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RESEARCH ARTICLE

Distributed Cooperative Synchronization and Tracking for Heterogeneous Platoons With Mixed-Order Nonlinear Vehicle Dynamics

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ABSTRACT This paper studies the synchronization and tracking issue for the connected and automated vehicle (CAV) platoon with mixed-order nonlinear vehicle dynamics. Previous researches have demonstrated that vehicle platoons can improve traffic efficiency and minimize the incidence of traffic collisions. However, vehicles within the same platoon may have different-order dynamics in practical scenarios. The heterogeneity issue of a vehicle platoon caused by the different-order dynamics is difficult to address by existing platoon control methods. Therefore, the platoon synchronization and tracking control problem for the heterogeneous CAV platoon which consists of mixed-order vehicles with a hybrid of second- and third-order dynamics is formulated in this paper. The proposed novel distributed sliding mode (DSM) control protocols for vehicles with second- and third-order dynamics aim to guarantee the convergence of vehicle state errors and the stability of the heterogeneous platoon. The distributed neural network-based adaptive estimation technique are developed in this paper to approximate the unknown uncertainties and compensate for the model nonlinearities of the platoon system. The uniform ultimate boundedness of errors and string stability are substantiated through the Lyapunov theory and infinity-norm technique. The efficacy of the developed platoon control strategies can be verified by the numerical examples.

INDEX TERMS CAV, mixed-order nonlinear dynamics, string stability, heterogeneous platoon, neural network.

I. INTRODUCTION

With the growing concerns over traffic congestion, environmental contamination, and car incidents escalating, there has been a considerable focus on various research directions in intelligent transportation systems (ITS) [1]. The connected and automated vehicle (CAV) platoon control, which requires that vehicles within the platoon follow their adjacent vehicles with a predetermined desired inter-vehicle spacing along the same driving path, is acknowledged as a key research focus within the realm of ITS. The investigation of vehicle platoon control has attracted numerous researchers, and the extensive explorations within the field have been carried out over the last several decades. Recent years, the implementation of vehicle platoon control has demonstrated its feasibility for improving road safety, enhancing overall traffic flow, and diminishing fuel expenditure [2]. The effectiveness of vehicle platoon control has been validated through some well-known projects in the real world, such as KONVOI in Germany, PATH in USA, and SARTRE in Europe [3], [4], [5]. A great number of key aspects in this field have been extensively investigated, including string stability [6], dynamic homogeneity and heterogeneity [7], platoon communication topology [8], etc.

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Many effective control techniques have been utilized to deal with the platoon control problem, including feedback linearization [9], model predictive control [10], sliding mode control [11], etc.

Recently, there have been a great number of articles focusing on vehicle platoon control, specifically on the powertrain dynamics modeling [12]. These researches have discussed a wide range of various vehicle dynamic models [13], [14], [15]. To simplify the vehicle dynamic model and facilitate the controller design, some earlier studies have investigated the homogeneous linear vehicle platoon [14], [15], [16]. Nevertheless, in practical scenarios, vehicle dynamics usually exhibit nonlinearity and can be influenced by uncertainties and external disturbances. Therefore, many studies made investigations on platoons with nonlinear vehicle dynamics [17], [18], [19], [20], [21].

According to existing literature, the second- and third-order dynamics for vehicles are commonly utilized in the existing works of the nonlinear vehicle platoon [20], [21], [22]. Based on the second-order vehicle dynamics, the coupled sliding mode control strategy are developed to achieve the platoon control objective and reduce the impact of external disturbances in [20]. In [21], a hierarchical platoon controller design framework based on the feedback linearization control and decentralized bidirectional control is established for the vehicle platoon with third-order dynamics. However, in practical applications, vehicle platoons often exhibit heterogeneity which reflects the differences among the vehicle dynamics and can have a substantial influence on the platoon stability and control effectiveness [7]. Therefore, the heterogeneity within a platoon should be taken into consideration, and the control protocols should be designed to effectively handle these differences. Some existing literature have dedicated to the investigation of the heterogeneous platoon [23], [24], [25], [26], [27]. The distributed control framework and the topological reaching law are developed for a nonlinear heterogeneous vehicle platoon to regulate vehicles to keep desired inter-vehicle distances and a uniform velocity in [23]. In [25], a robust coordinated control strategy is presented for a nonlinear and heterogeneous platoon to ensure the platoon stability and realize the control goal. The above-mentioned studies concerning heterogeneous platoons primarily focus on the heterogeneity resulting from the differences in vehicle model parameters. Nevertheless, the heterogeneity problem arising from differences in the orders of vehicle dynamics have not been involved. In real traffic scenarios, vehicles within the same platoon can exhibit heterogeneities and may have different-order dynamics such as second- or third-order dynamics, and this heterogeneity problem can be difficult to address by the existing platoon control approaches. Hence, it is significant and challenging to explore the control problem for the mixed-order heterogeneous CAV platoon with the vehicles containing a hybrid of second-order and third-order vehicle dynamics.

As the fundamental platoon control objective is that all following vehicles can reach prescribed inter-vehicle distances and track the velocity of the leader, a critical issue of the objective is to ensure the individual vehicle stability and the string stability. Within the platoon control systems, string stability is a significant feature that indicates the characteristic whereby vehicle spacing errors are not amplified by their propagation in the platoon [28].

Many studies on the issue of string stability have been introduced in [29], [30], and [31]. In [30], the safety-extended distributed control method based on the model predictive control is used in order to fulfill the platoon control goal and guarantee string stability by exploiting vehicle communication. In [31], under the platoon communication topology, the H_∞ optimal controller is implemented for the platoon to realize the vehicle consensus and string stability.

Unknown uncertainties and model nonlinearities existing in the vehicle dynamics can have an impact on the whole platoon, which can also affect the platoon stability and control performance [32]. The impact of the unknown uncertainties and nonlinearities on vehicle platoon systems has attracted considerable attention [33], [34], [35], [36]. In [35], distributed network-based backstepping control schemes are designed to solve the issue of unknown model parameters within the vehicle platoon. For the platoon with uncertain vehicle dynamics, a robust H-infinity control approach is presented to enhance vehicle tracking performance and ensure string stability in [36]. It is noteworthy that almost all of the previously discussed works necessitate to obtain exact information on vehicle dynamics. Nevertheless, acquiring such specific details about vehicle dynamics presents great difficulties in practical platoon systems. In addition, the accurate acquisition of unknown uncertainties and model nonlinearities in real-time can also be hard to achieve. Therefore, it is imperative and of practical significance to address the control problem for the mixed-order heterogeneous CAV platoon with unknown uncertainties and model nonlinearities in the absence of exact information of vehicle dynamics parameters.

Following from the aforementioned discussions, this paper focuses on the synchronization and tracking issue for the heterogeneous CAV platoon with mixed-order vehicle dynamics. The platoon investigated in this article is subject to unknown uncertainties, model nonlinearities, and external disturbances. To fulfill the control objective and reach the consensus among vehicles, the novel distributed sliding mode (DSM) control strategies are proposed in this paper. The suggested strategies guarantee the vehicle error convergence in spacing, velocity, and acceleration and ensure string stability of the platoon. To deal with unknown uncertainties and model nonlinearities for the platoon system, the distributed neural network-based adaptive laws are formulated. The primary contributions are outlined below.

First, in practice, the actual vehicle platoon systems are typically heterogeneous and may consist of following vehicles with different-order dynamics. Compared with the above-mentioned works that investigate the homogeneous vehicle platoon [16] or focus on the problem of the heterogeneity caused by different model parameters [23], this article analyzes the control problem of the mixed-order heterogeneous CAV platoon consisting of vehicles with a hybrid of second- and third-order dynamics.

Second, obtaining the precise knowledge of vehicle dynamics in the platoon can be hardly achieved in a practical scenario. Unlike existing studies that make the assumption that the dynamics for all following vehicles are known within a platoon [34], [35], [36], in this study, the distributed neural network-based adaptive updating mechanism is developed for the mixed-order heterogeneous CAV platoon to approximate unknown uncertainties and compensate for model nonlinearities of the platoon system without acquiring exact knowledge of vehicle dynamics.

Third, in order to ensure the convergence of vehicle state errors and the string stability, the novel DSM control protocols based on the platoon communication topology are designed in this article for the mixed-order heterogeneous CAV platoon. The Lyapunov technique and infinity-norm method are used to demonstrate the boundedness of state errors and the string stability, respectively.

The arrangement of this paper is presented below. The description of the mixed-order heterogeneous platoon model and the platoon control problem are established in Section II. The mechanism of the neural network approximation is introduced in Section III. The control strategy design for the mixed-order heterogeneous CAV platoon is explored in Section IV. The analysis of the convergence of state errors and string stability is provided in Section V. In Section VI, the numerical examples of the heterogeneous platoon are presented. The conclusions and future research are outlined in Section VII. The abbreviations and notations utilized throughout this article are shown in Table 1.

TABLE 1. List of abbreviations and notations.

Abbreviation/Symbol	Definition
CAV	Connected and Automated Vehicle
DSM	Distributed Sliding Mode
RBF	Radial Basis Function
$tr\{\cdot\}$	Trace of a matrix
$P>0$	P is positive definite
$\sigma\{\cdot\}$	Set of singular values
$\underline{\sigma}\{\cdot\}$	Minimum of $\sigma\{\cdot\}$
$\overline{\sigma}\{\cdot\}$	Maximum of $\sigma\{\cdot\}$
$\lvert\cdot\rvert$	Absolute value
$\lVert\cdot\rVert$	Euclidean norm
$\lVert\cdot\rVert_F$	Frobenius norm
$0_{m\times l}$	Zero matrix in $\Re^{m\times l}$
$\Re^N$	Set of all N-dimensional real vectors
I_N	Identity matrix
$\mathbf{1}_N$	N-dimensional unit column vector
$diag\{...\}$	Diagonal matrix

II. DESCRIPTION OF MIXED-ORDER PLATOON MODEL AND PROBLEM FORMULATION

A. MIXED-ORDER VEHICLE DYNAMICS

This article considers a mixed-order heterogeneous CAV platoon with N followers and one leader along the horizontal driveline. Based on the majority of existing literature on platoon control [22], the followers are modeled by second- or third-order vehicle dynamics. The following vehicles with third-order vehicle dynamics are represented by $m_2 = \{1, 2, \ldots, l\}$ and others with second-order vehicle dynamics are represented as $m_1 = \{l+1, l+2, \ldots, N\}$. All following vehicles are denoted as $M = \{1, 2, \ldots, N\}$, and the leader of the platoon is labeled as 0. The third-order vehicle dynamic model is described as [23]

$$\begin{cases} \dot{x}_{i,1}(t) = x_{i,2}(t) \\ \dot{x}_{i,2}(t) = x_{i,3}(t) \\ \dot{x}_{i,3}(t) = \phi_i(\bar{x}_i) + \alpha_i(\bar{x}_i, t)u_i(t) + \zeta_i \end{cases}$$
$$i \in m_2 = \{1, 2, \ldots, l\} \tag{1}$$

where $x_{i,1}(t)$, $x_{i,2}(t)$, and $x_{i,3}(t)$ denote the position, velocity, and acceleration of the ith vehicle, respectively; $\bar{x}_i = [x_{i,1}(t), x_{i,2}(t), x_{i,3}(t)]^T$; $\phi_i(\cdot)$ and $\alpha_i(\cdot)$ denote the nonlinear unknown uncertainties and are locally Lipschitz; $u_i(t)$ is the control input; ζ_i represents the unknown external disturbance.

The second-order vehicle dynamic model is described as [37]

$$\begin{cases} \dot{x}_{i,1}(t) = x_{i,2}(t) \\ \dot{x}_{i,2}(t) = \phi_i(\bar{x}_i) + \alpha_i(\bar{x}_i, t)u_i(t) + \zeta_i \end{cases}$$
$$i \in m_1 = \{l+1, l+2, \ldots, N\} \tag{2}$$

where $\bar{x}_i = [x_{i,1}(t), x_{i,2}(t), 0]^T$. Without loss of generality, one can make the assumption that $\alpha_i(\bar{x}_i, t) \geq \underline{\alpha}_i > 0$, where $\underline{\alpha}_i$ is the minimum value of α_i.

Globally, the third-order vehicle dynamics in (1) can be expressed as

$$\begin{cases} \dot{x}_1^{m_2}(t) = x_2(t) \\ \dot{x}_2^{m_2}(t) = x_3(t) \\ \dot{x}_3(t) = \phi^{m_2}(\bar{x}^{m_2}) + \alpha^{m_2}u^{m_2}(t) + \zeta^{m_2} \end{cases} \tag{3}$$

and the second-order vehicle dynamics in (2) can be globally described as

$$\begin{cases} \dot{x}_1^{m_1}(t) = x_2(t) \\ \dot{x}_2^{m_1}(t) = \phi^{m_1}(\bar{x}^{m_1}) + \alpha^{m_1}u^{m_1}(t) + \zeta^{m_1} \end{cases} \tag{4}$$

where

$$(\cdot)^{m_1} = \begin{bmatrix} 0_l & 0 \\ 0 & I_{N-l} \end{bmatrix}(\cdot);\ (\cdot)^{m_2} = \begin{bmatrix} I_l & 0 \\ 0 & 0_{N-l} \end{bmatrix}(\cdot);$$
$$x_1(t) = [x_{1,1}(t), x_{2,1}(t), \ldots, x_{N,1}(t)]^T;$$
$$x_2(t) = [x_{1,2}(t), x_{2,2}(t), \ldots, x_{N,2}(t)]^T;$$
$$x_3(t) = [x_{1,3}(t), \ldots, x_{l,3}(t), 0_{1\times(N-l)}]^T;$$
$$u(t) = [u_1(t), u_2(t), \ldots, u_N(t)]^T;$$
$$\phi(\bar{x}) = [\phi_1(\bar{x}_1), \phi_2(\bar{x}_2), \ldots, \phi_N(\bar{x}_N)]^T;$$
$$\bar{x} = [\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_N]^T;$$

$$\alpha = diag\{\alpha_1(\bar{x}_1, t), \ldots, \alpha_N(\bar{x}_N, t)\};$$
$$\zeta = [\zeta_1, \zeta_2, \ldots, \zeta_N]^T.$$

Remark 1: Practically, the characteristics of vehicle dynamics may be affected by numerous elements, including frictional resistance, air resistance, road conditions, etc. Some certain dynamics parameters of vehicles are time-dependent, and the same parameters can exhibit variations among different vehicle dynamics within a platoon. Furthermore, in practice, the platoon may comprises vehicles with different order dynamics. Thus, the mixed-order vehicle platoon studied in this paper can be nonlinear, time-varying, and heterogeneous.

Given that the leader needs to provide state information of all orders as a reference for the follower vehicles, the leader is therefore considered to be a third-order system as

$$\begin{cases} \dot{x}_{0,1}(t) = x_{0,2}(t) \\ \dot{x}_{0,2}(t) = x_{0,3}(t) \\ \dot{x}_{0,3}(t) = \phi_0(\bar{x}_0, t) \end{cases} \quad (5)$$

where $x_{0,1}(t)$, $x_{0,2}(t)$, and $x_{0,3}(t)$ denote the position, velocity, and acceleration of the leader, respectively; $\bar{x}_0 = [x_{0,1}(t), x_{0,2}(t), x_{0,3}(t)]^T$; $\phi_0(\bar{x}_0, t)$ denotes the nonlinear unknown function which is locally Lipschitz.

B. PLATOON COMMUNICATION TOPOLOGY

In a vehicle platoon system, the platoon communication topology plays a key role as it governs the establishment of communication links between vehicles. The commonly adopted platoon communication topologies are mainly categorized into: predecessor following (PF), predecessor-leader following (PLF), bidirectional (BD), bidirectional-leader (BDL), two predecessor following (TPF) and two predecessor-leader following (TPLF) [23], which are shown in Fig. 1.

To illustrate the communication among vehicles in the platoon, the platoon communication topology is described by $G = \{\mathbb{V}, \mathbb{E}\}$. G represents a directed graph, where $\mathbb{V}$ is a nonempty node set and $\mathbb{E} \in \mathbb{V} \times \mathbb{V}$ is the edge set. $(j, i) \in \mathbb{E}$ signifies the connection of a edge from node j to node i, which means that node i acquires the information from j. The connectivity of G is represented by its adjacency matrix $A = \left[a_{ij}\right] \in \Re^{N\times N}$, where $a_{ij} > 0$ if $(j, i) \in \mathbb{E}$ and otherwise $a_{ij} = 0$. $\mathbb{N}_i = \{j\,|(j, i) \in \mathbb{E}\}$ indicates the neighboring nodes set of node i, which implies that node i can receive information from the nodes within $\mathbb{N}_i$, but not vice versa. The Laplacian matrix can be described as $L = \left[l_{ij}\right] \in \Re^{N\times N}$. The in-degree matrix is described by $D = diag\,\{d_1, d_2, \ldots, d_N\}$, where $d_i = \sum_{j=1}^{N} a_{ij}$ [38].

A spanning tree is referred to as a subgraph of G that encompasses every node within the graph [39]. If there exists a route from the root node to all other nodes in G, it indicates that G incorporates a spanning tree. Suppose that the leader can solely send information to followers without receiving any from them. The directional links between the leader and followers are often described by the pinning matrix. Then, the pinning matrix can be described as $B = diag\,\{b_1, b_2, \ldots, b_N\}$, where $b_i > 0$ if the edge from the leader to node i exists, and $b_i = 0$ otherwise.

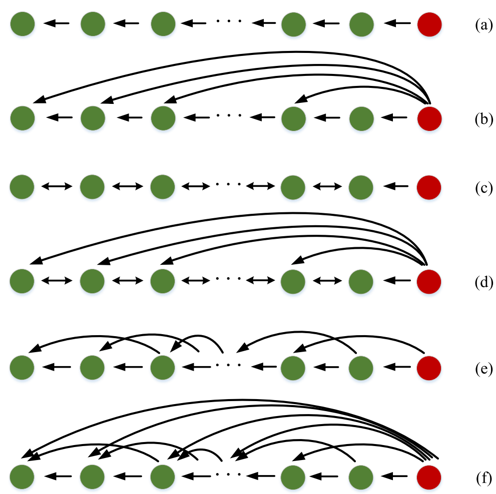

FIGURE 1. Commonly used platoon communication topologies. (a) PF; (b) PLF; (c) BD; (d) BDL; (e) TPF; (f) TPLF.

Assumption 1: We assume that G comprises at least a single spanning tree, which means that each follower within the vehicle platoon can directly or indirectly engage in communication with the leader [23].

Lemma 1: As B is a diagonal matrix with nonnegative elements, the matrix $L + B$ is positive definite if the platoon communication topology satisfies Assumption 1 [40].

C. CONTROL PROBLEM FORMULATION FOR MIXED-ORDER HETEROGENEOUS PLATOON

In order to ensure that each vehicle in the mixed-order heterogeneous CAV platoon maintains a prescribed spacing with its predecessor and tracks the trajectory of the leader, the platoon control objective for this study can be specifically outlined below.

(1) Design the novel DSM control protocols for all vehicles to ensure the error convergence within the mixed-order heterogeneous CAV platoon incorporating the vehicle spacing policy and platoon communication topology.

(2) Each following vehicle keeps a desired distance with its predecessors and synchronizes to all states of the leader.

(3) Design the distributed neural network-based adaptive laws to approximate unknown uncertainties and compensate for model nonlinearities of the heterogeneous platoon.

(4) Ensure the individual vehicle stability for each following vehicle and the string stability for the mixed-order heterogeneous CAV platoon.

The mixed-order heterogeneous CAV platoon studied in this article is illustrated in Fig. 2, comprising N followers and one leader. The spacing policy requires that each vehicle maintains a constant desired distance with its preceding vehicle, which is known as the constant spacing policy (CSP) [41]. Define the desired inter-vehicle spacing between neighboring followers in CSP as $d > 0$. Then, the ideal

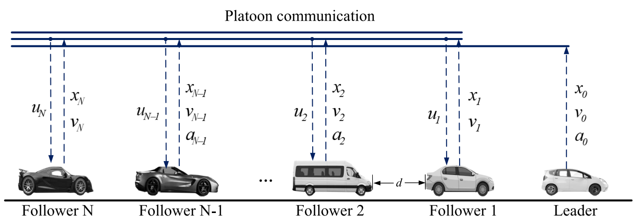

FIGURE 2. Mixed-order heterogeneous CAV platoon.

position for vehicle i is defined as

$$x_{ides}(t) = x_{0,1}(t) - i \times d \tag{6}$$

Then, the tracking errors of the ith vehicle are expressed as

$$\begin{cases} \mu_{i,1}(t) = x_{i,1}(t) - x_{ides}(t) \\ \mu_{i,2}(t) = x_{i,2}(t) - x_{0,2}(t) \end{cases}, \ i \in M$$
$$\mu_{i,3}(t) = x_{i,3}(t) - x_{0,3}(t) \quad i \in m_2 \tag{7}$$

where $\mu_{i,1}(t)$, $\mu_{i,2}(t)$, and $\mu_{i,3}(t)$ represent the position, velocity, and acceleration error, respectively.

Then, the final control objective of this study can be rephrased as

$$\begin{cases} \lim\limits_{t\to+\infty} \left\| x_{i,1} - x_{ides} \right\| = 0 \\ \lim\limits_{t\to+\infty} \left\| x_{i,2} - x_{0,2} \right\| = 0 \end{cases}, \ \forall i \in M$$
$$\lim_{t\to+\infty} \left\| x_{i,3} - x_{0,3} \right\| = 0 \quad \forall i \in m_2 \tag{8}$$

That is, the final control objective is that the spacing tracking error $\mu_{i,1}(t)$ and velocity tracking error $\mu_{i,2}(t)$ can reduce to zero for $i \in M = \{1, 2, \ldots, N\}$, and the acceleration tracking error $\mu_{i,3}(t)$ can converge to zero for $i \in m_2 = \{1, 2, \ldots, l\}$.

The platoon communication topology can directly impact the vehicle synchronization and string stability [23]. Based on the CSP and graph theory, define the synchronization errors of the heterogeneous platoon as

$$\begin{aligned} e_{i,1}(t) &= \sum_{j\in\mathbb{N}_i} a_{ij}[x_{j,1}(t) - x_{i,1}(t)] \\ &\quad + b_i[x_{ides}(t) - x_{i,1}(t)], \ i \in M \\ e_{i,2}(t) &= \sum_{j\in\mathbb{N}_i} a_{ij}[x_{j,2}(t) - x_{i,2}(t)] \\ &\quad + b_i[x_{0,2}(t) - x_{i,2}(t)], \ i \in M \\ e_{i,3}(t) &= \sum_{j\in\mathbb{N}_i^*} a_{ij}[x_{j,3}(t) - x_{i,3}(t)] \\ &\quad + b_i[x_{0,3}(t) - x_{i,3}(t)], \ i \in m_2 \end{aligned} \tag{9}$$

where $\mathbb{N}_i^*$ denotes the third-order neighbors of the ith vehicle. Then, the global synchronization errors can be represented as

$$\begin{aligned} e_1(t) &= -(L+B)[x_1(t) - x_{des}(t)] = -(L+B)\mu_1(t) \\ e_2(t) &= -(L+B)[x_2(t) - x_{0,2}(t)\mathbf{1}_N] = -(L+B)\mu_2(t) \\ e_3(t) &= -(L+B)[x_3^{m_2}(t) - x_{0,3}(t)\mathbf{1}_N^{m_2}] = -(L+B)\mu_3(t) \end{aligned} \tag{10}$$

where

$$\begin{aligned} e_1(t) &= [e_{1,1}(t), e_{2,1}(t), \ldots, e_{N,1}(t)]^T; \\ e_2(t) &= [e_{1,2}(t), e_{2,2}(t), \ldots, e_{N,2}(t)]^T; \\ e_3(t) &= [e_{1,3}(t), \ldots, e_{l,3}(t), 0_{1\times(N-l)}]^T; \\ \mu_1(t) &= [\mu_{1,1}(t), \mu_{2,1}(t), \ldots, \mu_{N,1}(t)]^T; \\ \mu_2(t) &= [\mu_{1,2}(t), \mu_{2,2}(t), \ldots, \mu_{N,2}(t)]^T; \\ \mu_3(t) &= [\mu_{1,3}(t), \ldots, \mu_{l,3}(t), 0_{1\times(N-l)}]^T; \\ x_{des}(t) &= [x_{1des}(t), x_{2des}(t), \ldots, x_{Ndes}(t)]^T. \end{aligned}$$

To simplify the exposition, we will not detail the time-dependence t of the parameters within (10). Since $x_1 = x_1^{m_1} + x_1^{m_2}, x_2 = x_2^{m_1} + x_2^{m_2}$, the differential of (10) can be expressed as

$$\begin{aligned} \dot{e}_1 &= -(L+B)(\dot{x}_1 - \dot{x}_{des}) = -(L+B)(x_2 - x_{0,2}\mathbf{1}_N) \\ \dot{e}_2 &= -(L+B)(\dot{x}_2 - \dot{x}_{0,2}\mathbf{1}_N) \\ &= -(L+B)(\dot{x}_2^{m_1} + \dot{x}_2^{m_2} - \dot{x}_{0,2}\mathbf{1}_N) \\ &= -(L+B)(\dot{x}_2^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}) + e_3 \\ \dot{e}_3 &= -(L+B)(\dot{x}_3 - \dot{x}_{0,3}\mathbf{1}_N^{m_2}) \\ &= -(L+B)[\phi^{m_2}(\bar{x}^{m_2}) + \alpha^{m_2}u^{m_2} + \zeta^{m_2} - \phi_0(\bar{x}_0)\mathbf{1}_N^{m_2}] \end{aligned} \tag{11}$$

Remark 2: The augmented graph is a graph that incorporates the leader node, and it can be described as $\overline{G} = \left\{\overline{\mathbb{V}}, \overline{\mathbb{E}}\right\}$, $\overline{\mathbb{E}} \in \overline{\mathbb{V}} \times \overline{\mathbb{V}}$. According to Lemma 1, the main diagonal elements of the matrix $L + B$ maintain nonnegative values, and its non-diagonal components keep nonpositive values.

Lemma 2: Assume graph $\overline{G}$ consists of at least one spanning tree. Then, according to (10), we have

$$\left\|\mu_j\right\| \le \left\|e_j\right\| / \underline{\sigma}(L+B), \ \ j = 1, 2, 3 \tag{12}$$

Proof: According to Lemma 1, $L+B$ is positive definite. In (10), we have $e_j = -(L+B)\mu_j$, $j = 1, 2, 3$. Then,

$$\mu_j = -(L+B)^{-1}e_j, \ \ j = 1, 2, 3 \tag{13}$$

Hence, the relationship between μ_j and e_j can be described as

$$\left\|\mu_j\right\| = \left\|(L+B)^{-1}e_j\right\| \le \left\|e_j\right\| / \underline{\sigma}(L+B), \ j = 1, 2, 3 \tag{14}$$

■

III. NEURAL NETWORK APPROXIMATION

Neural network is commonly employed for the estimation of unknown parameters and uncertainties in control systems. Radial basis function (RBF) neural network is a type of artificial neural network that use the radial basis function as the activation function, which is known for its simplicity, strong approximation capability, and efficient training process. The primary advantages of the RBF neural network include its ability to model nonlinear relationships effectively and its fast convergence during the training phase [42].

Therefore, the RBF neural network is used in this study to address the problem of unknown uncertainties and model nonlinearities. By designing the RBF neural network adaptive approximation mechanism, this article aims to estimate the nonlinear unknown uncertainties $\phi_i(\bar{x}_i)$ and $\alpha_i(\bar{x}_i, t)$. Assume $\phi_i(\bar{x}_i)$ and $\alpha_i(\bar{x}_i, t)$ are smooth and unknown. According to the neural network approximation method mentioned in [43], $\phi_i(\bar{x}_i)$ and $\alpha_i(\bar{x}_i, t)$ can be represented as

$$\begin{cases} \phi_i(\bar{x}_i) = W_{\phi_i}^T \varphi_{\phi_i}(\bar{x}_i) + \varepsilon_{\phi_i} \\ \alpha_i(\bar{x}_i, t) = W_{\alpha_i}^T \varphi_{\alpha_i}(\bar{x}_i) + \varepsilon_{\alpha_i} \end{cases}, \ i \in M \tag{15}$$

where W_{ϕ_i} and W_{α_i} denote the ideal neural network weights; $\varphi_{\phi_i}(\bar{x}_i)$ and $\varphi_{\alpha_i}(\bar{x}_i)$ represent the basis functions; $\bar{x}_i$ is the input of the network; ε_{ϕ_i} and ε_{α_i} are the approximation errors. $W_{\phi_i}, W_{\alpha_i}, \varphi_{\phi_i}(\bar{x}_i), \varphi_{\alpha_i}(\bar{x}_i) \in \Re^{o_i}$, o_i is the number of neurons for node i. The selection of basis functions may include options such as Gaussians, sigmoids, or hyperbolic tangents.

The basis function of the RBF neural network can be described as follows.

$$\varphi_k(\bar{x}_i) = \exp\left[-\frac{\left\|\bar{x}_i^T - c_k\right\|^2}{2b_k^2}\right] \tag{16}$$

where c_k denotes the coordinate value of center point; b_k represents the width value; $k \in \{1, 2, \ldots, o_i\}$.

Remark 3: According to the Weierstrass approximation theorem, there exist a sufficiently large positive o_i^* and a compact set Ω, such that for any $o_i > o_i^*$, it is always possible to find ideal W_{ϕ_i}, W_{α_i} and suitable $\varphi_{\phi_i}(\cdot)$, $\varphi_{\alpha_i}(\cdot)$, which makes (15) meet the requirement that $\max_{\bar{x}_i \in \Omega} \left|\varepsilon_{\phi_i}\right|$ and $\max_{\bar{x}_i \in \Omega} \left|\varepsilon_{\alpha_i}\right|$ are sufficiently small [44].

Define the approximation of $\phi_i(\bar{x}_i)$ and $\alpha_i(\bar{x}_i, t)$ as

$$\hat{\phi}_i(\bar{x}_i) = \hat{W}_{\phi_i}^T \varphi_{\phi_i}(\bar{x}_i), \ \ \hat{\alpha}_i(\bar{x}_i, t) = \hat{W}_{\alpha_i}^T \varphi_{\alpha_i}(\bar{x}_i) \tag{17}$$

where $\hat{\phi}_i(\bar{x}_i)$ and $\hat{\alpha}_i(\bar{x}_i, t)$ are the estimates of $\phi_i(\bar{x}_i)$ and $\alpha_i(\bar{x}_i, t)$; $\hat{W}_{\phi_i}$ and $\hat{W}_{\alpha_i}$ are the estimates of weights.

Then, the global $\phi(\bar{x})$ and α can be expressed as

$$\phi(\bar{x}) = W_\phi^T \varphi_\phi(\bar{x}) + \varepsilon_\phi, \ \ \alpha = W_\alpha^T \varphi_\alpha(\bar{x}) + \varepsilon_\alpha \tag{18}$$

where

$$\begin{aligned} W_\phi &= diag\{W_{\phi_1}, \ldots, W_{\phi_N}\}; \\ \varphi_\phi(\bar{x}) &= [\varphi_{\phi_1}^T(\bar{x}_1), \varphi_{\phi_2}^T(\bar{x}_2), \ldots, \varphi_{\phi_N}^T(\bar{x}_N)]^T; \\ \varepsilon_\phi &= [\varepsilon_{\phi_1}, \varepsilon_{\phi_2}, \ldots, \varepsilon_{\phi_N}]^T; \\ W_\alpha &= diag\{W_{\alpha_1}, \ldots, W_{\alpha_N}\}; \\ \varphi_\alpha(\bar{x}) &= diag\{\varphi_{\alpha_1}^T(\bar{x}_1), \ldots, \varphi_{\alpha_N}^T(\bar{x}_N)\}^T; \\ \varepsilon_\alpha &= diag\{\varepsilon_{\alpha_1}, \ldots, \varepsilon_{\alpha_N}\}. \end{aligned}$$

The global approximation $\hat{\phi}(\bar{x})$ and $\hat{\alpha}$ can be described by

$$\hat{\phi}(\bar{x}) = \hat{W}_\phi^T \varphi_\phi(\bar{x}), \ \ \hat{\alpha} = \hat{W}_\alpha^T \varphi_\alpha(\bar{x}) \tag{19}$$

where $\hat{\phi}(\bar{x}) = [\hat{\phi}_1(\bar{x}_1), \hat{\phi}_2(\bar{x}_2), \ldots, \hat{\phi}_N(\bar{x}_N)]^T$; $\hat{\alpha} = diag\{\hat{\alpha}_1(\bar{x}_1, t), \ldots, \hat{\alpha}_N(\bar{x}_N, t)\}$; $\hat{W}_\phi = diag\{\hat{W}_{\phi_1}, \ldots, \hat{W}_{\phi_N}\}$; $\hat{W}_\alpha = diag\{\hat{W}_{\alpha_1}, \ldots, \hat{W}_{\alpha_N}\}$.

Assumption 2: Define $W_{\phi_i M} = \left\|W_{\phi_i}\right\|_F$, $W_{\alpha_i M} = \left\|W_{\alpha_i}\right\|_F$, $\varphi_{\phi_i M} = \max_{\bar{x}_i \in \Omega} \left\|\varphi_{\phi_i}(\bar{x}_i)\right\|$, $\varphi_{\alpha_i M} = \max_{\bar{x}_i \in \Omega} \left\|\varphi_{\alpha_i}(\bar{x}_i)\right\|$. Then, there exist positive $W_{\phi M}$, $W_{\alpha M}$, $\varphi_{\phi M}$ and $\varphi_{\alpha M}$, such that $\left\|W_\phi\right\|_F \leq W_{\phi M}$, $\|W_\alpha\|_F \leq W_{\alpha M}$, $\left\|\varphi_\phi(\bar{x})\right\| \leq \varphi_{\phi M}$, $\|\varphi_\alpha(\bar{x})\|_F \leq \varphi_{\alpha M}$. ε_ϕ and ε_α are bounded by $\left\|\varepsilon_\phi\right\| \leq \varepsilon_{\phi M}$ and $\|\varepsilon_\alpha\|_F \leq \varepsilon_{\alpha M}$, where $\varepsilon_{\phi M} > 0$ and $\varepsilon_{\alpha M} > 0$ are the fixed bounds [45].

IV. DESIGN OF DISTRIBUTED CONTROL STRATEGY FOR MIXED-ORDER HETEROGENEOUS PLATOON

A. DSM CONTROL DESIGN

The design of the DSM control protocols for the mixed-order heterogeneous CAV platoon is presented in this section. Before proceeding with the strategy design, it is necessary to make the subsequent assumptions [46].

Assumption 3: Based on the leader dynamics in (5), the position, velocity, and acceleration are restricted by $\left|x_{0,1}\right| \leq x_{0,1M}$, $\left|x_{0,2}\right| \leq x_{0,2M}$, $\left|x_{0,3}\right| \leq x_{0,3M}$, where $x_{0,1M}, x_{0,2M}, x_{0,3M} > 0$. The unknown $\phi_0(\bar{x}_0, t)$ is bounded by $|\phi_0(\bar{x}_0, t)| \leq \phi_M$, where $\phi_M > 0$. The unknown disturbances ζ is bounded, such that $\|\zeta\| \leq \zeta_M$, where $\zeta_M > 0$.

There exist finite $\alpha_M > 0$ and $\hat{\alpha}_M > 0$, which are limited by $\|\alpha\| \leq \alpha_M$ and $\left\|\hat{\alpha}\right\| \leq \hat{\alpha}_M$.

The control input u is constrained by $\|u\| \leq u_M$, $u_M > 0$ [47].

Remark 4: Notably, the constraints specified in Assumption 2 and Assumption 3 will not be employed in the development of proposed control strategies but will be utilized only for the demonstration of the stability.

The uniform ultimate boundedness [48] will be utilized in the vehicle platoon system through the subsequent definition.

Definition 1: The tracking error $\mu_j(t)$ $(j = 1, 2, 3)$ is considered to be cooperatively uniformly ultimately bounded (CUUB) if there exists a compact set $\Omega_j \subset \Re^{N \times N}$, so that for $\forall \mu_j(t_0) \in \Omega_j$ there exists a bound $B_j > 0$ and time $T_j[B_j, \mu_1(t_0), \mu_2(t_0), \mu_3(t_0)]$ such that $\left\|\mu_j(t)\right\| \leq B_j, \forall t \geq t_0 + T_j$ [49]. If $\mu_1(t), \mu_2(t), \mu_3(t)$ are CUUB and $x_{i,1}(t), x_{i,2}(t), x_{i,3}(t)$ are bounded, the platoon consensus can be realized within a limited time.

Define the distributed sliding mode surface for vehicle i as [50]

$$s_i(t) = \begin{cases} \xi_{1,i} e_{i,1}(t) + e_{i,2}(t), & i \in m_1 \\ \xi_{2,i} e_{i,1}(t) + \xi_{3,i} e_{i,2}(t) + e_{i,3}(t), & i \in m_2 \end{cases} \tag{20}$$

where $\xi_{1,i}, \xi_{2,i}, \xi_{3,i} > 0$ are the design sliding mode parameters and are selected such that the polynomial $\gamma^2 + \xi_{3,i}\gamma + \xi_{2,i}$ is Hurwitz for $\forall i \in M = \{1, 2, \ldots, N\}$.

Remark 5: It is imperative to highlight that $\xi_{1,i}, \xi_{2,i}, \xi_{3,i}$ can be nonidentical for vehicles with mixed-order dynamics. By choosing the parameter $\xi_{1,i}, \xi_{2,i}, \xi_{3,i}$ for different vehicles, the flexibility in adjusting the performance of the mixed-order heterogeneous CAV platoon can be enhanced and the stability of the whole platoon can be guaranteed.

Globally, the sliding mode surface can be described as

$$\begin{aligned} s(t) &= s^{m_1}(t) + s^{m_2}(t) \\ s^{m_1}(t) &= \Lambda_1 e_1^{m_1}(t) + e_2^{m_1}(t) \\ s^{m_2}(t) &= \Lambda_2 e_1^{m_2}(t) + \Lambda_3 e_2^{m_2}(t) + e_3(t) \end{aligned} \tag{21}$$

where

$$\begin{aligned} \Lambda_1 &= diag\{0_{1\times l}, \xi_{1,l+1}, \ldots, \xi_{1,N}\}; \\ \Lambda_2 &= diag\{\xi_{2,1}, \ldots, \xi_{2,l}, 0_{1\times(N-l)}\}; \\ \Lambda_3 &= diag\{\xi_{3,1}, \ldots, \xi_{3,l}, 0_{1\times(N-l)}\}. \end{aligned}$$

To simplify the exposition, we will not detail the time-dependence t of the parameters within (21). Differentiating (21), one can obtain that

$$\begin{aligned} \dot{s} &= \dot{s}^{m_1} + \dot{s}^{m_2} \\ \dot{s}^{m_1} &= \Lambda_1 \dot{e}_1^{m_1} + \dot{e}_2^{m_1} \\ &= \Lambda_1 e_2^{m_1} - (L+B)(\dot{x}_2^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}) \\ &= \Lambda_1 e_2^{m_1} - (L+B)[\phi^{m_1}(\bar{x}^{m_1}) + \alpha^{m_1}u^{m_1} \\ &\quad + \zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\ \dot{s}^{m_2} &= \Lambda_2 \dot{e}_1^{m_2} + \Lambda_3 \dot{e}_2^{m_2} + \dot{e}_3 \\ &= \Lambda_2 e_2^{m_2} + \Lambda_3 e_3 \\ &\quad - (L+B)[\phi^{m_2}(\bar{x}^{m_2}) + \alpha^{m_2}u^{m_2} + \zeta^{m_2} - \phi_0(\bar{x}_0)\mathbf{1}_N^{m_2}] \end{aligned} \tag{22}$$

Define

$$\begin{aligned} E_1 &= \begin{bmatrix} e_1 & e_2 \end{bmatrix} \\ E_2 &= \dot{E}_1 = \begin{bmatrix} \dot{e}_1 & \dot{e}_2 \end{bmatrix} \end{aligned} \tag{23}$$

Since $e_1 = e_1^{m_1} + e_1^{m_2}$, $e_2 = e_2^{m_1} + e_2^{m_2}$, we have

$$\begin{aligned} \dot{E}_1^{m_1} &= \begin{bmatrix} \dot{e}_1^{m_1} & \dot{e}_2^{m_1} \end{bmatrix} \\ &= \begin{bmatrix} e_2^{m_1} & \dot{e}_2^{m_1} \end{bmatrix} \\ &= \begin{bmatrix} e_1^{m_1} & e_2^{m_1} \end{bmatrix} \begin{bmatrix} 0 & I_N \\ -\Lambda_1 & -I_N \end{bmatrix}^T \\ &\quad + [s^{m_1} - (L+B)(\dot{x}_2^{m_1} - x_{0,3}\mathbf{1}_N^{m_1})] \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \\ \dot{E}_1^{m_2} &= \begin{bmatrix} \dot{e}_1^{m_2} & \dot{e}_2^{m_2} \end{bmatrix} \\ &= \begin{bmatrix} e_2^{m_2} & e_3 \end{bmatrix} \\ &= \begin{bmatrix} e_1^{m_2} & e_2^{m_2} \end{bmatrix} \begin{bmatrix} 0 & I_N \\ -\Lambda_2 & -\Lambda_3 \end{bmatrix}^T + s^{m_2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \end{aligned} \tag{24}$$

According to (23), one can obtain that

$$\begin{aligned} E_2^{m_1} &= \dot{E}_1^{m_1} = E_1^{m_1}\theta_1^T + (s^{m_1} - \varpi)l^T \\ E_2^{m_2} &= \dot{E}_1^{m_2} = E_1^{m_2}\theta_2^T + s^{m_2}l^T \end{aligned} \tag{25}$$

where $\theta_1 = \begin{bmatrix} 0 & I_N \\ -\Lambda_1 & -I_N \end{bmatrix}$; $\theta_2 = \begin{bmatrix} 0 & I_N \\ -\Lambda_2 & -\Lambda_3 \end{bmatrix}$; $\varpi = (L+B)(\dot{x}_2^{m_1} - x_{0,3}\mathbf{1}_N^{m_1})$; $l = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Since θ_1, θ_2 are Hurwitz, given any β_1, β_2, there exist positive definite matrixes P_1, P_2 that satisfy

$$\theta_k^T P_k + P_k \theta_k = -\beta_k I, \quad k = 1, 2 \tag{26}$$

Remark 6: According to Lemma 2, $\overline{G}$ comprises at least one spanning tree, which indicates that there exists a directed path from the leader to vehicles with both second-order and third-order dynamics. Thus, it is reasonable to make the assumption in this paper that $\overline{G}$ incorporates at least one spanning tree for both the second- and third-order states contained within the follower vehicles, respectively.

Lemma 3: According to Lemma 1, $L + B$ is positive definite. Define

$$\begin{aligned} q &= [q_1, q_2, \ldots, q_N]^T = (L+B)^{-1}\mathbf{1}_N, \\ P &= diag\{p_i\} = diag\{1/q_i\}. \end{aligned} \tag{27}$$

Then, P is positive definite. Define

$$Q = P(L+B) + (L+B)^T P, \tag{28}$$

Then, $Q > 0$ [40].

For the platoon system in (3),(4), the DSM control protocols are designed as

$$\begin{aligned} u_i(t) &= \frac{1}{\hat{\alpha}_i}[cs_i(t) - \hat{W}_{\phi_i}^T \varphi_{\phi_i}(\bar{x}_i)] \\ &\quad + [\xi_{1,i}e_{i,2}(t)]/\hat{\alpha}_i(d_i + b_i), \quad i \in m_1 \\ u_i(t) &= \frac{1}{\hat{\alpha}_i}[cs_i(t) - \hat{W}_{\phi_i}^T \varphi_{\phi_i}(\bar{x}_i)] \\ &\quad + [\xi_{2,i}e_{i,2}(t) + \xi_{3,i}e_{i,3}(t)]/\hat{\alpha}_i(d_i + b_i), \quad i \in m_2 \end{aligned} \tag{29}$$

where c denotes the control gain. Globally, (29) becomes

$$\begin{aligned} \hat{\alpha}^{m_1}u^{m_1}(t) &= cs^{m_1}(t) - (\hat{W}_\phi^{m_1})^T \varphi_\phi^{m_1}(\bar{x}^{m_1}) \\ &\quad + (D+B)^{-1}\Lambda_1 e_2^{m_1}(t) \\ \hat{\alpha}^{m_2}u^{m_2}(t) &= cs^{m_2}(t) - (\hat{W}_\phi^{m_2})^T \varphi_\phi^{m_2}(\bar{x}^{m_2}) \\ &\quad + (D+B)^{-1}[\Lambda_2 e_2^{m_2}(t) + \Lambda_3 e_3(t)] \end{aligned} \tag{30}$$

The control gain c meets the requirement of

$$c\underline{\sigma}(Q) > \frac{y^2}{2\eta_\phi} + \frac{\lambda^2}{2\eta_\alpha} + \frac{[\overline{\sigma}(P_m) + v]^2}{\underline{\beta}} + 2h \tag{31}$$

where $y = \varphi_{\phi M}\overline{\sigma}(P)\overline{\sigma}(A)$; $\lambda = \varphi_{\alpha M}u_M\overline{\sigma}(P)\overline{\sigma}(A)$; $h = \frac{\overline{\sigma}(P)\overline{\sigma}(A)}{\underline{\sigma}(D+B)}[\overline{\sigma}(\Lambda_1) + \overline{\sigma}(\Lambda_3)]$; $v = \frac{\overline{\sigma}(P)\overline{\sigma}(A)}{\underline{\sigma}(D+B)}[\overline{\sigma}(\Lambda_1^2) + \overline{\sigma}(\Lambda_2) + \overline{\sigma}(\Lambda_3)\overline{\sigma}(\chi)]$; $\overline{\sigma}(\chi) = \overline{\sigma}(diag\{\Lambda_2, \Lambda_3\})$; $\underline{\beta} = \min\{\beta_1, \beta_2\}$; $\overline{\sigma}(P_m) = \max\{\overline{\sigma}(P_1), \overline{\sigma}(P_2)\}$; η_ϕ is defined in (34) and η_α is determined in (36).

B. DESIGN OF DISTRIBUTED NEURAL NETWORK-BASED ADAPTIVE LAW

In this section, the distributed neural network-based adaptive laws and singularity-free updating laws are developed to approximate unknown uncertainties and compensate for model nonlinearities of the mixed-order heterogeneous CAV platoon system.

According to (15) and (17), define the approximation errors $\tilde{\phi}_i(\bar{x}_i)$ and $\tilde{\alpha}_i(\bar{x}_i, t)$ as

$$\tilde{\phi}_i(\bar{x}_i) = \phi_i(\bar{x}_i) - \hat{\phi}_i(\bar{x}_i)$$

$$
\begin{aligned}
&= (W_{\phi_i}^T - \hat{W}_{\phi_i}^T)\varphi_{\phi_i}(\bar{x}_i) + \varepsilon_{\phi_i} \\
&= \tilde{W}_{\phi_i}^T \varphi_{\phi_i}(\bar{x}_i) + \varepsilon_{\phi_i} \\
\tilde{\alpha}_i(\bar{x}_i, t) &= \alpha_i(\bar{x}_i, t) - \hat{\alpha}_i(\bar{x}_i, t) \\
&= (W_{\alpha_i}^T - \hat{W}_{\alpha_i}^T)\varphi_{\alpha_i}(\bar{x}_i) + \varepsilon_{\alpha_i} \\
&= \tilde{W}_{\alpha_i}^T \varphi_{\alpha_i}(\bar{x}_i) + \varepsilon_{\alpha_i} \qquad (32)
\end{aligned}
$$

Then, (32) can be globally described as

$$
\begin{aligned}
\tilde{\phi}(\bar{x}) &= \phi(\bar{x}) - \hat{\phi}(\bar{x}) = \tilde{W}_\phi^T \varphi_\phi(\bar{x}) + \varepsilon_\phi \\
\tilde{\alpha} &= \alpha - \hat{\alpha} = \tilde{W}_\alpha^T \varphi_\alpha(\bar{x}) + \varepsilon_\alpha \qquad (33)
\end{aligned}
$$

where

$$
\begin{aligned}
&\tilde{\phi}(\bar{x}) = [\tilde{\phi}_1(\bar{x}_1), \tilde{\phi}_2(\bar{x}_2), \ldots, \tilde{\phi}_N(\bar{x}_N)]^T; \\
&\tilde{\alpha} = diag\{\tilde{\alpha}_1(\bar{x}_1, t), \ldots, \tilde{\alpha}_N(\bar{x}_N, t)\}; \\
&\tilde{W}_\phi = diag\{\tilde{W}_{\phi_1}, \ldots, \tilde{W}_{\phi_N}\}; \tilde{W}_\alpha = diag\{\tilde{W}_{\alpha_1}, \ldots, \tilde{W}_{\alpha_N}\}.
\end{aligned}
$$

Assumption 4: Assume that ε_ϕ and ε_α are bounded by $\|\varepsilon_\phi\| \leq \varepsilon_{\phi M}$ and $\|\varepsilon_\alpha\| \leq \varepsilon_{\alpha M}$, where $\varepsilon_\phi, \varepsilon_\alpha$ are the fixed bounds and $\varepsilon_\phi, \varepsilon_\alpha > 0$. ε is restricted by $\|\varepsilon\| \leq \varepsilon_M$, where $\varepsilon_M = \varepsilon_{\phi M} + \varepsilon_{\alpha M}$ [48].

The neural network-based adaptive law for the ith vehicle is established as

$$
\dot{\hat{W}}_{\phi_i} = -G_{\phi_i}[\varphi_{\phi_i}(\bar{x}_i)s_i(t)p_i(d_i + b_i) + \eta_\phi \hat{W}_{\phi_i}], \quad i \in M \qquad (34)
$$

where $G_{\phi_i} > 0$ is the parameter to be designed; η_ϕ is the positive gain. Globally,

$$
\dot{\hat{W}}_\phi = -G_\phi[\varphi_\phi(\bar{x}_i)s^T(t)P(D + B) + \eta_\phi \hat{W}_\phi] \qquad (35)
$$

where $G_\phi = diag\{G_{\phi_1}, \ldots, G_{\phi_N}\}$.

Design the neural updating law as [44]

$$
\dot{\hat{W}}_{\alpha_i} = \begin{cases}
-G_{\alpha_i}[\varphi_{\alpha_i}(\bar{x}_i)u_i(t)s_i(t)p_i(d_i + b_i) + \eta_\alpha \hat{W}_{\alpha_i}], \\
\quad \text{if} \hat{\alpha}_i > \underline{\alpha}_i \\
-G_{\alpha_i}[\varphi_{\alpha_i}(\bar{x}_i)u_i(t)s_i(t)p_i(d_i + b_i) + \eta_\alpha \hat{W}_{\alpha_i}], \\
\quad \text{if} \hat{\alpha}_i = \underline{\alpha}_i \text{and} [\varphi_{\alpha_i}(\bar{x}_i)u_i(t)s_i(t)p_i(d_i + b_i) \\
\quad + \eta_\alpha \hat{W}_{\alpha_i}] < 0 \\
0, \ \text{if} \hat{\alpha}_i = \underline{\alpha}_i \text{and} [\varphi_{\alpha_i}(\bar{x}_i)u_i(t)s_i(t)p_i(d_i + b_i) \\
\quad + \eta_\alpha \hat{W}_{\alpha_i}] \geq 0
\end{cases} \qquad (36)
$$

where $G_{\alpha_i}, \eta_\alpha > 0$.

V. STABILITY ANALYSIS

A. ANALYSIS OF PLATOON SYNCHRONIZATION AND ERROR CONVERGENCE

Theorem 1: Consider the mixed-order heterogeneous CAV platoon system consisting of the following vehicles described by (3), (4) and the leader described by (5) under Assumption 1 - Assumption 4. Use the DSM control protocols (29), neural network-based adaptive laws (34), and updating laws (36) for the platoon control system. Then, there exists $\overline{o}_i > 0$ such that for $o_i > \overline{o}_i$, the synchronization errors $e_1(t)$, $e_2(t)$ are ultimately bounded for $\forall i \in M$, $e_3(t)$ are ultimately bounded for $\forall i \in m_2$; the state errors $\mu_1(t)$, $\mu_2(t)$, and $\mu_3(t)$ are CUUB. In addition, for any $t \geq t_0$, $x_{i,1}(t)$, $x_{i,2}(t)$ are bounded for $\forall i \in M$, and $x_{i,3}(t)$ is bounded for $\forall i \in m_2$. All followers can track and reach synchronization with the leader within the finite time.

Proof: To simplify the exposition, the explicit representation of $\bar{x}$ and t within the parameters will not be described. Then, (22) can be described as

$$
\begin{aligned}
\dot{s}^{m_1} &= \Lambda_1 e_2^{m_1} - (L + B)[\phi^{m_1} + \alpha^{m_1}u^{m_1} \\
&\quad + (\hat{\alpha}^{m_1}u^{m_1} - \hat{\alpha}^{m_1}u^{m_1})] \\
&\quad - (L + B)[\zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\
&= \Lambda_1 e_2^{m_1} - (L + B)[\phi^{m_1} + \tilde{\alpha}^{m_1}u^{m_1} + \hat{\alpha}^{m_1}u^{m_1}] \\
&\quad - (L + B)[\zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\
\dot{s}^{m_2} &= \Lambda_2 e_2^{m_2} + \Lambda_3 e_3 \\
&\quad - (L + B)[\phi^{m_2} + \alpha^{m_2}u^{m_2} + (\hat{\alpha}^{m_2}u^{m_2} - \hat{\alpha}^{m_2}u^{m_2})] \\
&\quad - (L + B)[\zeta^{m_2} - \phi_0\mathbf{1}_N^{m_2}] \\
&= \Lambda_2 e_2^{m_2} + \Lambda_3 e_3 - (L + B)[\phi^{m_2} + \tilde{\alpha}^{m_2}u^{m_2} + \hat{\alpha}^{m_2}u^{m_2}] \\
&\quad - (L + B)[\zeta^{m_2} - \phi_0\mathbf{1}_N^{m_2}] \qquad (37)
\end{aligned}
$$

Using (18), (30) and (33), the above equation (37) can be written as

$$
\begin{aligned}
\dot{s}^{m_1} &= \Lambda_1 e_2^{m_1} - (L + B)[(W_\phi^{m_1})^T\varphi_\phi^{m_1} + \varepsilon_\phi^{m_1} + \tilde{\alpha}^{m_1}u^{m_1}] \\
&\quad - (L + B)[cs^{m_1} - (\hat{W}_\phi^{m_1})^T\varphi_\phi^{m_1} + (D + B)^{-1}\Lambda_1 e_2^{m_1}] \\
&\quad - (L + B)[\zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\
&= -(L+B)[(\tilde{W}_\phi^{m_1})^T\varphi_\phi^{m_1} + \varepsilon_\phi^{m_1} + (\tilde{W}_\alpha^{m_1})^T\varphi_\alpha^{m_1}u^{m_1} + \varepsilon_\alpha^{m_1}] \\
&\quad - (L + B)[cs^{m_1} + \zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\
&\quad + A(D + B)^{-1}\Lambda_1 e_2^{m_1} \\
\dot{s}^{m_2} &= \Lambda_2 e_2^{m_2} + \Lambda_3 e_3 - (L + B)[(W_\phi^{m_2})^T\varphi_\phi^{m_2} + \varepsilon_\phi^{m_2} + \tilde{\alpha}^{m_2}u^{m_2}] \\
&\quad - (L + B)[cs^{m_2} - (\hat{W}_\phi^{m_2})^T\varphi_\phi^{m_2} + \zeta^{m_2} - \phi_0\mathbf{1}_N^{m_2}] \\
&\quad - (L + B)[(D + B)^{-1}(\Lambda_2 e_2^{m_2} + \Lambda_3 e_3)] \\
&= -(L+B)[(\tilde{W}_\phi^{m_2})^T\varphi_\phi^{m_2} + \varepsilon_\phi^{m_2} + (\tilde{W}_\alpha^{m_2})^T\varphi_\alpha^{m_2}u^{m_2} + \varepsilon_\alpha^{m_2}] \\
&\quad - (L + B)[cs^{m_2} + \zeta^{m_2} - \phi_0\mathbf{1}_N^{m_2}] \\
&\quad + A(D + B)^{-1}(\Lambda_2 e_2^{m_2} + \Lambda_3 e_3) \qquad (38)
\end{aligned}
$$

Define the Lyapunov candidate as

$$
\begin{aligned}
V &= \frac{1}{2}s^T Ps + \frac{1}{2}tr\{\tilde{W}_\phi^T G_\phi^{-1}\tilde{W}_\phi\} + \frac{1}{2}tr\{\tilde{W}_\alpha^T G_\alpha^{-1}\tilde{W}_\alpha\} \\
&\quad + \frac{1}{2}tr\{E_1^{m_1}P_1(E_1^{m_1})^T + E_1^{m_2}P_2(E_1^{m_2})^T\} \qquad (39)
\end{aligned}
$$

Let $V_s = \frac{1}{2}s^T Ps$, $V_a = \frac{1}{2}tr\{\tilde{W}_\phi^T G_\phi^{-1}\tilde{W}_\phi\} + \frac{1}{2}tr\{\tilde{W}_\alpha^T G_\alpha^{-1}\tilde{W}_\alpha\}$, $V_e = \frac{1}{2}tr\{E_1^{m_1}P_1(E_1^{m_1})^T + E_1^{m_2}P_2(E_1^{m_2})^T\}$. Then, the differential of V_s can be described as

$$
\begin{aligned}
\dot{V}_s &= s^T P\dot{s} \\
&= (s^{m_1})^T P^{m_1}\dot{s}^{m_1} + (s^{m_2})^T P^{m_2}\dot{s}^{m_2} \\
&= -(s^{m_1})^T P^{m_1}(L + B)[(\tilde{W}_\phi^{m_1})^T\varphi_\phi^{m_1} + \varepsilon_\phi^{m_1}] \\
&\quad - (s^{m_1})^T P^{m_1}(L + B)[(\tilde{W}_\alpha^{m_1})^T\varphi_\alpha^{m_1}u^{m_1} + \varepsilon_\alpha^{m_1}] \\
&\quad - (s^{m_1})^T P^{m_1}(L + B)[cs^{m_1} + \zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}]
\end{aligned}
$$

$$
\begin{aligned}
&- (s^{m_2})^T P^{m_2}(L+B)[(\tilde{W}_\phi^{m_2})^T \varphi_\phi^{m_2} + \varepsilon_\phi^{m_2}] \\
&- (s^{m_2})^T P^{m_2}(L+B)[(\tilde{W}_\alpha^{m_2})^T \varphi_\alpha^{m_2} u^{m_2} + \varepsilon_\alpha^{m_2}] \\
&- (s^{m_2})^T P^{m_2}[cs^{m_2} + \zeta^{m_2} - \phi_0 \mathbf{1}_N^{m_2}] \\
&+ (s^{m_1})^T P^{m_1} A(D+B)^{-1}\Lambda_1 e_2^{m_1} \\
&+ (s^{m_2})^T P^{m_2} A(D+B)^{-1}(\Lambda_2 e_2^{m_2} + \Lambda_3 e_3) \\
= &-cs^T P(L+B)s - s^T P(D+B)\tilde{W}_\phi^T \varphi_\phi + s^T PA\tilde{W}_\phi^T \varphi_\phi \\
&- s^T P(D+B)\tilde{W}_\alpha^T \varphi_\alpha u + s^T PA\tilde{W}_\alpha^T \varphi_\alpha u \\
&- (s^{m_1})^T P^{m_1}(L+B)[\varepsilon_\phi^{m_1} + \varepsilon_\alpha^{m_1} + \zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\
&- (s^{m_2})^T P^{m_2}(L+B)[\varepsilon_\phi^{m_2} + \varepsilon_\alpha^{m_2} + \zeta^{m_2} - \phi_0\mathbf{1}_N^{m_2}] \\
&+ (s^{m_1})^T P^{m_1} A(D+B)^{-1}\Lambda_1 e_2^{m_1} \\
&+ (s^{m_2})^T P^{m_2} A(D+B)^{-1}(\Lambda_2 e_2^{m_2} + \Lambda_3 e_3) \qquad (40)
\end{aligned}
$$

As a result of the fact that $x^T y = tr\{yx^T\}$, according to Lemma 3, one can obtain that

$$
\begin{aligned}
\dot{V}_s = &-\frac{1}{2}cs^T Qs - tr\{\tilde{W}_\phi^T \varphi_\phi s^T P(D+B)\} \\
&- tr\{\tilde{W}_\alpha^T \varphi_\alpha u s^T P(D+B)\} \\
&+ tr\{\tilde{W}_\phi^T \varphi_\phi s^T PA\} + tr\{\tilde{W}_\alpha^T \varphi_\alpha u s^T PA\} \\
&- (s^{m_1})^T P^{m_1}(L+B)[\varepsilon_\phi^{m_1} + \varepsilon_\alpha^{m_1} + \zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\
&- (s^{m_2})^T P^{m_2}(L+B)[\varepsilon_\phi^{m_2} + \varepsilon_\alpha^{m_2} + \zeta^{m_2} - \phi_0\mathbf{1}_N^{m_2}] \\
&+ (s^{m_1})^T P^{m_1} A(D+B)^{-1}\Lambda_1 e_2^{m_1} \\
&+ (s^{m_2})^T P^{m_2} A(D+B)^{-1}(\Lambda_2 e_2^{m_2} + \Lambda_3 e_3) \qquad (41)
\end{aligned}
$$

According to (33), differentiating V_a yields

$$
\begin{aligned}
\dot{V}_a &= tr\{\tilde{W}_\phi^T G_\phi^{-1}\dot{\tilde{W}}_\phi\} + tr\{\tilde{W}_\alpha^T G_\alpha^{-1}\dot{\tilde{W}}_\alpha\} \\
&= tr\{\tilde{W}_\phi^T G_\phi^{-1}(\dot{W}_\phi - \dot{\hat{W}}_\phi)\} + tr\{\tilde{W}_\alpha^T G_\alpha^{-1}(\dot{W}_\alpha - \dot{\hat{W}}_\alpha)\} \\
&= -tr\{\tilde{W}_\phi^T G_\phi^{-1}\dot{\hat{W}}_\phi\} - tr\{\tilde{W}_\alpha^T G_\alpha^{-1}\dot{\hat{W}}_\alpha\} \qquad (42)
\end{aligned}
$$

According to (25), the differential of V_e can be expressed as

$$
\begin{aligned}
\dot{V}_e &= tr\{E_2^{m_1} P_1 (E_1^{m_1})^T + E_2^{m_2} P_2 (E_1^{m_2})^T\} \\
&= tr\{[E_1^{m_1}\theta_1^T + (s^{m_1} - \varpi)l^T]P_1(E_1^{m_1})^T\} \\
&\quad + tr\{(E_1^{m_2}\theta_2^T + s^{m_2} l^T)P_2(E_1^{m_2})^T\} \\
&= tr\{E_1^{m_1}\theta_1^T P_1(E_1^{m_1})^T + E_1^{m_2}\theta_2^T P_2(E_1^{m_2})^T\} \\
&\quad + tr\{s^{m_1} l^T P_1(E_1^{m_1})^T + s^{m_2} l^T P_2 (E_1^{m_2})^T\} \\
&\quad - tr\{\varpi l^T P_1 (E_1^{m_1})^T\} \qquad (43)
\end{aligned}
$$

Then, combining (41), (42) and (43), $\dot{V}$ can be described as

$$
\begin{aligned}
\dot{V} = &-\frac{1}{2}cs^T Qs - tr\{\tilde{W}_\phi^T[\varphi_\phi s^T P(D+B) + G_\phi^{-1}\dot{\hat{W}}_\phi]\} \\
&- tr\{\tilde{W}_\alpha^T[\varphi_\alpha u s^T P(D+B) + G_\alpha^{-1}\dot{\hat{W}}_\alpha]\} \\
&+ tr\{\tilde{W}_\phi^T \varphi_\phi s^T PA\} + tr\{\tilde{W}_\alpha^T \varphi_\alpha u s^T PA\} \\
&- (s^{m_1})^T P^{m_1}(L+B)[\varepsilon_\phi^{m_1} + \varepsilon_\alpha^{m_1} + \zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\
&- (s^{m_2})^T P^{m_2}(L+B)[\varepsilon_\phi^{m_2} + \varepsilon_\alpha^{m_2} + \zeta^{m_2} - \phi_0\mathbf{1}_N^{m_2}] \\
&+ (s^{m_1})^T P^{m_1} A(D+B)^{-1}\Lambda_1 e_2^{m_1} \\
&+ (s^{m_2})^T P^{m_2} A(D+B)^{-1}(\Lambda_2 e_2^{m_2} + \Lambda_3 e_3) \\
&+ tr\{E_1^{m_1}\theta_1^T P_1(E_1^{m_1})^T + E_1^{m_2}\theta_2^T P_2(E_1^{m_2})^T\} \\
&+ tr\{s^{m_1} l^T P_1(E_1^{m_1})^T + s^{m_2} l^T P_2 (E_1^{m_2})^T\} \\
&- tr\{\varpi l^T P_1 (E_1^{m_1})^T\} \qquad (44)
\end{aligned}
$$

Using the adaptive law (34) and updating law (36), (44) becomes

$$
\begin{aligned}
\dot{V} = &-\frac{1}{2}cs^T Qs + tr\{\tilde{W}_\phi^T \varphi_\phi s^T PA\} + tr\{\tilde{W}_\alpha^T \varphi_\alpha u s^T PA\} \\
&- (s^{m_1})^T P^{m_1}(L+B)[\varepsilon_\phi^{m_1} + \varepsilon_\alpha^{m_1} + \zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\
&- (s^{m_2})^T P^{m_2}(L+B)[\varepsilon_\phi^{m_2} + \varepsilon_\alpha^{m_2} + \zeta^{m_2} - \phi_0\mathbf{1}_N^{m_2}] \\
&+ (s^{m_1})^T P^{m_1} A(D+B)^{-1}\Lambda_1 e_2^{m_1} \\
&+ (s^{m_2})^T P^{m_2} A(D+B)^{-1}(\Lambda_2 e_2^{m_2} + \Lambda_3 e_3) \\
&+ tr\{E_1^{m_1}\theta_1^T P_1(E_1^{m_1})^T + E_1^{m_2}\theta_2^T P_2(E_1^{m_2})^T\} \\
&+ tr\{s^{m_1} l^T P_1(E_1^{m_1})^T + s^{m_2} l^T P_2 (E_1^{m_2})^T\} \\
&- tr\{\varpi l^T P_1 (E_1^{m_1})^T\} + tr\{\tilde{W}_\phi^T \eta_\phi \hat{W}_\phi\} \\
&\begin{cases}
+tr\{\tilde{W}_\alpha^T \eta_\alpha \hat{W}_\alpha\}, & \text{if } \hat{\alpha}_i > \underline{\alpha}_i \\
+tr\{\tilde{W}_\alpha^T \eta_\alpha \hat{W}_\alpha\}, & \text{if } \hat{\alpha}_i = \underline{\alpha}_i \text{ and } \\
& [\varphi_{\alpha_i}(\bar{x}_i)u_i(t)s_i(t)p_i(d_i+b_i) + \eta_\alpha \hat{W}_{\alpha_i}] < 0 \\
-tr\{\tilde{W}_\alpha^T \varphi_\alpha u s^T P(D+B)\}, & \text{if } \hat{\alpha}_i = \underline{\alpha}_i \text{ and } \\
& [\varphi_{\alpha_i}(\bar{x}_i)u_i(t)s_i(t)p_i(d_i+b_i) + \eta_\alpha \hat{W}_{\alpha_i}] \geq 0
\end{cases} \qquad (45)
\end{aligned}
$$

Considering that $tr\{\eta_\alpha \hat{W}_\alpha\} \geq -tr\{\varphi_\alpha u s^T P(D+B)\}$, if $[\varphi_{\alpha_i}(\bar{x}_i)u_i(t)s_i(t)p_i(d_i+b_i) + \eta_\alpha \hat{W}_{\alpha_i}] \geq 0$, one can write

$$
\begin{aligned}
&tr\{\tilde{W}_\alpha^T \eta_\alpha \hat{W}_\alpha\} \geq -tr\{\tilde{W}_\alpha^T \varphi_\alpha u s^T P(D+B)\}, \text{ if } \hat{\alpha}_i = \underline{\alpha}_i \text{ and} \\
&\qquad [\varphi_{\alpha_i}(\bar{x}_i)u_i(t)s_i(t)p_i(d_i+b_i) + \eta_\alpha \hat{W}_{\alpha_i}] \geq 0 \qquad (46)
\end{aligned}
$$

Then, we have

$$
\begin{aligned}
\dot{V} \leq &-\frac{1}{2}cs^T Qs + tr\{\tilde{W}_\phi^T \varphi_\phi s^T PA\} + tr\{\tilde{W}_\alpha^T \varphi_\alpha u s^T PA\} \\
&+ \eta_\phi tr\{\tilde{W}_\phi^T (W_\phi - \tilde{W}_\phi)\} + \eta_\alpha tr\{\tilde{W}_\alpha^T (W_\alpha - \tilde{W}_\alpha)\} \\
&- (s^{m_1})^T P^{m_1}(L+B)[\varepsilon_\phi^{m_1} + \varepsilon_\alpha^{m_1} + \zeta^{m_1} - x_{0,3}\mathbf{1}_N^{m_1}] \\
&- (s^{m_2})^T P^{m_2}(L+B)[\varepsilon_\phi^{m_2} + \varepsilon_\alpha^{m_2} + \zeta^{m_2} - \phi_0\mathbf{1}_N^{m_2}] \\
&+ (s^{m_1})^T P^{m_1} A(D+B)^{-1}\Lambda_1 s^{m_1} \\
&+ (s^{m_2})^T P^{m_2} A(D+B)^{-1}\Lambda_3 s^{m_2} \\
&- (s^{m_1})^T P^{m_1} A(D+B)^{-1}\Lambda_1^2 E_1^{m_1}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\mathbf{1}_2 \\
&+ (s^{m_2})^T P^{m_2} A(D+B)^{-1} E_1^{m_2}\begin{bmatrix} 0 & 0 \\ 0 & \Lambda_2 \end{bmatrix}\mathbf{1}_2 \\
&- (s^{m_2})^T P^{m_2} A(D+B)^{-1}\Lambda_3 E_1^{m_2}\begin{bmatrix} \Lambda_2 & 0 \\ 0 & \Lambda_3 \end{bmatrix}\mathbf{1}_2 \\
&+ tr\{E_1^{m_1}\theta_1^T P_1(E_1^{m_1})^T + E_1^{m_2}\theta_2^T P_2(E_1^{m_2})^T\} \\
&+ tr\{s^{m_1} l^T P_1(E_1^{m_1})^T + s^{m_2} l^T P_2 (E_1^{m_2})^T\} \\
&- tr\{\varpi l^T P_1 (E_1^{m_1})^T\} \qquad (47)
\end{aligned}
$$

According to the boundedness in Assumption 2 - Assumption 4, taking norm on (47), one can obtain

$$\begin{aligned}\dot{V} \le & -\frac{1}{2}c\underline{\sigma}(Q)\|s\|^2 + \eta_\phi W_{\phi M}\left\|\tilde{W}_\phi\right\|_F - \eta_\phi\left\|\tilde{W}_\phi\right\|_F^2 \\ & + \eta_\alpha W_{\alpha M}\left\|\tilde{W}_\alpha\right\|_F - \eta_\alpha\left\|\tilde{W}_\alpha\right\|_F^2 \\ & + \varphi_{\phi M}\overline{\sigma}(P)\overline{\sigma}(A)\|s\|\left\|\tilde{W}_\phi\right\|_F \\ & + \varphi_{\alpha M}u_M\overline{\sigma}(P)\overline{\sigma}(A)\|s\|\left\|\tilde{W}_\alpha\right\|_F \\ & + \overline{\sigma}(P)\overline{\sigma}(L+B)F_M\|s\| \\ & + \frac{\overline{\sigma}(P)\overline{\sigma}(A)}{\underline{\sigma}(D+B)}[\overline{\sigma}(\Lambda_1)+\overline{\sigma}(\Lambda_3)]\|s\|^2 \\ & + \frac{\overline{\sigma}(P)\overline{\sigma}(A)}{\underline{\sigma}(D+B)}[\overline{\sigma}(\Lambda_1^2)+\overline{\sigma}(\Lambda_2)+\overline{\sigma}(\Lambda_3)\overline{\sigma}(\chi)]\|s\|\|E_1\|_F \\ & - \frac{1}{2}\underline{\beta}\|E_1\|_F^2 + \overline{\sigma}(P_m)\|s\|\|E_1\|_F \\ & + \overline{\sigma}(L+B)\overline{\sigma}(P_m)\left\|\phi_M+\alpha_M u_M+\zeta_M+x_{0,3M}\right\|\|E_1\|_F\end{aligned} \tag{48}$$

where $F_M = \varepsilon_M + \zeta_M + \phi_{0M} + x_{0,3M}$; $\varepsilon_M = \varepsilon_{\phi M} + \varepsilon_{\alpha M}$.

According to (31), (48) can be described as

$$\dot{V} \le -r^T\Psi r + \vartheta^T r = -V_r \tag{49}$$

where

$$\begin{aligned}\Psi &= \begin{bmatrix} \frac{1}{2}c\underline{\sigma}(Q)-h & -\frac{1}{2}y & -\frac{1}{2}\lambda & -\frac{1}{2}[\overline{\sigma}(P_m)+v] \\ -\frac{1}{2}y & \eta_\phi & 0 & 0 \\ -\frac{1}{2}\lambda & 0 & \eta_\alpha & 0 \\ -\frac{1}{2}[\overline{\sigma}(P_m)+v] & 0 & 0 & \frac{\underline{\beta}}{2}\end{bmatrix}; \\ r &= \left[\|s\| \ \left\|\tilde{W}_\phi\right\|_F \ \left\|\tilde{W}_\alpha\right\|_F \ \|E_1\|_F\right]^T; \\ \vartheta &= \left[\overline{\sigma}(P)\overline{\sigma}(L+B)F_M \ \eta_\phi W_{\phi M} \ \eta_\alpha W_{\alpha M} \ \psi\right]^T; \\ \psi &= \overline{\sigma}(L+B)\overline{\sigma}(P_m)\left\|\phi_M+\alpha_M u_M+\zeta_M+x_{0,3M}\right\|.\end{aligned}$$

Then, V_r is positive definite when the subsequent conditions are satisfied.

(1) Ψ is positive definite; (2) $\|r\| > \frac{\|\vartheta\|}{\underline{\sigma}(\Psi)}$.

According to Sylvester's theorem in [48], if Ψ is positive definite, the following inequations should be satisfied.

- $\frac{1}{2}c\underline{\sigma}(Q)-h > 0$
- $\eta_\phi[\frac{1}{2}c\underline{\sigma}(Q)-h] - \frac{1}{4}y^2 > 0$
- $\eta_\phi\eta_\alpha[\frac{1}{2}c\underline{\sigma}(Q)-h] - \frac{1}{4}\eta_\phi\lambda^2 - \frac{1}{4}\eta_\alpha y^2 > 0$
- $\frac{1}{2}\underline{\beta}\eta_\phi\eta_\alpha[\frac{1}{2}c\underline{\sigma}(Q)-h] - \frac{1}{8}\underline{\beta}\eta_\alpha y^2 - \frac{1}{8}\underline{\beta}\eta_\phi\lambda^2 - \frac{1}{4}\eta_\phi\eta_\alpha[\overline{\sigma}(P_m)+v]^2 > 0$ (50)

Solve the above inequations, we can get condition (31), which indicates that c does exist and the proper control gain can ensure the individual vehicle stability and the stability of the overall platoon.

Define Z_d as

$$Z_d = \frac{\|\vartheta\|_1}{\underline{\sigma}(\Psi)} = \frac{F_M\overline{\sigma}(P)\overline{\sigma}(L+B)+\eta_\phi W_{\phi M}+\eta_\alpha W_{\alpha M}+\psi}{\underline{\sigma}(\Psi)} \tag{51}$$

Then, if $\|r\| > \frac{\|\vartheta\|_1}{\underline{\sigma}(\Psi)} > \frac{\|\vartheta\|}{\underline{\sigma}(\Psi)}$ holds, under condition (31), one can obtain that

$$\dot{V} \le -V_r, \quad \forall\|r\| \ge Z_d \tag{52}$$

where V_r is positive definite.

According to the Lyapunov candidate (39) and Reference [49], we have

$$\underline{\sigma}(\Theta)\|r\|^2 \le V \le \overline{\sigma}(\Gamma)\|r\|^2 \tag{53}$$

where

$$\begin{aligned}\Theta &= diag\left\{\underline{\sigma}(P)/2, 1/2\overline{\sigma}(G_\phi), 1/2\overline{\sigma}(G_\alpha), \underline{\sigma}(P_m)/2\right\} \\ \Gamma &= diag\left\{\overline{\sigma}(P)/2, 1/2\underline{\sigma}(G_\phi), 1/2\underline{\sigma}(G_\alpha), \overline{\sigma}(P_m)/2\right\}\end{aligned} \tag{54}$$

Based on Theorem 4.18 in [51], it follows that for any $r(t_0)$ there exists a T_0 such that

$$\|r(t)\| \le \sqrt{\frac{\overline{\sigma}(\Gamma)}{\underline{\sigma}(\Theta)}}Z_d, \quad \forall t \ge t_0+T_0 \tag{55}$$

Define $\delta = \min_{\|r\|>Z_d} V_r$. Then, we can obtain

$$T_0 = \frac{V(t_0)-\overline{\sigma}(\Gamma)Z_d^2}{\delta} \tag{56}$$

Therefore, (55) implies that the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$ and approximation errors $\tilde{W}_\phi$, $\tilde{W}_\alpha$ are ultimately bounded. Meanwhile, according to Lemma 2, $\mu_1(t)$, $\mu_2(t)$, $\mu_3(t)$ are CUUB, which indicates that the platoon synchronization and the leader tracking for all vehicles is realized in the finite time.

Next, the boundedness of $x_{i,1}(t)$, $x_{i,2}(t)$, and $x_{i,3}(t)$ for $\forall t \ge t_0$ can be proved as follows. From (39), one can write

$$\dot{V} \le -\underline{\sigma}(\Psi)\|r\|^2 + \|\vartheta\|\|r\| \tag{57}$$

According to (53) and (57), it follows that

$$\frac{d(\sqrt{V})}{dt} \le -\frac{\underline{\sigma}(\Psi)\sqrt{V}}{2\overline{\sigma}(\Gamma)} + \frac{\|\vartheta\|}{2\sqrt{\underline{\sigma}(\Theta)}} \tag{58}$$

Then, under Corollary 1.1 in [52], $V(t)$ is bounded for $\forall t \ge t_0$. By (39), we have $\|s\|^2 \le 2V(t)/\underline{\sigma}(P)$, which denotes that $s(t)$ is bounded. Meanwhile, $x_{0,1}(t)$, $x_{0,2}(t)$, and $x_{0,3}(t)$ are bounded by $x_{0,1M}$, $x_{0,2M}$, and $x_{0,3M}$ in Assumption 3, respectively. Then, $x_{i,1}, x_{i,2}$ for $i \in M$ and $x_{i,3}$ for $i \in m_2$ are bounded $\forall t \ge t_0$. The proof of Theorem 1 is done. ∎

B. STRING STABILITY ANALYSIS

In this section, the string stability of the mixed-order heterogeneous CAV platoon is proved by employing the approach presented in [53]. Before string stability analysis, the necessary definition and lemma are presented as follows.

Lemma 4 (Barbalat Lemma): If $f(t)$: $\Re \rightarrow \Re$ is a uniformly continuous function for $t \geq 0$ and $\lim_{t\to\infty} \int_0^t f(\tau) d\tau < \infty$, then $\lim_{t\to\infty} f(t) = 0$ [54].

Definition 2: The platoon system in (1),(2) is string stable if given any $\varepsilon > 0$, there exists a $\iota > 0$ such that [29]

$$\left\|\mu_{i,1}(0)\right\|_\infty < \varepsilon \Rightarrow \sup_i \left\|\mu_{i,1}(\cdot)\right\|_\infty < \iota \tag{59}$$

Theorem 2: Consider the mixed-order heterogeneous CAV platoon consisting of the following vehicles with third-order dynamics in (1) and second-order dynamics in (2). Under the proposed control protocol (29), the heterogeneous platoon is string stable in accordance with Definition 2.

Proof: According to (7),

$$\mu_{i,1}(t) = x_{i,1}(t) - x_{ides}(t) = x_{i,1}(t) - x_{0,1}(t) + i \cdot d \tag{60}$$

Then, we can obtain

$$\ddot{\mu}_{i,1}(t) = \ddot{x}_{i,1}(t) - \ddot{x}_{ides}(t) = x_{i,3}(t) - x_{0,3}(t), \quad \forall i \in m_2 \tag{61}$$

Given the fact that $x_{i,3}(t)$ and $x_{0,3}(t)$ are bounded, it implies that $\ddot{\mu}_{i,1}(t) \in \mathcal{L}_\infty$ for $\forall i \in m_2$. And, we can also obtain that

$$\ddot{\mu}_{i,1}(t) = \ddot{x}_{i,1}(t) - \ddot{x}_{ides}(t) = \dot{x}_{i,2}(t) - \dot{x}_{0,2}(t), \quad \forall i \in m_1 \tag{62}$$

Given the fact that $x_{i,2}(t)$ and $x_{0,2}(t)$ are bounded, it follows that the derivations of $x_{i,2}(t)$ and $x_{0,2}(t)$ are bounded, which indicates that $\ddot{\mu}_{i,1}(t) \in \mathcal{L}_\infty$ for $\forall i \in m_1$. Therefore, $\dot{\mu}_{i,1}(t)$ is uniformly continuous for $\forall i \in M$.

Furthermore,

$$\int_0^\infty \left|\dot{\mu}_{i,1}(t)\right| dt = \left|\mu_{i,1}(\infty)\right| - \left|\mu_{i,1}(0)\right| < \infty \tag{63}$$

According to (63), one can obtain that $\dot{\mu}_{i,1}(t) \in \mathcal{L}_2$. Therefore, it has $\lim_{t\to\infty} \dot{\mu}_{i,1}(t) = 0$ according to Lemma 4. Thus, it has $\dot{\mu}_{i,1}(t) \in \mathcal{L}_\infty$. Likewise, since $\mu_{i,1}(0) = 0$, one can obtain that $\mu_{i,1}(t) \in \mathcal{L}_2$. Therefore, one can further obtain $\lim_{t\to\infty} \mu_{i,1}(t) = 0$ according to Lemma 4.

If $\iota > 0$, then $\left\|\mu_{i,1}(0)\right\|_\infty = \sup_i \left|\mu_{i,1}(0)\right| = 0 < \iota$. In addition, consider that $\lim_{t\to\infty} \mu_{i,1}(t) = 0$, $\mu_{i,1}(0) = 0$, and $\mu_{i,1}(t) \in \mathcal{L}_2$. Therefore, $\exists Y, \kappa > 0$, s.t. $\sup_{i, t\in[0,\infty)} \left|\mu_{i,1}(t)\right| = Y < \kappa$ for $t \in [0, \infty)$. Consequently, string stability of the mixed-order heterogeneous CAV platoon with the developed control protocol (29) can be guaranteed according to (59). The demonstration for Theorem 2 is completed. ■

TABLE 2. Parameters of the mixed-order heterogeneous CAV platoon.

Vehicle	τ_i(s)	M_i(kg)	K_{di}(N)	ς_i	d_{mi}	R_i(m)	f	g
1	0.55	1837	0.385	–	100	–	–	–
2	–	1495	0.43	0.85	–	0.28	0.02	9.8
3	0.52	1764	0.385	–	110	–	–	–
4	–	1545	0.43	0.85	–	0.29	0.04	9.8
5	0.48	1688	0.385	–	120	–	–	–

VI. NUMERICAL EXAMPLE

The numerical examples are utilized to validate the correctness and efficacy of the developed control strategies in this section. We consider a mixed-order heterogeneous CAV platoon including one leading vehicle and five following vehicles ($M = 5$), where followers 1, 3, 5 have third-order dynamics and followers 2, 4 have second-order dynamics. The vehicle dynamics of the leader contains full state information. All followers are required to track the position and velocity of the leader, and the third-order followers should additionally track the acceleration of the leader. It is clear that the existing cooperative platoon control methods can hardly address the control issue of this vehicle platoon. The dynamics of the second-order followers are described as

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \dot{x}_{i,2} = \phi_{sp2}(\bar{x}_i) + \alpha_{sp2}(\bar{x}_i)u_i + \zeta_{2,i} \end{cases} \quad i \in 2,\ 4$$

and the dynamics of the third-order followers are

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \dot{x}_{i,2} = x_{i,3} \\ \dot{x}_{i,3} = \phi_{sp3}(\bar{x}_i) + \alpha_{sp3}(\bar{x}_i)u_i + \zeta_{3,i} \end{cases} \quad i \in 1,\ 3,\ 5$$

where $\phi_{sp2}(\bar{x}_i)$, $\phi_{sp3}(\bar{x}_i)$, $\alpha_{sp2}(\bar{x}_i)$, $\alpha_{sp3}(\bar{x}_i)$, $\zeta_{2,i}$, and $\zeta_{3,i}$ are given by

$$\begin{aligned} \phi_{sp2}(\bar{x}_i) &= \frac{K_{di}}{M_i} x_{i,2}^2 \\ \phi_{sp3}(\bar{x}_i) &= -\frac{2K_{di}}{M_i} x_{i,2} x_{i,3} - \frac{K_{di}}{\tau_i M_i} x_{i,2}^2 - \frac{1}{\tau_i} x_{i,3} \\ \alpha_{sp2}(\bar{x}_i) &= \frac{\varsigma_i}{M_i R_i}, \quad \alpha_{sp3}(\bar{x}_i) = \frac{1}{\tau_i M_i} \\ \zeta_{2,i} &= f \cdot g, \quad \zeta_{3,i} = -\frac{d_{mi}}{\tau_i M_i} \end{aligned}$$

where τ_i, M_i, K_{di}, ς_i, d_{mi}, R_i, f, and g are the engine time lag, vehicle mass, aerodynamic drag coefficient, mechanical efficiency of the driveline, mechanical drag, radius of wheel, coefficient of rolling resistance, and acceleration due to gravity of the ith vehicle, respectively [23], [55]. Table 2 lists the model parameters of the heterogeneous CAV platoon with mixed-order dynamics. It is worth mentioning that all parameters for the mixed-order vehicles are considered to be unknown.

Fig. 3 shows the PLF platoon communication topology for the mixed-order heterogeneous CAV platoon. There is no doubt that the communication topology meets the conditions outlined in Assumption 1.

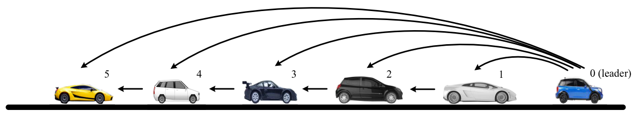

FIGURE 3. Communication topology for mixed-order heterogeneous CAV platoon.

The desired spacing d is 3.5m. The initial positions of the leader and followers are determined to $x_{0,1}(0) = 30$m, $x_{i,1}(0) = (30-i{\cdot}d)$m, $i \in M$, respectively. The initial weights of the neural network are randomly selected within the range of 0 and 0.1 [11].

Example 1: In this example, the leader of the platoon maintains a constant velocity and follows the trajectory outlined below.

$$x_{0,2}(t) = 2\text{m/s}, \qquad 0\text{s} \le t \le 80\text{s}$$

The initial state of followers are respectively denoted as $x_{i,1}(0) = (30-i \cdot d)$m, $x_{i,2}(0) = 3$m/s, $x_{i,3}(0) = 0$m/s^2, $i \in M$ [56].

Example 2: In this example, the acceleration of the leader is considered to be variable, and its trajectory is defined as follows.

$$x_{0,3}(t) = \begin{cases} 0\ \text{m/s}^2, & 0\text{s} \le t < 10\text{s} \\ 0.2t\ \text{m/s}^2, & 10\text{s} \le t < 20\text{s} \\ -0.2t\ \text{m/s}^2, & 20\text{s} \le t < 30\text{s} \\ 0\ \text{m/s}^2, & 30\text{s} \le t < 40\text{s} \\ -0.1t\ \text{m/s}^2, & 40\text{s} \le t < 50\text{s} \\ 0.1t\ \text{m/s}^2, & 50\text{s} \le t < 60\text{s} \\ 0\ \text{m/s}^2, & \text{otherwise.} \end{cases}$$

The initial state of followers are respectively defined as $x_{i,1}(0) = (30-i \cdot d)$m, $x_{i,2}(0) = 0$m/s, $x_{i,3}(0) = 0$m/s^2, $i \in M$ [56].

The DSM control protocols (29), distributed neural network-based adaptive laws (34), and neural updating laws (36) are implemented in both of the two examples. The parameters in the distributed mixed-order platoon control strategies are $c = 560$, $G_{\phi_i} = 10000$, $\eta_\phi = 0.0001$, $G_{\alpha_i} = 0.0001$, $\eta_\alpha = 30$. Then, the control results of the mixed-order heterogeneous CAV platoon are represented in Figs. 4-13.

For Example 1, as shown in Fig. 4 - Fig. 8, each vehicle within the platoon is able to keep a predefined spacing d with its adjacent vehicles and eventually achieve velocity synchronization with the leader. It is presented in Fig. 4 that the trajectories of all vehicles exhibit no intersections or overlaps, which indicates that there are no vehicle collisions occurring under both transient conditions and steady-state conditions. It is shown in Fig. 5 that the velocities eventually converge to that of the leader within 15s. For the third-order vehicles 1, 3, and 5, the accelerations of these vehicles can be measurable. In Fig. 6, given the difference in velocity between the followers 1, 3, 5 and the leader, there are significant accelerations in the initial stage for the third-order followers, and the initial accelerations rapidly decrease and converge to zero. Fig. 7 and Fig. 8 show the spacing and velocity errors of the followers, respectively. It can be noticed that the state errors in both spacing and velocity are capable of rapidly converging to zero within a finite time. Furthermore, Fig. 7 illustrates that the amplitude of the spacing errors $\mu_{i,1}(t)$ gradually decreases, which indicates that the mixed-order heterogeneous CAV platoon can achieve string stability.

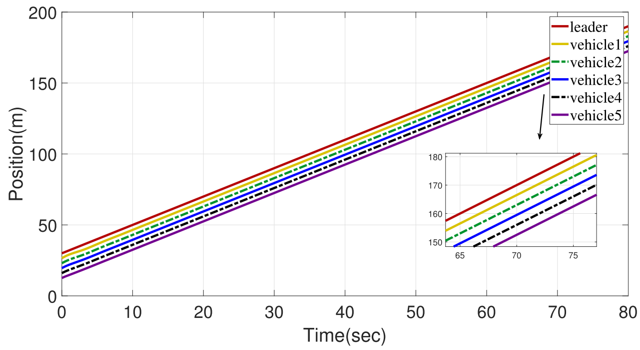

FIGURE 4. Position tracking of the platoon vehicles for Example 1.

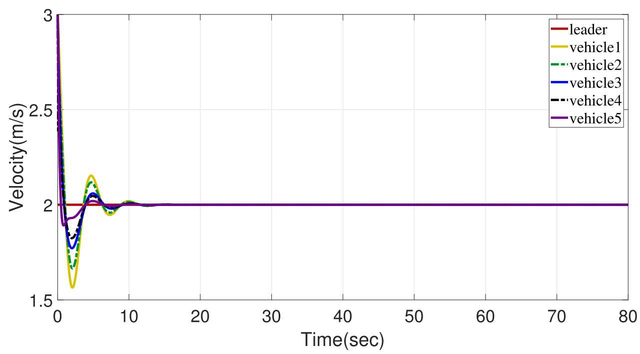

FIGURE 5. Velocity tracking of the platoon vehicles for Example 1.

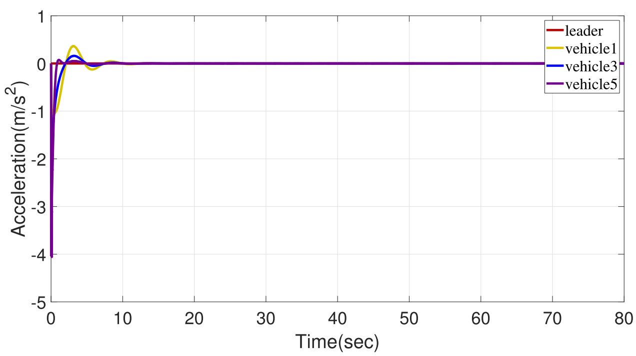

FIGURE 6. Acceleration tracking of the platoon vehicles for Example 1.

For Example 2, as shown in Fig. 9 and Fig. 13, the trajectory of the leader can be segmented into periods of acceleration and deceleration. Although the acceleration of the leader is variable, the developed control strategies can still

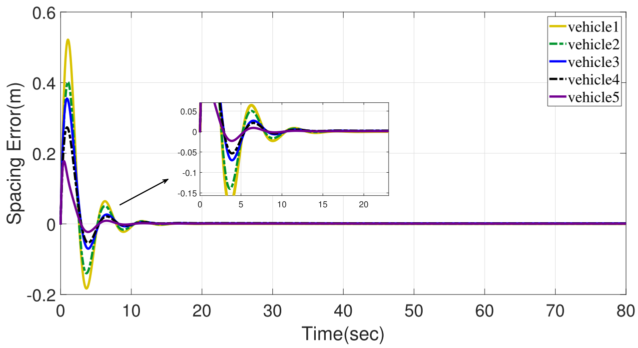

FIGURE 7. Spacing errors for Example 1.

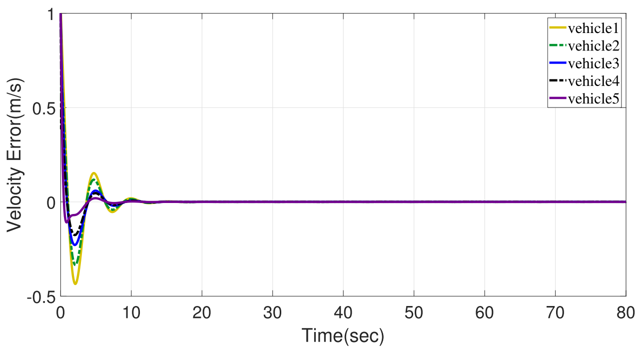

FIGURE 8. Velocity errors for Example 1.

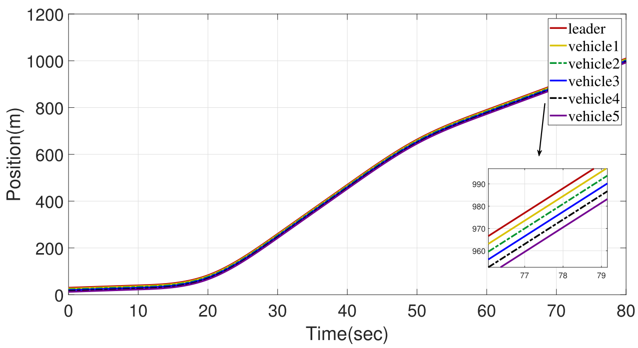

FIGURE 9. Position tracking of the platoon vehicles for Example 2.

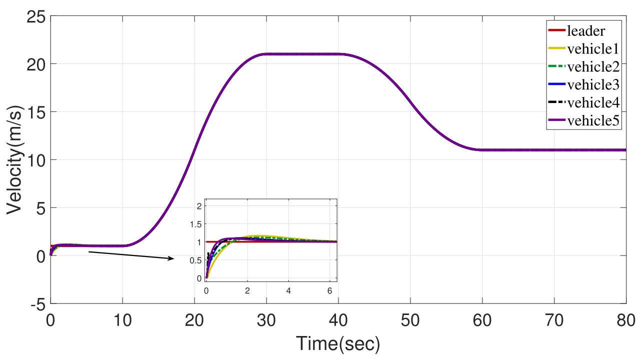

FIGURE 10. Velocity tracking of the platoon vehicles for Example 2.

realize the consensus of the mixed-order heterogeneous CAV platoon. It is shown in Fig. 9 and Fig. 10 that all following

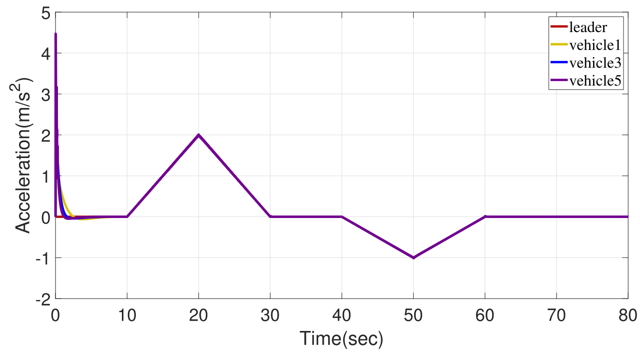

FIGURE 11. Acceleration tracking of the platoon vehicles for Example 2.

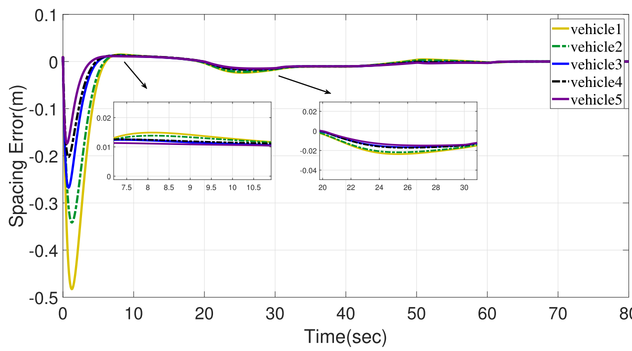

FIGURE 12. Spacing errors for Example 2.

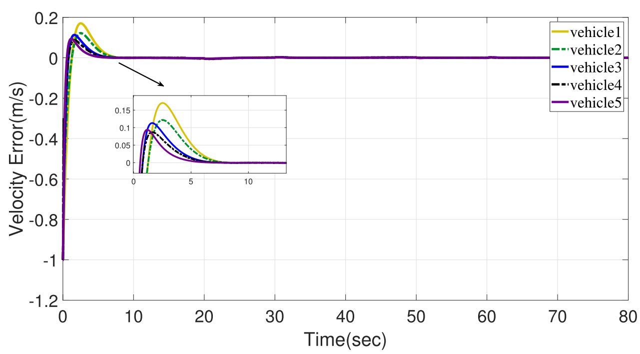

FIGURE 13. Velocity errors for Example 2.

vehicles are capable of effectively tracking the leading vehicle and keeping a desired spacing with neighboring vehicles in both the acceleration and deceleration phases. From Fig. 10 and Fig. 11, it is revealed that the velocity and acceleration of the third-order followers can rapidly converge to the corresponding states of the leading vehicle. The spacing errors and velocity errors of the followers are illustrated in Fig. 12 and Fig. 13. It is shown that for each vehicle in the platoon, the amplitudes of both the spacing and velocity error gradually decrease and are able to converge to the vicinity of zero in a short period of time. It is obvious in Fig. 12 that the mixed-order heterogeneous CAV platoon is string stable in Example 2.

To summarize, the novel DSM control protocols (29), the neural network-based adaptive laws (34), and the neural updating laws (36) can enable all followers within the mixed-order heterogeneous CAV platoon to track the trajectory of the leader and maintain the desired spacing. The proposed control strategies can realize the control objective and guarantee the error convergence and string stability.

VII. CONCLUSION

This study has resolved the synchronization and tracking problem of the mixed-order heterogeneous CAV platoon consisting of vehicles with second-order and third-order dynamics, whereas existing literature cannot resolve this problem. Compared to the existing studies, the novel distributed control methods have been developed for the heterogeneous CAV platoon comprising vehicles with mixed-order nonlinear dynamics, unknown uncertainties and external disturbances. To guarantee the error convergence in spacing, velocity, and acceleration of vehicles and string stability, the novel DSM control strategies based on the platoon communication topology are developed in this paper. The proposed control strategies do not need to acquire precise information of vehicle dynamics. In addition, the new distributed neural network-based adaptive updating mechanism is proposed to approximate unknown uncertainties and compensate for the model nonlinearities. The ultimate boundedness of state errors are proved by the Lyapunov approach. The infinity-norm method is utilized to demonstrate string stability of the heterogeneous platoon. To validate the effectiveness of the developed platoon control strategies, the numerical examples are presented in this paper. In future research, the influence of the switching communication topologies and the problems of fault-tolerant control for mixed-order platoon will be further investigated.

REFERENCES

[1] S. Chu and A. Majumdar, "Opportunities and challenges for a sustainable energy future," *Nature*, vol. 488, no. 7411, pp. 294–303, Aug. 2012.

[2] A. A. Alam, A. Gattami, and K. H. Johansson, "An experimental study on the fuel reduction potential of heavy duty vehicle platooning," in *Proc. 13th Int. IEEE Conf. Intell. Transp. Syst.*, Sep. 2010, pp. 306–311.

[3] S. Tsugawa, S. Jeschke, and S. E. Shladover, "A review of truck platooning projects for energy savings," *IEEE Trans. Intell. Vehicles*, vol. 1, no. 1, pp. 68–77, Mar. 2016.

[4] S. Tsugawa, "An overview on an automated truck platoon within the energy ITS project," *IFAC Proc. Volumes*, vol. 46, no. 21, pp. 41–46, 2013.

[5] S. E. Li, Y. Zheng, K. Li, Y. Wu, J. K. Hedrick, F. Gao, and H. Zhang, "Dynamical modeling and distributed control of connected and automated vehicles: Challenges and opportunities," *IEEE Intell. Transp. Syst. Mag.*, vol. 9, no. 3, pp. 46–58, Jun. 2017.

[6] S. Studli, M. M. Seron, and R. H. Middleton, "From vehicular platoons to general networked systems: String stability and related concepts," *Annu. Rev. Control*, vol. 44, pp. 157–172, Sep. 2017.

[7] G. Guo and W. Yue, "Hierarchical platoon control with heterogeneous information feedback," *IET Control Theory Appl.*, vol. 5, no. 15, pp. 1766–1781, Oct. 2011.

[8] Y. Li, K. Li, T. Zheng, X. Hu, H. Feng, and Y. Li, "Evaluating the performance of vehicular platoon control under different network topologies of initial states," *Phys. A, Stat. Mech. Appl.*, vol. 450, pp. 359–368, May 2016.

[9] A. Zakerimanesh, T. Qiu, and M. Tavakoli, "Heterogeneous vehicular platooning with stable decentralized linear feedback control," in *Proc. IEEE Int. Conf. Auto. Syst. (ICAS)*, Aug. 2021, pp. 1–5.

[10] P. Wang, H. Deng, J. Zhang, L. Wang, M. Zhang, and Y. Li, "Model predictive control for connected vehicle platoon under switching communication topology," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 7, pp. 7817–7830, Jul. 2022.

[11] X. Guo, J. Wang, F. Liao, and R. S. H. Teo, "Distributed adaptive sliding mode control strategy for vehicle-following systems with nonlinear acceleration uncertainties," *IEEE Trans. Veh. Technol.*, vol. 66, no. 2, pp. 981–991, Feb. 2017.

[12] Y. Liu, C. Zong, and D. Zhang, "Lateral control system for vehicle platoon considering vehicle dynamic characteristics," *IET Intell. Transp. Syst.*, vol. 13, no. 9, pp. 1356–1364, Sep. 2019.

[13] J. Shen, E. K. H. Kammara, and L. Du, "Fully distributed optimization-based CAV platooning control under linear vehicle dynamics," *Transp. Sci.*, vol. 56, no. 2, pp. 381–403, Mar. 2022.

[14] G. Guo and D. Li, "PMP-based set-point optimization and sliding-mode control of vehicular platoons," *IEEE Trans. Computat. Social Syst.*, vol. 5, no. 2, pp. 553–562, Jun. 2018.

[15] J. Chen, D. Bai, H. Liang, and Y. Zhou, "A third-order consensus approach for vehicle platoon with intervehicle communication," *J. Adv. Transp.*, vol. 2018, pp. 1–10, Jul. 2018.

[16] L. Xu, W. Zhuang, G. Yin, and C. Bian, "Distributed formation control of homogeneous vehicle platoon considering vehicle dynamics," *Int. J. Automot. Technol.*, vol. 20, no. 6, pp. 1103–1112, Dec. 2019.

[17] J. Shen, E. K. H. Kammara, and L. Du, "Nonconvex, fully distributed optimization based CAV platooning control under nonlinear vehicle dynamics," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 11, pp. 20506–20521, Nov. 2022.

[18] P. Wijnbergen, M. Jeeninga, and B. Besselink, "Nonlinear spacing policies for vehicle platoons: A geometric approach to decentralized control," *Syst. Control Lett.*, vol. 153, Jul. 2021, Art. no. 104954.

[19] H. Long, A. Khalatbarisoltani, and X. Hu, "MPC-based eco-platooning for homogeneous connected trucks under different communication topologies," in *Proc. IEEE Intell. Vehicles Symp. (IV)*, Jun. 2022, pp. 241–246.

[20] J. Wang, X. Luo, J. Yan, and X. Guan, "Distributed integrated sliding mode control for vehicle platoons based on disturbance observer and multi power reaching law," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 4, pp. 3366–3376, Apr. 2022.

[21] A. Ghasemi, R. Kazemi, and S. Azadi, "Stable decentralized control of a platoon of vehicles with heterogeneous information feedback," *IEEE Trans. Veh. Technol.*, vol. 62, no. 9, pp. 4299–4308, Nov. 2013.

[22] S. E. Li, Y. Zheng, K. Li, and J. Wang, "An overview of vehicular platoon control under the four-component framework," in *Proc. IEEE Intell. Vehicles Symp. (IV)*, Jun. 2015, pp. 286–291.

[23] Y. Wu, S. E. Li, J. Cortés, and K. Poolla, "Distributed sliding mode control for nonlinear heterogeneous platoon systems with positive definite topologies," *IEEE Trans. Control Syst. Technol.*, vol. 28, no. 4, pp. 1272–1283, Jul. 2020.

[24] D. Jia, D. Ngoduy, and H. L. Vu, "A multiclass microscopic model for heterogeneous platoon with vehicle-to-vehicle communication," *Transportmetrica B, Transp. Dyn.*, vol. 7, no. 1, pp. 311–335, Dec. 2019.

[25] G. Feng, D. Dang, and Y. He, "Robust coordinated control of nonlinear heterogeneous platoon interacted by uncertain topology," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 6, pp. 4982–4992, Jun. 2022.

[26] M. R. Hidayatullah and J.-C. Juang, "Centralized and distributed control framework under homogeneous and heterogeneous platoon," *IEEE Access*, vol. 9, pp. 49629–49648, 2021.

[27] Y. Li, Y. Zhao, and S. Tong, "Adaptive fuzzy control for heterogeneous vehicular platoon systems with collision avoidance and connectivity preservation," *IEEE Trans. Fuzzy Syst.*, vol. 1, no. 2, pp. 1–10, Jul. 2023.

[28] Y. Feng, S. Yu, H. Chen, Y. Li, S. Shi, J. Yu, and H. Chen, "Distributed MPC of vehicle platoons with guaranteed consensus and string stability," *Sci. Rep.*, vol. 13, no. 1, p. 10396, Jun. 2023.

[29] S. Feng, Y. Zhang, S. E. Li, Z. Cao, H. X. Liu, and L. Li, "String stability for vehicular platoon control: Definitions and analysis methods," *Annu. Rev. Control*, vol. 47, pp. 81–97, Aug. 2019.

[30] A. L. Gratzer, S. Thormann, A. Schirrer, and S. Jakubek, "String stable and collision-safe model predictive platoon control," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 10, pp. 19358–19373, Oct. 2022.

[31] J. Ploeg, D. P. Shukla, N. van de Wouw, and H. Nijmeijer, "Controller synthesis for string stability of vehicle platoons," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 2, pp. 854–865, Apr. 2014.

[32] X. Hu, L. Xie, L. Xie, S. Lu, W. Xu, and H. Su, "Distributed model predictive control for vehicle platoon with mixed disturbances and model uncertainties," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 10, pp. 17354–17365, Oct. 2022.

[33] Y. Zhu, J. Wu, and H. Su, "V2V-based cooperative control of uncertain, disturbed and constrained nonlinear CAVs platoon," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 3, pp. 1796–1806, Mar. 2022.

[34] Z. Ju, H. Zhang, and Y. Tan, "Distributed stochastic model predictive control for heterogeneous vehicle platoons subject to modeling uncertainties," *IEEE Intell. Transp. Syst. Mag.*, vol. 14, no. 2, pp. 25–40, Mar. 2022.

[35] Y. Zhu and F. Zhu, "Distributed adaptive longitudinal control for uncertain third-order vehicle platoon in a networked environment," *IEEE Trans. Veh. Technol.*, vol. 67, no. 10, pp. 9183–9197, Oct. 2018.

[36] F. Gao, S. E. Li, Y. Zheng, and D. Kum, "Robust control of heterogeneous vehicular platoon with uncertain dynamics and communication delay," *IET Intell. Transp. Syst.*, vol. 10, no. 7, pp. 503–513, Sep. 2016.

[37] J. Hu, P. Bhowmick, F. Arvin, A. Lanzon, and B. Lennox, "Cooperative control of heterogeneous connected vehicle platoons: An adaptive leader-following approach," *IEEE Robot. Autom. Lett.*, vol. 5, no. 2, pp. 977–984, Apr. 2020.

[38] H. Pei and Z. Lan, "Multi-agent consistent formation control operation optimization for high-speed trains," *IEEE Access*, vol. 11, pp. 139201–139210, 2023.

[39] H. Pei, "Group consensus of multi-agent systems with hybrid characteristics and directed topological networks," *ISA Trans.*, vol. 138, pp. 311–317, Jul. 2023.

[40] Z. Qu, *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*. Cham, Switzerland: Springer, 2009.

[41] X. Guo, J. Wang, F. Liao, and R. S. H. Teo, "Distributed adaptive integrated-sliding-mode controller synthesis for string stability of vehicle platoons," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 9, pp. 2419–2429, Sep. 2016.

[42] Z. Chen, F. Huang, W. Sun, J. Gu, and B. Yao, "RBF-neural-network-based adaptive robust control for nonlinear bilateral teleoperation manipulators with uncertainty and time delay," *IEEE/ASME Trans. Mechatronics*, vol. 25, no. 2, pp. 906–918, Apr. 2020.

[43] F. Lewis, S. Jagannathan, and A. Yesildirak, *Neural Network Control of Robot Manipulators and Non-linear Systems*. Boca Raton, FL, USA: CRC Press, 1998.

[44] X. Li and Y. Wang, "Distributed cooperative neural control of a class of nonlinear multi-agent systems with unknown time-varying control coefficient," *Int. J. Adapt. Control Signal Process.*, vol. 36, no. 1, pp. 2–16, Jan. 2022.

[45] X. Li, P. Shi, and Y. Wang, "Distributed cooperative adaptive tracking control for heterogeneous systems with hybrid nonlinear dynamics," *Nonlinear Dyn.*, vol. 95, no. 3, pp. 2131–2141, Feb. 2019.

[46] Y. Wang, H. Dong, and X. Li, "Distributed cooperative neural control for nonlinear heterogeneous platoon systems with unknown uncertainties," *Arabian J. Sci. Eng.*, vol. 1, pp. 1–14, Feb. 2024.

[47] X. Li, P. Shi, Y. Wang, and S. Wang, "Cooperative tracking control of heterogeneous mixed-order multiagent systems with higher-order nonlinear dynamics," *IEEE Trans. Cybern.*, vol. 52, no. 6, pp. 5498–5507, Jun. 2022.

[48] F. L. Lewis, H. Zhang, K. Hengster-Movric, and A. Das, *Cooperative Control of Multi-agent Systems: Optimal and Adaptive Design Approaches*. Berlin, Germany: Springer, 2013.

[49] H. Zhang and F. L. Lewis, "Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics," *Automatica*, vol. 48, no. 7, pp. 1432–1439, Jul. 2012.

[50] Y. Wang, P. Shi, and X. Li, "Event-triggered observation-based control of nonlinear mixed-order multiagent systems under input saturation," *IEEE Syst. J.*, vol. 18, no. 2, pp. 1392–1401, Jun. 2024.

[51] H. K. Khalil, *Nonlinear Systems*, 3rd ed., Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

[52] S. S. Ge and C. Wang, "Adaptive neural control of uncertain MIMO nonlinear systems," *IEEE Trans. Neural Netw.*, vol. 15, no. 3, pp. 674–692, May 2004.

[53] Y. Li, W. Chen, S. Peeta, and Y. Wang, "Platoon control of connected multi-vehicle systems under V2X communications: Design and experiments," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 5, pp. 1891–1902, May 2020.

[54] Y. Li, B. Yang, T. Zheng, Y. Li, M. Cui, and S. Peeta, "Extended-state-observer-based double-loop integral sliding-mode control of electronic throttle valve," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 5, pp. 2501–2510, Oct. 2015.

[55] X. Guo, J. Wang, F. Liao, and R. S. H. Teo, "Neuroadaptive quantized PID sliding-mode control for heterogeneous vehicular platoon with unknown actuator deadzone," *Int. J. Robust Nonlinear Control*, vol. 29, no. 1, pp. 188–208, Jan. 2019.

[56] Y. Li, Z. Qin, H. Zhu, S. Peeta, and X. Gao, "Platoon control of connected vehicles with heterogeneous model structures considering external disturbances," *Green Energy Intell. Transp.*, vol. 1, no. 3, Dec. 2022, Art. no. 100038.

[57] H. Pei, "Collaborative consensus tracking of heterogeneous group systems with switching topologies and input time delay," *Int. J. Control*, vol. 95, no. 2, pp. 319–329, Feb. 2022.

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