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RESEARCH ARTICLE

A Robust Seating Arrangement for Future Pandemics

GOKHAN KARAKOSE AND BAYRAM DUNDAR
Department of Industrial Engineering, School of Engineering, Architecture, and Design, Bartin University, Kutlubeyyazıcılar Campus, 74110 Bartin, Turkey
Corresponding author: Gokhan Karakose (gkarakose@bartin.edu.tr)

ABSTRACT Numerous studies have examined classroom seating arrangements to enhance student safety and resource utilization during COVID-19. These studies typically aimed to maximize the minimum distance between students for a given number of students to be assigned. This paper distinguishes itself from the existing literature by not only assigning students as far apart as possible but also focusing on maximizing the average distance between students. We call this new problem the maximum diversity social-distancing problem (MDPs), a novel variant of the maximum diversity problem (MDP). This problem is a two-phased problem, where the first phase involves producing the maximum of minimum distance (max-min) between students, and once that is resolved, the max-min distance is used in the second phase for having the highest dispersion. The first phase is here solved by a new algorithm, which effectively determines max-min distance for each student allocation scenario. For the second phase, three exact and one greedy approximation MDPs models are proposed. In computational testing, we observe that the greedy approximation MDPs model mostly returns optimal solutions across all tested classrooms in less than a second. More importantly, utilizing the greedy one significantly improves student pair spacing, increasing the average distance by over 40 centimeters compared to the max-min distance approach of the literature. Later, this effective approach is integrated into the pandemic management platform, which proactively assists the university administration in preparing for and effectively managing future infection outbreaks.

INDEX TERMS Sustainable education, maximum diversity problem, social distancing, seat assignment, mathematical modeling.

I. INTRODUCTION

During the COVID-19 pandemic, the economic situation of the countries has deteriorated, most workplaces, schools, and public places have been closed or they have continued to operate with less capacity. These have negatively affected societies in various areas, from social life to education. Therefore, governments and institutions have taken crucial steps to mitigate the impact of the pandemic and slow its spread; some of these include wearing masks, maintaining social distancing, and hygiene in crowded and closed environments. Many education institutions have switched to the online-learning to support the pandemic control in closed areas [1]. Alternatively, as face-to-face education has been preferred to online learning, different social distancing policies have been adopted by many countries in closed areas [2]. For example, Singapore, Germany, USA have enforced 1 meter (m), 1.5 m, and 2 m of physical distance in closed areas, respectively. In such countries, either the education institutions have significantly reduced their classroom capacity, or they have divided classes into a few sessions in order to facilitate continued in-person education. However, manually arranging students in classrooms while adhering to social distancing guidelines presents a formidable challenge for decision-makers. To address this issue, [3], for example, proposed several seat assignment models to determine the maximum number of students that could be fitted in a classroom while ensuring adequate physical distance between students. They noted that assigning students as far apart as possible could provide a safer space, reducing the risk of contracting the virus. So, recent studies such as [4] and [5], including the aforementioned study, have focused on

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maximizing the minimum distance (i.e., defined as max-min distance) among the students in the classroom, which actually corresponds to an application of the p-dispersion problem (pDp) in the context of pandemic control.

So far, the literature has focused on the pDp for seating plans in the context of pandemic control. Specifically, the pDp aims at assigning students as far apart as possible in a classroom setting in order to protect each student from being infected in the largest and equal way possible. Another related problem that can be used for seating arrangements is the maximum diversity problem (MDP). However, the seating plan generated by the MDP in a classroom setting does not prioritize protecting each student against virus risk as the goal is simply to maximize the overall distance between students, which may result in some students sitting in high-risk positions (i.e., some sitting very close to each other).

Our model MDPs combines both aforementioned approaches in order to generate a robust seating plan for all students by increasing the average distance between students and guaranteeing at least a certain level of protection for each student, thus further reducing the average probability of the virus spread. We call this new problem as the maximum diversity social-distancing problem (MDPs), a novel variant of the MDP. We observe that utilizing the MDPs for seating layout optimization significantly improves student pair spacing, increasing the average distance by over 40 centimeters (cm) compared to the pDp across all tested classroom scenarios. This demonstrates how effective the proposed solution procedure is in further improving student safety to better serve the sustainability of education. Accordingly, we expect this approach to better serve the ultimate purpose of slowing the spread of a pandemic and helping to flatten its peak, thus easing the burden on healthcare systems and sustaining continuing education.

The remaining of this study is organized as follows. Section II contains a literature review of research on Operations Research (OR) approaches to support pandemic control. Section III includes the proposed algorithm and optimization models developed to solve the problem in question. Section IV includes extensive computational testing discussing the performance of the examined models in various classroom instances. Section V introduces the pandemic management platform developed to help the decision process of authorized users on student seating placement. Finally, Section VI summarizes the paper with concluding remarks.

II. LITERATURE REVIEW

This section scrutinizes the studies that included OR-based modeling and solution approaches that can be well-adapted to respond to the pandemic from the perspective of social distancing in various sectors, including education. One of the earliest studies on MDP was [6], wherein the goal was to choose locations to site p facilities on a network in order to maximize the average distance between assigned facilities. Reference [7] developed a randomized greedy heuristic approach to solve the MDP, whereas [8] approached the MDP with respect to proposing the mathematical models with the objective function of equity in dispersion. Reference [9] reviewed the previously proposed integer programming models and developed a branch-and-bound algorithm for the MDP. Later, while [10] proposed an iterated greedy meta-heuristic algorithm, [11] developed a hybrid memetic algorithm that includes a constrained neighborhood tabu search procedure to efficiently solve the MDP. Reference [12] brought together all benchmark libraries and came up with a new library called MDPLIB. Reference [13] applied the t-linearization technique to solve the MDP in a short period of time. They also presented valid inequalities to reduce solution space within the concept of t-linearization. Reference [14] presented four mathematical models and compared them by using the input data from the MDPLIB. Reference [15] classified the history of the MDP into three eras; the early period, the expansion period, and the developed periods. They indicated the lack of the MDP application and suggested doing a more elaborate application from an OR perspective.

The studies that have applied OR techniques to the COVID-19 pandemic from the perspective of social distancing have also been reviewed. Choi [16] emphasized in their study that OR is a well-established field that develops the decision tools to assist decision-makers in a broad perspective, from resource allocation to humanitarian logistics. Barry et al. [17] developed a space allocation model that applies social distance constraints for workers returning to the office. The authors presented a graph-based approach and a linear model to optimize the workspace while considering the social-distancing constraint. Salari et al. [18] proposed a mixed-integer programming model applied to the airline industry in which the passengers are assigned to seats by taking into consideration social distancing. Kamga et al. [19] evaluated the bus capacity in terms of the maximum number of passengers that can be fitted on a transit bus based on both various social distancing and bus dimensions. The authors also evaluated the transportation capacity of the Newyork subways under different scenarios of social distances. They stated that with each inch of reduced social distance, the number of required trains drastically decreased. Chen et al. [20] applied optimization and machine learning methods to supply chain management, resource allocation, and along with some other areas to mitigate the impact of the pandemic on societies. Chen and Kong [21] implemented a social distancing policy in a hospital to compare three healthcare systems based on mortality rate and infection rate. Liu et al. [22] presented an evolutionary deep learning model that predicts the effect of social distancing on the spread of the virus depending on social distancing metrics. Fischetti et al. [23] designed an optimum layout that determines the location of seats and tables for service business while mitigating the risk of infection. Their mathematical model imitated the modeling approach used to locate wind turbines in an offshore area. Milne et al. [24] introduced three greedy methods to seat airplane passengers, separating those at higher risk of infection from those likely to be infectious.

They presented a procedure that categorizes passengers into four groups based on their likelihood of disease susceptibility and infectiousness. Tan et al. [25] proposed a non-linear programming model to provide a sustainable urban bus service for mitigating the speed of the epidemic in the post-pandemic era. Moore et al. [26] proposed a mixed integer programming model to assign passengers to seats based on the vehicle layout. Also, they considered household grouping in the seat assignment and presented a heuristic model to increase the utilization of seating of transit vehicles. Song et al. [27] used mobility data to compare the campus visitation between pre and post COVID-19 in the three largest universities in Texas. Their study showed a significant decrease in campus visitations for all universities and also found that the impact of COVID-19 was greater for those who live within a mile of the university. Dundar and Karakose [3] developed seat assignment models and a graph-based heuristic approach to maximize the number of students that can be fitted in a classroom while inter-seat distance is arranged based on social distance. Bortolete et al. [4], and Kudela [5] proposed the pDp models, which focused on finding the best seat assignment pattern by maximizing the minimum distance between individuals for various places such as classrooms.

III. PROBLEM FORMULATIONS

The problem at hand follows a two-phase framework. In the initial phase, the primary objective is to maximize the minimum distance among students, referred to as the "max-min" distance. Once this optimization is achieved, the computed max-min distance serves as the foundational parameter for the subsequent second phase, which aims to achieve the highest possible dispersion for a more robust seating layout. Algorithm 1 is employed effectively during the first phase, adeptly determining the max-min distance for each distinct student allocation scenario. As for the second phase, one can obtain the solution by using the proposed exact models or opting for the greedy approximation model.

A. MATHEMATICAL MODELS

Define a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ comprised of $\mathcal{V}$ nodes and $\mathcal{E}$ links, where $\mathcal{V}$ is a set of seats in a classroom and $\mathcal{E}$ represents the abstract links between pairwise seats. Let $\beth$ be the maximum of minimum distance between all pairs of students, which is effectively assessed by Algorithm 1 for each student allocation scenario Θ (i.e., the number of assigned students). Let d_{ij} be the distance between seat i and seat j, and finally, $\aleph$ be the least social distancing value that must be kept between each student pair ($\beth \geq \aleph$). We define x_i binary variable as 1 if a student is assigned to seat i, and 0 otherwise. Similarly, we define y_{ij} binary variable as 1 if both seat i and seat j are filled with students and 0 otherwise. The first exact MDPs model, 1e-MDPs, is provided below.

$$\max \sum_{i,j \in \mathcal{V}|j>i} d_{ij} y_{ij} \tag{1}$$

$$\text{s.t.} \quad y_{ij} \leq x_i, \quad \forall i,j \in \mathcal{V}|j > i \tag{2}$$

$$y_{ij} \leq x_j, \quad \forall i,j \in \mathcal{V}|j > i \tag{3}$$

$$x_i + x_j \leq y_{ij} + 1, \quad \forall i,j \in \mathcal{V}|j > i \tag{4}$$

$$y_{ij} \leq \frac{d_{ij}}{\beth}, \quad \forall i,j \in \mathcal{V}|j > i \tag{5}$$

$$\sum_{i \in \mathcal{I}} x_i = \Theta \tag{6}$$

$$x_i, y_{ij} \in \{0, 1\} \quad \forall i,j \in \mathcal{V} \tag{7}$$

In model 1e-MDPs, the objective (1) maximizes the total distance among the students. The constraints (2)-(4) enforce y_{ij} variable to take the value of 1 when both seat i and seat j are occupied. Constraint (5) guarantees that if the distance between seats i and j is not greater than or equal to $\beth$, both seats i and j cannot be filled together. The constraint (6) assigns Θ number of students to the classroom.

However, in our computational testing, for the sake of computational performance improvement, we relax y_{ij} variable of the model 1e-MDPs as given in the equation (8). Additionally, the constraint (5) is removed in the model 1e-MDPs and the constraint (9) is added instead in order to improve the performance of the model. Note that equation (9) still ensures that if the distance between seat i and seat j is less than $\beth$, then the corresponding variable y_{ij} takes the value of 0, as does the constraint (5).

$$y_{ij} \geq 0, \quad \forall i,j \in \mathcal{V}|j > i \tag{8}$$

$$y_{ij} = 0, \quad \forall i,j \in \mathcal{V}|j > i, d_{ij} < \beth \tag{9}$$

The second exact model, 2e-MDPs, has fewer constraints compared to the 1e-MDPs. The 2e-MDPs reads as follows:

$$\max \sum_{i,j \in \mathcal{V}|j>i} d_{ij} y_{ij} \tag{10}$$

$$x_i + x_j \leq y_{ij} + 1, \quad \forall i,j \in \mathcal{V}|j > i \tag{11}$$

$$\sum_{j \in \mathcal{V}} \left(y_{ij|j>i} + y_{ji|i>j} \right) = (\Theta - 1) x_i, \quad \forall i \in \mathcal{V} \tag{12}$$

$$x_i \in \{0, 1\}, \quad y_{ij} \geq 0 \quad \forall i,j \in \mathcal{V} \tag{13}$$

Here there is an implicit constraint in the 2e-MDPs that $y_{ij} = 0 \ \forall i,j \in \mathcal{V}|j > i, \ d_{ij} < \beth$. As noticed, constraints (2), (3) and (5) are now not included in the model 2e-MDPs. Specifically, removing (2) and (3) from the 2e-MDPs does not introduce any obstacle that would prevent the objective function (10) from taking the actual optimal value. In addition, if seat i is occupied, then the sum of assigned seat pairs $i - j$ from seat i must be equal to the number of assigned students minus one, given in constraint (12). As this constraint reduces the search space of the model 2e-MDPs, we expect that the 2e-MDPs return a solution in a shorter time compared to the first model. Note also that, for the sake of computational improvement, we found it better to include (6) in the model 2e-MDPs as a valid inequality constraint, since it generally further shortened the computational time of the 2e-MDPs.

In the third exact model, we introduce the addition of one new variable, w_i. Here, w_i represents the total distance from the seat i to the other seats. 3e-MDPs is formulated as follows.

The objective function (14) maximizes the sum of distances between occupied seats. If seat i is occupied, then constraint (15) becomes active and ensures that w_i takes at most the value of the total distance from seat i to the other seats whose distance from seat i is greater than or equal to $\daleth$. Similarly, constraint (16) ensures that w_i is the sum of the distance between the occupied seats if the distance between them holds the following equation $d_{ij} > \daleth, \quad \forall i, j \in \mathcal{V} | j > i$. Constraint (17) becomes active when the distance between two seats is less than $\daleth$. The seat pairs $i - j$ that activate Constraint (17) cannot be filled together. The constraint (18) is exactly the same as (6). Finally, the constraint (19) provides the nature of variables.

$$\max \sum_{i \in \mathcal{V}} w_i \tag{14}$$

$$w_i \leq x_i \left(\sum_{j \in \mathcal{V} | j > i, d_{ij} > \daleth} d_{ij} \right), \quad \forall i \in \mathcal{V} \tag{15}$$

$$w_i \leq \left(\sum_{j \in \mathcal{V} | j > i, d_{ij} > \daleth} d_{ij} x_j \right), \quad \forall i \in \mathcal{V} \tag{16}$$

$$x_i + x_j \leq 1, \quad \forall i, j \in \mathcal{V} | j > i, \quad d_{ij} < \daleth \tag{17}$$

$$\sum_{i \in \mathcal{V}} x_i = \Theta \tag{18}$$

$$x_i \in \{0, 1\}, \; w_i \geq 0 \quad \forall i \in \mathcal{V} \tag{19}$$

The last proposed model is the greedy approximation model denoted as a-MDPs. Recall that the previous models maximize the distance between occupied seats. In the a-MDPs, however, the objective function (20) includes the summation of two parts: (1) the double of the distance when both seats are occupied, and (2) the distance when one of the seats is occupied. As the objective (20) tends to greedily assign students to the furthest possible distance from each other, we will refer to the model a-MDPs as the greedy approximation model throughout the paper. In the such greedy assignment, part (2) is the one that may deviate the result of the a-MDPs from the optimal assignment because the objective should only take the distance into account between occupied seats. However, constraints (21) strictly narrow search space owing to $\daleth$ (recall that $\daleth$ is the largest possible minimum distance value between occupied seats), causing such deviation to be minimal. Hence, the a-MDPs showed remarkable performance in solving classroom instances in our computational testing, consistently returning optimal solutions (i.e., optimal allocation layouts for classrooms) in a very short period of time. The a-MDPs reads as follows.

$$\max \sum_{i,j \in \mathcal{V} | j > i} d_{ij}(x_i + x_j) \tag{20}$$

$$x_i + x_j \leq 1, \quad \forall i, j \in \mathcal{V} | j > i, \quad d_{ij} < \daleth \tag{21}$$

$$\sum_{i \in \mathcal{I}} x_i = \Theta \tag{22}$$

$$x_i \in \{0, 1\} \quad \forall i \in \mathcal{V} \tag{23}$$

B. A NOVEL ALGORITHM FOR OPTIMIZING THE STUDENT ALLOCATION

This section outlines the process for determining parameter $\daleth$ in the aforementioned models by utilizing Algorithm 1 for any given Θ. In Algorithm 1, lines 2-12 constitute the data preparation phase. Specifically, the Euclidean distance between the seats is calculated based on their positions on the x-y Cartesian plane. All d_{ij} data are placed in the $\mathcal{D}_s$ list in non-duplicated ascending order if d_{ij} between $\aleph$ and $\bar{\aleph}$. Remember that $\aleph$ is the least social distancing value that must be kept between each student pair as advised by authorities. We define $\bar{\aleph}$ as the maximum social distance value beyond which it would be unreasonable to examine (i.e., 4 m is selected in our computational testing). Lines 13-26 constitute the efficient frontier phase. Note that in this paper, the efficient frontier includes the best combination of the optimal seating distance and its corresponding maximum student assignment. Specifically, line 14 gives the maximum number of seats that can be filled according to the given social distance value $\mathcal{D}_s[t]$. Here, $\mathcal{D}_s[t]$ gives the element of $\mathcal{D}_s$ located at position t. Note that in any case, as the social distance increases (i.e., $\mathcal{D}_s[t]$ increases with each iteration t), either the number of assignments remains the same or decreases. Keeping this in mind that if γ, the number of occupied seats, calculated from line 15 is less than the number of occupied seats found in the previous iteration (i.e., γ_x), then we update the efficient frontier student allocation scenario list (i.e., γ_{List}), and it's corresponding the efficient frontier social distancing list (i.e., $\daleth_{List}$). From line 22, for $\gamma_x = \gamma$ situation, however, we leave γ_{List} as it is, but update the last entry of the current $\daleth_{List}$ (i.e., $\daleth_{List}[r]$) with $\mathcal{D}_s[t]$. That is, the model given in line (15) gives the same allocation value (γ) for both $\mathcal{D}_s[t]$ and $\daleth_{List}[r]$ social distance values. Since the aim is to maximize the social distance between the occupied seats, the largest social distance corresponding to the same number of student assignments is selected for that student allocation scenario Θ. To elaborate, let us assign Θ number of students to a classroom. Let not exist Θ value in the efficient frontier created by Algorithm 1. (If exists, then max-min distance, $\daleth$, corresponding to Θ is directly found from it). First, determine the values γ_i and γ_j such that they are closest to Θ, and at the same time smaller and larger than Θ, respectively (e.g., $\gamma_i < \Theta < \gamma_j$). Then, $\daleth$ max-min distance for Θ student allocation scenario is evaluated as the distance $\daleth_j$ corresponding to γ_j in the efficient frontier.

IV. COMPUTATIONAL RESULTS

The computational testing was performed on a PC with an Intel i7-11800H @ 2.3 GHz and 16 GB RAM. GAMS platform was used to code the formulations; CPLEX 12.6.2.0 was chosen as the solver to solve them; and Python

Algorithm 1 The Algorithm to Generate the Efficient Frontier for Each Classroom.

Inputs:
$\mathcal{V} = (b_{ir}, b_{jr})$ *where* $(i,j) \in V | i < j$ *and* $1 \leq r \leq q$
$\mathcal{D} = \{\}$
for $i \in V$ **do**
  for $j \in V$ **do**
    if $i < j$ **then**
      $$d_{ij} = \left[\sum_{1 \leq r \leq q} |b_{ir} - b_{jr}|^p \right]^{1/p}$$
      $\mathcal{D} = \mathcal{D} \cup d_{ij}$
    end if
  end for
end for
$\mathcal{D}_s$=sorting $\mathcal{D}$ in non-duplicated ascending order, such that $\aleph \leq d_{ij} \leq \aleph$
$r = 0, t = 0, \gamma_{List} = \{\}, \beth_{List} = \{\}, \gamma_x = inf, \beth_x = 0$
while $t < |\mathcal{D}_s|$ **do**
  $$\gamma = \begin{cases} \max \quad \sum_{i \in \mathcal{I}} x_i \\ s.t \\ \qquad x_i + x_j \leq 1, \quad \forall i,j \in \mathcal{V} | j > i, \\ \qquad d_{ij} < D_s[t] \\ \qquad x_i \in \{0,1\} \quad \forall i \in \mathcal{I} \end{cases}$$
  if $\gamma < \gamma_x$ **then**:
    $\gamma_x = \gamma$
    $\beth_x = D_s[t]$
    $\gamma_{List} = \gamma_{List} \cup \gamma_x$
    $\beth_{List} = \beth_{List} \cup \beth_x$
    $r = r + 1$
    $t = t + 1$
  else:
    $\beth_{List}[r] = \mathcal{D}_s[t]$
    $t = t + 1$
  end if
end while
draw the efficient frontier based on γ_{List} and $\beth_{List}$

3.10.1 was used to code Algorithm 1. The maximum allowed CPU time for computational testing is set to 10,000 seconds (s) for optimization models. Detailed computational tests were conducted for various classrooms at Bartin University.

A. IDENTIFYING THE CRITICAL SCENARIOS ON THE EFFICIENT FRONTIER

For illustrative purposes, we first choose a classroom identified as ED-B1-7, which has a seating capacity of 56. When Algorithm 1 was applied to ED-B1-7, it yielded an efficient frontier showcasing various seat assignment scenarios. Remarkably, this encompassed a total of 81 distinct seat distance values, with 11 pivotal points, referred to as critical scenarios, found along the efficient frontier, as visually represented in Figure 1. It is worth noting that within the range from 1 m to 4 m, the array of 81 different classroom seat distances included measurements such as 1 m, 1.06 m, 1.12 m, 1.14 m, and spanning up to 3.99 m. Since the seats in all examined classes are firmly anchored to the floor, parameter $\beth$ must reside within this range.

TABLE 1. Critical assignment scenarios for class ED-B1-7.

Social Distance(m)	# of assigned students
1.06	32
1.12	28
1.14	24
1.19	20
1.56	16
1.98	12
2.41	8
2.70	7
3.37	6
3.59	5
4.00	4

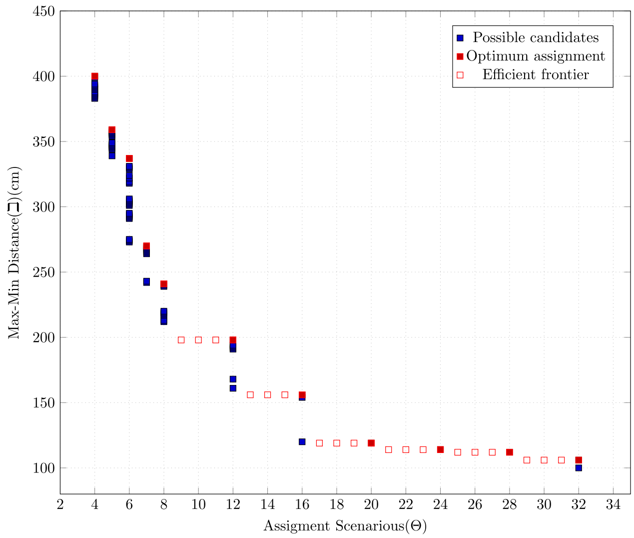

FIGURE 1. Efficient frontier for class ED-B1-7.

For the sake of clarity when discussing class ED-B1-7, the critical scenarios depicted in Figure 1 have been conveniently summarized in Table 1. According to Table 1, when we aim to allocate 16 students ($\Theta = 16$) to the ED-B1-7 classroom, the max-min distance $\beth$ between students is 1.56 m. However, in situations where the desired number of students (Θ) is not explicitly listed in Table 1, such as when we need to assign 17 students, we determine the value of $\beth$ by identifying the assignment closest to and greater than 17. To clarify, for the 17-student scenario, we find $\beth$ as 1.19 m, which corresponds to the assignment for 20 students in Table 1. Consequently, for the 17-student allocation scenario, the parameters Θ and $\beth$ in the MDPs models, such as in the model 1e-MDPs, are set as 17 and 1.19 m, respectively. This approach allows us to determine the max-min distance ($\beth$) for each assignment scenario Θ by referencing the efficient frontier, automatically performed by Algorithm 1.

In terms of computational efficiency, the process of constructing the efficient frontier as depicted in Figure 1 utilizing Algorithm 1 was completed in a quick 10.45 s for this class. Instead, if one applied the model developed by [3], the process of determining $\beth$ for all assigned student scenarios in the ED-B1-7 classroom would take much more than one hour. Another notable advantage of constructing an efficient frontier lies in its capacity to provide decision-makers with various options. For instance, when increasing the number of students in this classroom from 16 to 17, at least one student comes closer to their peers by 37 cm (i.e., the $\daleth$ value decreases from 1.56 m to 1.19 m). In this scenario, the decision-maker faces the choice of assigning 17 students to the classroom, or, if feasible, opting for 16 students to ensure greater spacing between them in terms of $\daleth$; if not feasible, he or she may consider evaluating other alternative classes that have a larger area where 17 students can be assigned. So, showing this flexibility enables decision-makers to make well-informed choices aligned with their specific priorities and constraints.

B. THE PERFORMANCES OF THE PROPOSED MODELS

The previous section includes a discussion of how $\daleth$ parameter is generated according to the given Θ assignment scenario. This section, however, introduces the performance of the proposed mathematical models. Specifically, Table 2 demonstrates the performance of three exact models (1e-MDPs, 2e-MDPs and 3e-MDPs) for small-size classes. These models were evaluated across a range of social distances, from 1 m to 4 m. For instance, the ED-B1-7 classroom has 29 feasible seat assignment scenarios within this range, accommodating between 4 to 32 students depending on social distancing. The models were tested for each scenario, and Table 2 presents the average computation times in seconds for each model. This analysis was extended to other classrooms in Table 2. The results show that the 3e-MDPs model consistently outperforms the other two models in terms of CPU efficiency. Conversely, the 1e-MDPs model exhibits the least favorable computation time performance across all classrooms. To illustrate, in the EA-K1-5 classroom, the 2e-MDPs model achieves the optimal solution approximately 21 times faster than the 1e-MDPs model, while the 3e-MDPs model attains the optimal solution about 1.4 times faster than the 2e-MDPs model.

Table 3 compares the performance of two exact models for mid-size classrooms. Observe that we exclude the model 1e-MDPs from Table 3, given its poor performance exhibited in Table 2. Notice in Table 3 that the model 3e-MDPs returns the solution in a shorter time than the 2e-MDPs for all classrooms examined. For example, for the EA-K1-3A class, the model 3e-MDPs provides an optimal solution in approximately 14 times less time than the 2e-MDPs, with average solution times of 35.62 s and 492.53 s, respectively. Although not detailed in Table 3, it is worth noting that across all 180 assignment scenarios encompassing all classes, the model 3e-MDPs consistently outperformed the model 2e-MDPs in terms of computational speed.

Table 4 presents a comparative assessment of the performance between the model 3e-MDPs and the greedy approximation model a-MDPs for large-size classes, defined as those with 100 or more seats. Observe again that we exclude the model 2e-MDPs from Table 4, given its poor performance for mid-size classrooms compared to the 3e-MDPs exhibited in Table 3. Observe in Table 4 that the 3e-MDPs was unable to provide solutions within the allocated time frame (i.e., 10,000s) for some student assignment scenarios (specifically, 4, 7, 3 and 42 unsolved student assignment scenarios out of a total of 53, 51, 60 and 79 scenarios for classrooms ED-Z-10, EA-Z-10, EA-K1-3B, and ED-K1-11/B, respectively). Note that the unsolved instances here mean that the solver attained a feasible solution that was not verified as optimal within 10,000s. In contrast, the approximation model a-MDPs successfully solved each of the 291 assignment scenarios in an exceptionally short time, averaging between 0.13 and 0.14 s of CPU time. Furthermore, Table 4 also compares the model a-MDPs and the 3e-MDPs in terms of the average distance (in centimeters) among assigned seats, revealing that both models provide nearly identical average distance values.

While MDPs models are primarily constructed based on the total distance objective function, calculating the average distance between students is a straightforward task once the assignment is completed. Note here that the average distance between students is calculated for the 3e-MDPs model, for which the solution is returned only within the specified time constraint (i.e. for 233 scenarios). For instance, in the case of the ED-Z-10 class, for 49 assignment scenarios (out of the total 53, excluding the four scenarios where the 3e-MDPs was unable to provide a solution within 10,000 s), the 3e-MDPs model yields an average distance between students of 588.30 cm, while the a-MDPs model produces a slightly lower value of 588.29 cm. In essence, if we employed the exact model, 3e-MDPs, instead of the a-MDPs model, for scenarios where the model 3e-MDPs could produce a solution within 10,000 s, the average distance between students would increase only a negligible amount of 0.01 cm, 0.12 cm, 0.07 cm, 0.03 cm for the classrooms according to the order provided in Table 4. Considering the assignment of 51 students in the ED-Z-10 classroom, a scenario not explicitly provided in Table 4, it is noteworthy that even though the a-MDPs demonstrated its weakest performance in this particular case, the 3e-MDPs achieved only a marginal increase of 0.33 cm in the average distance between students compared to the a-MDPs (i.e., the average distances of 562.50 cm and 562.17 cm for the models 3e-MDPs and a-MDPs, respectively). In addition, for the proven optimal layout scenarios (the scenarios that can be solved by the 3e-MDPs within 10,000 s), we observed that the model a-MDPs provided the optimal seating layout for 45, 40, 52, and 31 out of 49, 44, 57, and 37 assignment scenarios for classrooms ED-Z-10, EA-Z-10, EA-K1-3B, ED-K1-11/B, respectively. For the remaining scenarios, it consistently delivered results that were nearly optimal, typically deviating by less than a negligible amount of 0.33 cm from the optimal values.

C. COMPARING THE AVERAGE DISTANCE GENERATED BY THE OPTIMIZATION MODELS

This section merely compares the average distance between students found by the models 3e-MDPs, a-MDPs, and

TABLE 2. Comparing the CPU time of models 1e-MDPs, 2e-MDPs, and 3e-MDPs for small-size classrooms.

Class Code	# of seats	# of assignment scenarios	Average CPU time(s)		
			1e-MDPs	2e-MDPs	3e-MDPs
EO-K2-15	33	12	0.70	0.19	0.15
ED-Z-16	42	21	7.06	0.50	0.27
ED-B1-7	56	29	89.40	3.52	0.48
ED-Z-5	59	34	39.98	2.02	0.64
EA-K1-5	69	42	648.69	31.17	22.64

TABLE 3. Comparing the CPU time of models 2e-MDPs, and 3e-MDPs for mid-size classrooms.

Class Code	# of seats	# of student assignment scenarios	Average CPU time(s)	
			2e-MDPs	3e-MDPs
ED-B1-2	72	44	70.65	42.95
EA-K1-2	72	44	98.92	43.17
ED-Z-4B	84	43	333.09	35.04
EA-K1-3A	99	49	492.53	35.62

TABLE 4. Comparing models 3e-MDPs and a-MDPs for large-size classrooms in terms of the CPU time and the average distance.

Classroom code	Classroom capacity	# of assignment scenarios	3e-MDPs			a-MDPs	
			# of unsolved scenarios	Average CPU time (s)	Average distance (cm)	Average CPU time (s)	Average distance (cm)
ED-Z-10	110	53	4	331.43	588.30	0.13	588.29
EA-Z-10	116	51	7	751.16	604.71	0.13	604.59
EA-K1-3B	121	60	3	321.33	566.62	0.14	566.55
ED-K1-11/B	165	79	42	383.95	633.29	0.14	633.26

pDp of the literature across various assignment scenarios for all examined classrooms. Remember here that the pDp (or our developed Algorithm 1 alone) generates a seating layout that maximizes the smallest distance between students. Therefore, the average distance between students was derived by performing an additional calculation based on the seating arrangement provided by pDp. When Table 5 is examined, it is observed that the approximation model a-MDPs closely approximates the optimal values, consistent with what is observed in Table 4. For example, for class EA-K1- 5, for 42 possible student assignment scenarios, the optimal average distance between students is 447.30 cm, while the approximation model a-MDPs produced a fairly approximate value of 447.21 cm. In Table 5, we can also observe the excellent performance of the model a-MDPs for other classrooms. The model a-MDPs returned the optimal solutions between 0.13 and 0.15 s for each scenario. Despite this low running time, it was observed that it significantly increases the distance between students in the seating layout compared to the pDp of the literature. For example, the a-MDPs found the optimal value for the 12 scenarios in class EO-K2-5, and increased the average distance between the students by 21.89 cm compared to the pDp model (i.e., $370.09 - 348.20 = 21.89$ cm). Similarly, for other classes, the model a-MDPs also achieved an average distance increase ranging from 22.37 cm to 58.62 cm when compared to the pDp. For class EO-K2-5, the most favorable comparison of the a-MDPs, corresponding to the allocation scenario of the combination of $\Theta = 7$ and $\daleth = 1.19$m, the a-MDPs returned the average distance as 385.43 cm whereas the pDp returned the average distance of 326.81 cm, which corresponds to 58.62 cm distance improvement achieved by the a-MDPs. Similarly, for the most-favorable comparison of the a-MDPs among all examined student allocation scenarios of the remaining classes in Table 5, the a-MDPs significantly increased the average distance compared to the pDp, corresponding to the increase of 93.64 cm (i.e., 382.91-289.27 = 93.64 cm for the combination of $\Theta = 13$ and $\daleth = 125$ cm of ED-Z-16), 190.17 cm, 192 cm, 166.30 cm, 162.79 cm, 172.36 cm, 91.7 cm, 102.15 cm, 142.19 cm, 193.33 cm, 127.28 cm and 150.44 cm for the classes according to the order given in Table 5 (i.e., ED-Z-16, ED-B1-7, . . . , ED-K1-11/B), respectively.

D. INVESTIGATING THE PROBABILITY OF INFECTION

The rate of virus transmission will decrease with increasing social distance (e.g., [28], [29], [30]. For example, according to the UK's Scientific Advisory Group for Emergencies (SAGE), there may be a 2–10 fold increase in the likelihood of SARS-CoV-2 transmission at 1 m compared to 2 m [31]. While [28] states that the infection risk decreases monotonously with distancing, [29] developed a logarithmic function that captures the relationship between social distancing and the likelihood of infection.

$$p_a = \frac{-18.19 \ln(d_{ij}) + 43.276}{100} \tag{24}$$

TABLE 5. The comparison of average distance values found by 3e-MDPs, a-MDPs and pDp.

			3e-MDPs		a-MDPs	pDp
Classroom code	Classroom capacity	# of assignment scenarios	# of unsolved scenarios	Optimal average distance (cm)	Average distance (cm)	Average distance (cm)
EO-K2-5	33	12	-	370.09	370.09	348.20
ED-Z-16	42	21	-	383.35	383.26	360.89
ED-B1-7	56	29	-	444.28	444.23	396.26
ED-Z-5	59	34	-	438.39	438.29	402.53
EA-K1-5	69	42	-	447.30	447.21	392.91
ED-B1-2	72	44	-	447.92	447.87	395.40
EA-K1-2	72	44	-	449.55	449.46	394.54
ED-Z-4B	84	43	-	496.11	496.05	471.77
EA-K1-3A	99	49	-	511.46	511.41	480.77
ED-Z-10	110	53	4	588.30	588.29	536.29
EA-Z-10	116	51	7	604.71	604.59	545.65
EA-K1-3B	121	60	3	566.62	566.55	531.57
ED-K1-11/B	165	79	42	633.29	633.26	591.66

TABLE 6. The probability of virus transmission for each assignment scenario of classroom EO-K2-5.

Assignment scenario(Θ)	a-MDPs (p_w,p_a)	pDp (p_w,p_a)	Assignment scenario(Θ)	a-MDPs (p_w,p_a)	pDp (p_w,p_a)
4	0.169, 0.142	0.169, 0.142	10	0.395, 0.217	0.395, 0.230
5	0.254, 0.176	0.254, 0.187	11	0.395, 0.225	0.395, 0.225
6	0.285, 0.193	0.285, 0.205	12	0.401, 0.227	0.401, 0.245
7	0.313, 0.200	0.313, 0.229	13	0.401, 0.235	0.401, 0.238
8	0.313, 0.212	0.313, 0.218	14	0.420, 0.237	0.420, 0.254
9	0.346, 0.213	0.346, 0.213	15	0.420, 0.245	0.420, 0.247

To investigate the probability of virus transmission with respect to the potential values of social distance, we consider the smallest classroom, EO-K2-5, and run a-MDPs and pDp models for each assignment scenario of students as given in Table 6. The probability value in this table is calculated taking into account the mathematical model in [29] as indicated in Equation 24. In Table 6, p_w represents the probability of getting infection from a host based on the closest distance (which corresponds to $\beth$ found by pDp or Algorithm 1) while p_a represents the probability of getting infection based on the average distance. As known, p_w value must be the same for both a-MDPs and pDp models of each student allocation scenario, while p_a value of the a-MDPs is expected to be lower than p_a value of the pDp as the a-MDPs produces larger average distance value than do model the pDp. As shown in Table 6, the average virus spread rate (p_a) decreased from 22% to 21%, an improvement of 1% achieved when all scenarios were taken into account. An improvement of approximately 3% has been achieved in the transmission probability of the $\Theta = 7$ assignment scenario where p_a is the most reduced. Recall that scenario $\Theta = 7$ stands out as the assignment scenario in which a-MDP achieve the most substantial distance increase compared to pDp, which amounts to an increase of 58.62 cm for the class EO-K2-5. In addition, a reduction in the average probability of virus transmission was observed in other classrooms. For example, using the model a-MDPs instead of the pDp, it was observed that p_a value is reduced by up to approximately 4.75%, 5.48%, and 4.54% for $\Theta = 13$, $\Theta = 4$, and $\Theta = 23$ assignment scenarios of the ED-Z-16, ED-B1-7, and ED-Z-5 classrooms, respectively.

V. THE PANDEMIC MANAGEMENT PLATFORM

This section introduces a pandemic management platform developed to promptly and proactively assist university administration in preparing for and effectively managing future infection outbreaks. More specifically, this platform helps decision makers optimally utilize resources, assess risks, and implement strategic interventions in order to minimize the spread of infection within the university community. First, data are created for each classroom, including the surface area and the number of seats, doors, and windows. Second, the x-y coordinate of each seat was also determined to track the distance between occupied seats. Later, the collected data were incorporated into the developed decision platform, along with the integration of the greedy approximation model (a-MDPs). Note that we do not provide the exact MDPs options in the decision platform, as the computational testing demonstrates the superiority of the model a-MDPs over the exact MDPs models in terms of solution return time.

Figure 2 provides the developed decision support platform for Bartin University. Here, the authorized user first clicks the "First: Upload Data" button which facilitates the user in automatically accessing the data website and enables them to manually select the classrooms for examination. Once a classroom is selected, the relevant data is obtained to optimize student allocation. In Figure 3, the class highlighted with a yellow background represents the selected class. Subsequently, the user clicks the "Second: Generate Scenarios" button, which assists decision-makers in analyzing various student assignment scenarios. For example, Figure 4

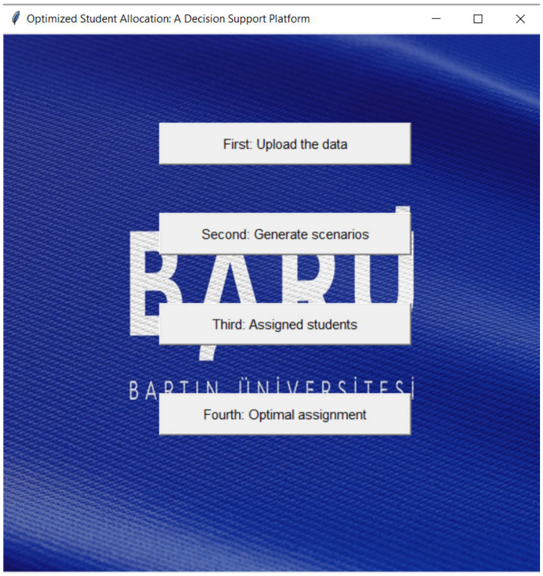

FIGURE 2. Optimized student allocation: A decision support platform.

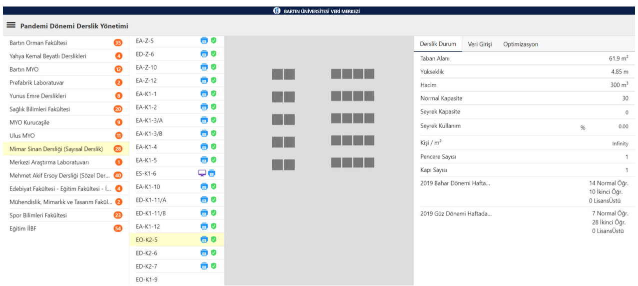

FIGURE 3. Classroom selection for examination.

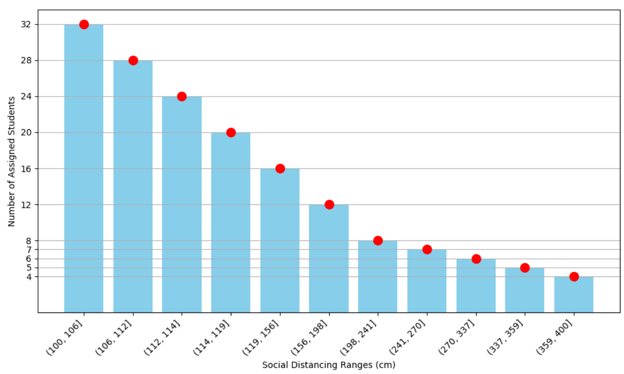

FIGURE 4. Classroom EO-K2-5: Number of assigned students vs. Social distancing ranges.

FIGURE 5. The number of assigned students for EO-K2-5.

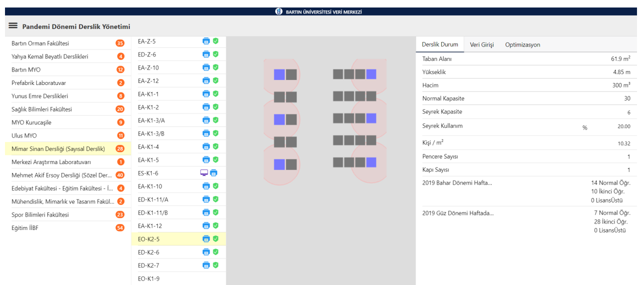

FIGURE 6. Classroom EO-K2-5: Optimal student allocation for classroom EO-K2-5.

explicitly illustrates the number of students assigned and their corresponding social distance values for the classroom EO-K2-5. Assume that the user aims to maintain a distance of at least 1.5 m between students in this classroom.

According to the Figure 4, it suggests that 16 students can be assigned to the classroom, with the minimum distance potentially expanding to 1.56 m. Alternatively, if the user intends to assign 30 students to this classroom, Figure 4 indicates that the distance between the students falls in the range of 1 m to 1.06 m. However, reducing the number of students by 2 (assigning 28 students instead) would allow for an increase of at least 6 cm between each pair of students, increasing the spacing between student pairs from 106 cm to 112 cm. This level of flexibility empowers decision makers to choose policies among various options, providing them with the options to make well-informed decisions. This approach develops a proactive decision-making process that goes beyond simply assigning a predetermined number of students or aiming for a specific social distancing value. Next, the user clicks the "Third: Assigned Students" button, which allows to manually enter the number of students assigned to the classroom, provided in Figure 5. The number in Figure 5 is determined by the user whose decision is based the Figure 4 created by Algorithm 1 for a given classroom. Finally, the user clicks the "Fourth: Optimal Assignment" button, initiating an automatic allocation of students to classrooms. This process utilizes a-MDPs to determine the optimal seating placement. In Figure 6, the positions of six students are represented by squares with a blue background for classroom EO-K-5.

Although Figure 6 might give the impression that the problem is easy to solve, it is, in fact, a challenging problem. The reason is basically due to the number of options to seat students. Namely, the number of potential seating placements

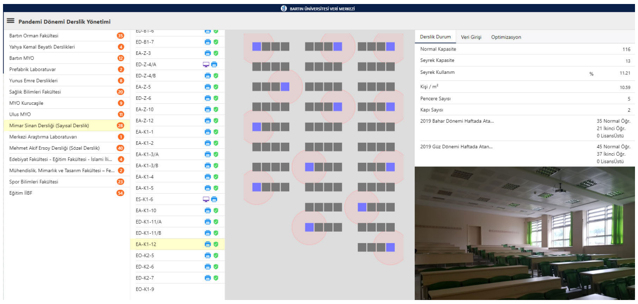

FIGURE 7. Classroom EA-K1-12: Optimal student allocation for classroom EA-K1-12.

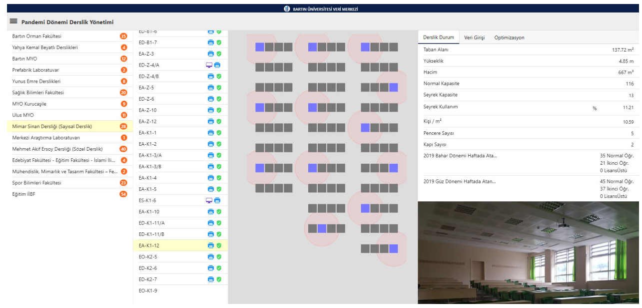

FIGURE 8. Classroom EA-K1-12: The best student allocation produced by MIS for classroom EA-K1-12.

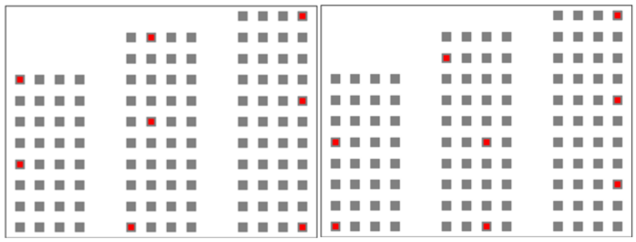

FIGURE 9. Classroom EA-K1-12: 8-student allocation scenario for classroom EA-K1-12.

is $\binom{30}{6} = 593,775$ for Figure 6, which is a very large number. Of course many student placements are likely unfeasible, violating the separation rule stipulated in the constraints of the optimization models, but technically all placements must be explored, implicitly or explicitly, so as to determine and decide that a solution is an optimal solution. More complex seating arrangements for a large classroom, which cannot be solved by mere human intuition, along with the related detailed discussions, are provided in the appendix A.

VI. CONCLUSION

This paper distinguishes itself from the existing literature by not only assigning students as far apart as possible but also focusing on maximizing the average distance between students. Hence, we first develop an algorithm to determine the max-min distance (i.e., $\beth$) for each student allocation scenario. Second, we introduce three exact models and one greedy approximation model, each of which uses $\beth$ input and generates a robust seating plan for students. Based on our computational testing, we observe that the proposed models produce a better seating layout than the one generated from the max-min distance approach of the literature (i.e.,pDp) by notably increasing the average distance between student pairs, thus further reducing the average probability of the virus spread among students. Later, we integrate the a-MDPs and Algorithm 1 into the pandemic management platform developed for Bartin University, which proactively assists the university administration in preparing for and effectively managing future infection outbreaks. Additionally, this platform can be used to design strategies for constructing healthy, sustainable buildings with pandemic-prevention capabilities.

APPENDIX A
ADDITIONAL EXPERIMENTS

This section includes more complex seating arrangements which are designed to showcase scenarios where human intuition falls short, thereby emphasizing the value of our proposed approach. Also, this section provides a detailed comparison of our method with a classical method, the maximum independent set (MIS) algorithm [32].

In Figure 7, the positions of 13 students are represented by squares with a blue background for classroom EA-K1-12, which has a seating capacity of 116. Observe that this seating arrangement is not solvable through human intuition, and there is no straightforward pattern in the optimal seating plan. In this seating plan, the minimum distance between any two students is 2.79 m. When averaging these minimum distances across all pairs, the average distance is 2.95 m. Additionally, the average distance between all pairs of students, considering all possible pairs, is 6.57 m. To place 13 students using the MIS algorithm, one would iterate the algorithm, incrementally adjusting distances to find a seating arrangement that maximizes the minimum distance between any pair of students, though it may not necessarily be the optimal seating arrangement. Figure 8 provides the best possible positions of 13 students produced by MIS algorithm for classroom EA-K1-12. In Figure 8, the minimum distance between students is 2.71 m, with an average of 2.87 m for these distances, and the total average distance is 6.37 m. Our method achieved a minimum distance increase of 8 cm, an average minimum distance increase of 8 cm and the total average distance increase of 20 cm when compared to the MIS.

We also provide an allocation of 8 students, illustrated with simple plots using squares with a red background. Figure 9 depicts the optimal seating layout on the left and the MIS layout on the right for this allocation. Observe that this seating plan is also not straightforward, given the 8 choose 116 scenarios. In the optimal seating plan given on the left of Figure 9, the minimum distance between any two students is 4.21 m. When averaging these minimum distances across all pairs, the average distance is 4.31 m. Additionally, the average distance between all pairs of students, considering all possible pairs, is 7.16 m. For the MIS layout on the right of Figure 9, the minimum distance between students is 4.07 m, with an average of 4.24 m for these distances, and the total average distance is 6.90 m. Our method achieved a minimum distance increase of 14 cm, an average minimum distance increase of 7 cm and the total average distance increase of 26 cm when compared to the MIS. Although not illustrated, our method achieved an average minimum distance increase of 9 cm and 10 cm for 15 and 30 student allocation scenarios, respectively.

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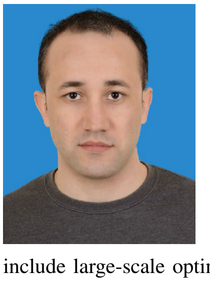

GOKHAN KARAKOSE received the B.S. degree from the Industrial Engineering Program, Eskisehir Osmangazi University, Eskisehir, Turkey, in 2009, and the M.S. and Ph.D. degrees from the Industrial and Manufacturing Systems Engineering Department, University of Missouri, Columbia, MI, USA, in 2013 and 2018, respectively. He is currently an Assistant Professor with the Department of Industrial Engineering, Bartin University, Turkey. His research interests include large-scale optimization, combinatorial optimization, and network optimization.

BAYRAM DUNDAR received the bachelor's degree from Selçuk University, Turkey, in 2008, and the M.S. degree in industrial engineering and the Ph.D. degree in industrial and manufacturing systems from the University of Missouri, in 2013. He is currently an Assistant Professor with the Department of Industrial Engineering, Bartin University, Turkey. His research interests include applying optimization techniques to healthcare, energy, and sustainability-related problems in the presence of uncertainty and include combinatorial optimization. Additionally, he has industry experience as a Production Planning Engineer. He is a member of the Institute for Operations Research and the Management Sciences (INFORMS) and the Institute of Industrial and Systems Engineers (IISE).