validate the control performance of the proposed control frameworks, the servo control system which is often used for comparison of the networked control performance is adopted. The model of the controlled system is presented as follows [11]:

$$\mathbb{G}(z^{-1}) = \frac{-0.0086z^{-1}+1.268227z^{-2}}{1-1.66168z^{-1}+0.6631z^{-2}}. \tag{11}$$

Suppose the random delay in the forward channel is described by Fig. 4, simulations are carried out under the three networked tracking control frameworks separately. Two kinds of reference signals are adopted. One is multi-sine signal and the other is step signal. The control performance in different control frameworks is shown in Fig. 5 and the control action comparison is presented in Fig. 6. In each figure, the left side subfigures are sine signal responses and the right side subfigures are step signal dynamics. In every subfigure, the red solid line is the reference signal, the blue dash line is the output of the fuzzy-inverse-model-based networked tracking controller, the orange dash dot line is the result of NPOTC method [7] and the green dot line is the PID result. From the two graphs, it can be seen that the three framework algorithms and NPOTC algorithm all exhibit timeliness characteristics. The NOPTC algorithm exhibits better performance in following step signals, but the amplitude of the control action is somewhat large. When the amplitude change of time-varying signal is small, the PID controller, although with poor control accuracy, could timely track dynamic changes. However, when the change in signal amplitude is large, the PID controller become more sluggish in response. For quantitative comparison, the cumulative error squared indicator is introduced. The comparison results are shown in Table 1. It can be seen that the performance of the framework 3 is the best for tracking the multi-sine signal, both the accuracy and the amplitude of the control action. Then is the NOPTC algorithm, result of the framework 2, output of the framework 1. The PID is the last. This is due to the adaptive strategy for its adaptation and the input-oriented scheme for its timeliness, which is consistent with our theoretical analysis. For tracking the step signal, NOPTC algorithm exhibits rapid response although the control action is a bit large. The other three algorithms are roughly equivalent. Therefore, we should compromise between control performance and control action.

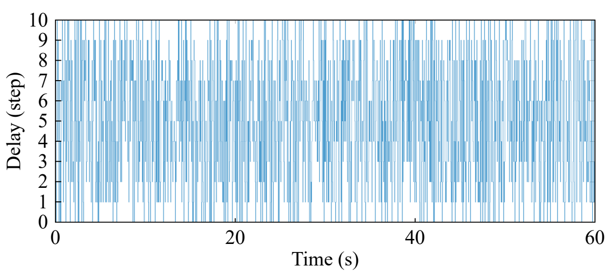

Fig. 4. Random delay in the forward channel.

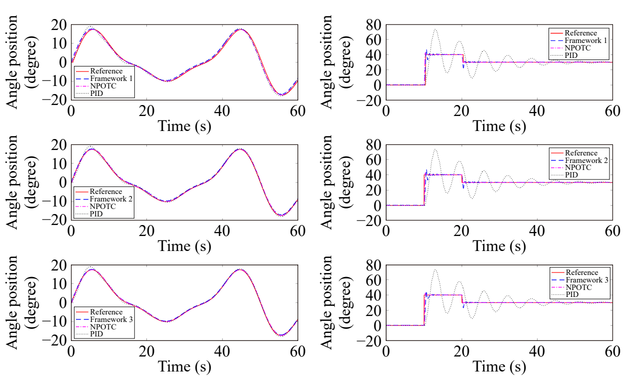

Fig. 5. Control performance comparison.

Conclusions: In this letter, three fuzzy-inverse-model-based networked tracking control frameworks have been presented. Of the three frameworks, two are outputs-oriented method and one is inputs oriented approach. The outputs-oriented method requires a fuzzy model predictor and a fuzzy inverse mode controller simultaneously and the parameters are adjusted according to the difference of outputs between the process and the built model. But the inputs-oriented approach only needs the fuzzy inverse model and the parameters are regulated on the basis of the dissimilarity of the control inputs from the fuzzy inverse models. Adaptive technology is an effective method for improving control performance. Simulations have proved the improvement of the control performance.

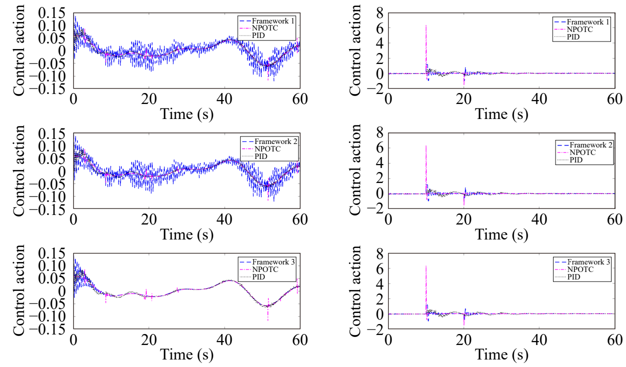

Fig. 6. Control action comparison.

Table 1. Control Performance Comparison

	Framework 1	Framework 2	Framework 3	NPOTC	PID
Sine	188.7096	188.1536	132.8641	151.1954	458.3568
Step	1.0553E+4	1.0562E+4	1.0399E+4	5.1136E+3	1.5051E+4

In the subsequent research, scholar can focus on the delay compensation in the feedback channel or consideration of both the forward and the feedback channels on the basis of these three frameworks. Some difficult points such as invertibility condition of different membership functions, except the triangular form of membership functions, extension from SISO to MIMO system are also concerned.

Acknowledgments: This work was partially supported by the Teaching Reform Project of BUU (JJ2022Z18) and the National Key R&D Program Project (2022YFB4601104).

References

[1] Q. Mao, Y. Xu, J. Chen, and T. T. Georgiou, "Implementation-oriented filtered PID control: Optimization of robustness margins," *Automatica*, vol. 152, p. 110974, 2023.

[2] I. Nașcu, N. A. Diangelakis, S. G. Muñoz, and E. N. Pistikopoulos, "Advanced model predictive control strategies for evaporation processes in the pharmaceutical industries," *Computers & Chemical Engineering*, vol. 173, p. 108212, 2023.

[3] V. C. Jorge, P. L. Andres, S. Víctor, F. H. Ramón, and Z. R. Arturo, "Continuous finite-time fuzzy control of mechanical systems via non-Lipschitz membership functions," *Engineering Applications of Artificial Intelligence*, vol. 126, p. 106804, 2023.

[4] S. Das, G. T. Khan, and N. P. Sing, "Adaptive neural network control of DC-DC power converter," *Expert Systems With Applications*, vol. 229, p. 120362, 2023.

[5] Z. Yan, X. Lai, Q. Meng, J. She, and M. Wu, "Time-varying disturbance-observer-based tracking control of uncertain flexible-joint manipulator," *Control Engin. Practice*, vol. 139, p. 105624, 2023.

[6] Z. Qu, Z. Du, Y. Kao, and Z. Wu, "Fuzzy event-triggered tracking control for nonlinear unreliable networked systems," *ISA Trans.*, vol. 133, pp. 205–217, 2022.

[7] Z. Pang, G. Liu, D. Zhou, and M. Chen, "Output tracking control for networked systems: A model-based prediction approach," *IEEE Trans. Industrial Electronics*, vol. 61, no. 9, pp. 4867–4877, 2014.

[8] X. Liu, X. Yu, G. Ma, and H. Xi, "On sliding mode control for networked control systems with semi-Markovian switching and random sensor delays," *Inform. Sciences*, vol. 337, pp. 44–58, 2016.

[9] R. Babuska, *Fuzzy Modeling for Control*, New York, USA: Springer Science+Business Media, LLC, 1998.

[10] S. Tong, D. Qian, N. Huang, G. Liu, J. Zhang, and G. Cheng, "Adaptive tracking design of NCS with time-varying signals using fuzzy inverse model," *Int. J. Control Autom. and Systems*, vol. 19, no. 11, pp. 3801–3811, 2021.

[11] S. Tong, D. Qian, X. Yan, J. Fang, and W. Liu, "Design of a networked tracking control system with a data-based approach," *IEEE/CAA J. Autom. Sinica*, vol. 6, no. 5, pp. 1261–1267, 2019.