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APPLIED RESEARCH

Decentralized Most Permissive Observer Architecture for Brake Leakage Diagnosis in Automotive Systems

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ABSTRACT Brake systems represent one of the most critical components contributing to the safety of modern automotive systems. They are commonly referenced as one of the most vehicle-related factors in fatal crashes. Usually, the faults in this type of component are provoked by brake fluid leaks affecting the performance and response of the entire system, which require manual maintenance procedures to locate the source of the problem. This study presents a novel Discrete Event System (DES) model capable of detecting brake fluid leakage faults using a decentralized Most Permissive Observer (MPO) architecture for fault diagnosis. The developments were verified via an enhanced virtual environment employing a simulation model of the braking system using Simscape and assessing the event-driven fault diagnosis scheme using Stateflow. A 'Design of Experiment' method is performed using a factorial design of 3^k, where k is the number of levels of a factor in the factorial experiment (in this paper k is 4), to conduct the assessment of test cases that combined edge values. The efficacy of the proposed DES model for fault diagnosis is demonstrated across a wide range of simulated scenarios, including fault locations, pressure drops, and single and multiple fault combinations. The proposed method consistently diagnoses the injected fault conditions, showcasing its efficiency under diverse operational conditions of the braking system.

INDEX TERMS Brake system, discrete event systems, decentralized diagnosed, most permissive observer.

I. INTRODUCTION

Modern automotive systems comprise electrical, hydraulic, mechanical, and computer systems which analyze such systems very complex particularly when it comes to detecting, identifying, and isolating faults, commonly referred to as fault diagnosis. Brake systems are examples of such complex systems, which represent one of the most critical components contributing to the safety of vehicles, enabling them to slow down or suddenly stop and avoid frontal collisions that could result in serious accidents. According to the National Highway Traffic Safety Administration, brake-related problems are typically the last fault in the causal chain of events leading to crashes, representing up to 22 percent of collisions involving light vehicles [1]. Moreover, according to a report by the Federal Motor Carrier Safety Administration, brake systems are the third most referenced vehicle-related factor in fatal crashes [2].

The most common causes of faults in brake systems include cracked braking components, broken lines, and low fluid levels in the reservoir of the master cylinder [3]. This type of fault results in brake fluid leaks that affect the performance and response of the braking system. Leak detection commonly occurs when the system is in a faulty condition, triggering a warning light that appears on the dashboard when the brake fluid level is already too low. Consequently, manual inspection is required by an experienced technician who locates the problem before proceeding to repair the system [4]. In this sense, the existence of a procedure that detects, isolates, and identifies

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TABLE 1. Comparison of fault diagnosis methods employed for braking systems.

Method: Description	Benefits	Drawbacks
Hardware-based: Uses hardware redundancy with multiple sensors and actuators to monitor key variables. Employs voting techniques and limit-checking procedures.	• Direct fault detection and isolation • Effective for out-of-range measurements • Can analyze fault in the spectral domain	• Requires additional equipment • High maintenance costs • Limited to detectable faults by sensors
History-based: Measures and interprets input/output data from testing setups. Uses signal processing and machine learning techniques for fault classification.	• Useful when modeling is not feasible • Can handle complex, non-linear systems • Adaptable to various fault types	• Requires significant experimental effort • Accuracy decreases at lower speeds/loads • Needs features to avoid misclassification
Model-based: Models the actual behavior of the system using mathematical relationships. Includes quantitative and qualitative approaches.	• Can confirm fault-free cases • Effective for root-cause analysis • Can handle complex system interactions	• Challenging to isolate faults in some cases • Requires accurate system modeling • Can be computationally intensive

leak-related faults would drastically reduce the time and cost of the performance recovery of brake systems. Fault diagnosis can also be performed in real-time as a part of the active safety systems of autonomous or semi-autonomous vehicles, prompting decisions about the faults.

Table 1 summarizes the key characteristics, benefits, and drawbacks of the three main fault diagnosis approaches for braking systems: hardware-based methods, history-based methods, and model-based methods, as classified by [5].

Hardware-based methods are traditionally used for fault diagnosis of dynamic systems. This approach typically involves using hardware redundancy with multiple sensors and actuators to monitor key variables [6], [7]. Voting techniques are then used to compare signals from parallel sensor groups, helping to identify faulty behavior in redundant devices. In braking systems, specialized hardware such as force, pressure, and motion sensors is required for data acquisition, enabling limit-checking procedures to detect faults from out-of-range measurements (i.e., signals exceeding established thresholds [7]). Moreover, analyzing specific fault types in the spectral domain facilitates direct isolation [8]. However, these fault diagnosis methods require additional equipment and incur high maintenance costs.

History-based methods involve measuring and interpreting input and output data from a testing setup, making them useful when analytical modeling of a process is not feasible. These methods typically monitor the condition of brake components by assessing signals. Signal processing for feature extraction and selection (e.g., using wavelets, statistics, etc.) aids in classifying and diagnosing fault conditions. Various classification algorithms, increasingly based on machine learning, are commonly used, including Fuzzy Logic [9], Neural Networks [10], [11], Principal Component Analysis [12], Statistical Methods [13], and Pattern Recognition [14]. To avoid misclassification, a minimum number of extracted and selected features is required from the acquired data [15]. However, the accuracy of history-based methods decreases with lower speeds and loads on the braking system, as the vibration signals become less distinguishable [16]. Implementing these methods for fault diagnosis also requires significant labor and experimental effort.

To address the significant drawbacks of previous methods, an alternative is to model the actual behavior of a dynamic system using model-based methods. These methods rely on the mathematical relationships between the components of a system. Quantitatively, alarms can be triggered by comparing the behaviors of the model and the actual braking system through direct signal comparison (i.e., residual generation) [17], consistency checks [18], state estimators with minimal estimation error [19], formal detectability analysis of combined indexes [20], or diagnostic observers [18]. While quantitative models can confirm fault-free cases, isolating faults remains challenging. Alternatively, some approaches use qualitative modeling, focusing on specific components of the braking system. These methods may involve decomposition to infer observed behavior [21], top-down structures that represent potential loss event pathways [22], and directed arcs from 'cause' to 'effect' nodes [23]. The Discrete Event System (DES) method is an effective qualitative technique for root-cause analysis of faults in a system [24], [25], [26].

DES provides an efficient framework for analyzing the event-driven behavior of complex systems. By providing abstract models based on event sequences, DES facilitates fault detection by analyzing system behavior and observable event patterns. This approach allows for the effective detection and identification of faults within the system's operational modes and transitions [27], [28], [29]. Commonly, DES fault diagnosis techniques are classified into static and dynamic approaches [30], in which, a set of sensors provide data about observed events, while in dynamic settings, using a sensor activation policy, the sensors are dynamically switched on/off. Although the dynamic approach helps to save energy, reduce the use of bandwidth, and improve security by activating only those sensors needed for fault diagnosis [31], such a strategy faces the challenge of synthesizing a policy for switching on/off sensors while obtaining observations.

A popular method for minimizing the activation sensors of the system under diagnosis is the Most Permissive Observer

(MPO) technique [32]. This approach employs an optimal policy for switching the sensors on/off by computing all sensor activation policies that satisfy the κ-diagnosability property. Centralized MPO-based architectures such as those presented in [32], incur high computational costs. By contrast, a decentralized MPO-based structure can handle the complexity of the design and implementation process [33], by employing a set of local MPOs, each of which observes a part of the system under diagnosis.

This study addressed the aforementioned challenges by developing a novel DES model to detect leakage faults in braking systems using a decentralized MPO architecture for fault diagnosis, which was later verified via an enhanced simulation environment. The contributions of this study are summarized as follows:

- Developing a comprehensive DES model for a braking system and its augmented form in which faults may arise in the form of brake fluid leaks
- Developing a novel decentralized MPO-based diagnosis structure for a braking system to diagnose a fault occurring during operation,
- Applying the developed fault diagnosis tool to a braking system in an enhanced simulation environment that captures the brake system components (e.g., master cylinder and calipers) as well as the inherent dynamic variables (e.g, force, pressure, position, and volumetric flow) for each component. The obtained results demonstrate the capability of the proposed method to correctly detect and isolate fluid leaks in braking systems.

The rest of the paper is organized as follows. In Section II, a high-level explanation of the braking system, its components, and its operation is provided. In Section III the DES model of the braking system is presented. Section IV explains the structure of the proposed decentralized dynamic MPO-based fault diagnosis. Section V discusses the simulation testing results of the braking system under normal and leakage-fault conditions. Finally, Section VI concludes the paper.

II. OVERVIEW OF BRAKE SYSTEM OPERATION

In this section, a high-level explanation of the components and operation of the braking system is presented. First, the hydraulics fundamentals for braking systems are briefly explained. Second, the principles of the operation of a braking system are detailed. Finally, the sensors that can be used for monitoring a braking system are discussed.

A. BRAKE SYSTEM HYDRAULICS

The braking system under investigation comprises several components that use brake fluid to transfer forces including a tandem master cylinder, two hydraulic circuits, and four calipers. Such a braking system is typically used for high-performance cars such as racing or sports cars. A diagram of the braking system indicating the essential components and sensors for one of the hydraulic circuits is shown in Fig.1, noting that both hydraulic circuits share the same setup.

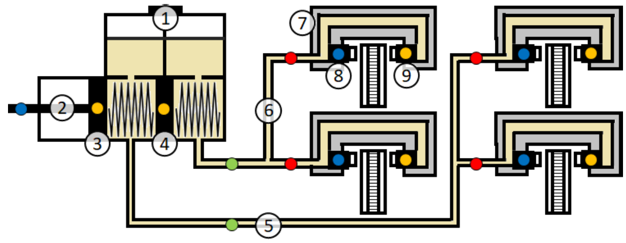

FIGURE 1. Diagram of the braking system including (1) master cylinder with (2) push-rod, (3) primary, and (4) secondary pistons, (5) primary and (6) secondary hydraulic circuits, and (7) fixed disc calipers with (8) inward and (9) outward pistons. Sensors are shown as circles including position (yellow), pressure (green), flow (red), and force (blue).

When the brake pedal is applied against the master cylinder, it relies on brake fluid to drive the pressure generated from the push rod to the braking mechanism at the wheels of the vehicle. The hydraulic system operates according to Pascal's Law which states that the pressure exerted on a confined incompressible fluid causes an increase in pressure at all points. In this sense, the pressure in the hydraulic circuit (P) generated by the piston of the master cylinder can be estimated by dividing the exerted force (F) by the area of the piston (A), meaning that the input pressure is inversely proportional to the size of the input piston (P=F/A). Consequently, the force created by the braking mechanism at the wheels is equal to the pressure time area, which means that the output force increases with the size of the output piston.

B. PRINCIPLES OF OPERATION

Inside the master cylinder, there are two chambers and two pistons in tandem. An inlet port connects each chamber with the brake fluid reservoir, and at the same time, an outlet port connects each chamber with a different hydraulic circuit. When brakes are applied, the push rod displaces (S) the primary piston blocking the inlet port to elevate the pressure of the primary chamber. This pressure then drove the second piston to block the inlet port of the secondary chamber. In both cases, the brake fluid (Q) is transferred through the outlet ports to each hydraulic circuit. However, when the brake pedal is released (i.e., no force is applied to the push rod), the primary and secondary pistons return to their initial positions owing to the pressure relief and spring forces.

In modern vehicle systems, transferring fluid from the master cylinder to the braking mechanisms in the wheels employs two separate hydraulic circuits. The driving application determines the most suitable hydraulic system configuration for installation. The front/rear (two front wheels and two rear wheels) and diagonal (left-front/right-rear wheels and right-front/left-rear wheels) configurations are the most common hydraulic circuit configurations. The idea behind these arrangements is to have the capability to stop the vehicle even if one of the hydraulic circuits fails.

The output of the hydraulic circuit was the pressure applied via calipers to the pads against the discs to slow down the wheels. This study used a fixed caliper with two pistons that received equal pressures. When the pressure in the hydraulic circuit increases, the brake fluid drives the pistons (and brake pads) toward the brake disc. Conversely, when the pressure in the hydraulic circuits decreased, the pistons returned to their initial positions letting the brake disc spin again.

C. BRAKE SYSTEMS SENSORS

The braking system relies on the proper execution of several steps. To monitor the status of multiple individual components during the braking process, different types of sensors were implemented, which are listed as follows:

1) POSITION SENSOR

The position sensor monitors the linear, angular, or rotary movement of the components. The position sensor aims for different purposes, such as directly activating the brake in by-wire systems, turning on brake lights, turning off cruise control if applied, optimizing regenerative braking, or starting the Anti-lock Braking System (ABS). Typically, a position sensor is mounted on the brake pedal to measure the brake stroke length. However, to increase the reliability of sensor readings, non-contact sensors are usually mounted outside any of the master cylinders or calipers.

2) PRESSURE SENSOR

It senses the brake fluid pressure at each hydraulic breaking circuit. In modern vehicles, it is located within the ABS pump and cannot be replaced separately from this component. It is usually employed as a data source for dynamic stability systems to apply and release pressure to the wheels, keeping the vehicle stable and safe during extreme driving conditions, such as excessive speeds while turning or preventing fish-tailing during emergency braking events.

This study assumes that this sensor is located at each hydraulic circuit to measure the brake fluid pressure plied by the master cylinder while checking the braking system status.

3) FLOW SENSOR

It measures the amount of brake fluid moving through a hydraulic circuit by measuring the volumetric flow rates. It is typically used for mapping flow profiles, leak testing, and maximum flow rate estimation within braking systems.

This study assumes that two of these sensors are placed in the hydraulic circuits before the inlet port of each caliper to measure the volumetric flow rate of each hydraulic circuit branch while checking the status of the braking system.

4) FORCE SENSOR

It monitors the forces acting on the braking system. It is designed to fit on the brake pedal or between the brake pad and disc in most by-wire systems. It is usually employed for high-performance action and safety in brake-by-wire closed-loop systems, considering the calibration results to detect aging, temperature, and other environmental variations.

This study considered this type of sensor located at the push-rod to monitor the behavior of the braking system.

III. DES MODELING

In this section, the DES modeling procedure for the braking system is presented. A DES model can be an appropriate method to provide an abstract yet effective description of the behaviors of the braking system in the form of a finite-state automaton that shows the sequences of event occurrences in the system by considering transitions and states. A finite-state automaton consists of the edges and nodes of a graph that represent the transitions and states of the system, respectively [27]. To develop a DES model of the braking system, the automata $G_i = (X_i, \Sigma_i, \delta_i, x_{0_i})$, $i = \{1, 2\}$ for two independent hydraulic circuits, where X_i is a finite set of states of the ith hydraulic circuit; Σ_i is the finite set of events consisting of two disjoint sets as $\Sigma_i = \Sigma_{o_i} \cup \Sigma_{uo_i}$, where Σ_{o_i} is the set of observable events, Σ_{uo_i} is the set of unobservable events, $\delta_i : X_i \times \Sigma_i \to X_i$ is the partial transition function. The statement $\delta_i(x, e) = x'$ indicates that there is a transition from state x to state x' when an event $e \in \Sigma_i$ occurs. Σ_i^* is the set of all finite strings of events in Σ_i as well as the zero-length string ε. The definition of δ_i to strings in Σ_i^* can be extended by recursively defining $\delta_i(x, s.e) = \delta_i(\delta_i(x, s), e)$, for any string $s \in \Sigma_i^*$ and any event $e \in \Sigma_i$, where $\delta_i(x, \varepsilon) = x$. Finally, x_{0_i} is the initial state of G_i. In the DES model of G_i, a faulty behavior can be modeled as an unobservable event, $f \in \Sigma_{uo_i} \in \Sigma_i$, whose occurrence must be diagnosed based on an observable event sequence (note that if the fault is observable, its diagnosis is trivial). In DES fault diagnosis, the aim is to diagnose the occurred fault from the limited number of system observations. Specifically, a discrete event system G_i is said to hold the $\mathcal{K}$-diagnosable property for a given fault $f \in \Sigma_{uo_i}$ if the occurrence of the fault f can always be reliably detected and isolated without ambiguity after the occurrence of $\mathcal{K}$ sequence of events. Mathematically, the $\mathcal{K}$-diagnosability can be defined as follows:

Definition 1: Let $\mathcal{K} \in \mathbb{N}$. The live language $L(G)$ is $\mathcal{K}$-diagnosable, if the following holds [24]:

$$(\forall s \in \Psi(f)), (\forall t \in L(G)/s), (|t| \geq \mathcal{K}) \Rightarrow$$
$$(\forall u \in L(G))[\mathcal{P}(u) = \mathcal{P}(st) \Rightarrow f \in u] \qquad (1)$$

where $\Psi(f) = \{t \mid t = s.f \in L(G)\}$ is the set of strings that end with the faulty event f, $|t|$ shows the length of string t, and $\mathcal{P}$ is the natural projection.

Fig. 2 shows the DES model of the braking system. In this model, $\Sigma_{o_i} = \{F > \beta, F < \beta, P_i > \alpha, P_i < \alpha, S_i, S_{ij}, Q_{ij} > \epsilon, Q_{ij} < \epsilon\}$, with $i, j = \{1, 2\}$, defines the set of observable events, where F is the force on the push-rod, P_i is the pressure on the ith hydraulic circuit, S_i is the displacement of the piston of the master cylinder in the ith hydraulic circuit, S_{ij} is the displacement of the piston of the jth caliper in the ith hydraulic circuit, and Q_{ij} is the volumetric flow in

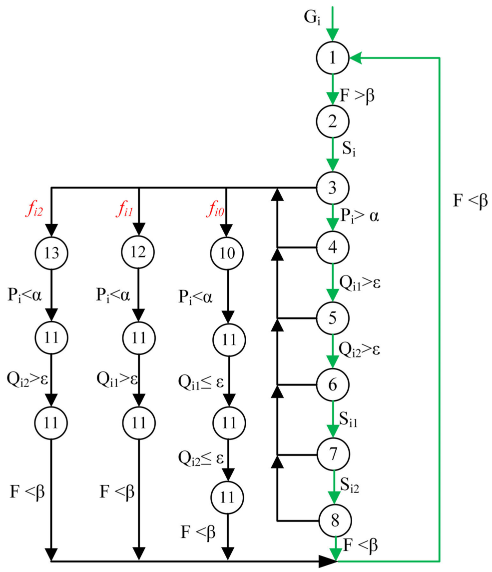

FIGURE 2. The DES model of the braking system comprising two hydraulic circuits, G_i, $i = \{1, 2\}$.

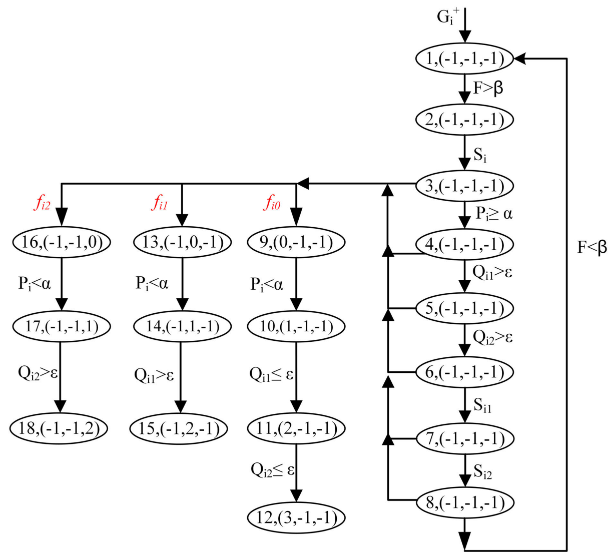

FIGURE 3. Augmented DES model G_i^+ constructed for hydraulic circuit G_i.

the jth caliper in the ith hydraulic circuit. For example, Q_{11} ($i = 1$ and $j = 1$) shows the volumetric flow of the brake fluid that is transferred from the primary piston to Caliper 1 in Hydraulic Circuit 1. In addition, β is a threshold value for the F to reach the level required to move the piston, α is the rate of change of P_i with respect to F, and ϵ is the rate of change of Q_{ij} with respect to the rate of change of F with respect to time (i.e., $\Delta F/\Delta t$).

The unobservable event set is $\Sigma_{uo_i} = \{f_{ij}\}$, where f_{i0} is a fault in the ith hydraulic circuit near the master cylinder, and f_{i1} and f_{i2} are faults in the 1st and 2nd caliper located in the ith hydraulic circuit.

A. NORMAL BRAKING MODE

The green branch in Fig. 2 shows the normal operation of the braking system. After pushing the push-rod, when $F > \beta$, in both hydraulic circuits, the pistons move forward, as captured by the event S_i. Moving the piston forward will increase the pressure in the ith hydraulic circuit until it becomes more than α, ($P_i > \alpha$), which subsequently increases the volumetric flow in the calipers. When the volumetric flow in the calipers becomes more than ϵ ($Q_{i1} > \epsilon, Q_{i2} > \epsilon$), the pistons in the calipers move forward, captured by events S_{i1} and S_{i2}. As soon as the push-rod is released ($F < \beta$), the system resets and returns to its first state.

B. FAULTY BRAKING MODE

One of the most common types of faulty behaviors in a braking system is leakage in a hydraulic circuit. When a leak in the braking system is present, the brake fluid drains out of the hydraulic circuit, therefore, there is not enough pressure to transmit the force from the pedal to the wheels. The location of this type of fault is usually near components such as master cylinders or calipers. These leaks are usually provoked by small orifices in the brake lines, with their size determining the severity of the leak. The extent of degradation is quantified by the pressure drop in the affected hydraulic circuit and the change in volumetric flow to the calipers. Consequently, three types of faults $\mathcal{F}_i = \{f_{i0}, f_{i1}, f_{i2}\}$ are defined, in the DES model as shown in Fig. 3 and described as follows:

- f_{i0}: This fault occurs when a leak exists near one of the pistons of the master cylinder (e.g., because of a sealing problem). In this case, the pressure in the corresponding hydraulic circuit becomes less than α, triggering the event $P_i < \alpha$, and consequently, the volumetric flow to both calipers becomes less than ϵ, ($Q_{ij} < \epsilon$), $j = 1, 2$.
- f_{i1}: This fault occurs when a leak exists near the first caliper in the ith hydraulic circuit. In this case, the pressure in the hydraulic circuit becomes less than α, ($P_i < \alpha$), and the volumetric flow to the caliper becomes more than ϵ, ($Q_{i1} > \epsilon$).
- f_{i2}: This fault occurs when a leak exists near the second caliper in the ith hydraulic circuit. In this case, the pressure in the hydraulic circuit is less than α, ($P_i < \alpha$), and the volumetric flow of brake fluid to the calipers is more than ϵ, ($Q_{i2} > \epsilon$).

The severity of degradation for faults f_{i0}, f_{i1}, and f_{i2} is characterized by substantial decreases in pressure and volumetric flow. For f_{i0}, both calipers are affected, while f_{i1} and f_{i2} impact the first and second calipers respectively. These changes lead to diminished braking force at the affected wheels. The threshold values α and ϵ are related to typical operating conditions and safety margins, allowing DES fault diagnosis methods to trigger events when significant deviations from normal operation occur.

IV. DES FAULT DIAGNOSIS

This section explains the DES model approach for fault diagnosis. This includes two steps: (1) constructing an enriched model, called an augmented model, which facilitates the tracking of fault information, and (2) developing decentralized MPO-based diagnosers to infer the fault information.

A. AUGMENTED MODEL

Given the DES model G_i, the augmented model $G_i^+ = (X_i^+, \Sigma_i, \delta_i^+, x_{0_i}^+)$ is constructed, where $X_i^+ = X_i \times \{\ell_{i0}, \ell_{i1}, \cdots, \ell_{iM}\}$, $x_{0_i}^+ = (x_{0_i}, -1, -1, \cdots, -1) \in X_i$, and $\delta_i^+ : X_i^+ \times \Sigma \rightarrow X_i^+$ [34]. The number of fault types in the system was $M+1$. Also, ℓ_{im} is a counter for tracking the number of post-fault event occurrences starting from -1 for a nonfaulty state. For any $x_i^+ = (x_i, \ell_{i0}, \ell_{i1}, \cdots, \ell_{iM}) \in X_i^+$ and $\sigma \in \Sigma_i$:

$$\delta_i^+((x_i, \ell_{i0}, \ell_{i1}, \cdots, \ell_{iM}), \sigma) = (\delta_i(x_i, \sigma), \ell_{i0}^{'}, \ell_{i1}^{'}, \cdots, \ell_{iM}^{'})$$

$$\text{such that} \begin{cases} \ell_{im}^{'} = \ell_{im}, & \text{if } \sigma \neq f_{im} \\ \ell_{im}^{'} = \ell_{im} + 1, & \text{if } \sigma = f_{im} \end{cases} \quad (2)$$

where $m \in \{0, 1, \cdots, M\}$, and f_{im} is the fault type corresponding to the counter ℓ_{im}.

In this study, $M = 2$ for the three types of faults under assessment include f_{i0}, f_{i1}, and f_{i2}. The augmented model G_i^+ constructed for DES model G_i is shown in Fig. 3.

B. DECENTRALIZED MPO-BASED DIAGNOSER

As a dynamic diagnoser, the MPO-based diagnoser requires partial observation of the system to detect the occurrence of faults. Therefore, the sensors are turned on/off dynamically by the diagnoser such that a limited set of events is observable at a given time. This limited set of events chosen to be observed is called the *sensing decision*, and is shown by γ. The MPO-based diagnoser utilizes a *sensor activation policy* which is shown by ω to generate a new sensing decision after each event observation. The sensor activation policy is $\omega_i : L(G_i) \rightarrow \Gamma_i$ where for any $s \in L(G_i)$, $\omega_i(s)$ is the set of events monitored after the occurrence of s, and $\Gamma_i = \{\gamma \in 2^{\Sigma_i} : \gamma \subseteq \Sigma_{o_i}\}$ is the set of all *admissible* sensing decision sets.

A decentralized MPO structure consists of multiple local MPOs, such that the occurrence of a fault in subsystem G_i can be diagnosed by only the local diagnoser associated with the subsystem. Consider local MPOs that as $MPO_i = (Y_i, Z_i, \Gamma_i, \Sigma_{o_i}, h_{YZ_i} \cup h_{ZY_i}, y_{0_i})$ [35]. Each MPO has two types of states: (1) Y-states which are known as decision states and are shown by rectangles, and (2) Z-states which are known as observation states and are shown by ellipses. The MPO can find an observation state using the function $h_{YZ_i} : Y_i \times \Gamma_i \rightarrow Z_i$ based on the sensing decision. Mathematically, it can be said that if $y \in Y_i$ and $\gamma \in \Gamma_i$ then $h_{YZ_i}(y, \gamma) = \{z = (\delta_i^+(x^+, t), \gamma) | x^+ \in y, t \in (\Sigma_i \setminus \gamma)^*\}$. Function $h_{YZ_i} : Y_i \times \Gamma_i \rightarrow Z_i$ transitions from an observation state (Z-state) to a decision state (Y-state). Mathematically, for any $z = (q, \gamma) \in Z_i$, where $q \in 2^{X^+}$ and $\gamma \in \Gamma_i$, exist $h_{ZY_i}(z = (q, \gamma), \sigma) = \{y = \delta_i^+(x^+, \sigma) | x^+ \in q, \sigma \in \gamma\}$. Then, the MPO can be constructed recursively, starting from $y_0 = (x_0, -1, -1, -1)$, and using the functions h_{YZ_i} to transit from Y-states to Z-states followed by h_{ZY_i} to transit from Z-states to Y-states.

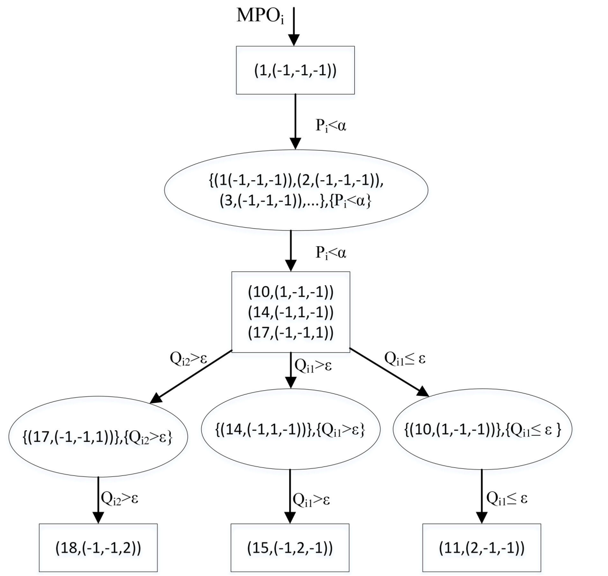

FIGURE 4. Local diagnoser $MPO_i, i = 1, 2$ for the system in Fig. 2.

Fig. 4 shows the local MPOS, MPO_i, $i = 1, 2$. There is no communication between local MPOs and each MPO detects the occurrence of the fault in its corresponding subsystem G_i based on its observation $\{P_i < \alpha, Q_{i1} \leq \epsilon, Q_{i1} < \epsilon, Q_{i2} > \epsilon\}$. This occurs by running the local MPOs in parallel with subsystems G_i, $i = 1, 2$. Mathematically, this can be thought of as the parallel composition of the local MPOs and subsystems, where they synchronously transit over shared/common events, while private events can occur asynchronously. The local MPO detects faults in the subsystems if ℓ_{i0}, ℓ_{i1}, or ℓ_{i2} become 1 or larger. For example, if the MPO transitions to state $(11, 2, -1, -1)$, $(15, -1, 2, -1)$, or $(18, -1, -1, 2)$, it implies the occurrence of faults f_{i0}, f_{i1}, or f_{i2}, respectively. The decentralized diagnoser makes decisions regarding fault occurrences in a disjunctive manner, where at least one of the local MPOs detects the fault event.

As it is shown in [33], the complexity of constructing a centralized MPO is $O(2^{|X|}.(K+2)^{|X|}.2^{|\Sigma|})$, where X and Σ are respectively the state space and the event set of the system under diagnosis G. However, the proposed decentralized MPO can effectively reduce the computational cost. By considering two DES models for the braking system with two hydraulic circuits, denoted as G_i for $i = 1, 2$, the number of states and events for each DES model is half of the entire braking system. Consequently, employing the decentralized method for this system leads to a significant reduction in the complexity of the diagnoser.

V. FAULT DIAGNOSIS SIMULATION

In this section, a simulation environment is presented to assess the braking system model under normal and faulty conditions. First, a time-driven virtual model of the braking system is developed using Simscape. Second, an MPO-based diagnoser is designed and implemented using StateFlow consistent with the derivations in Sections III-IV. Finally, the simulation results related to the fault injection and diagnosis procedures are discussed.

A. SIMULATION MODEL

The components of the braking system simulation model were developed using Simulink by extending an existing fixed caliper disc brake model enabled by Simscape [36]. The model was parameterized by employing numerical data defined in advance by the simulation model. In the developed Simulink model, the push-rod force acts as the input of the master tandem cylinder and creates pressure and swept volume from the piston to the front and rear calipers. The main components in the simulation model of the braking system, depicted in Fig. 5 include the master cylinder and caliper disk brakes.

1) MASTER CYLINDER

Hydraulic fluid accumulators were used in the primary and secondary circuits. The input of the model is the push-rod force from a mechanical source. The output of the model was the hydraulic pressure created in the primary and secondary hydraulic circuits. Two linear motion sensors were used to measure the global position of each one of the two pistons.

2) CALIPER DISC BRAKES

A fixed caliper disc brake model was employed for the four wheels. The fluid from the primary hydraulic circuit is divided between the two front wheels, and the fluid from the secondary hydraulic circuit is divided between the two rear wheels of the vehicle. The model does not have caliper discs connected to the rotating vehicle wheels. Instead, they were grounded in the model. Hence, the braking torque generated on the caliper discs could be neglected.

B. FAULT INJECTION

The function of the braking system depends on successive steps working properly (i.e., if any of the steps fail it will affect the integrity of the entire system). One of the most common faulty behaviors is leakage in the hydraulic circuit, where the brake fluid slowly drains out until there is not enough remaining fluid to transmit the pressure from the brake pedal to the wheels.

Because the location of this type of fault can be anywhere along the hydraulic circuit, Fig. 6 shows the fault injection strategy used to simulate the most common locations for leakage faults in the brake system model which include leakage near the master cylinder and calipers:

1) *Brake Fluid Leakage Near Master Cylinder:* A leakage can be located near the outlet port of the master cylinder. An internal fault of the master cylinder is another type of leak that resembles similar faulty behavior in the braking system, represented by f_{10} and f_{20}. This type of fault can be injected into Simscape by linking a pressure relief valve to a hydraulic circuit. The valve remained closed when the pressure was less than a specified value. When a certain pressure threshold is met or surpassed over the valve, the valve opens. The previous type of fault can be injected in any of the two hydraulic circuits between the master cylinder and the nearest flow sensor, as shown in Fig. 6.
2) *Brake Fluid Leakage Near Calipers:* A leakage can also be located in a hydraulic circuit near the inlet port of the caliper. Similarly, an internal fault of the caliper is another type of leak that resembles faulty behavior represented by f_{11}, f_{12}, f_{21}, and f_{22}. This type of fault can be injected in Simscape by linking a pressure relief valve to the hydraulic circuit, in the same way as in the previous case. The pressure relief valve was connected to any of the hydraulic circuits, between one caliper and the nearest flow sensor, as shown in Fig. 6.

Because the brake system model is symmetrical, similar results can be obtained by injecting faults at either of the calipers in the same hydraulic circuit.

C. FAULT DIAGNOSIS

The conventional procedure for identifying and locating leakages consists of looking for any visual cues indicating a leak, which requires the expertise and time of trained personnel to tackle the problem. The approach presented in this study can automatically diagnose faults and identify the exact location immediately after the fault occurs.

In our simulations, the DES model of the brake system presented in Fig. 2 is used, which observes events $\Sigma_{o_i} = \{F > \beta, F < \beta, P_i > \alpha, P_i < \alpha, S_i, S_{ij}, Q_{ij} > \epsilon, Q_{ij} \leq \epsilon\}$, with $i, j = \{1, 2\}$ to track the behaviors of the braking system while performing fault diagnosis related to brake fluid leakages in the braking system.

Depending on the source of the brake fluid leakage (e.g., failed seal, worn-out or damaged component), fault detection can occur at any point along the brake pedal range of movement (i.e., at any point along the pressure range exerted by the master cylinder in hydraulic circuits). Hence, to tune the diagnoser, a parameter identification procedure is employed in the fault diagnosis assessment, building a model for parameter values from measured data to estimate their existence. The identification procedure employed in this study is as follows.

1) *Parameter Identification for Force on Push-Rod:* Once the force is sufficiently large enough ($F > \beta$), the push-rod is pushed forward to achieve a positive hydraulic pressure in the braking system. The value of β should be greater than the minimum force required to

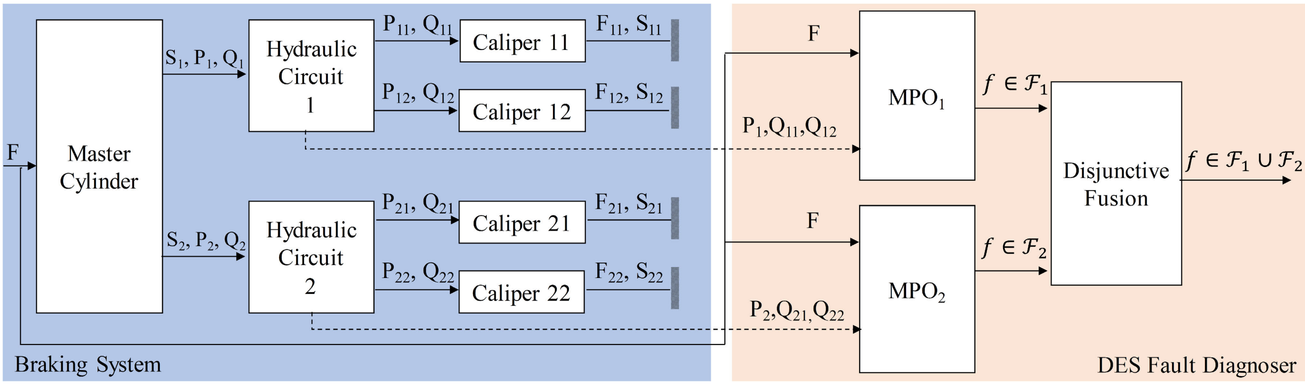

FIGURE 5. Simulation model of the braking system.

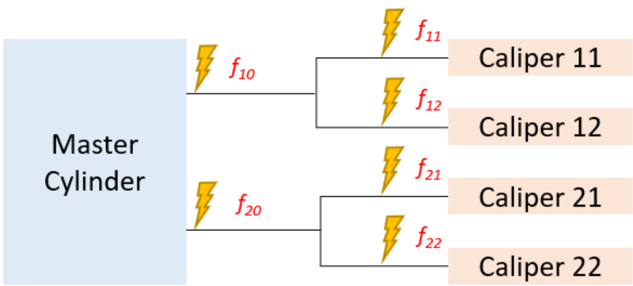

FIGURE 6. Fault injection test cases.

overcome the initial static condition of the mechanical components, which also corresponds to the nonlinear behavior in the P_i and Q_{ij} estimations.

2) *Parameter Identification for Pressure:* Once force is applied to the push-rod, the piston in the master cylinder moves forward, compressing the brake fluid within the hydraulic circuit which increases the pressure in the braking system. In a normal situation, the pressure exerted in the braking system is expected to be consistent with the applied force. More specifically, the pressure in the hydraulic circuits is expected to be greater than or equal to pressure α which is proportional to the applied force as follows:

$$\alpha = \theta F \tag{3}$$

3) *Parameter Identification for Volumetric Flow:* Once the pressure increases in the hydraulic circuit, the braking fluid moves forward, thereby increasing the volumetric flow in the braking system. In a normal situation, the resulting volumetric flow in the braking system is expected to be consistent with the rate of change in the applied force. More precisely, the volumetric flow in the hydraulic circuits is expected to be greater or equal to the volumetric flow ϵ which is proportional to the rate of change of the applied force as:

$$\epsilon = \zeta dF/dt \tag{4}$$

TABLE 2. Values for the identification of α and ϵ.

F	0.5	1	1.5	2	kN
	2.5	3.0	3.5	4.0	
P_i	0.36	0.94	1.52	2.10	MPa
	2.67	3.25	3.83	4.41	
dF/dt	5	10	15	20	kN/s
	25	30	35	40	
Q_{ij}	5.91	11.83	17.73	23.65	cm^3/s
	29.57	35.49	41.39	47.32	

D. RESULTS AND DISCUSSIONS

In this section, the outcomes of the tests performed are presented. This includes the procedure for parameter identification from simulated data, experimental design for evaluating the proposed fault diagnosis method by injecting different faults, and examination of the fault diagnosis results in scenarios involving single or multiple faults.

1) PARAMETER IDENTIFICATION

To determine the value of β, we observed the behavior of the pressure and volumetric flow parameters, P and Q, in the hydraulic circuits. Based on this observation, we can mark β=500 N, as the point at which the hydraulic pressure in the braking system becomes positive and assists in overcoming the initial static state of the mechanical components.

To identify the pressure and volumetric flow parameters, α and ϵ, sample data for F, P, dF/dt, and Q were previously collected, as depicted in Table 2. Then, using Eqs. 3 and 4 and using a simple linear regression technique, the values for α=1.16e^{3}F-2.28e^5 m^3/s and ϵ=1.18e^{-9}dF/dt N/s, were estimated as shown in Fig. 7.

Furthermore, as shown in Fig. 7, to avoid the Zeno phenomenon (infinite switches in finite time) in the developed model, we introduce the hysteresis thresholds $\Delta\alpha$ and $\Delta\epsilon$, which are set to small values 2e^5 Pa and 1.5e^{-5} m^3/s.

2) EXPERIMENTAL DESIGN

The experimental design was conceived as a factorial design comprising 4 factors (F1 $= P_1$, F2 $= P_2$, F3 $= f_{1j}$, and F4 $= f_{2j}$) and 3 levels (low='−', medium='o', and high='+') as depicted in Table 3. The factors and levels were selected to cover the entire range of braking conditions under normal and

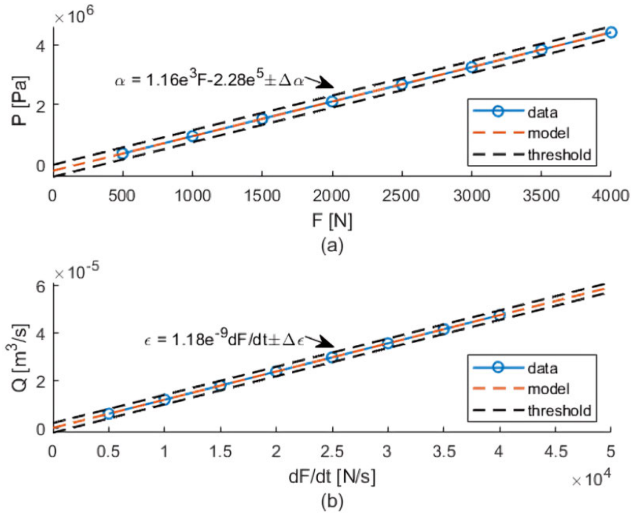

FIGURE 7. Linear regression result to identify α and ϵ.

TABLE 3. Design of Experiment.

Factor		Level		
		-	o	+
F1:	P_1 [MPa]	1.0	3.0	5.0
F2:	P_2 [MPa]	0.5	1.0	1.5
F3:	f_{1j}	f_{10}	f_{11}	f_{12}
F4:	f_{2j}	f_{20}	f_{21}	f_{22}

faulty behaviors. In this sense, the pressure at each hydraulic circuit, P_i, and the locations for the fault injection, f_{ij}, provide an orthogonal design for the factorial combination of 3^4.

Eighty-one simulation runs were conducted to assess the DES model fault diagnosis approach as shown in Table 4. Out of the eighty-one simulations, the first fifty-four were conducted under multiple fault conditions, while the remaining twenty-seven were conducted under single fault conditions. The single fault condition experiments were conducted by setting the F1 level to '+', which corresponds to a pressure drop of 5 MPa at the P_{1j}. This pressure drop magnitude does not affect the pressure of the primary hydraulic circuit of the braking system, as the maximum pressure exerted by the braking system is around 4.5 MPa. This allows us to inject a fault only in the secondary hydraulic circuit.

In order to evaluate the efficacy of the proposed fault diagnosis approach, the fault diagnosis responses were collected across eighty-one simulation runs. These responses were obtained by fusing the outcomes produced by the local diagnosis model MPO_i (as shown in Fig. 4), constructed for the extended DES model G_i^+ (as illustrated in Fig. 3). The responses acquired from the DES fault diagnosis approach were considered 'true' if they matched with the location of the injected faults, f_{ij}, in the hydraulic circuits.

3) FAULT ANALYSIS

The simulation environment described in Section V-A allows the verification of the proposed DES model to determine its capability to detect leakage faults in braking systems using an MPO-based diagnoser.

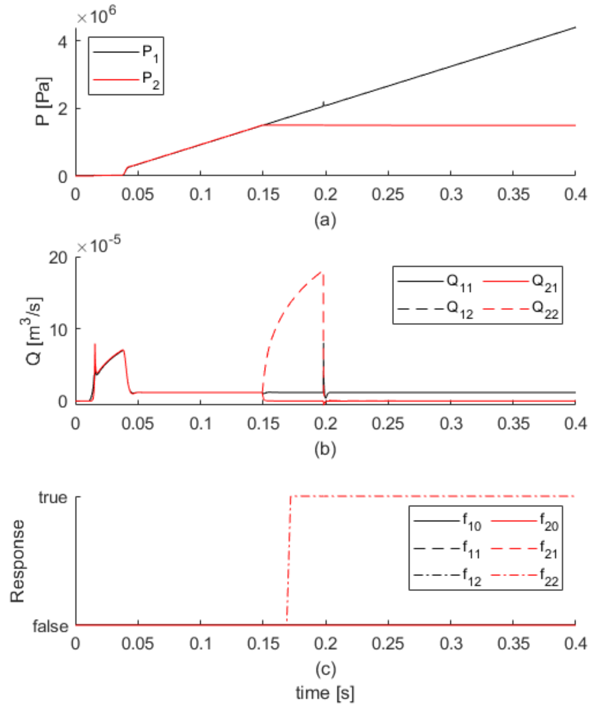

FIGURE 8. Single fault behavior at Run 81.

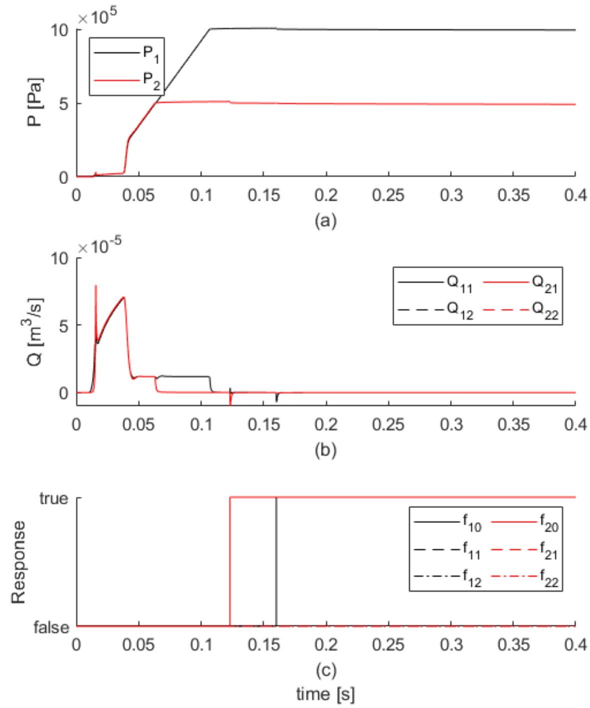

FIGURE 9. Multiple faults behavior at Run 1.

The results of runs 81 and 1, as listed in Table 4, are presented in Figs. 8-9. During each test, the input parameter F is consistently applied to the master cylinder push-rod, covering the range from 0 to 4000 N at a rate of 5000 N/s. The output parameters P_i and Q_{ij} are depicted over the entire simulation time, with the black and red lines corresponding

TABLE 4. Factorial design matrix and responses.

Run	F1	F2	F3	F4	Resp	Run	F1	F2	F3	F4	Resp	Run	F1	F2	F3	F4	Resp
1	-	-	-	-	true	28	o	-	-	-	true	55	+	-	-	-	true
2	-	-	-	o	true	29	o	-	-	o	true	56	+	-	-	o	true
3	-	-	-	+	true	30	o	-	-	+	true	57	+	-	-	+	true
4	-	-	o	-	true	31	o	-	o	-	true	58	+	-	o	-	true
5	-	-	o	o	true	32	o	-	o	o	true	59	+	-	o	o	true
6	-	-	o	+	true	33	o	-	o	+	true	60	+	-	o	+	true
7	-	-	+	-	true	34	o	-	+	-	true	61	+	-	+	-	true
8	-	-	+	o	true	35	o	-	+	o	true	62	+	-	+	o	true
9	-	-	+	+	true	36	o	-	+	+	true	63	+	-	+	+	true
10	-	o	-	-	true	37	o	o	-	-	true	64	+	o	-	-	true
11	-	o	-	o	true	38	o	o	-	o	true	65	+	o	-	o	true
12	-	o	-	+	true	39	o	o	-	+	true	66	+	o	-	+	true
13	-	o	o	-	true	40	o	o	o	-	true	67	+	o	o	-	true
14	-	o	o	o	true	41	o	o	o	o	true	68	+	o	o	o	true
15	-	o	o	+	true	42	o	o	o	+	true	69	+	o	o	+	true
16	-	o	+	-	true	43	o	o	+	-	true	70	+	o	+	-	true
17	-	o	+	o	true	44	o	o	+	o	true	71	+	o	+	o	true
18	-	o	+	+	true	45	o	o	+	+	true	72	+	o	+	+	true
19	-	+	-	-	true	46	o	+	-	-	true	73	+	+	-	-	true
20	-	+	-	o	true	47	o	+	-	o	true	74	+	+	-	o	true
21	-	+	-	+	true	48	o	+	-	+	true	75	+	+	-	+	true
22	-	+	o	-	true	49	o	+	o	-	true	76	+	+	o	-	true
23	-	+	o	o	true	50	o	+	o	o	true	77	+	+	o	o	true
24	-	+	o	+	true	51	o	+	o	+	true	78	+	+	o	+	true
25	-	+	+	-	true	52	o	+	+	-	true	79	+	+	+	-	true
26	-	+	+	o	true	53	o	+	+	o	true	80	+	+	+	o	true
27	-	+	+	+	true	54	o	+	+	+	true	81	+	+	+	+	true

to the results obtained from the primary and secondary hydraulic circuits, respectively. Furthermore, the MPO-based diagnoser output exhibits the time and location at which the faults were diagnosed in any of the hydraulic circuits, which is consistent with the behavior of P_i and Q_{ij}.

The simulation results of run 81, where a single fault was present, are shown in Figs. 8a-c. Fig. 8a shows that the pressure on both hydraulic circuits increases uniformly until P_2 becomes constant at 1.5 MPa due to a brake fluid leak, where the MPO_2 detects the occurrence of the fault f_{22} and transits from the state (1,-1,-1,-1) to the state (16,-1,-1,0), followed by the detection of the event $P_2 < \alpha$ which provokes the transit to (17,-1,-1,1). The behavior of P_1 is not affected by the pressure drop in P_2 as the primary and secondary hydraulic circuits are mechanically separated, which provokes that the MPO_1 remains in the state (1,-1,-1,-1). Fig. 8b illustrates that Q_{21} stops, and Q_{22} abruptly increases due to the brake fluid leak, while Q_{11} and Q_{12} remain constant and seemingly unaffected by the fault in the secondary hydraulic circuit. Finally, Fig. 8c depicts that the diagnoser warns about the occurrence of fault f_{22} immediately after P_2 stabilizes ($P_2 < \alpha$) and Q_{22} begins to increase, where the MPO_2 detects the event $\epsilon > Q_{22}$ and transits from the state (17,-1,-1,1) to (18,-1,-1,2). Since the observed behavior aligns with the test parameters specified in Table 4, the test response is defined as 'true', indicating that the injected fault has been successfully verified.

Figures 9a-c illustrate the outcomes of simulation run 1 conducted to examine the DES model fault diagnosis approach under the influence of multiple faults. Specifically, Fig. 9a shows that the pressure in both hydraulic circuits increases continuously, and then P_2 remains constant at 0.5 MPa, followed by a similar pattern in P_1 at 1 MPa. In this situation, the brake fluid leaks near the master cylinder in both the primary and secondary hydraulic circuits trigger the faults f_{10} and f_{20}, respectively. Correspondingly, both MPO_1 and MPO_2 transit from the state (1,-1,-1,-1) to the state (9,0,-1,-1), followed by the transition to the state (10,1,-1,-1) due to the occurrence of the faults $P_1 < \alpha$ and $P_2 < \alpha$. Consequently, as demonstrated in Fig. 9b, both Q_{21} and Q_{22} decrease due to the leakage, causing the fluid to flow to the brake calipers at a reduced rate, while Q_{11} and Q_{12} exhibit similar behavior when P_1 begins to remain constant. Finally, Fig. 9c displays the warning from the diagnoser about the occurrence of faults f_{10} and f_{20} after the Q_{11} and Q_{21} begin to decrease, where both MPO_1 and MPO_2 detect the events $Q_{11} < \epsilon$ and $Q_{21} < \epsilon$, and transit from the state (10,1,-1,-1) to the state (11,2,-1,-1). The observed behavior aligns with the test parameters defined in Table 4, and the test response is designated as 'true', indicating that the faults are detected correctly using the proposed MPO-based fault diagnosis approach.

As part of this work, a GitHub repository[1] was created to share the code and data with the community. It contains the simulation model of the braking system using Simscape and the event-driven fault diagnoser employing Stateflow.

Remark 1: Figs. 8b and 9b show the nonlinear behaviors at the beginning of both tests (when $F < \beta$). This is because of the static-to-dynamic transition of the mechanical components (e.g., mass-spring-damper model acting together with the hydraulic system model), as well as numerical inconsistencies produced by near-zero values of S_{ij}, which makes it difficult to estimate P_i and Q_{ij} in the simulation model.

[1] https://github.com/ACCESSLab/fault-detection-brake-system.git

Therefore, in our DES model in Section III, we considered $F > \beta$ as the minimum force to initiate the movement of the piston in the primary hydraulic circuit of the master cylinder.

VI. CONCLUSION

This paper addresses a novel DES-based fault diagnosis technique for brake systems, which represents one of the most critical components that contribute to the safety of modern automotive systems. One of the most common causes of faults in this kind of component is brake fluid leaks that affect the performance and response of the entire system, requiring manual inspections by experienced technicians to locate the source of the problem before proceeding to corrective maintenance. This study addresses the aforementioned challenge by developing a novel DES model and diagnoser to detect leakage faults in braking systems, using an MPO-based architecture for fault diagnosis. The developments have been verified via an enhanced virtual environment employing a simulation model of the braking system using Simscape and assessing the event-driven fault diagnosis scheme using Stateflow. To verify the proposed fault diagnosis method, an experimental approach was employed using a factorial design of 3^4. Based on these factorial combinations, we conducted eighty-one test cases, including single and multiple fault scenarios. The proposed DES fault diagnosis approach is shown to have a 'true' response in each simulation run, verifying its reliability over the entire range of operations.

This study represents a novel attempt to detect and isolate leakage faults in braking systems, filling a crucial gap in the existing literature, and offering an efficient solution to a critical safety issue in automotive systems.

Future investigations will explore probabilistic fault generation methods, evaluating the performance of the proposed DES model under various noise and disturbance scenarios, typical in real-world conditions. As additional brake leakage detection techniques emerge, we will conduct comparative analyses to benchmark our approach.

LIST OF SYMBOLS

Acronyms

MPO	Most Permissive Observer.
DES	Discrete Event System.

Subscripts

i	Hydraulic circuit number $\in \{1, 2\}$.
j	Caliper number $\in \{1, 2\}$.
k	MPO number $\in \{1, 2\}$.

Roman Symbols

F	Force on the push-rod.
P_i	Pressure.
S_i	Displacement of master cylinder piston.
S_{ij}	Displacement of caliper piston.
Q_{ij}	Volumetric flow.
f_{i0}	Leak fault near master cylinder.
f_{i1}	Leak fault near 1st caliper.
f_{i2}	Leak fault near 2nd caliper.
G_i	Discrete event system model.
X_i	Finite set of states.
x_{0_i}	Initial state of G_i.
x_i	State of G_i.
Y_{ik}	Set of decision states for MPO_{ik}.
Z_{ik}	Set of observation states for MPO_{ik}.
$y_{0_{ik}}$	Initial state of MPO_{ik}.
G_i^+	Augmented DES model.
$\mathcal{F}$	Set of faults.
$h_{YZ_{ik}}$	Transition function from Y_{ik} to Z_{ik}.
$h_{ZY_{ik}}$	Transition function from Z_{ik} to Y_{ik}.

Greek Symbols

ε	Zero-length string.
$\frac{dF}{dt}$	Force rate on the push-rod.
γ	Sensing decision events.
Γ_k	Sensing decision event set.
α	Pressure model parameter.
β	Force model parameter.
ϵ	Volumetric flow model.
θ	Vector with linear coefficients.
Σ_i	Finite set of events.
Σ_{o_i}	Finite set of observable events.
Σ_{uo_i}	Finite set of unobservable events.
δ_i	Partial transition function.
ω	Sensor activation policy.

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