IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received 17 July 2024, accepted 3 August 2024, date of publication 9 August 2024, date of current version 19 August 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3440629

RESEARCH ARTICLE

Robust Cooperative Fault-Tolerant Control for Discrete-Time Multi-Agent Systems With Uncertainties and Actuator Faults

JIANTAO SHI, (Senior Member, IEEE), XIANG CHEN, SHUANGQING XING, ANNING LIU, AND CHUANG CHEN, (Member, IEEE)
College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing 211816, China

Corresponding author: Chuang Chen (chenchuang@njtech.edu.cn)

This work was supported in part by the Postgraduate Research and Practice Innovation Program of Jiangsu Province under KYCX24_1600, in part by the National Natural Science Foundation (NNSF) of China under Grant 62373184 and Grant 62303217, in part by the Natural Science Foundation of Jiangsu Province under Grant BK20211502, and in part by the Natural Science Foundation of the Jiangsu Higher Education Institutions of China under Grant 23KJB510006.

ABSTRACT In this paper, we address a class of distributed robust cooperative fault-tolerant control problems for discrete-time multi-agent systems (MASs). The discrete-time MASs considered in this paper have both uncertainties and actuator faults, and a distributed intermediate variable estimator that does not require a fault estimation matching condition is designed for obtaining the estimation information of the actuator faults. In addition, to further obtain higher design freedom and better estimation performance, the intermediate variable estimator considers both centralised and distributed output estimation errors as feedback terms. A fault-tolerant control protocol is designed using the fault estimation information to ensure the reliable operation of discrete-time MASs. It is worth noting that in the method proposed in this paper, there is no need to know the boundary information about the faults and their changing rates, and the method is applicable to discrete-time MASs with directed communication topology. In order to ensure the solvability of the Linear Matrix Inequality (LMI) for better obtaining of the gain matrices of the designed fault estimators, further decoupling and dimensionality reduction of the LMI are carried out in this paper. Finally, a numerical simulation and a network of four one-link flexible joint manipulator systems simulation verify the truthfulness and effectiveness of the method proposed in this paper.

INDEX TERMS Discrete-time multi-agent systems, distributed intermediate variable estimator, cooperative fault tolerance control, directed communication topology.

I. INTRODUCTION

During the last decades of rapid technological development, industrial systems have become larger and more complex. As a powerful processing tool, MASs are often used to describe and analyze the behavior of complex systems. As a result, MASs have attracted a lot of attention from scholars and some remarkable results have been obtained [1], [2], [3], [4]. MASs have been used in various significant areas, such as satellite formation control [5], smart grids systems [6], and flight control systems [7].

MASs is a typically interconnected system which each agent exchanges information with its neighbors to accomplish the entire control task. It is this distributed cooperation that allows MASs to accomplish more complex control tasks with structurally simpler agents. However, a fault in one special agent can propagate to other normal agents through the network, so the probability of MASs fault naturally increases with the number of agents and the fault of a particular agent may seriously affect MASs, which may lead to agent instability or even whole system

The associate editor coordinating the review of this manuscript and approving it for publication was Jinquan Xu.

collapse [8], [9], [10]. Moreover, in the context of rapid technological development, MASs operate under increasingly complex and harsh conditions [11]. Therefore, it is essential to consider the reliable operation of MASs.

Ensuring reliability and safety is essential for the stable operation of the system. In recent years, fault diagnosis techniques for systems have been widely discussed and become a hot research topic. Fault diagnosis mainly consists of fault detection and isolation (FDI) and fault estimation (FE) [12], [13]. FDI are primarily used to detect the occurrence of a fault in the system and to determine its location so as to facilitate the further isolation of the fault. FE is utilized to obtain detailed information such as the shape and magnitude of the fault signal. In fact, FE can serve the purpose of FD-isolation and identification [14]. Fault-Tolerant Control (FTC) protocols can be designed based on the fault estimation information obtained through Fault Estimation (FE) to ensure the reliable operation of the system. In addition, it is generally recognized that the more detailed the fault information obtained, the better the performance of the designed FTC protocols [15]. There have been many excellent results in the field of FE, such as adaptive estimator, robust estimator, sliding mode estimator, etc [16], [17]. However, in most of the existing works, most of them only consider FE for centralized systems, and relatively few study the distributed FE problem for MASs.

For the leader-follower MASs, in order to obtain information on actuator and sensor fault estimation, a novel distributed unknown input observer (UIO) was designed in [18]. In [19], the problem of estimating actuator and sensor faults in MASs with directed graphs is considered based on intermediate variable observers. In [20], a FIR filter-based fault estimation scheme was proposed to estimate fault signals in MASs with stochastic nonlinearities. In [21], sliding-mode observers(SMO) are used to estimate actuator fault in MASs. For the MASs with directed communication topology, [22] designed a full-order fault estimation observers for fault estimation.

However, in some works including [21] and [22], the design of the fault estimator must satisfy the fault estimation matching condition. The limitation of this condition causes great inconvenience in the design of fault estimators for some practical systems. In order to overcome this restriction, [23] and [24] proposed a novel intermediate observers. The method does not need to satisfy neither the fault estimation.

Furthermore, in many of the existing works, including those mentioned above, the fault signal under consideration can only be a constant. Obviously, in the context of modern systems operating in increasingly complex and harsh environments, this assumption is very restrictive and even infeasible. In addition, in some work where time-varying fault signals are considered, it is necessary to know the bounds of the fault. Nevertheless, for most practical control systems, these conditions are highly restrictive. Therefore, the fault estimation of MASs must take into account the above situations.

Based on the fault estimation information, further FTC protocols can be designed. There have been many outstanding results on this aspect: For leader-followers MASs affected by denial-of-service attacks and actuator faults, [25] obtained fault estimation information by designing a novel adaptive edge-event-triggered observer and designed a novel resilient adaptive event-triggered fault-tolerant controller. For a five degree of freedom robotic manipulator, [26] obtains fault estimation information through an adaptive back-stepping methodology, and then designs an actuator and sensor FTC. [27] ingeniously combined fast integral terminal sliding mode control, robust exact differentiator observer, and feed-forward neural network based estimator, a state-of-the-art control algorithm was pioneered to solve the robust control and tuning of the robotic manipulators problem. For a class of MASs with directed and fixed topology, [28] proposed a novel FTC protocol and sidestepped the restriction of the zero initial condition. In order to solve the leader- following MASs consensus control problem, a distributed adaptive fault-tolerant scheme is given in [29]. In [30], a distributed FTC protocol is designed to ensure that discrete-time MASs are subject to link failures and actuator/sensor faults realize information consensus control.

It is important to note that in most of the existing FTC work, the mathematical model must be very accurate. However, uncertainties in the mathematical model of the control system are inevitable due to external disturbances present in the system's operating environment, errors in modeling the system, and so on. A large number of excellent results have emerged to address such problems: In [31], a FTC protocol for multi-area power systems subject to actuator faults was devised, considering both multiplicative and additive perturbations to compensate for uncertainty. In [32], a distributed FTC protocol was designed for leader-following MASs subject to the effects of actuator faults and uncertainties. To resolve the problem of dynamic uncertainties, [33] employed a fuzzy logic method for FTC protocol design and obtained better estimation performance. [34] designed a fault estimation method based on UIO for MASs, accounting for uncertainty effects. However, this method is only applicable to MASs with directed communication topology.

In summary, in most of the existing works, there are few studies on FTC of MASs with uncertainties, and in some SMO and UIO-based works, the control system is required to satisfy the so-called estimation matching condition, which is undoubtedly a restriction. What is more, some of the existing works require the assumption that the fault signal is a constant, or the bounds of the time-varying faults needs to be known. Obviously, the limitations of these conditions are highly restrictive and are not conducive to further promotion and practical application.

In order to solve the above problems, this paper proposes a novel fault estimation method and fault-tolerant control protocol for discrete-time MASs subject to uncertainties and actuator faults. The main contributions of this paper are as follows:

1. In order to avoid the restriction imposed by fault estimation matching condition, this paper designs an intermediate estimator for each agent in MASs for FE. Using this fault estimation information, an FTC protocol is designed to eliminate the adverse effects caused by actuator faults and uncertainties in MASs.

2. The method proposed in this paper is applicable to time-varying faults and does not require the upper bounds of the fault and its changing rate.

3. In the fault estimation method proposed in this paper, both centralised and distributed output estimation errors are considered as feedback terms for the intermediate estimator, so that it has the advantages of both distributed and centralized structures, which greatly enhances the design flexibility and makes the estimation performance further improved.

4. The work in this paper applies to MASs with directed communication topologies.

Notations: In this article, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. For a vector $l \in \mathbb{R}^n$, $\|l\|$ represents the 2-Norm. I_N denotes a diagonal matrix formed by N unit matrices. For a matrix $\mathcal{S}$, $He[\mathcal{S}] = \mathcal{S} + \mathcal{S}^T$, $\lambda_{\max}(\mathcal{S})$ denotes the maximum eigenvalues and $\lambda_{\min}(\mathcal{S})$ means minimum eigenvalues.

II. PROBLEM STATEMENT

In this paper, consider the following discrete-time MASs described by:

$$\begin{cases} x_i(k+1) = (A + \Delta A(k))x_i(k) + Bu_i(k) + Ef_i(k) \\ y_i(k) = Cx_i(k) \end{cases} \tag{1}$$

where $x_i(k) \in \mathbb{R}^n$ is the state of agent i, $u_i(k) \in \mathbb{R}^m$ is control protocol and $y_i(k) \in \mathbb{R}^q$ is and output information. The signal $f_i(k) \in \mathbb{R}^r$ indicates actuator faults when $E = B$. The perturbed matrix $\Delta A(k)$ represents the uncertainties considered in this paper and satisfies $\Delta A(k) = MF(k)N$, where $F^T(k)F(k) \leq I$. A, B, C, and E are system constant matrix of the system with appropriate dimensions.

Assumption 1: The bounds of the fault and its changing rates are unknown, i.e., the $\|\Delta f_i\| = \|f_i(k+1) - f_i(k)\| \leq \theta$, where θ is a positive real number.

Assumption 2: For every complex number s, the following equation holds:

$$rank \begin{bmatrix} A - sI & E \\ C & 0 \end{bmatrix} = n + r$$

Assumption 3: rank (B, E) = rank (B).

Assumption 4: (A, B) is stabilizable and (A, C) is observable.

Remark 1: In fact, Assumption 1 suggests that the faults are energy bounded, which is a very reasonable and natural assumption in physics and practical systems. For estimating time-varying signals, this assumption of bounds of energy is commonly used in FDI [12], [13] and FTC [15]. But notice that θ can be unknown, so the method proposed in this paper is more practical and natural than most traditional observers [16], [17], [21].

Remark 2: Assumption 2 means that the system (A, C, E) has a fixed number of zeros in the left-half and the matrix E is column-full rank. Assumption 2 is also common and natural in most existing works on FD, FI, and FE. It should be noted that Assumption 2 is not strictly constrained in real systems compared to the estimator matching condition.

Remark 3: Assumption 3 is common in FTC problems and implies the possibility that faults can be compensated for by the inputs of a fault-tolerant control protocol.

Remark 4: Assumption 4 is a common and essential requirement in FTC. Observability is necessary for the design of state observers and stabilizability is crucial for ensuring stable control of the system.

For further work, the following lemma is proposed:

Lemma 1 (Young's Inequality [19]): The following inequality always holds:

$$a^T b \leqslant \frac{1}{p}\alpha^p \|a\|^p + \frac{1}{q}\alpha^{-q} \|a\|^q$$

where $a, b \in \mathbb{R}^n$ and $\alpha > 0$, $p > 0$, $q > 0$, and $pq = p + q$.

For discrete-time MASs with uncertainties and actuator faults, this paper will design a novel intermediate fault estimator to obtain the fault estimation information so as to design an FTC protocol to ensure the reliable operation of MASs.

III. INTERMEDIATE ESTIMATOR DESIGN

In order to design an intermediate fault estimator for agent i, the following intermediate $\xi_i(k)$ variable is denoted:

$$\xi_i(k) = f_i(k) - \omega E^T x_i(k) \tag{2}$$

where ω is the intermediate constant.

Based on (1) and (2), we have:

$$\begin{aligned} \xi_i(k+1) = f_i(k+1) - \omega E^T((A + \Delta A)x_i(k) \\ + Bu_i(k) + E\xi_i(k) + \omega EE^T x_i(k)) \end{aligned} \tag{3}$$

And then, the our intermediate estimator is as shown below:

$$\begin{cases} \hat{x}_i(k+1) = A\hat{x}_i(k) + Bu_i(k) + E\hat{f}_i(k) \\ \qquad + \rho_1 L_1 \Gamma_{1i}(k) + \rho_2 L_2 \Gamma_{2i}(k) \\ \hat{\xi}_i(k+1) = -\omega E^T E\hat{\xi}_i(k) - \omega E^T(A\hat{x}_i(k) \\ \qquad + Bu_i(k) + \omega EE^T \hat{x}_i(k)) \\ \hat{f}_i(k) = \hat{\xi}_i(k) + \omega E^T \hat{x}_i(k) \\ \hat{y}_i(k) = C\hat{x}_i(k) \end{cases} \tag{4}$$

where $\hat{x}_i(k)$, $\hat{\xi}_i(k)$, $\hat{f}_i(k)$ and $\hat{y}_i(k)$ are the estimation of $x_i(k)$,$\xi_i(k)$,$f_i(k)$ and $y_i(k)$, respectively. In addition, $\Gamma_{1i}(k)$ and $\Gamma_{2i}(k)$ are defined as

$$\Gamma_{1i}(k) = y_i(k) - \hat{y}_i(k) \tag{5}$$

$$\Gamma_{2i}(k) = \sum_{j=1}^{N} a_{ij}[(y_i(k) - \hat{y}_i(k)) - (y_j(k) - \hat{y}_j(k))] \tag{6}$$

where $\Gamma_{1i}(k)$ and $\Gamma_{2i}(k)$ means centralized and distributed output estimation errors, respectively. $L_1, L_2 \in \mathbb{R}^n$ represent the respective gain matrices. Non-negative constants ρ_1 and ρ_2 represent the weights of each of the two feedback terms and satisfy $\rho_1 + \rho_2 = 1$.

Remark 5: From (4), it is not difficult to be found that the intermediate estimator designed in this paper has both distributed and centralised structural characteristics due to the introduction of the feedback terms $\Gamma_{1i}(k)$ and $\Gamma_{2i}(k)$. The intermediate estimator can be adjusted by the complementary regulation of ρ_1 and ρ_2. Through the complementary regulation of ρ_1 and ρ_2, the influence of neighbouring nodes on itself can be increased or decreased, and the exact size should be based on the operating conditions of the actual system. It should be noted that in practical applications, this design can greatly improve the flexibility and freedom of design, thus obtaining better estimation performance.

IV. THEORETICAL ANALYSIS

Next, we need to construct the global error dynamic system and analyse its convergence.

Define $e_{x_i}(k) = x_i(k) - \hat{x}_i(k)$, $e_{\xi_i}(k) = \xi_i(k) - \hat{\xi}_i(k)$, $e_{f_i}(k) = f_i(k) - \hat{f}_i(k)$. Since $e_{f_i}(k) = e_{\xi_i}(k) + \omega E^T e_{x_i}(k)$, we have:

$$\begin{aligned} e_{x_i}(k+1) &= (A - \rho_1 LC)e_{x_i}(k) + Ee_{\xi_i}(k) + \omega EE^T e_{x_i}(k) \\ &\quad + \Delta A x_i(k) - \rho_2 L_2 \Gamma_{2i}(k) \end{aligned} \tag{7}$$

$$\begin{aligned} e_{\xi_i}(k+1) &= -\omega E^T E e_{\xi_i}(k) - \omega E^T A e_{x_i}(k) - \omega^2 EE^T E e_{x_i}(k) \\ &\quad - \omega E^T \Delta A x_i(k) + \Delta f_i \end{aligned} \tag{8}$$

Based on Assumption 3, it implies the existence of a matrix B^* such that $(I - BB^*)E = 0$. Using the obtained fault estimation information $\hat{f}_i(k)$, we propose the following FTC protocol:

$$u_i(k) = -K\hat{x}_i(k) - B^* E \hat{f}_i(k) \tag{9}$$

where the matrix K must be such that$(A - BK)$ is a stable matrix.

Based on (9) and (1), one has

$$\begin{aligned} x_i(k+1) &= (A - BK)x_i(k) + \Delta A x_i(k) + BKe_{x_i}(k) \\ &\quad + Ee_{\xi_i}(k) + \omega EE^T e_{x_i}(k) \end{aligned} \tag{10}$$

From(7), (8) and (10), we can get the global error dynamic system:

$$\begin{aligned} x(k+1) &= (I_N \otimes (A - BK))x(k) + (I_N \otimes \Delta A)x(k) \\ &\quad + (I_N \otimes BK)e_x(k) + (I_N \otimes E)e_\xi(k) \\ &\quad + (I_N \otimes \omega EE^T)e_x(k) \end{aligned} \tag{11}$$

$$\begin{aligned} e_x(k+1) &= (I_N \otimes (A - \rho_1 L_1 C))e_x(k) + (I_N \otimes E)e_\xi(k) \\ &\quad + (I_N \otimes \omega EE^T)e_x(k) + (I_N \otimes \Delta A)x(k) \\ &\quad - (L \otimes \rho_2 L_2 C)e_x(k) \end{aligned} \tag{12}$$

$$\begin{aligned} e_\xi(k+1) &= -(I_N \otimes \omega E^T E)e_\xi(k) - (I_N \otimes \omega E^T A)e_x(k) \\ &\quad - (I_N \otimes \omega^2 EE^T E)e_x(k) - (I_N \otimes \omega E^T \Delta A)x(k) \\ &\quad + \Delta f \end{aligned} \tag{13}$$

where $x(k) = [x_1(k), x_2(k), \ldots, x_N(k)]^T$, $e_x(k) = [e_{x_1}(k), e_{x_2}(k), \ldots, e_{x_N}(k)]^T$, $e_\xi(k) = [e_{\xi_1}(k), e_{\xi_2}(k), \ldots, e_{\xi_N}(k)]^T$, $\Delta f = [\Delta f_1, \Delta f_2, \ldots, \Delta f_N]^T$

Theorem 1: If there actually exists matrix $P_1, P_2, P_3 > 0$, Q, and the constant $\delta > 0$ such that the following LMI holds:

$$\Pi^i = \begin{bmatrix} \tilde{\Pi}_{11}^i & \tilde{\Pi}_{12}^i \\ * & \tilde{\Pi}_{22}^i \end{bmatrix} < 0, i = 1, 2, \ldots, N \tag{14}$$

where $\tilde{\Pi}_{11}^i = \begin{bmatrix} \Pi_{11}^i & \Pi_{12}^i & \Pi_{13}^i \\ * & \Pi_{22}^i & \Pi_{23}^i \\ * & * & \Pi_{33}^i \end{bmatrix}$ *with* $\Pi_{11}^i = He[P_1 A - P_1 BK] + \frac{1}{\varepsilon_1}N^T N + \frac{1}{\varepsilon_2}N^T N + \frac{1}{\varepsilon_3}N^T N$, $\Pi_{12}^i = P_1 BK + \omega P_1 EE^T$, $\Pi_{13}^i = P_1 E$, $\Pi_{22}^i = He[(P_2 A - \rho_1 QC) + (\omega P_2 EE^T)] + \delta\rho_2\lambda_i C^T C$, $\Pi_{23}^i = P_2 E - \omega A^T EP_3 - \omega^2 EE^T EP_3$, $\Pi_{33}^i = -He[\omega P_3 E^T E]$,

$$\tilde{\Pi}_{22}^i = \operatorname{diag}\left\{-\frac{1}{\varepsilon_1}, -\frac{1}{\varepsilon_2}, -\frac{1}{\varepsilon_3}, -\frac{1}{\varepsilon_4}\right\},$$

$$\tilde{\Pi}_{12}^i = \begin{bmatrix} P_1 M & 0 & 0 & 0 \\ 0 & P_2 M & 0 & 0 \\ 0 & 0 & \omega P_3 E^T M & P_3 \end{bmatrix}.$$

and the observer gain matrices is designed to $L_1 = P_2^{-1} Q_2$, $L_2 = \delta P_2^{-1} C^T$. Then, based on Assumption 1-4, the intermediate estimator (4) make sure that the error dynamic system (11)-(13) is uniformly ultimately bounded for the given positive constants $\omega > 0$, $\varepsilon_i > 0 (i = 1, 2, 3, 4)$.

Proof: Consider the Lyapunov function shown below:

$$\begin{aligned} V(k) &= x^T(k)(I_N \otimes P_1)x(k) + e_x^T(k)(I_N \otimes P_2)e_x(k) \\ &\quad + e_\xi^T(k)(I_N \otimes P_3)e_\xi(k) \end{aligned} \tag{15}$$

From (11) - (13), we have

$$\begin{aligned} \Delta V &= V(k+1) - V(k) \\ &= x^T(k) He[I_N \otimes (P_1 A - P_1 BK)]x(k) \\ &\quad + 2x^T(I_N \otimes P_1 MF(k)N)x(k) \\ &\quad + 2x^T(I_N \otimes P_1 BK)e_x(k) \\ &\quad + 2x^T(k)(I_N \otimes P_1 E)e_\xi(k) \\ &\quad + 2x^T(k)(I_N \otimes \omega P_1 EE^T)e_x(k) \\ &\quad - 2e_\xi^T(k)(I_N \otimes \omega P_3 E^T A)e_x(k) \\ &\quad + e_x^T(k) He[I_N \otimes (P_2 A - \rho_1 P_2 L_1 C)]e_x(k) \\ &\quad + e_x^T(k) He[L \otimes (P_2 A - \rho_2 P_2 L_2 C)]e_x(k) \\ &\quad + 2e_x^T(k)(I_N \otimes P_2 E)e_\xi(k) \\ &\quad + 2e_x^T(k)(I_N \otimes \omega P_2 EE^T)e_x(k) \end{aligned}$$

$$
\begin{aligned}
&- 2\omega e_{\xi}^{T}(k)(I_N \otimes P_3E^TMF(k)N)x(k)\\
&+ 2e_x^T(k)(I_N \otimes P_2MF(k)N)x(k)\\
&- 2e_{\xi}^T(k)(I_N \otimes \omega P_3E^TE)e_{\xi}(k)\\
&- 2e_{\xi}^T(k)(I_N \otimes \omega^2P_3E^TEE^T)e_x(k)\\
&+ 2e_{\xi}^T(k)(I_N \otimes P_3)\Delta f
\end{aligned} \tag{16}
$$

Based on Lemma 1, the following inequalities hold for positive constants ε_1, ε_1, ε_3, and ε_4:

$$
\begin{aligned}
&2x^T(k)(I_N \otimes P_1M_1F_1(k)N_1)x(k)\\
&\le \varepsilon_1x^T(I_N \otimes P_1M_1)(I_N \otimes P_1M_1)^Tx(k)\\
&\quad + \frac{1}{\varepsilon_1}x^T(I_N \otimes N_1^TN_1)x(k)\\
&2e_x^T(k)(I_N \otimes P_2M_1F_1(k)N_1)x(k)\\
&\le \varepsilon_2e_x^T(k)(I_N \otimes P_2M_1)(I_N \otimes M_1^TP_2)e_x(k)\\
&\quad + \frac{1}{\varepsilon_2}x^T(k)(I_N \otimes N_1^TN_1)x(k)\\
&- 2e_{\xi}^T(k)(I_N \otimes \omega P_3E^TM_1F_1(k)N_1)x(k)\\
&\le \varepsilon_3e_{\xi}^T(k)(I_N \otimes \omega P_3E^TM_1)(I_N \otimes \omega M_1^TEP_3)e_{\xi}(k)\\
&\quad + \frac{1}{\varepsilon_3}x^T(k)(I_N \otimes N_1^TN_1)x(k)
\end{aligned} \tag{17}
$$

From Assumption 1, there will always exist a positive real number θ_N such that the following equation holds:

$$
\begin{aligned}
2e_{\xi}^T(k)(I_N \otimes P_3)\Delta f \le \frac{1}{\varepsilon_4}e_{\xi}^T(k)(I_N \otimes P_3)(I_N \otimes P_3)e_{\xi}(k)\\
+ \varepsilon_4\theta_N
\end{aligned} \tag{18}
$$

Furthermore, based on (17) and (18), we have

$$
\begin{aligned}
\Delta V \le\ & x^T(k)\left[He[I_N \otimes (P_1A - P_1BK)]\right]x(k)\\
&+ \varepsilon_1x^T(k)(I_N \otimes P_1M_1M_1{}^TP_1)x(k)\\
&+ \frac{1}{\varepsilon_1}x^T(k)(I_N \otimes N_1^TN_1)x(k)\\
&+ 2x^T(k)(I_N \otimes \omega P_1EE^T)e_x(k)\\
&+ 2x^T(I_N \otimes P_1BK)e_x(k)\\
&+ 2x^T(k)(I_N \otimes P_1E)e_{\xi}(k)\\
&+ e_x^T(k)\left[He[I_N \otimes (P_2A - \rho_1QC)]\right]e_x(k)\\
&+ e_x^T(k)\left[(L+L^T) \otimes (\delta\rho_2C^TC)\right]e_x(k)\\
&+ 2e_x^T(k)(I_N \otimes P_2E)e_{\xi}(k)\\
&+ 2e_x^T(k)(I_N \otimes \omega P_2EE^T)e_x(k)\\
&+ \varepsilon_2e_x^T(k)(I_N \otimes P_2M_1M_1^TP_2)e_x(k)\\
&+ \frac{1}{\varepsilon_2}x^T(k)(I_N \otimes N_1^TN_1)x(k)\\
&- 2e_{\xi}^T(k)(I_N \otimes \omega P_3E^TE)e_{\xi}(k)\\
&- 2e_{\xi}^T(k)(I_N \otimes \omega P_3E^TA)e_x(k)\\
&- 2e_{\xi}^T(k)(I_N \otimes \omega^2P_3E^TEE^T)e_x(k)\\
&+ \varepsilon_3e_{\xi}^T(k)(I_N \otimes \omega^2P_3E^TM_1M_1^TEP_3)e_{\xi}(k)\\
&+ \frac{1}{\varepsilon_3}x^T(k)(I_N \otimes N_1^TN_1)x(k)\\
&+ \frac{1}{\varepsilon_4}e_{\xi}^T(k)(I_N \otimes P_3P_3)e_{\xi}(k) + \varepsilon_4\theta_N
\end{aligned} \tag{19}
$$

where $Q = P_2L_1$, $L_2 = \delta P_2^{-1}C^T$. Obviously, (19) is equivalent to

$$
\Delta V \le \tilde{x}^T(k)\Phi\tilde{x}(k) + \varepsilon_4\theta_N \tag{20}
$$

where

$$
\tilde{x}(k) = \begin{bmatrix} x(k)\\ e_x(k)\\ e_{\xi}(k)\end{bmatrix}, \Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13}\\ * & \Phi_{22} & \Phi_{23}\\ * & * & \Phi_{33}\end{bmatrix} \tag{21}
$$

and $\Phi_{11} = He[I_N \otimes (P_1A - P_1BK)] + \varepsilon_1(I_N \otimes P_1MM^TP_1) + \frac{1}{\varepsilon_1}(I_N \otimes N^TN) + \frac{1}{\varepsilon_2}(I_N \otimes N^TN) + \frac{1}{\varepsilon_3}(I_N \otimes N^TN)$, $\Phi_{12} = (I_N \otimes P_1BK) + (I_N \otimes \omega P_1EE^T)$, $\Phi_{13} = (I_N \otimes P_1E)$, $\Phi_{22} = He[(I_N \otimes (P_2A - \rho_1QC)) + (I_N \otimes \omega P_2EE^T)] + (L+L^T) \otimes (\delta\rho_2C^TC) + (I_N \otimes P_2MM^TP_2)$, $\Phi_{23} = (I_N \otimes P_2E) - (I_N \otimes \omega A^TEP_3) - (I_N \otimes \omega^2EE^TEP_3)$, $\Phi_{33} = -He[(I_N \otimes \omega P_3E^TE)] + \varepsilon_3(I_N \otimes \omega^2P_3E^TMM^TEP_3) + \varepsilon_4(I_N \otimes P_3P_3)$.

According to (15), it can be obtained that

$$
\begin{aligned}
V(k) \le\ & \max\left[\lambda_{\max}(P_1), \lambda_{\max}(P_2), \lambda_{\max}(P_3)\right]\\
&\left(\| x(k) \|^2 + \|e_x(k)\|^2 + \left\|e_{\xi}(k)\right\|^2\right)\\
=\ & \max\left[\lambda_{\max}(P_1), \lambda_{\max}(P_2), \lambda_{\max}(P_3)\right]\|e_{\tilde{x}}(k)\|^2
\end{aligned} \tag{22}
$$

It yields that

$$
\|e_{\tilde{x}}(k)\|^2 \ge \frac{V(k)}{\max\left[\lambda_{\max}(P_1), \lambda_{\max}(P_2), \lambda_{\max}(P_3)\right]} \tag{23}
$$

From (20), we have

$$
\Delta V \le \lambda_{\max}(\Phi)\|e_{\tilde{x}}(k)\|^2 + \varepsilon_4\theta_N \tag{24}
$$

Due to $\lambda_{\max}(\Phi) < 0$, one has

$$
\Delta V \le \kappa V(k) + \alpha \tag{25}
$$

where $\kappa = \frac{\lambda_{\max}(\Phi)}{\max[\lambda_{\max}(P_1)\lambda_{\max}(P_2), \lambda_{\max}(P_3)]} < 0$, $\alpha = \varepsilon_4\theta_N$.

Define a set Ω:

$$
\begin{aligned}
\Omega = \{&\left(x(k), e_x(k), e_{\xi}(k)\right) \mid \lambda_{\min}(P_1)\|x(k)\|^2 + \lambda_{\min}(P_2)\\
&\|e_x(k)\|^2 + \lambda_{\min}(P_3)\|e_{\xi}(k)\|^2 \le -\frac{\alpha}{\kappa}\}
\end{aligned} \tag{26}
$$

And let $\bar{\Omega}$ represents the supplementary set of Ω. Obviously, if $\left(x(k), e_x(k), e_{\xi}(k)\right) \in \bar{\Omega}$, the following inequality holds:

$$
\begin{aligned}
V(k) \ge\ & \lambda_{\min}(P_1)\| x(k)\|^2 + \lambda_{\min}(P_2)\|e_x(k)\|^2\\
&+ \lambda_{\min}(P_3)\left\|e_{\xi}(k)\right\|^2\\
\ge\ & -\frac{\alpha}{\kappa}
\end{aligned} \tag{27}
$$

Based on (25) and (27), if $\left(x(k), e_x(k), e_{\xi}(k)\right) \in \bar{\Omega}$, one has

$$
\Delta V \le 0 \tag{28}
$$

Therefore, the error system $\big(x(k), e_x(k), e_\xi(k)\big)$ is uniformly bounded and will converge to the specified set Ω at a rate greater than $e^{\alpha t}$.

Clearly, as long as the $\Phi < 0$ holds, we can ensure that the global error (11)-(13) is asymptotically convergence. However, it is evident that $\Phi < 0$ exhibits high dimensionality and nonlinear characteristics. This is not beneficial to solving for LMI. Therefore, we need to make further decoupling and dimensionality reduction.

In (21), considering that the matrix $L + L^T$ is a real symmetric matrix, we have:

$$L + L^T = \bar{U}\bar{\Lambda}\bar{U}^T \tag{29}$$

where orthogonal matrix $\bar{U}$ is constructed from the eigenvectors of $L + L^T$, and $\bar{\Lambda} = \text{diag}\left\{\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_N\right\}$, $\bar{\lambda}_i\,(i = 1, 2, \ldots, N)$ are the corresponding eigenvalue of $L + L^T$. The following orthogonal matrix is proposed:

$$\mathcal{T} = \begin{bmatrix} \bar{U}^T \otimes I_n & 0 & 0 \\ 0 & \bar{U}^T \otimes I_n & 0 \\ 0 & 0 & \bar{U}^T \otimes I_r \end{bmatrix} \tag{30}$$

And then, by pre-multiplying and post-multiplying $\Phi < 0$ with $\mathcal{T}$ and its transpose matrix, one has

$$\tilde{\Phi} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ * & \tilde{\Phi}_{22} & \Phi_{23} \\ * & * & \Phi_{33} \end{bmatrix} < 0 \tag{31}$$

where $\tilde{\Phi}_{22} = He[(I_N \otimes (P_2A - \rho_1 Q_2 C)) + (I_N \otimes \omega P_2 E E^T)] + \bar{\Lambda} \otimes (\delta\rho_2 C^T C) + I_N \otimes P_2 M_1 M_1^T P_2$. It is noted that the remaining terms are identical to those in (21). Finally, utilizing the Schur complement lemma, we can get (14). In this way, we complete the proof of Theorem 1. □

Remark 6: In fact, an LMI with too high dimensions is not favourable for us to find its feasible solution, and may even lead to no solution for that LMI. Therefore, the dimensionality reduction of the LMI is necessary in order to obtain the estimator gain matrix successfully.

Remark 7: From (11)-(13), it is evident that there exists coupling among $x(k)$, $e_x(k)$, and $e_\xi(k)$. Therefore, the choice of a Lyapunov function(15) containing $x(k)$, $e_x(k)$, and $e_\xi(k)$ is very rigorous and crucial during the convergence analysis.

Remark 8: In the above proof process, we have utilised the spectral decomposition of the real symmetric matrix $L + L^T$ for decoupling and dimensionality reduction of the LMI, and it is not difficult to find out that it does not matter whether the matrix L is a symmetric matrix or not. Therefore, the method proposed in this paper is applicable to MASs with directed communication topology.

Remark 9: In the course of the above analysis, we obtain explicit bounds on convergence of the global error system, i.e., $-\frac{\alpha}{\kappa}$. Clearly, by adjusting the parameters K, ω, and ε_j ($j = 1, 2, 3, 4$), we can get a large enough range of convergence. The convergence rate is also given a quantitative process. Moreover, once K and ω are determined, we can obtain the observer gain matrix via the LMI. Conversely, if a feasible solution exists for LMI, the estimation performance can be improved by adjusting the parameters K and ω. Overall, the parameter design method provided in this paper is highly flexible and can lead to better estimation and control performance.

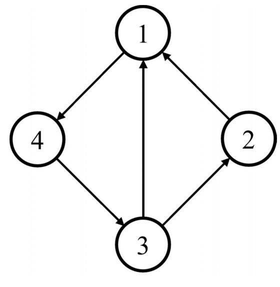
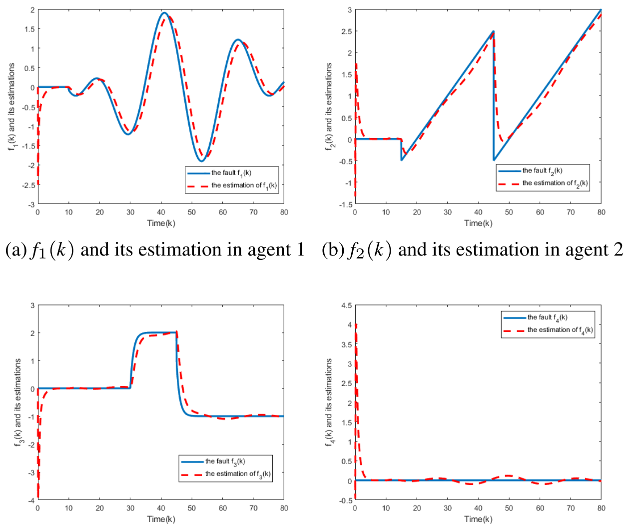


FIGURE 1. Topology of MASs.



(a) $f_1(k)$ and its estimation in agent 1 (b) $f_2(k)$ and its estimation in agent 2

(c) $f_3(k)$ and its estimation in agent 3 (d) $f_4(k)$ and its estimation in agent 4

FIGURE 2. Faults and its estimations in Example 1.

V. NUMERICAL SIMULATION

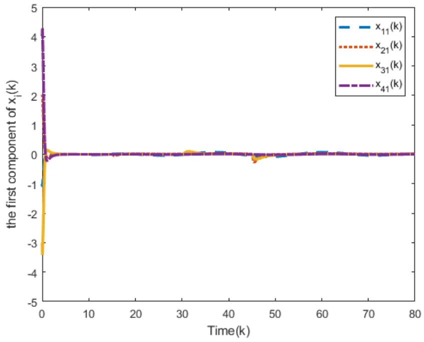
In this subsection, the veracity and effectiveness of the proposed method are verified through two simulation examples.

Example 1: In this example, MASs with four agents is considered. The topology of the MASs is shown in Fig.1. And then, we can get the adjacency matrix $\mathcal{A}$ and the Laplacian matrix $\mathcal{L}$:

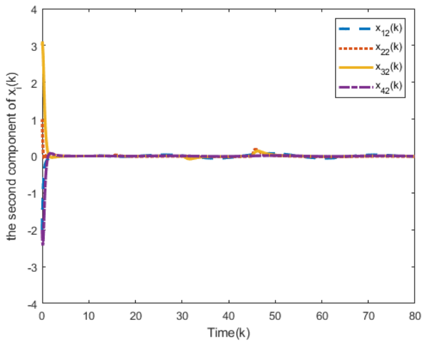
$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathcal{L} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Clearly, the Laplacian matrix $\mathcal{L}$ is not symmetric. Consider a system with the following parameters:

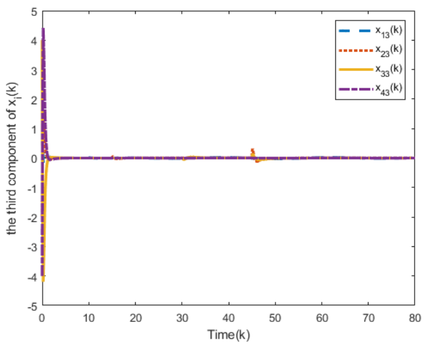
$$A = \begin{bmatrix} -2 & -1 & 3 \\ -3 & 2 & 1 \\ -2 & 1 & -1 \end{bmatrix}, B = E = \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}, C = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



(a) the first component of $x_i(k)$



(b) the second component of $x_i(k)$



(c) the third component of $x_i(k)$

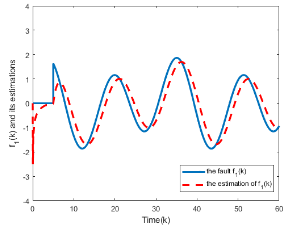
FIGURE 3. Component states of $x_i(k)$ in Example 1.

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, N = \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F(k) = \sin(k)$$

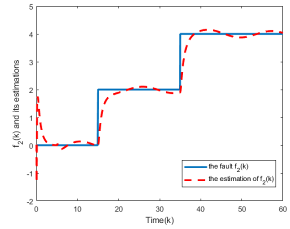
Apparently, rank(CE) $\neq$ rank(E), that is, the fault estimation matching condition is not satisfied. In order to make the matrices $(A - BK)$ to be stable, the gain matrix K is chosen as $K = \begin{bmatrix} 9.1843 & -6.6568 & -6.8801 \end{bmatrix}$.

And then, we consider the following faults in each agent separately:

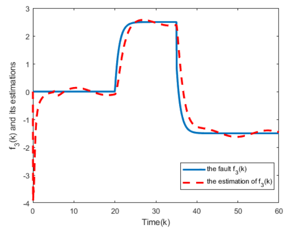
$$f_1(k) = \begin{cases} 0 & 0s \leqslant k \leqslant 10s \\ \sin(0.2k) + \cos(0.3k) & 10s < k \leqslant 80s \end{cases}$$



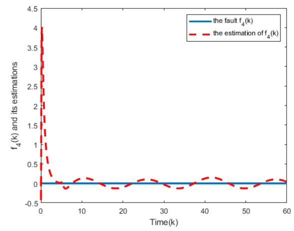
(a) $f_1(k)$ and its estimation in agent 1



(b) $f_2(k)$ and its estimation in agent 2



(c) $f_3(k)$ and its estimation in agent 3



(d) $f_4(k)$ and its estimation in agent 4

FIGURE 4. Faults and its estimations in Example 2.

$$f_2(k) = \begin{cases} 0 & 0s \leqslant k \leqslant 15s \\ 0.1k - 2 & 15s < k \leqslant 45s \\ 0.1k - 5 & 45s < k \leqslant 80s \end{cases}$$

$$f_3(k) = \begin{cases} 0 & 0s \leqslant k \leqslant 30s \\ 2\left(1 - e^{-(k-30)}\right) & 30s < k \leqslant 45s \\ 1 - 2\left(1 - e^{-(k-45)}\right) & 45s < k \leqslant 80s \end{cases}$$

$$f_4(k) = 0 \quad 0s \leqslant k \leqslant 80s$$

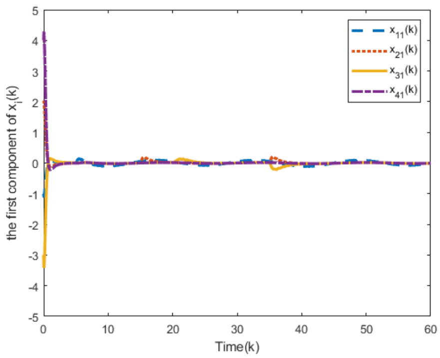
In addition, we choose sampling time as 0.001s and the weight value as $\rho_1 = 0.25$, $\rho_2 = 0.75$, *respectively. Let the intermediate constant* $\omega = 0.3$. *Form Theorem 1, we can get the gain matrices of fault estimator (4):*

$$L_1 = 10^3 * \begin{bmatrix} 17.5310 & -3.9315 \\ -32.1705 & 2.0079 \\ 69.3836 & -66.3622 \end{bmatrix}$$

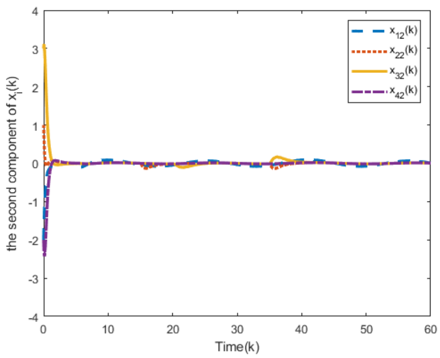
$$L_2 = 10^3 * \begin{bmatrix} -0.0261 & -0.3609 \\ 0.0882 & 0.4167 \\ -0.9831 & -2.3536 \end{bmatrix}$$

The simulation results are shown in Figs.2-3. Figs.2a-2d illustrate that the estimator proposed in this paper is satisfactory in terms of estimation performance. In these figures, the solid line represents the actual fault, while the dashed line represents its estimation. Since agent 4 is fault-free, its fault estimation is consistently near zero. Figs.3a-3c give the convergence of the states of each agent under the fault-tolerant control protocol proposed in this paper. It can be seen that the states of all the agents converge to a small 0 domain, even if affected by actuator faults and uncertainties. Obviously, the method proposed in this paper is realistic and effective.

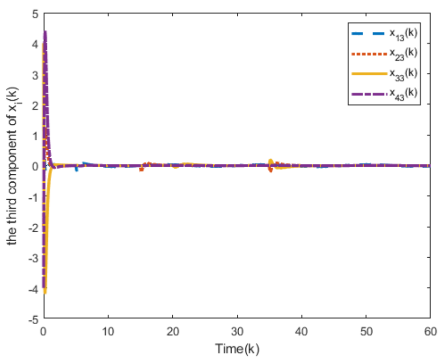
Example 2: In this example, a network of four one-link flexible joint manipulator systems [35] will be considered to validate the method proposed in this paper. The topology of the MASs is also shown in Fig.1.



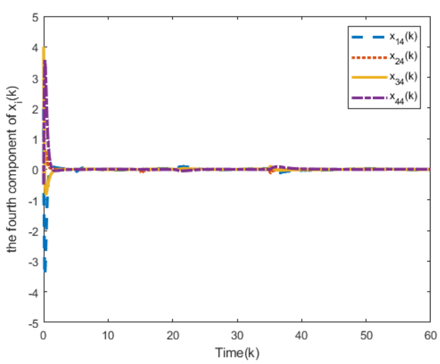
(a) the first component of $x_i(k)$



(b) the second component of $x_i(k)$



(c) the third component of $x_i(k)$



(d) the third component of $x_i(k)$

FIGURE 5. Component states of $x_i(k)$ in Example 2.

The system parameter matrices A, B, C, and E are shown as in [35], that is:

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ -48.6 & -1.25 & 48.6 & 0\\ 0 & 0 & 0 & 10\\ 1.95 & 0 & -1.951 & 0\end{bmatrix},$$

$$B=E=\begin{bmatrix}0\\21.6\\0\\0\end{bmatrix}, C=\begin{bmatrix}1\ 0\ 0\ 0\\0\ 0\ 1\ 0\\0\ 0\ 0\ 1\end{bmatrix}.$$

Consider the same M,N and $F(k)$ as in Example 1. Clearly, the observer matching condition is also not satisfied in this example. To ensure the stability of the matrix $(A-BK)$, let the gain matrix be: $K=\begin{bmatrix}3.5134 & -0.9144 & -2.9466 & -1.0852\end{bmatrix}$.

Choose the following faults in each of the four agents:

$$f_1(k)=\begin{cases}0 & 0s\leqslant k\leqslant 5s\\ 1.5\sin(0.4k)+0.5\cos(0.2k) & 5s<k\leqslant 60s\end{cases}$$

$$f_2(k)=\begin{cases}0 & 0s\leqslant k\leqslant 15s\\ 2 & 15s<k\leqslant 35s\\ 4 & 35s<k\leqslant 60s\end{cases}$$

$$f_3(k)=\begin{cases}0 & 0s\leqslant k\leqslant 20s\\ 2\left(1-e^{-(k-20)}\right) & 20s<k\leqslant 35s\\ 1-2\left(1-e^{-(k-35)}\right) & 35s<k\leqslant 60s\end{cases}$$

$$f_4(k)=0\ 0s\leqslant k\leqslant 60s$$

In addition, we choose sampling time as 0.001s and the weight value as $\rho_1=0.5$, $\rho_2=0.5$, respectively. Let the intermediate constant $\omega=0.22$. Form Theorem 1, we can get the gain matrices of fault estimator (4):

$$L_1=10^3*\begin{bmatrix}5.4394 & 4.7214 & 11.1519\\ -127.1607 & -113.6532 & -115.4788\\ -16.7119 & 23.8020 & 185.7841\\ -10.1021 & 71.1253 & 278.0609\end{bmatrix}$$

$$L_2=10^3*\begin{bmatrix}2.0216 & -0.1107 & -0.0041\\ -0.2015 & 1.1791 & 0.3599\\ 0.0786 & 0.2189 & 0.9389\\ 0.0821 & 0.7193 & 1.2491\end{bmatrix}$$

The simulation results are shown in Figs.4–5. The simulation results demonstrate that the proposed method remains accurate and effective for the four one-link flexible joint manipulator systems.

VI. CONCLUSION

For discrete-time MASs subject to uncertainties and actuator faults, we presents a novel robust distributed cooperative fault-tolerant control protocol in this paper. In contrast to many existing works, the method proposed in this paper does not require the fault estimation matching condition and does not require prior knowledge of the bounds of the fault and its changing rates. During the design of the fault estimator, both centralised and distributed architectures are considered, obtaining a higher degree of design freedom and better estimation performance. Based on the obtained fault estimation information, a fault tolerant control protocol is proposed.To handle the challenges of high dimensionality and coupling interference in LMI, coordinate transformation and Schur decomposition techniques are employed. These methods effectively reduce and decouple the LMI, ensuring the guarantee of the estimator's gain matrix.

The future direction of the work is as follows: 1) Considering that the fault-tolerant control protocol in this paper requires continuous communication between agents, the introduction of an event-triggering mechanism will be considered in future work [36].; 2) Consider the effect of both control input channel and measurement output channel uncertainties on the MASs FTC problem.

REFERENCES

[1] Y. Cao, W. Yu, W. Ren, and G. Chen, "An overview of recent progress in the study of distributed multi-agent coordination," *IEEE Trans. Ind. Informat.*, vol. 9, no. 1, pp. 427–438, Feb. 2013.

[2] Y.-T. Tang and Y.-G. Hong, "Hierarchical distributed control design for multi-agent systems using approximate simulation," *Acta Automatica Sinica*, vol. 39, no. 6, pp. 868–874, Mar. 2014.

[3] Z. Zuo, Q.-L. Han, B. Ning, X. Ge, and X.-M. Zhang, "An overview of recent advances in fixed-time cooperative control of multiagent systems," *IEEE Trans. Ind. Informat.*, vol. 14, no. 6, pp. 2322–2334, Jun. 2018.

[4] F. L. Lewis, H. Zhang, K. Hengster-Movric, and A. Das, *Cooperative Control of Multi-agent Systems: Optimal and Adaptive Design Approaches*. London, U.K.: Springer, 2013.

[5] M. A. Kamel, X. Yu, and Y. Zhang, "Formation control and coordination of multiple unmanned ground vehicles in normal and faulty situations: A review," *Annu. Rev. control*, vol. 49, pp. 128–144, Jan. 2020.

[6] V. Loia and A. Vaccaro, "Decentralized economic dispatch in smart grids by self-organizing dynamic agents," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 44, no. 4, pp. 397–408, Apr. 2014.

[7] J. Chang, Z. Guo, J. Cieslak, and D. Henry, "Robust fault accommodation strategy of the reentry vehicle: A disturbance estimate-triggered approach," *Nonlinear Dyn.*, vol. 103, no. 3, pp. 2605–2625, Feb. 2021.

[8] J. Qin, G. Zhang, W. X. Zheng, and Y. Kang, "Adaptive sliding mode consensus tracking for second-order nonlinear multiagent systems with actuator faults," *IEEE Trans. Cybern.*, vol. 49, no. 5, pp. 1605–1615, May 2019.

[9] H. Yang, Q.-L. Han, X. Ge, L. Ding, Y. Xu, B. Jiang, and D. Zhou, "Fault-tolerant cooperative control of multiagent systems: A survey of trends and methodologies," *IEEE Trans. Ind. Informat.*, vol. 16, no. 1, pp. 4–17, Jan. 2020.

[10] Y. Li, S. X. Ding, C. Hua, and G. Liu, "Distributed adaptive leader-following consensus for nonlinear multiagent systems with actuator failures under directed switching graphs," *IEEE Trans. Cybern.*, vol. 53, no. 1, pp. 211–221, Jan. 2023.

[11] M. Davoodi, N. Meskin, and K. Khorasani, "Simultaneous fault detection and consensus control design for a network of multi-agent systems," *Automatica*, vol. 66, pp. 185–194, Apr. 2016.

[12] S. X. Ding, *Advanced Methods for Fault Diagnosis and Fault-Tolerant Control*. Berlin, Germany: Springer, 2021.

[13] A. Varga, *Solving Fault Diagnosis Problems* (Studies in Systems, Decision and Control), vol. 84, 1st ed., Berlin, Germany: Springer, 2017, pp. 8–9.

[14] Z. Gao, "Fault estimation and fault-tolerant control for discrete-time dynamic systems," *IEEE Trans. Ind. Electron.*, vol. 62, no. 6, pp. 3874–3884, Jun. 2015.

[15] C. Liu, B. Jiang, R. J. Patton, and K. Zhang, "Hierarchical-structure-based fault estimation and fault-tolerant control for multiagent systems," *IEEE Trans. Control Netw. Syst.*, vol. 6, no. 2, pp. 586–597, Jun. 2019.

[16] S. Zhao, B. Huang, and C. Zhao, "Online probabilistic estimation of sensor faulty signal in industrial processes and its applications," *IEEE Trans. Ind. Electron.*, vol. 68, no. 9, pp. 8853–8862, Sep. 2021.

[17] Y. Shi, Y. Hua, J. Yu, X. Dong, and Z. Ren, "Fully data-driven robust output formation tracking control for heterogeneous multiagent system with multiple leaders and actuator faults," *IEEE Trans. Cybern.*, vol. 54, no. 5, pp. 3183–3196, May 2022.

[18] Y. Shi, C. Liu, and Y. Wang, "Asymptotically stable filter for MVU estimation of states and homologous unknown inputs in heterogeneous multiagent systems," *IEEE Trans. Autom. Sci. Eng.*, vol. 19, no. 2, pp. 884–894, Apr. 2022.

[19] J. Han, X. Liu, X. Gao, and X. Wei, "Intermediate observer-based robust distributed fault estimation for nonlinear multiagent systems with directed graphs," *IEEE Trans. Ind. Informat.*, vol. 16, no. 12, pp. 7426–7436, Dec. 2020.

[20] Y. Wu, Z. Mao, X. Yan, and B. Jiang, "Cooperative fault estimation for a class of heterogeneous multi-agents with stochastic nonlinearities based on finite impulse response filter," *Int. J. Robust Nonlinear Control*, vol. 32, no. 8, pp. 4696–4715, May 2022.

[21] P. P. Menon and C. Edwards, "Robust fault estimation using relative information in linear multi-agent networks," *IEEE Trans. Autom. Control*, vol. 59, no. 2, pp. 477–482, Feb. 2014.

[22] K. Zhang, B. Jiang, and V. Cocquempot, "Adaptive technique-based distributed fault estimation observer design for multi-agent systems with directed graphs," *IET Control Theory Appl.*, vol. 9, no. 18, pp. 2619–2625, Dec. 2015.

[23] J.-W. Zhu and G.-H. Yang, "Robust distributed fault estimation for a network of dynamical systems," *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 1, pp. 14–22, Mar. 2018.

[24] J.-W. Zhu, G.-H. Yang, W.-A. Zhang, and L. Yu, "Cooperative fault tolerant tracking control for multiagent systems: An intermediate estimator-based approach," *IEEE Trans. Cybern.*, vol. 48, no. 10, pp. 2972–2980, Oct. 2018.

[25] J. Zhang, D. Yang, H. Zhang, and H. Su, "Adaptive secure practical fault-tolerant output regulation of multiagent systems with DoS attacks by asynchronous communications," *IEEE Trans. Netw. Sci. Eng.*, vol. 10, no. 6, pp. 4046–4055, Dec. 2023.

[26] K. Ali, A. Mehmood, and J. Iqbal, "Fault-tolerant scheme for robotic manipulator—Nonlinear robust back-stepping control with friction compensation," *PLoS ONE*, vol. 16, no. 8, 2021, Art. no. e0256491.

[27] K. Ali, S. Ullah, A. Mehmood, H. Mostafa, M. Marey, and J. Iqbal, "Adaptive FIT-SMC approach for an anthropomorphic manipulator with robust exact differentiator and neural network-based friction compensation," *IEEE Access*, vol. 10, pp. 3378–3389, 2022.

[28] J.-N. Li and W. Ren, "Finite-horizon H_∞ fault-tolerant constrained consensus for multiagent systems with communication delays," *IEEE Trans. Cybern.*, vol. 51, no. 1, pp. 416–426, Jan. 2021.

[29] M. Khalili, X. Zhang, M. M. Polycarpou, T. Parisini, and Y. Cao, "Distributed adaptive fault-tolerant control of uncertain multi-agent systems," *Automatica*, vol. 87, pp. 142–151, Jan. 2018.

[30] B. Yan, C. Wu, and P. Shi, "Formation consensus for discrete-time heterogeneous multi-agent systems with link failures and actuator/sensor faults," *J. Franklin Inst.*, vol. 356, no. 12, pp. 6547–6570, Aug. 2019.

[31] J.-N. Li, H. Feng, Y. Wang, and G.-Y. Liu, "A novel failure-distribution-dependent non-fragile H_∞ fault-tolerant load frequency control for faulty multi-area power systems," *IEEE Trans. Power Syst.*, vol. 39, no. 2, pp. 2936–2946, May 2023.

[32] C. Liu, B. Jiang, and K. Zhang, "Distributed adaptive observers-based fault estimation for leader-following multi-agent linear uncertain systems with actuator faults," in *Proc. IEEE Chin. Guid., Navigat. Control Conf. (CGNCC)*, Aug. 2016, pp. 1162–1167.

[33] Y. Cui, X. Liu, X. Deng, and Q. Wang, "Observer-based adaptive fuzzy formation control of nonlinear multi-agent systems with nonstrict-feedback form," *Int. J. Fuzzy Syst.*, vol. 23, no. 3, pp. 680–691, Apr. 2021.

[34] Y. Liu and G.-H. Yang, "Integrated design of fault estimation and fault-tolerant control for linear multi-agent systems using relative outputs," *Neurocomputing*, vol. 329, pp. 468–475, Feb. 2019.

[35] K. Zhang, B. Jiang, and P. Shi, "Fast fault estimation and accommodation for dynamical systems," *IET Control Theory Appl.*, vol. 3, no. 2, pp. 189–199, Feb. 2009.

[36] J. Zhang, D. Yang, H. Zhang, Y. Wang, and B. Zhou, "Dynamic event-based tracking control of boiler turbine systems with guaranteed performance," *IEEE Trans. Autom. Sci. Eng.*, early access, Jul. 18, 2004, doi: 10.1109/TASE.2023.3294187.

JIANTAO SHI (Senior Member, IEEE) received the B.S. degree in electrical engineering and automation from Beijing Institute of Technology, Beijing, China, in 2011, and the Ph.D. degree in control science and engineering from Tsinghua University, Beijing, in 2016. From 2016 to 2021, he was an Associate Research Fellow with Nanjing Research Institute of Electronic Technology, Nanjing, China. He is currently a Full Professor with the College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing. His main research interests include optimal and learning control, distributed and cooperative control, fault diagnosis, and fault-tolerant control.

XIANG CHEN is currently pursuing the M.A.Sc. degree in control science and engineering with the College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing, China. His current research interest includes multi-agent system fault diagnosis.

SHUANGQING XING is currently pursuing the M.A.Sc. degree in control science and engineering with the College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing, China. His current research interest includes multi-agent system fault diagnosis.

ANNING LIU is currently pursuing the M.A.Sc. degree in control science and engineering with the College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing, China. Her current research interest includes multi-agent system fault diagnosis.

CHUANG CHEN (Member, IEEE) received the Ph.D. degree in control theory and control engineering from Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2022. From 2021 to 2022, he visited the Space Engineering Design Laboratory, York University, Toronto, Canada, as a joint Ph.D. Student. From 2022 to 2023, he was a Visiting Scholar with the State Key Laboratory of Industrial Control Technology, Zhejiang University, China. He is currently a Lecturer with the College of Electrical Engineering and Control Science, Nanjing Tech University, Nanjing, and also an in-service Postdoctoral Fellow with Shanghai Jiao Tong University. His current research interests include data-driven fault prognosis and health management.