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RESEARCH ARTICLE

Distributed Filtering for Networked Stochastic Nonlinear Systems With Fading Measurements and Random Packet Dropouts

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ABSTRACT In practical application scenarios, the phenomena of nonlinearity and missing data are commonly present in networked multi-sensor systems. Therefore, this paper investigates distributed filtering problems for networked stochastic nonlinear systems with fading measurements and random packet dropouts. Considering the statistical characteristics of sensors' fading measurements and random losses in transmitting state estimates of their neighbor nodes, a novel distributed Kalman filter (DKF) with multiple filter gains is proposed for each sensor, where multiple filter gains include one Kalman filter gain for measurements of sensor itself and different consensus filter gains for state estimates of its different neighbor nodes. Two compensation mechanisms are used for random packet losses among sensor nodes. Based on an inequality scaling method, an upper bound of the filtering error covariance matrix (UBFECM) dependent on a set of positive scalar parameters is derived, which can avoid calculating the cross-covariance matrices among sensor nodes and the state second moment matrix. Furthermore, multiple filter gains and scalar parameters are optimized by minimizing locally an UBFECM and using nonlinear optimization methods. The exponential boundedness in mean square of filtering error of DKF is proved, and the performance of DKF is also compared with local filter. Simulation results illustrate the effectiveness of the presented DKF algorithm.

INDEX TERMS Distributed Kalman filter, networked multi-sensor system, stochastic nonlinearities, fading measurements, random packet dropouts, boundedness analysis.

I. INTRODUCTION

In recent years, with the continuous innovation and development of science and technology, networked multi-sensor systems (NMSSs) based on intelligent sensors [1] have emerged, which have been widely applied in many fields [2], [3]. In NMSSs, each intelligent sensor can not only obtain local target information, but also obtain target information from its neighbor nodes through the network, aiming to further improve the performance of networked systems. However, considering the various network-induced phenomena such as nonlinear dynamics, missing data, and fading measurements in NMSSs [4], how to design a distributed estimation algorithm suitable for NMSSs to overcome the negative impact of network-induced phenomena on the performance of networked systems, which has become one of the hot topics studied by scholars [5], [6].

For the current research results on distributed estimation problems, three fusion techniques have been mainly developed [7], i.e., diffusion-based fusion, consensus-based fusion, and estimator-based fusion. In the consensus-based fusion, Olfati-Saber [8], [9] proposed a novel distributed Kalman consensus filter (KCF) on the basis of the standard Kalman filter, where KCF contains a consensus term with the difference in state estimation between sensor itself and its neighbor nodes. Obviously, in addition to inherit

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the merits of the standard Kalman filter, KCF also has advantages such as distributed structure, lower computing and communication burden, higher estimation accuracy, ease of fault diagnosis et al., which has received widespread attention in the research of distributed estimation algorithms.

In almost all practical systems, nonlinear phenomena are common in the dynamic or/and measurement process that might severely degrade the performance of networked systems. So, much work has been done on the distributed KCF problems for various nonlinear systems, see e.g., [10] and [11]. However, the estimation methods in the above literature are commonly used to solve nonlinear problems that occur in deterministic ways. Owing to sudden environment changes, the high maneuverability of the tracked target, intermittent network congestion et al., another kind of nonlinearities can be defined as stochastic nonlinearities, which may occur in a random manner [12], [13]. In fact, such stochastic nonlinearities include the state-dependent multiplicative noises as special cases, which have also been attracting extensive attention [14], [15]. Moreover, to meet the needs of practical applications, some literature has designed different distributed estimators for stochastic nonlinear systems, such as event-triggered multi-rate fusion estimation [16] and distributed state estimation with random delays and packet dropouts [17].

Besides, missing data often occur in NMSSs due to some uncertain factors such as sensor aging or failure, unreliable communication links, and external environmental interference. To reduce the impact of missing data on the performance of networked systems, according to the KCF framework, the distributed estimator designed in [18] depends on Bernoulli random variables that describe missing measurements, while the distributed estimator proposed in [19], as well as its filter gains and covariance matrices, depend entirely on the probabilities of missing measurements. Considering fading measurements as a more general form of missing measurements, the literature [20] proposed a distributed KCF based on fading measurement rates under the linear unbiased minimum variance criterion (LUMVC). For the phenomenon of random packet dropouts in networks, some distributed KCF algorithms mainly study measurement packet losses [21], transmission packet losses among sensor nodes [22], and mixture packet losses of the above two types [23].

In the above literature, there are still problems that the filtering algorithms rely on the arbitrary given or/and one common consensus filter gains, as well as free scalar parameters, which may lead to a decrease in estimation accuracy. So, some recent literature [24], [25] have developed distributed KCFs with optimal multiple filter gains and optimal scalar parameters, where the distributed filter proposed in [25] can avoid calculating the filtering error cross-covariance matrices (FECCM) by seeking the minimum upper bound of the filtering error covariance matrix. Unfortunately, for networked stochastic nonlinear systems, the design problems of distributed filters with multiple filter gains have been rarely taken into account, and the distributed filter proposed in [24] for missing data only considers the FECCM with significant computational burden, the above questions constitute the research motivations.

Motivated by the above discussion, combining with the KCF framework with multiple filter gains, we are interested in designing a distributed filtering algorithm for stochastic nonlinear systems with fading measurements and random packet dropouts in the present paper. For the proposed research topic, the following challenges that need to be solved: (1) how to design a novel distributed Kalman filter that can improve estimation accuracy for state estimates of different neighbor nodes? (2) how to reduce the impact of missing data from neighbor nodes on estimation performance? (3) how to design a distributed filtering algorithm with lower computing and communication burden to meet the needs of practical applications? (4) how to analyze the stability of the distributed filtering algorithm? Hence, the main contributions of this paper are summarized as follows:

- In response to the difficulty in determining whether data are missing or not in practice, based on statistical characteristics of fading measurements and random packet dropouts among sensor nodes, a novel distributed Kalman filter (DKF) with multiple filter gains is proposed to improve estimation accuracy, where multiple filter gains include one Kalman filter gain for measurements of sensor itself and different consensus filter gains for state estimates of its different neighbor nodes.
- For updating the state estimates at every time, the zero-input and the hold-input compensation mechanisms are used for missing estimates and missing covariance matrices of neighbor nodes, respectively.
- To avoid the calculation of the FECCM and the state second moment matrix, an upper bound of filtering error covariance matrix (UBFECM) containing a set of positive scalar parameters is sought by applying the linear matrix inequality technique. Furthermore, multiple filter gains and scalar parameters are optimized by minimizing locally an UBFECM and using nonlinear optimization methods.
- The exponential boundedness in mean square of filtering error of DKF is analyzed, and the estimation accuracy of DKF is also proven to be superior to that of local filter with fading measurements.

The rest of the paper is arranged as follows: In Section II, the distributed filtering problem is formulated. In Section III, the main results are derived for DKF. The stability of the proposed DKF is analyzed in Section IV. In Section V, an example is used to verify the feasibility of theoretical results. Finally, the conclusions are given.

Notation: $\Re^n$ denotes n dimensional Euclidean space. L is the number of sensors. E[•] stands for the expectation of a random variable "•". δ_{ij} is the Kronecker delta function. Pr ob{•} denotes occurring probability of an event "•". tr{ A }

and A^{T} refer to the trace and the transpose of a matrix A, respectively. diag[•] stands for a block-diagonal matrix of a general term "•". I_n stands for an n by n identity matrix. $\perp$ denotes uncorrelation or orthogonality. $(X)(\bullet)^{\mathrm{T}}$ is equivalent to $(X)(X)^{\mathrm{T}}$. $\|\bullet\|$ stands for Euclidean norm of a vector "•" or spectral norm of a matrix "•".

II. PROBLEM FORMULATION

Similar to [24], we can also use a graph $G = (\alpha, \beta)$ to describe the topological structure of NMSSs, where the vertex set $\alpha = \{1, 2, \ldots, L\}$ and the edge set $\beta \subseteq \alpha \times \alpha$ stand for all sensor nodes and communication channels, respectively. Especially in a directed graph, an arrow pointing from sensor l to sensor i means that sensor l is a neighbor node of sensor i. Moreover, the set of all neighbor sensor nodes of sensor i is represented as $N_i = \{l|(l, i) \in \beta, \forall l \neq i\}$, which can also be denoted as $l = i_1, i_2, \ldots, i_{r_i}$, and r_i is the number of neighbor nodes of sensor i.

Consider the following time-varying discrete-time system with stochastic nonlinearities and fading measurements:

$$x(t+1) = A(t)x(t) + f(x(t), \xi(t)) + B(t)w(t), \tag{1}$$

$$y_i(t) = \rho_i(t)H_i(t)x(t) + v_i(t), i = 1, 2, \ldots, L, \tag{2}$$

where the discrete time t is the natural number greater than zero. $x(t) \in \Re^n$ is the state vector and $y_i(t) \in \Re^{m_i}$ is the measurement vector of sensor i. $w(t) \in \Re^r$ and $v_i(t) \in \Re^{m_i}$, $i = 1, 2, \ldots, L$ are the process noise and measurement noises, respectively. The random variables $\rho_i(t)$ in (2), $i = 1, 2, \ldots, L$ are used to describe fading measurements of sensors, which are distributed over the interval [0, 1] with known mean $\bar{\rho}_i$ and variance σ_i^2. $A(t)$, $B(t)$, and $H_i(t)$, $i = 1, 2, \ldots, L$ are known matrices with suitable dimensions.

In addition, the function $f(x(t), \xi(t))$ in (1) denotes the stochastic nonlinearities of the state $x(t)$, which satisfies

$$\mathrm{E}\left[f(x(t), \xi(t))|x(t)\right] = 0, \tag{3}$$

$$\mathrm{E}\left[f(x(t), \xi(t))f^{\mathrm{T}}(x(k), \xi(k))|x(t)\right] = \sum_{\tau=1}^{o} \Psi_\tau(t)x^{\mathrm{T}}(t)\Phi_\tau(t)x(t)\delta_{tk}, \tag{4}$$

where $\xi(t)$ is a zero-mean white noise sequence independent of all other random variables, $\Psi_\tau(t)$ and $\Phi_\tau(t)$, $\tau = 1, 2, \ldots, o$ are known matrices with appropriate dimensions, and o is a known integer.

As we all know, the filter of sensor i can improve its estimation accuracy by using the one-step prediction estimates $\hat{x}_l(t|t-1)$, $l \in N_i$ of its neighbor nodes. However, packet dropouts often occur during data communication among any two sensor nodes, which can be described by using a group of Bernoulli distributed random variables $\gamma_{il}(t)$ with $\mathrm{Prob}\{\gamma_{il}(t) = 1\} = \bar{\gamma}_{il}$ and $\mathrm{Prob}\{\gamma_{il}(t) = 0\} = 1 - \bar{\gamma}_{il}$, i.e., $\gamma_{il}(t) = 1$ represents that sensor i can receive prediction estimates of its neighbor nodes l, otherwise it cannot, $i = 1, 2, \ldots, L$; $l \in N_i$. In this paper, we assume that the random variables $\rho_i(t)$ and $\gamma_{il}(t)$ are uncorrelated with each other and also independent of other variables.

In summary, the data used by the filter of sensor i are given as follows:

$$z_i(t) = \begin{pmatrix} y_i(t) \\ y_{ii_1}(t) \\ y_{ii_2}(t) \\ \vdots \\ y_{ii_{r_i}}(t) \end{pmatrix} = \begin{pmatrix} \rho_i(t)H_i(t)x(t) + v_i(t) \\ \gamma_{ii_1}(t)\hat{x}_{i_1}(t|t-1) \\ \gamma_{ii_2}(t)\hat{x}_{i_2}(t|t-1) \\ \vdots \\ \gamma_{ii_{r_i}}(t)\hat{x}_{i_{r_i}}(t|t-1) \end{pmatrix}, \quad i = 1, 2, \ldots, L. \tag{5}$$

Considering the practical applications of filter, it is difficult to obtain the values of random variables $\rho_i(t)$ and $\gamma_{il}(t)$, $l \in N_i$, a filter that depends on the statistical characteristics of these random variables can be developed. Then, based on (5) and the form of standard Kalman filter, we can use the methods in [24] and [25] to design the DKF of sensor i in the following form:

$$\hat{x}_i(t|t) = \hat{x}_i(t|t-1) + G_i(t)\left(y_i(t) - \bar{\rho}_i H_i(t)\hat{x}_i(t|t-1)\right) + \sum_{l \in N_i} C_{il}(t)\left(y_{il}(t) - \bar{\gamma}_{il}\hat{x}_i(t|t-1)\right), \tag{6}$$

where $\hat{x}_i(t|t)$ and $\hat{x}_i(t|t-1)$ denote filter and predictor at time t, respectively. $G_i(t) \in \Re^{n \times m_i}$ and $C_{il}(t) \in \Re^{n \times n}$, $l \in N_i$ are one Kalman filter gain for measurements and different consensus filter gains for state estimates of different neighbor nodes, respectively, which will be designed in the LUMVC.

In short, by minimizing an UBFECM of DKF $\hat{x}_i(t|t)$ in (6), our goal is to solve optimal solutions of filter gain matrices $G_i(t)$ and $C_{il}(t)$, $l \in N_i$.

The following assumptions will be used in this paper.

Assumption 1: $w(t)$ and $v_i(t)$ are uncorrelated white noises with mean zero and covariance matrices $\mathrm{E}\left[w(t)w^{\mathrm{T}}(t)\right] = Q_w(t) \geq 0$ and $\mathrm{E}\left[v_i(t)v_j^{\mathrm{T}}(t)\right] = R_i(t)\delta_{ij} > 0$, where $Q_w(t) \in \Re^{r \times r}$ and $R_i(t) \in \Re^{m_i \times m_i}$, $i, j = 1, 2, \ldots, L$.

Assumption 2: The initial state $x(0)$ with $\mathrm{E}[x(0)] = \mu_0$ and $\mathrm{E}\left[(x(0) - \mu_0)(x(0) - \mu_0)^{\mathrm{T}}\right] = P_0$ is uncorrelated with $w(t)$, $v_i(t)$, $\xi(t)$, $\rho_i(t)$, and $\gamma_{il}(t)$, $i = 1, 2, \ldots, L$; $l \in N_i$.

Remark 1: Obviously, the phenomenon of fading measurements has been included in (2), which can be transformed to the phenomenon of missing measurements when the random variables $\rho_i(t)$ describing the fading measurements only take two values 1 or 0. Therefore, the DKF in (6) proposed in this paper is also suitable for NMSSs with random missing data, where missing data are composed of measurements or/and transmission packets. Moreover, it follows from (5) that the measurement of sensor i depends on the number r_i of its neighbor nodes, and the missing data between sensor i and its neighbor nodes may affect the number of its neighbor nodes used in its filter. Based on the above analysis, it is known that random packet dropouts among sensor nodes described by the random variables $\gamma_{il}(t)$, $l \in N_i$ in (5) can also reflect the dynamically changing network topology structures.

Remark 2: Generally, the estimates and covariance matrices of every neighbor node of sensor i are packed into a data package at time t, which is used for transmission in networks. Moreover, for the problem of missing estimates of neighbor nodes, the zero-input compensation mechanism [26] is adopted in (5), i.e., the one-step prediction estimates of neighbor nodes at the filter are set to zero when $\gamma_{il}(t) = 0, l \in N_i$. However, considering the fact that the covariance matrices are usually greater than zero, the hold-input compensation mechanism [26] for missing covariance matrices of neighbor nodes is used to update the filter of sensor i, i.e., the filter holds at the last available one-step prediction error covariance matrices $P_l^*(t|t-1)$ of neighbor nodes, which are defined as $P_l^*(t|t-1) = \gamma_{il}(t)P_l(t|t-1)+(1-\gamma_{il}(t))\,P_l(t-\kappa|t-\kappa-1)$, $0 < \kappa \leq t; l \in N_i$. Note that the covariance matrices received from neighbor nodes should be immediately stored in sensors at every moment.

Remark 3: Similar to the design methods proposed in [24] and [25], different consensus filter gains $C_{il}(t)$, $l \in N_i$ are also developed for state estimates of different neighbor nodes of sensor i in (6), which are completely different from one common consensus filter gain in [10], [11], [18], [19], [20], [21], [22], and [23]. The main difference from one common gain is that $C_{il}(t)$ take into account the difference in accuracy among sensor nodes, i.e., emphasizing the role of high-precision sensors while neglecting the role of low-precision sensors, it is shown that this design method can improve estimation accuracy and reduce conservation for DKF in (6).

Remark 4: It is worth pointing out that stochastic nonlinearities described by (3) and (4) encompass the linear systems with the state-dependent multiplicative noises $Dx(t)\xi(t)$, where D and $\xi(t)$ are a matrix of appropriate dimensions and a zero mean Gaussian noise sequence, respectively [27].

III. DESIGN OF DKF

In this section, considering unknown whether data are missing or not at each time, a DKF algorithm that depends on the probabilities $\bar{\rho}_i$ and $\bar{\gamma}_{il}$ of random variables $\rho_i(t)$ and $\gamma_{il}(t)$, $l \in N_i$ is proposed in the LUMVC. The following lemmas are required before giving our main results.

Lemma 1 ([28]): The matrices X, A, and B with suitable dimensions satisfy the following derivation formula of traces of matrices product:

$$\frac{\partial}{\partial X}\mathrm{tr}\left\{AXB\right\} = A^{\mathrm{T}}B^{\mathrm{T}},$$
$$\frac{\partial}{\partial X}\mathrm{tr}\left\{AXBX^{\mathrm{T}}A^{\mathrm{T}}\right\} = A^{\mathrm{T}}AX\left(B+B^{\mathrm{T}}\right).$$

Lemma 2 ([29]): For the real matrices X and Y with appropriate dimensions, and a positive scalar θ, the following inequality holds:

$$XY^{\mathrm{T}} + YX^{\mathrm{T}} \leq \theta XX^{\mathrm{T}} + \theta^{-1}YY^{\mathrm{T}}.$$

To begin with, for sensor i, the filtering error and the one-step prediction error of DKF are denoted as respectively:

$$\tilde{x}_i(t|t) = x(t) - \hat{x}_i(t|t), \tag{7}$$
$$\tilde{x}_i(t|t-1) = x(t) - \hat{x}_i(t|t-1), \tag{8}$$

and their corresponding covariance matrices are denoted as respectively:

$$P_i(t|t) = \mathrm{E}\left[\tilde{x}_i(t|t)\tilde{x}_i^{\mathrm{T}}(t|t)\right], \tag{9}$$
$$P_i(t|t-1) = \mathrm{E}\left[\tilde{x}_i(t|t-1)\tilde{x}_i^{\mathrm{T}}(t|t-1)\right]. \tag{10}$$

To proceed, the following theorems give main results for our DKF algorithm.

Theorem 1: For systems (1) and (5) under Assumptions 1 and 2, based on any given positive scalar parameters $\theta_i(t)$, $\theta_{il}(t)$, $\varphi_i(t)$, and $\pi_{il}(t)$, $l \in N_i$, an UBFECM $P_i^u(t|t)$ of DKF in (6) of sensor i is computed by

$$\begin{aligned}P_i^u(t|t) &= \hbar_i(t)F_i(t)P_i^u(t|t-1)F_i^{\mathrm{T}}(t) + G_i(t)\bar{\Theta}_i(t)G_i^{\mathrm{T}}(t)\\ &\quad + C_i(t)\mathrm{diag}[\lambdabar_{il}(t)\bar{\chi}_{il}(t)]_{l\in N_i}C_i^{\mathrm{T}}(t),\end{aligned} \tag{11}$$

where

$$\begin{aligned}
\hbar_i(t) &= 1+\theta_i(t),\\
\lambdabar_{ii_k}(t) &= \left(1+\theta_i^{-1}(t)\right)\left(1+\theta_{ii_k}(t)\right)\prod_{s=1}^{k-1}\left(1+\theta_{ii_s}^{-1}(t)\right),\\
&\quad k = 1,2,\ldots,r_i;\ \theta_{ii_{r_i}}(t) = 0,\\
F_i(t) &= I_n - \bar{\rho}_iG_i(t)H_i(t) - C_i(t)\Gamma_i,\\
C_i(t) &= \left(C_{ii_1}(t),\ C_{ii_2}(t),\ldots,C_{ii_{r_i}}(t)\right)_{n\times nr_i},\\
\Gamma_i &= \left(\bar{\gamma}_{ii_1}I_n,\ \bar{\gamma}_{ii_2}I_n,\ldots,\bar{\gamma}_{ii_{r_i}}I_n\right)^{\mathrm{T}}_{nr_i\times n},\\
\bar{\chi}_{il}(t) &= \bar{\gamma}_{il}\left(1+\pi_{il}(t)\right)P_l^{u*}(t|t-1)\\
&\quad + \bar{\gamma}_{il}\left(1-\bar{\gamma}_{il}\right)\left(1+\pi_{il}^{-1}(t)\right)\bar{X}(t),\\
\bar{\Theta}_i(t) &= \sigma_i^2H_i(t)\bar{X}(t)H_i^{\mathrm{T}}(t) + R_i(t),\\
\bar{X}(t) &= \left(1+\varphi_i(t)\right)P_i^u(t|t-1)\\
&\quad + \left(1+\varphi_i^{-1}(t)\right)\hat{x}_i(t|t-1)\hat{x}_i^{\mathrm{T}}(t|t-1),
\end{aligned} \tag{12}$$

and $P_l^{u*}(t|t-1)$ are the last available upper bound of one-step prediction error covariance matrices of neighbor nodes of sensor i, which have already been explained in Remark 2.

Then, we adopt the following filter gain matrices to minimize the UBFECM $P_i^u(t|t)$ in (11) under given parameters:

$$\begin{aligned}G_i(t) &= \hbar_i(t)\bar{\rho}_i\left(I_n - \hbar_i(t)P_i^u(t|t-1)\Gamma_i^{\mathrm{T}}N_i^{-1}(t)\Gamma_i\right)\\ &\quad\times P_i^u(t|t-1)H_i^{\mathrm{T}}(t)M_i^{-1}(t),\end{aligned} \tag{13}$$

$$C_i(t) = \hbar_i(t)\left(I_n - \bar{\rho}_iG_i(t)H_i(t)\right)P_i^u(t|t-1)\Gamma_i^{\mathrm{T}}N_i^{-1}(t), \tag{14}$$

with

$$\begin{aligned}N_i(t) &= \hbar_i(t)\Gamma_iP_i^u(t|t-1)\Gamma_i^{\mathrm{T}} + \mathrm{diag}[\lambdabar_{il}(t)\bar{\chi}_{il}(t)]_{l\in N_i},\\ M_i(t) &= \hbar_i(t)\bar{\rho}_i^2H_i(t)P_i^u(t|t-1)H_i^{\mathrm{T}}(t) + \bar{\Theta}_i(t)\end{aligned}$$

$$
\begin{aligned}
&- \hbar_i^2(t)\bar{\rho}_i^2 H_i(t)P_i^u(t|t-1)\Gamma_i^{\mathrm{T}} N_i^{-1}(t)\Gamma_i \\
&\times P_i^u(t|t-1)H_i^{\mathrm{T}}(t).
\end{aligned}
$$

Proof: According to (7), (8), and (12), subtracting (6) from $x(t)$ yields the filtering error equation as

$$
\tilde{x}_i(t|t) = F_i(t)\tilde{x}_i(t|t-1) - G_i(t)\eta_i(t) - \sum_{l\in N_i} C_{il}(t)\zeta_{il}(t), \tag{15}
$$

where

$$
\eta_i(t) = (\rho_i(t) - \bar{\rho}_i)\, H_i(t)x(t) + v_i(t), \tag{16}
$$

$$
\zeta_{il}(t) = (\gamma_{il}(t) - \bar{\gamma}_{il})\, x(t) - \gamma_{il}(t)\tilde{x}_l(t|t-1),\ l \in N_i. \tag{17}
$$

Based on (9) and (15), the filtering error covariance matrix $P_i(t|t)$ of DKF in (6) can be derived as follows:

$$
\begin{aligned}
P_i(t|t) = &\ \mathrm{E}\left[\left(F_i(t)\tilde{x}_i(t|t-1) - \sum_{l\in N_i} C_{il}(t)\zeta_{il}(t)\right)(\cdot)^{\mathrm{T}}\right] \\
&+ G_i(t)\Theta_i(t)G_i^{\mathrm{T}}(t),
\end{aligned} \tag{18}
$$

where the fact of $\tilde{x}_i(t|t-1)\perp v_i(t)$, $\mathrm{E}\left[\rho_i(t)\right] = \bar{\rho}_i$, and $\mathrm{E}\left[\gamma_{il}(t)\right] = \bar{\gamma}_{il}$, $l \in N_i$ has been used. In term of $x(t)\perp v_i(t)$ and $\mathrm{E}\left[(\rho_i(t) - \bar{\rho}_i)^2\right] = \sigma_i^2$, substituting $\eta_i(t)$ in (16) into the definition $\Theta_i(t) = \mathrm{E}\left[\eta_i(t)\eta_i^{\mathrm{T}}(t)\right]$ yields

$$
\begin{aligned}
\Theta_i(t) &= \mathrm{E}\left[\left((\rho_i(t) - \bar{\rho}_i)\, H_i(t)x(t) + v_i(t)\right)(\cdot)^{\mathrm{T}}\right] \\
&= \sigma_i^2 H_i(t)X(t)H_i^{\mathrm{T}}(t) + R_i(t),
\end{aligned} \tag{19}
$$

and then based on (8), (10), and Lemma 2, the state second moment matrix $X(t)$ in (19) satisfies the following inequality:

$$
\begin{aligned}
X(t) &= \mathrm{E}\left[x(t)x^{\mathrm{T}}(t)\right] \\
&= \mathrm{E}\left[\left(\tilde{x}_i(t|t-1) + \hat{x}_i(t|t-1)\right)(\cdot)^{\mathrm{T}}\right] \\
&\le (1 + \varphi_i(t))\, P_i(t|t-1) \\
&\quad + \left(1 + \varphi_i^{-1}(t)\right) \hat{x}_i(t|t-1)\hat{x}_i^{\mathrm{T}}(t|t-1).
\end{aligned} \tag{20}
$$

Hence, it follows from the last equality of (20) that the upper bound $\bar{X}(t)$ in (12) of $X(t)$ is obtained. Moreover, it is easy to obtain the upper bound $\bar{\Theta}_i(t)$ in (12) of $\Theta_i(t)$ by substituting $\bar{X}(t)$ into (19).

Subsequently, on the basis of (9), (10), (12), (18), and Lemma 2, we have

$$
\begin{aligned}
&P_i(t|t) \\
&\le (1 + \theta_i(t))\, F_i(t)P_i(t|t-1)F_i^{\mathrm{T}}(t) + G_i(t)\Theta_i(t)G_i^{\mathrm{T}}(t) \\
&\quad + \left(1 + \theta_i^{-1}(t)\right) \\
&\quad \times \mathrm{E}\left[\left(C_{ii_1}(t)\zeta_{ii_1}(t) + \sum_{k=2}^{r_i} C_{ii_k}(t)\zeta_{ii_k}(t)\right)(\cdot)^{\mathrm{T}}\right] \\
&\le (1 + \theta_i(t))\, F_i(t)P_i(t|t-1)F_i^{\mathrm{T}}(t) + G_i(t)\Theta_i(t)G_i^{\mathrm{T}}(t) \\
&\quad + \left(1 + \theta_i^{-1}(t)\right)\left(1 + \theta_{ii_1}(t)\right) C_{ii_1}(t)\chi_{ii_1}(t)C_{ii_1}^{\mathrm{T}}(t) \\
&\quad + \left(1 + \theta_i^{-1}(t)\right)\left(1 + \theta_{ii_1}^{-1}(t)\right) \\
&\quad \times \mathrm{E}\left[\left(C_{ii_2}(t)\zeta_{ii_2}(t) + \sum_{k=3}^{r_i} C_{ii_k}(t)\zeta_{ii_k}(t)\right)(\cdot)^{\mathrm{T}}\right] \\
&\le (1 + \theta_i(t))\, F_i(t)P_i(t|t-1)F_i^{\mathrm{T}}(t) + G_i(t)\Theta_i(t)G_i^{\mathrm{T}}(t) \\
&\quad + \left(1 + \theta_i^{-1}(t)\right)\left(1 + \theta_{ii_1}(t)\right) C_{ii_1}(t)\chi_{ii_1}(t)C_{ii_1}^{\mathrm{T}}(t) \\
&\quad + \left(1 + \theta_i^{-1}(t)\right)\left(1 + \theta_{ii_1}^{-1}(t)\right)\left(1 + \theta_{ii_2}(t)\right) \\
&\quad \times C_{ii_2}(t)\chi_{ii_2}(t)C_{ii_2}^{\mathrm{T}}(t) \\
&\quad + \cdots + \left(1 + \theta_i^{-1}(t)\right)\left(1 + \theta_{ii_1}^{-1}(t)\right)\left(1 + \theta_{ii_2}^{-1}(t)\right) \\
&\quad \times \cdots \times \left(1 + \theta_{ii_{r_i-1}}^{-1}(t)\right) C_{ii_{r_i}}(t)\chi_{ii_{r_i}}(t)C_{ii_{r_i}}^{\mathrm{T}}(t) \\
&= \hbar_i(t)F_i(t)P_i(t|t-1)F_i^{\mathrm{T}}(t) + G_i(t)\Theta_i(t)G_i^{\mathrm{T}}(t) \\
&\quad + C_i(t)\mathrm{diag}[\lambda_{il}(t)\chi_{il}(t)]_{l\in N_i} C_i^{\mathrm{T}}(t).
\end{aligned} \tag{21}
$$

It is noted that the UBFECM $P_i^u(t|t)$ in (11) can be derived from (12) and the last equality in (21), where it follows from (10), (17), (20), Lemma 2, and $\mathrm{E}\left[\gamma_{il}^2(t)\right] = \bar{\gamma}_{il}$, $l \in N_i$ that

$$
\begin{aligned}
\chi_{il}(t) &= \mathrm{E}\left[\zeta_{ii_l}(t)\zeta_{ii_l}^{\mathrm{T}}(t)\right] \\
&= \mathrm{E}\left[\left((\gamma_{il}(t) - \bar{\gamma}_{il})\, x(t) - \gamma_{il}(t)\tilde{x}_l(t|t-1)\right)(\cdot)^{\mathrm{T}}\right] \\
&\le \bar{\gamma}_{il}\,(1 + \pi_{il}(t))\, P_l(t|t-1) \\
&\quad + \bar{\gamma}_{il}\,(1 - \bar{\gamma}_{il})\left(1 + \pi_{il}^{-1}(t)\right) X(t),
\end{aligned} \tag{22}
$$

and the upper bound $\bar{\chi}_{il}(t)$ in (12) of $\chi_{il}(t)$, $l \in N_i$ is also obtained by combining (22), $\bar{X}(t)$ in (12), and $P_l^*(t|t-1)$, $l \in N_i$ in Remark 2.

Finally, we will minimize the following performance index to solve filter gain matrices:

$$
J\left(G_i(t), C_i(t)\right) = \mathrm{tr}\left\{P_i^u(t|t)\right\}. \tag{23}
$$

According to (11), (23), and Lemma 1, we can let

$$
\begin{aligned}
&\frac{\partial}{\partial G_i(t)}\mathrm{tr}\left\{P_i^u(t|t)\right\} \\
&= G(t)\left(\hbar_i(t)\bar{\rho}_i^2 H_i(t)P_i^u(t|t-1)H_i^{\mathrm{T}}(t) + \bar{\Theta}_i(t)\right) \\
&\quad - \hbar_i(t)\bar{\rho}_i\left(I_n - C_i(t)\Gamma_i\right) P_i^u(t|t-1)H_i^{\mathrm{T}}(t) \\
&= 0, \\
&\frac{\partial}{\partial C_i(t)}\mathrm{tr}\left\{P_i^u(t|t)\right\} \\
&= C_i(t)\left(\hbar_i(t)\Gamma_i P_i^u(t|t-1)\Gamma_i^{\mathrm{T}} + \mathrm{diag}[\lambda_{il}(t)\bar{\chi}_{il}(t)]_{l\in N_i}\right) \\
&\quad - \hbar_i(t)\left(I_n - \bar{\rho}_i G_i(t)H_i(t)\right) P_i^u(t|t-1)\Gamma_i^{\mathrm{T}} \\
&= 0,
\end{aligned} \tag{24}
$$

and the filter gain matrices $G_i(t)$ in (13) and $C_i(t)$ in (14) to minimize (23) are obtained by solving (24).

Theorem 2: For systems (1) and (5) under Assumptions 1 and 2, the one-step predictor $\hat{x}_i(t+1|t)$ in (6) is computed by

$$
\hat{x}_i(t+1|t) = A(t)\hat{x}_i(t|t). \tag{25}
$$

Furthermore, the upper bound of one-step prediction error covariance matrix $P_i^u(t+1|t)$ in (11), $l \in N_i$ can be described by

$$P_i^u(t+1|t) = A(t)P_i^u(t|t)A^{\mathrm{T}}(t) + B(t)Q_w(t)B^{\mathrm{T}}(t) + \sum_{\tau=1}^{o}\Psi_\tau(t)\mathrm{tr}\left\{\Phi_\tau^{\mathrm{T}}(t)\bar{X}(t)\right\}. \quad (26)$$

Proof: Considering the linear space $\mathcal{L}(z_i(1), z_i(2), \ldots, z_i(t))$, we easily have (25) from (1).

Based on (7) and (8), subtracting (25) from (1) yields one-step prediction error equation of sensor i as

$$\tilde{x}_i(t+1|t) = A(t)\tilde{x}_i(t|t) + f(x(t), \xi(t)) + B(t)w(t). \quad (27)$$

By using (3), (4), (9), (10), (20), (27), $\tilde{x}_i(t|t)\perp w(t)$, and $\xi(t)\perp w(t)$, we have the one-step prediction error covariance matrix $P_i(t+1|t)$, i.e.,

$$\begin{aligned} P_i(t+1|t) &= A(t)P_i(t|t)A^{\mathrm{T}}(t) + B(t)Q_w(t)B^{\mathrm{T}}(t) \\ &\quad + \sum_{\tau=1}^{o}\Psi_\tau(t)\mathrm{E}\left[x^{\mathrm{T}}(t)\Phi_\tau(t)x(t)\right] \\ &= A(t)P_i(t|t)A^{\mathrm{T}}(t) + B(t)Q_w(t)B^{\mathrm{T}}(t) \\ &\quad + \sum_{\tau=1}^{o}\Psi_\tau(t)\mathrm{tr}\left\{\Phi_\tau^{\mathrm{T}}(t)X(t)\right\}, \end{aligned} \quad (28)$$

where the fact of $\mathrm{tr}\left\{ab^{\mathrm{T}}\right\} = a^{\mathrm{T}}b$ with the same dimensional vectors a and b has been used.

So, the $P_i^u(t+1|t)$ in (26) is easily got from (12) and (28).

Remark 5: From (13) and (14), it is not difficult to found that the filter gain matrices $G_i(t)$ and $C_i(t)$ in Theorem 1 depend on a set of arbitrarily given positive scalar parameters, and these gain matrices are not optimal and may even deteriorate the estimation accuracy of the filter. In order to overcome the above problem, by applying the fact that $\theta = \sqrt{\mathrm{tr}\left(YY^{\mathrm{T}}\right)/\mathrm{tr}\left(XX^{\mathrm{T}}\right)}$ in Lemma 2 is the minimum value to (12) and (20)–(22), the optimal expressions are derived for the scalar parameters in Theorem 1, i.e., the parameters $\theta_i(t)$ and $\theta_{il}(t)$, $l \in N_i$ are

$$\theta_{ii_k}(t) = \sqrt{\frac{\mathrm{tr}\left\{C_i(t)\mathrm{diag}[\eta_{il}(t)\bar{\chi}_{il}(t)]_{l\in N_i}C_i^{\mathrm{T}}(t)\right\}}{\mathrm{tr}\left\{C_{ii_k}(t)\bar{\chi}_{ii_k}(t)C_{ii_k}^{\mathrm{T}}(t)\right\}}},$$

$$\eta_{ii_s}(t) = \begin{cases} 0, \ s = 1, 2, \ldots, k, \\ \left(1+\theta_{ii_s}(t)\right)\prod_{m=k+1}^{s-1}\left(1+\theta_{ii_m}^{-1}(t)\right), \\ \quad s = k+1, k+2, \ldots, r_i, \end{cases}$$

$$k = r_i - 1, r_i - 2, \ldots, 1; \theta_{ii_{r_i}}(t) = 0, \quad (29)$$

$$\theta_i(t) = \sqrt{\frac{\mathrm{tr}\left\{C_i(t)\mathrm{diag}[\varepsilon_{il}(t)\bar{\chi}_{il}(t)]_{l\in N_i}C_i^{\mathrm{T}}(t)\right\}}{\mathrm{tr}\left\{F_i(t)P_i^u(t|t-1)F_i^{\mathrm{T}}(t)\right\}}},$$

$$\varepsilon_{ii_s}(t) = \left(1+\theta_{ii_s}(t)\right)\prod_{m=1}^{s-1}\left(1+\theta_{ii_m}^{-1}(t)\right), s = 1, 2, \ldots, r_i, \quad (30)$$

the parameters $\varphi_i(t)$ are

$$\varphi_i(t) = \sqrt{\mathrm{tr}\left\{\hat{x}_i(t|t-1)\hat{x}_i^{\mathrm{T}}(t|t-1)\right\}\Big/\mathrm{tr}\left\{P_i^u(t|t-1)\right\}}, \quad (31)$$

and the parameters $\pi_{il}(t)$, $l \in N_i$ are

$$\pi_{il}(t) = \sqrt{\bar{\gamma}_{il}\left(1-\bar{\gamma}_{il}\right)\mathrm{tr}\left\{\bar{X}(t)\right\}/\bar{\gamma}_{il}\mathrm{tr}\left\{P_l^{u*}(t|t-1)\right\}}. \quad (32)$$

Now, based on (13), (14), and (29)–(32), the optimal gain matrices $G_i^m(t)$ and $C_i^m(t)$ with optimal scalar parameters $\theta_i^m(t)$, $\theta_{il}^m(t)$, $\varphi_i^m(t)$, and $\pi_{il}^m(t)$, $l \in N_i$ can be solved. Furthermore, the minimum UBFECM $P_i^{mu}(t|t)$ for $P_i^u(t|t)$ in (11) can also be solved. However, according to (13), (14), (29), and (30), it can be found that the gain matrices and some scalar parameters are nonlinearly coupled, so we can use some nonlinear optimization methods to seek their approximate optimal numerical solutions, such as an iterative method used in this paper.

IV. STABILITY ANALYSIS OF DKF

In this section, the stability of the proposed DKF will be analyzed. To begin with, we will introduce the following useful lemma and assumption to ensure the stability analysis.

Lemma 3 ([30]): Suppose that there exists a stochastic process $V(\varsigma(t))$, real numbers $\underline{\kappa}$, $\bar{\kappa}$, $u > 0$, and $0 < \psi \leq 1$ such that

$$\underline{\kappa}\|\varsigma(t)\|^2 \leq V(\varsigma(t)) \leq \bar{\kappa}\|\varsigma(t)\|^2, \quad (33)$$

and

$$\mathrm{E}\left[V(\varsigma(t+1))|\varsigma(t)\right] \leq (1-\psi)V(\varsigma(t)) + u. \quad (34)$$

Then, the stochastic process $\varsigma(t)$ is exponentially bounded in mean square, i.e.,

$$\mathrm{E}\left[\|\varsigma(t)\|^2\right] \leq \frac{\bar{\kappa}}{\underline{\kappa}}(1-\psi)^t\mathrm{E}\left[\|\varsigma(0)\|^2\right] + \frac{u}{\underline{\kappa}}\sum_{i=1}^{t-1}(1-\psi)^i.$$

Assumption 3: There exist positive scalars a, b, c_{il}, g_i, h_i, ϖ, q_i, δ, and $\lambdabar$, $l \in N_i$ such that

$$\begin{aligned} &\|A(t)\| \leq a, \|B(t)\| \leq b, \|C_{il}(t)\| \leq c_{il}, \|G_i(t)\| \leq g_i, \\ &\|H_i(t)\| \leq h_i, Q_w(t) \leq \varpi I_n, R_i(t) \leq q_i I_{m_i}, \\ &x(t)x^{\mathrm{T}}(t) \leq x^{\mathrm{T}}(t)x(t)I_n \leq \delta I_n, \end{aligned}$$

and noticing the stochastic nonlinear function $f(x(t), \xi(t))$, one gets

$$\mathrm{E}\left[f(x(t), \xi(t))f^{\mathrm{T}}(x(t), \xi(t))|x(t)\right] \leq \lambdabar I_n.$$

Moreover, based on any given positive scalar parameters ϑ, the following condition holds:

$$\|\partial(t+1)\| < 1, \quad (35)$$

where $\partial(t+1)$ is defined as

$$\partial(t+1) = (1+\vartheta)\mathrm{E}\left[F^{\mathrm{T}}(t+1)F(t+1)\right]$$

$$
\begin{aligned}
&= (1+\vartheta)\Big(\bar{F}^{\mathrm{T}}(t+1)\bar{F}(t+1) \\
&\quad + \sum_{i=1}^{L}\sum_{j=1,\, j\neq i}^{L} \bar{\gamma}_{ij}\left(1-\bar{\gamma}_{ij}\right)\Omega_{ij}^{\mathrm{T}}(t+1)\Omega_{ij}(t+1)\Big),
\end{aligned} \tag{36}
$$

and $F(t+1)$ in (36) is an $nL \times nL$ block matrix, each block of which is an $n \times n$ matrix, the matrix element at the $(i,\ i)$-block place is $F_i(t+1)A(t)$ and the matrix element at the $(i,\ j)$-block place is $\gamma_{ij}(t+1)C_{ij}(t+1)A(t)$ if $j \in N_i$ and zero matrix otherwise, $\bar{F}(t+1)$ in (36) can be computed by replacing $\gamma_{ij}(t+1)$ with $\bar{\gamma}_{ij}$ in $F(t+1)$, $i, j = 1, 2, \ldots, L$. $\Omega_{ij}(t+1)$ in (36) is an $nL \times nL$ block matrix in which the block element at i row and j column is $C_{ij}(t+1)A(t), j \in N_i$, and otherwise the block elements are zero, $i, j = 1, 2, \ldots, L$.

Next, we will give main results for stability analysis of DKF.

Theorem 3: Under Lemma 3 and Assumption 3, the filtering error $\tilde{x}_i(t|t)$ in (15) of the DKF in Theorems 1-2 is exponentially bounded in mean square for bounded initial error.

Proof: Based on the augmented filtering error $\tilde{x}(t|t) = \left[\tilde{x}_1^{\mathrm{T}}(t|t), \tilde{x}_2^{\mathrm{T}}(t|t), \ldots, \tilde{x}_L^{\mathrm{T}}(t|t)\right]_{nL\times 1}^{\mathrm{T}}$, the Lyapunov function can be defined as

$$
V(\tilde{x}(t|t)) = \tilde{x}^{\mathrm{T}}(t|t)\tilde{x}(t|t). \tag{37}
$$

Obviously, if $\underline{\kappa} = \bar{\kappa} = 1$, then (37) satisfies (33).

Besides, it follows from (15)–(17) and (27) that the filtering error equation of sensor i can be obtained as

$$
\begin{aligned}
&\tilde{x}_i(t+1|t+1) \\
&= F_i(t+1)A(t)\tilde{x}_i(t|t) + F_i(t+1)f(x(t),\xi(t)) \\
&\quad + F_i(t+1)B(t)w(t) + \sum_{l\in N_i}\gamma_{il}(t+1)C_{il}(t+1)A(t)\tilde{x}_l(t|t) \\
&\quad + \sum_{l\in N_i}\gamma_{il}(t+1)C_{il}(t+1)f(x(t),\xi(t)) \\
&\quad + \sum_{l\in N_i}\gamma_{il}(t+1)C_{il}(t+1)B(t)w(t) - G_i(t+1)v_i(t+1) \\
&\quad - \left(\rho_i(t+1)-\bar{\rho}_i\right)G_i(t+1)H_i(t+1)x(t+1) \\
&\quad - \sum_{l\in N_i}\left(\gamma_{il}(t+1)-\bar{\gamma}_{il}\right)C_{il}(t+1)x(t+1).
\end{aligned} \tag{38}
$$

Introduce augmented vectors:

$$
G(t+1) = \mathrm{diag}[G_i(t+1)]_{nL\times \sum_{i=1}^{L} m_i},
$$

$$
\begin{aligned}
v(t+1) = \big[\, &v_1^{\mathrm{T}}(t+1)\ v_2^{\mathrm{T}}(t+1) \\
&\cdots\ v_L^{\mathrm{T}}(t+1)\,\big]^{\mathrm{T}}_{\sum_{i=1}^{L} m_i\times 1},
\end{aligned}
$$

$$
M(t+1) = \begin{bmatrix} I_n - \bar{\rho}_1 G_1(t+1)H_1(t+1) \\ I_n - \bar{\rho}_2 G_2(t+1)H_2(t+1) \\ \vdots \\ I_n - \bar{\rho}_L G_L(t+1)H_L(t+1) \end{bmatrix}_{nL\times n},
$$

$$
N(t+1) = \begin{bmatrix} \sum_{l\in N_1}\left(\gamma_{1l}(t+1)-\bar{\gamma}_{1l}\right)C_{1l}(t+1) \\ \sum_{l\in N_2}\left(\gamma_{2l}(t+1)-\bar{\gamma}_{2l}\right)C_{2l}(t+1) \\ \vdots \\ \sum_{l\in N_L}\left(\gamma_{Ll}(t+1)-\bar{\gamma}_{Ll}\right)C_{Ll}(t+1) \end{bmatrix}_{nL\times n},
$$

$$
\Lambda(t+1) = \begin{bmatrix} \left(\rho_1(t+1)-\bar{\rho}_1\right)G_1(t+1)H_1(t+1) \\ \left(\rho_2(t+1)-\bar{\rho}_2\right)G_2(t+1)H_2(t+1) \\ \vdots \\ \left(\rho_L(t+1)-\bar{\rho}_L\right)G_L(t+1)H_L(t+1) \end{bmatrix}_{nL\times n}. \tag{39}
$$

By using (1) and (39) to augment (38), it is achieved that

$$
\begin{aligned}
&\tilde{x}(t+1|t+1) \\
&= F(t+1)\tilde{x}(t|t) - \left(\Lambda(t+1)+N(t+1)\right)A(t)x(t) \\
&\quad + \left(M(t+1)-\Lambda(t+1)\right)f\left(x(t),\xi(t)\right) \\
&\quad + \left(M(t+1)-\Lambda(t+1)\right)B(t)w(t) - G(t+1)v(t+1),
\end{aligned} \tag{40}
$$

where $F(t+1)$ has been defined in (36).

By applying Lemma 1 and the fact of $\mathrm{tr}\{\, ab^{\mathrm{T}}\} = a^{\mathrm{T}}b$ for vectors a and b, it follows from (37) and (40) that

$$
\begin{aligned}
\mathrm{E}\left[V(\tilde{x}(t+1|t+1))|\tilde{x}(t|t)\right] &= \Xi_1(t+1) + \Xi_2(t+1) \\
&\quad + \Xi_3(t+1) + \Xi_4(t+1),
\end{aligned} \tag{41}
$$

where according to Lemma 3, Assumption 3, $\mathrm{E}\left[\rho_i(t)\right] = \bar{\rho}_i$, and $\mathrm{E}\left[\gamma_{il}(t)\right] = \bar{\gamma}_{il}$, $l \in N_i$, we have

$$
\begin{aligned}
&\Xi_1(t+1) \\
&= \mathrm{E}\Big[\left(F(t+1)\tilde{x}(t|t)\right. \\
&\qquad \left.-\left(\Lambda(t+1)+N(t+1)\right)A(t)x(t)\right)^{\mathrm{T}}(\bullet)\Big] \\
&\leq \tilde{x}^{\mathrm{T}}(t|t)\partial(t+1)\tilde{x}(t|t) \\
&\quad + \left(1+\vartheta^{-1}\right)\mathrm{tr}\left\{\mathrm{E}\left[\left(\Lambda(t+1)A(t)x(t)\right)(\bullet)^{\mathrm{T}}\right]\right\} \\
&\quad + \left(1+\vartheta^{-1}\right)\mathrm{tr}\left\{\mathrm{E}\left[\left(N(t+1)A(t)x(t)\right)(\bullet)^{\mathrm{T}}\right]\right\} \\
&\leq \tilde{x}^{\mathrm{T}}(t|t)\partial(t+1)\tilde{x}(t|t) + \left(1+\vartheta^{-1}\right)\sum_{i=1}^{L}\sigma_i^2 g_i^2 h_i^2 a^2 n\delta \\
&\quad + \left(1+\vartheta^{-1}\right)\sum_{i=1}^{L}\sum_{l\in N_i}\bar{\gamma}_{il}\left(1-\bar{\gamma}_{il}\right)c_{il}^2 a^2 n\delta,
\end{aligned} \tag{42}
$$

with $\partial(t+1)$ in (36),

$$
\begin{aligned}
&\Xi_2(t+1) \\
&= \mathrm{E}\left[\left(\left(M(t+1)-\Lambda(t+1)\right)f\left(x(t),\xi(t)\right)\right)^{\mathrm{T}}(\bullet)\right] \\
&= \mathrm{tr}\left\{\mathrm{E}\left[\left(\left(M(t+1)-\Lambda(t+1)\right)f\left(x(t),\xi(t)\right)\right)(\bullet)^{\mathrm{T}}\right]\right\} \\
&\leq \sum_{i=1}^{L}\left(\left(1+\bar{\rho}_i g_i h_i\right)^2 + \sigma_i^2 g_i^2 h_i^2\right) n\varkappa,
\end{aligned} \tag{43}
$$

and similar to (42) and (43), we also have

$$\Xi_3(t+1) = \mathrm{E}\left[\left((M(t+1)-\Lambda(t+1))\,B(t)w(t)\right)^{\mathrm{T}}(\bullet)\right] \le \sum_{i=1}^{L}\left((1+\bar{\rho}_i g_i h_i)^2 + \sigma_i^2 g_i^2 h_i^2\right) b^2 n\varpi, \quad (44)$$

$$\Xi_4(t+1) = \mathrm{E}\left[\left(G(t+1)v(t+1)\right)^{\mathrm{T}}(\bullet)\right] \le \sum_{i=1}^{L} g_i^2 m_i q_i. \quad (45)$$

Substituting (42)–(45) into (41) yields

$$\mathrm{E}\left[V(\tilde{x}(t+1|t+1))|\tilde{x}(t|t)\right] \le \tilde{x}^{\mathrm{T}}(t|t)\partial(t+1)\tilde{x}(t|t) + u, \quad (46)$$

where

$$u = \left(1+\vartheta^{-1}\right) a^2 n\delta \left(\sum_{i=1}^{L}\sigma_i^2 g_i^2 h_i^2 + \sum_{i=1}^{L}\sum_{l\in N_i}\bar{\gamma}_{il}\left(1-\bar{\gamma}_{il}\right)c_{il}^2\right) + n\left(\lambda + b^2\varpi\right)\sum_{i=1}^{L}\left((1+\bar{\rho}_i g_i h_i)^2 + \sigma_i^2 g_i^2 h_i^2\right) + \sum_{i=1}^{L} g_i^2 m_i q_i.$$

Then, it follows from (35), (36), and (46) that there are always positive scalars $0 < \psi \le 1$ and $u > 0$ such that (34) holds, where

$$\psi = 1 - \max_{G_i(t+1),\, C_{il}(t+1),\, \bar{\rho}_i,\, \bar{\gamma}_{il},\, \vartheta} \|\partial(t+1)\|.$$

To sum up, in line with Lemma 3, it is clear that the filtering error $\tilde{x}_i(t|t)$ in (15) is exponentially bounded in mean square.

Theorem 4: For systems (1) and (2) with Assumptions 1 and 2, the minimum UBFECM $P_i^{mu}(t|t)$ is bounded and satisfies the following inequality:

$$\mathrm{tr}\left\{P_i^{mu}(t|t)\right\} \le \mathrm{tr}\left\{P_i^{loc}(t|t)\right\},$$

where the local filtering error covariance matrix $P_i^{loc}(t|t)$ with fading measurements is described as

$$P_i^{loc}(t|t) = \left(I_n - \bar{\rho}_i K_i^{loc}(t)H_i(t)\right)P_i^{loc}(t|t-1)\left(I_n - \bar{\rho}_i K_i^{loc}(t)H_i(t)\right)^{\mathrm{T}} + K_i^{loc}(t)\bar{\Theta}_i(t)\left(K_i^{loc}(t)\right)^{\mathrm{T}},$$

and filter gain matrix $K_i^{loc}(t)$ is computed by

$$K_i^{loc}(t) = \bar{\rho}_i P_i^{loc}(t|t-1)H_i^{\mathrm{T}}(t) \times \left[\bar{\rho}_i^2 H_i(t)P_i^{loc}(t|t-1)H_i^{\mathrm{T}}(t) + \bar{\Theta}_i(t)\right]^{-1}.$$

Proof: The proof can be carried out along the similar line to the Theorem 3 in [25]. The detail is omitted to save space.

Remark 6: As is well known, the filtering algorithms in [14] and [24] dependent on the missing probabilities involve recursive calculation of the state second moment matrix, which inevitably require the system matrix $A(t)$ to be stable, leading to significant limitations in the application ranges. However, according to (11) and (12), we know that $P_i^u(t|t)$ depends on the upper bound $\bar{X}(t)$ of the state second moment matrix, which avoids the calculation of the state second moment matrix. Although the proposed DKF may reduce estimation accuracy, it does not require system matrix $A(t)$ to be stable, which can reduce computational burden and expand application ranges. Besides, as shown in Theorem 4, the proposed DKF with optimal filter gains and optimal scalar parameters can achieve better estimation accuracy than local filter with fading measurements by using data information of neighbor nodes.

Remark 7: At every moment, based on the data information of sensor i itself and its neighbor nodes, the update of state estimation for sensor i only may require one-step prediction estimates and the minimum upper bounds of one-step prediction error covariance matrices. Note that the zero-input and the hold-input compensation mechanisms are used in DKF. In a word, for sensor i, $i = 1, 2, \ldots, L$, the operations of the proposed DKF in Theorems 1-2, Remark 2, and Remark 5 can be summarized as in Algorithm 1.

Algorithm 1 DKF for Networked Stochastic Nonlinear Systems With Fading Measurements and Random Packet Dropouts

Initialization:

Loaded information includes $A(t)$, $B(t)$, $H_i(t)$, $\Psi_\tau(t)$, $\Phi_\tau(t)$, $Q_w(t)$, $R_i(t)$, $\bar{\rho}_i$, σ_i^2, $\bar{\gamma}_{il}$, and topological structure, $\tau = 1, 2, \ldots, o$; $l \in N_i$. Set the initial values $\hat{x}_i(0|-1) = \mu_0$, $P_i^{mu}(0|-1) = P_0$, and $P_l^{mu*}(0|-1) = P_0$, $l \in N_i$.

Sensor i iteratively updates the state estimation at each time t through the following four steps:

Receive data:

If sensor i receives the data packets of its neighbor nodes, the current received one-step prediction estimates $\hat{x}_l(t|t-1)$ and minimum upper bounds of one-step prediction error covariance matrix $P_l^{mu}(t|t-1)$ are used, $l \in N_i$, otherwise the compensation mechanisms in Remark 2 are adopted.

Filtering update:

Compute optimal filter gain matrices $G_i^m(t)$ and $C_i^m(t)$, and optimal scalar parameters $\theta_i^m(t)$, $\theta_{il}^m(t)$, $\varphi_i^m(t)$, and $\pi_{il}^m(t)$, $l \in N_i$ by Theorems 1-2 and Remark 5.

Compute the filter $\hat{x}_i(t|t)$ and the minimum UBFECM $P_i^{mu}(t|t)$ by (6) and (11), respectively.

Prediction update:

Compute the one-step prediction estimate $\hat{x}_i(t+1|t)$ and the minimum upper bound of one-step prediction error covariance matrix $P_i^{mu}(t+1|t)$ by (25) and (26), respectively.

Send data:

Send $\hat{x}_i(t+1|t)$ and $P_i^{mu}(t+1|t)$ to sensor j, $i \in N_j$; $j = 1, 2, \ldots, L$; $i \neq j$.

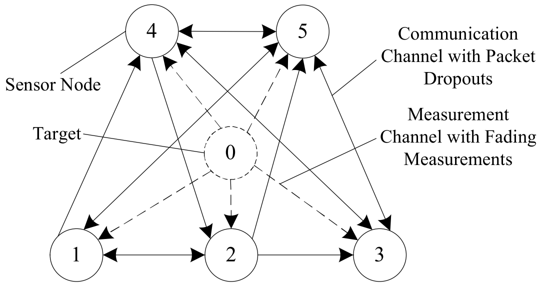


FIGURE 1. The directed topology with five sensors.

V. NUMERICAL EXAMPLE

In this section, considering NMSSs with five sensors shown in Figure 1, according to the definition of neighbor nodes, for instance, the set of neighbor nodes of sensor 3 can be represented as $N_3 = \{l\,|(l,\ 3) \in \beta,\ \forall l \neq 3\}$, i.e., $l = 2,\ 4,\ 5$ and $r_3 = 3$. In addition, we use an uninterruptible power system (UPS) in [24] as a simulation example to verify the effectiveness and feasibility of the proposed DKF algorithm. For the UPS with 1 kVA, the corresponding discrete-time model (1) with stochastic nonlinearities and fading measurements are derived based on the sampling time 10 ms at half-load operating point as follows:

$$x(t+1) = \begin{bmatrix} 0.9226 & -0.6330 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + f(x(t), \xi(t)) + \begin{bmatrix} 0.5 \\ 0 \\ 0.2 \end{bmatrix} w(t),$$

with the stochastic nonlinear function

$$\begin{aligned} &f(x(t), \xi(t)) \\ &= \begin{bmatrix} 0.2 & 0.3 & 0.3 \end{bmatrix}^{\mathrm{T}} \big[0.3\mathrm{sign}(x_1(t))x_1(t)\xi_1(t) \\ &\quad + 0.4\mathrm{sign}(x_2(t))x_2(t)\xi_2(t) + 0.5\mathrm{sign}(x_3(t))x_3(t)\xi_3(t)\big], \end{aligned} \tag{47}$$

where $x_i(t)$, $i = 1, 2, 3$ stand for the ith element of the system state $x(t)$. $\xi_i(t)$, $i = 1, 2, 3$ are uncorrelated Gaussian white noises with mean zero and variance one. It is easy to verify that the stochastic nonlinear function (47) satisfies (4) with

$$\begin{aligned} &\mathrm{E}\left[f(x(t), \xi(t))f^{\mathrm{T}}(x(t), \xi(t))|x(t)\right] \\ &= \begin{bmatrix} 0.2 \\ 0.3 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.3 \\ 0.5 \end{bmatrix}^{\mathrm{T}} x^{\mathrm{T}}(t)\mathrm{diag}\begin{bmatrix} 0.09 & 0.16 & 0.25 \end{bmatrix} x(t). \end{aligned}$$

The measurement equations satisfy (5) with $H_i = [23.738\ \ 20.287\ 0]$, $i = 1, 2, 4$ and $H_i = \begin{bmatrix} 0 & 20 & 23 \end{bmatrix}$, $i = 3, 5$. The measurement noises $v_i(t)$ of mean zero and variances $R_i = 0.1i$, $i = 1, 2, \ldots, 5$ is uncorrelated with the process noise $w(t)$ of mean zero and variance one. By setting the mean values $\bar{\gamma}_{il}$ of the random variable $\gamma_{il}(t)$, $l \in N_i$, i.e., the data reception probabilities, random packet dropouts among sensors and their neighbor nodes can be described in simulation, where the mean values $\bar{\gamma}_{ij}$, $i, j = 1, 2, \ldots, 5$ satisfy

$$(\bar{\gamma}_{ij})_{5\times5} = \begin{pmatrix} 0 & 0.78 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0.83 & 0 \\ 0 & 0.75 & 0 & 1 & 0.85 \\ 0.95 & 0 & 0.75 & 0 & 0.88 \\ 0.76 & 1 & 0.92 & 0.86 & 0 \end{pmatrix}, \tag{48}$$

and we can see from (48) that the data reception probabilities between sensor 3 and its three neighbor nodes are $\bar{\gamma}_{32} = 0.75$, $\bar{\gamma}_{34} = 1$, and $\bar{\gamma}_{35} = 0.85$, respectively. It should be noted that the value '0' means that there is no connection between a sensor and itself or other sensors, the value '1' means that a sensor can fully receive data from its neighbor nodes, and other values represent the data reception probabilities, which means that there are missing data between a sensor and its neighbor nodes. The probability density functions $p_i(s)$ over the interval $[0,\ 1]$ for $\rho_i(t)$, $i = 1, 2, \ldots, 5$ satisfy

$$p_1(s) = \begin{cases} 0.05, & s=0 \\ 0.10, & s=0.5 \\ 0.85, & s=1 \end{cases},\ p_2(s) = \begin{cases} 0.05, & s=0 \\ 0.10, & s=0.2 \\ 0.35, & s=0.6 \\ 0.50, & s=1 \end{cases},$$

$$p_3(s) = \begin{cases} 0.05, & s=0 \\ 0.15, & s=0.5 \\ 0.80, & s=1 \end{cases},\ p_4(s) = \begin{cases} 0.05, & s=0 \\ 0.10, & s=0.3 \\ 0.35, & s=0.7 \\ 0.50, & s=1 \end{cases},$$

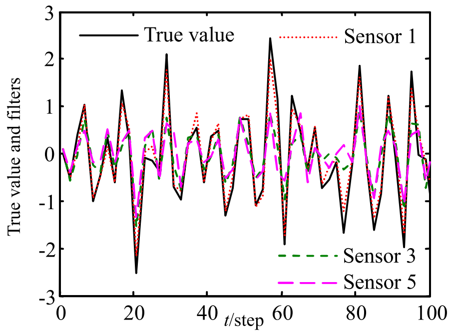
$$p_5(s) = \begin{cases} 0.05, & s=0 \\ 0.10, & s=0.2 \\ 0.20, & s=0.5 \\ 0.30, & s=0.8 \\ 0.35, & s=1 \end{cases},$$

with the expectations and variances can easily be calculated as $\bar{\rho}_1 = 0.9$, $\bar{\rho}_2 = 0.73$, $\bar{\rho}_3 = 0.875$, $\bar{\rho}_4 = 0.775$, $\bar{\rho}_5 = 0.71$, $\sigma_1^2 = 0.065$, $\sigma_2^2 = 0.0971$, $\sigma_3^2 = 0.0719$, $\sigma_4^2 = 0.0799$, and $\sigma_5^2 = 0.0919$, respectively. The initial values are $x(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ and $P_0 = 0.01I_3$. The 100 sampling data are used in simulation.

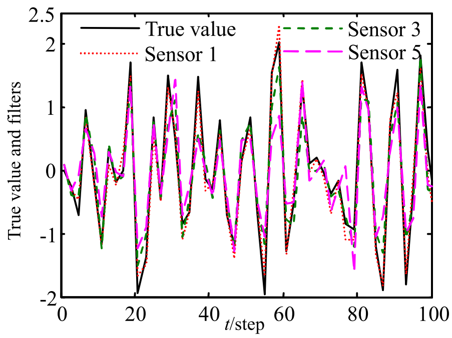
Firstly, according to the tracking performance of proposed DKF algorithm shown in Figure 2, we can see that sensors 1, 3, and 5 can track the real trajectory very well in the network shown in Figure 1, which shows that the proposed DKF algorithm is effective.

Secondly, from Figure 3, it can be seen that the mean square errors (MSEs) of all sensors are different, which means that the estimation accuracy of different sensors is different, mainly because each sensor has different simulation parameters. However, the MSEs of all sensors are bounded, which further verifies the correctness of Theorem 3. Note that the MSE can be defined by

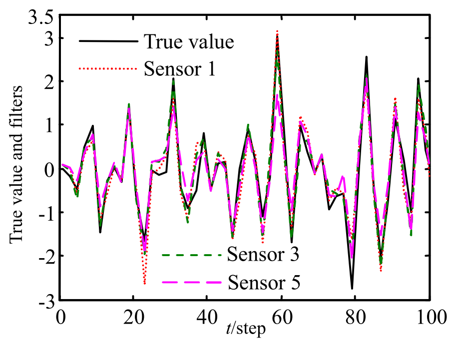
$$\mathrm{MSE}_{i,b}(t) = \frac{1}{M}\sum_{k=1}^{M}\left(x_b^{(k)}(t) - \hat{x}_{i,b}^{(k)}(t|t)\right)^2, \tag{49}$$



(a) The first state component.

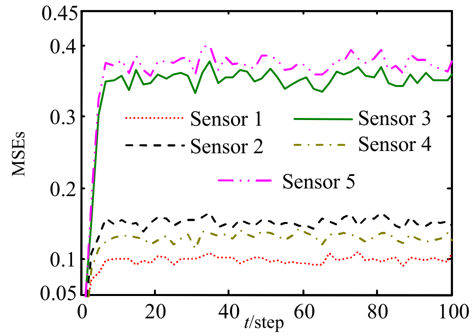


(b) The second state component.

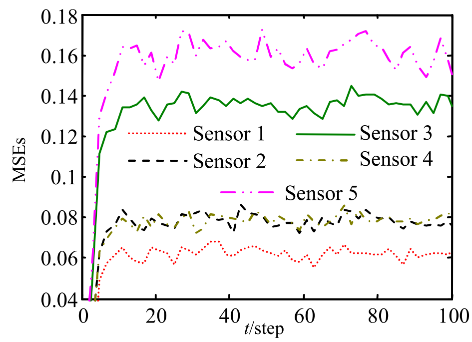


(c) The third state component.

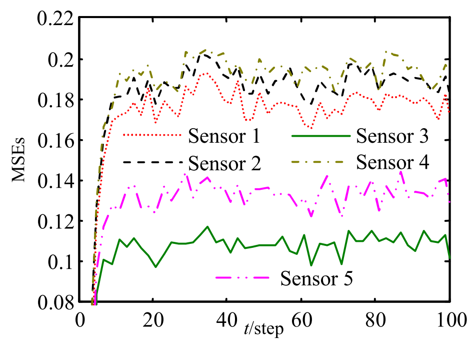
FIGURE 2. Tracking performance of DKFs of sensors 1, 3, and 5 in the directed topology.



(a) The first state component.



(b) The second state component.



(c) The third state component.

FIGURE 3. Comparisons of MSEs for DKFs of all sensors in the directed topology.

where i represents sensor node, $b = 1, 2, 3$ represent state components, M represents the number of Monte-Carlo tests and $M = 3000$ in simulation.

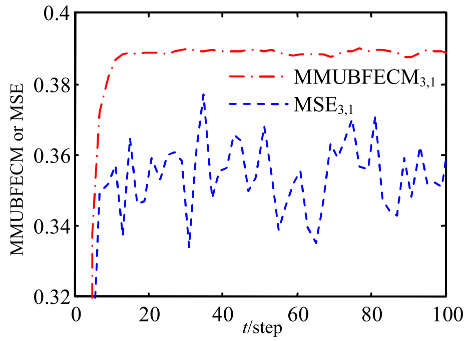
Thirdly, we compare the mean of minimum UBFECM (MMUBFECM) with the MSE for filter of sensor 3, where the MMUBFECM is computed by

$$\text{MMUBFECM}_{i,b}(t) = \frac{1}{M}\sum_{k=1}^{M} P_{i,b}^{mu(k)}(t|t),$$

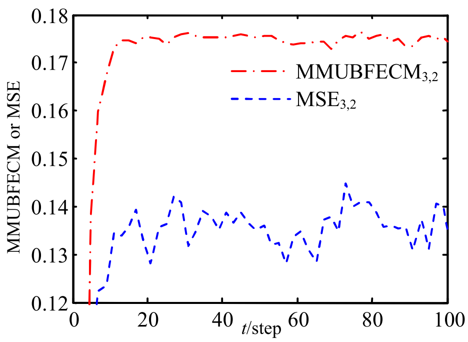
where the definitions of i, b, and M are the same as that of (49).

It follows from Figure 4 that the MSEs are almost below the MMUBFECMs for filter of sensor 3 in the directed topology, which indicates that the correctness of Theorems 1-2 and Remark 5.

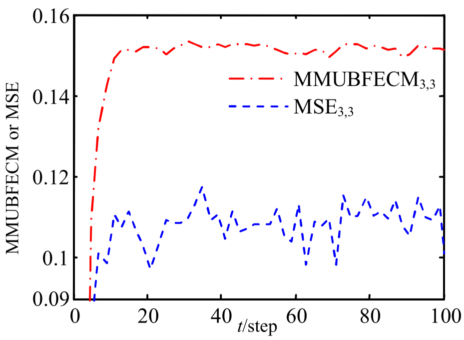
Fourthly, the effect of different fading measurements on the performance of the proposed DKF is studied. For sensor 3, three kinds of DKFs with different fading measurements are compared. One is the DKF with the mean $\bar{\rho}_3 = 0.875$ in Theorems 1-2 and Remark 5, the other is DKF_FM whose probability density function for $\rho_3(t)$ satisfies $p_3(0) = 0.85$, $p_3(0.5) = 0.1$, and $p_3(1) = 0.05$, where the mean $\bar{\rho}_3 = 0.1$ and the variance $\sigma_3^2 = 0.065$, the last one is DKF_WFM with the mean $\bar{\rho}_3 = 1$, which is the proposed DKF without fading measurements. Note that other parameters remain the same as above. The simulation results are given in Figure 5. From Figure 5, we can see that the



(a) The first state component.

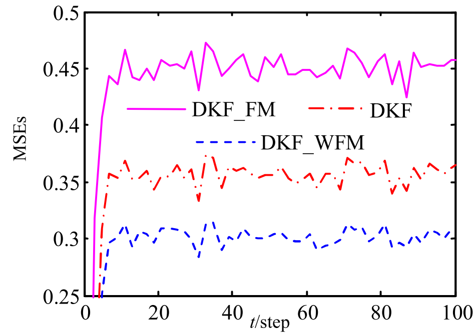


(b) The second state component.

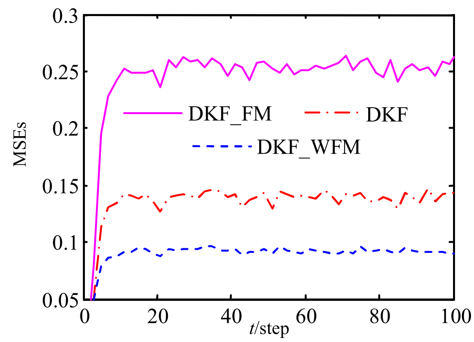


(c) The third state component.

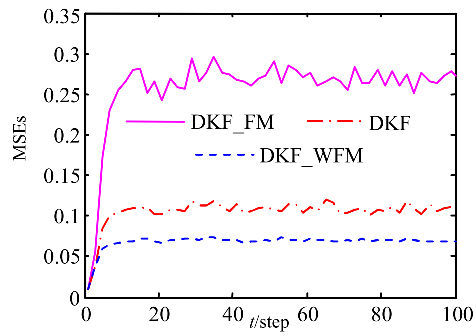
FIGURE 4. Comparisons of MSEs and MMUBFECMs for DKF of sensor 3 in the directed topology.



(a) The first state component.



(b) The second state component.



(c) The third state component.

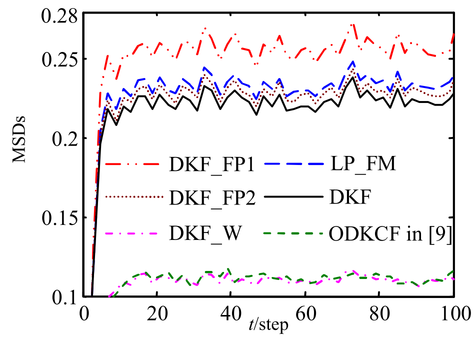
FIGURE 5. Comparisons of MSEs for DKFs of sensor 3 with different fading measurements in the directed topology.

performance of DKF is better than that of DKF_FM and worse than that of DKF_WFM, which demonstrates that the estimation accuracy becomes worse as the mean $\bar{\rho}_3$ of fading measurements decreases.

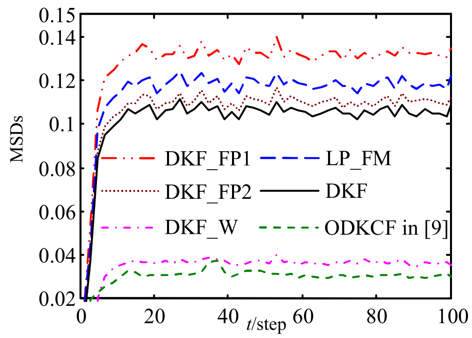
Finally, the performance of the following six filter algorithms will be compared: DKFs in Theorems 1-2 with fixed parameters, i.e., $\theta_i(t)$, $\theta_{il}(t)$, $\varphi_i(t)$, and $\pi_{il}(t)$, $l \in N_i$ are all 6 (DKF_FP1) or 1.8 (DKF_FP2); local filter with fading measurements in Theorem 4 (LF_FM); DKF in Theorems 1-2 and Remark 5; optimal distributed Kalman consensus filter (ODKCF) with one common optimal consensus filter gain and the FECCM in [9]; the proposed DKF without fading measurements and random packet dropouts (DKF_W). Note that ODKCF does not consider missing data, and requires the recursive calculation of the state second moment matrix. Other simulation parameters remain unchanged.

To compare estimation accuracy of different algorithms, the mean square deviation (MSD) for all sensors is used in simulation, i.e., $\mathrm{MSD}_b(t) = \frac{1}{L}\sum_{i=1}^{L} \mathrm{MSE}_{i,b}(t)$, where b and $\mathrm{MSE}_{i,b}(t)$ are defined in (49).

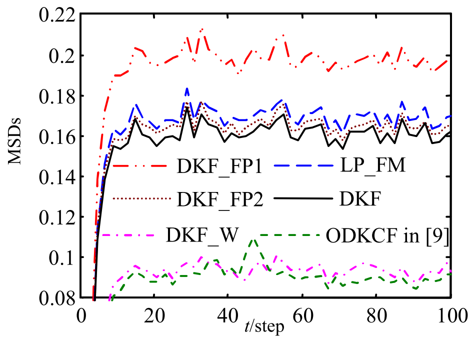
The simulation results are shown in Figure 6. From Figure 6, we can see that the performance of DKF outperforms that of LF_FM, which verifies the correctness of Theorems 1-4 and Remark 5. Especially, the performance of DKF_W is close to that of ODKCF, which further demonstrates the effectiveness of the proposed DKF with optimal multiple consensus filter gains and optimal scalar parameters. In addition, we can see that the performance of



(a) The first state component.



(b) The second state component.



(c) The third state component.

FIGURE 6. Comparisons of MSDs for different filter algorithms in the directed topology.

LF_FM outperforms that of DKF_FP1 and underperforms that of DKF_FP2, but the performance of both DKF_FP1 and DKF_FP2 is worse than that of DKF, which shows that inappropriate scalar parameters may lead to a decrease in estimation accuracy of DKF.

VI. CONCLUSION

Considering NMSSs with stochastic nonlinearities, fading measurements, and random packet dropouts, a novel DKF dependent on the probabilities of data loss has been presented, where multiple filter gain matrices are designed to improve estimation accuracy. For missing estimates and missing covariance matrices of neighbor nodes, the zero-input and the hold-input compensation mechanisms are used to update the state estimates of filter. Optimal Kalman filter gain, optimal different consensus filter gains, and optimal scalar parameters are obtained by minimizing locally an UBFECM and using a nonlinear optimization method. Moreover, it has proved that the filtering error of DKF has the exponential boundedness in mean square and the estimation accuracy of DKF outperforms that of local filter with fading measurements. The proposed DKF has low calculation and communication cost owing to avoiding the FECCM and the state second moment matrix. An UPS is used as a simulation example to illustrate the effectiveness of the presented DKF algorithm.

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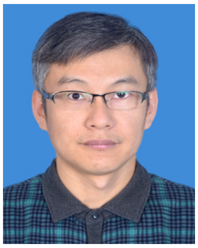

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