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RESEARCH ARTICLE

Data Source Selection for Integration in Data Sciences via Complex Hesitant Fuzzy Rough Multi-Attribute Decision-Making Method

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ABSTRACT Data sources are the raw information that is required for analysis and modeling in data sciences. They assist data scientists in making proper conclusions, proving hypotheses, and making rational decisions. It is always preferable if analytical results can be obtained from multiple reliable sources. Thus, it is essential to assess such data sources in data sciences about various critical characteristics and factors. Nevertheless, the application of all-inclusive multi-attribute decision-making methodologies for the selection of data sources for integration has not received adequate attention in the existing literature and research. Thus, this article explains a new multi-attribute decision-making method through the model of the complex hesitant fuzzy rough set, which is the complex hesitant fuzzy rough multi-attribute decision-making method. This methodology would handle the evaluated values of attributes that have uncertainty, hesitancy, and roughness altogether. Besides, this study introduces several properties of complex hesitant fuzzy rough sets and develops several aggregation operators in the framework of complex hesitant fuzzy rough set and their properties. Subsequently, a case study of data source selection in data science is explained to explain the relevance of the developed multi-attribute decision-making framework in data sciences. Finally, the comparison of the devised theory with prevailing theories is interpreted.

INDEX TERMS Data sciences, data source, complex hesitant fuzzy set, multi-attribute decision-making methodology.

I. INTRODUCTION

Data integration is also a significant process in data science since it determines the quality and accuracy of analytics results. It is crucial to select the proper data source for the integration; These sources can be raw data from social media or IoT devices, and organized databases from various industries. In the integration procedure, every source has its potential and threats that are unlike the other sources. Because they are a form of SQL-based question languages, conventional knowledge assets which include relational databases and spreadsheets are frequently easier to merge while there are large datasets or data from a couple of assets disparate values are integrating them is challenging. In such cases, the data pretreatment and normalization strategies are crucial to maintain uniformity and precision in the consolidated dataset. It is not uncommon to encounter barriers to unstructured or semi-established data assets, which are text, pictures, and video. These sources uncover other superior methods such as sign processing, picture processing, and language processing. Moreover, the absence of a default schema for unstructured records assets often poses challenges to integration and requires additional measures for data transformation and data cleaning.

External data sources alongside open datasets or third-party APIs may be hard to incorporate. Even though these websites are considered to be records, there can be limitations to access, poor quality data, and untrustworthy. To make sure that outside records are suitable for evaluation and to reduce the possibility of bias or mistakes,

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strict procedures of validation and integrity need to be followed in its integration. In recent years, there has also been an increased use of warehouses To ensure data quality and uniformity in such environments, records integration often includes information feed pipelines, data cleansing methods, and data management procedures. The theoretical perspective of data integration was studied by Lenzerini [1]. The teenage years of data integration were investigated by Halevy et al. [2]. In 2009, Dong et al. [3] analyzed the uncertainty and ambiguity of data integration. Voukelatou et al. [4] discussed the objective measuring and well-being of subjective related data sources and Haas et al. [5] analyzed a system for data integrated within data sources. The challenges and issues in biological data sources were devised by Davidson et al. [6]. Zipkin et al. [7] addressed issues related to data integration.

A mathematical assembly that goes beyond the traditional concept of a set is referred to as a fuzzy set (FS). A detail in classical set idea is unambiguously either a factor of a set or now not. Representing ambiguity or vagueness, but, is probably useful whilst dealing with subjective or faulty information. The solution to this issue is FS theory, invented by Zadeh [8] in 1965. Elements aren't actually in or out of a set; rather, they could belong to a set to a point according to FS's idea. A detail's degree of membership is indicated through quite a number between 0 and 1, wherein zero denotes the overall absence of membership inside the set, 1 denotes complete membership, and numbers in among indicate exceptional degrees of partial membership. The core idea of FS theory is the degree of membership. Every detail in the commonplace set is given a membership value that represents how much the detail belongs to the FS. This makes it possible to describe obscure or nebulous thoughts, like "tall" or "hot," in which membership is a continuum instead of a binary categorization. Abdulghafour et al. [9] investigated data integration based on fuzzy logic. The role of fuzzy decision-making (DM) approaches in data integration was devised by Wang et al. [10]. A modification of the FS concept, a hesitant FS (HFS) [11] offers circumstances wherein there could be doubt or reluctance concerning an element's membership in a set. Instead of genuinely one membership value, an HFS assigns many membership values to signify diverse feasible ranges of membership. HFSs are designed to symbolize the uncertainty or reluctance that arises all through DM processes when the element or object may be placed to a set in various degrees of membership and may not have a single, definitive value of membership. Decision-makers are better capable of expressing their hesitancy or doubt as a result. A complex fuzzy set (CFS) is the modification of typical FS, first devised by Ramot et al. [12], where they transformed the unit interval to a unit disc in a complex plane. This structure is in the model of the polar form of a complex number. After a few years, another model of CFS was deduced by Tamir et al. [13] and that structure is in the Cartesian framework of complex numbers and the range is a unit square instead of a unit disc. Based on these ideas, in 2021, Mahmood et al. [14] devised complex hesitant FS (CHFS) by considering the polar form of a CFS. Luqman et al. [15] devised the theory of hypergraphs within complex fuzzy information.

A. RESEARCH PROBLEM AND MOTIVATION

In today's society, the importance of data as a source for integration in data sciences cannot be overstated. A significant amount of diverse data has been produced via the digitalization of many areas of our lives, consisting of commercial enterprise, healthcare, finance, and different fields. For data scientists and analysts, this wealth of data is important because it allows them to get important insights, make smart judgments, and create predictive models. A complete knowledge of complex events is made feasible by the integration of data from numerous resources, which captures the linkages and interdependencies among and inside distinct datasets. Additionally, the incorporation of several data resources improves the robustness and dependability of evaluation, main to extra unique forecasts and useful outcomes. The capacity to seamlessly integrate data from diverse resources is critical for unleashing the whole ability of data sciences in addressing actual global problems and fostering innovation, in particular for companies seeking to benefit from a competitive area and make records-pushed picks. Further, to offer seamless collaboration and evaluation throughout numerous data resources, the records integration marketplace plays a critical role. The market's growth indicates how important it is to use unified data to make educated decisions and obtain a competitive advantage in the contemporary digital landscape. To achieve the intended result, the American English language with its vocabulary and expressions should be utilized. The market size of data integration from 2022 to 2032 is devised in Figure 1.

That's why these days, data integration is more important than ever, therefore choosing the right data sources is essential. Choosing a data source is a multi-attribute decision-making(MADM) problem that involves various attributes. Previous research has evaluated and chosen data sources using various MADM methods. Unfortunately, none of these methods take into account the hesitancy, ambiguity, or roughness of evaluation values, which leads to imprecise information and incomplete data. A MADM approach that can take these considerations into account is therefore desperately needed. As of right now, no mathematical framework or tool exists in the literature that can describe roughness, hesitation, and uncertainty all at once. For processing such data, conventional models such as RS, FS, FRS, HFS, HFRS, CFS, and the polar model of CHFS are inadequate. In this paper, we suggest a solution to this problem by including uncertainty, hesitation, and roughness in the MADM framework.

B. CONTRIBUTION

In light of the above research problem and motivation, this article interprets a novel method of MADM for the assessment and selection of data sources for integration in

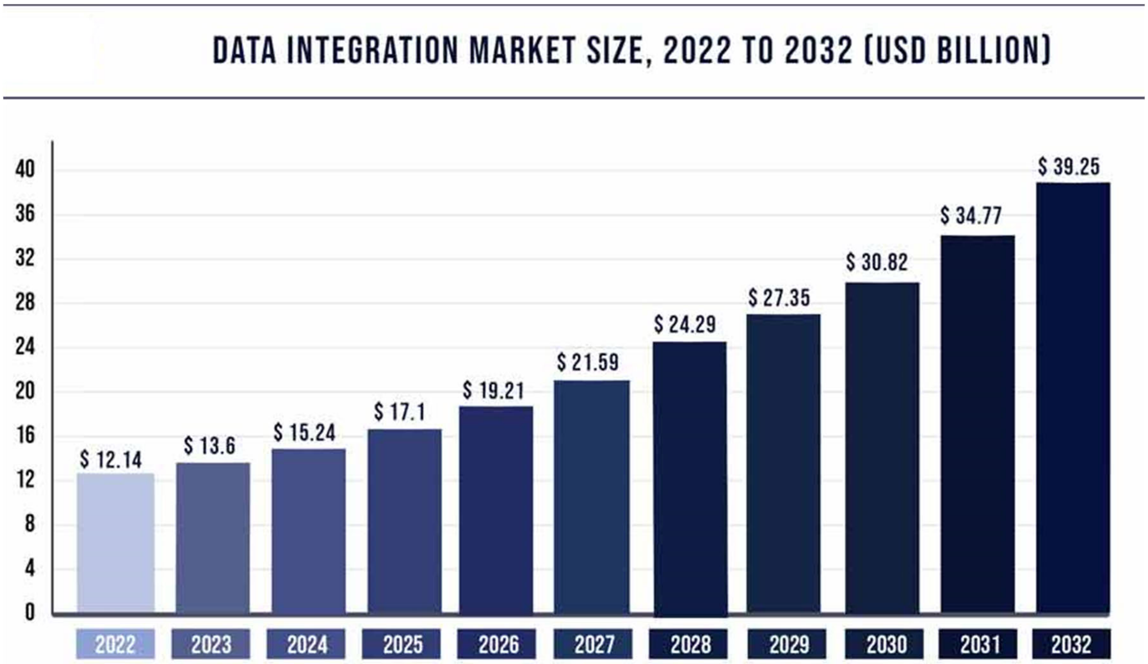

FIGURE 1. The market size of data integration.

data science. This MADM method deals with information that is roughness, uncertainty, and hesitancy and extra fuzzy information separately or simultaneously For this MADM method, this article first explains a new mathematical model complex hesitant fuzzy rough set (CHFRS) which is the modification of rough set, fuzzy set, fuzzy rough set, hesitant fuzzy set, hesitant fuzzy rough set, complex fuzzy set, and polar form of complex hesitant fuzzy set and deal with uncertainty, roughness and hesitancy and extra fuzzy information at once. The main contributions of this article are interpreted as follows

- To address the MADM dilemma of data source for integration in data science.
- To interpret a novel mathematical model with the name of CHFRS.
- To devise basic operational laws that complement, union, and intersection for the newly devised CHFRS.
- To deduce algebraic operational laws and score and accuracy function for CHFRS.
- To establish certain AOs along with their associated properties within the structure of CHFRS to aggregate complex hesitant fuzzy rough information.
- To illustrate a numerical example related to the data source selection for integration in data science to reveal the applicability of the proposed theory and MADM method in data sciences.
- To compare the devised work with certain prevailing theories to reveal the advantages and supremacy of the initiated theory.

C. CORE NOVELTY

Based on the above contribution, we have the underlying core novelty of this article.

- Construction of a novel mathematical structure "CHFRS" along with its basic operations.
- Construction of averaging and geometric AOs in the framework of CHFRS.
- Construction of MADM method for coping with MADM dilemma, particularly, the dilemma related to data source selection in data sciences.

D. CONSTRUCTION OF ARTICLE

In section, II we discuss, MADM and fuzzy MADM techniques. In section III, we state a problem that we need to handle in this script. In section IV, we review some prevailing concepts. In section V, we devise a novel concept of CHFRS and for that, we deduce CHFS in Cartesian form and complex hesitant fuzzy relation. We also analyze various operators of CHFS and CHFRS in section V. Further, in section V, we anticipate average/geometric AOs within the structure of CHFRS. In section VI, we anticipate an approach of MADM within CHFRS by utilizing the invented AOs and solving the considered problem in this section. Section VII contains the comparative analysis and section VIII contains the concluding remarks of this article.

II. LITERATURE REVIEW

Here, we discuss MADM and fuzzy MADM techniques.

A. MADM METHOD

MADM is a systematic procedure of evaluating and ranking the options with the help of many attributes or qualities. It is applied in many fields such as environmental science, business, engineering, and economics to decide on problems with multiple objectives or criteria. MADM methods help decision-makers to make quantitative assessments on options by attributes, which makes more objective and informed decisions essential to MADM's characteristic of providing a systematic framework for decision-making when it should take into consideration many factors. Such an environment required a more sophisticated approach to decision-making than the simple subjective evaluation or ranking of the options. Decision-makers can use MADM methods to analyze numerous trait traits in the 19th century. For instance, when choosing a supplier in the business world, the decision makers may be required to weigh such factors as price, quality, delivery time, etc. By applying MADM techniques, the decision-makers can evaluate the significance of each factor and come up with methods of evaluating the efficiency of the general board performance. Moreover, the weights assigned to the available resources are also clear in the MADM, which makes the decision-making process more transparent and traceable Transparency has two advantages: increased accountability and the internal reduction of bias or influence of opinion. Thus, MADM methods can also assist stakeholders with conflicting viewpoints and objectives to find a common ground Since MADM methods enable the comparison of various strategies and the evaluation of the problem from various angles, it fosters stakeholder cooperation and the establishment of shared goals and objectives. An issue relying on the selection of MADM was devised by Fishburn [16] and the theory of multi-attribute decision was investigated by Roldan et al. [17]. Aruldoss et al. [18] devised the MADM approach and its applications. For data integration, the multi-criteria DM was analyzed by Peng et al. [19].

B. FUZZY MADM METHOD

Fuzzy MADM is a decision-making method that incorporates fuzzy logic concepts into traditional MADM systems. When making decisions involving multiple criteria or attributes, each with different importance and ambiguity, this process can be used Fuzzy logic effectively handles uncertainty and uncertainty by membership maximum allowed, instead of using absolute mathematical values to express preferences and decision-makers, in a fuzzy MADM, decision makers can use linguistic terms. This is especially helpful in real complex situations when it may be difficult to quantify parameters or uncertainty in the decision process Fuzzy MADM methods handle this uncertainty and accuracy through fuzzy sets and fuzzy logic operations, providing decision-makers with flexible and useful tools Able to represent and manage existing ambiguities and uncertainties. Fuzzy MADM provides an approach to these challenges where decision-makers in industries such as engineering, finance, environmental management, and health are often confronted with multiple mutually exclusive value counts and inaccuracies and draw well-informed, uncertain conclusions. Furthermore, the use of fuzzy MADM methods gives decision-makers the advantage of including qualitative factors in decision-making that traditional quantitative methods may miss Fuzzy MADM methods make decisions more transparent and easier for decision-makers allowing them to communicate their priorities and research in natural language. Better stakeholder contribution, larger acceptance of choices, and eventually more successful strategy implementation can come from this. In 1979, Efstathiou [20] devised MADM by employing the theory of FS in his doctoral dissertation. Chen and Hwang [21] devised various fuzzy MADM approaches and Tzeng and Huang [22] discussed various applications.

III. PROBLEM STATEMENT

In the field of data sciences, the relevance, completeness, and accuracy of analyses and models depend on the choice of the best data sources to integrate. When presented with several possible data sources, each with different attributes, the MADM dilemma occurs. The goal is to create a framework for making decisions that efficiently assesses and ranks these data sources according to their attributes, finally determining the best combination that maximizes the integrated dataset's overall performance or utility for the intended analytical or modeling uses. Choosing a reliable and well-informed data source requires managing trade-offs between competing attributes, handling uncertainties, and taking stakeholder preferences into account. In this problem, the attributes and alternatives are described in Tables 1 and 2 respectively.

IV. PRELIMINARIES

Here, we revise the basic notion of HFS, CFS, and RS.

Tora [11] devised the notion of HFS, which is revealed as follows

Definition 1 [11]: The HFS over $\mathcal{Z}$ is a mapping $\mu_{\mathcal{T}_{HFS}} : \mathcal{Z} \to \mathbb{P}[0, 1]$, where $\mathbb{P}[0, 1]$ is a power set of $[0, 1]$.

After that, Alcantud and Torra [30] devised the theory of uniform HFS as A uniform HFS over $\mathcal{Z}$ would be devised as

$$\mathcal{T}_{HFS} = \left\{\sigma, \left(\mu_{\mathcal{T}_{HFS-j}}(\sigma)\right) : \sigma \in \mathcal{Z}, j = 1, 2, \ldots, n\right\}$$

Noted that $\mu_{\mathcal{T}_{HFS}}(\sigma)$ is a degree of membership of $\sigma \in \mathcal{Z}$.

Definition 2 [13]: The CFS would be devised as

$$\mathcal{T}_{CFS} = \left\{\sigma, \left(\mu^R_{\mathcal{T}_{CFS}}(\sigma) + \iota\mu^I_{\mathcal{T}_{CFS}}(\sigma)\right) : \sigma \in \mathcal{Z}\right\}$$

Noted that $\mu^R_{\mathcal{T}_{CFS}}(\sigma)$ and $\mu^I_{\mathcal{T}_{CFS}}(\sigma)$ signifies the degrees of membership and non-membership lie in a unit square of a complex plane.

A relation $E_{ER} \subseteq \mathcal{Z} \times \mathcal{Z}$ is devised as an equivalence relation if it is reflexive, symmetric, and transitive. An equivalence class concerning α is devised as

$$[\alpha] = \{\beta \in \mathcal{Z} : \beta \text{ is related to } \alpha\}$$

Definition 3 [29]: Let $E_{ER} \subseteq \mathcal{Z} \times \mathcal{Z}$ is an equivalence relation on $\mathcal{Z}$. Then $(\mathcal{Z}, E_{CHFR})$ would develop an approximation space. A $\emptyset \neq \mathcal{T} \subset \mathcal{Z}$ will be interpreted as definable if $\mathcal{T}$ can be written in the union of a few equivalence classes of $\mathcal{Z}$. If this is not the case, then $\mathcal{T}$ will be interpreted as not

TABLE 1. The attributes and their explanation.

ID of attributes	Name of attributes	Explanation
$\mathcal{A}_{art-1}$	Data Quality	This attribute assesses the dependability, precision, and comprehensiveness of the information supplied by every source. Improved data quality guarantees more reliable analysis and results in decision-making processes that rely on data.
$\mathcal{A}_{art-2}$	Data Relevance	Evaluating how well each source's data matches the particular goals or specifications of the study. Sources of pertinent data immediately contribute to the insights that are sought after, increasing the efficiency of further data processing and analysis.
$\mathcal{A}_{art-4}$	Data accessibility	Analyzing each source's availability and ease of access to data. Data integration timeliness and feasibility can be greatly impacted by accessibility issues, which include license agreements, possible constraints, and data retrieval procedures.
$\mathcal{A}_{art-5}$	Data Volume	Measuring the amount or quantity of information provided by every source. Greater amounts of data might present issues with processing, storage, and computing resource requirements, but they might also provide more thorough insights and statistical significance.

TABLE 2. The alternatives and their explanation.

ID of alternatives	Name of alternatives	Explanation
$\mathcal{X}_{alt-1}$	Internal Database	Information from the internal databases of a company can provide familiarity as well as perhaps excellent relevance, quality, and accessibility. Its reach, nevertheless, could be constrained in contrast to outside sources.
$\mathcal{X}_{alt-2}$	Third-Party Data Provider	Outside suppliers offer customized datasets that are pertinent to the investigation and may be of excellent quality. Cost and accessibility factors could differ.
$\mathcal{X}_{alt-3}$	Publicly Accessible Dataset	Publicly accessible datasets, also known as open data repositories, are frequently large and varied, offering insightful information at a reasonable cost. However, there may be differences in the relevance and quality of the data, and license and usage limitations may apply to accessibility.
$\mathcal{X}_{alt-4}$	Sensor Network	High amounts of timely data are available via real-time data streams gathered from IoT devices or sensor networks. However, due to proprietary systems or network problems, accessing may be difficult and data quality and relevancy may change over time.

definable. Then, $\mathfrak{T}$ will be written in definable subsets devised as lower and upper approximations.

$$\mathrm{E}_{ER}^{L}(\mathfrak{T}) = \left\{\left(\sigma \in \mathcal{Z} : [\sigma]_{E_{ER}} \subseteq A\right)\right\}$$
$$\mathrm{E}_{ER}^{U}(\mathfrak{T}) = \left\{\left(\mathfrak{T} \in \mathcal{Z} : \ [\sigma]_{E_{ER}} \cap A \neq \emptyset\right)\right\}$$

Then, the set $\left(\mathrm{E}_{ER}^{L}(\mathfrak{T}), \mathrm{E}_{ER}^{U}(\mathfrak{T})\right)$ would be devised as RS, where $\mathrm{E}_{ER}^{L}(\mathfrak{T}) \neq \mathrm{E}_{ER}^{U}(\mathfrak{T})$

V. COMPLEX HESITANT FUZZY ROUGHS SET

Here, we devise a novel concept of CHFRS and for that, we deduce CHFS in Cartesian form and complex hesitant fuzzy relation. We also analyze various operators of CHFS and CHFRS. Further, here, we anticipate average/geometric AOs within the structure of CHFRS. $\mathcal{Z}$ would be utilized as a universal set.

Definition 4: The CHFS would be devised as

$$\mathfrak{T}_{CHFS} = \left\{\sigma, \left(\mu_{\mathfrak{T}_{CHFS}}(\sigma)\right) : \sigma \in \mathcal{Z}\right\}$$
$$= \left\{\sigma, \left(\mu^{R}_{\mathfrak{T}_{CHFS}}(\sigma) + \iota\mu^{I}_{\mathfrak{T}_{CHFS}}(\sigma)\right) : \sigma \in \mathcal{Z}\right\}$$

Noted that, $\mu_{\mathfrak{T}_{CHFS}}(\sigma) = \mu^{R}_{\mathfrak{T}_{CHFS}}(\sigma) + \iota\mu^{I}_{\mathfrak{T}_{CHFS}}(\sigma)$ is a set of values in a complex plane's unit square that is $\mu_{\mathfrak{T}_{CHFS}}(\sigma) = \mu^R_{\mathfrak{T}_{CHFS}}(\sigma) + \iota\mu^I_{\mathfrak{T}_{CHFS}}(\sigma) = \left\{\mu^R_{\mathfrak{T}_{CHFS-j}}(\sigma) + \iota\mu^I_{\mathfrak{T}_{CHFS-j}}(\sigma), j = 1, 2, 3, .., n\right\}$, identifying the degree of membership of an element $\sigma \in \mathcal{Z}$ and $\mu^R_{\mathfrak{T}_{CHFS}}(\sigma)$ and $\mu^I_{\mathfrak{T}_{CHFS}}(\sigma)$ are real and unreal parts of the degrees of membership. $\mathfrak{T}_{CHFS} = \left(\mu_{\mathfrak{T}_{CHFS}}\right) = \left(\mu^R_{\mathfrak{T}_{CHFS}} + \iota\mu^I_{\mathfrak{T}_{CHFS}}\right)$ will be imagined as a CHF element (CHFE).

Also, the CHFS can be devised as

$$\mathfrak{T} = \left\{\sigma, \left(\mu_{\mathfrak{T}_j}(\sigma)\right) : \sigma \in \mathcal{Z}, j = 1, 2, \ldots, n\right\}$$
$$= \left\{\sigma, \left(\mu^R_{\mathfrak{T}_j}(\sigma) + \iota\mu^I_{\mathfrak{T}_j}(\sigma)\right) : \sigma \in \mathcal{Z}, j = 1, 2, \ldots, n\right\}$$

Noted that, $\mu_{\mathfrak{T}_j}(\sigma)$ is a degree of belongingness which contains real $\mu^R_{\mathfrak{T}_j}(\sigma)$ and unreal $\mu^I_{\mathfrak{T}_j}(\sigma)$ terms for every $\sigma \in \mathcal{Z}$. The $\mu_{\mathfrak{T}_j}(\sigma)$ is located in a complex plane's unit disc that is $\mu_{\mathfrak{T}_j} : \mathcal{Z} \to [0, 1] + \iota\ [0, 1]$. The gathering of all CHFS would be identified by *CHF* $(\mathcal{Z})$. The second definition can also be stated as a complex uniform hesitant fuzzy set.

Definition 5: Suppose $\mathfrak{T}_{CHFS-1} = \left(\mu_{\mathfrak{T}_{CHFS-1}}\right) = \left(\mu^R_{\mathfrak{T}_{CHFS-1}} + \iota\mu^I_{\mathfrak{T}_{CHFS-1}}\right)$, and $\mathfrak{T}_{CHFS-2} = \left(\mu_{\mathfrak{T}_{CHFS-2}}\right) = \left(\mu^R_{\mathfrak{T}_{CHFS-2}} + \iota\mu^I_{\mathfrak{T}_{CHFS-2}}\right)$ be two CHFEs, then

1. The complement of $\mathfrak{T}_{CHFS-1}$ is identified and deduced as

$$(\mathfrak{T}_{CHFS-1})^c = \left(\mu_{\mathfrak{T}_{CHFS-1}}\right)^c = \left(\bigcup_{y\ \in\ \mu_{CHFS-1}} \left\{\left(1 - y^R\right) + \iota\left(1 - y^I\right)\right\}\right)$$

2. The union of $\mathfrak{T}_{CHFS-1}$ and $\mathfrak{T}_{CHFS-2}$ is identified and deduced as

$$\mathfrak{T}_{CHFS-1} \cup \mathfrak{T}_{CHFS-2} = \left(\bigcup_{y_1\in\mu_{CHFS-1}, y_2\in\mu_{CHFS-2}} \left\{\begin{matrix}\max\left(y^R_1, y^R_2\right) \\ +\iota\max\left(y^I_1, y^I_2\right)\end{matrix}\right\}\right)$$

3. The intersection of $\mathfrak{T}_{CHFS-1}$ and $\mathfrak{T}_{CHFS-2}$ is identified and deduced as

$$\mathfrak{T}_{CHFS-1} \cap \mathfrak{T}_{CHFS-2} = \left(\bigcup_{y_1\in\mu_{CHFS-1}, y_2\in\mu_{CHFS-2}} \left\{\begin{matrix}\min\left(y^R_1, y^R_2\right) \\ +\iota\min\left(y^I_1, y^I_2\right)\end{matrix}\right\}\right)$$

Definition 6: A CHF subset of $\mathcal{Z} \times \mathcal{Z}$ would be deduced as CHF relation (CHFR) E_{CHFR} over $\mathcal{Z}$. Mathematically,

$$E_{CHFR} = \left\{\left((\sigma_1, \sigma_2), \mu_{\mathfrak{T}_{CHFR}}(\sigma_1, \sigma_2)\right) : (\sigma_1, \sigma_2 \in \mathcal{Z} \times \mathcal{Z})\right\}$$
$$= \left\{\left((\sigma_1, \sigma_2), \left(\mu^R_{\mathfrak{T}_{CHFR}}(\sigma_1, \sigma_2) + \iota\mu^I_{\mathfrak{T}_{CHFR}}(\sigma_1, \sigma_2)\right)\right) : (\sigma_1, \sigma_2 \in \mathcal{Z} \times \mathcal{Z})\right\}$$

Noted that $\mu_{\mathfrak{T}_{CHFR}}(\sigma_1, \sigma_2) = \left(\mu^R_{\mathfrak{T}_{CHFR}}(\sigma_1, \sigma_2) + \iota\mu^I_{\mathfrak{T}_{CHFR}}(\sigma_1, \sigma_2)\right)$ is any subset of the complex plane's unit square which identifies the degree of belongingness of the association between σ_1 and σ_2.

Definition 7: Suppose E_{CHFR} is a CHFR over $\mathcal{Z}$ and $(\mathcal{Z}, E_{CHFR})$ is a complex hesitant fuzzy approximation space (CHFAS), then $\forall\ \mathfrak{T}_{CHFS} \in CHF(\mathcal{Z})$, the lower and upper approximation concerning $(\mathcal{Z}, E_{CHFR})$ is identified and devised as

$$E^L(\mathfrak{T}_{CHFS}) = \left\{\sigma, \left(\mu_{E^L(\mathfrak{T}_{CHFS})}(\sigma)\right) : \sigma \in \mathcal{Z}\right\}$$
$$E^U(\mathfrak{T}_{CHFS}) = \left\{\sigma, \left(\mu_{E^U(\mathfrak{T}_{CHFS})}(\sigma)\right) : \sigma \in \mathcal{Z}\right\}$$

Which are CHFSs. Where,

$$\mu_{E^L(\mathfrak{T}_{CHFS})}(\sigma) = \mu^R_{E^L(\mathfrak{T}_{CHFS})}(\sigma) + \iota\ \mu^I_{E^L(\mathfrak{T}_{CHFS})}(\sigma) = \bigwedge_{y\ \in\ \mathcal{Z}}\left\{\left(\mu^R_{CHFR}(\sigma, y)\right)^c \vee \mu^R_{CHFS}(y)\right\} + \iota\bigwedge_{y\ \in\ \mathcal{Z}}\left\{\left(\mu^I_{CHFR}(\sigma, y)\right)^c \vee \mu^I_{CHFS}(y)\right\}$$

$$\mu_{E^U(\mathfrak{T}_{CHFS})}(\sigma) = \mu^R_{E^U(\mathfrak{T}_{CHFS})}(\sigma) + \iota\ \mu^I_{E^U(\mathfrak{T}_{CHFS})}(\sigma) = \bigvee_{y\ \in\ \mathcal{Z}}\left\{\mu^R_{CHFR}(\sigma, y) \wedge \mu^R_{CHFS}(y)\right\} + \iota\bigvee_{y\ \in\ \mathcal{Z}}\left\{\mu^I_{CHFR}(\sigma, y) \wedge \mu^I_{CHFS}(y)\right\}$$

If $E^L(\mathfrak{T}_{CHFS}) \neq E^U(\mathfrak{T}_{CHFS})$, then the pair $E_{CHFRS} = \left(E^L(\mathfrak{T}_{CHFS}), E^U(\mathfrak{T}_{CHFS})\right)$ would devised as CHFRS concerning $(\mathcal{Z}, E_{CHFR})$. For easiness, the complex hesitant fuzzy rough element (CHFRE) would be identified as $E_{CHFRS} = \left(E^L(\mathfrak{T}_{CHFS}), E^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E^{LR} + \iota E^{LI}\right), \left(E^{UR} + \iota E^{UI}\right)\right)$, where $E^{LR} = \bigwedge_{y\ \in\ \mathcal{Z}}\left\{\left(\mu^R_{CHFR}(\sigma, y)\right)^c \vee \mu^R_{CHFS}(y)\right\}$, $E^{LI} = \bigwedge_{y\ \in\ \mathcal{Z}}\left\{\left(\mu^I_{CHFR}(\sigma, y)\right)^c \vee \mu^I_{CHFS}(y)\right\}$, $E^{UR} = \bigvee_{y\ \in\ \mathcal{Z}}\left\{\mu^R_{CHFR}(\sigma, y) \wedge \mu^R_{CHFS}(y)\right\}$, and $E^{UI} = \bigvee_{y\ \in\ \mathcal{Z}}\left\{\mu^I_{CHFR}(\sigma, y) \wedge \mu^I_{CHFS}(y)\right\}$.

Definition 8: The complement, union, and intersection among two CHFRE $E_{CHFRS-1} = \left(E^L_1(\mathfrak{T}_{CHFS}), E^U_1(\mathfrak{T}_{CHFS})\right) = \left(\left(E^{LR}_1 + \iota E^{LI}_1\right), \left(E^{UR}_1 + \iota E^{UI}_1\right)\right)$ and $E_{CHFRS-2} = \left(E^L_2(\mathfrak{T}_{CHFS}), E^U_2(\mathfrak{T}_{CHFS})\right) = \left(\left(E^{LR}_2 + \iota E^{LI}_2\right), \left(E^{UR}_2 + \iota E^{UI}_2\right)\right)$ will be anticipated as

1.

$$(E_{CHFRS-1})^c = \left(\begin{matrix}\left(\bigcup_{E^L\ \in\ E^L_1(\mathfrak{T}_{CHFS})}\left\{\begin{matrix}\left(1 - E^{LR}\right) \\ +\iota\left(1 - E^{LI}\right)\end{matrix}\right\}\right), \\ \left(\bigcup_{E^U\ \in\ E^U_1(\mathfrak{T}_{CHFS})}\left\{\begin{matrix}\left(1 - E^{UR}\right) \\ +\iota\left(1 - E^{UI}\right)\end{matrix}\right\}\right)\end{matrix}\right)$$

2.

$$E_{CHFRS-1} \cup E_{CHFRS-2} = \left(\begin{array}{l} \left(\bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS}), E_2^L \in E_2^L(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \max\left(E_1^{LR}, E_2^{LR}\right) \\ +\iota \max\left(E_1^{LI}, E_2^{LI}\right) \end{array} \right\} \right), \\ \left(\bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS}), E_2^U \in E_2^U(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \max\left(E_1^{UR}, E_2^{UR}\right) \\ +\iota \max\left(E_1^{UI}, E_2^{UI}\right) \end{array} \right\} \right) \end{array} \right)$$

3.

$$E_{CHFRS-1} \cap E_{CHFRS-2} = \left(\begin{array}{l} \left(\bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS}), E_2^L \in E_2^L(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \min\left(E_1^{LR}, E_2^{LR}\right) \\ +\iota \min\left(E_1^{LI}, E_2^{LI}\right) \end{array} \right\} \right), \\ \left(\bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS}), E_2^U \in E_2^U(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \min\left(E_1^{UR}, E_2^{UR}\right) \\ +\iota \min\left(E_1^{UI}, E_2^{UI}\right) \end{array} \right\} \right) \end{array} \right)$$

Definition 9: For two CHFREs $E_{CHFRS-1} = \left(E_1^L(\mathfrak{T}_{CHFS}), E_1^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_1^{LR}+\iota E_1^{LI}\right), \left(E_1^{UR}+\iota E_1^{UI}\right)\right)$ and $E_{CHFRS-2} = \left(E_2^L(\mathfrak{T}_{CHFS}), E_2^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_2^{LR}+\iota E_2^{LI}\right), \left(E_2^{UR}+\iota E_2^{UI}\right)\right)$, with λ the operational laws are anticipated as

1.

$$E_{CHFRS-1} \oplus E_{CHFRS-2} = \left(\begin{array}{l} \left(\bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS}), E_2^L \in E_2^L(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} E_1^{LR}+E_2^{LR}-E_1^{LR}E_2^{LR} \\ +\iota\left(E_1^{LI}+E_2^{LI}-E_1^{LI}E_2^{LI}\right) \end{array} \right\} \right), \\ \left(\bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS}), E_2^U \in E_2^U(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} E_1^{UR}+E_2^{UR}-E_1^{UR}E_2^{UR} \\ +\iota\left(E_1^{UI}+E_2^{UI}-E_1^{UI}E_2^{UI}\right) \end{array} \right\} \right) \end{array} \right)$$

2.

$$E_{CHFRS-1} \otimes E_{CHFRS-2} = \left(\begin{array}{l} \left(\bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS}), E_2^L \in E_2^L(\mathfrak{T}_{CHFS})} \left\{ E_1^{LR}E_2^{LR} + \iota\left(E_1^{LI}E_2^{LI}\right) \right\} \right), \\ \left(\bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS}), E_2^U \in E_2^U(\mathfrak{T}_{CHFS})} \left\{ E_1^{UR}E_2^{UR} + \iota\left(E_1^{UI}E_2^{UI}\right) \right\} \right) \end{array} \right)$$

3.

$$\lambda\left(E_{CHFRS-1}\right) = \left(\begin{array}{l} \left(\bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} 1-\left(1-E_1^{LR}\right)^{\lambda} \\ +\iota\left(1-\left(1-E_1^{LI}\right)^{\lambda}\right) \end{array} \right\} \right), \\ \left(\bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} 1-\left(1-E_1^{UR}\right)^{\lambda} \\ +\iota\left(1-\left(1-E_1^{UI}\right)^{\lambda}\right) \end{array} \right\} \right) \end{array} \right)$$

4.

$$\left(E_{CHFRS-1}\right)^{\lambda} = \left(\left(\bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS})} \left\{ \left(E_1^{LR}\right)^{\lambda} + \iota\left(E_1^{LI}\right)^{\lambda} \right\} \right), \left(\bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS})} \left\{ \left(E_1^{UR}\right)^{\lambda} + \iota\left(E_1^{UI}\right)^{\lambda} \right\} \right) \right)$$

Definition 10: Take a CHFRE $E_{CHFRS} = \left(E^L(\mathfrak{T}_{CHFS}), E^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E^{LR}+\iota E^{LI}\right), \left(E^{UR}+\iota E^{UI}\right)\right)$, its score values would be anticipated as

$$\mathcal{S}\left(E_{CHFRS}\right) = \frac{1}{\ell} \sum_{j=1}^{\ell} \frac{E_j^{LR}+E_j^{IR}+E_j^{UR}+E_j^{IR}}{4}$$

where ℓ is the length of CHFRE.

A. AOs WITHIN COMPLEX HESITANT FUZZY ROUGH STRUCTURE

In this part of the script, we are going to develop some AOs within CHFRS that is CHFRWA, CHFROWA, CHFRWG, and CHFROWG operators.

Definition 11: Let $E_{CHFRS-k} = \left(E_k^L(\mathfrak{T}_{CHFS}), E_k^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR}+\iota E_k^{LI}\right), \left(E_k^{UR}+\iota E_k^{UI}\right)\right)$, $k = 1, 2, \ldots, n$ be a group of CHFREs and $\mho_{w} = \left(\mho_{w-1}, \mho_{w-2}, \ldots, \mho_{w-n}\right)$ be a weight vector that holds that $\mho_{w-k} \in [0, 1]$, and $\sum_{k=1}^{n} \mho_{w-k} = 1$. Then the CHFRWA operator is instigated as

$$CHFRWA\left(E_{CHFRS-1}, E_{CHFRS-2}, \ldots, E_{CHFRS-n}\right) = \bigoplus_{k=1}^{n} \mho_{w-k} E_{CHFRS-k}$$

Theorem 1: Let $E_{CHFRS-k} = \left(E_k^L(\mathfrak{T}_{CHFS}), E_k^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR}+\iota E_k^{LI}\right)\left(E_k^{UR}+\iota E_k^{UI}\right)\right)$, $k = 1, 2, \ldots, n$ be a group of CHFREs. Then the aggregated result will be a CHFRE after utilizing the CHFRWA operator, as shown at the bottom of the next page.

Proposition 1: Let $E_{CHFRS-k} = \left(E_k^L(\mathfrak{T}_{CHFS}), E_k^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR}+\iota E_k^{LI}\right), \left(E_k^{UR}+\iota E_k^{UI}\right)\right)$, $k = 1, 2, \ldots, n$ be a group of CHFREs. If $\forall\ k$, $E_{CHFRS-k} = E_{CHFRS}$ that is $E_k^{LR} = E^{LR}$, $E_k^{LI} = E^{LI}$, $E_k^{UR} = E^{UR}$, and $E_k^{UI} = E^{UI}$, then

$$CHFRWA\left(E_{CHFRS-1}, E_{CHFRS-2}, \ldots, E_{CHFRS-n}\right) = E_{CHFRS}$$

This is devised as the idempotency of the CHFRWA operator.

Proposition 2: Let $E_{CHFRS-k} = \left(E_k^L(\mathfrak{T}_{CHFS}), E_k^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR}+\iota E_k^{LI}\right), \left(E_k^{UR}+\iota E_k^{UI}\right)\right)$ and $E^*_{CHFRS-k} = \left(E_k^{L*}(\mathfrak{T}_{CHFS}), E_k^{U*}(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR*}+\iota E_k^{LI*}\right), \left(E_k^{UR*}+\iota E_k^{UI*}\right)\right)$, $k = 1, 2, \ldots, n$ be two groups of CHFREs. If $\forall\ k$, $E_{CHFRS-k} \leq E^*_{CHFRS-k}$ that is $E_k^{LR} \leq E_k^{LR*}$, $E_k^{LI} \leq E_k^{LI*}$, $E_k^{UR} \leq E_k^{UR*}$, and $E_k^{UI} \leq E_k^{UI*}$, then

$$CHFRWA\left(E_{CHFRS-1}, E_{CHFRS-2}, \ldots, E_{CHFRS-n}\right) \leq CHFRWA\left(E^*_{CHFRS-1}, E^*_{CHFRS-2}, \ldots, E^*_{CHFRS-n}\right)$$

This is devised as the monotonicity of the CHFRWA operator.

Proposition 3: Let $E_{CHFRS-k} = \left(E_k^L(\mathfrak{T}_{CHFS}), E_k^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR} + \iota E_k^{LI}\right), \left(E_k^{UR} + \iota E_k^{UI}\right)\right)$, $k = 1, 2, \ldots, n$ be a group of CHFREs. If

$$(E_{CHFRS})^- = \left(\begin{array}{c}\left(\min_k \left\{E_k^{LR}\right\} + \iota \min_k \left\{E_k^{LI}\right\}\right), \\ \left(\min_k \left\{E_k^{UR}\right\} + \iota \min_k \left\{E_k^{UR}\right\}\right)\end{array}\right),$$

and

$$(E_{CHFRS})^+ = \left(\begin{array}{c}\left(\max_k \left\{E_k^{LR}\right\} + \iota \max_k \left\{E_k^{LI}\right\}\right), \\ \left(\max_k \left\{E_k^{UR}\right\} + \iota \max_k \left\{E_k^{UR}\right\}\right)\end{array}\right),$$

ten

$$(E_{CHFRS})^- \leq CHFRWA\left(\begin{array}{c}E_{CHFRS-1}, \\ E_{CHFRS-2}, \ldots, \\ E_{CHFRS-n}\end{array}\right) = (E_{CHFRS})^+$$

This is devised as the boundedness of the CHFRWA operator.

Definition 12: Let $E_{CHFRS-k} = \left(E_k^L(\mathfrak{T}_{CHFS}), E_k^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR} + \iota E_k^{LI}\right), \left(E_k^{UR} + \iota E_k^{UI}\right)\right)$, $k = 1, 2, \ldots, n$ be a group of CHFREs, $\mathfrak{V}_{\mathcal{w}} = \left(\mathfrak{V}_{\mathcal{w}-1}, \mathfrak{V}_{\mathcal{w}-2}, \ldots, \mathfrak{V}_{\mathcal{w}-n}\right)$ be a weight vector that holds that $\mathfrak{V}_{\mathcal{w}-k} \in [0, 1]$, and $\sum_{k=1}^{n} \mathfrak{V}_{\mathcal{w}-k} = 1$ and $(\mathfrak{F}(1), \mathfrak{F}(2), \mathfrak{F}(3), \ldots, \mathfrak{F}(n))$ is a permutation of $(1, 2, .., n)$ such that $E_{CHFRS-\mathfrak{F}(k-1)} \geq E_{CHFRS-\mathfrak{F}(k)}\ \forall\ k$. Then the CHFROWA operator is instigated, as shown at the bottom of the next page.

Definition 13: Let $E_{CHFRS-k} = \left(E_k^L(\mathfrak{T}_{CHFS}), E_k^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR} + \iota E_k^{LI}\right), \left(E_k^{UR} + \iota E_k^{UI}\right)\right)$, $k = 1, 2, \ldots, n$ be a group of CHFREs and $\mathfrak{V}_{\mathcal{w}} = \left(\mathfrak{V}_{\mathcal{w}-1}, \mathfrak{V}_{\mathcal{w}-2}, \ldots, \mathfrak{V}_{\mathcal{w}-n}\right)$ be a weight vector that holds that $\mathfrak{V}_{\mathcal{w}-k} \in [0, 1]$, and $\sum_{k=1}^{n} \mathfrak{V}_{\mathcal{w}-k} = 1$. Then the CHFRWG operator is instigated as

$$CHFRWG\left(E_{CHFRS-1}, E_{CHFRS-2}, \ldots, E_{CHFRS-n}\right) = \bigotimes_{k=1}^{n} \left(E_{CHFRS-k}\right)^{\mathfrak{V}_{\mathcal{w}-k}}$$

Theorem 2: Let $E_{CHFRS-k} = \left(E_k^L(\mathfrak{T}_{CHFS}), E_k^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR} + \iota E_k^{LI}\right), \left(E_k^{UR} + \iota E_k^{UI}\right)\right)$, $k = 1, 2, \ldots, n$ be a group of CHFREs. Then the aggregated result will be a CHFRE after utilizing the CHFRWG operator, as shown at the bottom of the next page.

Definition 14: Let $E_{CHFRS-k} = \left(E_k^L(\mathfrak{T}_{CHFS}), E_k^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_k^{LR} + \iota E_k^{LI}\right), \left(E_k^{UR} + \iota E_k^{UI}\right)\right)$, $k = 1, 2, \ldots, n$ be a group of CHFREs, $\mathfrak{V}_{\mathcal{w}} = \left(\mathfrak{V}_{\mathcal{w}-1}, \mathfrak{V}_{\mathcal{w}-2}, \ldots, \mathfrak{V}_{\mathcal{w}-n}\right)$ be a weight vector that holds that $\mathfrak{V}_{\mathcal{w}-k} \in [0, 1]$, and $\sum_{k=1}^{n} \mathfrak{V}_{\mathcal{w}-k} = 1$ and $(\mathfrak{F}(1), \mathfrak{F}(2), \mathfrak{F}(3), \ldots, \mathfrak{F}(n))$ is a permutation of $(1, 2, .., n)$ such that $E_{CHFRS-\mathfrak{F}(k-1)} \geq E_{CHFRS-\mathfrak{F}(k)}\ \forall\, k$. Then the CHFROWG operator is instigated, as shown at the bottom of the next page.

Remark 1: The initiated CHFROWA, CHFRWG, and CHFROWG operators also hold idempotency, boundedness, and monotonicity properties.

VI. MULTI-ATTRIBUTE DECISION-MAKING APPROACH UNDER CHFRS

Let there be n alternatives that are $\{\mathcal{X}_{alt-1}, \mathcal{X}_{alt-2}, \ldots, \mathcal{X}_{alt-n}\}$ and m attributes that are $\{\mathcal{A}_{art-1}, \mathcal{A}_{art-2}, \ldots, \mathcal{A}_{art-m}\}$ in a MADM dilemma. This MADM dilemma aims to find the finest alternative by assessing the considered attributes. The decision expert would assess these alternatives and provide their evaluation values in the model of CHFRNs which is $E_{CHFRS-kj} = \left(E^L(\mathfrak{T}_{CHFS}), E^U(\mathfrak{T}_{CHFS})\right) = \left(\left(E_{kj}^{LR} + \iota E_{kj}^{LI}\right), \left(E_{kj}^{UR} + \iota E_{kj}^{UI}\right)\right)$. As every attribute can have significance in the view of the decision expert, thus the decision expert would provide the weight $\check{y} = \left(\check{y}_1, \check{y}_1, , ..\check{y}_m\right)^T$ such that $\check{y}_{\mathbb{1}} \in [0,1]$ for all j and $\sum_{j=1}^{m} \check{y}_j = 1$ to the attributes. To solve this dilemma of MADM, we have the following algorithm.

A. ALGORITHM OF THE MADM APPROACH

We have the underlying step in the algorithm of the MADM approach.

Step 1: Use the interpreted evaluation values of the decision expert and develop a complex hesitant fuzzy rough decision matrix $\mathfrak{D}$, where each entry contains a CHFRN.

$$CHFRWA\left(E_{CHFRS-1}, E_{CHFRS-2}, \ldots, E_{CHFRS-n}\right) = \left(\begin{array}{c}\bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS}),\ldots,E_n^L \in E_n^L(\mathfrak{T}_{CHFS})} \left\{\begin{array}{c}\left(1 - \prod_{k=1}^{n}\left(1 - E_k^{LR}\right)^{\mathfrak{V}_{\mathcal{w}-k}}\right) \\ +\iota\left(1 - \prod_{k=1}^{n}\left(1 - E_k^{LI}\right)^{\mathfrak{V}_{\mathcal{w}-k}}\right)\end{array}\right\}, \\ \bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS}),\ldots,E_n^U \in E_n^U(\mathfrak{T}_{CHFS})} \left\{\begin{array}{c}\left(1 - \prod_{k=1}^{n}\left(1 - E_k^{UR}\right)^{\mathfrak{V}_{\mathcal{w}-k}}\right) \\ +\iota\left(1 - \prod_{k=1}^{n}\left(1 - E_k^{UI}\right)^{\mathfrak{V}_{\mathcal{w}-k}}\right)\end{array}\right\}\end{array}\right)$$

Step 2: There are two kinds of attributes (benefit and cost kinds) involved in the MADM problem. Thus, normalization is required which will be performed by the formula.

$$\mathfrak{D}^{\mathbb{N}} = \begin{cases} \left(\left(E_{kj}^{LR} + \iota E_{kj}^{LI}\right), \left(E_{kj}^{UR} + \iota E_{kj}^{UI}\right)\right) & \textit{for benefit kind} \\ \left(\left(\left(E_{kj}^{LR} + \iota E_{kj}^{LI}\right), \left(E_{kj}^{UR} + \iota E_{kj}^{UI}\right)\right)\right)^{c} & \textit{for cost kind} \end{cases}$$

Step 3: The normalized matrix $\mathfrak{D}^{\mathbb{N}}$ would be aggregated by employing any of the deduced AOs to get the aggregated outcomes of the alternatives.

Step 4: The score values or accuracy values of the aggregated outcomes of the alternative would be anticipated in this step.

Step 5: With the assistance of score values or accuracy values, the ranking of alternatives would be portrayed.

$$CHFROWA\left(E_{CHFRS-1}, E_{CHFRS-2}, \ldots, E_{CHFRS-n}\right) = \underset{k=1}{\overset{n}{\oplus}} \mho_{\mathcal{w}-k} E_{CHFRS-\mathfrak{F}(k)}$$
$$= \left(\begin{array}{c} \bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS}),\ldots,E_n^L \in E_n^L(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \left(1-\prod_{k=1}^{n}\left(1-E_{\mathfrak{F}(k)}^{LR}\right)^{\mho_{\mathcal{w}-k}}\right) \\ +\iota\left(1-\prod_{k=1}^{n}\left(1-E_{\mathfrak{F}(k)}^{LI}\right)^{\mho_{\mathcal{w}-k}}\right) \end{array} \right\}, \\ \bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS}),\ldots,E_n^U \in E_n^U(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \left(1-\prod_{k=1}^{n}\left(1-E_{\mathfrak{F}(k)}^{UR}\right)^{\mho_{\mathcal{w}-k}}\right) \\ +\iota\left(1-\prod_{k=1}^{n}\left(1-E_{\mathfrak{F}(k)}^{UI}\right)^{\mho_{\mathcal{w}-k}}\right) \end{array} \right\} \end{array} \right)$$

$$CHFRWG\left(E_{CHFRS-1}, E_{CHFRS-2}, \ldots, E_{CHFRS-n}\right) = \left(\begin{array}{c} \bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS}),\ldots,E_n^L \in E_n^L(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \left(\prod_{k=1}^{n}\left(E_{k}^{LR}\right)^{\mho_{\mathcal{w}-k}}\right) \\ +\iota\left(\prod_{k=1}^{n}\left(E_{k}^{LI}\right)^{\mho_{\mathcal{w}-k}}\right) \end{array} \right\}, \\ \bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS}),\ldots,E_n^U \in E_n^U(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \left(\prod_{k=1}^{n}\left(E_{k}^{UR}\right)^{\mho_{\mathcal{w}-k}}\right) \\ +\iota\left(\prod_{k=1}^{n}\left(E_{k}^{UI}\right)^{\mho_{\mathcal{w}-k}}\right) \end{array} \right\} \end{array} \right)$$

$$CHFROWG\left(E_{CHFRS-1}, E_{CHFRS-2}, \ldots, E_{CHFRS-n}\right) = \underset{k=1}{\overset{n}{\otimes}} \left(E_{CHFRS-\mathfrak{F}(k)}\right)^{\mho_{\mathcal{w}-k}}$$
$$= \left(\begin{array}{c} \bigcup_{E_1^L \in E_1^L(\mathfrak{T}_{CHFS}),\ldots,E_n^L \in E_n^L(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \left(\prod_{k=1}^{n}\left(E_{\mathfrak{F}(k)}^{LR}\right)^{\mho_{\mathcal{w}-k}}\right) \\ +\iota\left(\prod_{k=1}^{n}\left(E_{\mathfrak{F}(k)}^{LI}\right)^{\mho_{\mathcal{w}-k}}\right) \end{array} \right\}, \\ \bigcup_{E_1^U \in E_1^U(\mathfrak{T}_{CHFS}),\ldots,E_n^U \in E_n^U(\mathfrak{T}_{CHFS})} \left\{ \begin{array}{c} \left(\prod_{k=1}^{n}\left(E_{\mathfrak{F}(k)}^{UR}\right)^{\mho_{\mathcal{w}-k}}\right) \\ +\iota\left(\prod_{k=1}^{n}\left(E_{\mathfrak{F}(k)}^{UI}\right)^{\mho_{\mathcal{w}-k}}\right) \end{array} \right\} \end{array} \right)$$

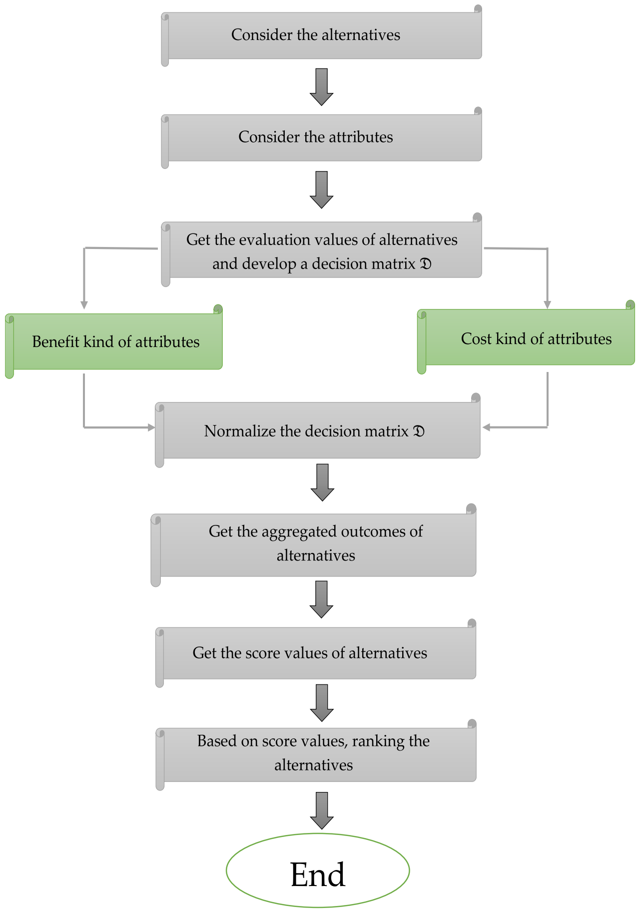

FIGURE 2. **The flowchart of the introduced MADM method within BHFRS.**

The flowchart of the devised MADM approach is revealed in Figure 2

B. ILLUSTRATIVE EXAMPLE

Here, we will show the applicability of the proposed work by investigating an example and solving the problem mentioned in Section III.

1) SOLVING THE PROBLEM

The decision expert would evaluate the data sources portrayed in Table 2, based on the attributes displayed in Table 1 to find out the finest data sources and would provide the evaluation values of these data sources in the model of complex hesitant fuzzy rough numbers. Further, as every attribute can have significance in the view of the decision expert, thus the decision expert would provide the weight (0.2, 0.25, 0.3, 0.25) to the attributes. Now to tackle this MADM dilemma, the proposed algorithm would be utilized as follows

Step 1: The complex hesitant fuzzy rough decision matrix consisting of evaluation values of the data sources based on the attributes is devised in Table 3.

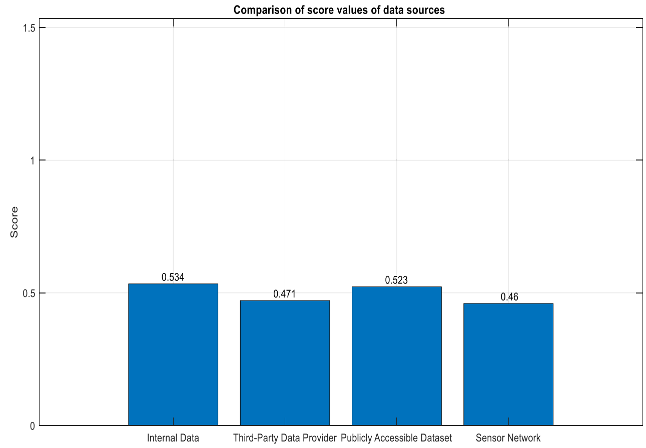

FIGURE 3. **The score values of data sources where the information is aggregated by CHFRWA operator.**

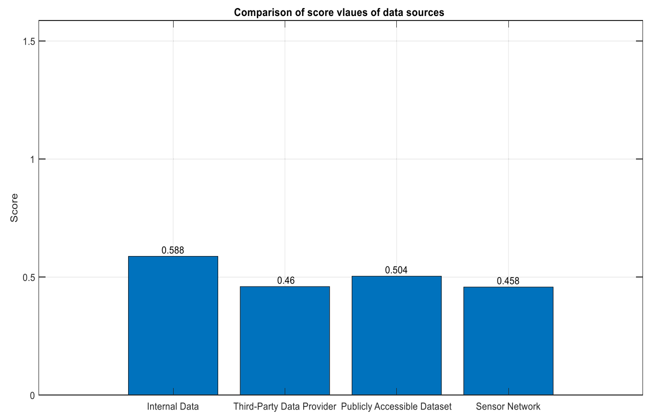

FIGURE 4. **The score values of data sources where the information is aggregated by CHFROWA operator.**

Step 2: As in this dilemma, the attributes are benefit kinds, so even after normalization, we would get the same decision matrix. Therefore, we escape this step.

Step 3: The aggregated results of data sources with the help of invented operators are anticipated in Table 4.

Step 4: The score values of data sources are devised in Table 5.

Step 5: With the assistance of score values of data sources, the ranking of data sources is devised in Table 6.

The score values and ranking of the data sources reveal that $\mathcal{X}_{alt-1}$(Internal Database) is the finest data source for integrating in data sciences. The graphical interpretation of score values of presented data sources by using deduced operators is displayed in Figures 3, 4, 5, and 6.

VII. COMPARATIVE STUDY

Here, the comparison of the initiated theory with underneath considered prevailing theories would be performed.

- The theory of hesitant fuzzy AOs and group decision-making (DM), was developed by Xia et al. [23].

TABLE 3. The evaluation values of the data sources are given by the expert.

	$\mathcal{A}_{art-1}$	$\mathcal{A}_{art-2}$	$\mathcal{A}_{art-3}$	$\mathcal{A}_{art-4}$
$\mathcal{X}_{alt-1}$	$\left(\left\{(0.126+\iota 0.283),(0.817+\iota 0.838),(0.977+\iota 0.896)\right\},\left\{(0.246+\iota 0.276),(0.786+\iota 0.819),(0.870+\iota 0.124)\right\}\right)$	$\left(\left\{(0.253+\iota 0.093),(0.367+\iota 0.828),(0.778+\iota 0.988)\right\},\left\{(0.096+\iota 0.145),(0.281+\iota 0.971),(0.910+\iota 0.241)\right\}\right)$	$\left(\left\{(0.122+\iota 0.453),(0.847+\iota 0.898),(0.732+\iota 0.952)\right\},\left\{(0.124+\iota 0.091),(0.612+\iota 0.980),(0.130+\iota 0.912)\right\}\right)$	$\left(\left\{(0.212+\iota 0.733),(0.097+\iota 0.837),(0.098+\iota 0.123)\right\},\left\{(0.010+\iota 0.710),(0.200+\iota 0.898),(0.121+\iota 0.513)\right\}\right)$
$\mathcal{X}_{alt-2}$	$\left(\left\{(0.242+\iota 0.364),(0.773+\iota 0.862),(0.787+\iota 0.091)\right\},\left\{(0.211+\iota 0.333),(0.001+\iota 0.777),(0.076+\iota 0.271)\right\}\right)$	$\left(\left\{(0.002+\iota 0.301),(0.017+\iota 0.809),(0.707+\iota 0.078)\right\},\left\{(0.132+\iota 0.271),(0.007+\iota 0.113),(0.009+\iota 0.111)\right\}\right)$	$\left(\left\{(0.129+\iota 0.307),(0.727+\iota 0.518),(0.917+\iota 0.892)\right\},\left\{(0.222+\iota 0.333),(0.717+\iota 0.983),(0.098+\iota 0.111)\right\}\right)$	$\left(\left\{(0.262+\iota 0.123),(0.717+\iota 0.893),(0.783+\iota 0.848)\right\},\left\{(0.112+\iota 0.157),(0.711+\iota 0.993),(0.333+\iota 0.444)\right\}\right)$
$\mathcal{X}_{alt-3}$	$\left(\left\{(0.221+\iota 0.033),(0.927+\iota 0.018),(0.037+\iota 0.118)\right\},\left\{(0.002+\iota 0.111),(0.198+\iota 0.555),(0.788+\iota 0.818)\right\}\right)$	$\left(\left\{(0.112+\iota 0.334),(0.700+\iota 0.863),(0.763+\iota 0.765)\right\},\left\{(0.222+\iota 0.333),(0.710+\iota 0.803),(0.734+\iota 0.888)\right\}\right)$	$\left(\left\{(0.112+\iota 0.233),(0.072+\iota 0.848),(0.729+\iota 0.208)\right\},\left\{(0.666+\iota 0.113),(0.777+\iota 0.889),(0.753+\iota 0.987)\right\}\right)$	$\left(\left\{(0.082+\iota 0.301),(0.037+\iota 0.038),(0.701+\iota 0.098)\right\},\left\{(0.261+\iota 0.113),(0.757+\iota 0.732),(0.333+\iota 0.848)\right\}\right)$
$\mathcal{X}_{alt-4}$	$\left(\left\{(0.132+\iota 0.253),(0.127+\iota 0.801),(0.387+\iota 0.827)\right\},\left\{(0.202+\iota 0.111),(0.787+\iota 0.096),(0.123+\iota 0.848)\right\}\right)$	$\left(\left\{(0.912+\iota 0.033),(0.197+\iota 0.823),(0.237+\iota 0.198)\right\},\left\{(0.113+\iota 0.333),(0.876+\iota 0.333),(0.811+\iota 0.808)\right\}\right)$	$\left(\left\{(0.211+\iota 0.367),(0.070+\iota 0.111),(0.109+\iota 0.123)\right\},\left\{(0.162+\iota 0.111),(0.298+\iota 0.888),(0.112+\iota 0.818)\right\}\right)$	$\left(\left\{(0.209+\iota 0.312),(0.273+\iota 0.808),(0.797+\iota 0.800)\right\},\left\{(0.262+\iota 0.111),(0.117+\iota 0.888),(0.123+\iota 0.009)\right\}\right)$

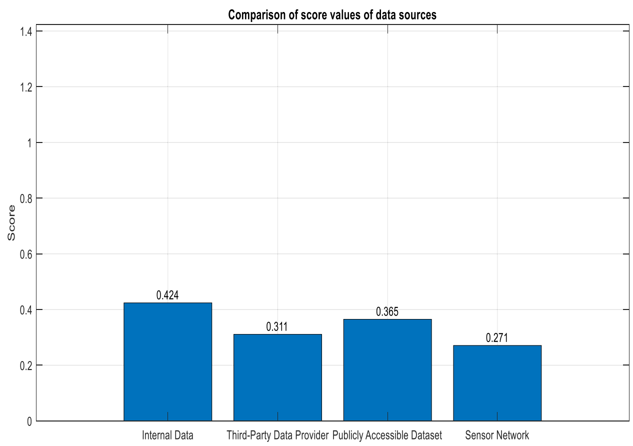

FIGURE 5. The score values of data sources where the information is aggregated by CHFRWG operator.

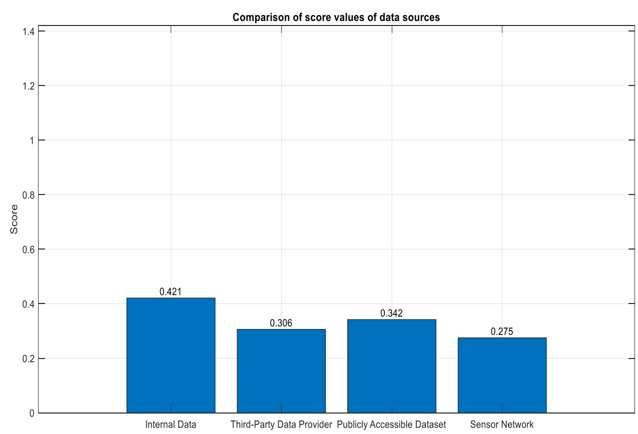

FIGURE 6. The score values of data sources where the information is aggregated by CHFROWG operator.

- Hesitant fuzzy power AOs and group DM under HFS, initiated by Zhang [24].
- The concept of HFS in MADM was devised by Lalotra and Singh [25].
- The notion of MADM under a complex fuzzy environment, based on probability AOs, was devised by Rehman [26].
- Arithmetic AOs within the polar structure of CFS, devised by Bi et al. [27]

TABLE 4. The aggregated values of the data sources after employing deduced operators.

Operators	$\mathcal{X}_{alt-1}$	$\mathcal{X}_{alt-2}$	$\mathcal{X}_{alt-3}$	$\mathcal{X}_{alt-4}$
CHFRWA	$\left(\begin{Bmatrix}(0.18+\iota 0.452),\\(0.647+\iota .857),\\(0.788+\iota 0.92)\end{Bmatrix},\begin{Bmatrix}(0.117+\iota 0.357),\\(0.518+\iota 0.971),\\(0.662+\iota 0.634)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.159+\iota 0.276),\\(0.634+\iota 0.796),\\(0.825+\iota 0.69)\end{Bmatrix},\begin{Bmatrix}(0.171+\iota 0.277),\\(0.499+\iota 0.939),\\(0.14+\iota 0.24)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.128+\iota 0.242),\\(0.575+\iota 0.659),\\(0.654+\iota 0.38)\end{Bmatrix},\begin{Bmatrix}(0.374+\iota 0.174),\\(0.686+\iota 0.789),\\(0.687+\iota 0.93)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.535+\iota 0.257),\\(0.168+\iota 0.7),\\(0.451+\iota 0.57)\end{Bmatrix},\begin{Bmatrix}(0.185+\iota 0.173),\\(0.62+\iota 0.734),\\(0.4+\iota 0.728)\end{Bmatrix}\right)$
CHFROWA	$\left(\begin{Bmatrix}(0.187+\iota 0.431),\\(0.618+\iota 0.849),\\(0.814+\iota 0.921)\end{Bmatrix},\begin{Bmatrix}(0.122+\iota 0.366),\\(0.518+\iota 0.967),\\(0.725+\iota 0.542)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.164+\iota 0.291),\\(0.642+\iota 0.806),\\(0.817+\iota 0.625)\end{Bmatrix},\begin{Bmatrix}(0.176+\iota 0.285),\\(0.432+\iota 0.917),\\(0.124+\iota 0.237)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.132+\iota 0.232),\\(0.604+\iota 0.587),\\(0.627+\iota 0.337)\end{Bmatrix},\begin{Bmatrix}(0.34+\iota 0.162),\\(0.668+\iota 0.771),\\(0.675+\iota 0.919)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.476+\iota 0.261),\\(0.167+\iota 0.72),\\(0.467+\iota 0.634)\end{Bmatrix},\begin{Bmatrix}(0.191+\iota 0.161),\\(0.632+\iota 0.701),\\(0.353+\iota 0.734)\end{Bmatrix}\right)$
CHFRWG	$\left(\begin{Bmatrix}(0.169+\iota 0.313),\\(0.397+\iota 0.853),\\(0.476+\iota 0.569)\end{Bmatrix},\begin{Bmatrix}(0.071+\iota 0.213),\\(0.4+\iota 0.945),\\(0.304+\iota 0.38)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.062+\iota 0.251),\\(0.287+\iota 0.735),\\(0.801+\iota 0.303)\end{Bmatrix},\begin{Bmatrix}(0.163+\iota 0.262),\\(0.06+\iota 0.547),\\(0.07+\iota 0.188)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.119+\iota 0.184),\\(0.179+\iota 0.181),\\(0.402+\iota 0.213)\end{Bmatrix},\begin{Bmatrix}(0.125+\iota 0.148),\\(0.574+\iota 0.751),\\(0.616+\iota 0.891)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.276+\iota 0.179),\\(0.144+\iota 0.447),\\(0.28+\iota 0.324)\end{Bmatrix},\begin{Bmatrix}(0.174+\iota 0.146),\\(0.375+\iota 0.445),\\(0.192+\iota 0.266)\end{Bmatrix}\right)$
CHFROWA	$\left(\begin{Bmatrix}(0.176+\iota 0.282),\\(0.38+\iota 0.846),\\(0.485+\iota 0.568)\end{Bmatrix},\begin{Bmatrix}(0.073+\iota 0.231),\\(0.39+\iota 0.937),\\(0.368+\iota 0.322)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.063+\iota 0.268),\\(0.289+\iota 0.752),\\(0.795+\iota 0.242)\end{Bmatrix},\begin{Bmatrix}(0.167+\iota 0.272),\\(0.031+\iota 0.534),\\(0.064+\iota 0.191)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.121+\iota 0.166),\\(0.176+\iota 0.128),\\(0.345+\iota 0.187)\end{Bmatrix},\begin{Bmatrix}(0.094+\iota 0.14),\\(0.538+\iota 0.73),\\(0.593+\iota 0.881)\end{Bmatrix}\right)$	$\left(\begin{Bmatrix}(0.245+\iota 0.195),\\(0.145+\iota 0.492),\\(0.306+\iota 0.383)\end{Bmatrix},\begin{Bmatrix}(0.182+\iota 0.138),\\(0.392+\iota 0.374),\\(0.175+\iota 0.267)\end{Bmatrix}\right)$

TABLE 5. The score values of the data sources for integration in data sciences.

Operators	$\mathcal{S}(\mathcal{X}_{alt-1})$	$\mathcal{S}(\mathcal{X}_{alt-2})$	$\mathcal{S}(\mathcal{X}_{alt-3})$	$\mathcal{S}(\mathcal{X}_{alt-4})$
CHFRWA	0.592	0.471	0.523	0.46
CHFROWA	0.588	0.46	0.504	0.458
CHFRWG	0.424	0.311	0.365	0.271
CHFROWA	0.421	0.306	0.342	0.275

TABLE 6. The ranking of the data sources for integration in data sciences.

Operators	Ranking
CHFRWA	$\mathcal{X}_{alt-1} > \mathcal{X}_{alt-3} > \mathcal{X}_{alt-2} > \mathcal{X}_{alt-4}$
CHFROWA	$\mathcal{X}_{alt-1} > \mathcal{X}_{alt-3} > \mathcal{X}_{alt-4} > \mathcal{X}_{alt-2}$
CHFRWG	$\mathcal{X}_{alt-1} > \mathcal{X}_{alt-3} > \mathcal{X}_{alt-2} > \mathcal{X}_{alt-4}$
CHFROWA	$\mathcal{X}_{alt-1} > \mathcal{X}_{alt-3} > \mathcal{X}_{alt-2} > \mathcal{X}_{alt-4}$

- Geometric AOs within the polar model of CFS, devised by Bi et al. [28].

The above-mentioned prevailing works are in the model of HFS, polar, and Cartesian model of CFS and we would try to solve the MADM dilemma (the stated problem) by using these concepts. For that, we would consider the information in Table 3 and the required result is devised in Table 7 and Table 8.

Tables 7 and 8 elucidate that prevailing concepts can't overcome this dilemma of MADM due to various reasons. The theory of HFS can't manage it because its AOs and DM techniques neither consider roughness nor the second dimension (extra hesitant fuzzy information). Thus, prevailing theories and ideas within HFS cannot solve and manage the considered problem. Also, the Cartesian structure of CFS misses both roughness and hesitancy, therefore it cannot manage the MADM problems containing hesitancy and roughness. Any MADM technique operator under the model of the Cartesian form of CFS will fail to overcome this sort of data. As for the polar model of CFS, it can't manage either roughness, hesitancy, or Cartesian CFS, hence it can't overcome the problem either. Furthermore, other than devised work, no structure in the literature can simulate roughness, hesitation, and the second dimension simultaneously. Advanced works that can manage and generalize all of the data included in these structures include FS, CFS, HFS, CHFS, and HFRS.

TABLE 7. The comparison of the revealing and invented work.

References	$\mathcal{S}(\mathcal{X}_{alt-1})$	$\mathcal{S}(\mathcal{X}_{alt-2})$	$\mathcal{S}(\mathcal{X}_{alt-3})$	$\mathcal{S}(\mathcal{X}_{alt-4})$
Xia et al. [23]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Zhang [24]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Lalotra and Singh [25]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Rehman [26]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Bi et al. [27]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Bi et al. [28]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
CHFRWA	0.592	0.471	0.523	0.46
CHFROWA	0.588	0.46	0.504	0.458
CHFRWG	0.424	0.311	0.365	0.271
CHFROWA	0.421	0.306	0.342	0.275

TABLE 8. The comparison of the revealing and invented work.

Operators	Ranking
Xia et al. [23]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Zhang [24]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Lalotra and Singh [25]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Rehman [26]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Bi et al. [27]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
Bi et al. [28]	$\dot{\times}\rightsquigarrow\dot{\times}\rightsquigarrow\dot{\times}$
CHFRWA	$\mathcal{X}_{alt-1} > \mathcal{X}_{alt-3} > \mathcal{X}_{alt-2} > \mathcal{X}_{alt-4}$
CHFROWA	$\mathcal{X}_{alt-1} > \mathcal{X}_{alt-3} > \mathcal{X}_{alt-4} > \mathcal{X}_{alt-2}$
CHFRWG	$\mathcal{X}_{alt-1} > \mathcal{X}_{alt-3} > \mathcal{X}_{alt-2} > \mathcal{X}_{alt-4}$
CHFROWA	$\mathcal{X}_{alt-1} > \mathcal{X}_{alt-3} > \mathcal{X}_{alt-2} > \mathcal{X}_{alt-4}$

VIII. CONCLUSION

In this article, we tackled a problem that is a selection of the finest data source for integration in data sciences as in the existing literature no model or tool can carry the hesitancy, extra fuzzy information, and roughness of the attributes of the data sources for integration in data sources. Therefore, to solve this dilemma, in this article, we first devised the notion of CHFS, CHFR, and CHFRS by using CHFR. We also discussed certain properties of the developed CHFRS. Then, we anticipated averaging/geometric AOs in the setting of CHFRS that is complex hesitant fuzzy rough weighted averaging, complex hesitant fuzzy rough ordered weighted averaging, complex hesitant fuzzy rough weighted geometric, and complex hesitant fuzzy rough ordered weighted geometric operators. Secondly, we analyzed a MADM approach that is a complex hesitant fuzzy rough MADM approach for tackling MADM dilemmas. After developing these theories, we solved the considered example by employing the invented MADM approach within CHFRS and got the finest data source for integration in data sciences. In solving this problem, the loss of data or information was very minimal because of the structure of CHFRS. In the end, we reveal the supremacy and dominance of the anticipated approach by comparing it with certain existing approaches.

In the future, we would try to solve more problems by employing some other mathematical models such as bipolar complex fuzzy set [31], bipolar complex fuzzy soft set [32], bipolar complex fuzzy linguistic set [33], and complex spherical fuzzy set [34], etc.

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