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RESEARCH ARTICLE

Does Investor Attention Matter for Carbon Market? New Insights From a Multi-Scale Quantile Causality Analysis

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ABSTRACT This paper firstly puts forward to explore the multi-scale, nonlinear quantile interactions between investor attention and the carbon market from January 2009 to December 2022. To achieve this goal, this study employs the adjusted CEEMDAN (complete ensemble empirical mode decomposition with adaptive noise) method, nonparametric quantile causality test, and quantile regression approach. The results illustrate that the Granger causality between investor attention and the carbon market is bidirectional and asymmetric across all scales. The explanatory power is stronger under extreme market conditions than in the case of normal markets at the short- and medium-term scales, whereas it is greater under normal markets in the long run. Furthermore, the marginal effect of investor attention on the carbon market is asymmetric across the whole quantiles. These evidences provide invaluable guidance for regulators to monitor risks in the carbon market and for investors to hedge risks aimed at different time horizons.

INDEX TERMS Carbon market, investor attention, GSVI, quantile causality test, multi-scale, quantile regression.

I. INTRODUCTION

With the rapid development of the global economy, greenhouse gas emissions have caused global climate change, posing a great threat to human survival and development. To cope with the frequent natural disasters brought about by climate change, carbon neutrality has become a global voice [1], even in the face of increasingly complex international political environments and increased downward pressure. By the end of December 2021, 136 countries had pledged to become carbon-neutral, including the European Union, China, the United Kingdom and Japan [2]. More importantly, a sound carbon market with a pricing function is essential for achieving carbon neutrality.

The carbon emissions trading market has become one of the most important carbon pricing mechanisms for the international community to mitigate climate change [3]. According to the *State and Trends of Carbon Pricing 2023*, 73 carbon pricing mechanisms are in operation worldwide. In 2022, the global carbon pricing revenue was approximately \$95 billion, of which the European Union Emissions Trading Scheme (EU ETS) contributed more than 40%. According to the European Commission [4], the EU ETS serves as the biggest carbon market globally and is the pioneering one. Moreover, the carbon emission trading price largely mirrors the price trends in the worldwide carbon market. Actually, influenced by multiple complex factors, carbon prices exhibit

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violent fluctuations. Since the start of 2022, the EU carbon prices have been fluctuating and increasing. Following the conflict between Russia and Ukraine, it dropped by 40% to 58 euros per ton. Nevertheless, the carbon price of the EU has risen by 22% since the beginning of this year, owing to increased natural gas consumption, changes in energy policy and other factors. As a result, the fluctuation of carbon prices does not only significantly affect the investment expectations in carbon markets, but also fundamentally affects the global carbon market development tendency. Under the carbon neutralization target, changes in carbon prices have attracted more and more attention from all sectors of society.

The massive trading volumes, significant price fluctuations, and stringent financial regulations have led to the EU carbon market being considered as a customary financial market for scrutinizing its financial features. In fact, influenced by many complex factors such as energy price, financial market, climate policy and uncertain events, carbon prices have conspicuous nonlinearity, uncertainty and space-time heterogeneity [5], [6], [7]. Simultaneously, with the gradual advancement of carbon market financialization and globalization, carbon market products have been added to the group of assets held by investors, thereby increasing the correlation among the carbon market, stock market, energy market, and others [8], [9]. This correlation, in turn, makes the volatility of the carbon market not only affected by fundamental factors, but also attracted increasing attention from investors. The rise of big data technology means that public concern has a crucial influence on carbon markets [10]. In contrast to the assumption of the rational asset pricing model, the attention of investors arising from behavior finance has a significant impact on the dynamics of carbon prices [11]. Nevertheless, there is limited research investigating the effect of investor attention on carbon prices, let alone the nonlinear and complex interactions between them.

There are various types of participants and investors in the actual carbon emissions trading market; therefore, significant heterogeneities and different behavioral cycles exist. For instance, institutional investors in the futures market, who solely engage in hedging, aim to only alleviate risks within the spot market. Thus, they seek a long-term equilibrium. Market regulators are primarily concerned with long-term carbon reduction policies. However, speculators tend to pay more attention to short-term returns. For regulated enterprises, in the long run, carbon dioxide released through industrial production has a demand-side effect on carbon prices [12]. From this point of view, the effects of investor attention on carbon prices differ across various frequency horizons. Hence, in order to depict a time-varying relationship pattern and attract investors throughout different investment periods, it is essential to conduct a time-frequency analysis.

Furthermore, in various market conditions (extreme values or tails), investors' attention may differ in terms of its direction, intensity, and content [13], [14]. For instance, in a bullish market, speculators are more interested in carbon futures prices than in a bearish market, as the former's higher price volatility signals more significant returns and vice versa. Thus, it is crucial to investigate the asymmetrical impact of investor attention on the carbon market under different market scenarios. Several scholars have examined how investors take note of stock returns [15], [16], crude oil price [17] and gold volatility [18] in diverse market scenarios. Nevertheless, based on our knowledge, there is no study that considers the asymmetries between investor attention and carbon price in their interactions.

This study aims to analyze the effects of investor attention on carbon prices and returns under different market conditions. This is accomplished by proposing a nonlinear multi-scale analysis approach that combines the improved complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN), nonparametric quantile causality test and quantile regression. The primary contributions of this study to the current literature are as follows:

① To estimate the interactions between investor attention and the carbon market for different frequency horizons, the CEEMDAN2014 (short for the improved CEEMDAN method) approach is adopted to realize the multi-scale decomposition of the original carbon price and investor attention signals. In comparison with other EMD-type models, such as the traditional EMD, EEMD and VMD (variational mode decomposition), the CEEMDAN model can reduce the mode reconstruction error and solve the mode mixing problem, thereby improving the noise reduction performance [19], [20]. However, the presence of residual noise in the modes and the presence of spurious modes remain two problems. In this context, the CEEMDAN2014 algorithm put forward by Colominas et al. [21] can obtain components with reduced noise, avoidance of spurious modes, and a clearer physical significance. ② The nonparametric quantile causality test proposed by Balcilar et al. [22] is implemented to capture the nonlinear causation between investor attention and the carbon market under various market conditions. It is meaningful to measure and explore the impact of investor attention on the carbon market only if a causal link exists between the two. In order to investigate the predictive ability of investor attention for carbon prices and returns under diverse market conditions, the overall distribution of these variables must be modeled instead of considering only their mean values. In contrast, the nonparametric causality-in-quantiles is robust in capturing the existence of causality at different quantiles of the variables distribution and can detect changes in causation immediately at each point of a given conditional distribution [14], [22]. Furthermore, we examine market conditions at different timescales through integrating the CEEMDAN2014 algorithm and the causality-in-quantiles approach. This process can not only bridge existing knowledge gaps but also facilitate the acquisition of more inclusive and accurate interactions between the two variables. ③ Quantile regression is employed to examine the impact of various market conditions. The objective is motivated by the limited information that a causal analysis may provide. The causal analysis may explain if investor attention can predict carbon prices or returns. However, investors and policymakers cannot deduce the exact features of the positive or negative relationships between variables using only the causality test. Thus, this paper employs quantiles regression to provide further insight into how investor attention might drive the carbon market and vice versa. Meanwhile, unlike the linear model, this approach ensures the robustness of the regression results given the biased variable distribution and avoids estimating biased parameters and giving poor investment decisions [17]. Furthermore, the CEEMDAN2014 method is utilized to consider different time-cycle factors. This enables the discovery of more complex relationships between diverse distributions of investor attention and the carbon market. ④ From a dynamic perspective, this study reveals the nonlinear interactions between investor attention and the carbon market at different timescales and different quantiles, which is conducive to comprehending European carbon market development and providing corresponding recommendations for market participants. Generally speaking, this article is the first to establish a direct correlation between the allocation of investor attention to carbon prices and returns, and so to explore the nonlinear quantile dependency over different time periods.

The remainder of this paper is structured into five parts. Section II provides a brief overview of the existing literature. Section III outlines the employed methodology. Section IV unveils the empirical findings and presents the utilized data. Finally, section V concludes the paper.

II. LITERATURE REVIEW

Various factors have been examined in previous studies to determine carbon prices, including macroeconomic situations [23], [24], [25], energy prices [5], [26], financial markets [27], [28], major events [29] and weather conditions [30], [31]. In addition to exploring influencing factors, numerous scholars have directed their attention towards investigating the volatility patterns and risk measurement associated with carbon prices. Their findings have uncovered several notable characteristics within the realm of carbon prices, including high non-stationarity, asymmetry, multiscale dynamics, fractal properties, and time-varying jump behaviors [32], [33], [34]. Based on carbon volatility features and its impact factors, many studies have concentrated on the carbon price forecasting models, which can be roughly divided into econometric methods [35], machine learning models [36], [37] and hybrid models [19], [20], [38]. Additionally, an expanding body of research delves into the interplay between the carbon market and various other markets, including but not limited to the stock market, energy market and commodity market [8], [39], [40], [41]. However, studies on the interactions have generated mixed and varied outcomes owing to data frequency, observation duration, and model configuration.

Regarding the correlation between the carbon market and other markets, the majority of studies have relied on econometric models, which can only yield outcomes within the time domain. Chevallier [42] employed the cointegration test, Granger causality test, and impulse response function analysis to reveal the long-term equilibrium relationship and interaction between EUA futures and spot prices, with the former demonstrating the price discovery function. Likewise, specific studies have employed these methods to investigate the nexus between the carbon market and energy or financial markets, documenting obvious two-way causative relations [43]. However, the connection between the carbon market and the energy and financial markets is confusing. Based on the dynamic conditional correlation GARCH (DCC-GARCH) model, Yu et al. [44] proved a positive time-varying volatility correlation between carbon prices and crude oil prices. In the same vein, Balcılar et al. [39] employed the Markov-switching DCC-GARCH approach to uncover that energy prices increase the carbon market fluctuations in regimes of low volatility. Different from this claim, Zhang and Sun [45] adopted the BEKK method to further established that there are no significant volatility spillovers between the carbon market and crude oil market. Furthermore, some scholars have applied the copula framework and quantile regression approach to capture tail dependence and connectedness across different quantiles. The results suggest that the uncertainty of financial markets and energy price fluctuations have asymmetric risk spillovers on carbon prices, which are more connected during extreme market movements [46], [47]. From the network microstructure perspective, Tan et al. [40] systematically examined the European carbon market spillover effects on energy and financial markets, and confirmed the stage difference in connections, with a more significant impact of financial risks. Also, the results of Xu et al. [48] presented a U-shaped trend in the different phases of the EU ETS.

In addition to the carbon market's interactions with other markets over time, some studies suggest exploring these connections across various frequency intervals. This methodology assumes that the carbon market's unions with other markets may differ over different time horizons. For example, Sadefo Kamdem et al. [49] analyzed how EUA and CER prices correlate during different periods by employing the wavelet method. There is substantial proof of their correlation at low frequencies, although the high- and mid-range frequencies show weakened linkages following the financial crisis. This is aligned with previous research on the association between carbon prices and other asset classes at different time scales. Using the EMD method alongside both linear and nonlinear Granger causality tests, scholars have concluded that the original data and the high frequency of EUA and crude oil prices are not correlated. However, there is evident mutual linear and nonlinear transmission in the intermediate term, along with a noteworthy reciprocal linear causation over the long haul [44]. In combination with the vector error correction model and MEMD approach, Zhu et al. [27] determined that electricity price impacts fluctuations in carbon prices from the short- to long-term, whereas coal, crude oil and natural gas only affect medium- and long-term variations. In order to reveal the directional connections at different time and frequency horizons, some studies adopted the spillover methods of Diebold and Yilmaz [50], as well as Barunik and Krehlik [51], which uncovered that the interactions between the carbon market and financial and energy markets are time-varying and frequency-varying [41], [52].

Behavioural finance has developed quickly in the past few years, highlighting the significance of investor attention in financial and commodity markets [53], [54], [55], [56]. Early literature typically used excess returns, trading volumes, and turnover rates as investor attention's proxy variables. However, these indicators do only display the trading features of the market and cannot directly reflect the strength and distribution of public attention. With the rapid development of the Internet, many studies have found that online big data based on keyword searches can accurately reflect changes in investor attention [57]. What's more, many works have applied the Google search volume (GSVI) or Baidu index as a surrogate variable for investor attention and discussed its relationship with other assets. For example, the research of Chen et al. [55] identified a nonlinear connection between investor attention and crude oil across different time frequencies. This was accomplished using the wavelet technique and a causality-in-quantiles approach, with the GSVI serving as the metric for measuring investor attention. The work of Gao et al. [58] explored the time-varying and asymmetric interactions between investor attention and Chinese green bonds and stocks markets using generalized forecast error variance decomposition model, and they took the Baidu index of "green finance" as the proxy for investor attention. Nevertheless, the involvement of investor attention in the emerging carbon market has been limited thus far. Recently, Zhang et al. [11] examined the non-negligible pricing factor of investor attention on carbon returns through VAR modeling analysis. Supporting this claim, Zhang and Xia [10] constructed a novel data-driven carbon price forecasting approach depending on Google Trends and online news data, resulting in a better performance when combined with the LSTM algorithm. However, these two studies have only concentrated on linear relations or discussed them in the time horizon, regardless of the interactions between carbon price and investor attention over the frequency interval as well as under different market conditions. Due to the inherent nonlinearity and unstationary features of carbon prices, Zhang et al. [56] adopted a nonparametric wavelet-based Granger causality test to investigate the connection between investor attention represented by the Baidu index and the carbon emission market in China. And they found there existed a bidirectional Granger causal relation between them, with the short-term cycle being the most common Granger causality. Nevertheless, there are still research gaps in the nonlinear interactions among the different market conditions between the carbon market and investor attention, which is crucial for market entities to make precise decisions.

To sum up, previous literature exemplifies multi-scale and inconclusive causalities between carbon and different markets, which provides valuable references for this paper. However, as far as we know, the potential influence of investor attention on the carbon market that analyzed through time-frequency techniques remains understudied. First of all, there exists inter-reactions between investor attention and the carbon market. On the one hand, in the process of promoting green emission reduction policy, carbon financial assets are increasingly favored by the markets. As one of the important reference factors in investors' investment processes, the fluctuation of carbon price and returns has a direct impact on investor attention. On the other hand, based on limited attention theory, investors' time and energy spent in the carbon market will gradually be transformed into actual capital investments. Therefore, investor attention has become the driver of their investment behavior, which has a critical impact on the carbon asset pricing, carbon price volatility and carbon returns. Secondly, as an important emerging factor of the carbon market, in varying market conditions and across diverse time horizons, the impact of investor' attention on carbon prices and investment returns can exhibit differences. Theoretically, on the one hand, it is necessary for market players to capture the relationship in extreme market environments between the two, as it could enable them to adopt better measures of risk aversion. On the other hand, since different market players differ in how they respond to and pay attention to information over different time horizons, understanding the frequency of dynamic interactions between investor attention and the carbon market is crucial. Motivated by this implication, our aim is to explore whether investor attention (carbon) is the Granger cause of carbon (investor attention). To achieve this, we conduct a blend of the CEEMDAN2014 technique and nonparametric quantile Granger causality examination. To obtain a more accurate assessment of the effects across various time and frequency ranges, we integrate CEEMDAN2014 and quantile regression approaches. This enabled us to compare the outcomes of different scales and quantiles. In this vein, this study addresses a gap in prior research.

III. METHODOLOGY

A. CONSTRUCTION OF INVESTOR ATTENTION INDEX

With the rapid development of the Internet, the main sources of information for investors have become online. However, only when it attracts investor attention will corresponding online search behavior occur. Meanwhile, data from Google Trends can effectively reflect the focus and degree of attention of global investors [17], [18]. Consistent with prior research, we choose the GSVI to represent investor attention, which can provide a highly diversified information set and avoid errors resulting from indirect proxies (such as trading volume and turnover rate) of investor attention [11]. Google Trends generates a trend chart with a standard quantitative range from 0 to 100, which indicates the ratio of actual searches to maximum searches for a given search term in a given time period and region. It should be noted that the value of GSVI doesn't represent absolute search volume, and 0 means that the search volume of keywords is lower than a certain threshold, whereas 100 indicates the highest point on a specified date [55].

Given different sensitivities of investors, their relevant search behaviors and keywords also vary. In order to reduce subjectivity and increase consistency with online search behavior, we first choose the keywords that are most likely to influence the changes in carbon price through expert review and discussion. Since there may be noise in these multiple carbon-related Google search terms, so that, we then employed the principal component analysis (PCA) method to reintegrate the information to construct a proxy for GSVI. PCA technique can convert the original search terms into new irrelevant variables, namely principal components, and each principal component is obtained as a linear combination of all search volumes. More importantly, a low-dimensional variable system can retain the complete or utmost information content of the variables, while reducing noise redundancy [59].

Supposing that x_{it} represents the i-th Google search term, and $\sum$ denotes the covariance matrix of variables along with their eigenvalues λ_i. Subsequently, we choose these eigenvectors e_i corresponding to eigenvalues exceeding 1, and whose variances add up to more than 80% as the principal components, as illustrated below.

$$\mathrm{PC}_{jt} = \sum\nolimits_{i=1}^{n} e_{ij} x_{it} \tag{1}$$

Here, PC_{jt} signifies the j-th principal component, while n refers to the overall count of the selected Google search terms.

Lastly, GSVI is constructed through assessing the proportionate contribution rate of the eigenvalue for each principal component. In order to standardize the original format, we refer to the research of Chen et al. [55] and normalize the GSVI and multiply it by 100.

$$\mathrm{GSVI}_t^* = \sum\nolimits_{j=1}^{m} \frac{\omega_j}{\sum_j j} \mathrm{PC}_{jt} \tag{2}$$

$$\mathrm{GSVI}_t = \frac{\mathrm{GSVI}_t^* - min\mathrm{GSVI}_t^*}{max\mathrm{GSVI}_t^* - min\mathrm{GSVI}_t^*} \times 100 \tag{3}$$

where ω_j is the j-th selected principal component eigenvalue and GSVI_t represents investor attention on the carbon market.

B. CEEMDAN2014 ALGORITHM

In fact, there are various market participants and investors in the carbon market, which may affect the interrelationships between investor attention and the carbon market in different time and frequency domains. Motivated by this, we utilize the decomposition method to decompose variables into various time scales, and then explore the time-frequency dynamic correlations between them. As for the frequency decomposition method, the CEEMDAN decomposition technology carried out by Torres et al. [60] has been proven to be an important improvement over EEMD. The CEEMDAN approach can solve the problem of matrix alignment and achieve negligible reconstruction error. Despite that, CEEMDAN still requires improvement, as its modes contain residual noise and there are spurious modes in the decomposition stage. Taking into account these two problems, this paper uses the CEEMDAN2014 method carried out by Colominas et al. [21] to decompose the carbon price and investor attention signals. The core of the CEEMDAN2014 model is replacing the estimation of modes with the estimation of local means. This replacement decreased the amount of noise present in the modes. In particular, the CEEMDAN2014 decomposition process does not directly utilize white noise but instead uses the mode obtained by EMD, so to reduce the overlapping problem. The CEEMDAN2014 algorithm is operated as follows:

Step 1. For every $i = 1, \ldots, N$, generate $x^{(i)}(t) = x(t) + \beta_0 E_1(w^{(i)}(t))$. After that, decompose each $x^{(i)}(t)$ by EMD, and estimate the local means of the realization to obtain the initial residue $r_1(t)$:

$$r_1(t) = \langle M(x^{(i)}(t))\rangle \tag{4}$$

where $x(t)$ represents the original carbon price and returns, or investor attention. The operator $E_k(\cdot)$ generates the k-th mode that has been obtained through EMD and $w^{(i)}$ refers to the realization of white Gaussian noise characterized by a means of 0 and a variance of 1. $M(\cdot)$ represents the signal's local mean that is applied, while $\langle\cdot\rangle$ refers to the averaging action throughout the realizations.

Step 2. At the first order ($k = 1$), calculate the first mode $d_1(t)$ using the following model:

$$d_1(t) = x(t) - r_1(t) \tag{5}$$

By doing so, a decrease in the level of noise can be achieved, compared with the first EEMD and CEEMDAN modes.

Step 3. In order to reduce the overlapping demonstrated in the formulation of CEEMDAN, we make no direct use of white noise but $E_k(w^{(i)}(t)$ to extract the k-th mode. And then, calculate the second residue $r_2(t)$ as the average of the realizations of local means:

$$r_2(t) = \left\langle M(r_1(t) + \beta_1 E_2(w^{(i)}(t)))\right\rangle \tag{6}$$

At the same time, get the second mode $d_2(t)$ from the following formula:

$$d_2(t) = r_1(t) - r_2(t) = r_1(t) - \langle M(r_1(t) + \beta_1 E_2(w^{(i)}(t)))\rangle \tag{7}$$

Step 4. For $k = 3, \ldots, K$, the following computation is used to estimate the k-th residue $r_k(t)$:

$$r_k(t) = \langle M(r_{k-1}(t) + \beta_{k-1} E_k(w^{(i)}(t)))\rangle \tag{8}$$

Step 5. Similar to Step 3, calculate the k-th mode $d_k(t)$:

$$d_k(t) = r_{k-1}(t) - r_k(t) \tag{9}$$

Step 6. Proceed to Step 4 for another k.

Continue iterating steps 4 - 6 until the remainder can no longer be decomposed using the EMD technique. Observe that, by the construction of CEEMDAN2014, the original signal can be expressed as:

$$x(t) = \sum_{k=1}^{K} d_k(t) + r_k(t) \tag{10}$$

K represents the total number of modes. Consistent with Colominas et al. [21], the constants $\beta_k = 0.2std(r_k(t))$ are determined to achieve the desired signal-to-noise ratio between the added noise and residue.

C. NONPARAMETRIC CAUSALITY- IN- QUANTILES TEST

Using the nonparametric causality-in-quantiles approach developed by Balcilar et al. [22], we examined the intricate relationship between investor attention and the carbon market under various market conditions. As financial data have frequently been observed to exhibit nonlinearity, structural changes and regime shifts, different nonlinear causality test techniques have been constructed to address these unfavorable statistical properties. Among which, the causality-in-quantiles test is advantageous in tackling broad time series dependencies and is robust against functional misspecification errors [41].

For the component of the k-th mode, assuming that a_t refers to investor attention and b_t denotes the carbon price or return, we say the lag vector of $\{b_{t-1}, \ldots, b_{t-p}; a_{t-1}, \ldots, a_{t-p}\}$ prevents a_t from causing b_t at the θ-thquantile if

$$\begin{aligned} &Q_\theta \left\{b_t | b_{t-1}, \ldots, b_{t-p}; a_{t-1}, \ldots, a_{t-p}\right\} \\ &= Q_\theta \left\{b_t | b_{t-1}, \ldots, b_{t-p}\right\} \end{aligned} \tag{11}$$

However, a_t is a prima facie cause in the θ-thquantile of b_t concerning $\{b_{t-1}, \ldots, b_{t-p}; a_{t-1}, \ldots, a_{t-p}\}$ if

$$\begin{aligned} &Q_\theta \left\{b_t | b_{t-1}, \ldots, b_{t-p}; a_{t-1}, \ldots, a_{t-p}\right\} \\ &\neq Q_\theta \left\{b_t | b_{t-1}, \ldots, b_{t-p}\right\} \end{aligned} \tag{12}$$

where $Q_\theta \{b_t|\cdot\}$ represents the θ-th quantile of b_t depending on t and $0 < \theta < 1$. Let $\boldsymbol{B}_{t-1} \equiv (b_{t-1}, \ldots, b_{t-p})$, $\boldsymbol{A}_{t-1} \equiv (a_{t-1}, \ldots, a_{t-p})$, and $\boldsymbol{C}_{t-1} = (\boldsymbol{A}_{t-1}, \boldsymbol{B}_{t-1})$, which denote a past set of information. Hence, $F_{b_t|\boldsymbol{C}_{t-1}}(b_t|\boldsymbol{C}_{t-1})$ and $F_{b_t|\boldsymbol{B}_{t-1}}(b_t|\boldsymbol{B}_{t-1})$ indicate the conditional distribution functions of b_t given $\boldsymbol{C}_{t-1}$ and $\boldsymbol{B}_{t-1}$, respectively. In addition, for almost all $\boldsymbol{C}_{t-1}$, $F_{b_t|\boldsymbol{B}_{t-1}}(b_t|\boldsymbol{B}_{t-1})$ is supposed to be absolutely continuous in b_t. If we further define that $Q_\theta(\boldsymbol{C}_{t-1}) \equiv Q_\theta(b_t|\boldsymbol{C}_{t-1})$ and $Q_\theta(\boldsymbol{B}_{t-1}) \equiv Q_\theta(b_t|\boldsymbol{B}_{t-1})$, we can then obtain that $F_{b_t|\boldsymbol{C}_{t-1}}\{Q_\theta(b_t|\boldsymbol{C}_{t-1})\} = \theta$ with a probability of one. Therefore, the assumptions to be tested for definitions (11) and (12) are:

$$H_0 : P\{F_{b_t|\boldsymbol{C}_{t-1}}\{Q_\theta(b_t|\boldsymbol{C}_{t-1})\} = \theta\} = 1 \tag{13}$$

$$H_1 : P\{F_{b_t|\boldsymbol{C}_{t-1}}\{Q_\theta(b_t|\boldsymbol{C}_{t-1})\} = \theta\} < 1 \tag{14}$$

The null hypothesis, defined in Equation (13), will be true only if $F[1\{b_t \leq Q_\theta(\boldsymbol{B}_{t-1})|\boldsymbol{C}_{t-1}\}] = \theta$ or, likewise, $1\{b_t \leq Q_\theta(\boldsymbol{B}_{t-1})\} = \theta + \varepsilon_t$, where $1\{\cdot\}$ is an indicator function and the regression error ε_t satisfies $F[\varepsilon_t|\boldsymbol{C}_{t-1}] = 0$. Conversely, the distance measure is defined as the following form:

$$J = E[\{F_{b_t|\boldsymbol{C}_{t-1}}\{Q_\theta(b_t|\boldsymbol{C}_{t-1})\} - \theta\}^2 f_{\boldsymbol{C}}(\boldsymbol{C}_{t-1}) \tag{15}$$

where $f_{\boldsymbol{C}}(\boldsymbol{C}_{t-1})$ represents the marginal density function of $\boldsymbol{C}_{t-1}$. It is worth noting that only if the null in Equation (13) is true, $J = 0$; otherwise, $J > 0$ will occur under the alternative hypothesis in the Equation of (14). Furthermore, the feasible kernel-based test statistic for J is founded on the following method:

$$\hat{J}_T = \frac{1}{T(t-1)h^{2m}} \sum_{t=m+1}^{T} \sum_{s=m+1, s\neq t}^{T} K_{ts}\hat{\varepsilon}_t\hat{\varepsilon}_s \tag{16}$$

where $K_{ts} = K[(\boldsymbol{C}_{t-1} - \boldsymbol{C}_{s-1})/h]$ refers to the kernel function using a bandwidth parameter h, T represents the sample size, m refers to the lag order. And $\hat{\varepsilon}_t = 1\{b_t \leq \hat{Q}_\theta(\boldsymbol{B}_{t-1})\} - \theta$ defines the estimation of the regression error, where $\hat{Q}_\theta(\boldsymbol{B}_{t-1})$ is estimated by $\hat{F}^{-1}_{b_t|\boldsymbol{B}_{t-1}}(\theta|\boldsymbol{B}_{t-1})$. The Nadaraya-Watson kernel estimator represented as $\hat{F}^{-1}_{b_t|\boldsymbol{B}_{t-1}}(b_t|\boldsymbol{B}_{t-1})$ is calculated by:

$$\hat{F}^{-1}_{b_t|\boldsymbol{B}_{t-1}}(b_t|\boldsymbol{B}_{t-1}) = \frac{\sum_{s=m+1,s\neq t}^{T} L((b_{t-1}b_{t-s})/h)1(b_s \leq b_t)}{\sum_{s=m+1,s\neq t}^{T} L((b_{t-1}b_{t-s})/h)} \tag{17}$$

with $L(\cdot)$ denoting the kernel function and h representing the bandwidth.

Following Balcilar et al. [22], we use the least-squares cross-validation technique to choose the most appropriate bandwidth, and the Gaussian-type kernels are utilized for $K(\cdot)$ and $L(\cdot)$. In a VAR model consisting of investor attention, carbon prices and returns within the k-th frequency domain, we employ the Schwarz information criterion to ascertain the appropriate lag order.

D. THE PROPOSED MULTI-SCALE CAUSALITIES BETWEEN INVESTOR ATTENTION AND CARBON MARKET

In this study, a CEEMDAN2014-based causality-in-quantiles test approach is suggested to explore the dynamic interactions between investor attention and the carbon market at different timescales and conditions, as displayed in Figure 1. This method includes four main steps: ① Multi-scale decomposition. The investor attention and carbon prices or returns are decomposed into several IMFs by the CEEMDAN2014 algorithm respectively, from high to low frequency. ② Nonparametric tests for quantile causality. The nonparametric causality- in-quantile tests were performed to investigate the interactions between the IMFs of each pair. ③ Quantile regression. We resort to the concept of quantile regression to assess the impact of investor attention (carbon market) on carbon prices or returns (investor attention). ④ Dynamic analysis. We analyze the empirical results of different quantiles as well as different time horizons, so to obtain dynamic interactions and contrastive analysis.

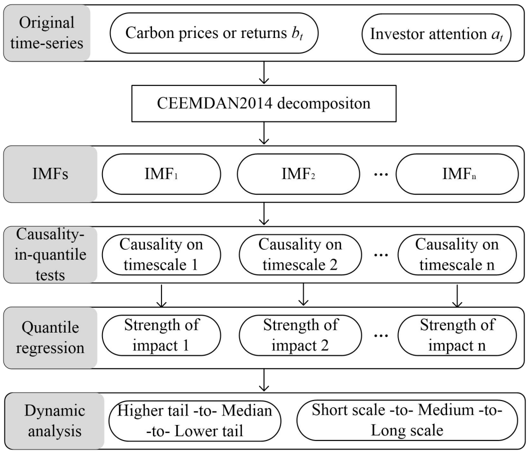

FIGURE 1. Framework of the proposed multi-scale quantile causalities analysis.

IV. EMPIRICAL ANALYSIS

A. DATA DESCRIPTION

Considering the mature price mechanism of the carbon futures market and active trading volume, this paper concentrates on the most important and largest carbon market, namely, the EU ETS carbon market. EUA futures prices (EUAP) and returns (EUAR) are employed because of their higher quality and practicability for market players [8], [19]. The logarithmic difference of EUA futures prices are used to obtain the respective EUA returns. The sample data for this study consists of 727 weekly[1] observations collected between 4th January 2009 and 4th December 2022. Our empirical investigation omits Phase I of the EU ETS (2005-2007) since it was still in a trial-running phase and was not mature enough. Notably, this sample range includes the global financial crisis, political uncertainties over the post-Kyoto period, the outbreak of COVID-19 and other important events and their impact on the carbon market, which implies the importance of exploring extreme impacts. The EUA prices are from the European Energy Exchange (https://www.eex.com/en/).

Internet search is a link between information resources and user needs. Only when investor attention is aroused will corresponding online search behaviors occur. Specially, Google is the biggest and most widely used search engine globally, which can effectively reflect investor attention to the carbon market. In line with previous studies [10], [11], this paper selected the GSVI from Google Trends (http://www.Google.com/trends) as the proxy variable for the widespread attention of investors on the carbon market. In view of the background of carbon emission reduction and associated words in Google Correlate, after several rounds of discussion, expert review and removing insufficient search volume, we select 19 keywords closely associated with the carbon market, namely, "EU ETS", "carbon market", "carbon emission", "carbon trading", "emission trading", "carbon footprint", "Kyoto protocol", "carbon neutral", "global warming", "climate change", "green development", "green economy", "sustainable development", "green investment", "emission reduction", "low carbon", "environmental protection", "carbon finance", "UNFCCC". Among them, some terms are directly related to the supply and demand of carbon market, such as "carbon trading", "emission trading" and "carbon footprint", and the terms "EU ETS", "Kyoto protocol" and "UNFCCC" are related to carbon trading policies and platforms, and others are in connection with low carbon development environment, such as "sustainable development", "climate change" and "green economy". Since the initial intention to establish carbon market is to furnish a market regulation means for emission reduction, and environmental issues are the core targets of the carbon market, with a typical policy-driven feature. Therefore, the Google search index we selected could reflect investor attention in the carbon market more directly and accurately. With a global region and a large time span, the GSVI time series viewed were weekly. And then, in order to retain as much information as possible about all entries and reduce noise, the PCA method was applied to extract equivalent information from these key search terms. Table 1 displays the components derived through the PCA approach.

[1]Due to the GSVI data availability, and to strike a balance between utilizing potentially noisy daily data and having a relatively short time span with monthly data, we choose weekly data of EUA futures prices.

TABLE 1. Eigenvalues and explained variances of the principal components.

Component	Eigenvalue	Proportion explained	Cumulative proportion
1	16.7461	0.8814	0.8814
2	0.3816	0.0201	0.9015
3	0.2533	0.0133	0.9148
4	0.2204	0.0116	0.9264
5	0.2172	0.0114	0.9378
6	0.1720	0.0091	0.9469
7	0.1546	0.0081	0.9550
8	0.1476	0.0078	0.9628
9	0.1313	0.0069	0.9697
10	0.1208	0.0063	0.9760
11	0.0981	0.0052	0.9812
12	0.0833	0.0044	0.9856
13	0.0713	0.0038	0.9894
14	0.0593	0.0031	0.9925
15	0.0421	0.0022	0.9947
16	0.0303	0.0016	0.9963
17	0.0271	0.0014	0.9977
18	0.0239	0.0013	0.9990
19	0.0196	0.0010	1.0000

TABLE 2. Descriptive statistics of variables.

Variable	Mean	Maximum	Minimum	Std.Dev.	Skewness	Kurtosis	Jarque-Bera	$Z_{BDS(10)}$
EUAP	20.3368	98.0100	3.9200	20.9741	2.0375	6.3093	834.7259[a]	106.4965[a]
EUAR	0.1021	10.6639	-16.8443	2.8941	-0.7554	7.1120	581.32[a]	10.4723[a]
GSVI	42.0087	100.0000	0.0000	16.1639	0.5061	3.5615	40.585[a]	65.8389[a]

Note: [a] Significance under the 1% level.

Based on Equation (1), we get the principle components. As we can see in Table 1, only the first component PC_{1t} with eigenvalue $\omega_1 = 16.7461$ greater than 1 can explain approximately 88% of the information. Therefore, we select the first component PC_{1t} as the main component, and then obtain the GSVI through applying formulas (2) and (3).

Table 2 presents the basic descriptive characteristics of the carbon prices, returns and investor attention. As can be seen, the wide range of carbon prices and returns demonstrates a high risk of carbon emission trading, with a weak positive value for average carbon returns. Moreover, the carbon prices skewness is above 0, and the kurtosis is higher than 3, suggesting a right-skewed distribution with a high peak. However, the carbon futures returns with skewness less than 0 are the opposite, showing a leftward skew. Furthermore, the significance of the JB statistics shows that carbon prices and returns and investor attention do not obey the normal distribution. Differently, the skewness of investor attention is slightly greater than 0 with the kurtosis close to 3. As for the nonlinear test, the respective p-values of the Z-statistics of all three variables are less than 0.01, demonstrating the nonlinear nature of these signals. This suggests that Granger causality tests are likely to suffer from misspecification when based on a linear framework.

Figure 2 illustrates time series trend graphs for each variable. As seen from Figure 2, there is some resemblance period for EUA prices and returns towards the conclusion of Phase II and during the outbreak of COVID-19, demonstrating the sensitivity of the carbon market to the external environment. However, the carbon price shows a rising trend due to the global movement towards achieving "carbon neutrality", with the pricing mechanism of the carbon market greatly improved. It is worth mentioning that the GSVI has shown a tendency to increase to a certain extent whenever carbon prices or returns experience a significant drop. This finding suggests that investor attention may have an adverse feedback impact on the carbon market.

B. MODE CHARACTERISTICS ANALYSIS

In order to explore the impact of investor attention on carbon prices and returns in different frequency domains, we conduct the CEEMDAN2014 method to decompose the original signals and analyze the suggested sequences. Figure 3 - 5 show the volatility trends of each mode component. As illustrated in Figures 3-5, with the signals changing from IMF1 to IMF7 or IMF8 and the residuals, the frequency changes from strong to weak.

In terms of the timescales of each mode reported in Table 3, these IMFs can be classified into three groups from high- to medium- and low-frequencies. IMF1, demonstrating a time period within one month, reflects short-term fluctuations owing to random variables, such as market noise and speculative trading [8], [27]. IMF2 to IMF5, with a timescale from one month to one year, interpret the influences of external heterogeneous environments on the carbon market and investors, such as economic crises, climate negotiations and extreme events. The remaining modes and residuals with a time horizon greater than a year, describe the long-term trends of these signals. Considering the frequency period and fluctuation of each component, we superimposed three groups of signals respectively. Therefore, quantile Granger causality is conducted at three different timescales, namely short-scale, medium-scale and long-scale, to discover the potential nonlinear Granger causality relation between investor attention and the carbon market. To conserve space, the reconstructed figures of the decomposition modes (Figure 9 to Figure 10) are supplied in the Appendix.

C. MULTI-SCALE QUANTILE GRANGER CAUSALITY ANALYSIS

Through a mode characteristics analysis, we find that the carbon market and investor attention have different features in different frequency domains. As mentioned, previous literature [11] relying on the mean linear Granger test illustrated that investor attention is the Granger cause of carbon returns, and short-term cycle length is the most common causality [56]. However, no studies have explored the dynamic nexus between the two factors throughout the entire time period while also taking into account different market conditions. Given the non-stationarity and nonlinearity characteristics of the decomposed components, thus, this article develops a method for quantile Granger testing rather than the traditional linear causality test on the basis of CEEMDAN2014 decomposition. This method is employed to examine the quantile causality of investor attention and the carbon market across various time horizons, and so to uncover possible nonlinear Granger causality.

In this context, we chose 19 quantiles (from 0.05 to 0.95, with an interval of 0.05). In line with the work of Chen et al. [55], the quantiles distributions are divided into three market

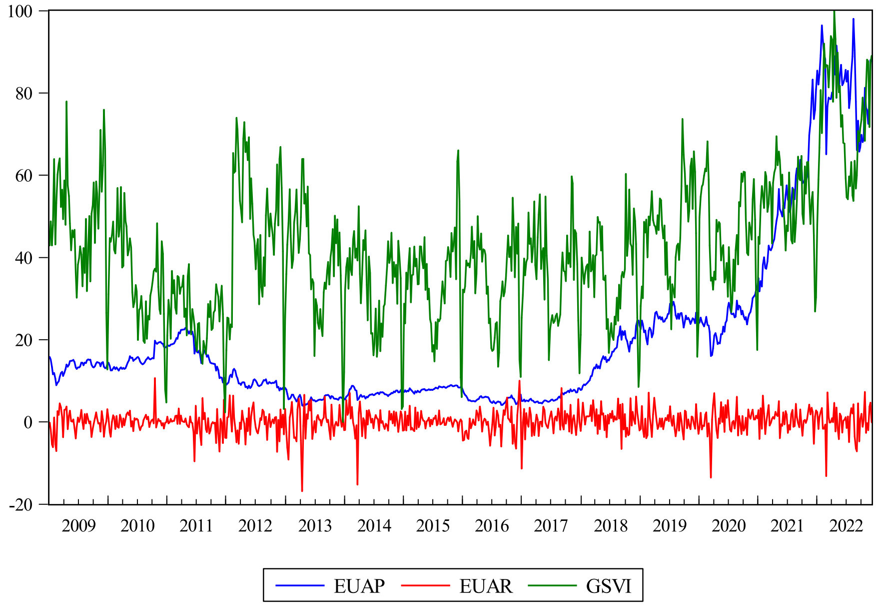

FIGURE 2. Time series of carbon prices and returns with investor attention.

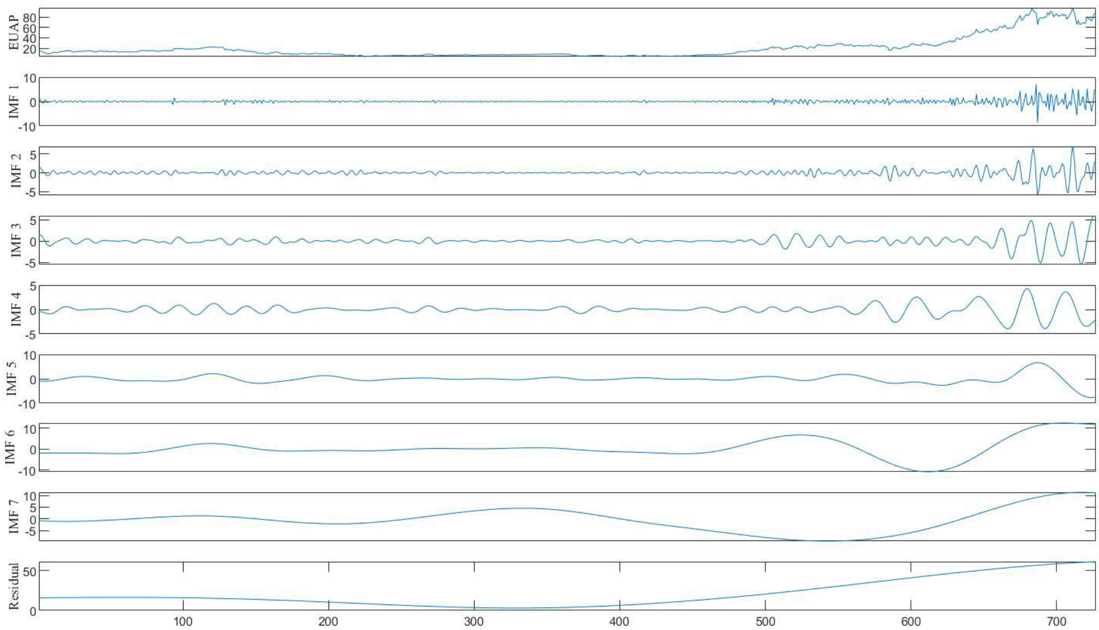

FIGURE 3. Decomposition results of original carbon price.

conditions: bear market (0.05-0.25), normal market (0.3-0.7), and bull market (0.75-0.95). Subsequently, Figure 6 and 7 visually represent the degree of causality and predictability between investor attention and the carbon market across different quantiles, where (a) - (c) correspond to the short, medium and long timescales, respectively.

As illustrated in Figure 6, GSVI is basically the Granger cause of carbon prices and returns across different frequency scales as well as quantiles, whereas the predictive strength is not the same. The curves in Figure 6 (a) and (b) are two-humped, indicating that the degree of causality in the short and medium terms intensifies towards the bear market or bull market conditions; however, this is opposite to that of Figure 6 (c) with one-humped curves over the long timescale. The implication is that the causality from investor attention to the carbon market is strong around the median of the carbon price and returns distribution, but weakens towards the quantiles far from the medium. In contrast, the long-term causality from GSVI to the carbon market is larger and more significant. That is to say, the causality from investor

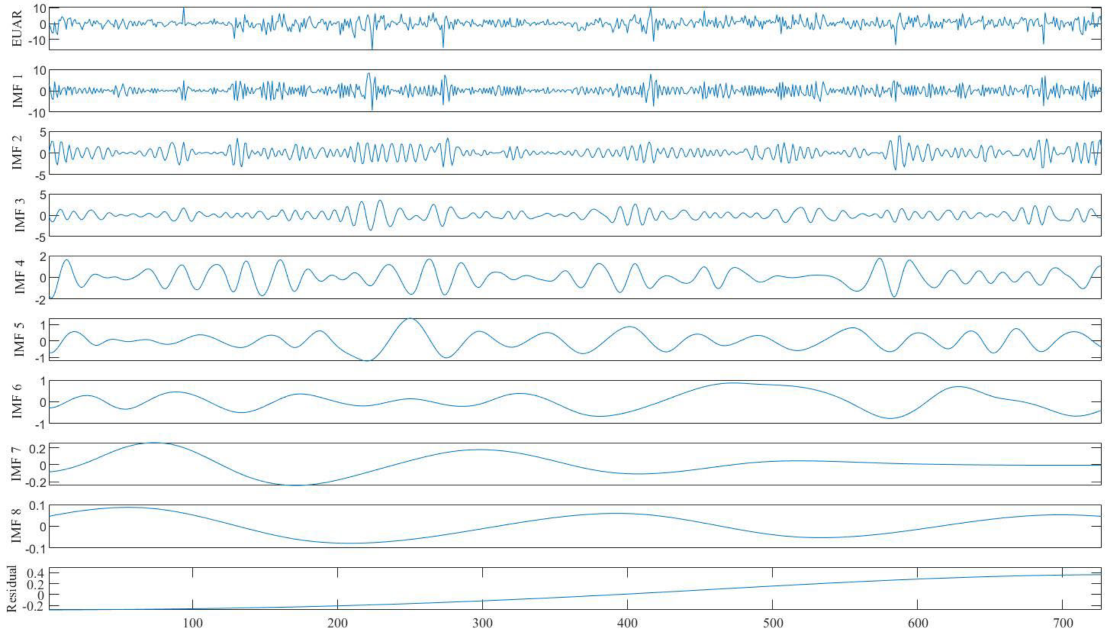

FIGURE 4. Decomposition results of carbon futures returns.

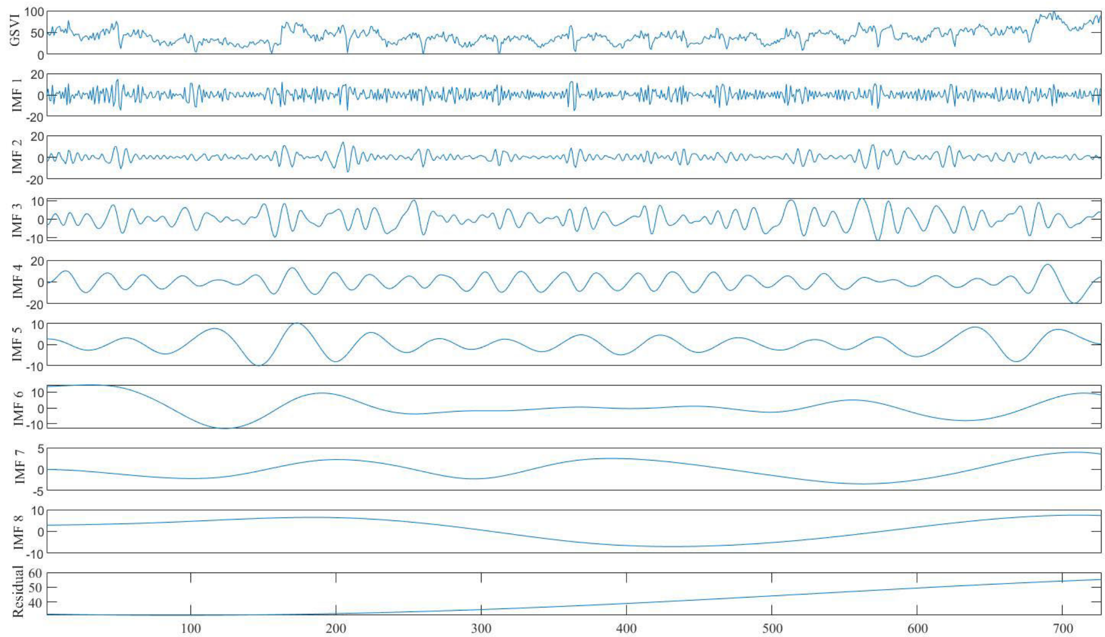

FIGURE 5. Decomposition results of investor attention.

TABLE 3. Timescale of the correspondence IMF modes (week).

Variable	IMF_1	IMF_2	IMF_3	IMF_4	IMF_5	IMF_6	IMF_7	IMF_8
EUAP	2.96	6.16	11.73	25.07	51.93	103.86	242.33	——
EUAR	2.81	5.68	10.69	22.72	42.76	103.86	242.33	363.5
GSVI	2.81	5.86	12.75	26.93	51.93	103.86	242.33	363.5

attention to the carbon market is asymmetric, which reaffirms the utilization of the quantile Granger causality test over the conventional linear test. ① Regarding the short-term scale perspective, investor attention is considered as a Granger cause of carbon prices at the 10% level of significance for most quantiles except for the lowest, median, and highest tails (i.e., $\theta = 0.05, 0.4, 0.45, 0.5, 0.95$). Moreover, carbon market investors pay more attention to the returns, as the causality of GSVI on carbon returns is stronger and more profound for almost all quantiles at the 5% significance level other than the lowest tails ($\theta = 0.05$). This finding indicates that, at the short term, the increase in investors attention promotes their understanding of carbon market information, enhances the demand for carbon trading products, and drives changes in carbon prices and returns, especially under bearish and bullish market conditions. But, investor attention cannot predict carbon price changs in a normal market when the carbon market is stable. ② However, in the case of the medium scale, little evidence of Granger causality from GSVI to carbon returns is observed at the 5% level for $\theta = 0.15, 0.2, 0.45$, and 0.75. For carbon prices, Granger causality exists for almost all distributions apart from the extreme bearish and bullish market conditions ($\theta = 0.05, 0.95$). One possible reason is that, due to the influenced of external heterogeneous environments and major events, the causality from GSVI to the carbon market is relatively complex among the medium scales. What's more, the time-lag effect of the external events and information acquisition may amplify the spillover effect from investor attention to the carbon market. Another plausible explanation may be attributed to the nonlinear and multi-scale characteristics of the causal connection between investor attention and carbon market. As shown in Table 5, the GSVI Granger causes carbon prices and returns across the scales of IMF3 to IMF5, and the predictive power become more and more greater. But for IMF2, causality is relatively weak except for a few moderate market phases. Therefore, at the time period of about 6 weeks, market players could not refer to investor attention to predict changes in carbon prices under normal market conditions or when the carbon futures return is extraordinarily low. ③ While in view of the long-term scale, GSVI to the cause of carbon prices and returns are basically equal, and both are significant and profound at the level of 1% across all quantiles and present an inverted U-shape. This indicates that the long-term causality is strong around the medium term and weakens towards the far tails of the carbon prices and returns distribution. In other words, investors place greater emphasis on carbon assets under normal market conditions in the long run. Possible explanation could be that carbon emissions trading is dominated by hedgers or emission control companies, who typically engage in long-term trading. However, speculators with low market share tend to make short-term investment decisions, which have a small impact on carbon prices and returns. In addition, the hump-shaped causality curve again verifies the asymmetric causal relationship between the GSVI and carbon market.

On the other hand, as seen from Figure 7, fluctuations in carbon prices and returns appear to be the dominant Granger factor in relation to investor attention, as observed across all frequency levels and on most quantiles, except for the extreme downturn/upturn markets at short timescales and

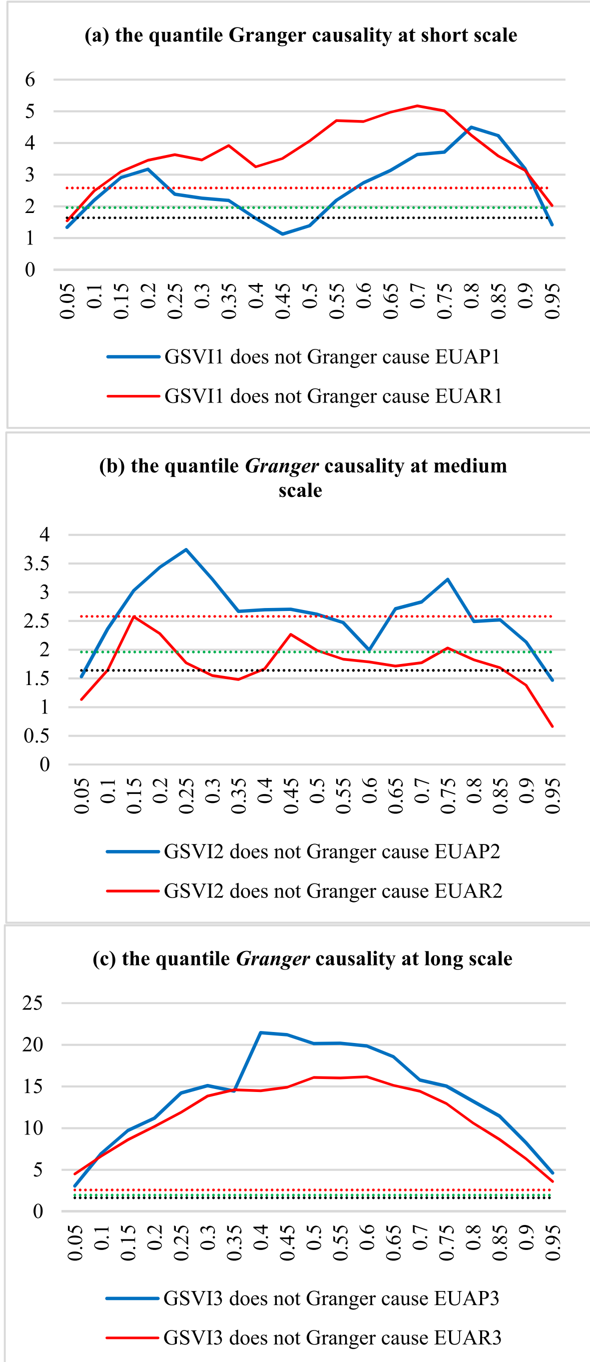

FIGURE 6. **Causality-in-quantiles test results for carbon market.**
Note: **The red, green and black dotted lines represent the critical values at the significance level of 1%, 5% and 10%, respectively. GSVI1, GSVI2 and GSVI3 are the reconstructed short-term, medium-term and long-term series respectively, and are the same as EUAP and EUAR.**

medium horizons. At the same time, the causality from carbon market shocks to investor attention intensifies over time increase.

① In the short term, carbon prices and returns show a Granger causality relationship with the GSVI at a significance level of 10% for all quantiles, apart from the lowest and highest tails ($\theta = 0.05, 0.95$). Moreover, the curve is smoother under normal market conditions, which implies that the explanatory power of carbon prices and returns does not differ greatly. Nevertheless, the curve becomes steeper under bullish market conditions, suggesting a stronger impact on investor attention. ② With regard to the medium-term scale, Granger causality indicates that there is a significant level of 5% for the flow from carbon prices and returns to GSVI, except for the outer tails of the conditional distribution ($\theta =$ 0.05, 0.95). Specifically, Granger causality is stronger under normality markets and reaches a maximum for $\theta = 0.6$. As we know, carbon futures returns are more symbolic of the risk of the carbon market than carbon prices. When impacted by external extreme events, the changes in carbon returns can attract more attention from investors, thus increasing their search volume online and showing greater prediction power than carbon prices. In this sense, investors should focus more on fluctuations in carbon returns. ③ In the case of the long-term timescale, Granger causality becomes stronger and also exhibits an inverted U-shaped curve. The two curves are similar and both are significant at the level of 1% under any market conditions. Therefore, we can deduce that the variations in carbon prices and returns easily affect the decrease or increase in the GSVI, which is mainly spread across different frequency scales under normal market conditions and weakens towards extremely bearish or bullish market phases. The reason might be that the variations in investor attention under extreme market conditions are remarkably affected by major events and emergencies besides carbon market volatility, thus demonstrating a weaker causality from carbon prices and returns around the quantiles far from the medium. Similarly, the hump-shaped causality curve affirms the asymmetric causal relation from carbon market to the attention of investors.

Accordingly, combined with Figure 6 and Figure 7, the results validate the hypothetical claims of our research that there are nonlinear and bi-directional causal connections between investor attention and the carbon market. When carbon prices and returns change, market players will take the initiative to select suitable keywords for Internet searches relying on recent information on the carbon market, further influencing the trend of the Google search index. Before making investment decisions and risk management decisions, carbon market participants will also take the initiative to care about online information, and changes in Google search data provide a reference for decision-making behavior, which will then affect the variations in carbon asset prices. Because different market players differ in how they respond to and pay attention to information over different time horizons, the interactions between them are multi-scaled, which falls in line with the results of Zhang et al. [56]. However, they did not find that the Granger causal relation behaved differently around different market conditions. As revealed by our results from the multi-scale quantile Granger causality tests, the hump-shaped causality curves imply that the long-term causality is stronger around normal market conditions and weaker towards bullish and bearish markets. Besides, the

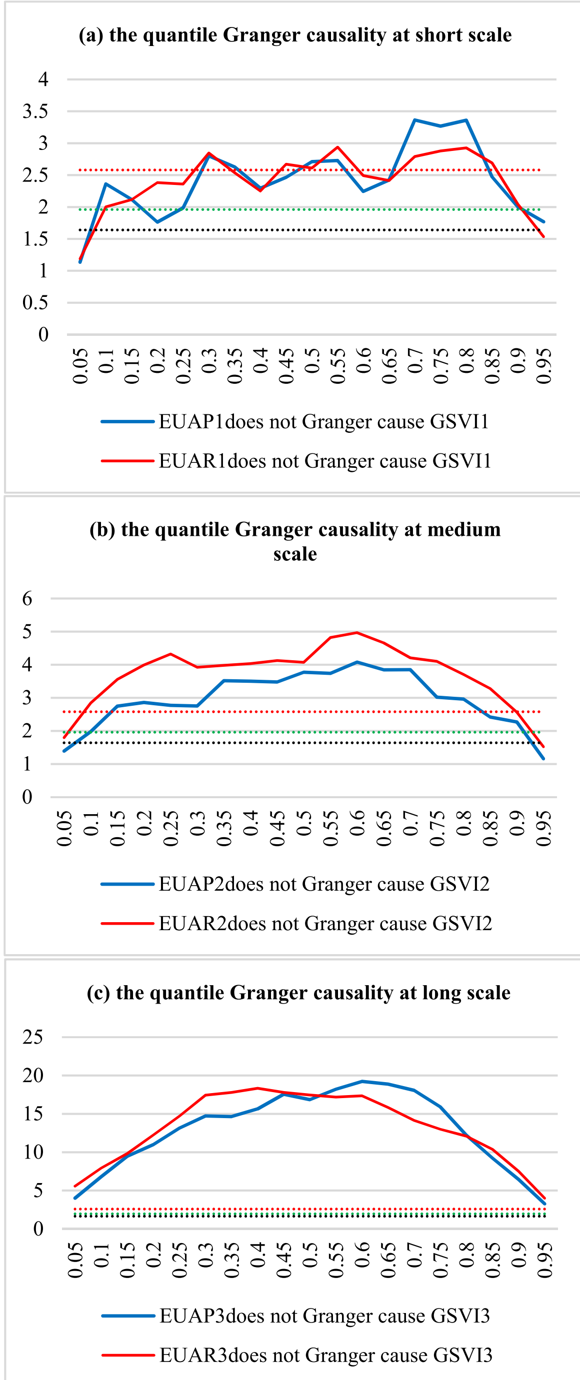

FIGURE 7. Causality-in-quantiles test results for investor attention.

short-term and medium-term causality is insignificant for the extreme downturn and upturn markets.

D. THE QUANTILE DEPENDENCE RESULTS ACROSS SCALES

The above results illuminate in detail about the strength of the causality-in-quantiles between investor attention and the carbon market across various time and frequency horizons. Notwithstanding, we could not determine the magnitude and direction of these impacts among them. As a consequence, this article sets up a CEEMDAN2014-based quantile regression method that can more accurately account for the nonlinear interactions between investor attention and the carbon market. The effects of investor attention on carbon prices and returns are shown in Figure 8 and Table 6.

As demonstrated in Figure 8 and Table 6, this study observes several important features of the relationship between investor attention and the carbon market. In terms of the magnitude of influence, it illustrates different significance across both time and frequency domains. Additionally, changes in carbon prices and returns display a distinctive asymmetric response to attention of investors at every level, with the exception of long-term carbon returns. This illustrates the necessity of analyzing the relationship between investor attention and the carbon market in different time domains and under various market conditions. ① For example, at the short-term scale, it captures a beneficial impact on the low quantiles of EUAP and EUAR but an adverse impact on the high quantile; and the marginal effect is more significant and stronger on EUAR, which is in accordance with the result demonstrated in Figure 6 (a). One major cause of this phenomenon might be the investors' risk hedging behaviour. During periods of high carbon prices and returns volatility, investors may rely on carbon futures as hedging techniques to be added as their portfolio investment to reduce upside or downside risks. Investor attention can promote carbon assets trading when the carbon futures price is low; otherwise, the attention of investors could hinder the price during times of a high-price period. It is understandably easy to interpret that during a collapse in carbon futures prices, the growing interest from investors impels them to purchase carbon assets; and then the increased demand could induce more carbon emission trading and higher prices, which becomes more obvious at the long-term scale. By comparison, when carbon prices are in a surge stage, investor attention may originate from short sellers, resulting in enhanced supply and lower prices. As the carbon returns are more vulnerable to market factors at the short-term scale, the influence of investor attention is still asymmetric between the low and high quantiles. ② However, in the long run, carbon returns are very much dictated by supply and demand fundamentals and are relatively unaffected by external forces, and the magnitude of the effect caused by investor attention decreases and homogenizes. It is worth noting that as the carbon market develops and we strive towards carbon neutrality, investor attention will shift to increasing carbon returns in response to the bullish tendency. As we can see from Figure 8 (c), the estimated coefficient is strengthened at the right tail of the carbon returns distribution. ③ Turning to the medium-term scale, the signs of the coefficients are opposite to those of the short-term and long-term scales. Under bearish market conditions, the GSVI negatively impacts carbon prices and returns. However, around bullish market conditions, variations in GSVI are positively correlated with the carbon market. The inefficiency of the carbon market and bounded rationality of investors can explain this finding. In the medium-term, investors are inclined to behave with greater rationality and calmness than short-term investors but are more proactive than long-term investors in the bear market. When the economic actor has limited rationality, according to prospect theory, they will make actual decisions in the ''noise trading'' of the inefficient market based on their own wealth expectation [61]. In view of this, the traders who are irrational seem to be more inclined towards risk aversion than revenue generation. As a result, they intend to maintain their positions or even liquidate them amidst market panic, leading to a decline in carbon prices and returns. Operations in a bull market may be the opposite of those in a bear market. Moreover, as can be seen from Table 6, the marginal effect is not significant at the medium term, owing to the external complex factors influence and multi-scale features of the variables, which is similar to the results in Figure 6 (b). Overall, investor attention has a nonlinear relationship with the carbon market, with stronger effects at the lower and upper quantiles. The carbon market is unstable and vulnerable as the carbon price or return is extraordinarily high or low. Hence, even a slight shock from investor attention can result in obvious fluctuations in the carbon market. In addition, facing the pressure of extreme carbon market conditions, regulators tend to impose more policies to maintain carbon market stability, which inevitably increases the attention of investors and amplifies the impact of GSVI on carbon assets.

Likewise, the carbon market has an asymmetric impact on investor attention, and presents a difference between carbon prices and returns (see Table 4). By contrast, carbon returns seem to have a more significant and greater impact on investor attention, whether at the short-term scale, medium-term scale or long-term scale. ① As for the short scale, the frequent fluctuations in carbon prices and returns imply noise trading and hedging effect behavior of investors, leading to lower carbon trading yields, which is more obvious for low- or high-search phases. At the same time, high-frequency oscillations of carbon returns indicate a high risk in the carbon market, causing investors to shift their investment decisions towards other commodity markets, and then reduce attention to the carbon market. ② Nevertheless, in terms of the medium-term scale, greater fluctuations in carbon prices and higher carbon returns lead to increased investor attention. As mentioned in Section IV-B, the medium-scale characteristics of carbon prices and returns are mainly attributed to the impact of external heterogeneous environments and major events. With lower fluctuation amplitude and frequency of carbon assets, the marginal effect of the carbon market on investor attention becomes positive. But the signs of the statistics of quantile regression are almost insignificant across the investor attention distribution, which might be owing to the impulses from external events. This phenomenon is also consistent with the result in Table 6, which suggests that we are unable to obtain valid reference information from their nexus at the medium timescale. ③ Meanwhile, the relationship becomes weaker over long-term timescales. This is probably because the effect

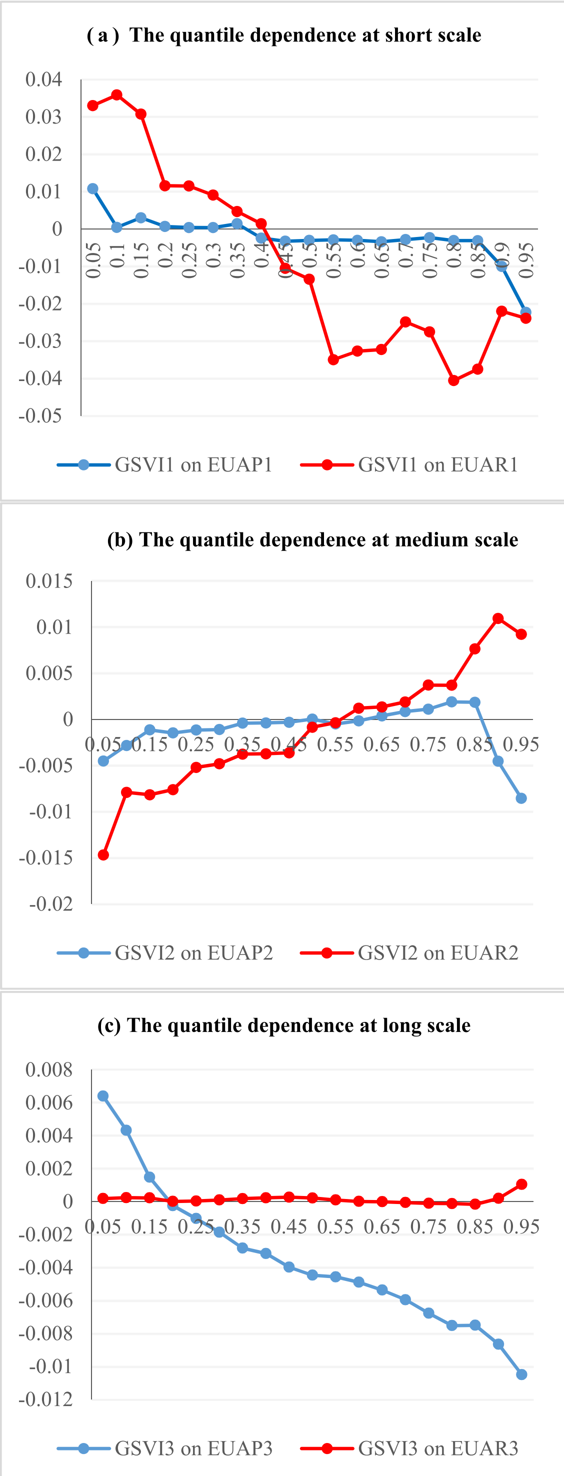

FIGURE 8. Estimates of the quantile dependence from investor attention to carbon market.

of carbon price and return changes on investor attention will be gradually assimilated over time. Notwithstanding, we can also find that the signs of the coefficients are positive across almost all investor attention distributions but negative at the right tail. Under the vision of carbon neutrality and the long-term growth trend of carbon prices and carbon returns

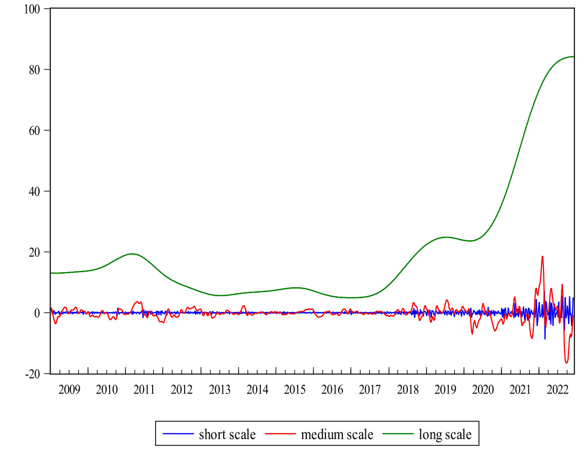

FIGURE 9. The reconstruction diagram of carbon prices decomposition sequence.

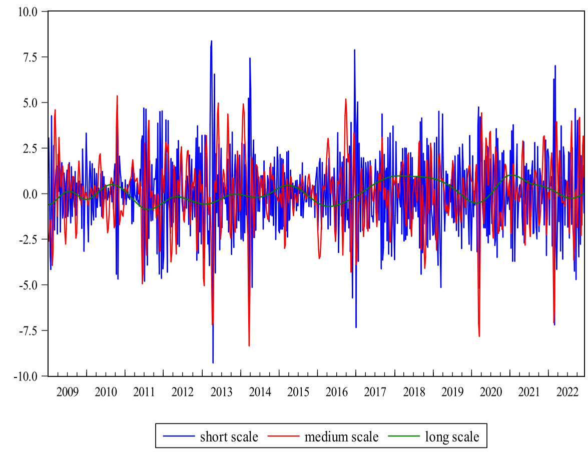

FIGURE 10. The reconstruction diagram of carbon returns decomposition sequence.

(see Figures 3 and 4), investor attention to the carbon market presents an increasing trend. However, under bullish market conditions, changes in carbon prices and returns may increase the market risk of carbon futures, since carbon futures serve as essential commodity assets, thereby resulting in a decrease in investor attention based on their risk hedging behaviour. Furthermore, our findings once again highlight the nonlinear impacts between investor attention and the carbon market under various distributions and among different timescales, which is consistent with our theoretical derivation proposed in Section II.

Interestingly, comparing Tables 4 and 6, we can observe that the impact of the carbon market on investor attention is significantly greater than that of investor attention on the carbon market. Under carbon neutrality targets, the carbon trading system is designed mainly to address global climate and environmental issues. Different from traditional financial commodities, carbon assets have weak financial attributes and long transaction cycles [8], and they are vulnerable to the impact of regulations on emission reduction, changes in energy prices and macroeconomic shocks. Therefore,

TABLE 4. The quantile dependence results of carbon market on investor attention.

θ	The impacts of carbon prices on investor attention			The impacts of carbon returns on investor attention		
	Short scale	Medium scale	Long scale	Short scale	Medium scale	Long scale
0.05	0.601142	-0.033421	0.006000***	-0.060257	-0.192323*	-0.027247
0.1	-0.759268*	-0.100882	0.005904***	-0.252181**	-0.022794	0.13477***
0.15	-0.547925	-0.022668	0.004586***	-0.356342***	0.052555	0.186608***
0.2	-0.451659	0.025055	0.003617**	-0.404851***	0.094689	0.098954***
0.25	-0.438048	0.00335	0.003229***	-0.425309***	0.063651	0.058877***
0.3	-0.30877	0.025244	0.002575***	-0.345214***	0.056758	0.031487*
0.35	-0.196338	0.032317	0.002195***	-0.333142***	0.134157***	0.032408***
0.4	-0.368077	0.019482	0.002215***	-0.279858***	0.112173**	0.043835***
0.45	-0.504509	0.04968	0.002261***	-0.285876***	0.111569**	0.058289***
0.5	-0.341428	0.053764	0.002665***	-0.321368***	0.123847**	0.082189***
0.55	0.07409	0.054566	0.003366***	-0.328469***	0.113852*	0.098285***
0.6	0.199836	0.060788*	0.004302***	-0.336763***	0.101864*	0.111412***
0.65	0.421679	0.084251**	0.005589***	-0.285521***	0.100639*	0.120062***
0.7	0.202342	0.090911**	0.007032***	-0.201545***	0.096689	0.132603***
0.75	0.232996	0.078476**	0.008453***	-0.170275**	0.046617	0.136083***
0.8	0.172400	0.049155	0.009807***	-0.193951**	0.076942	0.134196***
0.85	-0.048384	0.045623	0.010435***	-0.291135***	0.101119	0.051629*
0.9	0.359241	0.047854	-0.000592	-0.156514	0.058023	-0.125584***
0.95	-0.477262	0.153582*	-0.006459***	-0.302296	0.032538	-0.36497***

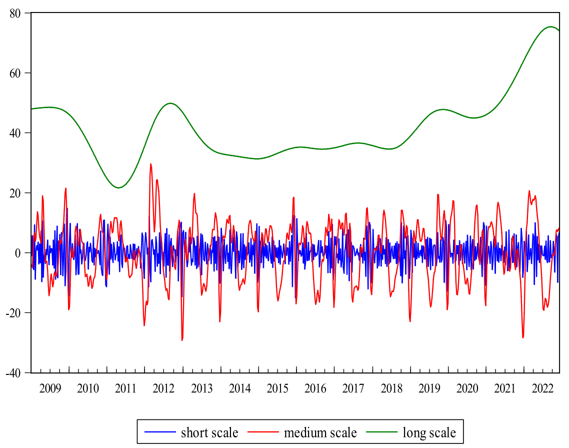

FIGURE 11. The reconstruction diagram of investor attention decomposition sequence.

investor attention may not be the most critical factor influencing the carbon market, while stable and sound development may bring an important effect to investors.

V. CONCLUSION

The carbon emissions trading market is considered as one of the most effective approaches to achieving carbon neutrality, and has been receiving increasing attention. Compared with other traditional financial markets, the carbon market exhibits noticeable nonlinearity and is sensitive to shocks in climate policy and economic environmental changes. From the standpoint of behavioral finance, an interaction between carbon markets and investor attention is possible. However, most of the existing research has concentrated solely on the interactions between the carbon market and the energy, finance, and commodity markets, and has overlooked the nonlinear interactions between investor attention and the carbon market, especially the spillover and connectedness between them under different market conditions and timescales. In this context, we adopt a multi-scale perspective and construct a novel CEEMDAN2014-based quantile Granger causality approach and quantile regression method, and so to explore the nonlinear quantile dependency over different time periods. The primary conclusions are encapsulated as follows:

We initially employ the CEEMDAN2014 method to decompose investor attention and carbon price and return sequences, which demonstrates the advantages of describing the multi-scale features of these signals. After decomposition, we reorganized the IMFs and residuals into three time horizons, namely short scale (within one month), medium scale (above one month but below one year), and long scale

TABLE 5. Quantile granger causality results for medium terms (IMF2 to IMF5).

θ	GSVI does not Granger cause EUAP				GSVI does not Granger cause EUAR			
	IMF_2	IMF_3	IMF_4	IMF_5	IMF_2	IMF_3	IMF_4	IMF_5
0.05	0.9166	1.7641*	1.4444	2.4320**	0.5560	2.1920**	1.4956	1.9779**
0.1	1.8105*	2.7377***	2.4454**	3.4739***	1.3786	2.9150***	2.1535**	2.8613***
0.15	2.1383**	3.4165***	2.6828***	4.6131***	1.4008	3.3702***	2.5636**	3.8579***
0.2	3.2137***	4.2649***	3.4604***	6.1808***	1.9296*	4.3273***	3.6946***	5.1425***
0.25	2.9220***	4.2013***	3.7287***	8.4203***	1.9208*	4.4357***	4.0660***	6.6799***
0.3	2.8169***	4.0290***	4.0610***	8.7911***	1.8032*	4.5282***	3.6533***	6.6990***
0.35	2.3323**	3.3510***	3.5790***	9.8084***	1.8564*	5.2062***	3.8225***	6.8679***
0.4	1.4712	3.5234***	2.9951***	11.0304***	1.6109	5.4750***	3.8835***	7.1143***
0.45	1.0722	2.9562***	2.5450**	11.1387***	1.7741*	5.3805***	3.7838***	7.7645***
0.5	0.6373	3.0028***	2.3994**	10.8007***	1.8571*	4.9844***	4.0325***	7.8903***
0.55	0.7169	2.9973***	2.4725**	9.6624***	1.9320*	5.1028***	4.3357***	8.3184***
0.6	0.9739	2.7489***	3.3305***	8.7835***	1.8517*	4.7226***	4.7165***	8.2492***
0.65	1.9253*	3.1572***	3.4878***	7.8286***	2.3614**	4.7943***	4.2443***	7.2103***
0.7	2.4851**	3.1494***	3.7255***	6.7193***	2.3742**	4.4233***	4.6739***	6.2576***
0.75	2.3477**	4.0150***	3.9129***	7.3794***	1.8610*	4.2128***	4.3669***	5.2442***
0.8	2.5231**	3.2034***	3.4813***	7.6745***	1.7620*	4.0285***	4.5150***	4.0827***
0.85	2.6830***	3.4420***	2.6282***	6.8835***	1.7034*	3.2803***	3.2743***	3.2223***
0.9	1.7939*	2.4897**	2.5679**	5.0313***	0.8727	2.9614***	2.3791**	2.2323**
0.95	1.1438	1.4746	1.7653*	2.5422**	0.3404	2.2569**	1.5584	1.7155*

Note: The *, ** and *** represent critical values at significance levels of 1%, 5%and 10%, respectively.

(above one year). On the one hand, this paper explores the predictability of investor attention towards carbon prices at various time intervals utilizing the nonparametric quantile Granger causality test approach. On the other hand, this study also examines the effects of different market conditions on this predictability. The results indicate an asymmetric and dynamic connection between investor attention and the carbon market across all timescales.

In the short term, carbon prices and returns have drawn the interest of investors across nearly all quantiles except for the medium one ($\theta = 0.4, 0.45, 0.5$). Investor attention is a significant and larger driver of carbon return. But in view of the medium-term, especially for the scale of IMF2 (about a month and a half), the Granger causality is relatively weak except for a few moderate market phases ($\theta = 0.2, 0.45, 0.75$). While as for the long-term scale, investor attention appears to be the Granger cause of carbon prices and returns at all quantiles, and the explanatory power is larger and more significant. The hump-shaped curve reveals that investors pay more attention towards carbon assets in normal markets. What's more, the carbon market causes investor attention across all timescales and quantiles, with the exception of the extreme downturn and upturn markets ($\theta = 0.05, 0.95$) at short and medium scales. As investor attention may generate interest, the results indicate that carbon assets can be treated as speculative assets in the short run and hedging assets in the long run, thus confirming their financial attributes of carbon assets. However, we may be unable to obtain valuable information from medium-term interactions, for the complex and inconspicuous causality results between investor attention and the carbon market.

After knowing the causality of investor attention on carbon prices and returns, it is necessary to further quantify these influences. Our proposed method consists of a combined approach using multi-scale analysis and the quantile regression method, which is robust to outliers and structural breaks in a dataset. Empirical findings reveal an observable asymmetrical impact of investor attention on carbon prices and returns across different time frames. At the short-term scale, the influence of the lower percentiles of carbon prices and returns is positive (about 0.01 for $\theta = 0.05$), whereas it is negative for the higher quantiles (nearly -0.02 for $\theta = 0.95$), with a stronger impact on carbon returns. Nevertheless, its impact on carbon returns diminishes in the long-term (approximately 0.0002). Additionally, contradictory results are obtained from the medium scale, which could

TABLE 6. The quantile dependence results of investor attention on carbon market.

θ	The impacts of investor attention on carbon prices			The impacts of investor attention on carbon returns		
	Short scale	Medium scale	Long scale	Short scale	Medium scale	Long scale
0.05	0.010800	-0.004510**	0.006401***	0.033048	-0.014688**	0.000189**
0.1	0.000450	-0.002813*	0.004329***	0.035898*	-0.007896*	0.000244**
0.15	0.003022	-0.001126*	0.001482**	0.030750*	-0.008161**	0.000227*
0.2	0.000690	-0.001459*	-0.000243	0.011591	-0.007603**	0.000017**
0.25	0.000409	-0.001140	-0.001023**	0.011529	-0.005190*	0.000039**
0.3	0.000391	-0.001090*	-0.001859***	0.009087	-0.004802**	0.000105**
0.35	0.001413	-0.000400	-0.002817***	0.004700	-0.003757*	0.000187*
0.4	-0.002422*	-0.000368	-0.003142***	0.001419	-0.003715	0.000236**
0.45	-0.003258**	-0.000304	-0.003975***	-0.010501	-0.003611	0.000272***
0.5	-0.003045*	0.000047	-0.004461***	-0.013429*	-0.000839	0.000222***
0.55	-0.002895***	-0.000485	-0.004563***	-0.034965***	-0.000353	0.000102**
0.6	-0.003023**	-0.000143	-0.004886***	-0.032675***	0.001228	0.000014***
0.65	-0.003402***	0.000390	-0.005362***	-0.032251***	0.001365	-0.000006***
0.7	-0.002817*	0.000862*	-0.005949***	-0.024851***	0.001901	-0.000050***
0.75	-0.002299*	0.001118*	-0.006770***	-0.027521***	0.003734	-0.000102***
0.8	-0.003083	0.001910**	-0.007515***	-0.040558***	0.003711	-0.000113***
0.85	-0.003093	0.001869*	-0.007498***	-0.037485***	0.007657*	-0.000161*
0.9	-0.010026**	-0.004521	-0.008648***	-0.022002	0.010952*	0.000199***
0.95	-0.022314***	-0.008536	-0.010495***	-0.023894	0.009228	0.001050***

be attributed to external uncertainties and illogical trade. As for the influence of the carbon market on investor attention, the findings show that carbon returns have a more pronounced effect. Moreover, the strength of the impact decreases with increasing timescales (less than 0.1 at the long-term scale). It should be noted that investor attention is influenced more by the carbon market. All in all, the outcome is consistent with our theoretical derivation.

Consequently, examining and measuring the interactions between the carbon market and investor attention across various frequency domains and diverse market situations hold several practical implications. Firstly, carbon price prediction is of paramount importance for market regulators and participants as it enables them to monitor and mitigate risks, particularly in challenging market conditions. Thus, policymakers should devote sufficient attention to GSVI to maintain a stable and efficient carbon market. Considering the confusing connectedness between investor attention and the carbon market at the medium scale, policymakers should make their policies transparent, thus reducing the time and information costs of policy transmission. Meanwhile, market regulators should enhance the defense mechanism of the carbon market in response to external event impacts, and should be fully aware of the importance of reducing irrational components in carbon market volatility. Secondly, by tracking the asymmetric impacts of investor attention on carbon assets at various quantiles, investors can improve the timing of their investments in carbon emission trading. Since investor attention exerts a more significant impact on the carbon market on extreme quantiles of the carbon assets distribution, investors should pay more attention to the GSVI when carbon prices or returns are incredibly high or low. Furthermore, to achieve better profits and hedge effectively, individual investors in the carbon market may consider the significant factor of investor attention, especially its multi-scale effects. The varied spillovers from the GSVI to the carbon market in different time domains indicate that the short-, medium-, and long-term carbon pricing mechanisms are not the same, especially in the presence of GSVI. This finding provides new insights into the formulation of accurate carbon pricing models and suggests that investors should consider short-, medium-, and long-term factors when adjusting their investment plans and asset portfolios. For instance, carbon financial assets can be speculative in short term, given that their price changes often correlate with investor attention. Hedgers can devise long-term investment strategies based on the optimistic development prospects of the carbon market, while market returns might decrease in the medium-term. Lastly, academics and researchers need to take into attention the dynamic and asymmetric influencing of investor attention on the carbon market. The approach proposed in our study could comprehensively illustrate how do they affect each other under whole distributions and among different timescales. Neglecting these analyses could result in losing valuable information that is useful for investors and policymakers.

Accordingly, this article contributes to the existing literature from the following aspects: First and foremost, this study is the first to estimate the spillovers and connectedness between investor attention and the carbon market from a multi-scale and quantile perspective. Moreover, our findings will promote research on the impact of investor attention and expand the theoretical methods for studying carbon financial asset factors, which will help us better understand the carbon pricing mechanism. Second, the multi-scale analysis and quantile dependency provide new insights into the nexus between investor attention and the carbon market, promoting investors to optimize their investment plans and asset portfolios and helping policymakers to formulate accurate and effective policies to reduce carbon emissions. Third, by combining the three different methods, namely the improved CEEMDAN technique, quantile Granger causality test and quantile regression method, we provide more scientific and comprehensive results over the interactions between investor attention and the carbon market. However, the existing research using the traditional VAR model [11] and Granger causality tests [56] can only obtain their linear relations, which are unable to provide more accurate and specific reference views to carbon market participants.

Future research should focus on exploring the information transmission between investor attention and the carbon market, in addition to investigating the influence of other factors. As in the medium term, the effects between investor attention and the carbon market become slightly weaker and confusing. The primary factor driving this phenomenon is that the article only takes into account the interplay between the carbon market and investor attention, disregarding any effects that stem from other unaccounted factors. Especially under extreme market conditions, external uncertainties are also likely to have an impact on the carbon market and investor attention. The existing literature demonstrates that external factors of uncertainty also impact the carbon market, such as the macroeconomic environment, energy price, and economic policy uncertainty [6], [25], [41]. Then, we believe that the carbon market and investor attention are not the only factors that influence each other. They may be connected to other external factors, leading to structural change interactions. Moreover, although we obtained comprehensive and abundant results from our proposed method, there are still some limitations need further improvement. For example, the nonparametric quantile causality test analyzes only the causal relationship between two variables, and does not consider the impact of other factors on their connections. In this context, we can dig out the relevant factors that affect changes in their relationships, and then add them to the model to improve explaining ability. Moreover, since it is difficult to separate the rich information carried in highly complex IMFs using the CEEMDAN technique alone, we can further adopt secondary decomposition to extract more details from the sequences effectively. In addition, this paper mainly relied on subjective judgment and linear dimensionality reduction methods to obtain a proxy for the GSVI. In the future, we can use text mining and word cloud methods to fully mine related keywords of the carbon market, and combine more accurate nonlinear dimension reduction methods, such as the Isomap (Isometric feature mapping) approach, to get a more representative indicator.

APPENDICES

See Figures 9–11 and Tables 5 and 6.

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