IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 27 June 2024, accepted 20 August 2024, date of publication 27 August 2024, date of current version 4 September 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3450583

RESEARCH ARTICLE

Optimal Decisions for Medical Equipment Remanufacturing Under Carbon Price Fluctuation With Carbon Options

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This work was supported in part by the Fundamental Research Funds for the Central Universities under Grant JUSRP124047, and in part by the General Project of Philosophy and Social Science Research in Colleges and Universities in Jiangsu Province under Grant 2024SJYB0637.

ABSTRACT This study examines the effects of carbon options in hedging against the risk of carbon price fluctuation in a remanufacturing setting. Specifically, we develop an optimization model for a medical equipment enterprise that produces new products in the first period, and in the second period manufactures both new and remanufactured ones under a cap-and-trade mechanism. Since the carbon price is uncertain, carbon options are introduced for the enterprise to trade for carbon credits. We first study the scenario where carbon options are excluding as a benchmark, and then investigate the carbon options scenario. We show that for a given carbon emission reduction rate, the introduction of carbon options can decrease the remanufactured quantity in comparison to the case without carbon options. Additionally, the carbon emission reduction strategy of the enterprise can lower the optimal selling prices of the two products over these two periods. When the carbon emission reduction rate is given, our numerical analysis shows that introducing carbon options can enhance the overall consumer surplus over the two periods, but the first-period consumer surplus remains unchanged. After introducing carbon options, with the increased volatility in the carbon price, the optimal emission abatement rate decreases, while the total profits, consumer surplus, and environmental impact over the two periods increase and are higher than those in the scenario without carbon options.

INDEX TERMS Cap-and-trade regulation, carbon options, carbon price fluctuation, medical equipment remanufacturing.

I. INTRODUCTION

In recent years, the environmental issues caused by carbon emissions have attracted widespread attention from society and the public. The issue of carbon emissions has also become increasingly pressing due to the escalating global climate crisis [1]. Industries across the spectrum are under scrutiny to reduce their carbon footprint and contribute to a more sustainable future [2]. To address these issues, governments around the world have taken different measures to encourage businesses to reduce carbon emissions [1]. For example, carbon cap-and-trade mechanism is a fundamental regulation based on the carbon trade market. The carbon trade market is a dynamic platform where emissions reduction commitments are translated into tradable assets. Under the cap-and-trade mechanism, companies can buy and sell carbon credits, which represent the right to emit carbon dioxide [3], [4], [5]. This mechanism provides an effective financial incentive to encourage enterprises to reduce their carbon emissions below the cap as much as possible.

To save associated costs, companies are also looking for ways to reduce carbon emissions, among which

The associate editor coordinating the review of this manuscript and approving it for publication was Mohammad Hossein Moaiyeri.

remanufacturing is considered an important low-carbon strategy has made significant progress on enterprise agenda [6], [7], [8]. Recently, in the field of medical equipment, with the continuous progress of medical technology and the widespread application of medical equipment, a series of new challenges have also emerged [9]. These challenges include high equipment procurement and maintenance costs, an increase in medical equipment waste, and difficulties in accessing modern medical equipment in areas with insufficient resources. In this context, the remanufacturing of medical equipment has emerged as an innovative way to address these issues. Medical equipment remanufacturing is a process of repairing, updating, and improving abandoned, scrapped, or obsolete medical equipment to restore it to its original specifications and performance standards. This field covers a wide range of medical equipment types, including but not limited to medical imaging equipment, surgical equipment, monitoring equipment, and laboratory instruments. In China, the Action Plan for Promoting Large scale Equipment Renewal and Consumer Goods Exchange for New issued by the State Council points out the promotion of iterative upgrading and remanufacturing of equipment and facilities in medical and health institutions. The process of remanufacturing includes equipment disassembly, cleaning, maintenance, component replacement, performance testing, and repackaging to ensure that the equipment can be safely and effectively put back into use. Recently, Shang and Li [9] have demonstrated the feasibility of the remanufacturing of medical equipment, such as X-ray machines, ultrasound instruments, pacemakers, surgical instruments, etc. In practice, there are many actual medical enterprises that implement remanufacturing. For instance, Wuxi Exanovo Medical Instrument Co.,Ltd (Exanovo, for short) specializes in magnetic resonance medical equipment. As well as producing new equipment, Exanovo focuses on the remanufacturing of end-of-life magnetic resonance medical equipment.

However, while pursuing sustainability, enterprises like Exanovo also face the challenge of carbon emissions. This is because the remanufacturing process involves factors directly related to carbon emissions such as energy consumption, logistics transportation, and material use [10]. For the medical equipment remanufacturing enterprises, the carbon trade market serves as a dynamic platform where they seek to balance their carbon emissions through the buying and selling of carbon credits. Within this marketplace, the price of carbon credits is not static; rather, it experiences fluctuations influenced by a myriad of factors, including evolving environmental policies, economic conditions, regulatory changes, and technological advancements. Under the cap-and-trade mechanism, carbon price is a significant indicator to measure the cost of carbon emissions, and its volatility poses a threat to the growth of the carbon market [11].

In addition, the fluctuation of carbon prices directly affects the economic costs of enterprises, especially for industries with high carbon emissions, such as the medical equipment remanufacturing industry. Moreover, it also influences the development of the carbon market [11]. This may generate impacts on the operations and decisions of the enterprise [8]. Therefore, understanding this fluctuation and its impact on the cost-effectiveness of emissions reduction projects is essential for governments and enterprises participating in carbon trading. Recently, a director of Exanovo highlighted that the enterprise faces risks from fluctuating carbon prices, leading to uncertain production costs and affecting operational efficiency and profitability.

Therefore, how to optimize the operational decisions of medical equipment remanufacturing enterprises under carbon price fluctuations has become an urgent issue. Within the realm of carbon trading, carbon options have emerged as a versatile and innovative financial instrument. The concept of carbon options, a financial derivative instrument that provides the holder with the right (not obligation) to buy or sell carbon credits at a predetermined price, has gained traction as a risk management and financial strategy [8], [12]. Moreover, carbon options provide market participants with the flexibility to hedge against potential carbon price volatility or speculate on future price movements. In the context of medical equipment remanufacturing, the use of carbon options can offer flexibility and cost control in response to carbon price fluctuations. This innovative approach is essential for optimizing decision-making processes in the face of uncertain carbon pricing dynamics.

Motivated by the practices of medical remanufacturing enterprise, we aim to examine the effects of carbon price fluctuation and study how to mitigate this risk. Additionally, we seek to provide managerial insights for medical remanufacturing enterprises and other stakeholders, such as the government, particularly regarding the introduction of carbon options. If so, when would be the optimal time for medical remanufacturing enterprises to introduce carbon options? We are also interested in whether these enterprises should implement carbon emission reduction strategies and aim to discuss the impacts of carbon options on these strategies. Consequently, we address the following issues:

(i) How do carbon price fluctuations influence the medical equipment remanufacturing enterprise?

(ii) What are the effects of carbon options in hedging against the risk of carbon price fluctuations?

(iii) How does the carbon emission reduction strategy affect the optimal production quantities and the selling prices of both products?

To address these issues, we develop a nonlinear decision-making optimization model for the medical equipment enterprise under the carbon price fluctuations to examine the role of carbon options. More specifically, we first study the scenarios without carbon options and with carbon options, and the optimal solutions are derived under the two scenarios. Subsequently, we compare the optimal solutions under these two scenarios, and we also examine the effects of carbon abatement strategy. Additionally, we discuss how carbon options affect consumer surplus and environmental benefits.

Finally, we use numerical examples to discuss how carbon price fluctuations influence the optimal emission abatement rate, the optimal production quantity, the optimal expected profit, consumer surplus and the environment.

Our contributions are as follows. First, we develop an integrated optimization decision-making model for medical equipment remanufacturing enterprises that considers carbon price fluctuations, carbon emissions, and economic costs. Although prior studies have extensively examined remanufacturing and carbon emissions reduction decisions under the cap-and-trade mechanism, they often ignore carbon price fluctuations. In this study, we consider carbon price fluctuations and study the optimal remanufacturing and carbon emissions reduction decisions. This allows us to provide actionable decision recommendations to help remanufacturing enterprises balance sustainability and economic efficiency. Second, we introduce carbon options to hedge against the risk of carbon price fluctuations. To our knowledge, the role of carbon options has not been examined in remanufacturing while considering carbon price fluctuations and carbon emission reductions. Through this study, we explore the nuanced applications of carbon options within the carbon trade market, highlighting their role in risk management for medical equipment remanufacturing enterprises and their potential implications for sustainable healthcare practices.

The rest of this paper is organized as follows. Section II reviews related studies. Section III describes our model framework. Section IV provides the models with and without carbon options. Section V conducts comparative analysis. Section VI presents numerical examples. Finally, Section VII concludes this paper.

II. LITERATURE REVIEW

Our work is mostly relevant to four streams of literature: production decision with remanufacturing, cap-and-trade mechanism, emissions reduction investment, and carbon options.

A. PRODUCTION DECISION WITH REMANUFACTURING

In recent years, many researchers have investigated the production decision with remanufacturing. Sitcharangsie et al. [13] and Li et al. [14] provide a systematic review on the decision makings in remanufacturing practice. Considering the homogeneity between new and remanufactured products, multi-period production planning models are developed by Ferrer and Swaminathan [15]. Further, the models are extended by considering the heterogeneity of the two products [16]. Wei et al. [17] investigate the pricing and production decisions while considering two reverse channels. They show that the presence of an online platform benefits the manufacturer compared with that without the online platform. Li et al. [18] study the production and remanufacturing decisions without considering the competition between both new and remanufactured products. Under a remanufacturing setting, Qin et al. [19] compare the integrated, outsourcing, and authorization remanufacturing modes, and the conditions under which the original equipment manufacturer can obtain more profits are obtained. In a similar vein, the researches of production decisions with remanufacturing are also conducted by Wang and Wang [7], Ma and Chen [12], Kenne et al. [20], Assid et al. [21], Wang and Wang [22]. In the field of medical equipment, Shang and Li [9] study the remanufacturing strategy for the medical equipment to meet the demand for urgency. Based on Q-learning and dual deep Q-networks, they develop two reinforcement learning frameworks to find the optimal recycling scheme for the medical equipment company.

B. CAP-AND-TRADE MECHANISM

The second stream of the literature is related to cap-and-trade mechanism. In recent years, many researches have focused on the cap-and-trade mechanism in operations management (e.g., Benjaafar et al. [1]; Chen et al. [23]; Du et al. [24]; He et al. [25]). Under the mechanism, some other researchers investigate the production decisions with remanufacturing. For instance, Chang et al. [26] consider a monopolist manufacturer producing both new and remanufactured products in a two-period setting under the limitation of cap-and-trade mechanism. They show how the cap-and-trade mechanism affects production decisions, and prove that the carbon price is more effective in controlling productions and emissions compared with the carbon cap. Wang et al. [27] investigate the interaction of capital and emissions constraints on production decisions with remanufacturing. They find that the carbon emission constraint can motivate the firm to implement remanufacturing. Under the cap-and-trade mechanism, Sun and Liu [28] explore the production decisions with remanufacturing in the presence of consumer education and government subsidies. They show the benefits of the mechanism for remanufacturing, and their results show that government subsidies are more effective on the original equipment manufacturer than the independent remanufacturer. In this regards, similar studies of production decisions with remanufacturing under this mechanism are conducted by Liu et al. [3], Chai et al. [4], Miao et al. [29], and Zhu et al. [30].

C. EMISSIONS REDUCTION INVESTMENT

The third research stream focuses on emissions reduction investments, a topic of growing interest in academia. Scholars have explored optimal green technology and investment decisions for emissions reduction. Bai et al. [31] investigate emission reduction decisions and supply chain coordination under a cap-and-trade mechanism in a make-to-order context. Chen et al. [32] analyze optimal emissions reduction investments in non-cooperative and cooperative multi-level supply chains. Recently, Jauhari et al. [33] study closed-loop supply chain coordination, linking market demand to green technology levels. Adam et al. [34] consider government incentives and energy-saving levels to address coordination and emissions reduction issues. Jauhari et al. [35] focus on sustainable vendor-buyer inventory models, offering insights for stakeholders. Additionally, researchers have examined emissions reduction investments in remanufacturing contexts. Ding et al. [10] study production and carbon emission reduction decisions under carbon tax and take-back legislation, discussing the impact of carbon emission reduction strategies. Wang and Wang [7] investigate production and emissions reduction decisions under differentiated carbon tax regulations for new and remanufactured products. However, they overlook carbon price fluctuations. Recently, Ding [36] explores how carbon price fluctuations and risk aversion affect remanufacturing decisions using the mean-variance criterion, highlighting that emissions reduction investment strategies may not always maximize environmental benefits.

D. CARBON OPTIONS

Finally, this paper is related to the literature on carbon options. Chevallier et al. [11] show the negative impacts of the carbon price fluctuations and demonstrate how the introduction of options trading affects volatility. Wang et al. [37] study three optimization models for a manufacturer to purchase carbon credits from green organizations only through forward contract, carbon option contract, and combination contract (forward contract and carbon option contract), considering government regulations on carbon emissions per unit product. They obtain and compare the optimal procurement strategies under these three models, and find that the combination contract can cope with price fluctuations. On this basis, Wang and Choi [38], [39] further consider government restrictions on a manufacturer total carbon emissions and the implementation of carbon reduction strategies by a manufacturer. They analyze the optimal decisions of the manufacturer considering carbon option contract and discuss the impacts of carbon option contract. In a remanufacturing setting, Ding et al. [8] study the procurement decisions of carbon financial instruments. By considering demand uncertainty, they use carbon options to hedge against the risk of this enterprise under the cap-and-trade mechanism. In their study, three optimization models are developed under three contracts: pure wholesale price contracts, pure carbon option contracts, and portfolio contracts. Finally, they demonstrate the value of introducing carbon options. Following Ding et al. [8], Ma and Chen [12] consider the unidirectional and the bidirectional carbon option. They find that the carbon option contract can improve the firm's profits.

E. SUMMARY

In the literature, numerous studies have examined production decisions involving remanufacturing under the cap-and-trade mechanism. However, most assume a fixed carbon price, neglecting carbon price fluctuations. Therefore, mitigating the risks associated with carbon price fluctuations becomes a pivotal issue in operations management. This paper addresses this gap by investigating the impact of carbon options on hedging risks related to carbon price fluctuations in remanufacturing contexts. Inspired by practices in medical equipment remanufacturing, we analyze a monopolistic enterprise operating under uncertainty in carbon pricing within the cap-and-trade system. Carbon options are introduced to enable the enterprise to purchase carbon credits, and their role is subsequently discussed.

III. PROBLEM DESCRIPTION AND ASSUMPTION

In this study, we examine an optimization problem for a carbon-dependent monopolistic medical equipment enterprise, and develop a nonlinear decision-making optimization model. Specifically, in this model, the enterprise produces only new products in the first period, and manufactures both new products and remanufactured ones in the second period. The enterprise decides on the production quantity of new products in the first period and the quantities of both new and remanufactured products in the second period. For example, Exanovo produces new equipment in the first period. After consumer usage, Exanovo retrieves its own end-of-life equipment for remanufacturing. Figure 1 illustrates the graphical representation of this system.

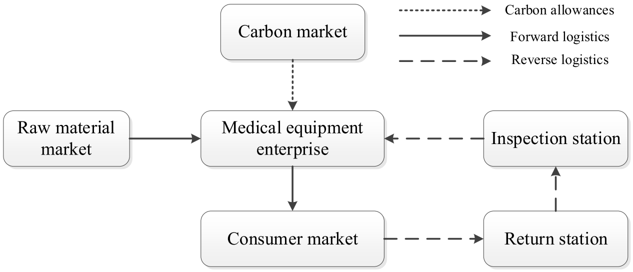

FIGURE 1. The proposed remanufacturing system.

Under the carbon cap-and-trade mechanism, a certain quantity of free carbon allowances, denoted by E_g, are allocated to this enterprise in both periods. If the free carbon allowances are insufficient to cover production, the enterprise purchases the necessary carbon emission permits from the carbon spot market; conversely, it sells additional carbon emission permits. Due to the fluctuations of carbon prices, denoted by s, this study assumes that s follows a random distribution with a mean of μ and a standard deviation of σ within the region $(\underline{s}, \overline{s})$, with cumulative distribution and probability density functions denoted as $F(\cdot)$ and $f(\cdot)$, respectively. In a two-period setting, the enterprise must pre-purchase carbon options at a unit price, denoted by o, in the first period, and then secure carbon allowances by comparing the actual carbon price with the exercise price, denoted by w, of the carbon options in the second period. Hence, the actual price paid for unit carbon credit is determined by $\min(w, s)$.

In addition, this medical equipment enterprise invests in emission abatement technologies to reduce actual carbon emissions. As well as determining the production quantities, the enterprise also makes decisions regarding emission reduction rate. In this study, we assume that the investment

TABLE 1. **Notations and descriptions.**

Notation	Description
Decision variables	
q_{1n}	Production quantity of new products in the first period (unit)
q_{2n}, q_{2r}	Production quantities of new and remanufactured products in the second period (unit)
p_{1n}	Selling price of new products in the first period (CNY/unit)
p_{2n}, p_{2r}	Selling prices of new and remanufactured products in the second period (CNY/unit)
e	Emission reduction rate
Parameters	
e_n, e_r, e_d	Carbon emission quantities of new, remanufactured and unrecycled products without carbon emission reduction strategies (ton/unit)
E_n, E_r, E_d	Carbon emission quantities of new, remanufactured and unrecycled products under carbon emission reduction strategies (ton/unit)
s	Carbon price (CNY/ton)
o	Option price (CNY/ton)
w	Exercise price (CNY/ton)
φ	Marginal impact of carbon reduction strategies on the carbon emissions of remanufactured products
E_g	The quantity of free carbon allowances in each period (ton)
a	Potential market size (unit)
α	Discount coefficient for the consumer value of remanufactured products
λ	Lagrange multiplier
Functions	
π	Profit function
$E(\pi)$	Expected profit function
L	Lagrange function

cost is $ke^2/2$, where k represents carbon emission reduction efficiency and e is the emission reduction rate [10], [40]. Consequently, after implementing carbon reduction strategies, the actual carbon emissions per unit for new and remanufactured products are $E_n = (1-e)e_n$ and $E_r = (1-\varphi e)e_n$, respectively. Herein, φ represents the marginal impact of carbon reduction strategies on the emissions of remanufactured products, and the actual carbon emissions for both products satisfy $E_n > E_r$ [10].

The notations are as shown in Table 1.

The fundamental assumptions are as follows:

Assumption 1: The carbon emissions of the two products satisfy $e_r < e_n$ [10]. For the sake of analysis, this paper does not consider any other operational costs [41]. Therefore, under this assumption, the production costs for the two products satisfy $se_n > se_r$. The relationship of costs aligns with the fundamental assumptions commonly used in literature on remanufacturing operations management, such as Ferguson and Toktay [42].

Assumption 2: We employ consumer utility theory derive the inverse demand functions for new and remanufactured products. We assume that consumer willingness to pay for new products, denoted by v, exhibits heterogeneity and follows a uniform distribution within the interval $[0, a]$. Additionally, the willingness to pay for remanufactured products is a fraction α of that to new ones, and α satisfies $0 \leq \alpha < 1$. Consequently, the net utilities for consumers choosing to buy new products and remanufactured products are $u_n = v - p_n$ and $u_r = \alpha v - p_r$, respectively. When $u_n > u_r$ and $u_n > 0$ ($u_r > u_n$ and $u_r > 0$), consumers opt for new (remanufactured) products. Consequently, the inverse demand functions for new and remanufactured products can be derived as $p_n = a - q_n - \alpha q_r$ and $p_r = \alpha(a - q_n - q_r)$, respectively. One can refer to Ferguson and Toktay [42] and Ferrer and Swaminathan [15] for detailed derivations.

Assumption 3: The product life cycle-based approach is employed to measure environmental impacts. Life cycle assessment is a crucial method for assessing environmental impacts, as it considers the environmental effects at various stages, including raw material processing, manufacturing, remanufacturing, product usage, recycling, and disposal. In comparison to new products, remanufactured ones consume fewer energy, resulting in lower carbon emissions. Therefore, we assume that the carbon emissions of new products over the entire life cycle are higher than remanufactured ones, consistent with Assumption 1. Consequently, the environmental impacts of new and remanufactured products are E_n and E_r, respectively. Additionally, not all waste products are recovered and remanufactured, and such used products may be casually discarded by consumers, leading to higher environmental impacts. Therefore, it is assumed that the unit carbon emissions of these disposed waste products are E_d, satisfying $E_d > E_n$, and therefore the environmental impact of disposed waste products is $E_d(q_n - q_r)$. Consequently, based on the life cycle assessment approach, the overall environmental impact of all products is $\Gamma = E[E_n q_n + E_r q_r + E_d(q_n - q_r)]$ [40].

Assumption 4: The government's allocation of free carbon allowances is insufficient to meet the production requirements of the medical equipment enterprise, i.e., $E_g < \min(E_n q_{1n}, E_n q_{2n} + E_r q_{2r})$. Otherwise, carbon options become an investment tool for the enterprise rather than hedging against carbon price fluctuations risks.

IV. MODEL SETUP

Addressing the two-period production and carbon reduction decision problem for medical equipment remanufacturing companies in the presence or absence of carbon options, this section constructs a decision optimization model to investigate the impact of introducing carbon options on the enterprise's two-period production and carbon reduction decisions under carbon price fluctuations. This problem falls under the category of a three-stage optimization problem, with the decision events as below: first, the enterprise determines the level of carbon reduction; second, the enterprise decides on the production quantity for new products at the first stage; finally, the enterprise determines the production quantities of both new and remanufactured products at the second stage.

A. THE SCENARIO WITHOUT CARBON OPTIONS

In the absence of carbon options, the enterprise can only buy additional carbon allowances at the prevailing spot price on the carbon market. Consequently, the enterprise's profits in the first and second periods are as follows:

$$\pi_1 = p_{1n}q_{1n} - s[(1-e)e_n q_{1n} - E_g] - \frac{1}{2}ke^2 \tag{1}$$

$$\begin{aligned}\pi_2 &= p_{2n}q_{2n} + p_{2r}q_{2r} - s[(1-e)e_n q_{2n} \\ &\quad + (1-\varphi e)e_r q_{2r} - E_g] - c\rho q_{1n}\end{aligned} \tag{2}$$

Next, we solve this three-stage optimization problem using a backward induction approach.

(1) Optimization analysis of the third stage

In the third stage, the enterprise makes decisions for producing new and remanufactured products to maximize its expected profit of the second period. Therefore, the optimization objective and corresponding constraints for this stage are as follows:

$$\begin{aligned}\max \pi_2(q_{2n}, q_{2r}\,|q_{1n}, e) &= [p_{2n} - \mu(1-e)e_n]q_{2n} + [p_{2r} \\ &\quad - \mu(1-\varphi e)e_r]q_{2r} + \mu E_g \\ &\quad - c\rho q_{1n}\end{aligned} \tag{3}$$

$$s.t. \quad q_{2r} \le \rho q_{1n} \tag{4}$$

Among these, ρ represents the recycling rate, and Constraint (4) states that the quantity of remanufactured products does not exceed the quantity of recycled products at the second period. Note that not all recovered used products can be remanufactured, we focus on the case of $q_{2r} < \rho q_{1n}$ to investigate the characteristics of enterprises engaged in remanufacturing.

By constructing the Lagrangian function and applying Karush-Kuhn-Tucker (KKT) conditions to solve this nonlinear optimization problem, the optimal production quantities for new and remanufactured products are as shown in Lemma 1. Note that the superscript *SN* and *SO* denote the scenarios without and with carbon options, respectively. All the proofs of lemmas are presented in Appendix A.

Lemma 1: In the scenario without carbon options, for given q_{1n} and e, the optimal production quantities for new and remanufactured products in the second period are $q_{2n}^{SN*} = \frac{(1-\alpha)a-\mu[(1-e)e_n-(1-\varphi e)e_r]}{2(1-\alpha)}$ and $q_{2r}^{SN*} = \frac{\mu[\alpha(1-e)e_n-(1-\varphi e)e_r]}{2\alpha(1-\alpha)}$, respectively.

Lemma 1 demonstrates the optimal production quantities of the second period for the medical equipment enterprise under the scenario without carbon options. An analysis of this optimal solution reveals that the production quantities are independent on the production quantity of new products at the first period, i.e., q_{1n}. The primary reason for this is that when $\lambda = 0$, the constraints between q_{1n} and q_{2r} are relatively loose. In such cases, the enterprise can make production decisions based on the principle of maximizing its own profit, which results in the production quantities of both products being independent of q_{1n}. Furthermore, for given values of q_{1n} and e, the production quantities are also independent of the recycling rate ρ. This implies that regardless of the actual recycling rate, when the emission abatement rate is fixed, the enterprise will not alter the production quantities of both products.

(2) Optimization analysis of the second stage

In the second stage, the enterprise makes production decisions for new products in the first period to maximize the total expected profit over the two periods. Consequently, the optimization objective for this stage is as follows

$$\begin{aligned}\max \pi_t(q_{1n}\,|e) &= [p_{1n} - \mu(1-e)e_n]q_{1n} + \mu E_g \\ &\quad - \frac{1}{2}ke^2 + \pi_2^*\end{aligned} \tag{5}$$

Solving Eq. (5) yields Lemma 2.

Lemma 2: Under the scenario without carbon options, for a given e, the optimal production quantity of new products in the first period is $q_{1n}^{SN*} = \frac{a-\mu(1-e)e_n-c\rho}{2}$.

Lemma 2 demonstrates the optimal production quantity of new products in the first period under the scenario without carbon options. For a given emission abatement rate, the production quantity decreases as the recycling rate increases. The primary reason for this is that the quantity of raw materials for remanufacturing (i.e., waste products) increases as the recycling rate. Consequently, the enterprise has an incentive to reduce the production quantity, thereby lowering production costs. Furthermore, compared to the scenario without emissions reduction, implementing carbon reduction strategies increases the production quantity. This is because the implementation of carbon reduction strategies effectively reduces the enterprise's carbon emissions costs, leading to a reduction in its total production costs and motivating an increase in production quantities.

(3) Optimization analysis of the first stage

In the first stage, the enterprise makes decisions regarding the emission reduction rate to maximize the total expected profit over the two periods. Therefore, the optimization objective for this stage is as follows

$$\begin{aligned}\max \pi_t(e) &= [p_{1n}^{SN*} - \mu(1-e)e_n]q_{1n}^{SN*} + \mu E_g \\ &\quad - \frac{1}{2}ke^2 + \pi_2^*\end{aligned} \tag{6}$$

Solving Eq. (5) yields Lemma 3.

Lemma 3: Under the scenario without carbon options, the optimal solution for the carbon emission reduction rate is, as shown in the equation at the bottom of the next page.

Lemma 3 demonstrates the optimal solution for the emission abatement rate under the scenario in the absence of carbon options. An analysis of this optimal solution reveals that $\frac{\partial e^{SN*}}{\partial \rho} < 0$, indicating that with an increase in the recycling rate, the emission reduction effort decreases. The reason for this is that as the recycling rate increases, the enterprise increases the production quantity in the first period. This leads to an increase in the enterprise's total production costs (including production costs and recycling costs). Therefore, the enterprise reduces the carbon emission reduction rate to minimize its overall costs. Despite the fact that implementing carbon reduction strategies reduces the carbon emission costs for both products, the total investment cost associated with carbon reduction strategies, coupled with the increase in production costs, compels the enterprise to lower the emission reduction level.

Next, we analyze the scenario where an enterprise adopts carbon options in the presence of carbon price fluctuations.

B. THE SCENARIO WITH CARBON OPTIONS

In this scenario, in the first period, the medical equipment enterprise acquires carbon options to obtain the right to trade carbon allowances. In the second period, it trades carbon options to obtain carbon allowances, thereby meeting its production needs. Consequently, the carbon allowances required by the enterprise in the first period must be purchased at the carbon spot price, whereas in the second period, carbon allowances are obtained through trading carbon options. On the basis, the enterprise's profits in the first and second periods are as follows

$$\pi_1 = p_{1n}q_{1n} - s[(1-e)e_nq_{1n} - E_g] - o[(1-e)e_nq_{2n} + (1-\varphi e)e_rq_{2r} - E_g] - \frac{1}{2}ke^2 \tag{7}$$

$$\pi_2 = p_{2n}q_{2n} + p_{2r}q_{2r} - \min(w,s)[(1-e)e_nq_{2n} + (1-\varphi e)e_rq_{2r} - E_g] - c\rho q_{1n} \tag{8}$$

Similar to the scenario without carbon options, the optimization problem for these three stages is solved using backward induction. For convenience, let $t(w) = w - \int_{\underline{s}}^{w} F(x)dx$, i.e., $t(w) = E\min(w,s)$.

(1) Optimization analysis of the third stage

In the third stage, the enterprise makes decisions regarding q_{2n} and q_{2r} to maximize the expected profit at the second period. Therefore, the optimization objective and corresponding constraints for this stage are as follows

$$\max \pi_2(q_{2n}, q_{2r} \,|q_{1n}, e) = [p_{2n} - t(w)(1-e)e_n]q_{2n} + [p_{2r} - t(w)(1-\varphi e)e_r]q_{2r} + t(w)E_g - c\rho q_{1n} \tag{9}$$

$$s.t. \quad q_{2r} \le \rho q_{1n} \tag{10}$$

The solution of this nonlinear optimization problem yields Lemma 4.

Lemma 4: In the context of carbon options, for given values of q_{1n} and e, the optimal second-period production quantities of new and remanufactured products are $q_{2n}^{SO*} = \frac{(1-\alpha)a - t(w)[(1-e)e_n - (1-\varphi e)e_r]}{2(1-\alpha)}$ and $q_{2r}^{SO*} = \frac{t(w)[\alpha(1-e)e_n - (1-\varphi e)e_r]}{2\alpha(1-\alpha)}$, respectively.

Lemma 4 demonstrates the optimal production quantities of both products for enterprises in the context of carbon options during the second period. Analogous to the scenario without carbon options, this optimal solution is independent of the first-period production quantities and recovery rate.

(2) Optimization analysis of the second stage

In the second stage, the enterprise makes decisions regarding the first-period production quantity of new products to maximize the expected profit over the two periods. Therefore, by substituting q_{2n}^{SO*} and q_{2r}^{SO*} into Eq. (9) and combining it with the profit function from the first period, the optimization objective for this stage is obtained as follows

$$\max \pi_t(q_{1n} \,|e) = [p_{1n} - \mu(1-e)e_n]q_{1n} + o[(1-e)e_nq_{2n}^{SO*} + (1-\varphi e)e_rq_{2r}^{SO*} - E_g] + (\mu + o)E_g - \frac{1}{2}ke^2 + \pi_2^* \tag{11}$$

Solving Eq. (11) yields Lemma 5.

Lemma 5: In the context of carbon options, for a given e, the optimal solution for the first-period production quantity is $q_{1n}^{SO*} = \frac{a - \mu(1-e)e_n - c\rho}{2}$.

Lemma 5 demonstrates the optimal production quantity of new products for enterprises in the context of carbon options. Upon analyzing this optimal solution, it becomes evident that with the introduction of carbon options, the production quantity is independent of both the carbon option price and the exercise price.

(3) Optimization analysis of the first stage

In this stage, following the principle of profit maximization, the enterprise makes decisions regarding the carbon emission reduction rate. By substituting into Eq. (11), the optimization objective for this stage is expressed by

$$\max \pi_t(q_{1n} \,|e) = [p_{1n}^{SO*} - \mu(1-e)e_n]q_{1n}^{SO*} + o[(1-e)e_nq_{2n}^{SO*} + (1-\varphi e)e_rq_{2r}^{SO*} - E_g] + (\mu + o)E_g - \frac{1}{2}ke^2 + \pi_2^* \tag{12}$$

Solving Eq. (12) yields Lemma 6.

Lemma 6: In the context of carbon options, the optimal emission reduction rate is, as shown in the equation at the bottom of the next page.

Lemma 6 illustrates the optimal solution for the emission reduction effort in the context of carbon options. Analyzing this optimal solution reveals that $\frac{\partial e^{SO*}}{\partial \rho} < 0$, it aligns with the conclusion in the absence of carbon options. That is, the emission reduction level decreases as the recovery rate increases.

Next, we proceed with a comparative analysis to discuss the impacts of carbon options.

$$e^{SN*} = \frac{\mu[\alpha e_n(1-\alpha)(2a - \mu e_n - c\rho) + \mu[\varphi e_r(\alpha e_n - e_r) - \alpha e_n(e_n - e_r)]]}{2k\alpha(1-\alpha) - 2\alpha\mu^2 e_n(e_n - \varphi e_r) + \mu^2(\alpha^2e_n^2 - \varphi^2e_r^2)}.$$

V. COMPARATIVE ANALYSIS

In this section, we first compare and analyze the optimal solutions of the two scenarios: with and without carbon options. Next, we explore the impact of implementing carbon emission abatement strategies. Finally, we analyze the effects of carbon options on consumer surplus and the environment.

A. COMPARISONS ON OPTIMAL SOLUTIONS

First, by comparing the production quantities in the scenarios with and without carbon options, Proposition 1 can be derived. Note that all the proofs of propositions are presented in Appendix B.

Proposition 1: For a given e, we have $q_{2n}^{SO*} > q_{2n}^{SN*}$ and $q_{2r}^{SO*} < q_{2r}^{SN*}$.

Proposition 1 implies that, for a given emission abatement rate, the production quantity of new products in the second period under the carbon options scenario exceeds that of the scenario without carbon options, while the remanufactured quantity is lower than the scenario without carbon options. In the second period, compared to the scenario without carbon options, the introduction of carbon options allows the enterprise to incur only the cost of exercising carbon options, and this cost is lower than the expected value of carbon prices. To be specific, the introduction of carbon options during this period effectively reduces the operational costs for companies. Compared with remanufactured products, the introduction of carbon options leads to a greater reduction in carbon emission costs for new products due to their higher actual unit carbon emissions. Consequently, companies are motivated to produce more new products. This competition between the two products leads to a decrease in the remanufactured quantity.

In addition, analyzing the total production quantity in the two scenarios reveals that the introduction of carbon options can increase the total production quantity. The reason for this lies in the fact that, compared to purchasing additional carbon emissions allowances at the current carbon spot market prices, the use of carbon options effectively reduces the enterprise's carbon emission costs, thereby reducing its total operational costs. Hence, companies are incentivized to enhance their total production quantity.

By substituting the production quantities under both scenarios into the inverse demand functions, Proposition 2 characterizes the optimal sale prices under the two scenarios can be derived.

Proposition 2: For a given e, we have $p_{2n}^{SO*} < p_{2n}^{SN*}$ and $q_{2r}^{SO*} < q_{2r}^{SN*}$.

According to Proposition 2, for a given carbon emission reduction rate, the optimal selling prices in the carbon option scenario are lower than in the scenario without carbon options. With the introduction of carbon options, the carbon emissions cost for the enterprise decreases. Consequently, the enterprise is motivated to reduce the sale prices of both products to attract consumers, thereby enhancing its profits.

Next, by comparing the production quantities at the first period in both scenarios, Proposition 3 can be established.

Proposition 3: For a given e, we have $q_{1n}^{SO*} = q_{1n}^{SN*}$.

According to Proposition 3, whether carbon options are introduced or not, the medical equipment enterprise's production quantities of the first period remain unchanged. The reason is that the enterprise purchases carbon quotas at the carbon spot price in the first period, implying that the emission costs are the same under both scenarios. On the other hand, combining the optimal solutions in the second period reveals that the production quantities of both products are independent on the production quantity of the first period. Therefore, even though the enterprise incurs carbon option costs in the carbon option scenario, these costs are influenced only by the second-period production quantities, not by the first-period production quantity. Additionally, combining the inverse demand function, we show that the selling prices at the first period are the same in both scenarios.

B. EFFECTS OF CARBON EMISSION REDUCTION STRATEGY

In this subsection, we compare the optimal solutions with and without emission reduction strategies and investigate how implementing carbon emission reduction strategies affects the performance of the enterprise. For ease of exposition, an overbar is used to represent the optimal solution without emission reduction strategies. First, we compare the production quantities at the second period under the scenario without carbon emission reduction strategies, leading to Proposition 4.

Proposition 4: i) $q_{1n}^{j*} > \bar{q}_{1n}^{j*}$; ii) if $\varphi > e_n/e_r$ ($\varphi < e_n/e_r$), then $q_{2n}^{j*} < \bar{q}_{2n}^{j*}$ ($q_{2n}^{j*} > \bar{q}_{2n}^{j*}$); if $\varphi > \alpha e_n/e_r$ ($\varphi < \alpha e_n/e_r$), then $q_{2r}^{j*} > \bar{q}_{2r}^{j*}$ ($q_{2r}^{j*} < \bar{q}_{2r}^{j*}$); where $j = SN, SO$.

Proposition 4 demonstrates that the implementation of emission abatement strategies consistently enhances the production quantity of the first period, and the quantity is increases with the carbon emission reduction rate. This is attributed to the effective reduction of carbon emissions and, consequently, costs through the implementation of carbon emission reduction strategies, incentivizing companies to increase new product production. From a second-period perspective, whether carbon options are introduced or not, the impacts of emission abatement strategies on the production quantities of both products in the second period are

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$$e^{SO*} = \frac{\alpha e_n(1-\alpha)[a(\mu+o+t)-\mu(c\rho+\mu e_n)]+t(2o+t)[\alpha e_n(e_r(1+\alpha)-e_n)-\varphi e_r^2]}{\alpha(1-\alpha)(2k-\mu^2 e_n^2)+t(2o+t)[\alpha e_n(2\varphi e_r-e_n)-\varphi^2 e_r^2]}$$

identical. Specifically, when the marginal impact of carbon emission abatement strategies on remanufactured product carbon emissions is significant, indicating a larger influence on remanufactured product emissions, the implementation of carbon emission reduction increases remanufactured product production but decreases new product production. Conversely, when the marginal impact of emission abatement strategies on remanufactured product carbon emissions is minor, indicating a smaller influence on remanufactured product emissions, the implementation of carbon emission reduction increases new product production. Due to new products' cannibalization effect on remanufactured products, the enterprise would reduce remanufactured product production. Furthermore, a higher carbon emission rate results in a lower quantity of remanufactured products.

Analyzing the impacts of implementing emission abatement strategies on the sale prices of both products over two periods yields Proposition 5.

Proposition 5: i) $p_{1n}^{j*} < \bar{p}_{1n}^{j*}$; ii) $p_{2n}^{j*} < \bar{p}_{2n}^{j*}$, $p_{2r}^{j*} < \bar{p}_{2r}^{j*}$; where $j = SN, SO$.

According to Proposition 5, regardless of whether carbon options are introduced, when an enterprise implements carbon emission reduction strategies, the optimal sale prices for both products over the two periods decrease. This is because the implementation of carbon emission reduction strategies can reduce the carbon emissions of both products, subsequently lowering their costs. Consequently, companies are incentivized to lower the selling prices to stimulate market demand, thereby enhancing their profitability. Furthermore, if the carbon emission rate is higher, the optimal sale prices for both products over the two periods are lower.

C. ANALYSIS ON CONSUMER SURPLUS

In this subsection, we discuss the influences of carbon options from the perspective of consumers. Specifically, we begin by examining consumer surplus in the first and second periods separately, followed by an analysis of the total consumer surplus over the two periods. According to research by Esenduran et al. [43] and Yenipazarli [2], the consumer surplus in the first and second periods is represented as follows

$$CS_1^j = \int_0^{q_{1n}^j} (a - q_{1n}^j)dq - p_{1n}^j q_{1n}^j \tag{13}$$

$$\begin{aligned} CS_2^j &= \int_{\frac{p_{2n}^j - p_{2r}^j}{1-\alpha}}^{a} (v - p_{2n}^j)dv + \int_{\frac{p_{2r}^j}{\alpha}}^{\frac{p_{2n}^j - p_{2r}^j}{1-\alpha}} (\alpha v - p_{2r}^j)dv \\ &= \frac{\alpha q_{2r}^{j2} + q_{2n}^{j2} + 2\alpha q_{2n}^j q_{2r}^j}{2} \end{aligned} \tag{14}$$

where $j = SO, SN$ represents the scenarios with and without carbon options.

Consequently, the total consumer surplus over the two periods can be determined as

$$CS_t^j = CS_1^j + CS_2^j \tag{15}$$

By substituting the optimal solutions with and without carbon options into Eqs. (13) and (14) and comparing them, Proposition 6 can be derived. For ease of exposition, let $\Omega = \frac{(1-\varphi e)e_r}{(1-e)e_n}$, where Ω represents the relative cost of remanufacturing.

Proposition 6: For a given e, i) $CS_1^{SN*} = CS_1^{SO*}$; ii) if $t_1^{SN} < \Omega < \max(t_1^{SN}, t_2^{SN})$, then $CS_2^{SN*} > CS_2^{SO*}$ and $CS_t^{SN*} > CS_t^{SO*}$; if $\max(t_1^{SN}, t_2^{SN}) < \Omega$, then $CS_2^{SN*} < CS_2^{SO*}$ and $CS_t^{SN*} < CS_t^{SO*}$, where $t_1^{SN} = \alpha - \rho\alpha(1-\alpha)[\frac{a-c\rho}{\mu(1-e)e_n} - 1]$ and $t_2^{SN} = \alpha - \sqrt{\frac{\alpha(1-\alpha)[a-\mu(1-e)e_n]}{\mu(1-e)e_n}}$.

Proposition 6 demonstrates that for a given emission abatement effort, the consumer surplus in the first period remains unchanged, regardless of the introduction of carbon options. This is because in both scenarios, companies purchase additional carbon quotas at the carbon spot price in the first period. In the carbon option scenario, although companies incur the cost of purchasing carbon options, it does not alter the enterprise's optimal production quantity in the first period, and thus, its optimal selling price remains unaffected. Therefore, the consumer surplus remains the same in both scenarios.

From the perspective of the second period and the overall two-period analysis, we find that the consumer surplus in both scenarios is influenced by the relative cost of remanufacturing. Specifically, if the relative cost of remanufacturing is higher, introducing carbon options can increase consumer surplus compared to the scenario without carbon options. Conversely, if the relative cost of remanufacturing is lower, introducing carbon options may reduce consumer surplus. In particular, if the thresholds for the relative cost of remanufacturing satisfy $t_1^{SN} > t_2^{SN}$, then the introduction of carbon options consistently enhances consumer surplus. Therefore, if enterprises are concerned about the interests of consumers, the relative cost of remanufacturing must be taken into account when introducing carbon options. This insightful finding suggests that enterprises can strategically utilize carbon options, particularly when the relative cost of remanufacturing is high, to enhance consumer surplus.

The above analysis indicates that the value of carbon options is influenced by the presence of remanufacturing, particularly by the relative cost associated with it.

Next, by comparing consumer surplus with and without carbon emission reduction scenarios, Proposition 7 can be derived.

Proposition 7: $CS_1^{j*} > C\bar{S}_1^{j*}$, $CS_2^{j*} > C\bar{S}_2^{j*}$, $CS_t^{j*} > C\bar{S}_t^{j*}$, where $j = SN, SO$.

Proposition 7 reveals that whether or not carbon options are introduced, implementing carbon emission reduction strategies consistently enhances consumer surplus in both the first and second periods. In the first period, carbon emission reduction reduces the enterprise's carbon emission costs, motivating the enterprise to increase production and lower product prices, ultimately benefiting consumers. In the second period, combined with Propositions 4 and 5, implementing carbon emission reduction strategies leads to an increase in the total production of both products and a reduction in their selling prices, consequently increasing consumer surplus. Therefore, from the consumer's perspective, implementing carbon emission reduction strategies is advantageous in a two-period environment. Specially, if the carbon emission rate is higher, the consumer surpluses over the two periods are higher.

D. ANALYSIS ON ENVIRONMENTAL IMPACT

In this subsection, we discuss the environmental impacts of adopting carbon options in a two-period environment. According to Assumption 3 and based on a life cycle analysis approach, the environmental impact over two periods is as follows:

$$\Gamma^j = E_n(q_{1n}^j + q_{2n}^j) + E_r q_{2r}^j + E_d(q_{1n}^j - q_{2r}^j) \qquad (16)$$

where $j = SO, SN$ represents the scenarios with and without carbon options.

For ease of analysis, let $x = \frac{E_r}{E_n}$ and $y = \frac{E_d}{E_n}$. Then, x and y represent the relative environmental impacts of remanufactured products and unrecycled products, respectively. Based on Assumption 3, we have $x < 1$ and $y > 1$. Eq. (16) can then be further expressed as

$$\Gamma^j = E_n[q_{1n}^j + q_{2n}^j + x q_{2r}^j + y(q_{1n}^j - q_{2r}^j)] \qquad (17)$$

By comparing the environmental impacts under the scenarios with and without carbon options, Proposition 8 can be derived.

Proposition 8: For a given e, we have $\Gamma^{SO*} > \Gamma^{SN*}$.

Proposition 8 indicates that for a given emission abatement effort, introducing carbon options increases the environmental impact. Compared to the base scenario, although introducing carbon options in the first period does not change the production quantity of new products, it does increase the second-period production quantity of new products. Despite the decrease in the remanufactured quantity, the total production quantity increases, which is one of the primary reasons. Additionally, the decrease in the remanufactured quantity implies an increase in the quantity of unrecycled products, and these unrecycled products have a relatively significant environmental impact, leading to an overall increase in environmental impact.

Furthermore, for a given carbon emission reduction rate, since the optimal decisions over the two periods are independent of the carbon option price, environmental benefits are not influenced by carbon option prices. However, as the exercise price increases, the environmental impact in the carbon option scenario decreases. This is because the increase in the exercise price reduces the total second-period production quantity, thereby lowering the environmental impact.

The above conclusions were obtained under the assumption of a given emission abatement level. Since the optimal carbon emission reduction rates are complex in both scenarios, we proceed with numerical examples to analyze the carbon emission reduction rate.

VI. NUMERICAL EXAMPLES

In this section, we employ numerical examples to study the effects of introducing carbon options on the optimal carbon emission abatement rate. Furthermore, we analyze the impacts of carbon price fluctuations on the enterprise's profits, consumers, and the environment. Herein, the parameter values are adopted from the work of Ding [36]. Additionally, Exanovo provided us with data on the average price of carbon options, where the option price and exercise price are 20 and 70 CNY per ton, respectively. To visually illustrate our results, we applied dimensionless processing to these data. Therefore, the parameters are set as follows: $a = 1$, $\alpha = 0.5$, $e_n = 0.6$, $e_r = 0.2$, $e_d = 1$, $\varphi = 0.6$, $c = 0.1$, $k = 0.8$, $o = 0.2$, $w = 0.7$, $\rho = 0.8$ and $E_g = 0.01$. Besides, based on the real-time data from Shanghai Environment and Energy Exchange, it is assumed that the carbon price follows a uniform distribution with a mean of $\mu = 1$. In fact, our results remain robust even if the carbon price can follow any random distribution, because $t(w)$ is positively affected by the lower limit of the distribution region, i.e., $\underline{s}$.

A. ANALYSIS ON OPTIMAL CARBON EMISSION REDUCTION RATE

We first explore the impact of carbon price fluctuations on the optimal emission abatement rates under both scenarios. The results obtained are shown in Figure 1.

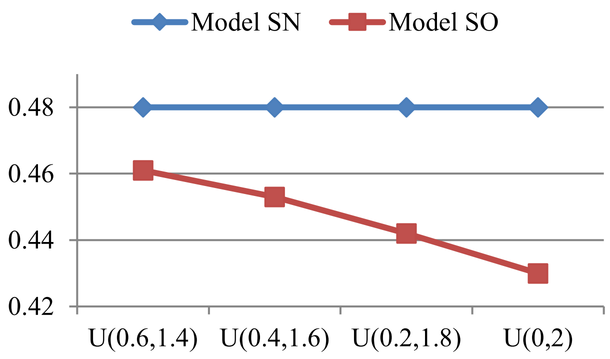

FIGURE 2. **The impact of carbon price fluctuations on the optimal emission reduction rate.**

From Figure 2, it can be observed that as carbon price volatility increases, the optimal emission abatement level unchanged under the scenario without carbon options, but decreases under the scenario with carbon options. In the absence of carbon options, because the medical equipment enterprise purchases additional emission rights based on the mean of carbon price, the fluctuation in carbon prices does not affect their optimal decisions. However, under the carbon option scenario, high carbon price volatility implies increased uncertainty in operating costs for companies. However, the enterprise can use carbon option trading to reduce actual carbon emission costs and hedge against the risks posed by uncertain carbon prices, thereby

TABLE 2. Optimal emission reduction rates under different carbon option prices and exercise prices.

o	0.15			0.20			0.25		
w	0.5	0.6	0.7	0.5	0.6	0.7	0.5	0.6	0.7
e^{SN*}	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480	0.480
e^{SO*}	0.391	0.415	0.436	0.408	0.432	0.453	0.427	0.450	0.471

reducing their enthusiasm for investing in carbon reduction technologies.

Additionally, through comparison, it is evident that the optimal emission abatement level under the carbon option scenario is consistently lower than that under the scenario without carbon options. With the introduction of carbon options, the enterprise can, by paying a certain price in the first period, gain the right to trade carbon options in the second period. By comparing the actual carbon price with the exercise price, the enterprise can obtain carbon emission rights at a lower cost, signifying that carbon options can reduce an enterprise's costs. Therefore, in comparison to the scenario without carbon options, the reduction in operational costs due to the introduction of carbon options leads to a reduction in an enterprise's emission levels.

Furthermore, assuming that carbon prices follow a uniform distribution within the interval (0.4, 1.6), we analyze the influences of carbon option prices and exercise prices on the optimal emission abatement levels under both scenarios. The results obtained are presented in Table 2.

From Table 2, it can be observed that when the exercise price is constant, as carbon option prices increase, the optimal emission abatement level under the carbon option scenario also increases. The increase of the carbon option price raises actual operational costs for companies, prompting them to raise carbon emission reduction rates to lower actual carbon emissions and consequently reduce costs. Furthermore, when carbon option prices are fixed, as the exercise price increases, the optimal emission abatement level increases under the carbon option scenario. The reason for this is that the increase in carbon option exercise prices raises companies' expected costs, thereby motivating them to increase carbon emission reduction levels to lower their own costs.

B. ANALYSIS ON OPTIMAL PRODUCTION QUANTITY AND EXPECTED PROFIT

In this subsection, we analyze the influences of carbon price fluctuations on the production quantities and expected profits of the enterprise under both scenarios. Combining with Proposition 3, it can be deduced that the effects of carbon price fluctuations on the optimal production quantities in the first period are the same as their impact on the optimal emission abatement levels. That is, as the carbon price volatility increases, the optimal production quantities of the first period decrease and are consistently lower than those in the scenario without carbon options. Therefore, based on the inverse demand functions for the first period, it can also be inferred that as carbon price volatility increases, the optimal selling prices in the first period increase and are consistently higher than the production quantities in the scenario without carbon options.

Next, we investigate the impact of carbon price fluctuations on the production quantities and expected profits in the second period under both scenarios. The results obtained are presented in Table 3.

Based on Table 3, it is evident that the production quantities of both new and remanufactured products in the carbon option scenario increase as carbon price volatility increases. Combining the preceding analysis, it can be deduced that introducing carbon options can reduce the operational costs for the enterprise compared to the scenario without carbon options. Moreover, these operational costs decrease as carbon price volatility rises, motivating the enterprise to increase the production quantities, thereby enhancing their expected profits. Additionally, a separate comparison of production quantities in both scenarios reveals that in the carbon option scenario, the production quantities of remanufactured products (new products) are consistently lower (higher) than in the scenario without carbon options, which aligns with the conclusion under the assumption of a given emission abatement level. The results also reveal that as carbon price volatility increases, the optimal selling price for new products decreases in the carbon option scenario, while that for remanufactured products increases. Furthermore, comparing the optimal selling prices under the two scenarios shows that in the carbon option scenario, the optimal selling prices for both product types are lower than in the scenario without carbon options. Combining the production quantities of the two products, it is evident that the introduction of carbon options reduces an enterprise's costs, prompting them to increase production by reducing selling prices, thus increasing expected profits.

From a profitability perspective, as carbon price volatility increases, the expected profits of both the first and second periods for companies in the carbon option scenario increase, highlighting the value of carbon options in the face of carbon price fluctuations. From the perspective of the first period, although the enterprise increases production quantity in the second period, the reduction in the emission abatement effort to some extent can lower the cost of carbon options. Additionally, due to the enterprise raising the optimal sale price of the first period, the expected profits in that period

TABLE 3. The impact of carbon price fluctuations on production quantities and expected profits.

	$U(0.6,1.4)$	$U(0.4,1.6)$	$U(0.2,1.8)$	$U(0,2)$
q_{2n}^{SN*}	0.330	0.330	0.330	0.330
q_{2n}^{SO*}	0.376	0.379	0.383	0.388
q_{2r}^{SN*}	0.027	0.027	0.027	0.027
q_{2r}^{SO*}	0.023	0.024	0.025	0.026
p_{2n}^{SN*}	0.656	0.656	0.656	0.656
p_{2n}^{SO*}	0.612	0.609	0.604	0.599
p_{2r}^{SN*}	0.321	0.321	0.321	0.321
p_{2r}^{SO*}	0.300	0.298	0.296	0.293
π_1^{SN*}	0.035	0.035	0.035	0.035
π_1^{SO*}	0.014	0.015	0.016	0.017
π_2^{SN*}	0.104	0.104	0.104	0.104
π_2^{SO*}	0.134	0.136	0.140	0.144
π_t^{SN*}	0.139	0.139	0.139	0.139
π_t^{SO*}	0.148	0.151	0.156	0.161

increase. At the second period, as carbon price volatility increases, the decrease in the emission abatement level and the increase in production quantity leads to higher expected profits for the enterprise. Furthermore, comparing the expected profits in both scenarios, it is evident that the expected profit in the first period in the carbon option scenario is lower than in the base scenario because the enterprise needs to pay for carbon options in the first period. However, the second-period expected profit in the carbon option scenario is significantly larger than in the scenario without carbon options, resulting in higher total expected profits. The above analysis shows that introducing carbon options allows companies to better manage risks associated with carbon price fluctuations, and the expected profits increase with rising carbon price volatility. For the enterprise, the value of introducing carbon options is higher when carbon price volatility is significant.

C. ANALYSIS ON OPTIMAL PRODUCTION QUANTITY AND EXPECTED PROFIT

Firstly, assuming $e = 0.7$, we compare the consumer surplus thresholds t_1^{SN} and t_2^{SN} in Propositions 6, as illustrated in Figure 3.

From Figure 3, it can be observed that regardless of how the discount coefficient for the consumer value of remanufactured products varies, $t_1^{SN} > t_2^{SN}$ always holds. Combining with Proposition 6, it is evident that the total consumer surplus in the scenario without carbon options is lower than in the scenario with carbon options. Considering the consumer surplus in the first period for both scenarios, it is clear that the consumer surplus in the carbon option scenario is larger than in the basic scenario. This implies that, for a given carbon emission reduction rate, introducing carbon options can always enhance the total consumer surplus, regardless of the relative cost of remanufacturing. This finding complements Proposition 6 and further underscores the positive impact of carbon options on consumer.

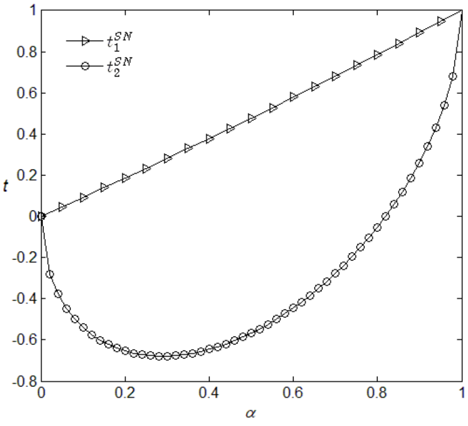

FIGURE 3. The comparison of consumer surplus thresholds.

Analyzing the impacts of carbon price fluctuation on consumer surplus, we show them in Figure 4.

From Figure 4, it is evident that as carbon price variability increases, consumer surplus in the carbon option scenario rises and is higher than in the scenario without carbon options, which is consistent with the conclusion under the assumption of a given carbon emission reduction rate. Combining with Table 3, as carbon price volatility increases, even though the production quantity of the first period decreases, the total production quantity in the second period increases. Additionally, the optimal selling prices for both products in this period decrease, ultimately leading to an increase in the consumer surplus. It appears that introducing carbon options benefits consumers, regardless of whether the emission abatement level is given or not.

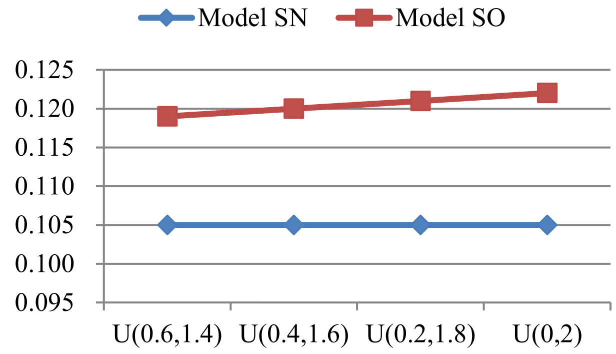

FIGURE 4. Impact of carbon price fluctuation on consumer surplus.

Next, we explore the influences of carbon price fluctuation on the environment. The results are presented in Figure 5.

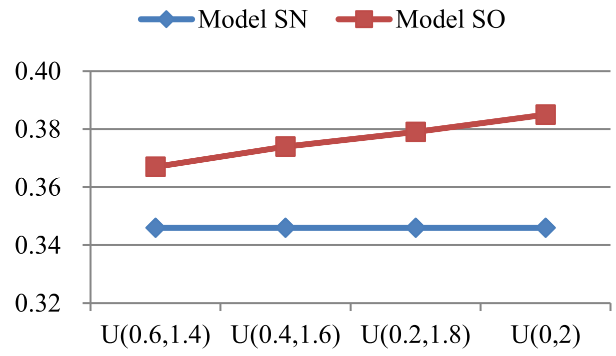

FIGURE 5. Impact of carbon price volatility on the environment.

From Figure 5, it is evident that as carbon price volatility increases, the environmental impact (total carbon emissions) in the carbon option scenario increases and is higher than in the scenario without carbon options, which is consistent with the conclusion under the assumption of a given carbon emission reduction rate. Combining with Figure 1 and Table 3, as carbon price volatility increases, the motivation for the medical equipment enterprise to invest in emission abatement technology decreases, leading to a lower optimal carbon reduction rate. Additionally, despite the decrease in production quantity of the first period, the production quantities of both types of products increase in the second period, ultimately resulting in an overall increase in environmental impact. Therefore, from an environmental perspective, the medical equipment enterprise should focus on reducing their environmental impact while introducing carbon options to increase their expected profits, achieving a win-win situation for both economic and environmental benefits.

TABLE 4. Optimal emission reduction rates under different discount coefficient.

o	0.3	0.4	0.5	0.6	0.7
e^{SN*}	0.507	0.502	0.480	0.425	0.261
e^{SO*}	0.491	0.488	0.471	0.436	0.348

D. SENSITIVITY ANLYSIS OF DISCOUNT COEFFICIENT

In this subsection, we perform sensitivity analysis on the discount coefficient for consumer valuation of remanufactured products in optimal decision-making. Given a fixed carbon emission rate, analysis of optimal production quantities with and without carbon options reveals that as the discount coefficient increases, the quantity of new products remains unchanged in the first period, while in the second period, the quantity of new (remanufactured) products decreases (increases). This observation is straightforward to understand, as a higher discount coefficient consistently incentivizes enterprises to produce more remanufactured products.

Herein, we examine the influence of the discount coefficient on the optimal carbon emissions reduction rate in scenarios both with and without carbon options. The numerical solutions are presented in Table 4.

Table 4 demonstrates that as the discount coefficient for consumer valuation of remanufactured products increases, the optimal carbon emission reduction rates decrease under both scenarios. This trend arises because a higher discount coefficient indicates greater consumer willingness to purchase remanufactured products, prompting enterprises to reduce investment in carbon emission reduction to lower costs. Furthermore, comparing the optimal carbon emission reduction rates between scenarios reveals that with a relatively low discount coefficient, the rate is lower when carbon options are introduced compared to without, consistent with our previous analysis. Conversely, with a higher discount coefficient, the optimal reduction rate is higher with carbon options than without. Additionally, the analysis indicates that the sensitivity of optimal carbon emission reduction rates to the discount coefficient is less pronounced when carbon options are introduced compared to when they are not. In essence, introducing carbon options mitigates this sensitivity amidst carbon price fluctuations.

The aforementioned analysis suggests that in order to improve carbon emissions reduction rate, the medical equipment company can consider introducing carbon options when the discount coefficient for the consumer value of remanufactured products is relatively large.

VII. MANAGERIAL INSIGHTS

This study provides timely managerial insight into managing carbon price fluctuations in the context of remanufacturing from the perspectives of various stakeholders.

From the perspective of decisions makers, i.e., medical equipment companies, the introduction of carbon options under carbon price fluctuations can enhance expected profits, with higher volatility in carbon prices translating into greater value from these options. Therefore, medical equipment companies should actively acquire carbon allowances through the purchase of carbon options to hedge against price fluctuations and increase profitability. This finding holds significant implications for medical equipment companies. We observe that carbon options provide substantial value, particularly when carbon price variability is high. Hence, under conditions of significant price variability, introducing carbon options proves more advantageous for medical equipment companies. However, it is important to note that higher carbon option and exercise prices may diminish enterprise profits. Therefore, enterprises can negotiate with carbon option suppliers to moderate these costs. Furthermore, to boost carbon emission reduction rates, enterprises can strategically employ carbon options, especially when the discount coefficient for consumer valuation of remanufactured products is high.

From the perspectives of consumers and the environment, introducing carbon options introduces a trade-off compared to scenarios without carbon options: consumer surplus increases while environmental benefits decrease. Additionally, we find that implementing carbon emission reduction strategies consistently increases consumer surplus but may adversely affect the environment due to increased total production quantities of new and remanufactured products. These findings indicate that both the use of carbon options and the implementation of emission reduction strategies have similar impacts on consumers and the environment. Therefore, if enterprises prioritize either consumer interests or environmental concerns, the choice to adopt these measures becomes straightforward. However, balancing the trade-off between these two objectives is crucial when enterprises aim to address both consumer and environmental interests.

From a governmental perspective, it is imperative to actively encourage medical equipment companies to acquire carbon allowances through the purchase of carbon options, while carefully adjusting carbon option prices and exercise prices to effectively increase the quantity of remanufactured products and mitigate environmental impact. Consistent with previous analyses, we find that governments face the dilemma of incentivizing medical equipment companies to use carbon options and implement carbon emission reduction strategies. One viable approach could involve setting relatively higher option-related prices to offset environmental impacts. Furthermore, our findings indicate that increased variability in carbon prices enhances consumer surplus but diminishes environmental benefits. During periods of significant carbon price fluctuations, governments can promote the use of carbon options.

VIII. CONCLUSION

In this paper, we focus on a medical equipment remanufacturing enterprise under carbon price fluctuations. More specifically, we investigate the two-period production decisions and emission reduction decisions with carbon options. Firstly, we analyze the scenarios with and without carbon options, and obtain the optimal production decisions under a given carbon emission reduction rate for the two scenarios. Subsequently, a further analysis was performed to examine the impact of introducing carbon options on consumer surplus and assess the environmental impact based on the life cycle assessment method. Finally, through numerical examples, we analyze the influences of carbon price fluctuations on the optimal emission abatement rate, production quantities, selling prices, expected profits, consumer surplus, and the environment. Compared with previous studies, we obtain many valuable conclusions regarding the use of carbon option and the implementation of carbon emission reduction strategies, and the main findings are as follows:

(i) For a given carbon emission reduction rate, compared to the basic scenario, in the carbon option scenario, the production quantity of new products in the second period increases, while that of remanufactured products decreases, but the production quantity of new products in the first period remains unchanged. In other words, using carbon option can enhance the enterprise's production incentives. In addition, introducing carbon options can reduce the optimal sale prices of both types of products in the second period, which is beneficial for consumers, but it will not change the optimal sale price of new ones at the first period. Although introducing carbon options does not alter consumer surplus in the first period, it can enhance consumer surplus in the second period and the overall consumer surplus over the two periods, which further demonstrate the positive role of carbon option for consumers.

(ii) Compared to the scenario without carbon emission reduction strategies, implementing carbon emission reduction strategies consistently enhances the quantity of new products at the first period and, under certain conditions, can also simultaneously increase the production quantities of both products in the second period. Moreover, the implementation of carbon emission reduction strategies can lead medical equipment companies to lower the sale prices of both products over the two periods. This suggests that the carbon emission reduction strategy can benefit consumers, consistent with Ding [36], but we extend this research by incorporating the consideration of carbon options.

(iii) With the increased volatility in carbon prices, the optimal emission abatement rate remains unchanged in the scenario without carbon options, while in the carbon option scenario, the optimal emission abatement rate decreases and is lower than that in the absence of carbon options if the discount coefficient for the consumer value of remanufactured products is relatively low. Additionally, the total profits, consumer surplus, and environmental impact over the two periods for medical equipment companies increase and are higher than in the scenario without carbon options. This finding reveals the negative effect of fluctuation in carbon prices, but also demonstrates the positive role of carbon option for enterprises and consumers, which provide feasible solutions for managing this risk.

Our study contributes to the operations management literature by introducing carbon options to manage the fluctuation of carbon price in a remanufacturing setting. As mentioned earlier, most of previous studies ignore the fluctuation of carbon price when taking into carbon emissions consideration in their models (e.g., Liu et al. [3], Chai et al. [4], Chang et al. [26], Miao et al. [29], and Zhu et al. [30]). Although Ding [36] examine the influences of carbon price fluctuation and associated risk aversion, effective risk management strategies have not yet been addressed. Our conclusions provide practical insights into hedging against the risks posed by carbon price fluctuations. These findings not only assist enterprises in managing carbon price volatility but also offer valuable guidance for governments in developing carbon markets and financial instruments.

This paper assumes that the enterprise monopolizes the entire market. Future research directions could consider competition between two or more companies or the introduction of upstream and downstream enterprises, expanding from individual companies to a supply chain level. In our model, we focus on a medical equipment enterprise, but extending our analysis to encompass the broader remanufacturing industry could yield additional insights for decision-making and guide managerial practices. Additionally, the type of carbon options in this study is call options, and future research can consider put options or the portfolio of two options. More importantly, the fluctuation of carbon price is usually affected by the market demand of products. Therefore, it is significant to consider both demand uncertainty and carbon price uncertainty, and then examine other influencing factors, such as the variability in market demand. This holistic approach can offer valuable insights for introducing various types of carbon options. Finally, we conduct a theoretic study in this paper. In future research, we can conduct empirical researches to reveal the impacts of carbon price fluctuation and the efficacy of carbon options.

APPENDIX A

Proof of Lemma 1:

For a given q_{1n} and e, we first analyze the concavity and convexity of π_2 with respect to q_{2n} and q_{2r}. Since $\frac{\partial^2\pi_2}{\partial q_{2n}^2} = -2$, $\frac{\partial^2\pi_2}{\partial q_{2n}\partial q_{2r}} = -2\alpha$, $\frac{\partial^2\pi_2}{\partial q_{2r}\partial q_{2n}} = -2\alpha$ and $\frac{\partial^2\pi_2}{\partial q_{2r}^2} = -2\alpha$, the Hessian matrix of π_2 is $H = \begin{bmatrix} -2 & -2\alpha \\ -2\alpha & -2 \end{bmatrix}$. Given that $H_1 = -2 < 0$ and $H_2 = 4(1-\alpha^2) > 0$, it follows that π_2 is a jointly concave function of q_{2n} and q_{2r}.

Next, introducing the Lagrange multiplier λ, the Lagrangian function for this optimization problem and its corresponding KKT conditions are as follows:

$$\begin{aligned} L^{SN} &= [p_{2n} - \mu(1-e)e_n]q_{2n} + [p_{2r} - \mu(1-\varphi e)e_r]q_{2r} \\ &\quad + \mu E_g - c\rho q_{1n} + \lambda(\rho q_{1n} - q_{2r}) \end{aligned}$$

$$s.t. \begin{cases} a - 2q_{2n} - 2\alpha q_{2r} - \mu(1-e)e_n = 0 \\ \alpha(a - 2q_{2n} - 2q_{2r}) - \mu(1-\varphi e)e_r - \lambda = 0 \\ \lambda_1(\rho q_{1n} - q_{2r}) = 0 \\ \lambda \geq 0 \end{cases}$$

We will now discuss λ, analyzing the following two scenarios: $\lambda = 0$ and $\lambda > 0$. To study the characteristics of enterprises engaged in remanufacturing, we will only focus on the case of $\lambda = 0$. When $\lambda = 0$, combining with the KKT conditions, it is known that $q_{2r} < \rho q_{1n}$. Substituting into the KKT conditions and solving them simultaneously yields $q_{2n}^{SN*} = \frac{(1-\alpha)a-\mu[(1-e)e_n-(1-\varphi e)e_r]}{2(1-\alpha)}$ and $q_{2r}^{SN*} = \frac{\mu[\alpha(1-e)e_n-(1-\varphi e)e_r]}{2\alpha(1-\alpha)}$. Q.E.D.

Proof of Lemma 2:

For a given e, by substituting $q_{2n}^{SN*} = \frac{(1-\alpha)a-\mu[(1-e)e_n-(1-\varphi e)e_r]}{2(1-\alpha)}$ and $q_{2r}^{SN*} = \frac{\mu[\alpha(1-e)e_n-(1-\varphi e)e_r]}{2\alpha(1-\alpha)}$ into Eq. (5) and then conducting analysis, we can obtain $\frac{\partial\pi_t^2(q_{1n}|e)}{\partial q_{1n}^2} = -2$. Consequently, it can be concluded that $\pi_t(q_{1n}|e)$ is a concave function with respect to q_{1n}, and solving it yields $q_{1n}^{SN*} = \frac{a-\mu(1-e)e_n-c\rho}{2}$. Q.E.D.

Proof of Lemma 3:

Taking the second partial derivative of Eq. (6) yields $\frac{\partial\pi_t^2(e)}{\partial e^2} = \frac{2k\alpha(1-\alpha)-2\alpha\mu^2e_n(e_n-\varphi e_r)+\mu^2(\alpha^2e_n^2-\varphi^2e_r^2)}{-2\alpha(1-\alpha)}$. To ensure the existence of an optimal solution for the emission reduction rate, we assume that the carbon reduction coefficient satisfies $2k\alpha(1-\alpha) - 2\alpha\mu^2e_n(e_n-\varphi e_r) + \mu^2(\alpha^2e_n^2 - \varphi^2e_r^2) > 0$. Based on this assumption, let $\frac{\partial\pi_t(e)}{\partial e} = 0$, and solving for this equation yields $e^{SN*} = \frac{\mu[\alpha e_n(1-\alpha)(2a-\mu e_n-c\rho)+\mu A]}{2k\alpha(1-\alpha)-2\alpha\mu^2e_n(e_n-\varphi e_r)+\mu^2(\alpha^2e_n^2-\varphi^2e_r^2)}$, where $A = \varphi e_r(\alpha e_n - e_r) - \alpha e_n(e_n - e_r)$. Q.E.D.

Proof of Lemma 4:

The constructed Lagrangian function and KKT conditions for this nonlinear problem are analogous to the case without carbon options. The Lagrangian function and KKT conditions are as follows

$$\begin{aligned} &L^{SO} \\ &= [p_{2n} - t(w)(1-e)e_n]q_{2n} + [p_{2r} - t(w)(1-\varphi e)e_r]q_{2r} \\ &\quad + t(w)E_g - c\rho q_{1n} + \lambda(\rho q_{1n} - q_{2r}) \end{aligned}$$

$$s.t. \begin{cases} a - 2q_{2n} - 2\alpha q_{2r} - t(w)(1-e)e_n = 0 \\ \alpha(a - 2q_{2n} - 2q_{2r}) - t(w)(1-\varphi e)e_r - \lambda = 0 \\ \lambda_1(\rho q_{1n} - q_{2r}) = 0 \\ \lambda \geq 0 \end{cases}$$

Herein, we specifically consider the scenario where $\lambda = 0$. Therefore, solving the KKT conditions simultaneously yields $q_{2n}^{SO*} = \frac{(1-\alpha)a-t(w)[(1-e)e_n-(1-\varphi e)e_r]}{2(1-\alpha)}$ and $q_{2r}^{SO*} = \frac{t(w)[\alpha(1-e)e_n-(1-\varphi e)e_r]}{2\alpha(1-\alpha)}$. Q.E.D.

Proof of Lemma 5:

The proof process for this lemma is the same as that of Lemma 2, and therefore we omit it. Q.E.D.

Proof of Lemma 6:

The proof process for this lemma is the same as that of Lemma 3, and therefore we omit it. Q.E.D.

APPENDIX B

Proof of Proposition 1:

For a given e, the comparison of the production quantities of new products under two scenarios yields $q_{2n}^{SO*} - q_{2n}^{SN*} = \frac{[\mu - t(w)][(1-e)e_n - (1-\varphi e)e_r]}{2(1-\alpha)}$. Due to $o + t(w) < \mu$, we deduce $q_{2n}^{SO*} - q_{2n}^{SN*} > 0$, and then $q_{2n}^{SO*} > q_{2n}^{SN*}$ can be proved.

In a similar way, by comparing the yields of remanufactured products under two scenarios, we obtain $q_{2r}^{SO*} - q_{2r}^{SN*} = \frac{[t(w) - \mu][\alpha(1-e)e_n - (1-\varphi e)e_r]}{2\alpha(1-\alpha)}$. In conjunction with $o + t(w) < \mu$, we can conclude $q_{2r}^{SO*} - q_{2r}^{SN*} < 0$, and then $q_{2r}^{SO*} < q_{2r}^{SN*}$ can be proved. Q.E.D.

Proof of Proposition 2:

By substituting q_{2n}^{SO*} and q_{2n}^{SN*} into the inverse demand functions, we obtain $p_{2n}^{SO*} = \frac{a + t(w)(1-e)e_n}{2}$ and $p_{2n}^{SN*} = \frac{a + \mu(1-e)e_n}{2}$. By comparing these optimal selling prices, we obtain $p_{2n}^{SO*} - p_{2n}^{SN*} = \frac{[t(w) - \mu](1-e)e_n}{2}$. In conjunction with $o + t(w) < \mu$, we obtain $p_{2n}^{SO*} < p_{2n}^{SN*}$. Similarly, the selling prices of remanufactured products are determined as $p_{2r}^{SO*} = \frac{\alpha a + t(w)(1-\varphi e)e_r}{2}$ and $p_{2r}^{SN*} = \frac{\alpha a + \mu(1-\varphi e)e_r}{2}$, and by comparison, we deduce $p_{2r}^{SO*} < p_{2r}^{SN*}$. Q.E.D.

Proof of Proposition 3:

By combining Lemma 2 and Lemma 4, it is straightforward to conclude $q_{1n}^{SO*} = q_{1n}^{SN*}$. Q.E.D.

Proof of Proposition 4:

We prove this proposition by conducting the following analysis:

i) In the scenario without carbon options, the production quantity of new products in the first period is $q_{1n}^{SN*} = \frac{a - \mu(1-e)e_n - c\rho}{2}$. Correspondingly, in the absence of emission abatement strategies, the production quantity of new products is $\bar{q}_{1n}^{SN*} = \frac{a - \mu e_n - c\rho}{2}$. By comparison, we obtain $q_{1n}^{SN*} - \bar{q}_{1n}^{SN*} = \frac{\mu e e_n}{2}$, which leads to $q_{1n}^{SN*} > \bar{q}_{1n}^{SN*}$. Similarly, by comparing the optimal solutions for the production quantity in the first period with or without considering carbon emission reduction strategies in the carbon option scenario, we arrive at the same conclusion.

ii) In the scenario without carbon options, the production quantities of new products and remanufactured products in the second period are $q_{2n}^{SN*} = \frac{(1-\alpha)a - \mu[(1-e)e_n - (1-\varphi e)e_r]}{2(1-\alpha)}$ and $q_{2r}^{SN*} = \frac{\mu[\alpha(1-e)e_n - (1-\varphi e)e_r]}{2\alpha(1-\alpha)}$, respectively. Let $e = 0$, we can derive the optimal solutions for new product and remanufactured product production quantities without emission reduction, i.e., $\bar{q}_{2n}^{SN*} = \frac{(1-\alpha)a - \mu(e_n - e_r)}{2(1-\alpha)}$ and $\bar{q}_{2r}^{SN*} = \frac{\mu(\alpha e_n - e_r)}{2\alpha(1-\alpha)}$. By comparing the optimal solutions for the production quantity of new product with and without carbon emission reduction strategies, we obtain $q_{2n}^{SN*} - \bar{q}_{2n}^{SN*} = \frac{\mu e(e_n - \varphi e_r)}{2(1-\alpha)}$. Consequently, when $\varphi > \frac{e_n}{e_r}$, we obtain $q_{2n}^{SN*} < \bar{q}_{2n}^{SN*}$; when $\varphi < \frac{e_n}{e_r}$, we obtain $q_{2n}^{SN*} > \bar{q}_{2n}^{SN*}$. In a similar vein, by comparing the optimal solutions for the quantity of remanufactured products with and without carbon emission reduction strategies, we obtain $q_{2n}^{SN*} - \bar{q}_{2n}^{SN*} = \frac{\mu e(e_n - \varphi e_r)}{2(1-\alpha)}$. Therefore, when $\varphi > \frac{\alpha e_n}{e_r}$, we obtain $q_{2r}^{SN*} > \bar{q}_{2r}^{SN*}$; when $\varphi < \frac{\alpha e_n}{e_r}$, we obtain $q_{2r}^{SN*} < \bar{q}_{2r}^{SN*}$. Similarly, by comparing the optimal solutions for the production quantities in the second period with or without considering emission abatement strategies in the carbon option scenario, we arrive at the same conclusion. Q.E.D.

Proof of Proposition 5:

In the scenario without carbon options, the production quantities of new products in the first period with and without carbon emission reduction strategies are $p_{1n}^{SN*} = \frac{a + \mu(1-e)e_n + c\rho}{2}$ and $\bar{p}_{1n}^{SN*} = \frac{a + \mu e_n + c\rho}{2}$, respectively. By comparison, we obtain $p_{1n}^{SN*} - \bar{p}_{1n}^{SN*} = \frac{-\mu e e_n}{2}$, and consequently $p_{1n}^{SN*} < \bar{p}_{1n}^{SN*}$.

Similarly, by comparing the optimal solutions for the selling prices of new products in the first period with or without considering carbon emission reduction strategies in the carbon option scenario, we arrive at the same conclusion. Q.E.D.

Proof of Proposition 6:

By substituting the optimal solutions under both scenarios with and without carbon options into Eq. (13), we obtain $CS_1^{SN*} = CS_1^{SO*} = \frac{[a - \mu(1-e)e_n - c\rho]^2}{8}$, and consequently $CS_1^{SN*} = CS_1^{SO*}$.

Substituting the optimal solutions in the scenario without carbon options into Eq. (14), we obtain $CS_2^{SN*} = \frac{[a - \mu(1-e)e_n]^2}{8} + \frac{[\mu(\alpha(1-e)e_n - (1-\varphi e)e_r)]^2}{8\alpha(1-\alpha)}$. Calculating the first partial derivative of with CS_2^{SN*} respect to μ yields $\frac{\partial CS_2^{SN*}}{\partial \mu} = \frac{\mu[\alpha(1-e)e_n - (1-\varphi e)e_r]^2}{4\alpha(1-\alpha)} - \frac{[a - \mu(1-e)e_n](1-e)e_n}{4}$, and consequently $\frac{\partial^2 CS_2^{SN*}}{\partial \mu^2} > 0$. Let $\frac{\partial CS_2^{SN*}}{\partial \mu} = 0$, we obtain $(1-\varphi e)e_r = \alpha(1-e)e_n + \sqrt{\frac{\alpha(1-\alpha)[a - \mu(1-e)e_n](1-e)e_n}{\mu}}$ and $(1-\varphi e)e_r = \alpha(1-e)e_n - \sqrt{\frac{\alpha(1-\alpha)[a - \mu(1-e)e_n](1-e)e_n}{\mu}}$. Combining with $q_{2r} < \rho q_{1n}$, we get $\Omega > t_1^{SN}$, where $t_1^{SN} = \alpha - \rho\alpha(1-\alpha)[\frac{a - c\rho}{\mu(1-e)e_n} - 1]$. Therefore, when $t_1^{SN} < \Omega < \max(t_1^{SN}, t_2^{SN})$, we have $\frac{\partial CS_2^{SN*}}{\partial \mu} > 0$; when $\max(t_1^{SN}, t_2^{SN}) < \Omega$, we have $\frac{\partial CS_2^{SN*}}{\partial \mu} < 0$, where $t_2^{SN} = \alpha - \sqrt{\frac{\alpha(1-\alpha)[a - \mu(1-e)e_n]}{\mu(1-e)e_n}}$. Due to $o + t(w) < \mu$, when $t_1^{SN} < \Omega < \max(t_1^{SN}, t_2^{SN})$, we have $CS_2^{SN*} > CS_2^{SO*}$; when $\max(t_1^{SN}, t_2^{SN}) < \Omega$, we have $CS_2^{SN*} < CS_2^{SO*}$.

Because the consumer surplus in the first period is the same under the scenarios with and without carbon options, the relationship between the total consumer surplus over two periods in both scenarios is the same as the relationship with the consumer surplus in the second period. Q.E.D.

Proof of Proposition 7:

In the scenario without carbon options, the consumer surplus in the first period is $CS_1^{SN*} = \frac{[a - \mu(1-e)e_n - c\rho]^2}{8}$, and consequently $\frac{\partial CS_1^{SN*}}{\partial e} > 0$. When carbon emission reduction strategies are not considered, i.e., $e = 0$, we have $CS_1^{SN*} > C\bar{S}_1^{SN*}$.

Similarly, analyzing the consumer surplus in the first period in the carbon option scenario leads to the same conclusion. In the second period, when carbon options are not considered, due to $\frac{\partial CS_2^{SN*}}{\partial e} = \frac{(1-e)\mu e_n}{4\alpha(1-\alpha)} \left\{ \begin{array}{l} \frac{\alpha(1-\alpha)e_n}{\alpha e_n - \varphi e_r} \left[\frac{a}{(1-e)\mu e_n} - 1 \right] \\ -\alpha + \frac{(1-\varphi e)e_n}{(1-e)e_n} \end{array} \right\}$ and $\frac{(1-\varphi e)e_r}{(1-e)e_n} > \alpha - \rho\alpha(1-\alpha)[\frac{a-c\rho}{\mu(1-e)e_n} - 1]$ $(\Omega > t_1^{SN})$, it follows that $\frac{\partial CS_2^{SN*}}{\partial e} > 0$. Likewise, analyzing the consumer surplus in the second period in the carbon option scenario leads to the same conclusion. Furthermore, due to $CS_t^j = CS_1^j + CS_2^j$, we can obtain $CS_t^{j*} > C\bar{S}_t^{j*}$. Q.E.D.

Proof of Proposition 8:

Substituting the optimal production solutions under the scenarios with and without carbon options into Eq. (17), we obtain Γ^{SN*} and Γ^{SO*}. Consequently, through a comparison, we can get the following result:

$$\begin{array}{c} \Gamma^{SO*} - \Gamma^{SN*} = E_n[(1+y)(q_{1n}^{SO*} - q_{1n}^{SN*}) + q_{2n}^{SO*} - q_{2n}^{SN*} \\ +(x-y)(q_{2r}^{SO*} - q_{2r}^{SN*})] \end{array}.$$

In conjunction with Propositions 1 and 3, and $x < y$, we can obtain $\Gamma^{SO*} > \Gamma^{SN*}$. Q.E.D.

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