

## RESEARCH ARTICLE

# Event-Triggered Asynchronous Dissipative Control for Markov Jump Systems With Unknown Probabilities

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**ABSTRACT** This paper investigates the issue of the event-triggered asynchronous dissipative control for Markov jump systems with unknown probabilities and packet losses. A hidden Markov model is used to describe the asynchronous problem. A Bernoulli model and an event-triggered scheme are adopted to address the packet loss and reduce the transmission rate of sampled signals, respectively. Furthermore, there exists unknown probability information of the controller. Then, Lyapunov functions are constructed to obtain sufficient conditions for the stochastic stability and dissipative performance of the closed-loop system. Moreover, a design method for controller parameters and event-triggered matrices is proposed. Finally, the effectiveness of the proposed approach is verified through a three-modal numerical example.

**INDEX TERMS** Markov jump systems, asynchronous control, event-triggered mechanism, packet losses, unknown probabilities.

## I. INTRODUCTION

In recent years, Markov jump systems (MJSs) have been widely concerned in various fields. These systems consist of multiple subsystems, which dynamic characteristics change with the alterations in the external environment or the internal structure of the system. At any given time, only one subsystem operates with stochastic jumps occurring among subsystems based on specific transition probabilities. Researchers have not only explored the theoretical aspects of MJSs, but also applied them to practical engineering controls [1], [2]. In [3], MJS was applied to mobile communication, ensuring the continuity of communication. In [4], the adaptive strategy of MJSs under mixed attack was used in the mass-spring model. In addition, MJSs have also been used in the automotive industries and biological

systems, reflecting that MJSs have been applied in different fields in recent years, especially in the control field.

With the in-depth integration of control theory and communication technology, the networked control system has been developed rapidly. At the same time, the continuous expansion of the control scale has brought many new problems such as communication delays and packet losses, which inevitably lead to asynchronous patterns. However, previous researches [5], [6], [7] mainly focused on modal independence or modal complete synchronization, ignoring that achieving complete synchronization in real-world systems is very difficult. Aware of these limitations, scholars focused on the research of asynchronous control. The hidden Markov model (HMM) in [8], [9], and [10] was favored by many scholars for its excellent performance in modeling asynchronous phenomena. Subsequently, researchers explored the sliding mode control of discrete semi-MJSs under asynchronous conditions. They described the asynchronous

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relationship between the controller and system modes by using the HMM in [11]. Another work [12] expressed the advantage of using the HMM by researching the generalized non-fragile asynchronous controller in the neutral MJS. Similarly, in T-S fuzzy MJSs, an HMM in [13] was established to characterize the asynchronous modes between the controller and system modes, effectively addressing the issue of incomplete premise matching in output tracking asynchronous control.

On the other hand, due to the large scale and complex structure of modern industrial control, researchers have focused on the analysis of limited network bandwidth in recent years. Event-triggered data transmission methods were proposed to reduce communication time, avoid network communication congestion and make reasonable use of network resources. These methods were extensively applied in [14], [15], and [16]. The proposal of a fixed lower bound event-triggered scheme in [17] has avoided the Zeno problem caused by external interference. Researchers further introduced the zero-order holder and adopted Lyapunov functions to reject DoS interference attacks and maintain the control performance of the closed-loop system. The proposed event-triggered sliding-mode control method was applied to aircraft engine systems in [18], minimizing the convergence region in the vicinity of the sliding surface. In [19], the dynamic event-triggered mechanism was studied for singular MJS, and the traffic was further reduced by introducing a dynamic auxiliary variable.

As a critical parameter of mode-switching in MJSs, the probability information inevitably affects the performance of the system. Many previous studies often assumed that conditional probabilities in MJSs are completely known in [20], [21], and [22]. However, many practical engineering applications are located in complex network environments, which makes it difficult to obtain accurate probability information. To deal with this issue, control problems with partially unknown probabilities in MJSs were considered in [23], [24], and [25]. For example, in [23], the sliding mode control method was applied to a Markov jump system with partially known transition probabilities under the premise of time-varying actuator failures. The stability problem of MJS with partially unknown transition probabilities was investigated in [25] under the presence of time-varying delays and deceptive attacks.

Over the past decades, the packet loss issue in network control systems has been extensively focused on by many scholars, and different handling methods have been proposed in [26], [27], [28], [29], [30], and [31]. For example, the asynchronous dynamic system in [26] regarded the packet loss as an event to design a filter with  $H_\infty$  performance. The packet loss issue in [29] was modeled as a binary exchange sequence, with its values being governed by a Bernoulli probability distribution. For the fuzzy networked system in [30], the packet loss was described by a random variable that obeys a Bernoulli distribution. By constructing

suitable Lyapunov functions, the sufficient conditions for the existence of fuzzy filters were proposed. In [31], packet loss and reception were described by a Markov chain, better illustrating the influence of the packet loss situation at the current moment in a real network on that at the next moment.

To our knowledge, many studies have separately considered the cases of asynchrony, the packet loss, and the unknown probability. However, there are few works that simultaneously address all three scenarios, which is the motivation of this paper. The primary contributions of this paper include the following:

- 1) In this paper, a more practical model is established. On this basis, a dissipative control scheme based on the event-triggered mechanism is proposed.
- 2) Using Lyapunov functions and relaxation matrix technology, the collaborative design method of the controller and event trigger is further simplified. Compared with the existing work [27], a more reasonable packet loss model is adopted to effectively avoid the long-term zero input phenomenon between the controller and the actuator.

The subsequent content of this paper is structured as follows. Section II describes a series of studied models. Section III provides sufficient conditions and related parameter settings for the closed-loop system to meet two performance requirements. Section IV presents simulation results to verify the effectiveness of the scheme. Section V summarizes the work of this paper and provides prospects for further work.

## A. NOTATIONS

Please refer to Table 1 for the symbols and their meanings.

TABLE 1. Symbols and their meanings.

Symbol	Meaning
$\mathbb{R}^n$	N-dimensional set of real valued vectors
$\mathbb{R}^{m \times n}$	$m \times n$ -dimensional set of real valued vectors
$A^T$	The transposition of matrix $A$
$A^{-1}$	The inverse of matrix $A$
$\Pr\{x\}$	The probability of event $x$ occurring
$E\{x\}$	Expectations of random variable $x$

## II. PRELIMINARIES

In this paper, consider the following Markov jump plant model:

$$\begin{cases} x_{k+1} = A_{i_k}x_k + B_{1i_k}u_k + B_{2i_k}w_k \\ y_k = C_{i_k}x_k + D_{1i_k}u_k + D_{2i_k}w_k \end{cases} \quad (1)$$

where  $x_k \in \mathbb{R}^{n_x}$ ,  $y_k \in \mathbb{R}^{n_y}$ ,  $u_k \in \mathbb{R}^{n_u}$  indicate the system state, the controlled output, the control input, respectively, and  $w_k \in \mathbb{R}^{n_w}$  refers to the disturbance input with  $w_k \in l[0, +\infty)$ .  $A_{i_k}$ ,  $B_{1i_k}$ ,  $B_{2i_k}$ ,  $C_{i_k}$ ,  $D_{1i_k}$ ,  $D_{2i_k}$  are known real matrices with appropriate dimensions. The Markov jump

process of system (1) is described by the modal parameter  $\iota_k$  ( $\iota_k \in L, L = \{1, 2, \dots, l\}$ ) and conforms to the transition probability matrix  $\Theta = [\theta_{st}]$ , which the transition probability  $\theta_{st}$  is defined as:

$$\Pr\{\iota_{k+1} = t | \iota_k = s\} = \theta_{st} \quad (2)$$

apparently,  $\theta_{st} \in [0, 1]$  and  $\sum_{t=1}^L \theta_{st} = 1$ , for  $\forall s, t \in L$ .

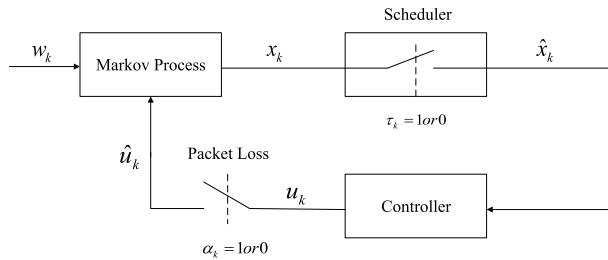


FIGURE 1. Asynchronous control structure block diagram.

In practice, the limitation of network communication bandwidth may cause network congestion. In this paper, the event trigger is introduced to reduce the transmission frequency of sampled signals. The main event-triggered mechanism is given by:

$$(x_{\iota_k} - x_k)^T G(x_{\iota_k} - x_k) \leq \varepsilon x_k^T G x_k \quad (3)$$

we attempt to compare the previously transmitted state  $x_{\iota_k}$  with the current system state  $x_k$ . If the relationship between  $x_{\iota_k}$  and  $x_k$  conforms to (3), the current system state  $x_k$  will not be transmitted, otherwise it will be transmitted. Among them,  $\varepsilon > 0$  stands for the trigger threshold parameter and  $G$  is the positive definite weighting matrix to be designed.

By utilizing a zero-order holder, the signal received by the controller is ensured to meet the following requirements.

$$\hat{x}_k = x_{\iota_k}, k = [t_k, t_k + 1, \dots, t_{k+1}) \quad (4)$$

The transmission error is defined as

$$e_k = \hat{x}_k - x_k \quad (5)$$

The inequality (3) can be stated as

$$e_k^T G e_k \leq \varepsilon x_k^T G x_k \quad (6)$$

*Remark 1:* The trigger's transmission time  $\{t_0, t_1, t_2, \dots\}$  falls within the sampling time  $\{0, 1, 2, \dots\}$ . By utilizing the event-triggered mechanism (3), data transmission occurs only at certain moments, effectively reducing the frequency of data transmission. Especially, when  $\varepsilon = 0$ , the sampled signal will be transmitted periodically. Besides, a performance indicator, denoted as the data transmission rate  $TP = t_S/t_T \times 100\%$ , is appointed to evaluate communication performance, where  $t_S$  and  $t_T$  are the transmission times of sampled data when using the event-triggered mechanism and without using it, respectively. A lower data transmission rate indicates better communication performance.

By utilizing the theory of HMM, the following asynchronous controller is employed:

$$u_k = K_{\sigma_k} \hat{x}_k \quad (7)$$

where  $K_{\sigma_k}$  represents the controller gain that needs to be determined. The controller operates asynchronously in comparison to the original system mode. The controller mode  $\sigma_k$  is affected by the mode  $\iota_k$  of system (1) and meets the conditional probability matrix  $\Phi = [\phi_{sg}]$ . The conditional probability  $\phi_{sg}$  is defined as:

$$\Pr\{\sigma_k = g | \iota_k = s\} = \phi_{sg} \quad (8)$$

which indicates the probability of the controller operating in mode  $g$  when the system (1) operates in mode  $s$ . Apparently,  $\phi_{sg} \in [0, 1]$  and  $\sum_{g=1}^L \phi_{sg} = 1$ , for  $\forall s, g \in L$ .

*Remark 2:* In this paper, the asynchronous problem between the controller modes and the system modes is described by HMM. Their modal transitions are controlled by  $\iota_k$  and  $\sigma_k$ , respectively, where the controller modal parameter  $\sigma_k$  is indirectly affected by the original system modal parameter  $\iota_k$  through the conditional probability matrix  $\Phi$ . It should be noted that the HMM asynchronous model also encompasses both synchronous (i.e.  $\Phi = I$ ) and pattern-independent (i.e.  $\sigma_k \in \{1\}$ ) in [32], so the asynchronous controller under the HMM scheme covers a wider range.

Note that the controller (7) and the original system (1) are transmitted information through the network. Deliberating issues such as unstable network signals and blockages, the system status information may not be successfully transmitted to the controller, resulting in the packet loss. Therefore, this paper introduces a Bernoulli stochastic process to characterize the packet loss process:

$$\hat{u}_k = \alpha_k u_k + (1 - \alpha_k) \hat{u}_{k-1} = \alpha_k K_{\sigma_k} \hat{x}_k + (1 - \alpha_k) \hat{u}_{k-1} \quad (9)$$

where  $\alpha_k$  indicates the Bernoulli process, when  $\alpha_k = 1$ , namely, successful transmission,  $\hat{u}_k = u_k$ ; When  $\alpha_k = 0$ , namely, transmission failed,  $\hat{u}_k = \hat{u}_{k-1}$ . we assume that  $\alpha_k$  satisfies

$$\Pr\{\alpha_k = 1\} = \alpha, \Pr\{\alpha_k = 0\} = 1 - \alpha \quad (10)$$

it can be concluded that

$$E\{\alpha_k\} = \alpha, E\{\alpha_k^2\} = \alpha \quad (11)$$

Further defining  $\bar{\alpha}_k = \alpha_k - \alpha$ , therefore

$$E\{\bar{\alpha}_k\} = 0, E\{\bar{\alpha}_k^2\} = \bar{\alpha}^2 \quad (12)$$

where  $\bar{\alpha} = \sqrt{\alpha - \alpha^2}$ .

*Remark 3:* Note that in most literature, the packet loss model  $\hat{u}_k = \alpha_k u_k$  sets the input of the actuator to zero when the controller output is lost. However, in this paper, the final input is not set to zero, which is a more reasonable approach. Specifically, in cases of continuous packet losses, setting the actuator input to zero for a long time will inevitably

degrade system performance. Therefore, the packet loss model adopted in this paper is worth studying.

Deliberating the complexity of the actual system, we assume that conditional probabilities are partially unknown, that is, in the following form:

$$\Phi = \begin{bmatrix} \phi_{11} & ? & ? \\ ? & ? & \phi_{23} \\ ? & \phi_{32} & ? \end{bmatrix} \quad (13)$$

where “?” means the probability is unknown. For  $\forall s \in L$ , we define  $\ell = \ell_K^s + \ell_U^s$ , in which

$$\begin{cases} \ell_K^s = \{g : \phi_{sg} \text{ is known} \} \\ \ell_U^s = \{g : \phi_{sg} \text{ is unknown} \} \end{cases} \quad (14)$$

*Remark 4:* In the field of asynchronous control of Markov jump systems based on HMM, most research assumes that conditional probabilities are known. However, obtaining complete information about the conditional probability is extremely challenging. Hence, this paper explores a more complex scenario where conditional probabilities are partially unknown. It is worth mentioning that under this framework, there are two special cases: (1) Conditional probabilities are completely known (i.e.  $\ell_U^s = \emptyset$ ); (2) Conditional probabilities are completely unknown (i.e.  $\ell_K^s = \emptyset$ ).

For the following, we define  $\iota_k = s$ ,  $\iota_{k+1} = t$ ,  $\sigma_k = g$  to express conveniently.

Combining (1), (5), (7) and (9), the following closed-loop dynamic system is gained:

$$\begin{cases} x_{k+1} = \bar{A}_{sg}x_k + \alpha_k B_{1s}K_g e_k + (1 - \alpha_k)B_{1s}\hat{u}_{k-1} \\ \quad + B_{2s}w_k \\ y_k = \bar{C}_{sg}x_k + \alpha_k D_{1s}K_g e_k + (1 - \alpha_k)D_{1s}\hat{u}_{k-1} \\ \quad + D_{2s}w_k \end{cases} \quad (15)$$

where  $\bar{A}_{sg} = A_s + \alpha_k B_{1s}K_g$ ,  $\bar{C}_{sg} = C_s + \alpha_k D_{1s}K_g$ .

To foster the work of this paper, some vital lemmas and definitions are listed as follows.

*Definition 1 [33]:* The system (15) is stochastically stable, if  $w_k \equiv 0$  and the following condition is fulfilled for the arbitrary initial condition  $(x_0, \iota_0)$

$$\mathbb{E} \left\{ \sum_{k=0}^{\infty} \|x_k\|^2 | x_0, \iota_0 \right\} < \infty \quad (16)$$

*Definition 2 [33]:* For a scalar  $\gamma > 0$ , matrices  $\mu \leq 0$ ,  $\nu$  and symmetric  $\varpi$ , the closed-loop system (15) is strictly  $(\mu, \nu, \varpi) - \gamma$ -dissipative, for any positive integer  $N$ , when  $w_k \in l[0, +\infty)$  and the following condition is satisfied under the zero initial condition

$$\sum_{k=0}^N \mathbb{E} \{G(w_k, y_k)\} \geq \gamma \sum_{k=0}^N w_k^T w_k \quad (17)$$

where  $G(w_k, y_k) = y_k^T \mu y_k + 2y_k^T \nu w_k + w_k^T \varpi w_k$  and  $\mu = -U_1^T U_1$  is negative semi-definite.

The main focus of this paper is to design an asynchronous controller in the form of (7) in order to ensure that the closed-loop control system (15) satisfies both conditions (16) and (17). This will make the closed-loop system (15) stochastically stable and strictly  $(\mu, \nu, \varpi) - \gamma$ -dissipative.

### III. MAIN RESULTS

Based on the principles of the stochastic stability and dissipation, we attempt to study two types of performance of the closed-loop system (15) and derive the corresponding sufficient conditions as follows.

*Theorem 1:* The closed-loop system (15) is fulfilled for Definition 1 and Definition 2, if there contains a matrix  $K_g \in \mathbb{R}^{n_u \times n_x}$ , positive definite matrices  $P_s \in \mathbb{R}^{n_x \times n_x}$ ,  $G \in \mathbb{R}^{n_x \times n_x}$ ,  $H_{sg} \in \mathbb{R}^{n_x \times n_x}$ , for  $\forall s \in L$ ,  $g \in \ell_U^s$ , satisfying

$$\Pi_s^K + (1 - \phi_s^K) H_{sg} < P_s \quad (18)$$

and for  $\forall s, g \in L$ , satisfying

$$\begin{bmatrix} \bar{\phi}^{-1} & \vartheta_{sg} \\ * & \pi_{sg} \end{bmatrix} < 0 \quad (19)$$

where  $\Pi_s^K = \sum_{g \in \ell_K^s} \phi_{sg} H_{sg}$ ,  $\phi_s^K = \sum_{g \in \ell_K^s} \phi_{sg}$ ,

$$\begin{aligned} \bar{\phi}_s^{-1} &= \text{diag} \left\{ -\bar{P}_s^{-1}, -\bar{P}_s^{-1}, -I, -I \right\}, \\ \vartheta_{sg} &= \begin{bmatrix} \bar{A}_{sg}^* & \alpha B_{1s}K_g & (1 - \alpha) B_{1s} & B_{2s} \\ f B_{1s}K_g & f B_{1s}K_g & -f B_{1s} & 0 \\ U_1 \bar{C}_{sg}^* & \alpha U_1 D_{1s}K_g & (1 - \alpha) U_1 D_{1s} & U_1 D_{2s} \\ f U_1 D_{1s}K_g & f U_1 D_{1s}K_g & f U_1 D_{1s} & 0 \end{bmatrix}, \\ \pi_{sg} &= \begin{bmatrix} \varepsilon G - H_{sg} & 0 & 0 & -\bar{C}_{sg}^{*T} \nu \\ * & -G & 0 & -(\alpha D_{1s}K_g)^T \nu \\ * & * & -I & -((1 - \alpha) D_{1s})^T \nu \\ * & * & * & Q_s \end{bmatrix}, \\ \bar{A}_{sg}^* &= A_s + \alpha B_{1s}K_g, \bar{C}_{sg}^* = C_s + \alpha D_{1s}K_g, \\ Q_s &= -D_{2s}^T \nu - \nu^T D_{2s} + \gamma I - \varpi, \end{aligned}$$

$$f = \sqrt{\alpha - \alpha^2}, \bar{P}_s = \sum_{t=1}^l \theta_{st} P_t.$$

*Proof:* First, we derive some useful conditions from (18) and (19). From (18), we have

$$\sum_{g=1}^l \phi_{sg} H_{sg} - P_s < 0 \quad (20)$$

when  $\phi_s^K < 1$

$$\begin{aligned} &\sum_{g=1}^l \phi_{sg} H_{sg} - P_s \\ &= \Pi_s^K + (1 - \phi_s^K) \sum_{g \in \ell_U^s} \frac{\phi_{sg}}{1 - \phi_{sg}} H_{sg} - P_s \\ &= \sum_{g \in \ell_U^s} \frac{\phi_{sg}}{1 - \phi_{sg}} \left\{ \Pi_s^K + (1 - \phi_s^K) H_{sg} - P_s \right\} \end{aligned} \quad (21)$$

and when  $\phi_s^K = 1$ , apparently, (18) is equivalent to (20).

Next, adopting Schur complement to (19), we have

$$\begin{cases} \Upsilon_{1sg} \triangleq \wp_{sg}^1 - \vartheta_{sg}^T (\bar{\wp}_s^1)^{-1} \vartheta_{sg} < \tilde{H}_{sg} \\ \Upsilon_{2sg} \triangleq \wp_s^2 - \tilde{h}_{sg}^T (\bar{\wp}_s^2)^{-1} \tilde{h}_{sg} < \hat{H}_{sg} \end{cases} \quad (22)$$

where  $\hat{H}_{sg} = \text{diag}\{H_{sg}, 0, 0\}$ ,  $\tilde{H}_{sg} = \text{diag}\{H_{sg}, 0, 0, 0\}$ ,

$$\begin{aligned} \wp_{sg}^1 &= \begin{bmatrix} \varepsilon G & 0 & 0 & -\tilde{C}_{sg}^{*T} \nu \\ * & -G & 0 & -(\alpha D_{1s} K_g)^T \nu \\ * & * & -I & -((1-\alpha) D_{1s})^T \nu \\ * & * & * & Q_s \end{bmatrix}, \\ \wp_s^2 &= \begin{bmatrix} \varepsilon G & 0 & 0 \\ * & -G & 0 \\ * & * & -I \end{bmatrix}, \bar{\wp}_s^2 = \text{diag}\{-\bar{P}_s^{-1}, -\bar{P}_s^{-1}\}, \\ \tilde{h}_{sg} &= \begin{bmatrix} \bar{A}_{sg}^* & \alpha B_{1s} K_g & (1-\alpha) B_{1s} \\ f B_{1s} K_g & f B_{1s} K_g & -f B_{1s} \end{bmatrix}. \end{aligned}$$

Subsequently, we introduce the following modal-dependent Lyapunov-Krasovskii function

$$V_k = x_k^T P_k x_k \quad (23)$$

Introducing  $\varsigma_{1k} = [x_k^T \ e_k^T \ \hat{u}_{k-1}^T]^T$ ,  $\varsigma_k = [\varsigma_1^T \ w_k^T]^T$  and indicating  $\Delta V_k$  which is the forward difference of  $V_k$ , we have

$$\begin{aligned} E\{\Delta V_k\} &= \{V_{k+1} - V_k | x_k, t_k = s\} \\ &= E\{x_{k+1}^T P_t x_{k+1} - x_k^T P_s x_k\} \\ &= E\left\{\sum_{t=1}^l \sum_{g=1}^l \theta_{st} \phi_{sg} x_{k+1}^T P_t x_{k+1} - x_k^T P_s x_k\right\} \\ &= E\left\{\sum_{g=1}^l \phi_{sg} \varsigma_k^T \begin{bmatrix} \tilde{h}_{sg}^T \\ B_{2s}^T \end{bmatrix} \bar{P}_s \begin{bmatrix} \tilde{h}_{sg} \\ B_{2s} \end{bmatrix} \varsigma_k - x_k^T P_s x_k\right\} \end{aligned} \quad (24)$$

Based on the event-triggered mechanism (6), we know

$$\varepsilon x_k^T G x_k - e_k^T G e_k \geq 0 \quad (25)$$

Substituting (25) into (24), we have

$$\begin{aligned} E\{\Delta V_k\} &\leq E\left\{\sum_{g=1}^l \phi_{sg} \varsigma_k^T \begin{bmatrix} \tilde{h}_{sg}^T \\ B_{2s}^T \end{bmatrix} \bar{P}_s \begin{bmatrix} \tilde{h}_{sg} \\ B_{2s} \end{bmatrix} \varsigma_k \right. \\ &\quad \left. + \varsigma_{1k}^T \wp_s^2 \varsigma_{1k} - x_k^T P_s x_k\right\} \end{aligned} \quad (26)$$

Noticing that  $w_k \equiv 0$  in the definition of stochastic stability, thus combining (26), it is easy to gain that

$$\begin{aligned} E\{\Delta V_k\} &\leq E\left\{\sum_{g=1}^l \phi_{sg} \varsigma_{1k}^T \Upsilon_{2sg} \varsigma_{1k} - x_k^T P_s x_k\right\} \\ &< E\left\{\varsigma_{1k}^T \sum_{g=1}^l \phi_{sg} \hat{H}_{sg} \varsigma_{1k} - x_k^T P_s x_k\right\} \\ &< E\left\{x_k^T \left(\sum_{g=1}^l \phi_{sg} H_{sg} - P_s\right) x_k\right\} \end{aligned}$$

$$\leq \delta E\{x_k^T x_k\} \quad (27)$$

where “<” is gained from (22), and  $\delta = \lambda_{\max}\left(\sum_{s \in L} \sum_{g=1}^l \phi_{sg} H_{sg} - P_s\right)$ . Hence

$$E\left\{\sum_0^\infty \Delta V_k\right\} = E\{V_\infty - V_0\} \leq \delta E\left\{\sum_0^\infty x_k^T x_k\right\} \quad (28)$$

From (18) and (20), we get that  $\delta < 0$ ; therefore,

$$E\left\{\sum_0^\infty x_k^T x_k\right\} < \infty \quad (29)$$

which conforms to Definition 1; namely, the stochastic stability of the system (15) is verified.

Besides, the system (15) will be proven to be strictly  $(\mu, \nu, \varpi) - \gamma$ -dissipative. The performance indicator  $J$  is described as below:

$$J = \sum_{k=0}^\infty E\left\{w_k^T (\gamma I - \varpi) w_k - y_k^T \mu y_k - 2y_k^T \nu w_k\right\} \quad (30)$$

Under the zero initial condition, we have

$$\begin{aligned} J &\leq \sum_{k=0}^\infty E\{w_k^T (\gamma I - \varpi) w_k - y_k^T \mu y_k - 2y_k^T \nu w_k \\ &\quad + \Delta V_k + \varepsilon x_k^T G x_k - e_k^T G e_k\} \\ &\leq \sum_{k=0}^\infty E\left\{\sum_{g=1}^l \phi_{sg} \varsigma_k^T \Upsilon_{1sg} \varsigma_k - x_k^T P_s x_k\right\} \\ &< \sum_{k=0}^\infty E\left\{\varsigma_k^T \sum_{g=1}^l \phi_{sg} \tilde{H}_{sg} \varsigma_k - x_k^T P_s x_k\right\} \\ &= \sum_{k=0}^\infty E\left\{x_k^T \left(\sum_{g=1}^l \phi_{sg} H_{sg} - P_s\right) x_k\right\} < 0 \end{aligned} \quad (31)$$

The two “<” can be gained from (22) and (20), respectively. Moreover, by Definition 2, condition (17) is met. Thus, this proof is validated.

*Remark 5:* By utilizing the sufficient condition provided in Theorem 1, we can ensure that the closed-loop system (15) is stochastically stable and strictly  $(\mu, \nu, \varpi) - \gamma$ -dissipative. However, the nonlinear term and a high-dimensional gain matrix make the operation complex, and we simplify the conditions by introducing a relaxation matrix. Among them, linearization processing is more convenient for controller design.

Next, we present a solution method for determining the parameters of the event-triggered asynchronous controller based on Theorem 1, using a relaxation matrix. This method is detailed in Theorem 2.

*Theorem 2:* The closed-loop system (15) is fulfilled for stochastically stable and strictly  $(\mu, \nu, \varpi) - \gamma$ -dissipative, if there contain matrices  $\bar{K}_g \in \mathbb{R}^{n_u \times n_x}$ ,  $E \in \mathbb{R}^{n_x \times n_x}$ , positive

definite matrices  $\bar{P}_s \in \mathbb{R}^{n_x \times n_x}$ ,  $\bar{H}_{sg} \in \mathbb{R}^{n_x \times n_x}$ , for  $\forall s \in L$ ,  $g_j \in g \in \ell_K^s$ ,  $\bar{g}_j \in \bar{g} \in \ell_U^s, j = 1, 2, \dots, L$ , satisfying

$$\begin{bmatrix} -\bar{P}_s & \Gamma_{1sg} & \Gamma_{2s\bar{g}} \\ * & \Omega_{sg} & 0 \\ * & * & \Omega_{s\bar{g}} \end{bmatrix} < 0 \quad (32)$$

and for  $\forall s, g \in L$ , satisfying

$$\begin{bmatrix} \Xi_{sg} & N_{sg} & M_{sg} \\ * & -I & 0 \\ * & * & \partial \end{bmatrix} < 0 \quad (33)$$

where

$$\begin{aligned} \Gamma_{1sg} &= [\sqrt{\phi_{sg1}}\bar{P}_s \cdots \sqrt{\phi_{sgj}}\bar{P}_s \cdots \sqrt{\phi_{sl}}\bar{P}_s] \\ \Gamma_{2s\bar{g}} &= [\Gamma_{2s\bar{g}_1} \cdots \Gamma_{2s\bar{g}_j}] \\ \Gamma_{2s\bar{g}_j} &= \sqrt{1 - \sum \phi_{sgj}}\bar{P}_s \\ \Omega_{sg} &= \text{diag}\{-\bar{H}_{sg1}, \dots, -\bar{H}_{sgj}, \dots, -\bar{H}_{sl}\} \\ \Omega_{s\bar{g}} &= \text{diag}\{-\bar{H}_{s\bar{g}_1}, \dots, -\bar{H}_{s\bar{g}_j}, \dots, -\bar{H}_{sl}\} \\ \tilde{C}_{sg} &= C_s E + \alpha D_{1s} \bar{K}_g \\ \Xi_{sg} &= \begin{bmatrix} \varepsilon \bar{G} + \bar{H}_{sg} - E^T - E & 0 & 0 & -\tilde{C}_{sg}^T \nu \\ * & -\bar{G} & 0 & -(\alpha D_{1s} \bar{K}_g)^T \nu \\ * & * & -I & -((1-\alpha)D_{1s})^T \nu \\ * & * & * & Q_s \end{bmatrix}, \\ N_{sg} &= \begin{bmatrix} U_1 \tilde{C}_{sg}^* & \alpha U_1 D_{1s} \bar{K}_g & (1-\alpha) U_1 D_{1s} & U_1 D_{2s} \\ f U_1 D_{1s} K_g & f U_1 D_{1s} K_g & -f U_1 D_{1s} & 0 \end{bmatrix}^T, \\ M_{sg} &= [\sqrt{\theta_{s1}} \hat{M}_{sg}^T \quad \sqrt{\theta_{s2}} \hat{M}_{sg}^T \cdots \sqrt{\theta_{sl}} \hat{M}_{sg}^T], \\ \bar{M}_{sg} &= \begin{bmatrix} A_s E + \alpha B_{1s} \bar{K}_g & \alpha B_{1s} \bar{K}_g & (1-\alpha) B_{1s} & B_{2s} \\ f B_{1s} \bar{K}_g & f B_{1s} \bar{K}_g & -f B_{1s} & 0 \end{bmatrix}, \\ \partial &= \text{diag}\{-\bar{P}_1, -\bar{P}_2, \dots, -\bar{P}_l\}. \end{aligned}$$

When there is a feasible solution to (32) and (33), the controller gain and the event-triggered weighting matrix can be expressed as bellow:

$$K_g = \bar{K}_g E^{-1}, G = (E^T)^{-1} \bar{G} E^{-1} \quad (34)$$

*Proof:* First, we define

$$\bar{P}_s = P_s^{-1}, \bar{H}_{sg} = H_{sg}^{-1}, \bar{K}_g = K_g E, \bar{G} = E^T G E \quad (35)$$

where  $E$  is an invertible slack matrix. Adopting a congruence conversion to (32) by  $\text{diag}\{P_s, I, \dots, I\}$ , we have

$$\begin{bmatrix} -\bar{P}_s & \bar{\Gamma}_{1sg} & \bar{\Gamma}_{2sg} \\ * & \Omega_{sg} & 0 \\ * & * & \Omega_{s\bar{g}} \end{bmatrix} < 0 \quad (36)$$

where  $\bar{\Gamma}_{1sg} = [\sqrt{\phi_{sg1}}I \cdots \sqrt{\phi_{sgj}}I \cdots \sqrt{\phi_{sl}}I]$ ,  $\bar{\Gamma}_{2s\bar{g}} = [\bar{\Gamma}_{2s\bar{g}_1} \cdots \bar{\Gamma}_{2s\bar{g}_j}]$ ,  $\bar{\Gamma}_{2s\bar{g}_j} = \sqrt{1 - \sum \phi_{sgj}}I$ .

Next, adopting Schur complement, we know that (36) is equivalent to (18). Besides, the following inequality holds:

$$(\bar{H}_{sg} - E)^T \bar{H}_{sg}^{-1} (\bar{H}_{sg} - E) \geq 0 \quad (37)$$

hence

$$-E^T \bar{H}_{sg}^{-1} E \leq \bar{H}_{sg} - E^T - E \quad (38)$$

Then, (33) is written as

$$\begin{bmatrix} \tilde{\Xi}_{sg} & N_{sg} & M_{sg} \\ * & -I & 0 \\ * & * & \partial \end{bmatrix} < 0 \quad (39)$$

where

$$\tilde{\Xi}_{sg} = \begin{bmatrix} \varepsilon \bar{G} - E^T \bar{H}_{sg} E & 0 & 0 & -\tilde{C}_{sg}^T \nu \\ * & -\bar{G} & 0 & -(\alpha D_{1s} \bar{K}_g)^T \nu \\ * & * & -I & -((1-\alpha)D_{1s})^T \nu \\ * & * & * & Q_s \end{bmatrix}$$

Ordering  $\Psi = \text{diag}\{(E^T)^{-1}, (E^T)^{-1}, I, \dots, I\}$ , and using a congruence conversion to (39) by  $\Psi$ , we get

$$\begin{bmatrix} \tilde{\Xi}_{sg} & \tilde{N}_{sg} & \tilde{M}_{sg} \\ * & -I & 0 \\ * & * & \partial \end{bmatrix} < 0 \quad (40)$$

where

$$\begin{aligned} \tilde{\Xi}_{sg} &= \begin{bmatrix} \varepsilon G - H_{sg} & 0 & 0 & -\tilde{C}_{sg}^{*T} \nu \\ * & -G & 0 & -(\alpha D_{1s} K_g)^T \nu \\ * & * & -I & -((1-\alpha)D_{1s})^T \nu \\ * & * & * & Q_s \end{bmatrix}, \\ \tilde{N}_{sg} &= \begin{bmatrix} U_1 \tilde{C}_{sg}^* & \alpha U_1 D_{1s} K_g & (1-\alpha) U_1 D_{1s} & U_1 D_{2s} \\ f U_1 D_{1s} K_g & f U_1 D_{1s} K_g & -f U_1 D_{1s} & 0 \end{bmatrix}^T, \\ \tilde{M}_{sg} &= [\sqrt{\theta_{s1}} \hat{M}_{sg}^T \quad \sqrt{\theta_{s2}} \hat{M}_{sg}^T \cdots \sqrt{\theta_{sl}} \hat{M}_{sg}^T], \\ \hat{M}_{sg} &= \begin{bmatrix} A_s + \alpha B_{1s} K_g & \alpha B_{1s} K_g & (1-\alpha) B_{1s} & B_{2s} \\ f B_{1s} K_g & f B_{1s} K_g & -f B_{1s} & 0 \end{bmatrix}. \end{aligned}$$

By using Schur complement and (40), we derive (19). Thereby, this proof is verified.

*Remark 6:* In Theorem 2, we introduce a relaxation matrix  $E$ , and then use inequality transformation techniques such as matrix scaling to obtain linear matrix inequalities, so as to solve the nonlinear problem in Theorem 1 with the LMI toolbox of Matlab.

*Remark 7:* The parameter  $\gamma$  reflects the dissipative performance of the system. The larger the parameter, the better the system's dissipative performance. Therefore, we can optimize the parameter  $\gamma$  to obtain the optimal performance  $\gamma^*$ :

$$\begin{cases} \min & -\gamma \\ \text{s.t.} & (32), (33) \end{cases} \quad (41)$$

It should be noted that the analysis of dissipative performance encompasses two special properties:

(1)  $H_\infty$ : let  $\mu = -I, \nu = 0, \varpi = (\gamma^2 + \gamma)I$  in (32) and (23).

(2) Passivity: when  $\mathbb{R}^{n_v} = \mathbb{R}^{n_w}$ , let  $\mu = 0, \nu = I, \varpi = 2\gamma I$  in (32) and (33).

*Remark 8:* The algorithm proposed in this paper is as follows:

**Algorithm 1.** Co-Optimization Algorithm

- 1: **Input:** System parameter, trigger threshold  $\varepsilon$ , packet loss rate  $\alpha$ , transition probability  $\phi_g$
- 2: **Output:** Optimal performance parameters  $\gamma^*$ , controller gain  $K_g$ , the data transmission rate  $TP$  and trigger matrix  $G$
- 3: **Initialization:** Dissipative performance parameters  $(\mu, \nu, \varpi)$
- 4: **Step 1:** Use the convex optimization of (41) to solve the optimal performance parameters  $\gamma^*$ , controller gain  $K_g$ , and trigger matrix  $G$
- 5: **Step 2:** The value of  $TP$  was calculated based on the triggered times and the total time

**IV. NUMERICAL EXAMPLE**

In this section, we will demonstrate the effectiveness of the proposed design method by using the example with three modes.

$$\begin{aligned}
 A_1 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
 A_3 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, \\
 B_{13} &= \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, B_{22} = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}, \\
 B_{23} &= \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}, C_1 = [0.17 \ 0.18], \\
 C_2 &= [0.42 \ 0.9], C_3 = [0.12 \ 0.5], \\
 D_{11} &= D_{12} = D_{13} = 0, D_{21} = 0.1, D_{22} = 0.8, D_{23} = 0.3.
 \end{aligned}$$

Letting the system transition matrix  $\Theta$  as below:

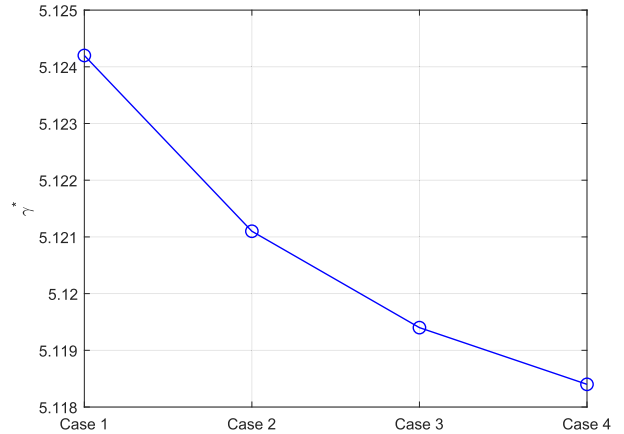
$$\Theta = \begin{bmatrix} 0.85 & 0.5 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.5 & 0.1 & 0.4 \end{bmatrix}$$

Then, we assume the dissipative parameters  $\mu = -1, \nu = 1, \varpi = 5$ , separately, the parameter  $\alpha = 0.9$ , and the error threshold  $\varepsilon = 0.1$ .

Besides, four different situations of the conditional probability matrix will be simulated and compared.

$$\begin{aligned}
 \text{Situation 1: } \Phi &= \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.1 & 0.9 & 0 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}, \\
 \text{Situation 2: } \Phi &= \begin{bmatrix} 0.9 & ? & ? \\ 0.1 & 0.9 & 0 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}, \\
 \text{Situation 3: } \Phi &= \begin{bmatrix} 0.9 & ? & ? \\ ? & 0.9 & ? \\ 0.1 & 0.1 & 0.8 \end{bmatrix}, \\
 \text{Situation 4: } \Phi &= \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}.
 \end{aligned}$$

Observing from Situation 1 to Situation 4, we notice a gradual decrease in the amount of known conditional probabilities.



**FIGURE 2.** Dissipative performance  $\gamma^*$  under four cases.

First, based on Theorem 2 and Remark 7, we can simulate the above four cases and obtain the corresponding dissipative performance parameter as shown in Figure 2. It can be seen from Figure 2 that the more information about the unknown conditional probability, the worse the dissipative performance of the system. In addition, the controller solved in Case 4 is equivalent to the controller that is strictly mode independent, that is, under any mode, the controller gain is:

$$K_g = [1.5286 \ 0.9644]$$

Next, we will further investigate Case 3, and obtain the event-triggered weighting matrix by solving linear matrix inequalities in Theorem 2:

$$G = \begin{bmatrix} 1.0553 & 0.6845 \\ 0.6845 & 0.4468 \end{bmatrix}$$

The controller gain:

$$\begin{aligned}
 K_1 &= [1.5170 \ 0.9871] \\
 K_2 &= [1.3660 \ 0.8944] \\
 K_3 &= [1.3626 \ 0.8864]
 \end{aligned}$$

We consider that the initial value  $x_0 = [0.3 \ 0.2]^T$  of the system and disturbance input  $w_k = 0.9^k \sin(k)$ , and subsequently, we apply the obtained feasible solution to the proposed event-triggered asynchronous control scheme for simulation. The response curves of the system state, output, and control input are shown in Figure 3 and the time interval of the event-triggered mechanism is shown in Figure 4. As we can observe, these curves gradually tend to equilibrium, indicating that the closed-loop system is stochastically stable. Meanwhile, the data transmission amount is significantly reduced, showing a reduction in communication consumption. Furthermore, Figure 5 illustrates the asynchronous behavior between the modes of the system and the modes of the controller.

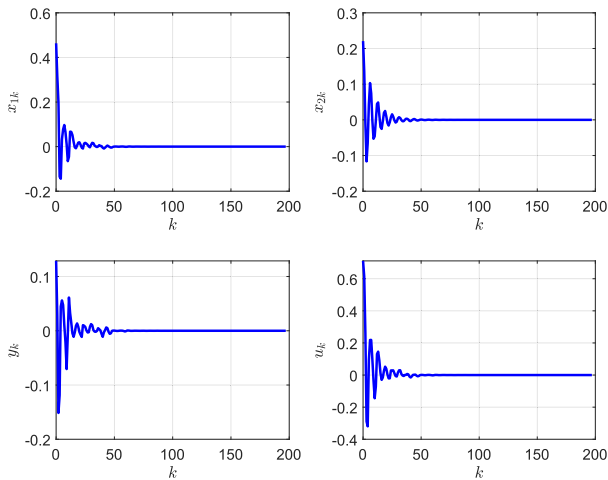


FIGURE 3. System status, outputs, and control input.

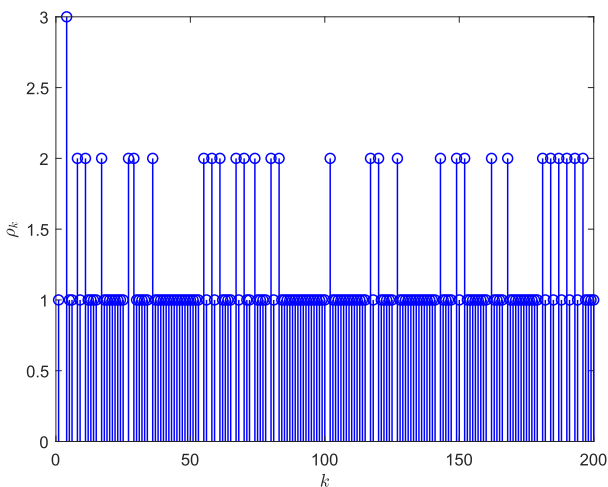


FIGURE 4. Event-triggered transmission interval.

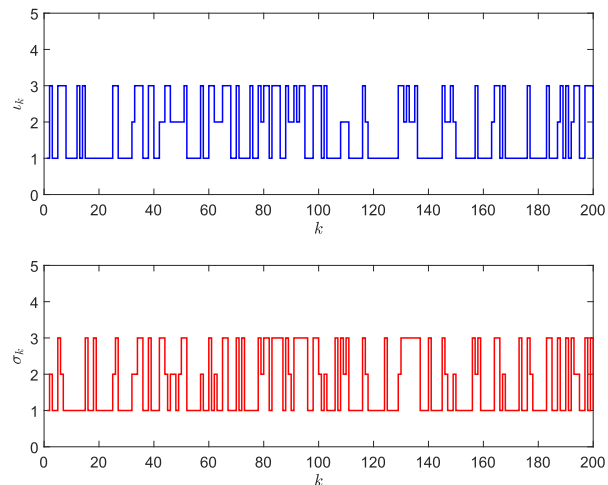


FIGURE 5. Original system and controller modes.

Furthermore, we analyze the influence of the event-triggered threshold  $\varepsilon$  on both system performance and

TABLE 2. Dissipative performance  $\gamma^*$  and data transmission rate.

$\varepsilon$	0	0.1	0.2	0.3	0.4
$\gamma^*$	5.1298	5.1194	5.1159	5.1136	5.1118
$TP(\%)$	100	82.5	76.5	69.5	65

communication performance by varying its value. Based on Theorem 2, we conduct simulations and present the results in Table 2. It is evident that as the threshold  $\varepsilon$  increases, the dissipative performance decreases slightly, while the communication performance improve significantly. Therefore, in practical application, considering the balance between two kinds of performance, an appropriate event-triggered threshold can be selected to obtain better communication performance and more satisfactory comprehensive performance under the premise of ensuring that the system meets the actual dissipation requirements.

TABLE 3. Dissipative performance  $\gamma^*$  under different packet loss rates.

$\alpha$	1	0.9	0.8	0.7
$\gamma^*$	5.1198	5.1194	5.1189	5.1182

Next, we change  $\alpha$  to study the impact of packet loss rate on system performance, where the smaller  $\alpha$ , the higher packet loss rate. As shown in Table 3, when  $\alpha = 1$  indicates no packet loss, the dissipative performance of the system is optimal. With the increase of packet loss rate, the dissipative performance decreases. The dissipative performance of the two packet loss models is shown in Figure 6. It can be seen that the dissipative performance based on the packet loss model given in this paper is better than that in [27].

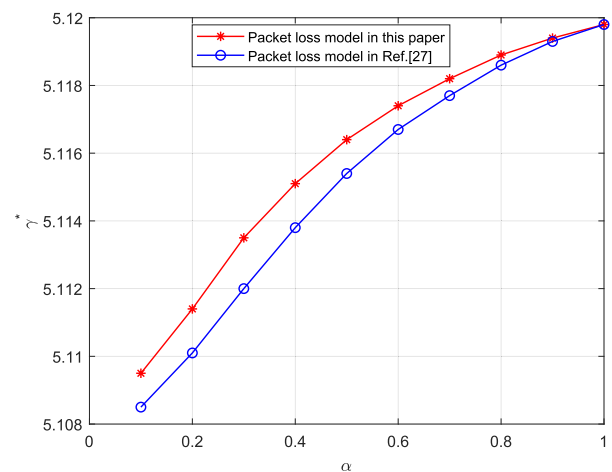


FIGURE 6. Comparison of dissipative performance  $\gamma^*$  between two packet loss models.

V. CONCLUSION

This paper has addressed the problem of asynchronous and event-triggered control for discrete Markov jump systems



with packet losses and partially unknown conditional probabilities. Firstly, an event-triggered mechanism is introduced to reduce the data transmission rate of the network channel. Additionally, a Bernoulli packet loss model with non-zero actuator input is considered to handle packet loss issues. Then, by utilizing Lyapunov stability theory and dissipation theory, sufficient conditions for the stochastic stability and strict dissipation of the closed-loop system have been derived, even there exist the packet loss and partially unknown conditional probabilities. The controller gain is obtained in the form of linear matrix inequalities. Finally, the designed controller has been validated through an example with three modes, demonstrating its compliance with stability theory and dissipation theory. For issues such as time-delay, quantization, and dynamic event-triggered mechanisms, further researches in the future will be worth considering.

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