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RESEARCH ARTICLE

A Quantum Algorithm for Boolean Functions Processing

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ABSTRACT Detecting junta variables is a critical issue in Boolean function analysis, circuit design optimization, and machine learning feature selection. In this paper, we investigate a novel quantum computation algorithm based on the Mz operator. The algorithm takes in an unknown oracle concealing a Boolean function with n variables and an unknown input state, which can be quantum or classical. The input state can be complete or incomplete quantum basis states, and it can be a weighted or uniform superposition of basis states. The proposed approach determines whether a given variable is a junta with a time cost of $O(2/\epsilon^2)$ and a memory cost of $2n+6$. The algorithm is analyzed and experimentally implemented using the Qiskit simulator and IBM's real quantum computer. Experimental results show that the proposed approach achieved a quantum supremacy ratio 6300% higher than that of the classical method when verifying junta variables for Boolean functions with 12 variables. The results suggest that the proposed quantum method can verify junta variables in scenarios beyond the capabilities of current classical or quantum methods.

INDEX TERMS Quantum algorithm, junta problem, Mz, entanglement, Boolean functions.

I. INTRODUCTION

Quantum computers are complex devices based on the concepts of quantum physics, a branch of research dedicated to understanding the actions and properties of atoms and particles. Quantum computers work by analyzing and regulating the behavior of these particles. This operates entirely differently than with supercomputers or traditional computers [1]. Quantum computing is a cross-disciplinary domain that incorporates elements of information science, computer science, physics, and mathematics. It utilizes quantum mechanics to tackle complicated issues quicker than traditional computers. This field has grown rapidly and made remarkable advancement across a variety of scientific fields, which include quantum machine learning [2], quantum communication [3], quantum IoT [4], quantum cryptography [5], [6], quantum computation for power systems [7], [8], computational biology [9], drug discovery [10], protein structure prediction [11], [12], quantum chemistry [13], materials science [14], and other areas [15]. In accordance with the principles of quantum physics, quantum computing is a potentially revolutionary technology that utilizes quantum mechanical concepts such as superposition and entanglement. This allows for the resolution of traditional and intractable problems that are exceedingly challenging for classical computers [16], enabling the tackling of issues that are beyond the capabilities of traditional computing systems [17], [18]. Researchers continually attempt to develop creative quantum algorithms that surpass their conventional equivalents as technology advances. Recently, it was developed a quantum computing model that solves problems by measuring the strength of entanglement using the concurrence measure [16]. Based on the capability of this model to reduce the cost of computations from exponential time to polynomial time, this model was applied [19] to solve the problem of hamming distance, and compared to

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the classical deterministic computations, it might require exponentially less time.

The junta problem is a major problem that arises throughout numerous domains. Based only on the variables, chosen uniformly at random from samples, the junta problem is the task of learning an unknown Boolean function. These variables are sometimes referred to as the junta for a Boolean function. The junta problem is currently extensively utilized within the field of machine learning [20] and computational learning theory [21]. In computational biology, when studying the relationship between a genetic attribute and a long DNA sequence, it is anticipated that only a small, unidentified portion of the sequence affects this attribute [22].

A k-junta is defined as a Boolean function $f : \{0, 1\}^n \rightarrow \{0, 1\}$ which is dependent solely on k of the n variables. Evaluating accurately a Boolean function f for identifying the k dependent variables is the goal of the k-junta problem. As quantum computers evolve, numerous researchers are utilizing algorithms of quantum learning to tackle the problem of k-junta and the function operation U_f is built around a Boolean function f. Based on Fourier sampling and $O(\frac{k}{\delta})$ function operations, a quantum property detector is introduced by Atıcı and Servedio [23] to identify k-junta using $O(\frac{k}{\delta})$ quantum queries. They also provided an algorithm of quantum learning to determine the variables that are dependent with an accuracy of δ, using $O(2^k \log \delta^{-1})$ random examples and $O(\delta^{-1} k \log k)$ quantum examples. A quantum learning technique is presented by Floess et al. [24] for solving the problem of k-junta that utilizes the amplitude amplification technique [25] and the algorithm of Bernstein-Vazirani [26]. Particularly, the Boolean function contains linear, quadratic, or cubic terms, with each variable appearing only once. When k approaches a big number, and for a Boolean function that is a product of k input variables, the Floess et al.'s algorithm needs to perform $O(2^k)$ operations of function to locate at least one dependent variable. This algorithm achieves this task with a high success probability close to 0.96. Ambainis et al. [27] proposed an algorithm of quantum learning that employs $\sqrt{\delta^{-1} k}$ function operations and tackles the group testing problem. El-Wazan et al. [28] proposed a quantum approach applicable to any Boolean function. They also devised another black box function that incorporates two function operations. To identify variables that are dependent with a probability of at least $\frac{2}{3}$, the explained quantum-based approach uses $O(2^{\frac{n}{2}})$ function operations. However, no exact quantum learning algorithm presented for the problem of k-junta. For the exceptional case of 2-junta, the Boolean function $f(x) = x_g x_h$, where $0 \leq g, h \leq n-1$ is devised by Floess et al. [24]. They suggested an algorithm for quantum computers that uses t function operations to find variables that are dependent with a probability of $1 - (0.25)^t$. Chen [29] presented a quantum algorithm to tackle the problem of 2-junta with certainty. In the worst scenario, Chen's algorithm identifies two dependent variables through applying $O(\log_2 n)$ times. Furthermore, on average, 3.82 function operations are required. To find the three dependent variables of the Boolean function $f(x) = x_g x_h x_k$, where $0 \leq g, h, k \leq n-1$, Chen [30] utilized a similar concept to provide an exact quantum algorithm. On average, 7.23 function operations are required when $n \geq 16$. He showed that the explained algorithm fails to solve the problem of k-junta with a single uncomplemented product for $4 \leq k < 2^{-1} n$. Chen [31] introduced an exact quantum learning approach to tackle the problem of 2-junta, involving $O(\log_2 n)$ function operation, in the worst-case scenario. When $n \geq 8$, this approach [31] requires on average, 5.3 function operations. Chen and An [32] suggested an exact algorithm of quantum learning for identifying two dependent variables to tackle the problem of 2-junta. In the worst-case scenario, using modified black-box function that is presented by El-Wazan et al. [28], this algorithm takes only three function operations. Later, to solve the problem of 3-junta, Chen [33] proposes an exact algorithm of quantum learning to find three dependent variables. This algorithm solves the 3-junta problem using a modified black-box function. The quantum algorithm necessitates $O(\log_2 n)$ function operations, with an average at most 3.41 in the worst-case scenario.

Although there have been significant prior efforts, there is still a need for further investigations to address and tackle the issue of junta determination. Specifically, one open research problem pertains to detecting junta variables in scenarios where the oracle and inputs are both unknown and the inputs are provided through an unknown weighted or uniform distribution of incomplete or complete superposition input states. Therefore, this paper explores a new quantum approach to handle this open research problem. The approach is based on the operator Mz and involves evaluating the degree of entanglement. The time and memory costs associated with this proposed approach are also extensively discussed. Furthermore, this new approach is implemented empirically using both IBM's quantum computer simulator and IBM's real quantum computer.

The remaining paper is organized as follows: Section II presents the problem statement. Section III explains the classical algorithm for junta verification. In Section IV, the methodology and the quantum Mz operators are discussed. Section V provides a comprehensive explanation of the proposed quantum approach, including an analysis of the proposed algorithm through a case study. This section also addresses the memory and time complexities of both the proposed algorithm and its traditional counterpart. Section VII focuses on presenting the experimental results and discussing the proposed algorithm in detail. Section VIII comprises the main results of this paper.

II. PROBLEM STATEMENT

The problem statement is formulated by two cases as follows: (i) Assuming it is required to learn a classical machine learning model on a Boolean expression of a very large number of Boolean variables n, e.g., $n \geq 50$. Thus, it is needed to generate a dataset for this expression to train the model, but this type of dataset cannot be processed using current classical main computer memories (RAM). One approach that could help reduce the size of the dataset is by checking whether this logical expression has a junta variable or not. If a Boolean expression has m junta variables, this will reduce the dataset to 2^m rows in the dataset rather than 2^n, where $m < n$.

(ii) Assuming that an unknown Boolean expression is provided via an oracle and is applied to an unknown input in a weighted and complete/incomplete superposition state, such that the input state is received through a quantum communication channel [34]. It is required to check whether there are junta variables for this oracle and input or not. These two cases can be is articulated as follows:

Given: An oracle U_f that hides an unknown Boolean expression acts on a given unknown or a prepared input state of n variables defined as follows:

$$|\Gamma\rangle = |x_1, x_2, \ldots, x_n\rangle = \sum_j w_j |j\rangle, \text{ where } K \leq 2^n, \quad (1)$$

such that K is the number of basis states.

Goal: Check whether a given variable x_i, where $i = 1, 2, \ldots, n$, is a junta or not in polynomial time in terms of a predefined error of ϵ.

If the input state $|\Gamma\rangle$ is prepared with all the basis states $K = 2^n$ and all coefficients $w_j's$ are equal to $2^{\frac{-n}{2}}$ in Eq. (1), then this corresponds to case (i). On the other hand, if the input state $|\Gamma\rangle$ is given and unknown such that $K < 2^n$, or all coefficients $w_j's$ are not equal in Eq. (1), then this corresponds to case (ii).

III. CLASSICAL COMPUTING APPROACH

In the domain of function testing, particularly within the context of machine learning, when faced with an extensive array of input features, the investigation focuses on the challenges associated with managing a substantial number of input variables. The goal is to determine if there is a limited set of characteristics that influence another property within the dataset [35]. In this scenario, the features are treated as coordinates, and the elements of the original problem are encoded through a dictation process. This process systematically represents the problem components as follows: Encode a certain $i \in [n]$ by providing the truth table of the function $f : \{0, 1\}^n \rightarrow \{0, 1\}$, where $f(x) = x_i$. The process requires $log(N)$ bits to specify i and 2^n bits for the truth table of the Boolean function f. Junta testing is applied to input functions for Boolean functions $f : \{0, 1\}^n \rightarrow \{0, 1\}$. This can be verified classically using Algo. 1 [36], [37].

Algorithm 1 : Classical Algorithm of Checking Junta Variables [37]

Step 1: Any Boolean function f in positive polarity Reed-Muller form of n input variables, and $N = 2^n$ can be written as follows: $f = f(x_0, x_1, \ldots\ldots, x_{n-1})$.

Step 2: Define $\overline{f}$ the function when flipping the x_i bit in f:

$$\overline{f} = f(x_0, x_1, x_2 \ldots.., x_{i-1}, \overline{x_i}, x_{i+1}, \ldots\ldots, x_{n-3}, x_{n-2}, x_{n-1})$$

Step 3: Check if the function $\overline{f} = f$ by comparing the outputs of each function.

Step 4: To decide the result of testing x_i:

(a) If x_i affects the function then $\overline{f} \neq f$ and x_i is junta.

(b) If x_i does not affect the function then $\overline{f} = f$ and x_i is not junta.

IV. METHODOLOGY

The quantum entanglement phenomenon denotes a state wherein subsystems are correlated to a certain degree such that measuring the state of a subsystem affects the state of the remaining subsystem no matter what, irrespective of their spatial separation [38]. In situations where direct detection of entanglement is not feasible, for example, due to quantum operations, entanglement measures are employed to quantify the degree of entanglement within a quantum system. Specifically, in the context of a two-qubit system, the strength of entanglement is assessed through the utilization of the concurrence measure [19], which is estimated as $C(|lh\rangle) = 2|a_0a_3|$, where $C \in [0, 1]$, only if the two-qubit system is in the state $|lh\rangle = a_0|00\rangle + a_3|11\rangle$ [16]. Recently, a new quantum computing model [16] was discussed to perform quantum computing operations based on the actual/virtual degree of entanglement. Fig. 1 shows the schematic structure of this model, the main operations and techniques of this model are summarized. This model involves creating two decoupled replicas. On each replica, an oracle U_f is applied independently to an input state $|\Psi(t)\rangle$ and an output qubit $|l\rangle$. An additional qubit $|h\rangle = |0\rangle$, is attached to this system. Subsequently, the Mz operator is applied to the qubits $|l\rangle \otimes |h\rangle$, which performs two operations: (1) creation of entanglement and (2) measurement of entanglement.

1-Entanglement creation:
By applying a CNOT gate on the two ancillary qubits, with $|l\rangle$ as the control qubit and $|h\rangle$ as the target qubit, we can create either actual or virtual quantum entanglement between the qubits $|lh\rangle$ to varying degrees depending on the state of the qubit $|l\rangle$. We define 'actual entanglement' as a situation where the state of two qubits $|lh\rangle$ cannot be factorized into individual qubit states across all possible orthonormal basis sets. On the other hand, if the state of two qubits cannot be factorized only within the computational basis set, we refer to this as 'virtual entanglement'.

2-Entanglement measurement:
In this operation, a measurement in the computational basis set is conducted on the four qubits of the two replicas, denoted as $|lhlh\rangle$. To determine the virtual or actual concurrence value C between the qubits $|lh\rangle$, one

of the three formulas in Eq.(2) is used [16]:

$$C = \sqrt{2(P_{0011} + P_{1100})} = 2\sqrt{P_{1100}} = 2\sqrt{P_{0011}}, \tag{2}$$

Algorithm 2 The Proposed Quantum Algorithm for Verifying Junta Variables

Step 1: If an unknown input state $|\Gamma\rangle$ is received and its state is defined by Eq. (1) then set the qubit $|C_t\rangle = |0\rangle$, and go to Step 2.
Else{
(i) Set the qubit $|C_t\rangle = |1\rangle$.
(ii) Apply n controlled Hadamard gates, CH, on the control qubit $|C_t\rangle$ and each qubit $|0_r\rangle$ in the register $|\Gamma\rangle$ as a target qubit, where $r = 0, 1, 2, \ldots, n-1$, to create a uniform complete superposition of all possible computational basis states as follows:

$$|C_t\rangle \otimes |\Gamma\rangle = \prod_{r=0}^{n-1} CH_{\{C_t\Gamma_r\}}(|C_t\rangle|0\rangle)^{\otimes n}) = \sum_{x=0}^{K-1} w_x|x\rangle,$$

where $w_x = 2^{\frac{-n}{2}}$, and $K = 2^n$}.
Step 2: Initialize a closed quantum system composed of the qubit $|C_t\rangle$, the register $|\Gamma\rangle$, and three ancillary qubits $|slh\rangle = |0\rangle^{\otimes 3}$. The state of this system is described by Eq. (3).

$$|\psi_0^{S_1}\rangle = |C_t\rangle \otimes |\Gamma\rangle \otimes |slh\rangle = |C_t\rangle \sum_x w_x|x\rangle|000\rangle. \tag{3}$$

Step 3: To generate the output of the unknown Boolean expression on the qubit $|s\rangle$, the oracle U_f is applied to the qubits $|\Gamma\rangle|s\rangle$ of the system $|\psi_0^{S1}\rangle$ as follows:

$$|\psi_1^{S_1}\rangle = (I \otimes U_f \otimes I^{\otimes 2})|\psi_0^{S_1}\rangle = |C_t\rangle \sum_x w_x|x\rangle|f(x)\rangle|00\rangle. \tag{4}$$

Step 4: Apply the quantum negation gate X on the qubit with index i in the register $|\Gamma\rangle$ as explained in Eq. (5).

$$\begin{aligned}|\psi_2^{S_1}\rangle &= (I^{\otimes(2+1+2+\ldots+(i-1))} \otimes X \otimes I^{\otimes((i+1)+\ldots+(n-1)+3)})|\psi_1^{S_1}\rangle \\ &= |C_t\rangle \sum_x w_x|x'\rangle|f(x)\rangle|00\rangle, \\ &\quad \text{where } x' = (x_1, x_2, \ldots, \neg x_i, \ldots, x_n).\end{aligned} \tag{5}$$

Step 5: Repeat Step 3 again by applying the oracle U_f on the register $|\Gamma\rangle$, and the qubit $|s\rangle$.

$$\begin{aligned}|\psi_3^{S_1}\rangle &= (I \otimes U_f \otimes I^{\otimes 2})|\psi_2^{S_1}\rangle \\ &= |C_t\rangle \sum_x w_x|x'\rangle|f(x) \oplus f(x')\rangle|00\rangle.\end{aligned} \tag{6}$$

Step 6: Apply the controlled Hadamard gate CH gate on the control qubit $|s\rangle$, and the target qubit $|l\rangle$,

$$|\psi_4^{S_1}\rangle = (I^{\otimes(n+1)} \otimes CH \otimes I)|\psi_3^{S_1}\rangle. \tag{7}$$

Step 7: Apply Steps 1-6 in parallel to create an additional replica. This step does not violate the no-cloning theorem [39] because, upon repeating these steps, a new closed decoupled version of the same state $|\psi_4^{S_2}\rangle = |\psi_4^{S_1}\rangle$ is created. Therefore, the state of the whole system is $|\psi_5\rangle = |\psi_4^{S_1}\rangle \otimes |\psi_4^{S_2}\rangle$.
Step 8: Apply M_z operator on the four qubits $|lh\rangle \otimes |lh\rangle$, and estimate the virtual/actual concurrence value C using Eq. (2):

(a) If $C > 0$ then
x_i is junta.
(b) If $C = 0$ then
x_i is not junta.

where P_{0011} and P_{1100} represent the probabilities of the basis states $|0011\rangle$ and $|1100\rangle$, respectively. Subsequently, the solution to the problem is derived using one of two approaches. The first approach involves utilizing the formula solution $C = 2\frac{\sqrt{t_1(N-t_1)}}{N} = 2|a_0a_3|$, while the second technique determines the solution based on identifying the presence or absence of actual/virtual entanglement between the two qubits [16].

V. THE PROPOSED QUANTUM ALGORITHM AND ANALYSIS

A. PROPOSED QUANTUM ALGORITHM FOR JUNTA VARIABLES

To handle the two cases of the stated problem described in Section II, a novel quantum algorithm based on the computing model explained in Section IV is suggested. The detailed steps of this new quantum algorithm are explained in Algo. 2, and Fig. 2 shows the quantum circuit of this algorithm.

B. ANALYSIS OF THE PROPOSED ALGORITHM: CASE STUDY

Here, Algorithm 2 is analyzed extensively through a case study ensuring that the analysis remains generalizable. In this particular case study, we assume that we have received an input state via a quantum teleportation channel [34], [40] in the following form:

$$\begin{aligned}|\Gamma\rangle = |x_0x_1x_2\rangle &= \sqrt{\frac{3}{20}}e^{\frac{-8\pi i}{3}}|010\rangle + \sqrt{\frac{6}{20}}|011\rangle \\ &+ \sqrt{\frac{2}{20}}|101\rangle + \sqrt{\frac{9}{20}}|110\rangle,\end{aligned} \tag{8}$$

where $K = 4$. Additionally, we have an oracle U_f that conceals a Boolean function $f(x_0, x_1, x_2)$ with the following mappings:

$$\begin{aligned}&f(001) = 1,\ f(010) = 0,\ f(011) = 1, \\ &f(101) = 1,\ f(110) = 0,\ f(111) = 0.\end{aligned} \tag{9}$$

The objective of this case study is to determine whether the variable x_0 is a junta or not. Therefore, by applying the steps of this algorithm, we begin with Step 1. In this case study, since the received inputs $|\Gamma\rangle$ are defined by Eq. (8), we proceed to Step 2. Hence, this case study represents case (ii) as described in Section II. However, in another case study, if the state of the inputs is not given, it corresponds to case (i) in Section II. In such scenario, it can be simulated on a quantum computer and solved by applying three Hadamard gates $H^{\otimes 3}$ on three empty qubits $|0\rangle^{\otimes 3}$, producing the state $|\Gamma\rangle = 8^{-0.5}\sum_{x=0}^{7}|x\rangle$. In Step 2, the state is incorporated into a larger subsystem comprising 6 qubits. This subsystem consists of the three qubits from the register $|\Gamma\rangle$ and additional extra qubits denoted as $|slh\rangle = |0\rangle^{\otimes 3}$. The state

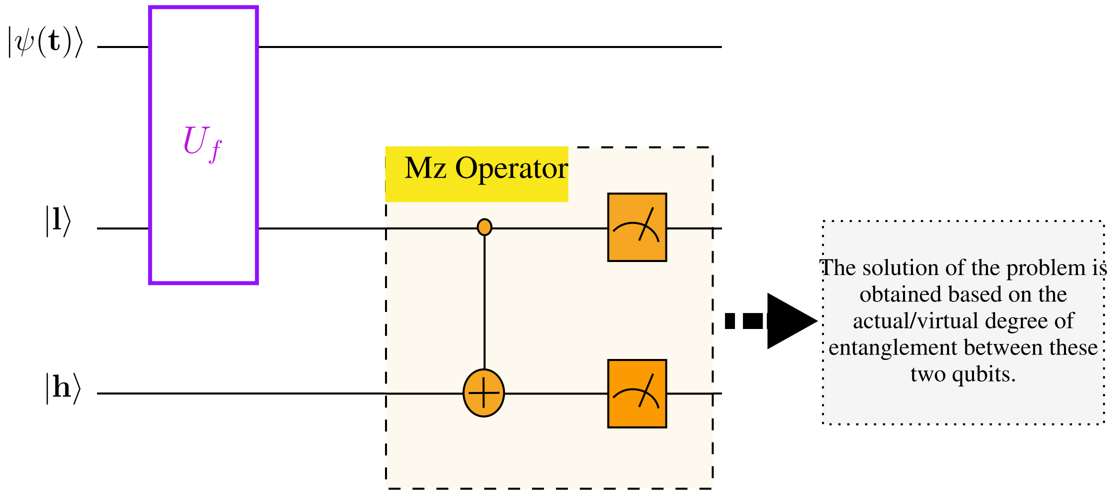

FIGURE 1. The abstract structure of the *Mz* operator-based quantum computing model when operating on a single replica.

of this subsystem is given by:

$$\begin{aligned} |\psi_0^{S_1}\rangle = |C_t\rangle|\Gamma\rangle|slh\rangle &= \sqrt{\frac{3}{20}}e^{\frac{-8\pi i}{3}}|0, 010, 000\rangle \\ &+ \sqrt{\frac{6}{20}}|0, 011, 000\rangle + \sqrt{\frac{2}{20}}|0, 101, 000\rangle \\ &+ \sqrt{\frac{9}{20}}|0, 110, 000\rangle. \end{aligned} \tag{10}$$

In Step 3, the oracle U_f that contains the operations of the Boolean function is applied to the register $|\Gamma\rangle$ and the qubit $|s\rangle$, resulting in the state defined by Eq. (11).

$$\begin{aligned} |\psi_1^{S_1}\rangle = (I \otimes U_f \otimes I^{\otimes 2})|\psi_0^{S_1}\rangle &= \sqrt{\frac{3}{20}}e^{\frac{-8\pi i}{3}}|0, 010, 000\rangle \\ &+ \sqrt{\frac{6}{20}}|0, 011, 100\rangle + \sqrt{\frac{2}{20}}|0, 101, 100\rangle \\ &+ \sqrt{\frac{9}{20}}|0, 110, 000\rangle. \end{aligned} \tag{11}$$

This state indicates that the outputs of the oracle U_f are generated in the qubit $|s\rangle$, which is entangled with the register $|\Gamma\rangle$. In Step 4, the quantum negation gate X is applied to the variable under investigation, which in this case study is x_0. The resulting state is defined by Eq. (12).

$$\begin{aligned} |\psi_2^{S_1}\rangle &= (I \otimes X \otimes I^{\otimes 5})|\psi_1^{S_1}\rangle \\ &= \sqrt{\frac{3}{20}}e^{\frac{-8\pi i}{3}}|0, 110, 000\rangle + \sqrt{\frac{6}{20}}|0, 111, 100\rangle \\ &+ \sqrt{\frac{2}{20}}|0, 001, 100\rangle + \sqrt{\frac{9}{20}}|0, 010, 000\rangle. \end{aligned} \tag{12}$$

Step 5 involves repeating Step 3, resulting in the state defined by Eq. (13).

$$\begin{aligned} |\psi_3^{S_1}\rangle &= (I \otimes U_f \otimes I^{\otimes 2})|\psi_2^{S_1}\rangle \\ &= \sqrt{\frac{3}{20}}e^{\frac{-8\pi i}{3}}|0, 110, (0 \oplus 0)00\rangle \\ &+ \sqrt{\frac{6}{20}}|0, 111, (1 \oplus 0)00\rangle \\ &+ \sqrt{\frac{2}{20}}|0, 001, (1 \oplus 1)00\rangle \\ &+ \sqrt{\frac{9}{20}}|0, 010, (0 \oplus 0)00\rangle. \\ &= \sqrt{\frac{3}{20}}e^{\frac{-8\pi i}{3}}|0, 110, 000\rangle + \sqrt{\frac{6}{20}}|0, 111, 100\rangle \\ &+ \sqrt{\frac{2}{20}}|0, 001, 000\rangle + \sqrt{\frac{9}{20}}|0, 010, 000\rangle. \end{aligned} \tag{13}$$

The result of this step is obtained due to the XOR operation between the output of the oracle U_f from Step 3 and the output of the oracle U_f after negating the Boolean input variable x_0 in Step 4. Then, in Step 6, the controlled Hadamard gate CH is applied to the control and target qubits $|s\rangle$ and $|l\rangle$, respectively, to transform the state as described by Eq. (14).

$$\begin{aligned} |\psi_4^{S_1}\rangle &= (I^{\otimes 4} \otimes CH \otimes I)|\psi_3^{S_1}\rangle \\ &= \sqrt{\frac{3}{20}}e^{\frac{-8\pi i}{3}}|0, 110\rangle|0\rangle^{\otimes 3} + \sqrt{\frac{2}{20}}|0, 001\rangle|0\rangle^{\otimes 3} \\ &+ \sqrt{\frac{9}{20}}|0, 010\rangle|0\rangle^{\otimes 3} \\ &+ \sqrt{\frac{6}{20}}|0, 111\rangle\frac{(|100\rangle + |110\rangle)}{\sqrt{2}}. \end{aligned} \tag{14}$$

In this formula:

$$\text{Let } |\zeta_1\rangle = \sqrt{\frac{3}{20}}e^{\frac{-8\pi i}{3}}|0,110\rangle|0\rangle + \sqrt{\frac{2}{20}}|0,001\rangle|0\rangle + \sqrt{\frac{9}{20}}|0,010\rangle|0\rangle, \text{ and } |\zeta_2\rangle = \sqrt{\frac{6}{20}}|0,111\rangle|1\rangle. \tag{15}$$

Then, Eq. (14) can be rewritten as:

$$|\psi_4^{S_1}\rangle = (I^{\otimes 4} \otimes CH \otimes I)|\psi_3^{S_1}\rangle = |\zeta_1\rangle|00\rangle + |\zeta_2\rangle\frac{(|00\rangle + |10\rangle)}{\sqrt{2}}. \tag{16}$$

In parallel, Step 7 executes Steps 1-6 to construct another independent replica of the system, denoted as $|\psi_4^{S_2}\rangle = |\psi_4^{S_1}\rangle$, without violating the no-cloning theorem [39]. Step 7 determines the state of the whole system, which is described by Eq. (17).

$$\begin{aligned}|\psi_5\rangle &= |\psi_4^{S_1}\rangle \otimes |\psi_4^{S_2}\rangle \\ &= (|\zeta_1\rangle^{\otimes 2} + \frac{1}{\sqrt{2}}|\zeta_1\rangle|\zeta_2\rangle + \frac{1}{\sqrt{2}}|\zeta_2\rangle|\zeta_1\rangle + 0.5|\zeta_2\rangle^{\otimes 2})|0\rangle^{\otimes 4} \\ &+ (\frac{1}{\sqrt{2}}|\zeta_1\rangle|\zeta_2\rangle + \frac{1}{2}|\zeta_2\rangle^{\otimes 2})|0\rangle^{\otimes 2}|10\rangle \\ &+ (\frac{1}{\sqrt{2}}|\zeta_2\rangle|\zeta_1\rangle + \frac{1}{2}|\zeta_2\rangle^{\otimes 2})|1\rangle|0\rangle^{\otimes 3} \\ &+ 0.5|\zeta_2\rangle^{\otimes 2})|10\rangle^{\otimes 2}|10\rangle^{\otimes 2}.\end{aligned} \tag{17}$$

Finally, in step 8, the operator Mz is applied by using two CNOT gates on the four qubits $|lh\rangle \otimes |lh\rangle$, where $|l\rangle$ is the control qubit and $|h\rangle$ is the target qubit. This yields the state described in Eq. (18).

$$\begin{aligned}|\psi_6\rangle &= (I^{\otimes 10} \otimes CNOT_{\{lh\}} \otimes CNOT_{\{lh\}})|\psi_5\rangle \\ &= (|\zeta_1\rangle^{\otimes 2} + \frac{1}{\sqrt{2}}|\zeta_1\rangle|\zeta_2\rangle + \frac{1}{\sqrt{2}}|\zeta_2\rangle|\zeta_1\rangle + 0.5|\zeta_2\rangle^{\otimes 2})|0\rangle^{\otimes 4} \\ &\quad + (\frac{1}{\sqrt{2}}|\zeta_1\rangle|\zeta_2\rangle + \frac{1}{2}|\zeta_2\rangle^{\otimes 2})|0011\rangle \\ &\quad + (\frac{1}{\sqrt{2}}|\zeta_2\rangle|\zeta_1\rangle + \frac{1}{2}|\zeta_2\rangle^{\otimes 2})|1100\rangle + 0.5|\zeta_2\rangle^{\otimes 2})|1\rangle^{\otimes 4}.\end{aligned} \tag{18}$$

Subsequently, this operator assesses the success probabilities of obtaining the basis states $|0011\rangle$ and $|1100\rangle$ by measuring the four qubits $|lh\rangle \otimes |lh\rangle$ to ascertain the value of the virtual/actual concurrence C using one of the formulas in Eq. (2). By considering the states $|\zeta_1\rangle$ and $|\zeta_2\rangle$ in Eq. (15), alongside Eqs. (18) and (2), it is deduced that $C > 0$. Consequently, it is inferred that the variable x_0 constitutes a junta.

VI. COMPLEXITY ANALYSIS

Here, the efficiency of the proposed algorithm is investigated in terms of memory space cost, time space cost, and the problem domain in comparison to the classical algorithm. In the classical approach (see Section III), examining whether a given Boolean variable x_i is a junta requires generating 2^n rows in the truth table, where n is the number of Boolean variables. Consequently, this truth table is stored in 2^n locations in the main memory. Then, the oracle U_f that encodes a given unknown Boolean function is invoked 2^n times to evaluate each row in the truth table. After that, the Boolean values of the variable x_i are negated, and the oracle U_f is invoked an additional 2^n times, with the outcomes stored in memory. Finally, the 2^n truth values obtained from invoking the oracle U_f before negating the variable are compared with the truth values obtained after invoking the same oracle after negating the Boolean function. Therefore, the memory cost is $2 \cdot 2^n = 2^{n+1}$, and the time cost is also $2 \cdot 2^n = 2^{n+1}$ when using classical computers. Furthermore, in terms of the problem domain, the classical algorithm can be used to handle case (1), but it cannot handle case (2) in the proposed problem statement (see Section II). Case (2) cannot be solved using classical computers because unknown, incomplete, and weighted superposition states cannot be implemented or processed via classical computing systems. On the other hand, for case (1) in the proposed problem statement: The proposed quantum algorithm examines whether a Boolean variable x_i is a junta or not, based on whether the virtual/actual concurrence value $C > 0$ or not. The value C is computed by estimating the probabilities of the basis states $|0011\rangle$ and $|1100\rangle$ using Eq. (2). The upper bound error of evaluating the actual/virtual concurrence is $\epsilon_{\max} = \frac{1}{\sqrt{2M}}$, where M is the number of oracle recalls. In the proposed algorithm, a given oracle U_f is recalled twice (total queries are $4M$ see Algo. 2) in each replica, so the maximum complexity of the proposed approach is given by Eq. (19).

$$M = 2\epsilon^{-2}. \tag{19}$$

Consequently, it is evident that the time complexity is significantly reduced from the exponential computation time of 2^{n+1} using the classical approach to the polynomial time of $2\epsilon^{-2}$ in the proposed quantum approach. Fig. 3 illustrates the comparison between the complexity of the classical algorithm and the proposed quantum approach for examining whether a given Boolean variable x_i. Also, It is evident from this figure that as the number of variables n increases, the proposed algorithm achieves exponential speedup compared to the classical approach. Additionally, the proposed algorithm significantly outperforms the quantum approach explained in Ref. [28], which has an exponential time cost of $O(\sqrt{2^n})$. Moreover, the proposed algorithm tackles case (2) in the proposed problem statement, with a time cost of $O(\epsilon^{-2})$, which current quantum approaches [28], [30], [31], [32], [33] have been unable to solve. Regarding the proposed quantum approach, it requires a memory register of size n qubits to store the basis states of an input state, along with an additional three ancillary qubits $|s\rangle$, $|l\rangle$, and $|h\rangle$ for each replica. Since the propose algorithm needs two replicas, the total memory

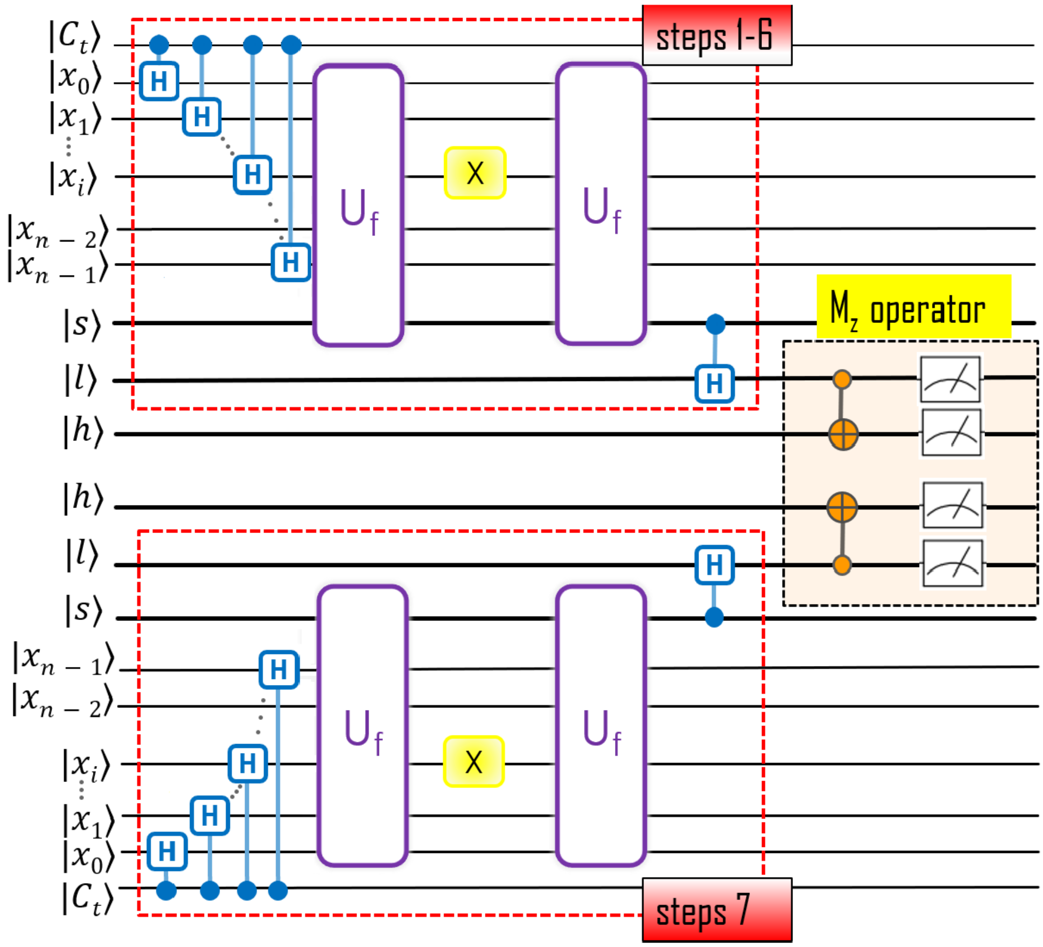

FIGURE 2. The detailed elementary quantum circuit of the proposed approach that classifies variables of unknown logical functions that receive unknown quantum input states into junta/not junta classes.

space required for the proposed approach is $2n + 6$ qubits. It should be noted that the qubit $|C_t\rangle$ can be insulated from the system after the first step. Therefore, it is evident that the memory space is reduced from exponential space 2^{n+1} in the classical approach to linear space in the proposed quantum approach.

VII. EXPERIMENTAL REALIZATION OF THE PROPOSED ALGORITHM

The proposed quantum approach is implemented experimentally via eight experiments. These experiments verify whether given oracles are junta for the number of inputs: $n = 2$, $n = 3$, $n = 12$, and $n = 60$. The simulation results are obtained using a machine with v.0.40.0 platform characteristics, running on a local PC with 12 GB of RAM and a 2.4 GHz CPU. Then, all of these experiments are conducted on IBM's real quantum computers. Ten trials are conducted for each experiment, and the average probabilities are provided. The supposition behind these experimental configurations is that the oracles are kept secret from everyone except the experiment creator. The statistical fidelities of both the simulation results and real quantum computer results are computed as $F_{\text{sim}} = \sum_{j=0}^{2^n-1} \sqrt{p_j^{sim} p_j^{th}}$ and $F_{\text{rc}} = \sum_{j=0}^{2^n-1} \sqrt{p_j^{rc} p_j^{th}}$. Here, p_j^{th}, p_j^{sim}, and p_j^{rc} are the success probabilities for the basis states obtained from theoretical calculations, simulation results, and real quantum computer results, respectively. In each experiment, the oracle receives different Boolean input variables whose states are known only to the experiment's creator. The proposed algorithm is empirically implemented in four different sets of experiments. Each experimental set consists of two experiments tested for different predefined numbers of variables: $n = 2$, $n = 3$, and $n = 12$, with errors $\epsilon = \frac{2}{\sqrt{20{,}000}} \approx 0.01$ (see section VI), where the number of shots is 20, 000 for experiments 1-4 and experiments 7-8. The results of all experiments are plotted in Figs. 9-16. In these figures, the red bars represent the theoretical probabilities of the basis states, while the green bars and the blue bars represent the average probabilities of the simulation results and the real quantum computer for 10 trials, respectively.

In the first experiment, we assume that the number of Boolean variables is $n = 2$, and the oracle U_{f_1} implements the

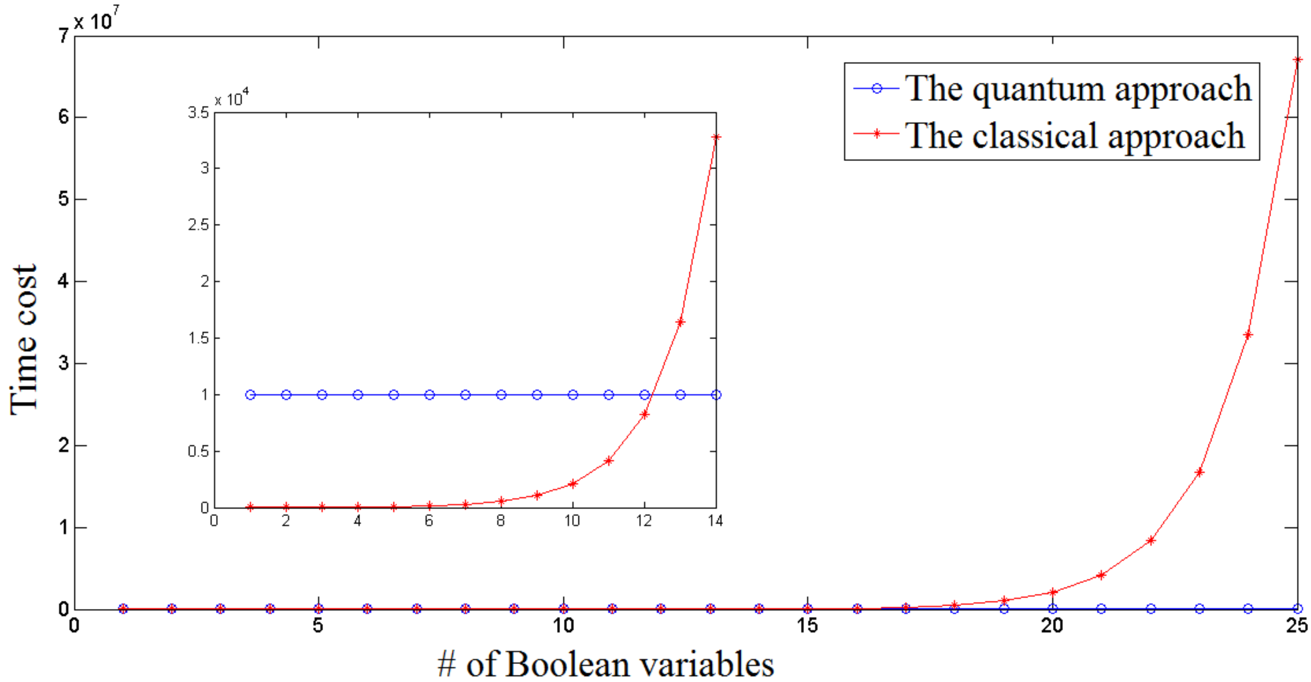

FIGURE 3. Comparison between the time cost for the proposed quantum approach for unknown oracles on uniform superposition states with a predefined error of $\epsilon = 10^{-2}$ and the classical approach.

TABLE 1. Comparing the advantages and disadvantages of testing junta variables using the classical and the quantum approaches.

Approach Name:	Proposed quantum algorithm	Classical algorithm
Input(s):	unknown Boolean expressions and unknown quantum input states.	Unknown Boolean expressions only.
Method:	Quantum computation	Boolean logic.
Memory Space:	$2n + 6$.	2^{n+1}.
Time Complexity:	$2\epsilon^{-2}$.	2^{n+1}.
Advantages:	(i) The number of input variables n does not affect computation time. (ii) Solves problems can not be handled using . current classical/quantum algorithms	Accurate.
Limitations:	Not deterministic (it depends on a predefined error ϵ).	(i) Expoentianl costs in time and memory space. (ii) Can not be used for quantum inputs

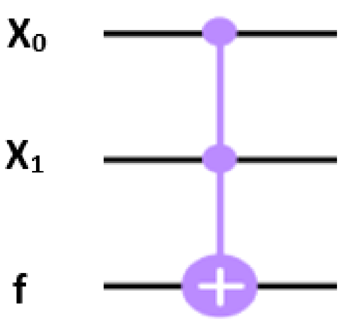

FIGURE 4. Representation of the Boolean function $f(x_0, x_1) = x_0x_1$ by unitary transformations in the quantum circuit model.

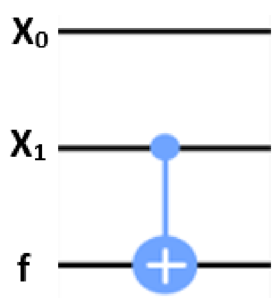

FIGURE 5. Representation of the Boolean function $f(x_0, x_1) = x_1$ by unitary transformations in the quantum circuit model.

Boolean function $f(x_0, x_1) = x_0x_1$, which is described by the quantum circuit shown in Fig. 4. The objective is to determine whether the variable x_0 is a junta or not. Fig. 9 presents the results obtained for theoretical, simulation, and real quantum computer outcomes. Theoretically, after applying the quantum circuit of the proposed quantum approach (see Fig. (2)), the estimated probabilities for the basis states

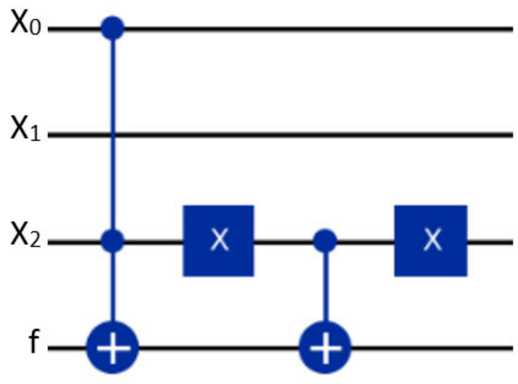

FIGURE 6. Representation of the Boolean function $f(x_0, x_1, x_2) = x_0x_2 + \overline{x_2}$ by unitary transformations in the quantum circuit model.

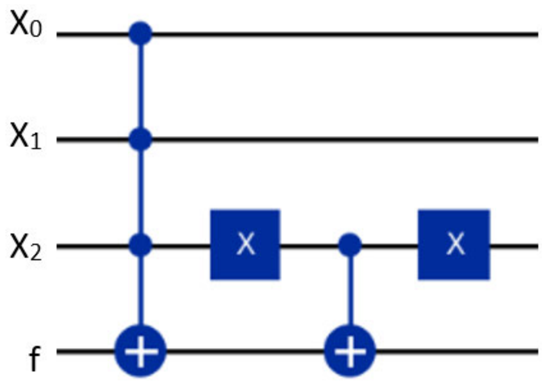

FIGURE 7. Representation of the Boolean function $f(x_0, x_1, x_2) = x_2x_0x_1 + \overline{x_2}$ by unitary transformations in the quantum circuit model.

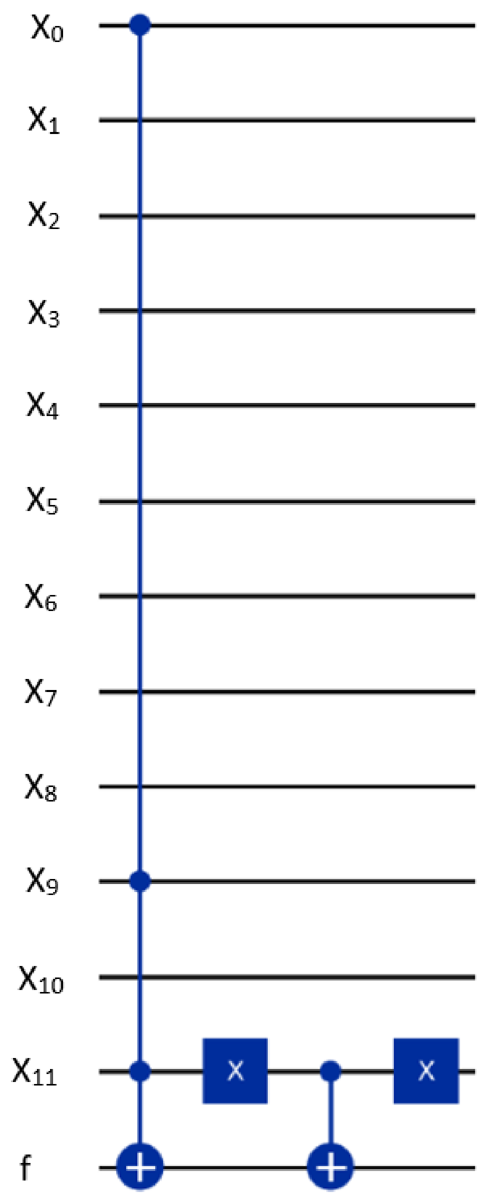

FIGURE 8. Representation of the Boolean function $f(x_0, x_1, \ldots, x_{11}) = x_0x_9x_{11} + \overline{x_{11}}$ by unitary transformations in the quantum circuit model.

$|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are 0.5625, 0.1875, 0.1875, and 0.0625, respectively. Therefore, according to Eq. (2), the theoretical value of the virtual concurrence is $C_{th} = 0.866025404$, indicating that $C_{th} > 0$. According to Step 8 of the proposed algorithm (see Algo. 2), since C_{th} is greater than 0, theoretically, x_0 is a junta. The simulation results for the average probabilities of these basis states, $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$, are obtained as 0.562463379, 0.186132813, 0.187597656 and 0.063806152, respectively. According to Eq. (2), the simulation value of the virtual concurrence is $C_{sim} = \sqrt{2(P_{0011} + P_{1100})} = 0.864558233$, indicating that $C_{sim} > 0$. According to Step 8 of the proposed algorithm, since C_{sim} is greater than 0, experimentally, x_0 is a junta. The results produced by the quantum circuit, Fig. (2), via the real quantum computer for the average probabilities of these basis states, $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$, are 0.544415, 0.16163, 0.167595 and 0.04899, respectively. Additionally, the sum of the average probabilities for the basis states $|0001\rangle$, $|0010\rangle$, $|0100\rangle$, and $|1000\rangle$ is 0.07737. According to Eq. (2), the real quantum computer value of the virtual concurrence is $C_{rc} = \sqrt{2(P_{0011} + P_{1100})} = 0.811449$, indicating that $C_{rc} > 0$. According to Step 8 of the proposed algorithm, since C_{rc} is greater than 0, experimentally, x_0 is a junta. The statistical fidelity of the simulation results and the real quantum computer results are $F_{sim} \approx 1$, and $F_{rc} = 0.96072$. In the second experiment, it is assumed that the number of Boolean variables is $n = 2$, and the oracle U_{f_2} implements the Boolean function $f(x_0, x_1) = x_1$. The quantum circuit of this oracle is shown in Fig. 5. The objective is to decide whether the variable x_0 is junta or not. Fig. 10 presents the results obtained for theoretical, simulation, and real quantum computer outcomes. Theoretically, after applying the quantum circuit of the proposed quantum approach (see Fig. (2)), the estimated probability for the basis state $|00\rangle|00\rangle$ is 1, and the probabilities of the basis states $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are 0. Therefore, according to Eq. (2), the theoretical value of the virtual concurrence is $C_{th} = 0$.

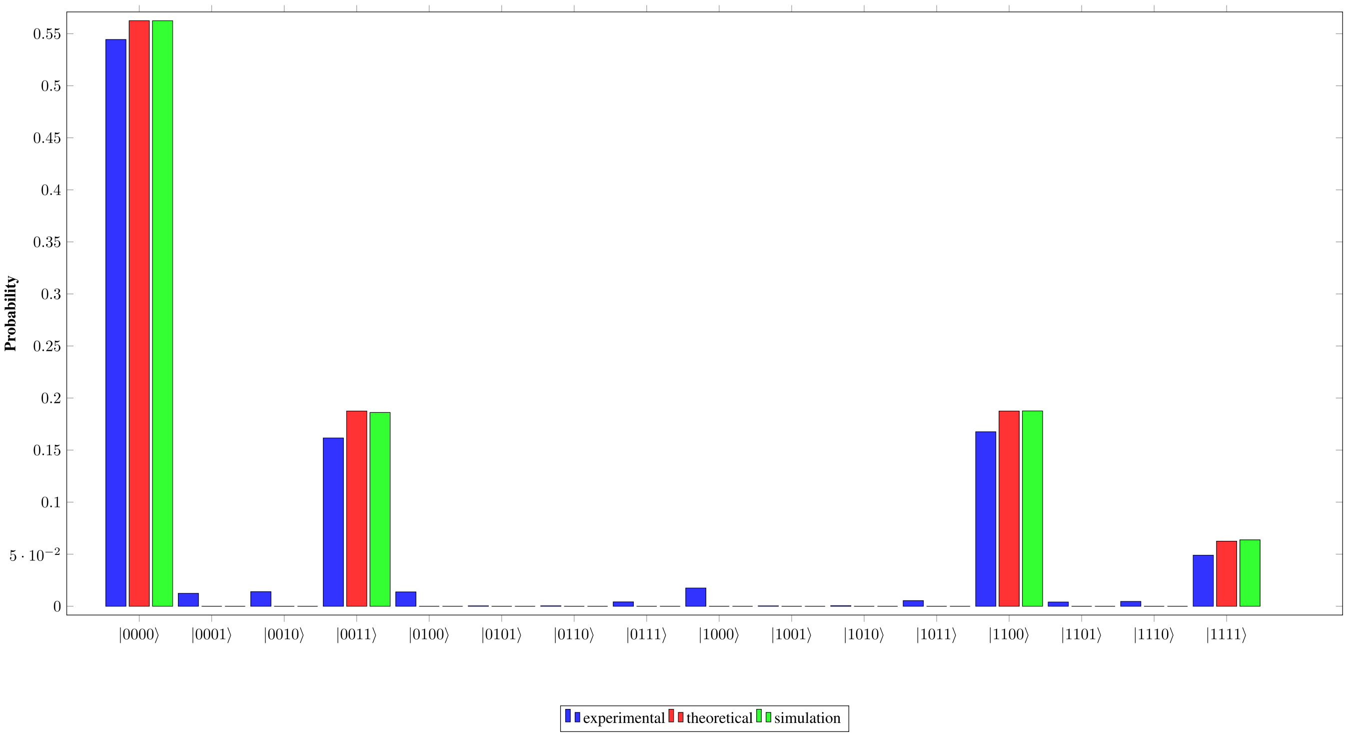

FIGURE 9. Experimental, theoretical, and simulation results for the Boolean function $f(x_0, x_1) = x_0.x_1$ to check whether the variable x_0 is junta.

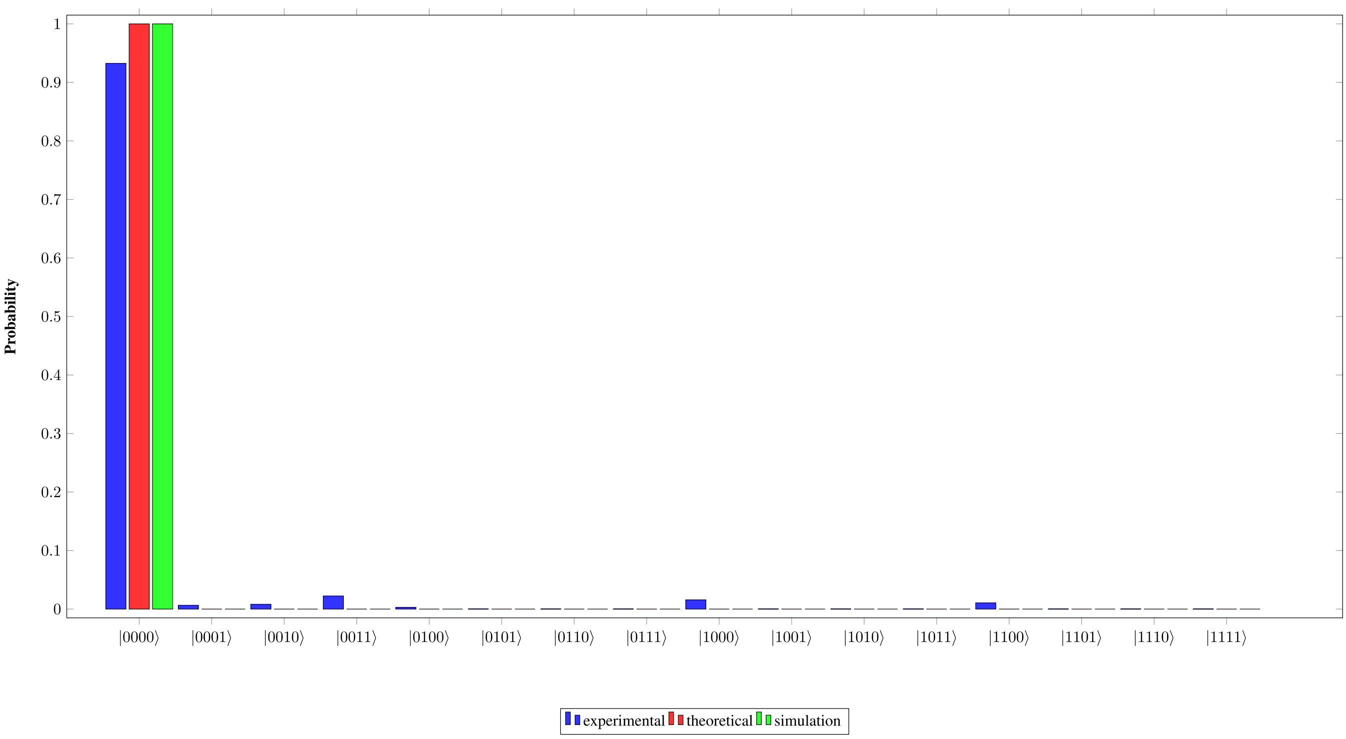

FIGURE 10. Experimental, theoretical, and simulation results for the Boolean function $f(x_0, x_1) = x_1$ to check whether the variable x_0 is not junta.

According to Step 8 of the proposed algorithm (see Algo. 2), theoretically, x_0 is not junta. The simulation results for the average probabilities of the basis state $|00\rangle|00\rangle$ are 1, and the probabilities of the basis states $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are 0. According to Eq. (2), the simulation value of the virtual concurrence is $C_{sim} = 0$. According to Step 8 of

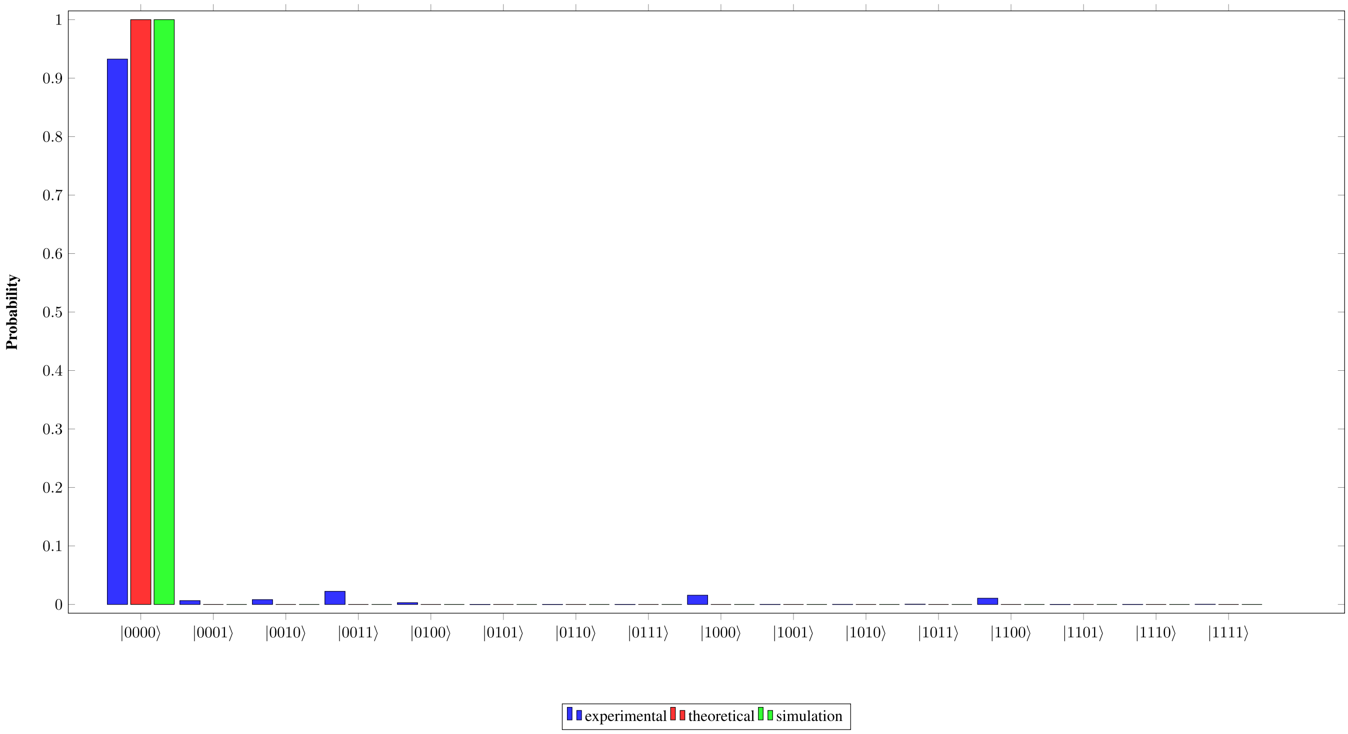

FIGURE 11. Experimental, theoretical, and simulation results for the Boolean function $f(x_0, x_1, x_2) = x_0x_2 + \overline{x_2}$ to check whether the variable x_1 is not junta.

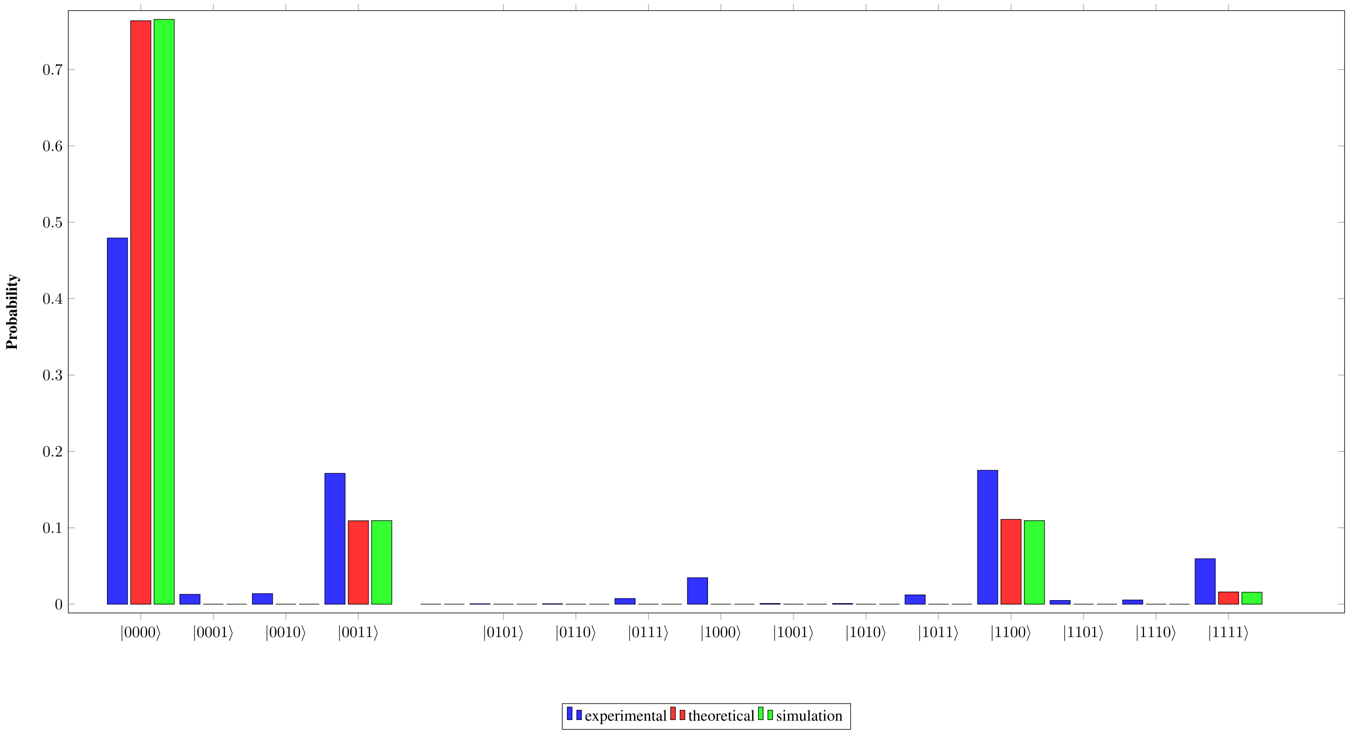

FIGURE 12. Experimental, theoretical, and simulation results for the Boolean function $f(x_0, x_1, x_2) = x_2x_0x_1 + \overline{x_2}$ to check whether the variable x_1 is junta.

the proposed algorithm, experimentally, x_0 is not junta. The results obtained from executing the proposed algorithm in this experiment using a real quantum computer for the average probabilities of the basis states, namely $|00\rangle|00\rangle$, $|00\rangle|11\rangle$,

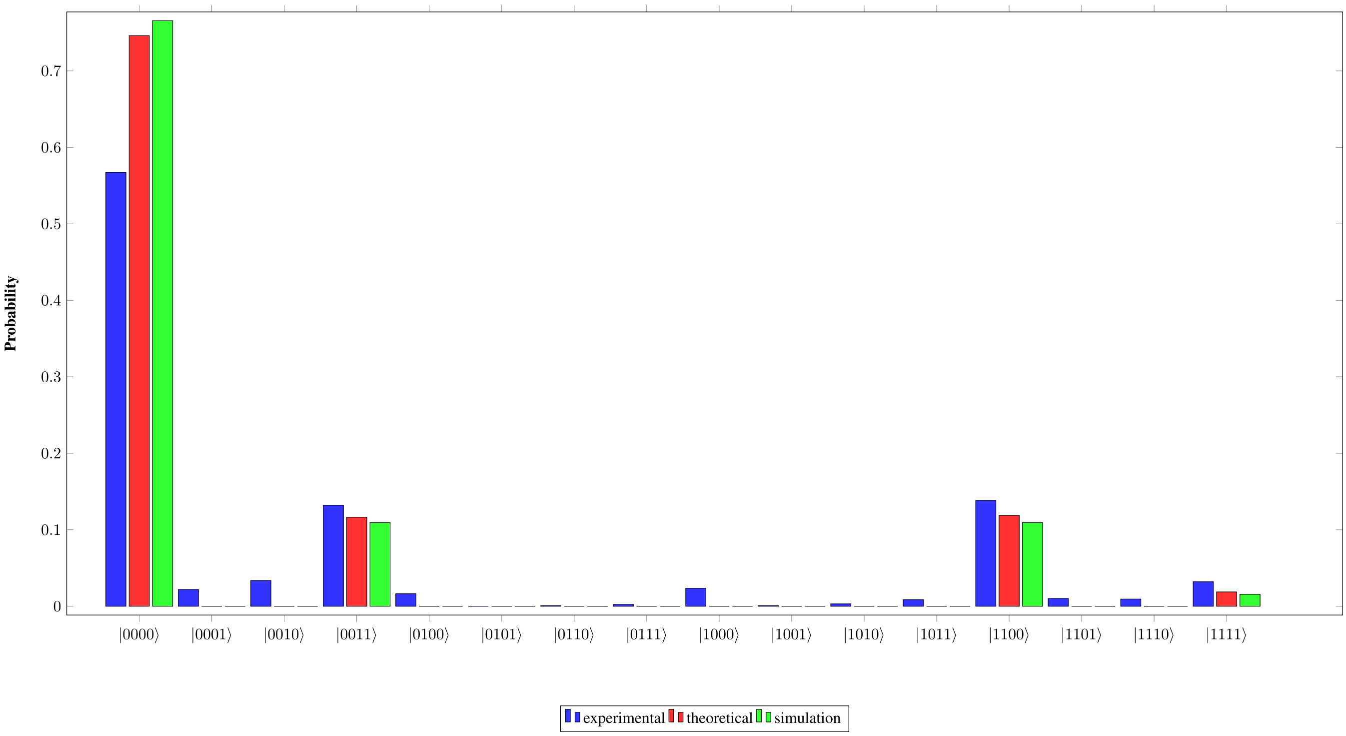

FIGURE 13. **Experimental, theoretical, and simulation results for the Boolean function $f(x_0, x_1, \ldots, x_{11}) = x_0x_9x_{11} + \overline{x_{11}}$, giveh the input stats in the form $|X\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)^{\otimes 12}$ to test if the variable x_0 is junta or not with allowed error 0.125.**

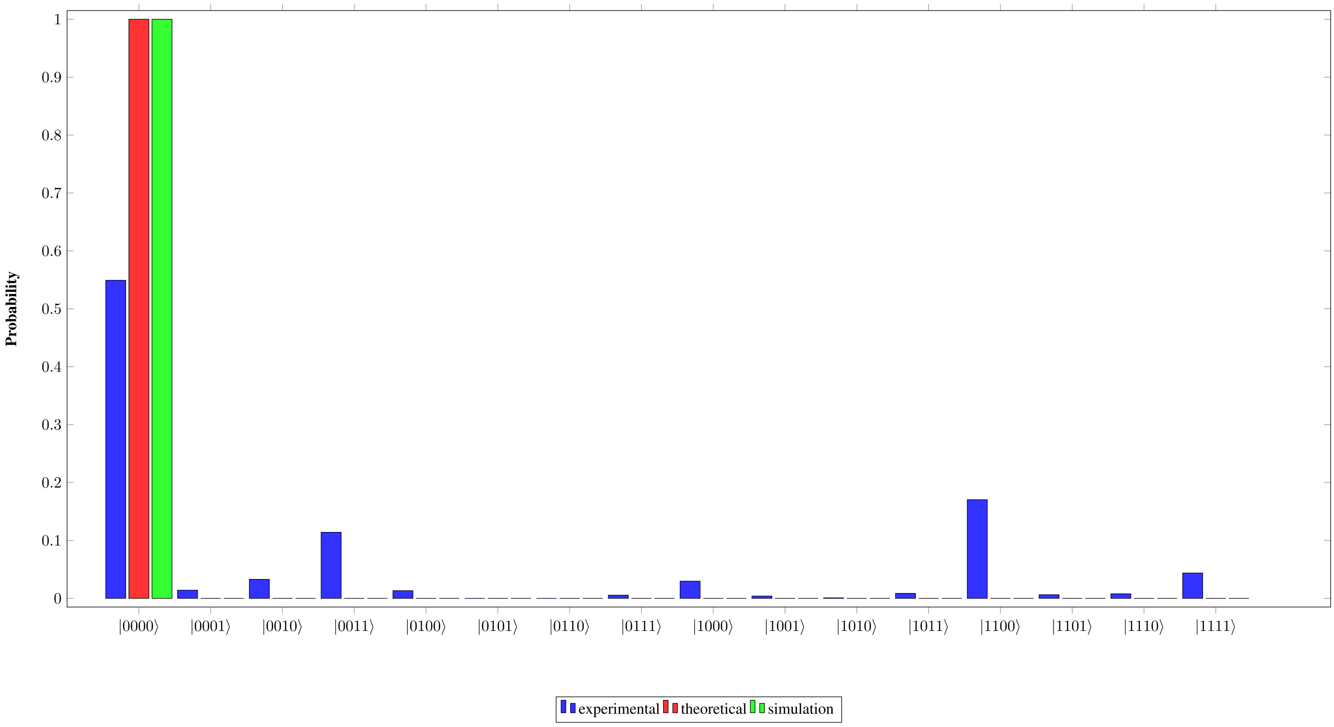

FIGURE 14. **Experimental, theoretical, and simulation results for the Boolean function $f(x_0, x_1, \ldots, x_{11}) = x_0x_9x_{11} + \overline{x_{11}}$, giveh the input stats in the form $|X\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)^{\otimes 12}$ to test if the variable x_1 is junta or not with allowed error 0.125.**

$|11\rangle|00\rangle$, and $|11\rangle|00\rangle$, are 0.93253, 0.02237, 0.01058, and 0.000265, respectively. Additionally, the sum of the average probabilities for the basis states $|0001\rangle$, $|0010\rangle$, $|0100\rangle$, and $|1000\rangle$ is 0.034255. Thus, the outcomes of the quantum computer simulator and the actual quantum computer differ slightly. According to Eq. (2), the value of the virtual

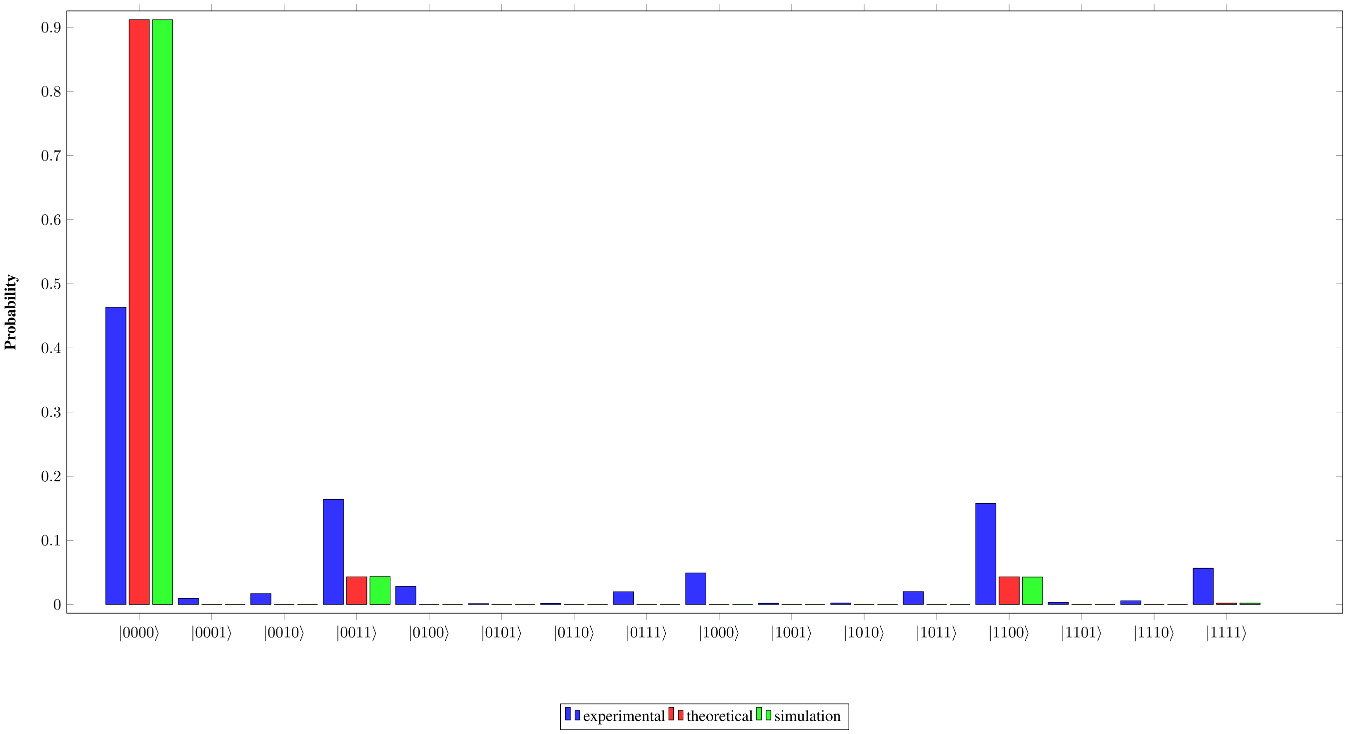

FIGURE 15. Experimental, theoretical, and simulation results for the Boolean function $f(x_0, x_1, \ldots, x_{11}) = x_0x_9x_{11} + \overline{x_{11}}$, giveh the input stats in the form $|X\rangle = \left(\sqrt{0.7}e^{\frac{j\pi}{4}}\,|0\rangle + \sqrt{0.3}\,e^{\frac{j\pi}{3}}\,|1\rangle\right)^{\otimes 12}$ to test if the variable x_0 is junta or not.

concurrence on the real quantum computer, denoted as C_{rc}, is calculated as $C_{rc} = 0.257$. The statistical fidelity of the simulation results and the real quantum computer results are $F_{sim} = 1$ and $F_{rc} = 0.0.965675929$.

In the third experiment, we assume that the number of Boolean variables is $n = 3$, and the oracle U_{f_3} implements the Boolean function $f(x_0, x_1, x_2) = x_0x_2 + \overline{x_2}$. The quantum circuit for this oracle is illustrated in Fig. 6. The objective is to determine whether the variable x_1 is a junta or not. The results obtained for the theoretical, simulation, and real quantum computer outcomes are presented in Fig. 11. Theoretical calculations indicate that after applying the quantum circuit of the proposed quantum approach (see Fig. 2), the estimated probability for the basis state $|00\rangle|00\rangle$ is 1, and the estimated probabilities for the basis states $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are 0. Thus, according to Eq. (2), the theoretical value of the virtual concurrence is $C_{\text{th}} = 0$. Based on Step 8 of the proposed algorithm (Algorithm 2), since C_{th} theoretically vanishes, it implies that x_1 is not a junta. The simulation results for the average probabilities of these basis states, $|00\rangle|00\rangle$, $\{|0011\rangle, |11\rangle|00\rangle, |1111\rangle\}$, are obtained as 1 and 0, respectively. According to Eq. (2), the simulation value of the virtual concurrence is $C_{\text{sim}} = 0$. According to Step 8 of the proposed algorithm, since C_{sim} is 0, experimentally, x_1 is not a junta. The results obtained from executing the proposed algorithm in this experiment using a real quantum computer yield the following average probabilities for the basis states: $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ with values of 0.679825, 0.08959, 0.11443, and 0.015325, respectively. Furthermore, the sum of the average probabilities for the basis states $|0001\rangle$, $|0010\rangle$, $|0100\rangle$, and $|1000\rangle$ amounts to 0.10083. Thus, the outcomes of the quantum computer simulator and the actual quantum computer differ slightly. The statistical fidelity of the simulation results, and real quantum computer results are $F_{sim} = 1$, and $F_{rc} = 0.824515$. In the fourth experiment, we assume that the number of Boolean variables is $n = 3$, and the oracle U_{f_4} implements the Boolean function $f(x_0, x_1, x_2) = x_2x_0x_1 + \overline{x_2}$. The quantum circuit for this oracle is illustrated in Fig. 7. The objective is to determine whether the variable x_1 is a junta or not. The results obtained for the theoretical, simulation, and real quantum computer outcomes are presented in Fig. 12. Theoretical calculations indicate that after applying the quantum circuit of the proposed quantum approach (see Fig. 2), the estimated probabilities for the basis states $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are 0.765625, 0.109375, 0.109375, and 0.015625, respectively. Thus, according to Eq. (2), the theoretical value of the virtual concurrence is $C_{\text{th}} = 0.661437828$, indicating that $C_{\text{th}} > 0$. According to Step 8 of the proposed algorithm (see Algo. 2), since C_{th} is greater than 0, theoretically, x_1 is a junta. The simulation results for the average probabilities of these basis states, $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$, are obtained as 0.76385498, 0.109204102, 0.110974121, and 0.015966797, respectively. According to Eq. (2), the simulation value of the virtual concurrence is calculated as $C_{\text{sim}} = 0.663593585$, indicating that $C_{\text{sim}} > 0$. Following Step 8 of the proposed algorithm, since C_{sim} is greater than 0, it can be experimentally concluded that x_1 is a junta. The

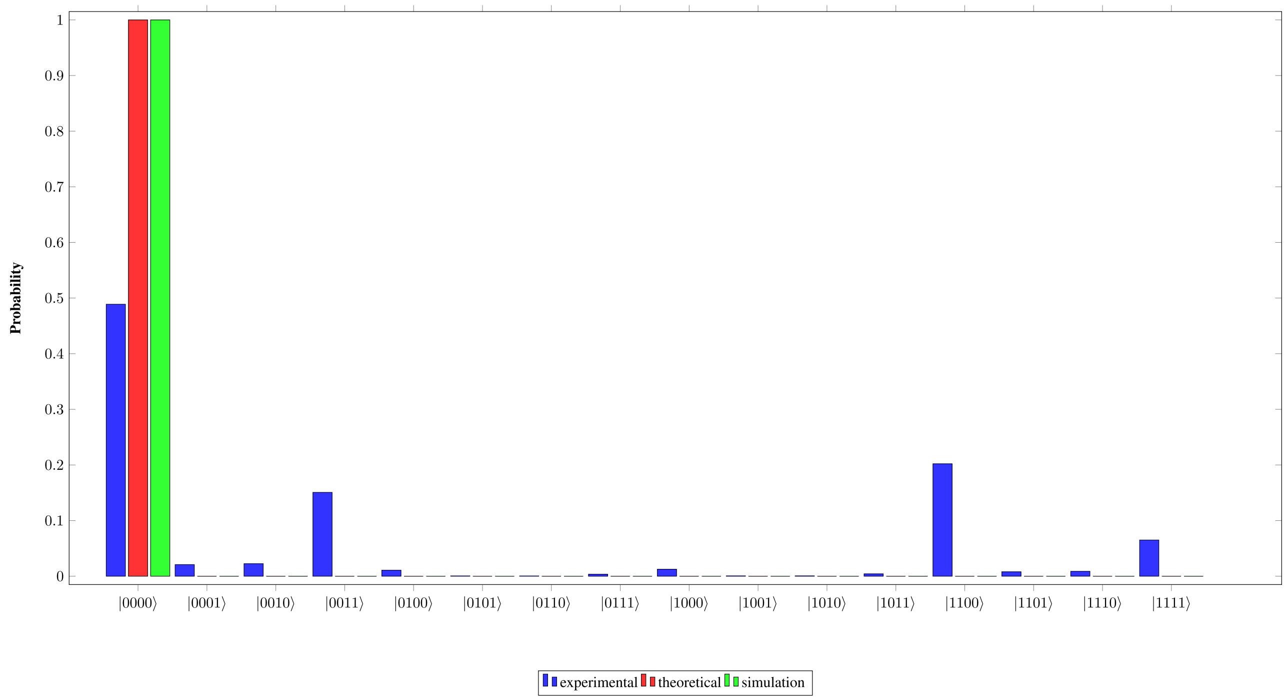

FIGURE 16. **Experimental, theoretical, and simulation results for the Boolean function $f(x_0, x_1, \ldots, x_{11}) = x_0x_9x_{11} + \overline{x_{11}}$, giveh the input stats in the form $|X\rangle = \left(\sqrt{0.7}e^{\frac{j\pi}{4}}\,|0\rangle + \sqrt{0.3}\,e^{\frac{j\pi}{3}}\,|1\rangle\right)^{\otimes 12}$ to test if the variable x_1 is junta or not.**

results obtained from executing the proposed algorithm in this experiment using a real quantum computer for the average probabilities of the basis states, namely $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$, are 0.47937, 0.171275, 0.17527, and 0.059475, respectively. Additionally, the sum of the average probabilities for the basis states $|0001\rangle$, $|0010\rangle$, $|0100\rangle$, and $|1000\rangle$ is 0.11461. According to Eq. (2), the value of the virtual concurrence on the real quantum computer, denoted as C_{rc}, is calculated as $C_{\mathrm{rc}} = 0.83252027$. The statistical fidelity of the simulation results and real quantum computer results are $F_{\mathrm{sim}} = 0.999973$ and $F_{\mathrm{rc}} = 0.9116298180885183$, respectively.

In the fifth experiment, we assume that the number of Boolean variables is $n = 12$, and the oracle U_{f_7} implements the Boolean function in the form of $f(x_0, x_1, \ldots, x_{11}) = x_0x_9x_{11}+\overline{x_{11}}$. The quantum circuit for this oracle is illustrated in Figure 8. Additionally, we assume that the state of these inputs is $|X\rangle = \left(\frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\right)^{\otimes 12}$. In this experiment, we need to determine whether the variable x_0 is a junta with a predetermined error $\epsilon = 0.125$. This requires invoking the oracle 128 times according to Eq. (19). Thus, the number of shots performed in both the simulator and the real quantum computer is 128. The results obtained for the theoretical, simulation, and real quantum computer outcomes are presented in Fig. 13. Theoretical calculations indicate that after applying the quantum circuit of the proposed quantum approach (see Figure 2), the estimated probabilities for the basis states $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are 0.765625, 0.109375, 0.109375, and 0.015625, respectively. Thus, according to Eq. (2), the theoretical value of the virtual concurrence is $C_{th} = 0.661437828$, indicating that $C_{th} > 0$. According to Step 8 of the proposed algorithm (see Algo. 2), since C_{th} is greater than 0, theoretically, x_0 is a junta. The simulation results for the average probabilities of these basis states, $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$, are obtained as 0.74609375, 0.11640625, 0.11875, and 0.01875, respectively. According to Eq. (2), the simulation value of the virtual concurrence is calculated as $C_{sim} = 0.685793$, indicating that $C_{sim} > 0$. Following Step 8 of the proposed algorithm, since C_{sim} is greater than 0, it can be experimentally concluded that x_0 is a junta. The results obtained from executing the proposed algorithm in this experiment using a real quantum computer for the average probabilities of the basis states, namely $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$, are 0.5671875, 0.13203125, 0.13828125, and 0.03203125, respectively. Additionally, the sum of the average probabilities for the basis states $|0001\rangle$, $|0010\rangle$, $|0100\rangle$, and $|1000\rangle$ is 0.13046875. According to Eq. (2), the value of the virtual concurrence on the real quantum computer, denoted as C_{rc}, is calculated as $C_{rc} = 0.735272058$. The statistical fidelity of the simulation results and real quantum computer results are $F_{sim} = 0.996432$ and $F_{rc} = 0.9267478422371156$, respectively. In the sixth experiment, we assume that the number of Boolean variables is $n = 12$, and the oracle U_{f_6} implements the Boolean function in the form of $f(x_0, x_1, \ldots, x_{11}) = x_0x_9x_{11} + \overline{x_{11}}$. Furthermore, we assume that the state of these inputs is $|X\rangle = \left(\frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle\right)^{\otimes 12}$. In this experiment, the goal is to determine whether the variable x_1 is a junta with the same error value as in the previous experiment. Thus, the same number of shots, 128, are performed in both the simulator and the real quantum computer. The results obtained for the theoretical, simulation, and real quantum computer outcomes are presented in Figure 14. Theoretical calculations indicate that after applying the quantum circuit of the proposed quantum approach (see Figure 2), the estimated probabilities for the basis states $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are 1, 0, 0, and 0, respectively. Therefore, according to Eq. (2), the theoretical value of the virtual concurrence is $C_{th} = 0$. Following Step 8 of the proposed algorithm (see Algo. 2), it can be theoretically concluded that x_1 is not a junta. The simulation results for the average probabilities of the basis states $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are also 1, 0, 0, and 0, respectively. According to Eq. (2), the simulation value of the virtual concurrence is $C_{\text{sim}} = 0$. Following Step 8 of the proposed algorithm, since $C_{\text{sim}} = 0$, it can be experimentally concluded that x_1 is not a junta. The results obtained from executing the proposed algorithm in this experiment using a real quantum computer for the average probabilities of the basis states, namely $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$, are 0.54921875, 0.1140625, 0.1703125, and 0.04375, respectively. Additionally, the sum of the average probabilities for the basis states $|0001\rangle$, $|0010\rangle$, $|0100\rangle$, and $|1000\rangle$ is 0.12265625. Thus, the outcomes of the quantum computer simulator and the actual quantum computer differ slightly. The statistical fidelity of the simulation results and real quantum computer results are $F_{sim} = 1$ and $F_{rc} = 0.7100915433942302$, respectively.

In the seventh experiment, we assume that the number of Boolean variables is $n = 12$, and the oracle U_{f_5} implements the Boolean function in the form of $f(x_0, x_1, \ldots, x_{11}) = x_0x_9x_{11} + \overline{x_{11}}$. The quantum circuit for this oracle is illustrated in Figure 8. Additionally, we assume that unknown given inputs are obtained through the quantum teleportation protocol or a quantum communication channel [40]. The state of these inputs is $|X\rangle = \left(\sqrt{0.7}e^{\frac{j\pi}{4}}\ |0\rangle + \sqrt{0.3}\ e^{\frac{j\pi}{3}}\ |1\rangle\right)^{\otimes 12}$, which is only known to the experiment's creator and unknown to everyone else. The objective of this experiment is to determine whether the variable x_0 is a junta or not. The results obtained for the theoretical, simulation, and real quantum computer outcomes are presented in Figure 15. Theoretical calculations indicate that after applying the quantum circuit of the proposed quantum approach (see Figure 2), the estimated probabilities for the basis states $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are 0.912, 0.043, 0.043, and 0.002, Thus, according to Eq. (2), the theoretical value of the virtual concurrence is $C_{th} = 0.414728827$. According to Step 8 of the proposed algorithm (see Algo. 2), since C_{th} is greater than 0, it can be theoretically concluded that x_0 is a junta. The simulation results for the average probabilities of the basis states $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$ are obtained as 0.9119, 0.04338, 0.04274, and 0.00198, respectively. According to Eq. (2), the simulation value of the virtual concurrence is calculated as $C_{sim} = \sqrt{2(P_{0011} + P_{1100})} = 0.415018072$, indicating that $C_{sim} > 0$. Following Step 8 of the proposed algorithm, since C_{sim} is greater than 0, it can be experimentally concluded that x_0 is a junta. The results obtained from executing the proposed algorithm in this experiment using a real quantum computer for the average probabilities of the basis states $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$, are 0.463385, 0.16389, 0.157545, and 0.05639, respectively. Additionally, the sum of the average probabilities for the basis states $|0001\rangle$, $|0010\rangle$, $|0100\rangle$, and $|1000\rangle$ is 0.15879. According to Eq. (2), the value of the virtual concurrence on the real quantum computer, denoted as C_{rc}, is calculated as $C_{rc} = 0.80179$. The statistical fidelity of the simulation results and real quantum computer results are $F_{sim} = 0.999979$ and $F_{rc} = 0.9267478422371156$. In the eighth experiment, it is assumed that the number of Boolean variables is $n = 12$, and the oracle U_{f_6} implements the Boolean function in the form of $f(x_0, x_1, \ldots, x_{11}) = x_0x_9x_{11} + \overline{x_{11}}$. The quantum circuit for this oracle is illustrated in Figure 8. Moreover, it is assumed that unknown given inputs are obtained through the quantum teleportation protocol or a quantum communication channel [40]. The state of these inputs is $|X\rangle = \left(\sqrt{0.7}e^{\frac{j\pi}{4}}\ |0\rangle + \sqrt{0.3}\ e^{\frac{j\pi}{3}}\ |1\rangle\right)^{\otimes 12}$, which is only known to the experiment's creator and unknown to everyone else. The objective is to determine whether the variable x_1 is a junta or not. The results obtained for the theoretical, simulation, and real quantum computer outcomes are presented in Figure 16. Theoretical calculations indicate that after applying the quantum circuit of the proposed quantum approach (see Figure 2), the estimated probability for the basis state $|00\rangle|00\rangle$ is 1, and for the basis states $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, $|11\rangle|11\rangle$ is 0. Thus, according to Eq. (2), the theoretical value of the virtual concurrence is calculated as $C_{th} = 0$. Following Step 8 of the proposed algorithm (see Algo. 2), it can be theoretically concluded that x_1 is not a junta. The simulation results for the average probability for the basis state $|00\rangle|00\rangle$ is 1, and the estimated probabilities for the basis states $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, $|11\rangle|11\rangle$ are 0. According to Eq. (2), the simulation value of the virtual concurrence is calculated as $C_{\text{sim}} = 0$. Following Step 8 of the proposed algorithm, since $C_{\text{sim}} = 0$, it can be experimentally concluded that x_1 is not a junta. The results obtained from executing the proposed algorithm in this experiment using a real quantum computer for the average probabilities of the basis states, namely $|00\rangle|00\rangle$, $|00\rangle|11\rangle$, $|11\rangle|00\rangle$, and $|11\rangle|11\rangle$, are 0.488855, 0.150655, 0.2021, and 0.064995, respectively. Additionally, the sum of the average probabilities for the basis states $|0001\rangle$, $|0010\rangle$, $|0100\rangle$, and $|1000\rangle$ is 0.093395. Thus, the outcomes of the quantum computer simulator and the actual quantum computer differ slightly. The statistical fidelity of the simulation results and real quantum computer results is $F_{\text{sim}} = 0.999979$ and $F_{\text{rc}} = 0.9267478422371156$.

The outcomes of experiments $1-8$ show a disparity between the results obtained from the real quantum computer and both the theoretical outcomes and simulation results. This disparity can be attributed to the susceptibility of real computers to various types of noise and errors, including decoherence and environmental interference. These factors remain open challenges in the research field [41]. On the other hand, it is evident that both the theoretical predictions and simulations of the proposed algorithm on quantum computers align with the initial expectations and preparations made by the experiment's designer. This confirms the successful experimental realization of the proposed algorithm. Table 1 presents both the advantages and disadvantages of the proposed algorithm compared to other current approaches for testing whether a given variable is a junta or not. The proposed algorithm achieves a time complexity of $O(\epsilon^{-2})$ and a memory cost of $2n+6$. In experiments $5-6$, it is clear that the proposed algorithm efficiently decides whether the variable underpinning is a junta or not for an oracle of $n = 12$ Boolean variables by recalling with a time complexity of 128 times when the inputs are represented in the uniform complete superposition. When testing the same variables for the same oracle using the classical approach, it requires a time cost of $2^{n+1} = 2^{12+1}$ (see Section VI). Thus, the proposed algorithm achieves a quantum supremacy ratio of $(\frac{\text{classical approach time}}{\text{quantum approach time}} - 1) \times 100 = (\frac{2^{n+1}}{2\epsilon^{-2}} - 1) \times 100 = (\frac{2^{13}}{2\epsilon^{-2}} - 1) \times 100 = 6300\%$ in terms of time cost. Furthermore, experiments $7-8$ cannot be implemented using classical computers, and the current quantum approaches [28], [30], [31], [32], [33] also cannot solve the issues conducted in experiments $7-8$ by the proposed algorithm. However, the proposed approach successfully accomplished these experiments.

Overall, it is evident that the proposed algorithm can not only outperform the classical approach in terms of time complexity and memory space, but also it addresses a form of the junta problem that cannot be solved using current classical or other existing quantum approaches.

VIII. CONCLUSION

This study investigated a new quantum approach to determine whether a given variable is a junta or not given an unknown oracle. The proposed approach has a time cost of $2\epsilon^{-2}$, achieving exponential speed up compared to classical approaches as the number of Boolean variables increases. Additionally, the memory cost of the presented approach scales linearly with the number of variables compared to the classical approach, which scales exponentially. Moreover, the proposed quantum approach can solve the same problem with the same memory space and time cost when both the oracle is unknown and the inputs are incomplete and weighted superposition of some basis states. This task is intractable for both classical computers and the state-of-the-art quantum approaches but achievable using the proposed approach. Eight experiments were conducted using IBM's real quantum computer and the Qiskit simulator to practically realize the suggested algorithm.

CONFLICT OF INTEREST

All authors declare that there is no conflict of interest.

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