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# Fixed-Time Sliding Mode Control for Vehicle Platoon with Input Dead-Zone and Prescribed Performance

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**ABSTRACT** This article studies the fixed-time sliding mode control (FTSMC) issue for vehicle platoon with prescribed performance, input dead-zone (IDZ), unknown nonlinearities, and external disturbances. Unlike the traditional funnel-shaped prescribed performance function, a new tunnel fixed-time prescribed performance function (TFTPPF) is proposed, which defines more compact tunnel-shaped performance boundaries that can accelerate convergence speed and reduce overshoot of tracking error. The convergence time of TFTPPF is predefined and is independent of control parameters, which enhances the design flexibility of performance function. Furthermore, Chebyshev neural network (CNN) is adopted to approximate unknown nonlinearities, and new adaptive mechanisms are designed to estimate IDZ slope and external disturbances. Utilizing the proposed TFTPPF, CNN, and adaptive mechanisms, a novel FTSMC strategy for vehicle platoon is proposed to ensure string stability and achieve predefined tracking performance within a fixed time. Finally, comparative numerical examples are employed to validate the validity and advantages of the proposed strategy.

**INDEX TERMS** Vehicle platoon control, input dead-zone, fixed-time sliding mode, unknown nonlinearities, prescribed performance control.

## I. INTRODUCTION

Recently, the rapid increase in the number of automobiles has caused numerous traffic issues, including traffic congestion, energy consumption, and exhaust emissions [1]–[4]. Vehicle platoon control is receiving widespread attention for its benefits in enhancing traffic efficiency, improving driving safety, optimizing resource utilization efficiency, and reducing carbon emissions [5]–[7]. Many control issues of vehicle platoon have been studied, including heterogeneity in dynamics [8], information flow topology [9], spacing policy [10], etc.

Vehicles are inevitably impacted by external disturbances and unknown nonlinearities, potentially compromising the control performance of vehicle platoon [11]–[14]. To mitigate the impact of external disturbances and unknown nonlinearities on system performance, several methods have been developed, for example iterative proportional-integral observer [15], [16],  $H_\infty$  control [17], model predictive control [18], and disturbance observer [19], [20], etc. In these studies, due to the fact that sliding mode control (SMC) possesses

strong robustness and insensitivity to nonlinearities and disturbances, it is widely applied in vehicle platoon control [21], [22]. In [21], a distributed adaptive SMC strategy is designed for vehicle platoon with unknown nonlinearities to guarantee that follower vehicles track the leader well. In [22], an integral SMC is designed to ensure that vehicle maintains predefined spacing from the adjacent vehicles. However, one issue with the aforementioned traditional SMC is that they are unable to predesign convergence time of tracking error for vehicle platoon, which may lead to delayed system response, increased accident risks, and reduced road safety in complex traffic environments [23], [24]. Therefore, it is necessary to develop a new SMC strategy for vehicle platoon to ensure convergence of tracking error within a fixed time.

The input dead-zone (IDZ) frequently occurs in the actuators of vehicle platoon, which reduces the system precision and may even result in system instability [25]–[27]. To mitigate the negative impacts of IDZ on the vehicle platoon, researchers have conducted extensive research on it [28], [29].

In [28], by assuming that IDZ slope is a positive constant, an adaptive strategy is designed to achieve velocity synchronization with the leader. In [29], a feedback linearization strategy is proposed to achieve control objectives based on the assumption that IDZ slope is strictly monotonic. It is worth noting that the practical applications of the above methods are constrained by the assumption of IDZ slope. Additionally, the lack of performance constraints in the vehicle platoon may lead to deviations between the actual system performance and expectations, or even result in failure to meet the expected performance indicators.

To ensure that the vehicle platoon possesses predetermined performance, prescribed performance control (PPC) is developed, which utilizes the predetermined performance indicators as controller parameters to guarantee that the system achieves the desired performance [30], [31]. In [30], a distributed PPC strategy for vehicle platoon is developed to guarantee that tracking error does not exceed a predetermined boundary, the convergence speed is no less than a given lower bound, and the steady-state error converges within a predefined range. In [31], PPC method is applied in vehicle platoon with input delay to ensure that the system possesses predefined performances. The convergence time of prescribed performance function (PPF) in traditional PPC approaches infinity. However, in most applications, it is desirable for system state to reach a predefined range within a predetermined time [32]. In [33], a fixed-time PPC strategy is designed to guarantee that tracking error of vehicle platoon converges to a predefined range within a fixed time. It is worth noting that the performance boundaries of the aforementioned PPC are distributed on different sides of the coordinate system and present a loose funnel shape, which cannot effectively ensure that the overshoot of tracking error is small or zero. Therefore, it is essential to develop a new PPC strategy to reduce overshoot, while enhancing system performance.

In light of the aforementioned discussion, a novel control strategy for vehicle platoon with prescribed performance, IDZ, unknown nonlinearities, and external disturbances is developed to guarantee string stability and achieve predefined tracking performance within a fixed time. Chebyshev neural network (CNN) is utilized to approximate unknown nonlinearities and new adaptive mechanisms are designed to estimate IDZ slope and external disturbances, thereby effectively reducing the negative impacts of unknown nonlinearities, IDZ, and external disturbances on the system performance. The advantages of this scheme are outlined below.

- 1) A new tunnel fixed-time prescribed performance function (TFTPPF) is proposed, which defines performance boundaries that are located on the same side of the coordinate system and are more compact, thereby reducing the overshoot of vehicle platoon and enhancing the tracking performance.
- 2) CNN is employed to approximate unknown nonlinearities in vehicle platoon, and adaptive mechanisms are designed to estimate IDZ slope and external disturbances, which effectively mitigate the adverse impacts of un-

known nonlinearities, IDZ, and external disturbances on system performance.

- 3) Based on the proposed TFTPPF, CNN, and adaptive mechanisms, a novel fixed-time sliding mode control (FTSMC) strategy is designed to ensure string stability of vehicle platoon and achieve predetermined tracking performance within a fixed time.

The remainder of this paper is organized as follows. Section II introduces the dynamic model of vehicle and PPC. Section III gives the design process of the controller and the stability analysis. In Section IV, the numerical examples illustrate the validity and superiority of the designed strategy. Section V summarizes the conclusions.

## II. PROBLEM STATEMENT

### A. VEHICLE DYNAMICS

As depicted in Figure. 1, a vehicle platoon is comprised of one leader (vehicle 0) and  $N$  follower vehicles, and each vehicle can obtain information from adjacent vehicles. The dynamic model of the following vehicle  $i$  ( $i = 1, 2, \dots, N$ ) is as follow

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= a_i(t) \\ \dot{a}_i(t) &= Dz(u_i(t)) + D_i(t) - \frac{1}{m_i\tau_i} \left[ \rho_a A_i C_{di} \left( \frac{1}{2} v_i^2(t) \right) \right. \\ &\quad \left. + \tau_i v_i(t) a_i(t) + \wp_i \right] - \frac{1}{\tau_i} a_i(t) \\ &= Dz(u_i(t)) + D_i(t) + f_i(v_i(t), a_i(t)) \end{aligned} \quad (1)$$

where  $x_i(t)$ ,  $v_i(t)$ , and  $a_i(t)$  denote the  $i$ th vehicle's position, velocity, and acceleration, respectively;  $m_i$  represents mass;  $\tau_i$  is the engine time constant;  $\rho_a$  denotes air density;  $A_i$  represents the cross-sectional area; the road slope term  $\wp_i = m_i g \sin \phi + m_i g \ell_i \cos \phi$ , with  $g$  being the acceleration of the gravity,  $\phi$  represents the angle of the road slope, and  $\ell_i$  denotes the rolling resistance coefficient;  $C_{di}$  denotes the air drag coefficient; the external disturbances  $D_i(t)$  are bounded as  $|D_i(t)| \leq \bar{D}_i$ , with  $\bar{D}_i > 0$ . In practice, since  $\rho_a, A_i, C_{di}, \tau_i, \ell_i$ , and  $\phi$  are often difficult or even impossible to obtain directly, the nonlinearities  $f_i(v_i(t), a_i(t))$  are assumed to be unknown.  $Dz(u_i(t))$  is the control input as

$$Dz(u_i(t)) = \begin{cases} z_{i,r}(u_i(t) - \gamma_{i,r}), & u_i(t) \geq \gamma_{i,r} \\ 0, & -\gamma_{i,l} < u_i(t) < \gamma_{i,r} \\ z_{i,l}(u_i(t) + \gamma_{i,l}), & u_i(t) \leq \gamma_{i,l} \end{cases} \quad (2)$$

where  $\gamma_{i,r}$  and  $\gamma_{i,l}$  denote the breakpoints of IDZ,  $z_{i,r}$  and  $z_{i,l}$  are the IDZ slopes. To streamline the notation, we adopt  $\bullet$  as a representation for  $\bullet(\cdot)$ .

The leader can be described as

$$\begin{aligned} \dot{x}_0 &= v_0 \\ \dot{v}_0 &= a_0 \end{aligned} \quad (3)$$

To ensure traffic safety, a quadratic spacing policy (QSP) is designed as

$$e_i = d_i - w_i - h_1 v_i - h_2 v_i^2 - \mu \quad (4)$$

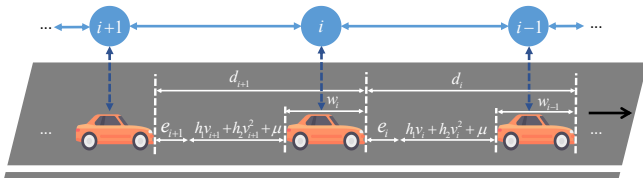


FIGURE 1: The communication topology of vehicle platoon.

where  $d_i = x_{i-1} - x_i$ ,  $e_i$  is the tracking error,  $w_i$  represents the length of vehicle,  $h_1 > 0$ ,  $h_2 > 0$ , and  $\mu$  denotes the standstill spacing.

## B. PRESCRIBED PERFORMANCE

For traditional PPC, the performance boundaries are given by [34]

$$\begin{cases} -\Omega \Xi_i \leq e_i \leq \Xi_i, e_i(0) \geq 0 \\ -\Xi_i \leq e_i \leq \Omega \Xi_i, e_i(0) < 0 \end{cases} \quad (5)$$

where  $\Xi_i = (\Xi_{i,0} - \Xi_{i,\infty})e^{-\partial_i t} + \Xi_{i,\infty}$  denotes PPF;  $\partial_i > 0$ ,  $\Omega$  is a design parameter with  $0 < \Omega \leq 1$ ,  $\Xi_{i,\infty} = \lim_{t \rightarrow \infty} \Xi_i(t) > 0$ , and  $\Xi_{i,0} = \Xi_i(0) > e_i(0) \geq 0$ .

According to (5), the tracking trajectories of  $e_i$  and PPF are presented in Figure. 2. From Figure. 2, although traditional PPC can ensure that  $e_i$  converges within the range defined by the performance boundaries, there are still some problems. 1) The performance boundaries are situated on different sides of the coordinate system, presenting a loose funnel shape, which is not conducive to achieving small overshoot or zero overshoot of  $e_i$ . 2) From (5), different initial conditions may lead to controller redesign, which is detrimental to the controller design. 3) The convergence time of traditional PPF tends to infinity. However, in most practical applications, it is desired that the system state can reach a predefined range within a predetermined time.

To address the aforementioned issues, the new performance boundaries are proposed as

$$\rho_{i,l} < e_i < \rho_{i,r} \quad (6)$$

where  $\rho_{i,l} = [\kappa_1 \text{sign}(e_i(0)) - \varsigma_1] \delta_i - \delta_{i,tf} \text{sign}(e_i(0))$  and  $\rho_{i,r} = [\kappa_2 \text{sign}(e_i(0)) + \varsigma_2] \delta_i - \delta_{i,tf} \text{sign}(e_i(0))$  are the TFTPFF, with  $0 < \kappa_1 < 1$ ,  $0 < \kappa_2 < 1$ ,  $0 < \varsigma_1 < 1$ ,  $0 < \varsigma_2 < 1$ ;  $\delta_i$  is defined as

$$\delta_i = \begin{cases} (\delta_{i,0} - \delta_{i,tf})e_i^{-\varphi_i t} + \delta_{i,tf} & 0 < t < t_f \\ \delta_{i,tf} & t \geq t_f \end{cases}$$

where  $\delta_{i,0} = \lim_{t \rightarrow 0} \delta_i > 0$ ,  $\varphi_i > 0$ , and  $\delta_{i,tf} = \lim_{t \rightarrow t_f} \delta_i > 0$ ;  $t_f > 0$  is the convergence time. Based on (6), the tracking trajectories of  $e_i$  and TFTPFF are shown in Figure. 3. Unlike (5), (6) is more concise in form without considering the sign of  $e_i$ , which simplifies the controller design. From Figure. 3, the proposed tunnel performance boundaries are more compact, which reduces overshoot of tracking error and enhances system performance. Furthermore,  $t_f$  can be predefined, independent of initial conditions or control parameters, which improves the design flexibility of PPF.

As given in (6),  $e_i$  is constrained, making it unable to be directly applied in the controller design. Therefore,  $e_i$  is transformed into an equivalent and unconstrained  $\omega_i$  as

$$\omega_i = \ln \left( \frac{\theta_i}{1 - \theta_i} \right) \quad (7)$$

where  $\theta_i = [e_i - \rho_{i,l}] / [\rho_{i,r} - \rho_{i,l}]$ .

*Theorem 1:* If  $\omega_i$  is bounded, then  $e_i$  can converge to the predefined range defined by TFTPFF.

*Proof:* (7) can be rewritten as

$$e^{\omega_i} = \frac{\theta_i}{1 - \theta_i} \quad (8)$$

Then, we have

$$\theta_i = \frac{e^{\omega_i}}{1 + e^{\omega_i}} \quad (9)$$

As  $\omega_i$  is bounded, there exists  $\bar{\omega}_i > 0$  such that  $-\bar{\omega}_i \leq \omega_i \leq \bar{\omega}_i$  holds. Then, one can obtain

$$0 < \frac{e^{-\bar{\omega}_i}}{1 + e^{-\bar{\omega}_i}} \leq \theta_i \leq \frac{e^{\bar{\omega}_i}}{1 + e^{\bar{\omega}_i}} < 1 \quad (10)$$

Based on (7) and (10), one has

$$0 < \frac{e_i - \rho_{i,l}}{\rho_{i,r} - \rho_{i,l}} < 1 \quad (11)$$

Then the inequality (6) holds. The proof is completed.

## C. CONTROL OBJECTIVES

The purpose of this paper is to design a strategy for vehicle platoon to achieve the following objectives.

- Individual stability [35]: The vehicle maintains the desired spacing from the adjacent vehicles, and achieves the synchronization of velocity and acceleration with the leader within a fixed time.
- String stability [36]:  $e_i$  does not amplify along the vehicle platoon, that is

$$\left| \frac{E_{i+1}(s)}{E_i(s)} \right| \leq 1, \quad i = 1, 2, \dots, N \quad (12)$$

where  $E_i(s)$  represents the Laplace transform of  $e_i$ .

- Prescribed tracking performance [37]: The vehicle platoon achieves predefined tracking performance, i.e.,  $\rho_{i,l} < e_i < \rho_{i,r}$ .

*Remark 1:* One of the objectives in this paper is to ensure that  $e_i$  evolves within the range defined by performance boundaries and ultimately converges to a neighborhood of zero, i.e., for  $t \geq 0$ ,  $\rho_{i,l}(t) < e_i(t) < \rho_{i,r}(t)$ . Therefore, when  $t = 0$ , the selected parameters should ensure that  $e_i(0)$  is within the performance boundaries. As  $t$  tends to infinity, the chosen parameters should ensure that the performance boundaries approach zero. Specifically,  $\rho_{i,l}(\infty)$  and  $\rho_{i,r}(\infty)$  are small constants and satisfy the conditions that  $\rho_{i,l}(\infty) < 0$  and  $\rho_{i,r}(\infty) > 0$ .

*Remark 2:* According to (5) and Figure. 2, we can conclude that the performance boundaries of traditional PPF are conservative and the convergence time cannot be predefined, which is detrimental to enhancing system performance. Therefore,

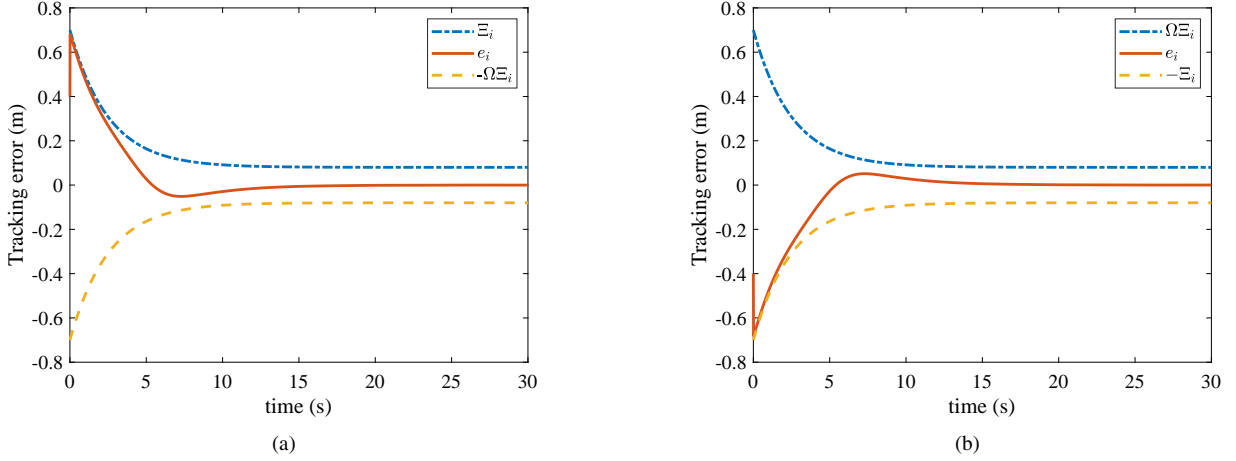


FIGURE 2:  $e_i$  under traditional PPC. (a)  $e_i(0) \geq 0$ . (b)  $e_i(0) < 0$ .

the TFTPFF is developed, which features more compact performance boundaries and allows for preset convergence time, thereby reducing tracking error overshoot, improving convergence speed, and enabling better transient and steady-state performance for vehicle platoon.

Before designing the controller, we introduce in advance some useful lemmas.

**Lemma 1:** [38] Consider the following system

$$\dot{x} = -\alpha \text{sig}^{\bar{\lambda}_2}(x) - \beta \text{sig}^{\bar{h}_2}(x) \quad (13)$$

where  $\alpha > 0, \beta > 0$ ;  $\bar{\lambda}_1, \bar{\lambda}_2, \bar{h}_1, \bar{h}_2$  satisfy  $\bar{\lambda}_1 > \bar{\lambda}_2 > 0, \bar{h}_2 > \bar{h}_1 > 0$ ;  $\text{sig}^{\mathfrak{S}}(x) = |x|^{\mathfrak{S}} \text{sign}(x)$  with  $\mathfrak{S} > 0$ .

Then, (13) is fixed-time stable (FTS) and the convergence time

$$T_1 \leq \bar{T}_1 = \frac{1}{\alpha} \frac{\bar{\lambda}_2}{\bar{\lambda}_1 - \bar{\lambda}_2} + \frac{1}{\beta} \frac{\bar{h}_1}{\bar{h}_1 - \bar{h}_2} \quad (14)$$

**Lemma 2:** [22] For  $\tau \in \mathbb{R}$  and  $\forall \mu \geq 0$ , the following inequality holds

$$0 \leq |\mu| - \mu \tanh\left(\frac{\mu}{\tau}\right) \leq \kappa \tau \quad (15)$$

where  $\kappa = 0.2785$ .

**Lemma 3:** [39] Consider  $\tau_1, \tau_2, \dots, \tau_H > 0$ , then

$$\sum_{i=1}^H \tau_i^y \geq \left(\sum_{i=1}^H \tau_i\right)^y, 0 < y \leq 1 \quad (16)$$

$$\sum_{i=1}^H \tau_i^y \geq H^{1-y} \left(\sum_{i=1}^H \tau_i\right)^y, 1 < y < \infty$$

where  $y > 0$ .

**Lemma 4:** [40] If there is a Lyapunov function  $V(x)$  satisfies

$$\dot{V}(x) \leq -\chi V^{o_1}(x) - \eta V^{o_2}(x) + \sigma \quad (17)$$

where  $o_1 > 1, 0 < o_2 < 1, \chi > 0, \eta > 0$ , and  $\sigma > 0$ .

Then, the system is FTS and the convergence time  $T_2$  satisfies

$$T_2 \leq \bar{T}_2 = \frac{1}{\chi(o_1 - 1)U} + \frac{1}{\eta(1 - o_2)U} \quad (18)$$

where  $0 < U < 1$ .

**Lemma 5:** [33] The following inequality holds

$$p_1 p_2 \leq \frac{\vartheta^{g_1}}{g_1} |p_1|^{L_1} + \frac{1}{L_2 \vartheta^{L_2}} |p_2|^{L_2} \quad (19)$$

where  $\vartheta > 0, p_1 > 0, p_2 > 0, g_1 > 1, g_2 > 1$ , and  $(g_1 - 1)(g_2 - 1) = 1$ .

**Lemma 6:** [41] The CNN is a functional connectivity network based on Chebyshev polynomials (CP), which has been proven capable of approximating nonlinear systems. Therefore, for a continuous nonlinear function  $f(Z): \mathbb{R}^n \rightarrow \mathbb{R}, f(Z)$  can be described by CNN as

$$f(Z) = W^* \zeta(Z) + \varepsilon \quad (20)$$

where  $Z \in \mathbb{R}^n$  denotes the input vector;  $\varepsilon$  is approximation error;  $W^* \in \mathbb{R}^{1 \times (nM_1 + 1)}$  represents the optimal weight vector with  $M_1$  being the order of CP;  $\zeta(Z) \in \mathbb{R}^{(nM_1 + 1) \times 1}$  denotes basis function, which is described as

$$\zeta(Z) = [1, \zeta_1(z_1), \dots, \zeta_{M_1}(z_1), \dots, \zeta_1(z_n), \dots, \zeta_{M_1}(z_n)]^T$$

where  $\zeta_k(z_j)$  ( $k = 1, \dots, M_1; j = 1, \dots, n$ ) can be derived through the following two-term recursive formula

$$\zeta_{k+1}(z_j) = 2z_j \zeta_k(z_j) - \zeta_{k-1}(z_j)$$

$$\zeta_0(z_j) = 1, \zeta_1(z_j) = z_j, z_j \in \mathbb{R}$$

**Lemma 7:** [42] The following inequality holds

$$G(M - G)^y \leq \frac{y}{y+1} (G^{y+1} - M^{y+1}), y \geq 1 \quad (21)$$

$$G(M - G)^{y+1} \leq \frac{y+1}{y+2} (G^{y+2} - M^{y+2}), 0 < y < 1$$

where  $G \geq M \geq 0$ , and  $y > 0$ .

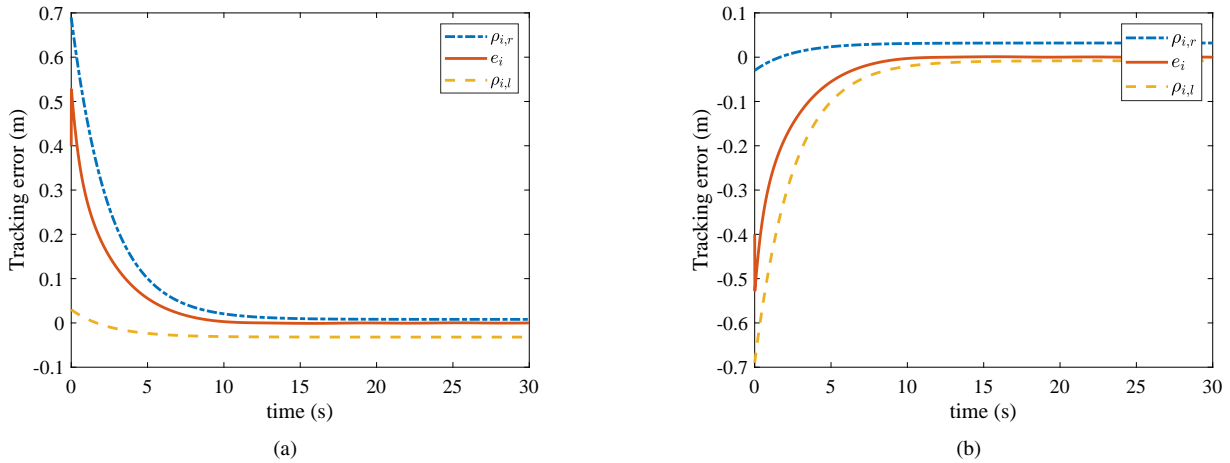


FIGURE 3:  $e_i$  under TFTPFF. (a)  $e_i(0) \geq 0$ . (b)  $e_i(0) < 0$ .

### III. CONTROLLER DESIGN AND STABILITY ANALYSIS

In this section, the vehicle platoon strategy is designed to achieve aforementioned objectives. Before controller design, (2) is converted into an input-related function as

$$Dz_i(u_i) = z_i u_i + \gamma_i \quad (22)$$

where

$$z_i = \begin{cases} z_{i,r} u_i \geq 0 \\ z_{i,l} u_i < 0 \end{cases} \quad \text{and} \quad \gamma_i = \begin{cases} -z_{i,r} \gamma_{i,r}, & u_i \geq \gamma_{i,r} \\ -z_i u_i, & -\gamma_{i,l} < u_i < \gamma_{i,r} \\ z_{i,l} \gamma_{i,l}, & u_i \leq -\gamma_{i,l}. \end{cases}$$

According to (22), we can obtain

$$|\gamma_i| \leq \bar{\gamma}_i \quad (23)$$

where  $\bar{\gamma}_i = \max\{|z_{i,r} \gamma_{i,r}|, |z_{i,l} \gamma_{i,l}|\}$ .

Define a sliding mode surface as

$$s_i = \dot{\omega}_i + \alpha_1 \text{sig}^{\frac{\bar{m}_1}{n_1}}(\omega_i) + \alpha_2 \text{sig}^{\frac{\bar{m}_2}{n_2}}(\omega_i) \quad (24)$$

where  $\alpha_1 > 0$ ,  $\alpha_2 > 0$ ,  $\bar{m}_1 > \bar{n}_1 > 0$ , and  $\bar{n}_2 > \bar{m}_2 > 0$ .

To ensure string stability, a coupled sliding mode surface is developed as

$$S_i = \begin{cases} q s_i - s_{i+1}, & i = 1, \dots, N-1 \\ q s_i, & i = N \end{cases} \quad (25)$$

where  $0 < q \leq 1$  is coupled weight factor and  $s_{N+1} = 0$ .

Then, we have

$$S = Qs \quad (26)$$

where

$$Q = \begin{bmatrix} q & -1 & \cdots & 0 & 0 \\ 0 & q & -1 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & \cdots & q & -1 \\ 0 & 0 & \cdots & 0 & q \end{bmatrix}$$

$$s = [s_1 \quad s_2 \quad \cdots \quad s_N]^T$$

$$S = [S_1 \quad S_2 \quad \cdots \quad S_N]^T.$$

Differentiating (26) yields

$$\begin{aligned} \dot{S}_i &= q \dot{s}_i - \dot{s}_{i+1} \\ &= q \left[ \frac{\bar{m}_1}{\bar{n}_1} \alpha_1 \text{sig}^{\frac{\bar{m}_1 - \bar{n}_1}{\bar{n}_1}}(\omega_i) \dot{\omega}_i + \frac{\bar{m}_2}{\bar{n}_2} \alpha_2 \text{sig}^{\frac{\bar{m}_2 - \bar{n}_2}{\bar{n}_2}}(\omega_i) \dot{\omega}_i \right. \\ &\quad \left. + \dot{A}_i (\dot{e}_i + \Pi_i) + A_i (\ddot{e}_i + \dot{\Pi}_i) \right] - \dot{s}_{i+1} \end{aligned} \quad (27)$$

where  $\dot{\omega}_i = A_i [\dot{e}_i + \Pi_i]$ ,  $\Pi_i = \frac{e_{i,l} \dot{e}_{i,r} - \dot{e}_{i,l} e_{i,r} - e_i (\dot{\rho}_{i,r} - \dot{\rho}_{i,l})}{e_{i,r} - e_{i,l}}$ , and

$$A_i = \frac{1}{(1-\theta_i)(\rho_{i,r} - \rho_{i,l})\theta_i} > 0.$$

For  $i = N$ , we have

$$\begin{aligned} \dot{S}_N &= q \dot{s}_N \\ &= q \left[ \frac{\bar{m}_1}{\bar{n}_1} \alpha_1 \text{sig}^{\frac{\bar{m}_1 - \bar{n}_1}{\bar{n}_1}}(\omega_N) \dot{\omega}_N + \frac{\bar{m}_2}{\bar{n}_2} \alpha_2 \text{sig}^{\frac{\bar{m}_2 - \bar{n}_2}{\bar{n}_2}}(\omega_N) \dot{\omega}_N \right. \\ &\quad \left. + \dot{A}_N (\dot{e}_N + \Pi_N) + A_N (\ddot{e}_N + \dot{\Pi}_N) \right] \end{aligned} \quad (28)$$

Define the reaching law as

$$\dot{S}_i = -\beta_1 \text{sig}^{r_1}(S_i) - \beta_2 \text{sig}^{l_1}(S_i) \quad (29)$$

where  $\beta_1 > 0$ ,  $\beta_2 > 0$ ,  $r_1 > 1$ , and  $0 < l_1 < 1$ .

According to Lemma 6,  $f_i(v_i(t), a_i(t))$  can be approximated by CNN as

$$f_i(v_i(t), a_i(t)) = W_i^* \zeta_i(Z_i) + \varepsilon_i \quad (30)$$

where  $W_i^*$  denotes the optimal weight vector;  $\zeta_i(Z_i)$  denotes basis function;  $\varepsilon_i$  is approximation error satisfying  $|\varepsilon_i| \leq \bar{\varepsilon}_i$  with  $\bar{\varepsilon}_i > 0$ ;

As  $z_i$ ,  $v_i$ , and  $\varpi_i$  are often difficult or even impossible to obtain directly in practice, the adaptive mechanisms are developed to estimate them as

$$\begin{aligned} \dot{\hat{z}}_i &= v_{zi} (-F_i S_i u_i - k_1 \hat{z}_i^{r_1} - k_2 \hat{z}_i^{l_1+1}) \\ \dot{\hat{l}}_i &= v_{li} (F_i S_i \tanh\left(\frac{S_i}{\xi_i}\right) - k_3 \hat{l}_i^{r_1} - k_4 \hat{l}_i^{l_1+1}) \\ \dot{\hat{\varpi}}_i &= v_{\varpi i} (F_i^2 S_i^2 \frac{\vartheta_i^2}{2} \zeta_i^T \zeta_i - k_5 \hat{\varpi}_i^{r_1} - k_6 \hat{\varpi}_i^{l_1+1}) \end{aligned} \quad (31)$$

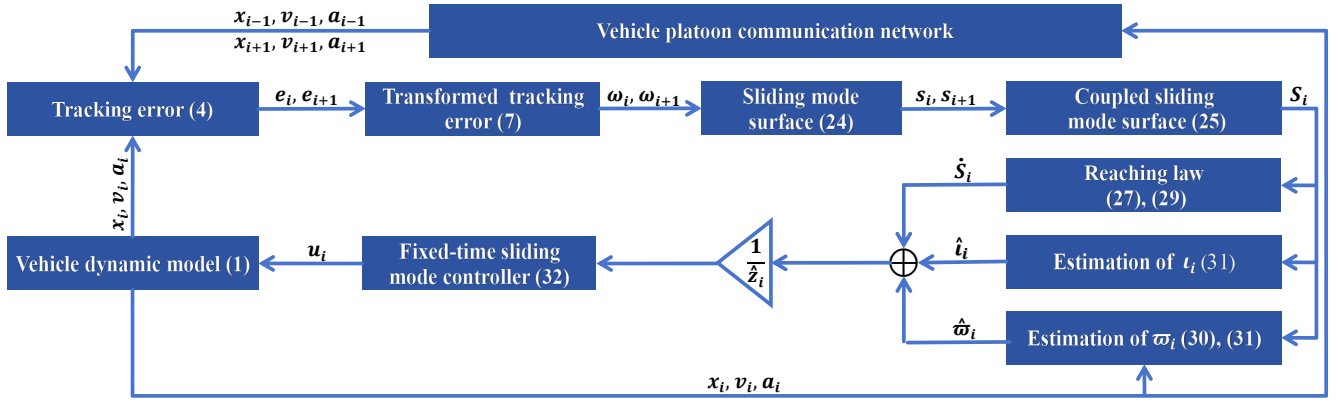


FIGURE 4: The design block diagram of the controller.

where  $F_i = qA_i(h_1 + 2h_2v_i)$ ,  $\varpi_i = \|W_i^*\|^2 = W_i^{*T}W_i^*$ ,  $\iota_i \geq |\bar{D}_i + \bar{\gamma}_i + \bar{\varepsilon}_i|$ ;  $\vartheta_i, \xi_i, v_{zi}, v_{li}$ , and  $v_{\varpi i}$  are positive constants;  $k_1, k_2, k_3, k_4, k_5, k_6$  are small constants.

The novel fixed-time sliding mode controller is designed as

$$u_i = \frac{1}{F_i \zeta_i} \left[ \beta_1 \text{sig}^{r_1}(S_i) + \beta_2 \text{sig}^{l_1}(S_i) + F_i^2 S_i \frac{\vartheta_i^2}{2} \hat{\varpi}_i \zeta_i^T \zeta_i + F_i \hat{l}_i \tanh\left(\frac{S_i}{\xi_i}\right) + Y_i \right] \quad (32)$$

where

$$Y_i = \begin{cases} qA_i(a_{i-1} - a_i - 2h_2a_i^2) + q\left(\frac{\bar{m}_1}{\bar{n}_1}\alpha_1 \text{sig}^{\frac{\bar{m}_1 - \bar{n}_1}{\bar{n}_1}}(\omega_i)\dot{\omega}_i + \frac{\bar{m}_2}{\bar{n}_2}\alpha_2 \text{sig}^{\frac{\bar{m}_2 - \bar{n}_2}{\bar{n}_2}}(\omega_i)\dot{\omega}_i + \dot{A}_i(\dot{e}_i + \Pi_i)\right) + qA_i\ddot{\Pi}_i - \dot{s}_{i+1}, i = 1, \dots, N-1. \\ qA_i(a_{i-1} - a_i - 2h_2a_i^2) + q\left(\frac{\bar{m}_1}{\bar{n}_1}\alpha_1 \text{sig}^{\frac{\bar{m}_1 - \bar{n}_1}{\bar{n}_1}}(\omega_i)\dot{\omega}_i + \frac{\bar{m}_2}{\bar{n}_2}\alpha_2 \text{sig}^{\frac{\bar{m}_2 - \bar{n}_2}{\bar{n}_2}}(\omega_i)\dot{\omega}_i + \dot{A}_i(\dot{e}_i + \Pi_i)\right) + qA_i\ddot{\Pi}_i, i = N. \end{cases}$$

*Remark 3:* A disadvantage of CNN control method is that the number of parameters to be estimated grows rapidly as the number of nodes increases, potentially resulting in excessive computational load. Inspired by [43], this paper adopts the minimum learning parameter technique, where the number of parameters depends only on the number of vehicles, thereby effectively addressing the aforementioned issue.

*Theorem 2:* For vehicle platoon (1) with unknown nonlinearities, IDZ, and external disturbances, the adaptive mechanisms (31) and the controller (32) can ensure that  $e_i$  converges to a neighborhood of zero within a fixed time, and the individual stability can be guaranteed. Furthermore, if  $0 < q \leq 1$  holds, the string stability can be ensured.

To more clearly explain the structure of the controller (32), we present its design block diagram as shown in Figure 4.

*Proof:* Define the Lyapunov function as

$$V_i = \frac{1}{2}S_i^2 + \frac{1}{2v_{z_i}}\tilde{z}_i^2 + \frac{1}{2v_{l_i}}\tilde{l}_i^2 + \frac{1}{2v_{\varpi_i}}\tilde{\varpi}_i^2 \quad (33)$$

where  $(\tilde{\cdot}) = (\cdot) - (\cdot)$ . Differentiating (33) yields

$$\dot{V}_i = S_i\dot{S}_i - \frac{1}{v_{z_i}}\tilde{z}_i\dot{\tilde{z}}_i - \frac{1}{v_{l_i}}\tilde{l}_i\dot{\tilde{l}}_i - \frac{1}{v_{\varpi_i}}\tilde{\varpi}_i\dot{\tilde{\varpi}}_i \quad (34)$$

According to (27) and (32), one has

$$\begin{aligned} S_i\dot{S}_i &= F_i S_i(-z_i u_i - \gamma_i - f_i(v_i, a_i) - D_i) + Y_i S_i \\ &= F_i S_i(-(\tilde{z}_i - \tilde{z}_i)u_i - \gamma_i - f_i(v_i, a_i) - D_i) + Y_i S_i \\ &\leq F_i S_i(-\tilde{z}_i u_i + \bar{\gamma}_i - W_i^{*T} \zeta_i + \bar{\varepsilon}_i + \bar{D}_i) \\ &\quad + Y_i S_i - (F_i^2 S_i^2 \frac{\vartheta_i^2}{2} \hat{\varpi}_i \zeta_i^T \zeta_i \\ &\quad + F_i \hat{l}_i S_i \tanh\left(\frac{S_i}{\xi_i}\right) + Y_i S_i \\ &\quad + \beta_1 S_i^{r_1+1} + \beta_2 S_i^{l_1+1}) \end{aligned} \quad (35)$$

Based on Lemma 5, we have

$$-F_i S_i W_i^{*T} \zeta_i \leq F_i^2 S_i^2 \frac{\vartheta_i^2}{2} \hat{\varpi}_i \zeta_i^T \zeta_i + \frac{1}{2\vartheta_i^2} \quad (36)$$

According to (35) and (36), we can obtain

$$\begin{aligned} S_i\dot{S}_i &\leq F_i S_i(-\tilde{z}_i u_i + F_i S_i \frac{\vartheta_i^2}{2} \hat{\varpi}_i \zeta_i^T \zeta_i + \iota_i) \\ &\quad + \frac{1}{2\vartheta_i^2} S_i F_i - (F_i \hat{l}_i S_i \tanh\left(\frac{S_i}{\xi_i}\right) \\ &\quad + \beta_1 S_i^{r_1+1} + \beta_2 S_i^{l_1+1}) \end{aligned} \quad (37)$$

By combining (31), (34), and (37), we have

$$\begin{aligned} \dot{V}_i &\leq -\beta_1 S_i^{r_1+1} - \beta_2 S_i^{l_1+1} + k_1 \tilde{z}_i \tilde{z}_i^{r_1} \\ &\quad + k_2 \tilde{z}_i \tilde{z}_i^{l_1+1} + k_3 \tilde{l}_i \tilde{l}_i^{r_1} + k_4 \tilde{l}_i \tilde{l}_i^{l_1+1} \\ &\quad + k_5 \tilde{\varpi}_i \tilde{\varpi}_i^{r_1} + k_6 \tilde{\varpi}_i \tilde{\varpi}_i^{l_1+1} + F_i \iota_i (S_i \\ &\quad - S_i \tanh\left(\frac{S_i}{\xi_i}\right)) + \frac{1}{2\vartheta_i^2} \end{aligned} \quad (38)$$

Based on Lemma 7, one can obtain

$$\begin{aligned}
 k_1 \tilde{z}_i^{r_1+1} &= k_1 \tilde{z}_i (z_i - \tilde{z}_i)^{r_1} \\
 &\leq \frac{k_1 r_1}{r_1 + 1} (z_i^{r_1+1} - \tilde{z}_i^{r_1+1}) \\
 k_2 \tilde{z}_i^{l_1+1} &= k_2 \tilde{z}_i (z_i - \tilde{z}_i)^{l_1+1} \\
 &\leq \frac{k_2 (l_1 + 1)}{l_1 + 2} (z_i^{l_1+2} - \tilde{z}_i^{l_1+1}) \\
 k_3 \tilde{l}_i^{r_1} &= k_3 \tilde{l}_i (l_i - \tilde{l}_i)^{r_1} \\
 &\leq \frac{k_3 r_1}{r_1 + 1} (l_i^{r_1+1} - \tilde{l}_i^{r_1+1}) \\
 k_4 \tilde{l}_i^{l_1+1} &= k_4 \tilde{l}_i (l_i - \tilde{l}_i)^{l_1+1} \\
 &\leq \frac{k_4 (l_1 + 1)}{l_1 + 2} (l_i^{l_1+2} - \tilde{l}_i^{l_1+1}) \\
 k_5 \tilde{\omega}_i^{r_1} &= k_5 \tilde{\omega}_i (\omega_i - \tilde{\omega}_i)^{r_1+1} \\
 &\leq \frac{k_5 r_1}{r_1 + 1} (\omega_i^{r_1+1} - \tilde{\omega}_i^{r_1+1}) \\
 k_6 \tilde{\omega}_i^{l_1+1} &= k_6 \tilde{\omega}_i (\omega_i - \tilde{\omega}_i)^{l_1} \\
 &\leq \frac{k_6 (l_1 + 1)}{l_1 + 2} (\omega_i^{l_1+2} - \tilde{\omega}_i^{l_1+1})
 \end{aligned} \tag{39}$$

According to Lemma 2, one has

$$F_{i,l_i} \left( S_i - S_i \tanh\left(\frac{S_i}{\xi_i}\right) \right) \leq 0.2785 F_{i,l_i} \xi_i \tag{40}$$

By combining (38), (39), and (40), we can obtain

$$\begin{aligned}
 \dot{V}_i &\leq -\beta_1 S_i^{r_1+1} - \frac{k_1 r_1}{r_1 + 1} z_i^{r_1+1} - \frac{k_3 r_1}{r_1 + 1} l_i^{r_1+1} \\
 &\quad - \frac{k_5 r_1}{r_1 + 1} \tilde{\omega}_i^{r_1+1} - \beta_2 S_i^{l_1+1} - \frac{k_2 (l_1 + 1)}{l_1 + 2} z_i^{l_1+1} \\
 &\quad - \frac{k_4 (l_1 + 1)}{l_1 + 2} l_i^{l_1+1} - \frac{k_6 (l_1 + 1)}{l_1 + 2} \tilde{\omega}_i^{l_1+1} + \frac{k_1 r_1}{r_1 + 1} z_i^{r_1+1} \\
 &\quad + \frac{k_2 (l_1 + 1)}{l_1 + 2} z_i^{l_1+2} + \frac{k_3 r_1}{r_1 + 1} l_i^{r_1+1} + \frac{k_4 (l_1 + 1)}{l_1 + 2} l_i^{l_1+2} \\
 &\quad + \frac{k_5 r_1}{r_1 + 1} \omega_i^{r_1+1} + \frac{k_6 (l_1 + 1)}{l_1 + 2} \omega_i^{l_1+2} + \frac{1}{2\vartheta^2} \\
 &\quad + F_{i,l_i} 0.2785 \xi_i \\
 &\leq -\beta_1 (S_i^2)^{\frac{r_1+1}{2}} - \frac{k_1 r_1}{r_1 + 1} (z_i^2)^{\frac{r_1+1}{2}} - \frac{k_3 r_1}{r_1 + 1} (l_i^2)^{\frac{r_1+1}{2}} \\
 &\quad - \frac{k_5 r_1}{r_1 + 1} (\tilde{\omega}_i^2)^{\frac{r_1+1}{2}} - \beta_2 (S_i^2)^{\frac{l_1+1}{2}} - \frac{k_2 (l_1 + 1)}{l_1 + 2} (z_i^2)^{\frac{l_1+1}{2}} \\
 &\quad - \frac{k_4 (l_1 + 1)}{l_1 + 2} (l_i^2)^{\frac{l_1+1}{2}} - \frac{k_6 (l_1 + 1)}{l_1 + 2} (\tilde{\omega}_i^2)^{\frac{l_1+1}{2}} \\
 &\quad + \frac{k_1 r_1}{r_1 + 1} z_i^{r_1+1} + \frac{k_2 (l_1 + 1)}{l_1 + 2} z_i^{l_1+2} + \frac{k_3 r_1}{r_1 + 1} l_i^{r_1+1} \\
 &\quad + \frac{k_4 (l_1 + 1)}{l_1 + 2} l_i^{l_1+2} + \frac{k_5 r_1}{r_1 + 1} \omega_i^{r_1+1} + \frac{k_6 (l_1 + 1)}{l_1 + 2} \omega_i^{l_1+2} \\
 &\quad + F_{i,l_i} 0.2785 \xi_i + \frac{1}{2\vartheta^2}
 \end{aligned} \tag{41}$$

Then, (41) can be rewritten as

$$\begin{aligned}
 \dot{V}_i &\leq \chi_i \left( \left(\frac{1}{2} S_i^2\right)^{\frac{r_1+1}{2}} + \left(\frac{1}{2v_{z_i}} z_i^2\right)^{\frac{r_1+1}{2}} + \left(\frac{1}{2v_{l_i}} l_i^2\right)^{\frac{r_1+1}{2}} \right. \\
 &\quad \left. + \left(\frac{1}{2v_{\tilde{\omega}_i}} \tilde{\omega}_i^2\right)^{\frac{r_1+1}{2}} \right) + \eta_i \left( \left(\frac{1}{2} S_i^2\right)^{\frac{l_1+1}{2}} + \left(\frac{1}{2v_{z_i}} z_i^2\right)^{\frac{l_1+1}{2}} \right. \\
 &\quad \left. + \left(\frac{1}{2v_{l_i}} l_i^2\right)^{\frac{l_1+1}{2}} + \left(\frac{1}{2v_{\tilde{\omega}_i}} \tilde{\omega}_i^2\right)^{\frac{l_1+1}{2}} \right) + \sigma_i
 \end{aligned} \tag{42}$$

where

$$\begin{aligned}
 \chi_i &= \min \left\{ 2^{\frac{1+r_1}{2}} \beta_1, (2v_{z_i})^{\frac{1+r_1}{2}} \frac{k_1 r_1}{r_1 + 1}, (2v_{l_i})^{\frac{1+r_1}{2}} \frac{k_3 r_1}{r_1 + 1}, \right. \\
 &\quad \left. (2v_{\tilde{\omega}_i})^{\frac{1+r_1}{2}} \frac{k_5 r_1}{r_1 + 1} \right\} \\
 \eta_i &= \min \left\{ 2^{\frac{1+l_1}{2}} \beta_2, (2v_{z_i})^{\frac{1+l_1}{2}} \frac{k_2 (l_1 + 1)}{l_1 + 2}, \right. \\
 &\quad \left. (2v_{l_i})^{\frac{1+l_1}{2}} \frac{k_4 (l_1 + 1)}{l_1 + 2}, (2v_{\tilde{\omega}_i})^{\frac{1+l_1}{2}} \frac{k_6 (l_1 + 1)}{l_1 + 2} \right\} \\
 \sigma_i &= \frac{k_1 r_1}{r_1 + 1} z_i^{r_1+1} + \frac{k_2 (l_1 + 1)}{l_1 + 2} z_i^{l_1+2} + \frac{k_3 r_1}{r_1 + 1} l_i^{r_1+1} \\
 &\quad + \frac{k_4 (l_1 + 1)}{l_1 + 2} l_i^{l_1+2} + \frac{k_5 r_1}{r_1 + 1} \omega_i^{r_1+1} \\
 &\quad + \frac{k_6 (l_1 + 1)}{l_1 + 2} \omega_i^{l_1+2} + F_{i,l_i} 0.2785 \xi_i + \frac{1}{2\vartheta^2}.
 \end{aligned}$$

According to Lemma 3, we have

$$V \leq -\chi_i 4^{\frac{1-r_1}{2}} V_i^{\frac{r_1+1}{2}} - \eta_i V_i^{\frac{l_1+1}{2}} + \sigma_i \tag{43}$$

By Lemma 4, we can obtain that  $V_i$  is FTS. It means that  $S_i$ ,  $\tilde{z}_i$ ,  $\tilde{l}_i$ , and  $\tilde{\omega}_i$  can converge to a neighborhood of zero within convergence time  $T_{i,1}$ .  $T_{i,1}$  is given as

$$T_{i,1} \leq \bar{T}_{i,1} = \frac{1}{\chi_i 4^{\frac{1-r_1}{2}} \left(\frac{r_1-1}{2}\right) M_i} + \frac{1}{\eta_i \left(\frac{1-l_1}{2}\right) M_i} \tag{44}$$

where  $0 < M_i < 1$ .

Based on (7), (24), (25), and Lemma 1,  $s_i$ ,  $\omega_i$ , and  $e_i$  can converge to a neighborhood of zero within convergence time  $T_{i,2}$  when  $S_i = 0$ .  $T_{i,2}$  is described as

$$T_{i,2} \leq \bar{T}_{i,2} = \frac{1}{\alpha_1} \frac{\bar{n}_1}{\bar{m}_1 - \bar{n}_1} + \frac{1}{\alpha_2} \frac{\bar{n}_2}{\bar{n}_2 - \bar{m}_2} \tag{45}$$

According to (44) and (45), the maximum value of the convergence time as

$$T_{i,max} = T_{i,1} + T_{i,2} \tag{46}$$

It means that the vehicle platoon (1) can achieve the individual stability within  $T_{i,max}$ . The proof of string stability is similar to the proof process in [34]. As  $S_i = q s_i - s_{i+1}$ , when  $S_i$  converges to a neighborhood of zero, we have

$$\begin{aligned}
 q \left[ \dot{\omega}_i + \alpha_1 \text{sig}^{\frac{\bar{m}_1}{n_1}}(\omega_i) + \alpha_2 \text{sig}^{\frac{\bar{m}_2}{n_2}}(\omega_i) \right] \\
 \approx \left[ \dot{\omega}_{i+1} + \alpha_1 \text{sig}^{\frac{\bar{m}_1}{n_1}}(\omega_{i+1}) + \alpha_2 \text{sig}^{\frac{\bar{m}_2}{n_2}}(\omega_{i+1}) \right]
 \end{aligned} \tag{47}$$

By taking the Laplace transform of equation (47), we can get the following two cases.

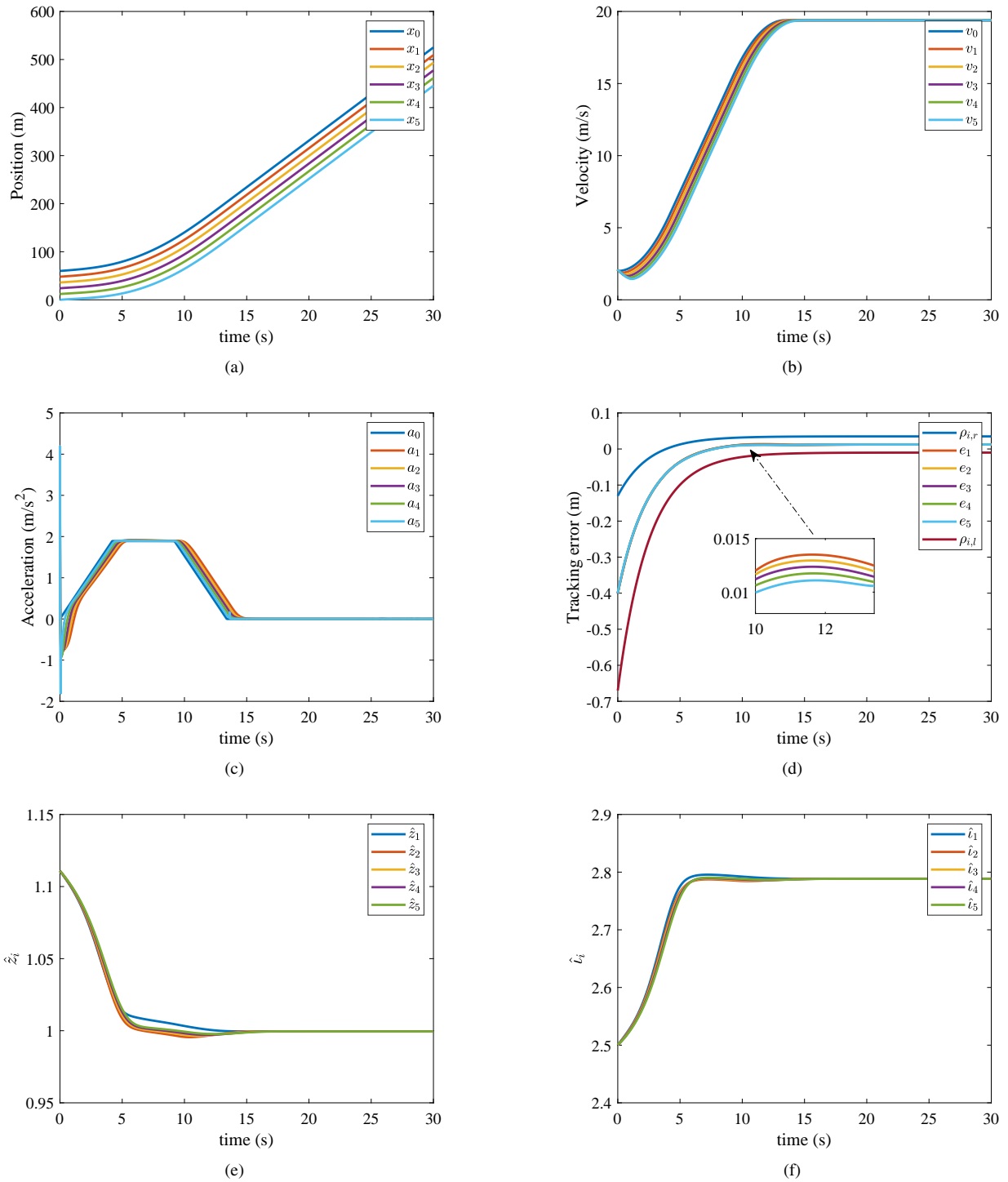


FIGURE 5: The results of the controller with TFTPFF. (a) Position  $x_i$ . (b) Velocity  $v_i$ . (c) Acceleration  $a_i$ . (d) Tracking error  $e_i$ . (e) Estimation of  $z_i$ . (f) Estimation of  $l_i$ .



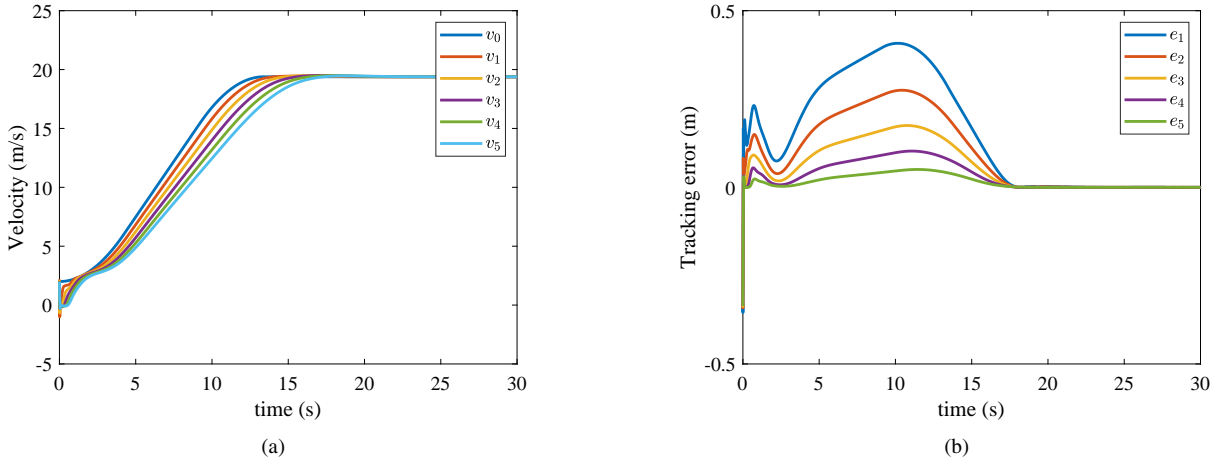


FIGURE 6: The results of the controller without PPF. (a) Velocity  $v_i$ . (b) Tracking error  $e_i$ .

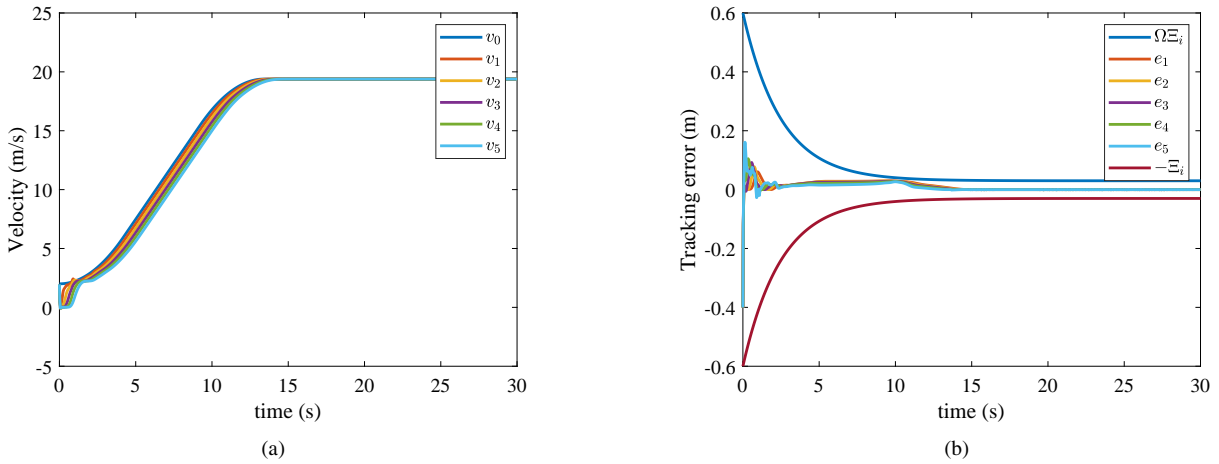


FIGURE 7: The results of the controller with traditional PPF. (a) Velocity  $v_i$ . (b) Tracking error  $e_i$ .

Case 1: if  $\omega_i \geq 0$ , we have

$$\begin{aligned} q \left[ s \bar{W}_i(s) + \alpha_1 \bar{W}_i^{\frac{\bar{m}_1}{n_1}-1}(s) + \alpha_2 \bar{W}_i^{\frac{\bar{m}_2}{n_2}-1}(s) \right] \\ \approx s \bar{W}_{i+1}(s) + \alpha_1 \bar{W}_{i+1}^{\frac{\bar{m}_1}{n_1}-1}(s) + \alpha_2 \bar{W}_{i+1}^{\frac{\bar{m}_2}{n_2}-1}(s) \end{aligned} \quad (48)$$

Then

$$\frac{\bar{W}_{i+1}(s)}{\bar{W}_i(s)} \approx \frac{q \left[ s + \alpha_1 \bar{W}_i^{\frac{\bar{m}_1}{n_1}-1}(s) + \alpha_2 \bar{W}_i^{\frac{\bar{m}_2}{n_2}-1}(s) \right]}{s + \alpha_1 \bar{W}_{i+1}^{\frac{\bar{m}_1}{n_1}-1}(s) + \alpha_2 \bar{W}_{i+1}^{\frac{\bar{m}_2}{n_2}-1}(s)} \approx q \quad (49)$$

Case 2: if  $\omega_i < 0$ , we can obtain

$$\begin{aligned} q \left[ s \bar{W}_i(s) - \alpha_1 \bar{W}_i^{\frac{\bar{m}_1}{n_1}-1}(s) - \alpha_2 \bar{W}_i^{\frac{\bar{m}_2}{n_2}-1}(s) \right] \\ \approx s \bar{W}_{i+1}(s) - \alpha_1 \bar{W}_{i+1}^{\frac{\bar{m}_1}{n_1}-1}(s) - \alpha_2 \bar{W}_{i+1}^{\frac{\bar{m}_2}{n_2}-1}(s) \end{aligned} \quad (50)$$

Then

$$\frac{\bar{W}_{i+1}(s)}{\bar{W}_i(s)} \approx \frac{q \left[ s - \alpha_1 \bar{W}_i^{\frac{\bar{m}_1}{n_1}-1}(s) - \alpha_2 \bar{W}_i^{\frac{\bar{m}_2}{n_2}-1}(s) \right]}{s - \alpha_1 \bar{W}_{i+1}^{\frac{\bar{m}_1}{n_1}-1}(s) - \alpha_2 \bar{W}_{i+1}^{\frac{\bar{m}_2}{n_2}-1}(s)} \approx q \quad (51)$$

where  $\bar{W}_i(s)$  is the Laplace transform of  $\omega_i$ . According to (7),  $\omega_i$  and  $e_i$  have the same monotonicity. Therefore, when  $0 < q \leq 1$ , string stability can be guaranteed. The proof is completed.

#### IV. NUMERICAL EXAMPLES

In this section, we will illustrate the effectiveness and advantages of the proposed strategy through numerical examples.

Consider a vehicle platoon consisting of one leader and five following vehicles. The parameters of vehicle platoon are the same as those in [15]: the air density  $\rho_a = 0.2$ , the mass  $m_i = [1500, 1600, 1550, 1650, 1600]$ , the engine time constant  $\tau_i =$

[0.15, 0.3, 0.2, 0.25, 0.2], the angle of the road slope  $\phi = 0$ , the rolling resistance coefficient  $\ell_i = 0.1$ , the cross-sectional area  $A_i = 2.2$ , the acceleration of the gravity  $g = 9.8$ , the air drag coefficient  $C_{di} = 0.35$ ,  $h_1 = 0.2$ ,  $h_2 = 10^{-5}$ , the external disturbances  $D_i = 0.1 \sin(t)$ , the length of vehicle  $w_i = 2$ , and the standstill spacing  $\mu_i = 10$ .

### A. RESULTS OF DESIGNED CONTROLLER STRATEGY

In this subsection, using the adaptive mechanisms (31) and the controller (32), numerical examples are carried out to verify the effectiveness of the proposed strategy. The parameters of the controller are  $z_{i,r} = 1$ ,  $z_{i,l} = 0.95$ ,  $\gamma_{i,r} = 2$ ,  $\gamma_{i,l} = 1.9$ ,  $\vartheta_i = 0.1$ ,  $\delta_{i,0} = 0.6$ ,  $\delta_{i,\infty} = 0.05$ ,  $\varphi_i = 0.4$ ,  $t_f = 20$ ,  $\kappa_1 = 0.9$ ,  $\kappa_2 = 1.2$ ,  $\varsigma_1 = 0.3$ ,  $\varsigma_2 = 0.9$ ,  $\xi_i = 0.1$ ,  $v_{zi} = v_{\varpi i} = v_{\omega i} = 0.01$ ,  $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 0.001$  and  $q = 0.9$ .

The results are presented in Figure. 5. The position profiles are presented in Figure. 5(a). As there are no intersections or overlaps, it is indicated that the designed strategy for vehicle platoon can ensure no collision between vehicles. Figure. 5(b) shows that vehicle platoon achieves synchronization of velocity. According to Figure. 5(c), we can obtain that the acceleration of the following vehicle can track the acceleration of the leader. It is evident from Figure. 5(d) that before time  $t_f$ ,  $e_i$  converges to a neighborhood of zero, indicating that the designed strategy can ensure the vehicle platoon maintains the desired spacing. In addition, as  $e_i$  consistently remains within the predefined region, we can conclude that the designed strategy can ensure the vehicle platoon achieves the prescribed tracking performance. Figures. 5(e) and (f) demonstrate that the designed adaptive mechanisms can effectively estimate  $z_i$  and  $l_i$ .

### B. RESULTS OF COMPARATIVE NUMERICAL EXAMPLE

In this subsection, we present two comparative numerical examples to demonstrate the advantages of the designed strategy. The controller parameters are the same as in example 1.

Case:1 Comparison with the control strategy without PPF.

The results are presented in Figure. 6. Figure. 6(a) shows that the velocity of the following vehicle can track the velocity of the leader. It is evident from Figure. 6(b) that  $e_i$  can eventually converge to the neighborhood of zero. By comparing Figure. 5 with Figure. 6, we can obtain that the designed strategy can accelerate the convergence rate, reduce the overshoot of  $e_i$ , and decreases the velocity fluctuation, thereby enabling the vehicle platoon to possess better transient and steady-state performance.

Case:2 Comparison with the control strategy with traditional PPF. The traditional PPF is selected as [9]:  $\Xi_i = (\Xi_{i,0} - \Xi_{i,\infty})e^{-\partial_i t} + \Xi_{i,\infty}$ , where  $\Xi_{i,0} = 0.6$ ,  $\Xi_{i,\infty} = 0.05$ , and  $\partial_i = 0.5$ .

The results are shown in Figure. 7. By comparing Figure. 5 with Figure. 7, we can see that the designed strategy can effectively reduce the overshoot of  $e_i$  and decreases the velocity fluctuation. Therefore, the designed strategy can better improve performance of vehicle platoon.

## V. CONCLUSION

A FTSMC problem for vehicle platoon with prescribed performance, IDZ, unknown nonlinearities, and external disturbances is investigated. Compared with the traditional funnel-shaped PPF, the designed TFTPFF defines tunnel-shaped performance boundaries that can enhance the convergence rate of  $e_i$  and reduce the system overshoot. The convergence time of TFTPFF can be predetermined and is not affected by the initial conditions. CNN is applied to approximate nonlinearities in vehicle platoon, while the novel adaptive mechanisms are designed to estimate IDZ slope and external disturbances. Based on the proposed TFTPFF, CNN, and adaptive mechanisms, a new FTSMC strategy is proposed to ensure string stability and achieve predefined tracking performance within a fixed time. The results of the comparative numerical examples demonstrate that the proposed strategy can effectively reduce overshoot of  $e_i$ , accelerate convergence rate, and thus improve system performance. Our future work will be dedicated to researching the control issues of the vehicle platoon with prescribed performance and system failures.

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