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RESEARCH ARTICLE

A Macro-Micro Integrated Modeling Approach for Urban Day-to-Day Travel Based on Complex System Dynamics in Information Environment

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ABSTRACT Urban day-to-day travel system is a complex nonlinear system, its current status depends not only on the status of the whole urban transportation system at the previous point in time, but also on the travel decision made by each traveler. Moreover, information environment generated by various intelligent terminals like smart phones, has even increased the complexity of urban day-to-day travel system, for micro level route choice decision made by each traveler can be affected by macro level road traffic flow information at any time, and micro route choice behavior can generate new macro traffic information. Nowadays, there is a lack of macro-micro integrated models to explain urban day-to-day travel system, especially its development in information environment. In this paper, complex system dynamics has been proposed and used to explain urban day-to-day travel system in a macro-micro integrated way. The main contribution of this paper is twofold: in micro level, each traveler in urban day-to-day travel system in information environment is endowed with unique physical properties, and the time correlation effect of individual travelers is described; in macro level, the traffic flow fluctuations under the influence of individual micro decisions, the stylized facts of urban day-to-day travel has been deduced, and an effective description of urban day-to-day travel phenomena has been formed.

INDEX TERMS Urban transportation, day-to-day travel, macro-micro integrated modeling, complex system dynamics, information environment.

I. INTRODUCTION

A. BACKGROUND

Researchers and scholars have always been trying to make the most accurate description of urban day-to-day travel, so as to give urban transportation management department more valuable references in making governance decisions [1], [2]. However, making such a description is somehow a tricky issue, as the very urban day-to-day travel system can be recognized as a complex nonlinear system, whose current status depends not only on the status of the whole urban transportation system at the previous point in time, but also on the travel decision made by each traveler [3]. Moreover, placed in information environment formed by intelligent connected vehicle, smartphones, and other types of intelligent terminals [4], in microscopic point of view, the decision

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made by each traveler can be affected by traffic information at any time, and the implementation of decision will sure generates new traffic information [5], [6], [7], [8]; while in macroscopic point of view, the past changing process of road traffic flow will also produce new feedback from successive travel decisions made by travelers on each traveler's next travel behavior [9], [10], [11]. Therefore, it is of great necessity to carry out macro-micro integrated model for urban day-to-day travel system in information environment, it can help people to better grasp the information influencing factors of decision-making and the decision-implementation process of each individual traveler, stylized traffic flow fluctuation performance of macro urban road network, as well as the travel information generation and their micro causes.

B. RESEARCH MOTIVATION AND MAIN CONTRIBUTIONS OF THIS PAPER

Scholars have already made a lot effort in dealing with urban day-to-day travel problems and have tried to build theoretical interpretable models, however, there are still some problems in relevant research which can be concluded from the literature review in Section II: firstly, there is a lack of macro-micro integrated models to explain urban day-to-day travel, most existing research is only based on macro road traffic flow distribution or micro travel decisions; secondly, in the current era where intelligent smartphones and other intelligent terminals are widely used by travelers, information factors have brought tremendous changes to urban day-to-day travel system, but there is a lack of research on the penetration of information environment into urban day-to-day travel behavior. The above issues form the research motivation of this paper, which is to attempt to establish a macro-micro integrated model for the complex urban day-to-day travel system in information environment. As in information environment, only analyzing micro travel choices cannot observe the feedback effect of urban road traffic flow on individual travel, resulting in the study of travel decisions being limited to micro level game analysis, which cannot fully explore the role and inducing factors of macro road traffic information in micro decision-making; while only analyzing the traffic flow of the macro road network cannot observe the micro causes behind macro phenomena, leading to the isolation of the analysis of macro traffic flow changes. In this paper, complex system dynamics has been proposed and used to explain urban day-to-day travel system in a macro-micro integrated way. The main contribution of this paper is twofold: in micro level, each traveler in urban day-to-day travel system in information environment is endowed with unique physical properties, and the time correlation effect of individual travelers is described; in macro level, the traffic flow fluctuations under the influence of individual micro decisions, the stylized facts of urban day-to-day travel has been deduced, and an effective description of urban day-to-day travel phenomena has been formed.

C. ORGANIZATION OF THIS PAPER

The rest of this paper will be organized as follows: in Section II, a comprehensive literature review will be given; in Section III, the micro route choice problem in urban day-to-day travel is defined and described by using complex system dynamics methods; in Section IV, the definition of travel risk in information environment that reflects the time correlation effect of micro route choices by travelers is discussed, and a macro description of urban day-to-day travel system based on the definition of basic micro travelers is constructed; in Section V, theoretical verification and validation have been made to verify the specific macro-micro transportation integrated model proposed can effectively display the stylized facts in unban day-to-day travel in information environment; in Section VI, the modeling effectiveness and performance of the proposed model are evaluated by numerical analysis; finally in Section VII, the entire research is summarized, and the comments on future work is made.

II. LITERATURE REVIEW

A. MICRO DECISION OF URBAN DAY-TO-DAY TRAVEL

From the traditional micro decision model of urban day-to-day travel, it can be found that the models proposed by scholars for decades can be divided into two categories: deterministic ones and stochastic ones. Among them, the deterministic models originated earlier. Yang and Zhang have pointed out that the deterministic models can be further divided into five categories: gravity flow model, percentage conversion model, network trial and error model, projection dynamic system model and evolutionary traffic dynamic system model [12]. Specifically, the gravity flow model mainly relies on the theory of gravity flow to provide route choice calculation methods, represented by Smith et al., but the premise assumption of such model is a bit too simple [13]; the percentage conversion model simulates the allocation of travel choices, represented by Smith et al., who have quantified travel costs and have described the transition from high to low travel costs, achieving a more rational expression of micro travel decisions than the gravity flow model [14], this model has been further optimized and improved by Zhu et al. and Alibabai and Mahmassani later on [15], [16]; the network trial and error model mainly describes the reverse impact of dynamic changes in route traffic on route choice decision-making, represented by Friesz et al., who have described the dynamic changes in micro demand under the influence of path traffic, and have proposed a trial and error adjustment strategy for micro route choice [17], this model was also further optimized by Jin and Guo and Huang in later stage [18], [19]; the projection dynamic system model follows the concept of travel cost in the percentage conversion model, represented by Nagurney and Zhang, who have considered the characteristics of urban day-to-day travel as always in daily units and in time periods, and have established a projection dynamic analysis method based on variational inequalities for discrete time axes [20]; the evolutionary traffic dynamic system model uses the concept of dynamic system to simulate the dynamic route trajectory of continuous traffic, represented by the research of Bertsekas and Gafni [21], afterwards, Sandholm and Ye et al. have developed a more complex dynamic system model based on game theory as well as route preference and time-varying sensitivity of travelers later on [22], [23].

Compared with deterministic models, the development of stochastic models started relatively later. Helzelton have pointed out in an earlier stage that using Markov process to describe the discontinuity and randomness of route adjustment of urban day-to-day travelers has unique advantages [24]. Subsequently, Hazelton and Watling have established a macro traffic flow distribution model based on micro route choices which has also employed Markov analysis methods [25]; After reinforcement learning (RL) algorithm received wide attention, based on Markov process and RL, Wahba and Shalaby and Wahba and Shalaby have introduced the consideration of discretization departure time to the microscopic route choice model of urban day-to-day travel [26], [27]. Huang et al. have also used Markov processes to study the spatiotemporal transformation law of individual micro travel decisions of urban private car users, and have demonstrated the superiority of such model in describing the mobility of urban road network [28].

In addition to the traditional deterministic and stochastic micro decision models for day-to-day travel, there are also other models which should be discussed. Among them, the micro travel decision model based on stated preference (SP) has been developed for a long time. Abdel-Aty et al. have provided an SP based micro choice preference survey test in an earlier stage, attempting to identify the internal relationship between travel time and route choice [29]; Cheng et al. have also analyzed the classification influencing factors and route choice influencing factors of urban day-to-day travelers based on SP in recent years [30]. The relatively new models to analyze the micro decision process for day-to-day travel are usually based on multi-agent and RL during the past decades. Lahkar and Seymour and Wei et al. have used RL to define the difference between expected time and perceived actual travel time as positive and negative feedback, and have attempted to describe the perception of road sections for different travelers and influence their micro travel decisions and based on human memory characteristics [31], [32]. In the past few years, some scholars have analyzed micro decision behavior based on smart card data (SCD), but most of them are based on route choice for rail transit commuting, like Cui and Long and Kim who have generated a series of clusters and classifications of travelers based on identifying passenger groups, and discussed changes in micro travel patterns based on regularity and contingency [33], [34].

It can be seen that for micro decision of urban day-to-day travel, both the earlier deterministic models, as well as the subsequent stochastic models and other models, are constantly developing due to the complexity of urban day-to-day travel system. It can be foreseen that in the context of the deep impact of the information environment on urban day-to-day travel system, such models will inevitably have new development trends.

B. MACRO TRAFFIC FLOW OF URBAN DAY-TO-DAY TRAVEL

For urban day-to-day travel, at the macroscopic level, many relevant studies have acknowledged that the changes in macro road traffic come from the influence of micro route choice, and can be seen as the aggregation of micro route choice decision-making behavior. Friesz and Shah have pointed out that data acquisition has a significant impact on analyzing macro traffic flow, as it is micro decisions that affect the final macro performance [35]; to address data issues, Naveh and Kim have utilized bus SCD and Bluetooth (BT) detectors for road testing to obtain potential spatiotemporal motion patterns of urban day-to-day traffic and describe traveler movements on different dates within a week [36]; Sirmatel and Geroliminis have proposed a nonlinear moving horizon estimation (NMHE) method that can infer future macro traffic flow demand based on current driving origin-destination (OD) information [37]; Li et al. have fully considered the diversity of micro route choices and have proposed a multimodal Logit kernel (MLK) model, which represents the stochastic user equilibrium problem in multimodal transportation networks as a fixed point problem and explains the macro allocation of travel routes [38]; Ma et al. have captured the macro traffic performance by introducing the city trip speed performance index (CTSPI), providing quantitative indicators of macro congestion levels on different roads [39].

Of course, if the influence of micro travel route choice is disregarded, in the field of transportation engineering, the more traditional discussion of macro traffic flow in urban day-to-day travel mainly focuses on studying the user equilibrium (UE) state it can achieve. Wardrop have proposed the concept of equilibrium state in transportation systems [40]; Guo and Liu and Guo have effectively developed the basic macro traffic equilibrium state, specifically assuming that the research premise is based on the actual situation of the macro road network under restricted conditions, and the UE is finally formed by travelers as rational individuals [41], [42]; Site has proposed the concept of state dependent random UE (SDSUE), pointing out that it is related to the fixed point state of the Markov assignment process for route choice, and has provided its calculation method [43]; Torkjazi et al. have investigated the differences and significance between ordinary UE and pessimistic UE (PUE) by collecting actual data from the city of Tehran, and have found that the PUE-based model is more accurate in estimating urban road traffic flow by applying it to three major cities in Iran [44].

At present, there is relatively little research on the integration of macro and micro modeling in analyzing macro traffic flow of urban day-to-day travel by comparing with other research areas [45], [46]. In recent years, only a few studies have generally started from macro traffic flow models, introducing a description of the influencing factors of micro route choices. Zhang et al. have established an optimized macro basic diagram model, which maps the micro completion rate of travel to the accumulation of vehicle traffic flow on macro road sections, and defines the parameter of route travel cost for micro travelers [9]; although Kazhamiakin et al. have conducted a more specific modeling of the game process of micro travel decisions made by travelers, however, the macro traffic flow in the model has only been used for presentation, without achieving macro-micro integrated analysis for urban day-to-day travel [10]; based on the real demand for attribute analysis of traffic flow on urban roads in traffic demand management (TDM), Zhang et al. have proposed an analysis method that integrates micro travel demand, transforming TDM into a supply-demand interaction analysis problem for road traffic, but the abstraction of micro travel in the model is still relatively rough [11].

As discussed in Part A in this section, the model interpretation of macro traffic flow in urban day-to-day travel is also constantly developing. Moreover, it can be seen that the integration of macro and micro modeling is an important development trend [4], and the main contribution of this paper is also a beneficial attempt in this integration modeling.

C. URBAN DAY-TO-DAY TRAVEL IN INFORMATION ENVIRONMENT

The emergence of information technology has profoundly changed modern transportation systems, and of course, it has also had a huge impact on urban day-to-day travel. As intelligent terminals like smartphones have been widely integrated into everyday lives of people, traditional transportation models for travelers to make travel decisions cannot effectively explain the travel behavior in information environment, thus, corresponding optimization and innovation for those models are needed. Mahmasani and Liu have pointed out that the emergence of the concept of intelligent transportation systems (ITS) has greatly changed the day-to-day travel behavior of urban people [47], while Ratti et al. have further pointed out that with the widespread use of smartphones and location-based services (LBS) related applications, personalized information push will significantly change the intensity of day-to-day travel activities for urban travelers and the evolution of day-to-day travel throughout the city [48].

It should be pointed out that the emergence of any technology may have two sides. Information technology and the information environment it creates can have both positive and negative impacts on urban day-to-day travel. From the perspective of travelers, they can choose to either trust the pushed information or ignore it. On the positive side, Ben-Eila and Shiftan have pointed out that travelers are often willing to accept the risk of high cost under information prompts [49]; through a survey of the attitudes of urban travelers towards intelligent terminals, Tsirimpa has found that real-time information obtained by mobile devices has a significant impact on the rearrangement of personal activities of travelers [50]; Lila and Anjaneyulu have pointed out that the introduction of information and communication technology can reduce the number of ineffective driving kilometers and delay time of vehicles, and play a positive role in energy conservation and emission reduction [51]. While on the negative side, Wu and Huang have pointed out that travelers may not necessarily change their travel strategies according to the information prompted by advanced traveler information systems (ATIS), and at the same time, the optimal state of macroscopic road network traffic may not be achieved under the guidance of ATIS [52]; Fusco et al. have found that travelers with similar OD often have similar average travel times, and even though there may be significant differences in time costs when choosing different routes, travelers still tend to prefer their commonly used routes, and the informative effect is not significant [53].

Although the positive impact of information environment on travelers remains to be discussed, it cannot be denied that the impact of information environment on urban day-to-day travel cannot be ignored. There are many sources of information that affect the travel decisions of travelers, including global information prompts such as variable information signs (VMS) or map applications on intelligent terminals, and local information only known by a certain range of travelers under specific situations or through specific channels. Jiang et al., Farahani et al., and Diop et al. have studied the induction effect of VSM and route guidance systems on urban traffic flow, and have also pointed out that the responses of travelers to VMS depend on the quality of information and the attributes of alternative roads [54], [55], [56]; whereas Zhou et al. and Meneguzzer have assigned different model definitions to different strategies of day-to-day travelers with different attributes in the case of local information permeability, and have examined the mixed equilibrium state that the system ultimately converges [57], [58]. In addition, personal travel feelings and the information from social media applications on intelligent terminals acquired by travelers are also important sources of information in urban day-to-day travel. For example, Ye and Ukkusuri have discussed the impact of self-awareness on subsequent travel for urban travelers under information influence [59], while Wei et al. and Zhang et al. have focused on analyzing the impact of travelers' participation in information interaction in social media on their travel decisions and have pointed out that such interaction does not necessarily provide travelers with better route choices [60], [61].

It can be seen that the emergence of information factors has made the already complex urban day-to-day travel system even more complex. The duality of information (i.e. the coexistence of positive and negative impacts on the travel system) and the duality of travelers towards information (i.e. positive and negative attitudes) have further increased this complexity. The macro-micro integrated modeling of urban day-to-day travel system in information environment will become an interesting research direction.

III. SYSTEM DYNAMICS DESCRIPTION OF URBAN DAY-TO-DAY TRAVEL SYSTEM

A. SINGLE-PERIOD ROAD TRAFFIC FLOW FLUCTUATION UNDER THE INFLUENCE OF MICRO TRAVEL DECISION-MAKING

It is of no doubt that the macro traffic flow increment of any road section in urban area is formed by the aggregation of micro travel decisions. For simplicity in this section, it is assumed that the total number of urban travelers does not change, and the day-to-day ODs of travelers are fixed. In the absence of the information environment influence, it is assumed that for each micro traveler, the decision of whether to choose a certain road section to pass through is made by a "coin toss" in the optional road sections between their ODs. That is, there is a half chance of bringing a positive increment to the macro traffic flow basic value of the road section in a discrete time period, and a half chance of bringing a negative increment. In this way, the urban traffic flow changes on a certain road section can be regarded as identically and independently distributed, that is, in the i-th discrete time period, if the probability distribution function of traffic flow changes Δx_i is expressed as $p_i[\Delta x_i]$, then $\Delta x_i \equiv \Delta x$ ($i = 1, 2, \ldots$) can be used to represent the traffic flow changes at any discrete time period, and its probability distribution function can be denoted as $p[\Delta x]$ in any discrete time period.

Thus, the mean value of the probability distribution function $f[\Delta x]$ of the traffic flow changes in the road section can be defined as:

$$\begin{aligned}\overline{f[\Delta x]} &= E\left[f[\Delta x]\right] = \sum_{\Delta x} f[\Delta x]p[\Delta x] \\ &= \int_{-\infty}^{+\infty} f[\Delta x]p[\Delta x]\mathrm{d}(\Delta x)\end{aligned} \tag{1}$$

Generally, the mean value of Δx, a discrete value is taken. Based on the relevant basic knowledge of probability theory, the mean value can be calculated by summing the discrete probability distribution function $p[\Delta x]$.

For equation (1), the mean value of Δx represents the 1st order moment of Δx, and the 2nd order moment of Δx describes the variance of Δx:

$$\sigma_{i,i-1}^2 = \overline{(\Delta x - \overline{\Delta x})^2} \tag{2}$$

For the micro travel decision on a certain road section brought about by "coin toss", the mean value of the increment on the very road section is 0, which is not actually a possible result for a certain traveler in practice. Therefore, in the explanation of urban day-to-day travel system, the concept of mean value cannot describe the objective impact of micro travel decisions on macro traffic flow. Nevertheless, the mean value 0 can bring some special attributes to the "coin toss" decision, like σ, which is the standard deviation of macro flow changes of the certain road section under the influence of micro route choice decisions, and it can also be called fluctuation rate. Even if there are infinitely many order of moments of Δx, the fluctuation rate of traffic flow on the road section only involves the calculation of the 1st and 2nd order moments. In general, moments of all orders contain all important information of $p[\Delta x]$, for urban day-to-day travel system, due to the small frequency of significant traffic flow changes occurring on a certain road section, $p[\Delta x]$ will decay monotonically to 0 when $|\Delta x|$ approaches ∞. In urban day-to-day travel system, there is no experience or pattern indicating that at what speed will $p[\Delta x]$ tend towards 0, so there might be a "thick tail" phenomenon, which means that the tail might holds a certain distribution weight. In this case, for a larger order value, the corresponding moment will be large, thus, if the "thick tail" phenomenon is not fully considered in mathematical analysis, important information related to large traffic flow changes may be lost. That is to say, if only the 1st and 2nd order moments have been considered in discussing the effects and travel risk of implementing micro travel decisions, it is likely to give misleading conclusions about the macro traffic analysis. This is just the characteristic of the complex urban day-to-day travel system, and to be specific, it is difficult to describe the evolution law of the system solely based on the mean value, but it is necessary to accurately answer the problems related to the possible travel time, cost, and risk in the system. According to the general understanding of risk, it should refer to the fact that it takes much more time than expected to pass through a specific route, in which case the variation in road traffic flow is usually far from the mean value.

B. MULTI-PERIOD ROAD TRAFFIC FLOW FLUCTUATION UNDER THE INFLUENCE OF MICRO TRAVEL DECISION-MAKING

In this part, in order to approach the stylized fluctuation facts of traffic flow on macro road section under the influence of micro travel decisions, the statistical properties of the moments of the traffic flow changes on a certain road section over multi discrete time periods will be discussed. As previously defined, the traffic flow changes on a certain road section during the i-th time period can be represented as $\Delta x_i = x_i - x_{i-1}$, therefore, the traffic flow changes on the road section between the 0th and the n-th discrete time periods can be written as:

$$\Delta x_{n,0} = \sum_{j=1}^{n} \Delta x_j = x_n - x_0 \tag{3}$$

The mean value of traffic flow changes between the 0th and the n-th discrete time periods is:

$$\overline{\Delta x_{n,0}} = \sum_{j=1}^{n} \overline{\Delta x_j} \tag{4}$$

Regardless of traffic flow changes Δx_j is identically and independently distributed or not, equation (4) holds, which means that "the mean of the sum is equal to the sum of the mean''. At the special case of each mean value is the same, it can be obtained that

$$\overline{\Delta x_{n,0}} = \sum_{j=1}^{n} \overline{\Delta x_j} = n\overline{\Delta x} \tag{5}$$

Regardless of the factors of urban development, for the macro traffic flow changes on the road section over multi-period composed of micro ''coin toss'' decisions, the mean value of Δx is always 0, thus, the mean value of $\Delta x_{n,0}$ is 0. The variance of traffic flow changes between the 0th and the n-th discrete time periods is:

$$\begin{aligned}
\sigma_{n,0}^2 &= \overline{(\Delta x_{n,0} - \overline{\Delta x_{n,0}})^2} = \overline{(\Delta x_{n,0})^2} - (\overline{\Delta x_{n,0}})^2 \\
&= \overline{\left(\sum_{j=1}^{n} \Delta x_j\right)^2} - \left(\overline{\sum_{j=1}^{n} \Delta x_j}\right)^2 \\
&= \overline{\sum_{i=1}^{n}\sum_{j=1}^{n} \Delta x_i \Delta x_j} - \left(\overline{\sum_{j=1}^{n} \Delta x_j}\right)^2 \\
&= \sum_{i=1}^{n} \overline{(\Delta x_i)^2} + \sum_{i \neq j} \overline{\Delta x_i \Delta x_j} - \sum_{i=1}^{n} \left(\overline{\Delta x_i}\right)^2 \\
&\quad - \sum_{i \neq j} \overline{\Delta x_i} \cdot \overline{\Delta x_j}
\end{aligned} \tag{6}$$

If there are no correlations between each Δx_i, when $i \neq j$, the mean value of $\Delta x_i \Delta x_j$ equals the product of the mean values of Δx_i and Δx_j. Thus, equation (6) can be simplified as:

$$\begin{aligned}
\sigma_{n,0}^2 &= \sum_{i=1}^{n} \overline{(\Delta x_i)^2} - \sum_{i=1}^{n} (\overline{\Delta x_i})^2 \\
&= \sum_{i=1}^{n} \left\{\overline{(\Delta x_i)^2} - (\overline{\Delta x_i})^2\right\} = \sum_{i=1}^{n} \sigma_{i,i-1}^2
\end{aligned} \tag{7}$$

Equation (7) means that for unrelated random variables, the variance of sum is equal to the sum of variances. For special cases where the variance is the same for each discrete time period, such as identically and independently distributed variables, there is $\sigma_{i,i-1}^2 = \sigma^2$, and even $\sigma_{i,i-n}^2 = \sigma_{n,0}^2 = n\sigma^2$. Thus, the standard deviation of traffic flow changes, namely the fluctuation rate of traffic flow changes over n consecutive discrete time periods can be described as

$$\sigma_{i,i-n} = n^{\frac{1}{2}}\sigma \tag{8}$$

So it can be seen that the fluctuation rate of traffic flow changes increases by a multiple of the square root of time increment n. If the opposite assumption is made, that is, all traffic flow changes Δx_i are interrelated and have the same value and sign, namely $\Delta x_i = \Delta x$, then

$$\begin{aligned}
\sigma_{i,i-n}^2 &= \overline{(\Delta x_{n,0} - \overline{\Delta x_{n,0}})^2} = \overline{(\Delta x_{n,0})^2} - (\overline{\Delta x_{n,0}})^2 \\
&= \overline{\left(\sum_{j=1}^{n} \Delta x_j\right)^2} - \left(\overline{\sum_{j=1}^{n} \Delta x_j}\right)^2 \\
&= \overline{(n\Delta x)^2} - (\overline{n\Delta x})^2 = n^2\left[\overline{(\Delta x)^2} - (\overline{\Delta x})^2\right] \\
&= n^2\sigma^2
\end{aligned} \tag{9}$$

Likewise, there is:

$$\sigma_{i,i-n} = n\sigma \tag{10}$$

It can be seen from equation (10) that, unlike the randomness of discrete ''coin toss'' decisions, this situation can be recognized as walking along a straight line at a constant speed, with a standard deviation proportional to n. In the more general case of finite but non zero positive correlation, the response expression of the standard deviation is between the unrelated case described in equation (8) and the correlated case described in equation (10). If the traffic flow changes Δx_i are anti-correlated (the correlation between them is negative), the dependency relationship will be very similar to n^0. It can be proven that after n discrete time periods, the fluctuation rate of the real urban day-to-day traffic flow on certain road sections increases proportionally to $n^{1/\mu}\sigma$, where μ is determined by factors such as the specific development situation of the city and the infiltration of information into travelers.

C. EXPLANATION OF THE PROBABILITY DISTRIBUTION OF ROAD TRAFFIC FLOW FLUCTUATION BASED ON COMPLEX SYSTEM DYNAMICS

As discussed above, $\Delta x_{n,0}$ provides the traffic flow changes from the 0th to the n-th discrete time periods, assuming the traffic flow changes for each discrete time period Δx_j are all identically and independently distributed. For the mean value of $\Delta x_{n,0}$, due to its identity as n times the mean value of traffic flow changes in a single discrete time period, there is

$$\frac{\Delta x_{n,0} - n\overline{\Delta x_{n,0}}}{n} = \frac{\Delta x_{n,0} - n\overline{\Delta x}}{n} \tag{11}$$

When n approaches ∞, for the convenience of discussion, rewriting the left hand side of equation (11) in the exponential function, and by employing equation (1), as in equation (12), shown at the bottom of the next page, can be acquired.

By employing Taylor expansion, the left hand side of equation (12) can be calculated as in (13), shown at the bottom of the next page.

Applying Fourier transform to equation (13), the probability distribution function can be gotten as in (14), shown at the bottom of the next page.

When n approaches ∞, it can be concluded that the probability distribution function of road traffic changes tends to be Gaussian, and it is independent of the specific form of the probability distribution function $p[\Delta x]$ of traffic flow changes on the road section over a single time period. Therefore, equation (14) can be rewritten as:

$$p[\Delta x_{n,0} - \overline{\Delta x_{n,0}}] = \left[\frac{1}{2\pi\sigma_{n,0}^2}\right]^{\frac{1}{2}} \exp\left[-\frac{(\Delta x_{n,0} - \overline{\Delta x_{n,0}})^2}{2\sigma_{n,0}^2}\right] \tag{15}$$

where

$$\sigma_{n,0} = n^{\frac{1}{2}}\sigma \tag{16}$$

In other words, the probability distribution function of traffic flow changes $\Delta x_{n,0}$ has the following Gaussian form:

$$p[\Delta x_{n,0}] = \left[\frac{1}{2\pi\sigma_{n,0}^2}\right]^{\frac{1}{2}} \exp\left[-\frac{(\Delta x_{n,0} - \overline{\Delta x_{n,0}})^2}{2\sigma_{n,0}^2}\right] \tag{17}$$

where $\sigma_{n,0}$ is the standard deviation.

IV. QUANTIFICATION OF URBAN DAY-TO-DAY TRAFFIC RISK IN INFORMATION ENVIRONMENT

A. INFORMATION ENVIRONEMNT AND ITS IMPACT ON DAY-TO-DAY TRAVEL

As discussed in Section II-C, the emergence of information and communication technology (ICT) and various intelligent terminals have greatly changed the behavior of urban day-to-day travel and have built an information environment for urban day-to-day travel systems. ICT provides more accurate positioning and road network state information and higher quality travel route suggestions for day-to-day travelers [4]. Thus, urban day-to-day travel system in information environment can provide rational traffic information release strategies for travelers. Under the impact of traffic information released, day-to-day travelers may change their micro travel route choice strategies and even adjust their travel purpose during certain time period, resulting in great macro traffic flow changes on urban road networks. To some extent, information release also represents a management method for the stability of the macro road network operation by transportation regulatory authorities. For example, when there are traffic accidents or other unexpected situations on certain road sections, the release of congestion information and the broadcasting of ramp flow control information are all implementation means of transportation regulatory to ensure the orderly and stable operation of the macro road network in urban day-to-day travel system.

B. MICRO TRAVEL DECISION-MAKING IN INFORMATION ENVIRONEMNT

Even if the influence of traffic information factors is not considered, it is always for sure that each urban day-to-day traveler hopes to spend the lowest time cost to achieve the purpose of commuting. Therefore, time saved in travel means the actual profit acquired by each traveler, while increase in time cost means that the travel risk becomes a reality. In urban day-to-day travel system, each traveler exists as a micro route choice decision-maker. On the basis of micro route choice, travelers are not only responsible for executing the final choice behavior, but play a role in personal time management as well.

For the micro travel decision of each traveler, it is possible to make a "to pass through" or "not to pass through" decision for a certain discrete time period and a certain road section. In actual urban day-to-day travel, the personalities and travel demands of the travelers are different, thus, although they ultimately implement travel from the macro point of view, the micro differences between them are significant. Some travelers may not need to consider the macro road traffic conditions and insist on adopting the same travel strategy to implement their travel plans; some travelers, on the other hand, need to consider the traffic conditions of the certain road section they pass through in information environment, and only implement their strategies when the information shows that the certain road section they need to pass through is not congested. Moreover, for each traveler, a certain amount of physical time is required from the formulation of micro route choice strategies to their final travel implementation. Although this physical time is short, it is not zero. During this period, for travelers who rely on information factors to develop micro route choice strategies, the relevant road traffic flow information on which their strategies are based may have changed. In other words, the traffic information learned by travelers when making micro route choices may not necessarily represent the traffic information during the travel implementation process. For example, the traffic information published by applications on smartphone or other

$$\overline{e^{ink\frac{\Delta x_{n,0}-n\overline{\Delta x}}{n}}} = \overline{e^{ik[\Delta x_{n,0}-\overline{\Delta x_{n,0}}]}} = \int_{-\infty}^{+\infty} e^{ik[\Delta x_{n,0}-\overline{\Delta x_{n,0}}]} p[\Delta x_{n,0} - \overline{\Delta x_{n,0}}] d(\Delta x_{n,0}) \tag{12}$$

$$\begin{aligned}\overline{e^{ik[\Delta x_{n,0}-\overline{\Delta x_{n,0}}]}} &= \overline{e^{ik\frac{[\Delta x_1-\overline{\Delta x}]+[\Delta x_2-\overline{\Delta x}]+\ldots+[\Delta x_n-\overline{\Delta x}]}{n}}} = \left(\overline{e^{ik\frac{\sigma}{n}}}\right)^n = \left(1 + ik\frac{\sigma}{n} - \frac{1}{2!}k^2\frac{\sigma^2}{n^2} + \ldots\right)^n \\ &\approx \left(1 - \frac{k^2\sigma^2}{2n^2}\right)^n = e^{-\frac{k^2\sigma^2}{2n}}\end{aligned} \tag{13}$$

$$p[\Delta x_{n,0} - \overline{\Delta x_{n,0}}] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-\frac{k^2\sigma^2}{2n}} \exp\left[-ik(\Delta x_{n,0} - \overline{\Delta x_{n,0}})\right] \mathrm{d}k = \left[\frac{1}{2\pi n\sigma^2}\right]^{\frac{1}{2}} \exp\left[-\frac{(\Delta x_{n,0} - \overline{\Delta x_{n,0}})^2}{2n\sigma^2}\right] \tag{14}$$

media may have delays, or the attributes of the traffic information seen by travelers may be the average traffic flow over a period of time, or the traffic information received may be incorrect, all of the above factors may prevent travelers from obtaining true information of the macro urban road network.

In the mathematical modeling process of micro route choice, as described in Section III, it is often necessary to divide a day-to-day travel between certain OD points into several shorter discrete time periods for the convenience of analysis. It is assumed that at the beginning of each discrete time period, all travelers who are preparing to implement travel strategies at that time "aggregate" together and have an impact on the flow of road sections in the macro urban transportation network. In this process, there may be an "excessive travel demand" for the capacity of the road section, which will change the traffic flow characteristics of the road section in the next discrete time period, thereby affecting the micro travel decisions of travelers involved in the road section in the next discrete time period. The description of the implementation of real-time micro strategies throughout the entire process is crucial for building a complex system dynamics model of urban day-to-day travel. Figure 1 can be used to describe the process of travelers determining their real-time travel plans of whether "to pass through" a certain road section or not at the beginning of a certain discrete time period.

The mechanism by which travelers influence macro road traffic during the specific implementation of their micro travel decisions is also an important feature of the complex urban day-to-day travel system, similar to the "market mechanism" that considers the contradiction between demand and supply. As mentioned before, the micro travel decision-making of travelers can be roughly divided into two types. For certain discrete time period of a certain road section, one type of decision will definitely choose "to pass through", while the other type will determine whether to choose "to pass through" based on the traffic information at the beginning of the discrete time period. These two different micro travel decision-making methods determine their impact on macro section flow. Assuming that a_i represents a willingness of travelers to choose the road section to travel, the micro travel decision $a_i[t]$ made at time t will be a function of the section flow $x[t–1]$ during the implementation of the decision. Thus, the micro travel strategy $a_i[t, x[t–1]]$ can be mathematically defined based on the preferences of travelers, and according to traffic information, the micro strategy of choosing "to pass through" only when the traffic flow on the road section is lower than a certain value r_i (for example, when the traffic information shows that the road section is "unblocked", rather than "with high traffic volume" or "blocked") can be expressed as:

$$a_i[t, x[t-1]] = a_i[t]u[x[t-1] - r_i] \tag{18}$$

where $u[t]$ is a unit step function, namely for $t \geq 0$, there is $u[t] = 1$, while for $t < 0$, there is $u[t] = 0$.

In most cases, the formation of urban day-to-day travel system is jointly influenced by micro decisions such as "to pass through" a certain road section definitely and "to pass through" depending on the situation. The mathematical expression of the very micro travel decision-making of "to pass through" depending on the situation has been given by equation (18), and can be described as choosing "to pass through" at that discrete time period when the traffic flow on the road section is lower than r_i, and choosing "not to pass through" when the traffic flow is higher than r_i.

C. MACRO TRAVEL OBSERVATION IN INFORMATION ENVIRONMENT

From the literature review in Section II, it can also be seen that for travel systems, the observable variables are all about the traffic flows of different discrete time periods and road sections. Thus, it can be imagined that although there are numerous variables driving the complex system of day-to-day travel to varying degrees, only a very limited number of variables can truly observe the evolution of the urban day-to-day travel system from the macro point of view. Of course, since traffic flow as a function of time is a fundamental experimental output variable and can also be observed for all road sections in urban transportation network, it is necessary to discuss it in detail in this section.

Consider the typical situation where, as an observer of macro traffic conditions in urban day-to-day travel systems, a set of high-frequency traffic flow data for a specific time period can be obtained. Within the specific time period, the travel route of each micro traveler is recorded, and the recorded data can also be seen as a set of data points $x[t_1]$, $x[t_2]$, $x[t_3]$, …that appear in discrete time, separated by irregular time intervals. For more complex situations, such as holidays and weekends, travel behavior undergoes significant changes as it no longer falls under the category of "day-to-day travel". In addition, on weekdays after a long holiday, urban day-to-day travel system may also exhibit a situation that is different from the macro flow distribution on continuous working days, and some inherent macro flow distribution characteristics may also be changed. Therefore, in the complex system of urban day-to-day travel, it is often difficult to model and fit data, and there is even the possibility of obtaining false dynamic trends of macro traffic flow. Even if only day-to-day travel on weekdays has been studied, it is difficult to find the exact method of linking the flow values of certain discrete time period and road section on the previous Friday and the following Monday. There is no unique answer to the explanation of such time interval relationships. In order to avoid unintentionally introducing correlations due to different methods, any methodological exploration for the complex urban day-to-day travel system must be carefully conducted.

Of course, as macro observers of micro travel decisions in urban day-to-day travel, it is necessary to consider what kind of traffic flow function should be used in order to statistically test the observed data. Especially, it is hoped that

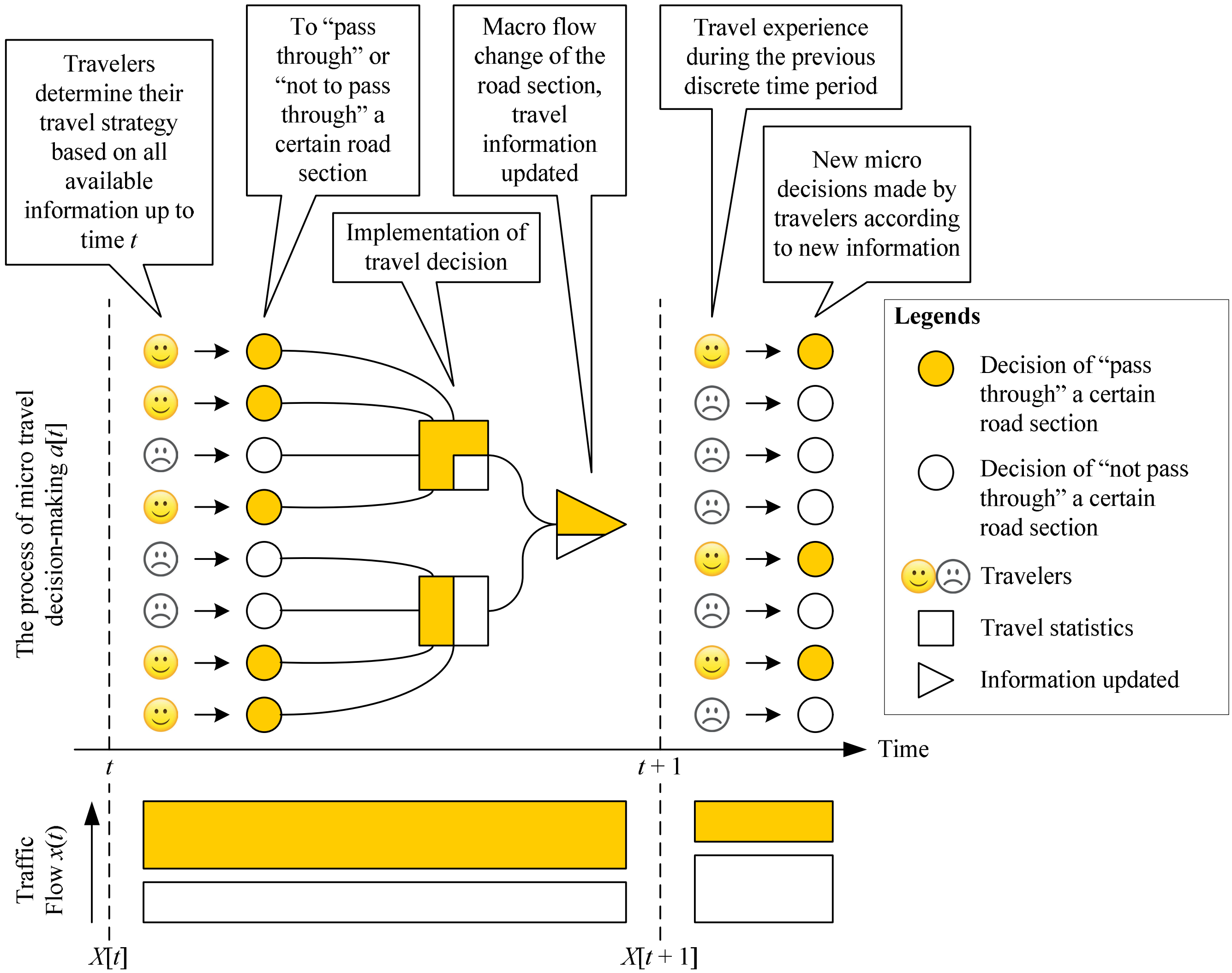

FIGURE 1. The process by which travelers determine their micro real-time travel plans.

through the selection of proper traffic flow function, as much meaningful information as possible can be extracted, and it is necessary to avoid introducing false correlations and biased understanding. When examining the traffic flow fluctuations, linear traffic flow variation:

$$\Delta x[t, t-\Delta t] = x[t] - x[t-\Delta t] \tag{19}$$

is always considered; moreover, in order to distinguish the relative importance of travel time cost and saved, it is necessary to discuss the relative changes in traffic flow. The definition of traffic flow change rate is

$$R[t, t-\Delta t] = \frac{\Delta x[t, t-\Delta t]}{x[t-\Delta t]} = \frac{(x[t] - x[t-\Delta t])}{x[t-\Delta t]} \tag{20}$$

In typical cases where traffic flow changes are much smaller than the traffic flow itself, namely when $\Delta x[t, t\text{–}\Delta t] \ll x[t, t\text{–}\Delta t]$, it can be assumed that $R[t, t\text{–}\Delta t]$ is proportionnal to $\Delta x[t, t\text{–}\Delta t]$.

It is of no doubt that as a complex system, it is often difficult to determine absolute "right" or "wrong" in the modeling of urban day-to-day travel. To select appropriate functions at appropriate times based on available data and the issues considered, would be rational in describing micro travel decisions and their corresponding macro traffic flow changes. Fortunately, in urban day-to-day travel system, due to the fact that the traffic flow change $\Delta x[t, t-\Delta t]$ is often much smaller than the traffic flow $x[t]$, or even $x[t\text{–}\Delta t]$, macro traffic flow functions often have similar statistical properties.

D. QUANTIFICATION OF TRAVEL RISK IN INFORMATION ENVIRONMENT

Corresponding to the diversity of models describing urban day-to-day travel, the travel risk in day-to-day travel can also be quantified via various possible means. As mentioned in Section III-B, the standard quantification of travel risk is usually based on fluctuation rate σ. By calculating the standard deviation of the empirical traffic flow distribution on a certain road section over a given discrete time period Δt, the empirical value of σ can be obtained. If the probability distribution function of these empirical traffic flow changes is indeed a Gaussian function, measuring travel risk by using σ has certain significance, as for the road section with an average flow change of 0, σ is the only parameter that can determine the shape of the Gaussian probability distribution function.

However, it should be pointed out that Gaussian function is not suitable for describing the tail part of the probability distribution function of traffic flow changes no matter how long the discrete time period Δt has been chosen, because the Central Limit Theorem only applies to the region of the distribution center. Thus, regarding the tail part of the probability distribution function, the parameter σ cannot effectively provide any information. In actual day-to-day travel systems, many uncertain risks for travelers are often contained at the tail part of the probability distribution function.

To avoid the above issues about σ, unexpected significant changes and their probabilities in road traffic flow should be focused on when quantifying the travel risk. Assuming there are n random variables correspond to traffic flow changes in n consecutive discrete periods, represented as $\{y_i\} = \{y_1, y_2, \ldots, y_n\}$, and assuming they are all identically and independently distributed with the same probability distribution $p[y]$, in this way, risk is composed of the maximum values contained in $\{y_i\} = \{y_1, y_2, \ldots, y_n\}$, The probability of $y_{\max} > \Omega$ can be calculated as

$$p[y_{\max} > \Omega] = 1 - [p_<[\Omega]]^n = 1 - [1 - p_>[\Omega]]^n \approx 1 - \exp[-np_>[\Omega]] \tag{21}$$

where Ω is the maximum increase in traffic flow set that the road section can withstand, $p_>[\Omega] \ll 1$, and its definition is

$$p_>[\Omega] = \int_\Omega^\infty p[y]\mathrm{d}y \tag{22}$$

For a series of n experiments, $p_>[y_{\max} > \Omega] \approx 1$–exp[–$n\cdot_>[\Omega]$] gives the probability of satisfying the condition $y_{\max} > \Omega$. For Gaussian probability distribution

$$p[y] = (1/\sqrt{2\pi\sigma^2})e^{-y^2/2\sigma^2} \tag{23}$$

Ω is proportional to σ, and even with the risk assessment method that focuses on the tail part of the distribution, the quantification of travel risk still indirectly depends on σ.

E. ESTABILISHMENT OF COMPLEX SYSTEM DYNAMICS FRAMEWORK FOR URBAN DAY-TO-DAY TRAVEL SYSTEM IN INFORMATION ENVIRONMENT

In section III-C, the properties of identically and independently distributed variables were discussed using probability theory, and it has been pointed out that in actual urban day-to-day travel system, the traffic flow changes often do not possess such properties. For the sake of completeness, in this section, the extension of probability theory to describe the case of non identically or independently distributed variables will be discussed, so as to establish a corresponding theoretical framework for complex system dynamics. For simplicity, assuming y_1 and y_2 are two traffic flow variables measured on the same road section at time t_1 and t_2, and to be specific, y_1 and y_2 are traffic flows or traffic flow increments on the same road section. The joint probability of y_1 appearing at t_1 and y_2 appearing at t_2 can be given by $p[y_2, t_2; y_1, t_1]$. If a set of traffic flow variables obtained sequentially, the joint probability can be described as $p[y_n, t_n; y_{n-1}, t_{n-1}; \ldots; y_1, t_1]$. For this joint probability, it is unknown in advance whether there is an implicit correlation between variables or how each variable is distributed at each moment. But with the help of conditional probability, it can be written as in equation (24), shown at the bottom of the next page. In equation (24), $p[y_i, t_i| y_{i-1}, t_{i-1}; \ldots; y_1, t_1]$ represents the conditional probability of seeing the result y_i at time t_i under the condition that historical values of $y_{i-1}, t_{i-1}; \ldots; y_1, t_1$ are known. If the conditional probability and past historical situation are known, the future value probability of the random variable can be determined. In a simple case, if for all n, $p[y_n, t_n; y_{n-1}, t_{n-1}; \ldots; y_1, t_1]$ remains unchanged at the starting point of any time,

$$\begin{aligned} p[y_n, t_n; y_{n-1}, t_{n-1}; \ldots; y_1, t_1] &= p[y_n, t_n|y_{n-1}, t_{n-1}; \ldots; y_1, t_1]p[y_{n-1}, t_{n-1}; \ldots; y_1, t_1] \\ &= p[y_n, t_n|y_{n-1}, t_{n-1}; \ldots; y_1, t_1]p[y_{n-1}, t_{n-1}|y_{n-2}, t_{n-2}; \ldots; y_1, t_1] \cdot p[y_{n-2}, t_{n-2}; \ldots; y_1, t_1] \\ &= \prod_{i=2}^{n} p[y_i, t_i|y_{i-1}, t_{i-1}; y_{i-2}, t_{i-2}; \ldots; y_1, t_1]p[y_1, t_1] \end{aligned} \tag{24}$$

$$p[y_i, t_i|y_{i-1}, t_{i-1}; y_{i-2}, t_{i-2}; \ldots; y_1, t_1] = p[y_i, t|y_{i-1}, t - T; y_{i-2}, t - 2T; \ldots; y_1, t - (i-1)T] \tag{25}$$

$$p[y_i, t_i|y_{i-1}, t_{i-1}; y_{i-2}, t_{i-2}; \ldots; y_1, t_1] = p[y_i, t|y_{i-1}, t - T; y_{i-2}, t - 2T; \ldots; y_{i-n}, t - (i-n)T] \tag{26}$$

$$p[y_t|y_{t-T}, y_{t-2T}, \ldots] = p[y_t|y_{t-T}, y_{t-2T}, \ldots, y_{t-nT}] \tag{27}$$

this process becomes a stationary process. If only the mean value and covariance are independent of time, this process is called a weakly stationary process or a generalized stationary process. At this point, the product term in equation (24) can be rewritten as in equation (25), shown at the bottom of the previous page, where T represents the duration of each discrete time period. If the conditional probability can be limited to a limited past history for all times, then as in equation (26), shown at the bottom of the previous page, can be given. Simplify y_i, t_i to y_t, the above equation can be rewritten as in equation (27), shown at the bottom of the previous page.

It can be seen that equation (27) describes an n-order Markov process. When $n = 1$, it only depends on the most recent historical value, at which point

$$p[y_t|y_{t-T}, y_{t-2T}, \ldots] = p[y_t|y_{t-T}] \tag{28}$$

usually referred to as a Markov process, this process is precisely the "coin toss" decision mentioned in section III-A, where y_{t-T} represents the road traffic at t–T and y_t represents the road traffic at t. If y_t and t are both discrete variables, $p[y_t|y_{t-T}]$ can be simply replaced by $p[y_t|y_{t-1}]$ to obtain a Markov chain. By using the following equation (29), the n-order Markov process of scalar can be transformed into a 1-order Markov process of an n-dimensional vector. Note that for Markov processes with M possible values, the M possible states are denoted as $y_{t,\alpha}$:

$$p[y_{t+1,\beta}] = \sum_{\alpha=1}^{M} p[y_{t+1,\beta}|y_{t,\alpha}]p[y_{t,\alpha}] \tag{29}$$

where $\alpha = 1, 2, \ldots, M$, $\beta = 1, 2, \ldots, M$. If the state probability vector of each component is defined as $\boldsymbol{p_t} = \{p[y_{t,1}], p[y_{t,2}], \ldots, p[y_{t,M}]\}$, and the transfer matrix is defined as $\boldsymbol{P} = p[y_{t+1}|y_{t,\beta}]$, then all states can be represented as $\boldsymbol{p_{t+1}} = P\cdot \boldsymbol{p_t}$. Considering that the transfer matrix is time independent, by repeatedly applying $p_{t+1} = \boldsymbol{P} \cdot p_t$, there is $\boldsymbol{p_{t+1}} = P^{\boldsymbol{n}}\cdot \boldsymbol{p_t}$. Therefore, the power of the transfer matrix determines the time evolution behavior of the system. If the system can reach any other state from each state, then the system has ergodicity of each state. Under steady-state conditions, there is $\boldsymbol{p_{t+1}} = p_t = \boldsymbol{p}$, thus, there is

$$\boldsymbol{p} = \boldsymbol{P} \cdot \boldsymbol{p} \tag{30}$$

By using equation (30), the steady-state probabilities of each state in all M states can be obtained.

Here in this section, a complex system dynamics method has been provided to describe the statistical properties of urban day-to-day travel system. The method is different from the macro traffic evolution model based on simple "coin toss" micro travel decision-making discussed in section III-B, but instead applies an n-order Markov model with $n > 1$. This complex system dynamics model can not only express the non identically distributed properties of traffic flow on road sections at different times, but also introduce time correlation and time dependence into the model to a certain extent.

V. THEORETICAL VERIFICATION AND VALIDATION

A. MODEL DESCRIPTION VERIFICATION OF MICRO TRAVEL STRATEGIES

Through the discussion in previous sections, it can be seen that for urban day-to-day travel system, there is a time correlation in the sequence of macro traffic flow changes. In this section, Markov process in the case of $n = 4$ will be discussed and calculated in detail in order to examine the impact of time correlation on the establishment of traffic models and the calculation of travel benefits (travel time saved) and risks (extra travel time cost).

If a traveler pays attention to both travel routes A and B simultaneously. At certain discrete time periods every day, it is necessary to make a decision to take the route A or B for travel. After making the decision, the travel plan is implemented; and after completing the travel plan, the day-to-day travel process is summarized and analyzed. The binary state of the micro route choice made is formed based on the length of the actual day-to-day travel time compared to the planned time. To quantify the specific effects of micro route choice, an integration mechanism is established here. Assuming that the travel time of a certain route choice is shorter than the planned time, the decision effect is considered successful, and a score of +1 is given; if the travel time chosen of a certain route choice is longer than the planned time, the decision effect is considered a failure, and a score of –1 is given.

For travel route A, when the planned travel time is rational, the probability of a good decision is generally $p_{\text{success,A}} = 0.5$. Therefore, on average, the micro route choice decision may be either successful or unsuccessful. If the development of cities leads to an increase in population and unexpected factors that may occur on roads, the actual probability of successful decision-making should be less than 0.5. The same conclusion also applies to travel route B. Therefore, for the two routes, regardless of the choice, on average, the probability of decision failure is slightly higher.

For simplicity, consider a simple scenario where the probability $p_{\text{success,A}}$ of the success in choosing route A is independent of history, while the probability $p_{\text{success,B}}$ of the success in choosing route B depends on previous situations. If $s[t]$ represents the cumulative score of a traveler at the beginning of the t-th day, and if the route choice decision is successful on that day, the cumulative score of the traveler becomes $s[t+1] = s[t]+1$; If the decision fails, its cumulative score becomes $s[t+1] = s[t]–1$. Since the success in choosing route B depends on the previous situations, its cumulative score is no longer a Markov process with $n = 1$. Assuming that $p_{\text{success,B}}$ depends on the results of the previous 3 days as mentioned at the beginning of this section, a $n = 4$ Markov chain can be constructed using states $s[t–2] – s[t–3]$, $s[t–1] – s[t–2]$, $s[t] – s[t–1]$, whose components are the cumulative score changes on day t–3, day t–2, and day t–1. The success and failure probabilities in choosing route A and route B can be summarized in Figure 2, and the cumulative score vector state transition of choosing route B for 3 consecutive days can be summarized in Figure 3.

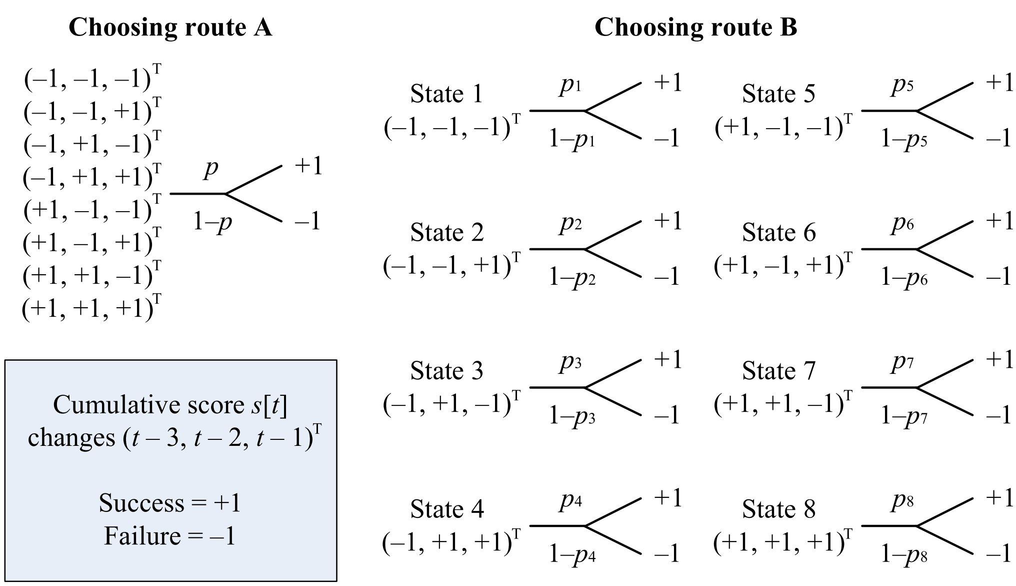

FIGURE 2. The success and failure probabilities in choosing route A and route B based on the results of cumulative score vector changes in the previous 3 days.

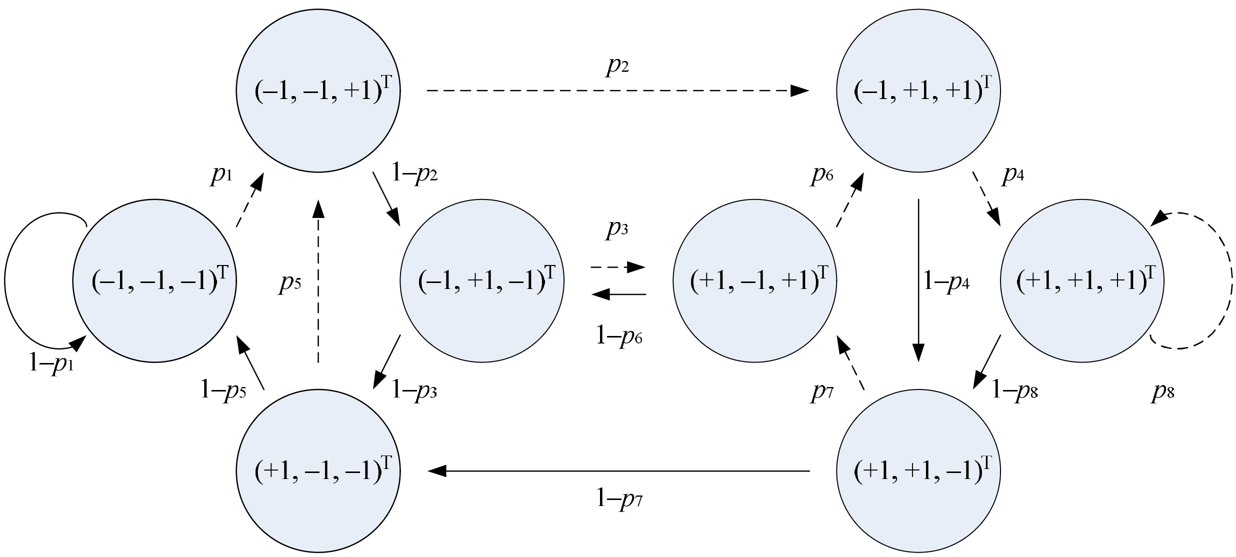

FIGURE 3. Cumulative score vector state transition diagram of choosing route B for 3 consecutive days.

For decisions of choosing route B with non-trivial time correlations, and assuming that travelers can only choose route B for travel, using the strategy evaluation cumulative score s_B owned by all travelers on decisions of choosing route B during the day t–3, day t–2, and day t–1, the following state vector can be defined as:

$$Y_B[t] = \begin{pmatrix} s_B[t] - s_B[t-1] \\ s_B[t-1] - s_B[t-2] \\ s_B[t-2] - s_B[t-3] \end{pmatrix} \tag{31}$$

which has 8 different states $(-1,-1,-1)^T$, $(+1,-1,-1)^T$, $(-1,+1,-1)^T$, $(+1,+1,-1)^T$, $(-1,-1,+1)^T$, $(+1,-1,+1)^T$, $(-1,+1,+1)^T$, $(+1,+1,+1)^T$.

If $\sigma_i[t]$ is used to represent the probability that the traveler will be in state i on day t, since there are only 8 possible i values, these probabilities can form an 8-dimensional vector $\boldsymbol{\sigma}[\boldsymbol{t}] = (\sigma_1[t], \sigma_2[t], \sigma_3[t], \sigma_4[t], \sigma_5[t], \sigma_6[t], \sigma_7[t], \sigma_8[t])^T$. With the help of the probability $p_{\text{success},B}$, the dynamic equation of probability vector evolution can be given:

$$\boldsymbol{\sigma}[t+1] = \boldsymbol{D} \cdot \boldsymbol{\sigma}[t] \tag{32}$$

where $\boldsymbol{D}$ can be as in equation (33), shown at the bottom of the next page, according to Figure 3. Therefore, as discussed in Section IV-E, a Markov chain can be constructed, and the steady-state of the system means where

$\sigma[t+1] = D\cdot \sigma[t] = \sigma[t]$, namely

$$(\boldsymbol{D}-\boldsymbol{E})\boldsymbol{\sigma} = 0 \tag{34}$$

where $\boldsymbol{E}$ is the identity matrix. In addition, there is

$$\sigma_1+\sigma_2+\sigma_3+\sigma_4+\sigma_5+\sigma_6+\sigma_7+\sigma_8 = 1 \tag{35}$$

Therefore, there is a solution as in equation (36), shown at the bottom of the page, shows, where

$$N = -1-3p_1+p_5+p_3+p_8+o(p) \tag{37}$$

where o(p) is a higher-order term formed by multiplying multiple probabilities less than 1. Generally, $p_1+p_2+p_3+p_4+p_5+p_6+p_7+p_8 \neq 1$. Therefore, for steady-state conditions, the probability $p_{\mathrm{success,B}}$ within a range of changes per unit discrete time period (usually one day in urban day-to-day travel system) is

$$\begin{aligned} p_{\mathrm{success,B}} &= \sum_{i=1}^{8}\pi_i p_i \approx \frac{p_1}{p_2}+p_1+\ldots \\ &= p_1(1+\frac{1}{p_2}+\ldots) \end{aligned} \tag{38}$$

where p_2 is for sure a value less than 1, so $(1+1/p_2+\ldots)$ should be a value greater than 2. This means that when p_1 does not need to take a very large value, $p_{\mathrm{success,B}}$ can be greater than 1/2, and the failure criterion for travelers when choosing route A is $1-p>p$.

Next, consider the scenario where travelers randomly switch between two routes for micro route choice. Assuming that this randomness also follows the "coin toss" decision, the analysis for this scenario will be the same as for the scenario where only path B can be chosen. Simply replace p_i with p'_i, and p'_i can be seen as $(p_i+p)/2$. Therefore, when randomly switching between two routes, the basis for determining the failure of a decision is

$$p_{\mathrm{success}} \approx \frac{p_1+p}{2}(1+\frac{2}{p_2+p}+\ldots) \tag{39}$$

From equation (39), it can be seen that if a traveler makes a random decision switching between the 2 routes A and B, the probability of decision failure exceeding the probability of success is very low for a period of time. The effect of success in failure is somehow contrary to the actual travel experience intuition, and the reason for such counterintuitive effects lies in the existence of time correlation effect. For urban day-to-day travel participants, this effect is undoubtedly very tempting, but there are also certain traps in it. Performing a switching operation between two routes with a high probability of decision failure may lead to systematic success, but correspondingly, it is also possible to ultimately lead to systematic failure during the switching between the routes that all travelers hope to achieve decision success, depending entirely on the time correlation effect between the 2 routes.

This section is based on the complex system dynamics model of urban day-to-day travel system established in Section IV, and discusses in detail its portrayal of the implementation of micro level travel decisions by travelers. Through the discussion, it can be seen that compared to simple "coin toss" decisions, complex system dynamics can better reflect the individual personalities of urban day-to-day travelers in making travel decisions, because they can evaluate travel profits and risks based on historical decisions and their effects, and then provide decisions for the next travel. Of course, this discussion is only based on $n=1$ (i.e. "coin toss" situation) and $n=4$ Markov process situations.

$$\boldsymbol{D} = \begin{pmatrix} 1-p_1 & 0 & 0 & 0 & 1-p_5 & 0 & 0 & 0 \\ p_1 & 0 & 0 & 0 & p_5 & 0 & 0 & 0 \\ 0 & 1-p_2 & 0 & 0 & 0 & 1-p_6 & 0 & 0 \\ 0 & p_2 & 0 & 0 & 0 & p_6 & 0 & 0 \\ 0 & 0 & 1-p_3 & 0 & 0 & 0 & 1-p_7 & 0 \\ 0 & 0 & p_3 & 0 & 0 & 0 & p_7 & 0 \\ 0 & 0 & 0 & 1-p_4 & 0 & 0 & 0 & 1-p_8 \\ 0 & 0 & 0 & p_4 & 0 & 0 & 0 & p_8 \end{pmatrix} \tag{33}$$

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \\ \sigma_7 \\ \sigma_8 \end{pmatrix} = \frac{1}{N}\begin{pmatrix} -(p_6p_7p_8+p_5p_6p_7-p_5p_6p_7p_8+p_3p_8-p_8p_6p_3-p_5p_6p_3+p_5p_8p_6p_3+1+p_6p_3-p_5p_3p_8 \\ \qquad -p_6p_7+p_5p_3-p_5-p_3-p_8+p_5p_8) \\ p_1(p_6p_7-p_6p_7p_8-p_6p_3+p_8p_6p_3-p_3p_8+p_3-1+p_8) \\ p_1 \\ -p_1(p_8-1)(-p_2-p_6p_3+p_2p_3) \\ p_1(p_6p_7-p_6p_7p_8-p_6p_3+p_8p_6p_3-p_3p_8+p_3-1+p_8) \\ -p_1(-p_2p_3-p_3p_8+-p_3p_2p_8+p_3-p_2p_7p_8+p_2p_7) \\ -p_1(p_8-1)(-p_2-p_6p_3+p_2p_3) \\ p_4p_1(-p_2-p_6p_3+p_2p_3) \end{pmatrix} \tag{36}$$

In practical model applications, n can be adjusted based on the characteristics of travelers.

B. THE EXPLANATORY EFFECT OF STYLZED MACRO TRAFFIC FLOW CHANGES ON ROAD SECTIONS

Based on the definition of travel decisions of travelers in the complex system dynamics model established earlier, a macro description of urban day-to-day travel system on specific road sections can also be constructed, forming a macro-micro integrated model. In this section, the modeling effect from macroscopic point of view, namely the explanatory effect of the model on the stylized facts in urban road traffic flow, is evaluated.

As a complex system, the drastic flow changes in road traffic indicate that the traffic flow continues to rise or fall over a period of time. Through general travel cognition, it can be found that sudden changes in traffic flow occur from time to time, and their duration and amplitude of increase and decrease are not fixed, relying on the properties of time correlation. As with Section V, complex system dynamics model based on $n = 4$ Markov process is considered, the change of traffic flow on a specific road section can be given by the following equation:

$$\begin{aligned}\Delta x[t, t-1] \equiv \Delta x[t] &= x[t] - x[t-1]\\ &= \varepsilon[t] + \varepsilon[t-1]\varepsilon[t-2] \qquad (40)\end{aligned}$$

where $\varepsilon[t]$ is a Gaussian white noise process with zero mean value and unit variance, namely $p[\varepsilon[t] = +1] = p[\varepsilon[t] = -1] = 1/2$. Note that choosing a specific road section for a certain traveler will only result in a traffic flow increment of 1 or 0, for the ease of calculation here, –1 instead of 0 is used. Therefore, the mean value of traffic flow changes on the road section is

$$\overline{\Delta x[t]} = \overline{\varepsilon[t]} + \overline{\varepsilon[t-1]\varepsilon[t-2]} \qquad (41)$$

where $\varepsilon[t–1]$ and $\varepsilon[t–2]$ take ± 1 with the same probability, so their mean values are also 0. And it is not difficult to see that the mean value of the product of traffic flow changes at different times, namely the mean value of the autocorrelation function $\Delta x[t]\cdot\Delta x[t']$ is also 0. Considering various situations comprehensively, it is not difficult to see that the mean value of $\Delta x[t]\cdot\Delta x[t']$ is also 0. As the mean value and the autocorrelation function of $\Delta x[t]$ are both 0, it seems like a "random process", which means that when only the lower-order time correlations has been considered, it may lead to a conclusion that the sequence of traffic flow changes is just like a "coin toss" decision.

However, by calculating higher-order time correlations, it can be found that non-zero higher-order correlations that do not exist in strict "coin toss" decision, can appear in the model presented in this paper, which means that standard linear statistical analysis tools may mask the actual existence of higher-order time correlations. If the 3rd-order correlation function $\Delta x[t–2]\cdot\Delta x[t–1]\cdot\Delta x[t]$ is considered, its value should be 0 in the "coin toss" decision model, but in the model presented in this paper, it is not zero and the condition mean value of "$\Delta x[t]|\Delta x[t–2]\cdot\Delta x[t–1]$" is always not 0. To be specific, the mean value of "$\Delta x[t]|\Delta x[t–2]\cdot\Delta x[t–1]$" should be proportional to "$\Delta x[t–2]\cdot\Delta x[t–1]$" and is not zero, so the flow sequence $x[t]$ has a certain degree of predictability, which is also a result of the memory that urban day-to-day travel systems usually have, that is, the expected value of traffic flow change in a future discrete time period, or a time step in the future, depends on the product of the results of the two time steps beforehand. Next, it is supposed to analyze in detail the consequences of higher-order time correlations on the magnitude and duration of drastic flow changes in urban road traffic. The magnitude and duration of drastic flow changes on a certain road section are defined as the magnitude and time experienced by the flow $x[t]$ from the local maximum to the local minimum, or the magnitude and time experienced by the flow $x[t]$ from the local minimum to the local maximum. By employing equation (40), the traffic flow changes for each discrete time period or time step can be obtained, as shown in Table 1.

TABLE 1. Traffic flow changes for each discrete time period in the model with higher order time correlations.

$\varepsilon[t–2]$	$\varepsilon[t–1]$	$\varepsilon[t–1]$	$\Delta x[t]$
+1	+1	+1	+3
+1	+1	–1	+1
+1	–1	+1	+1
+1	–1	–1	–1
–1	+1	+1	+1
–1	+1	–1	–1
–1	–1	+1	–1
–1	–1	–1	–3

From Table 1, it can be seen that the maximum flow variation for 3 discrete time periods or time steps is 3 (+3 or –3); in addition, some information about the maximum amplitude of traffic changes and their duration can also be seen from the table, such as random sequences "–1,–1,–1", and "+1, +1, +1" can generate a traffic sequence with a maximum duration of 3 steps and a maximum traffic variation of +3 and –3. This means that the maximum sequence of traffic changes is hidden in the standard statistical calculation conclusions of the model, but the time correlation that only appears in the form of higher-order correlation can be manifested in traffic flow fluctuations. Compare the $\Delta x[t]$ corresponding to the "coin toss" decision

$$\Delta x[t] = x[t] - x[t-1] = \varepsilon[t] \qquad (42)$$

It can be observed that there is no difference between the models given by equation (40) and (42) in terms of mean value and autocorrelation coefficient, but in fact, they are very different. In special cases, the model given in equation (42) can generate drastic flow change with infinite time period and amplitude, which corresponds to an infinitely long random "coin toss" decision that always yields positive results. And in terms of quantitative calculation, it can be found that the probability of the occurrence of a drastic flow change with the amplitude of at least n is 0.5^n. Therefore, in the macro road network of urban day-to-day travel systems, there is a very small and limited possibility of generating a drastic flow change with arbitrary duration and amplitude on specific road sections. This conclusion is significantly different from the time correlation given in equation (40). Therefore, the time correlation in traffic flow changes has a dramatic impact on transportation system dynamics, and the existence of higher-order time correlation limits the duration and magnitude of traffic flow fluctuations. On some level, higher-order time correlations have an important impact on the analysis of micro route choices in urban day-to-day travel systems.

Through the theoretical verification and validation presented in this section, it can be seen that under the influence of micro level travel decisions made by urban day-to-day travelers, the traffic flow changes in specific road section in the macro level road network can be described by the complex system dynamics model proposed in this paper. More specifically, the results (success or failure) of micro level travel decisions, as well as travel decisions made in the subsequent discrete time period after obtaining the information of macro level road traffic flow and past decision results, can also be described through the proposed model, thus achieving a full description of the personalities of urban day-to-day travelers. Of course, in the theoretical verifications and validations made in this section, only $n = 1$ and $n = 4$ Markov process situations in the proposed complex system dynamics model for whether to choose a specific road section in day-to-day travel have been discussed. However, it is evident that by changing the value of n, the model can achieve effective description of not only personalized micro travel decision-making process, but the macro road flow changing process affected by micro travel decision-making in information environment as well.

VI. NUMERICAL ANALYSIS

In this section, in order to conduct a more comprehensive performance evaluation of the established complex system dynamics model that includes micro route choice game between travelers, and to examine its performance in explaining the dynamics of urban day-to-day travel systems, numerical analysis is conducted on the established model to observe whether it can exhibit system dynamics and stylized facts similar to macro road flow fluctuations. In the discussion, without considering the daily increase in urban traffic flow due to the increasing number of vehicles, the reference value $L[t]$ of the flow change on the selected road section is always 0. That is to say, for each day-to-day traveler, when choosing a specific road section to travel, if the increment of traffic flow on this road section is less than $L[t] = 0$ compared to the reference traffic flow value, then the micro strategy $s[t + 1]$ for choosing this road section will be added 1 point; on the contrary, $s[t + 1]$ will be deducted 1 point. However, considering the inherent characteristics of the urban day-to-day travel system, in numerical simulation, the value of the rating threshold R that travelers should reach when making micro route choice decisions is set to a relatively smaller value of 3, so as to avoid affecting the interpretation of the rigid demand in day-to-day travel of the model. That is to say, as discussed in Section V-A, a certain route will only be selected when $s[t] \geq 3$.

Figure 4 shows the situation that local day-to-day travelers $P = 500$, and the flow rate for ensuring normal traffic on a certain road section during a specific discrete time period is $P/2 = 250$. As mentioned earlier, with $L[t] = 0$, each traveler has a strategy number $s = 2$ related to $Y_B[t]$ that discussed in Section V-A, and the memory time for strategy cumulative score is 10 (i.e. considering the actual experience of the same type of travel in the past 10 days). As discussed above, in the numerical experiment, the confidence index is set as $R = 3$. To traverse the memory length of all travelers, the mastery of the past information length m of travelers is taken as 3 to 10. The numerical simulation results for $m = 3$ and $m = 10$ are presented in Figure 4 respectively.

From Figure 4, it can be seen that the difference in the memory length of transportation information for day-to-day travel significantly affects the results of micro route choice decision-making, thereby affects the evolution of macro traffic flow. Considering that the total number of states M in the state set is $M = 2^m$, a larger value of m corresponds to a larger M/P ratio, which means that the total number of combinations of the historical states on a certain road section has a greater ratio to the number of travelers participating in this road section related day-to-day travel. It can be seen that when $m = 3$, the M/P ratio is small, as mentioned above, each traveler has a strategy number $s = 2$, thus, the total number of available strategy combinations for travelers is $2^M = 2^8 = 256$, which is relatively small compared to 500 local day-to-day travelers. As a result, many travelers hold the same successful strategy R^*, and the score of $s[t]$ of strategy R^* is the same. At the beginning of the next discrete time period, these travelers will take the same actions according to R^* to form an aggregation effect; at the same time, only a small number of travelers may hold the anti-correlation strategy $\underline{R}^*$ of this successful strategy and take the opposite action, resulting in a small anti-aggregation effect. This high aggregation state of travelers due to the lack of available strategy combinations creates a huge fluctuation in road traffic in Figure 4(a). In contrast to the above situation, when $m = 10$, the M/P ratio is high, the total number of available strategy combinations is $2^M = 2^{1024}$, which is very large compared to the 500 local day-to-day travelers. It is difficult for any two travelers to hold exactly the same micro strategy combination, resulting in a small aggregation effect. The number of travelers holding any two anti correlation strategies is roughly the same, as a result, the flow fluctuation of the road section in Figure 4(b) is relatively small.

It can be seen that in the case of small m value, the proposed model based on complex system dynamics can reproduce some of the stylized facts that occur in real urban day-to-day travel system. From Figure 4(a), it can be observed that in the case of a small m value, the flow of the road section is

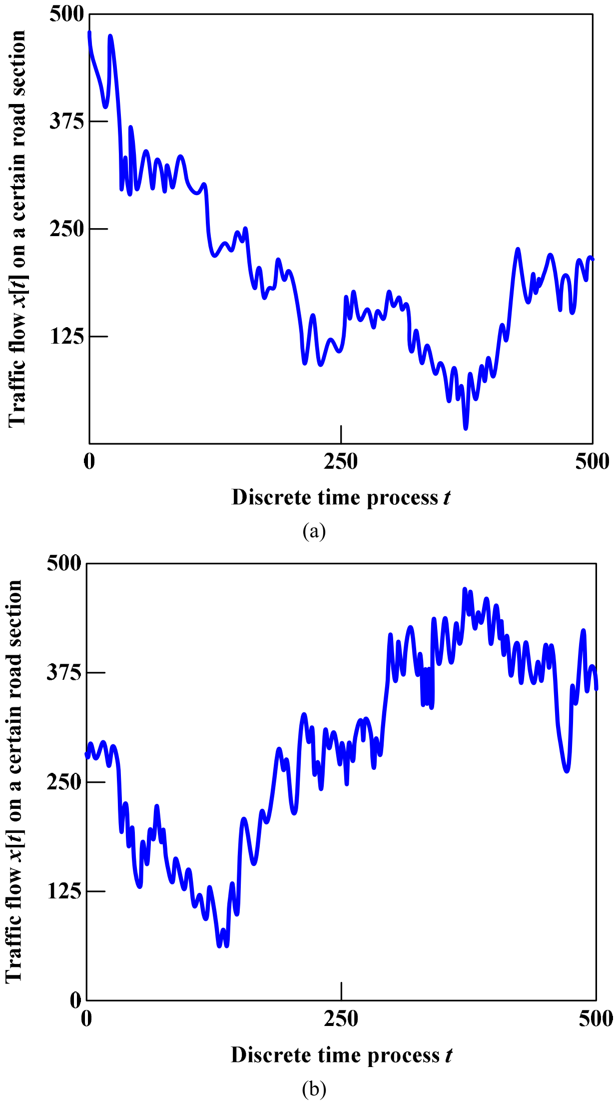

FIGURE 4. Numerical analysis of the changing process of traffic flow $x[t]$ on certain road section.

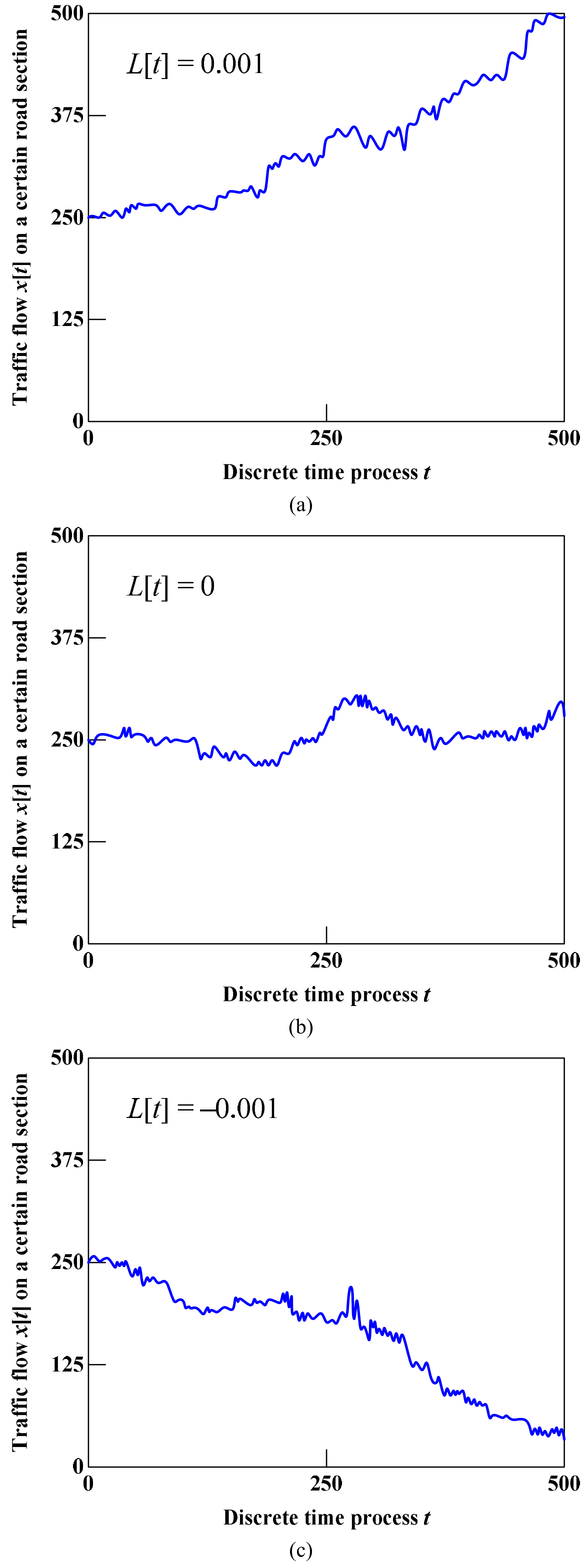

FIGURE 5. Numerical analysis of the evolution process of traffic flow $x[t]$ on certain road section with different L[t] values set.

relatively small compared to the situation in Figure 4(b), and cluster phenomena caused by aggregation effects often occur. From the evaluation point of view, as long as a large number of travelers take the same action, this strategy will be penalized, resulting in high-frequency traffic flow fluctuations on the road section, accompanied by the thick tail phenomenon discussed in Section V-B. In the case of large m value, the proposed model has a more random overall performance of the traffic flow sequence because many day-to-day travelers do not have the possibility of behavior aggregation due to holding the same strategy combination. As there is a fact that the aggregation behavior of day-to-day travelers during commuting often occurs due to OD similarity, models with smaller m value seem to be closer to the actual urban day-to-day travel situation.

Next, numerical analysis is conducted on the system evolution behavior with different $L[t]$ values in the situation when $m = 5$, as shown in Figure 5. It is obvious that for situations when $L[t] > 0$, the traffic flow on certain road section is supposed to increase gradually, which seems to be in line with the development situation of cities. When travelers generally expect a positive increase in traffic flow, choosing a road section with an appropriate amount of traffic flow increment seems to be a rational thing. Therefore, travelers tend to make decisions "to pass through that road section". For situations when $L[t] < 0$, the traffic flow on the road section is supposed to decrease gradually, this situation is applicable to simulating a period of time when the road section is limited by special reasons. In this situation, travelers tend to make decisions "not to pass through that road section". For situations when $L[t] = 0$, the traffic flow of the road section is in a stable and approximately non deviating process from its mean.

For different values of $L[t]$, travelers will have corresponding changes in the formulation of micro path selection strategies. In different time stages, different $L[t]$ values will trigger the urban day-to-day system to evolve into different states, forming a new supply-demand balance relationship between road sections and travelers in the macro transportation system. Although the traffic flow variation patterns shown in Figure 5 are in oscillatory states, their trends can be captured and understand by complex system dynamics. It can be seen that the expected change in $L[t]$ for the increase in road traffic volume will also generate a clustering effect among traveler, and this change is of great significance for urban transportation planners and managers to rationally evaluate the changing and evolution of day-to-day travel behavior caused by road traffic flow control, road closure, and new road construction, as well as the effectiveness of road planning and design.

VII. CONCLUSION, COMMENTS, AND FUTURE WORK

In this paper, the basic physical concepts of complex system dynamics involved in analyzing the micro route choice of urban day-to-day travel in information environment and the macro traffic flow under its influence have been discussed, focusing on macro-micro integrated modeling in explaining the macro traffic flow variable attributes, travel risk quantification, and time correlation effects in urban day-to-day travel under the influence of micro individuals. Specifically, this paper starts with basic probability knowledge and discusses the possibility and methods of explaining the traffic flow fluctuations in urban day-to-day travel system based on micro "coin toss" route choice decision-making related probability knowledge; on this basis, the dynamics impact of micro route choice decision-making on macro traffic flow is discussed, and thus a discussion on evaluating travel risk at the level of macro traffic flow changes is introduced; then, a theoretical framework of non identically or independently distributed variables has been established to discuss the correlation and independence of traffic flow fluctuations in different discrete time periods under the influence of micro route choice decision-making in information environment; in addition, further analysis is conducted on the methods for defining the success and failure of micro decision-making, and through numerical analysis, the possibility of higher-order time correlations in macro road traffic flow changes under the influence of micro decision-making evaluation, as well as whether the existence of such correlations would cause drastic flow changes in macro road traffic flow and to what extent, are all discussed and analyzed.

To be frank, the model discussed in this paper can be described as the prototype of a macro-micro integrated modeling methodology for urban day-to-day travel based on complex system dynamics in information environment. It is called the prototype because there are many micro factors that need to be considered to fully simulate urban day-to-day travel systems. The day-to-day travelers are influenced by the available time of their time changes in each discrete time period, as well as several unexpected situations in the macro transportation network, such as regulations and accidents. Of course, the original intention of modeling the micro route choice of urban day-to-day travel system is to use as few variables as possible to explain travel behavior and its macro performance to the greatest extent. The basic concepts of complex system dynamics discussed in this paper can serve as the basis for the discussion of micro route choice decision-making in urban day-to-day travel system. In the future, based on the model proposed in this paper, in-depth research will also be conducted on the interesting changes that may occur in micro route choice decision-making and its macro impact under the influence of different types of global information and local information that is only mastered by a few travelers in the information age.

APPENDIX A
NOMENCLATURE

x_i	Traffic flow in the i-th discrete time period.
Δx_i	Traffic flow changes in the i-th discrete time period.
Δx	Traffic flow changes in any discrete time period if Δx_i are identically and independently distributed.
$\Delta x_{n,0}$	Traffic flow changes between the n-th and the 0th discrete time periods.
$p[\Delta x_i]$	The probability distribution function of Δx_i.
$p[\Delta x]$	The probability distribution function of Δx_i.
$p[\Delta x_{n,0}]$	The probability distribution function of $\Delta x_{n,0}$.
$f[\Delta x]$	The probability distribution function of traffic flow changes of the certain road section discussed.
$\sigma_{i,i-1}$	The 2nd order moment of Δx_i (the variance of Δx_i).
σ	The 2nd order moment of Δx (the variance of Δx).
$\sigma_{n,0}$	The 2nd order moment of $\Delta x_{n,0}$ (the variance of $\Delta x_{n,0}$).
n	The number of consecutive discrete time periods, the order of a Markov process in describing urban day-to-day travel.
k	An intermediate variable set for deriving $p[\Delta x_{n,0}]$.

$a_i[t]$	The micro travel decision made by the i-th urban day-to-day traveler at time t.
$x[t]$	The traffic flow of the certain road section discussed at time t.
$a_i[t, x[t-1]]$	The micro travel strategy made by the i-th urban day-to-day traveler at time t by considering the traffic flow of the certain road section discussed at time $t-1$.
r_i	A certain reference value of the traffic flow of the certain road section discussed.
$u[t]$	A unit step function.
$R[t, t-\Delta t]$	The traffic flow change rate of the certain road section discussed between time t and $t-\Delta t$.
y_i	The traffic flow of the certain road section discussed in the i-th time period of n consecutive discrete time periods ($i = 1, 2, \ldots, n$).
Ω	The maximum increase in traffic flow set that the certain road section discussed can withstand.
t_i	The i-th time period of n consecutive discrete time periods ($i = 1, 2, \ldots, n$).
T	The duration of each discrete time period.
M	The number of all possible states in an n-order Markov process.
α, β	2 integer constants ranging from 1 to M.
$\boldsymbol{P_t}$	State probability vector.
$\boldsymbol{P}$	State transfer matrix.
$P_{\text{success},i}$	The probability of success (the travel time is shorter than planned time) in choosing road section i.
$s[t]$	Cumulative score of a traveler at time t.
$s_i[t]$	Cumulative score of choosing road section i of a traveler at time t.
$Y_i[t]$	State vector of cumulative scores formed by $s_i[t]-s_i[t-1]$, $s_i[t-1]-s_i[t-2]$, …, its dimension is determined by n.
$\sigma_i[t]$	The probability that a traveler will be in state i at time $t (i = 1, 2, \ldots, M)$.
$\boldsymbol{D}$	A specific state transfer matrix.
$\boldsymbol{E}$	An identity matrix.
p	The probability of using "coin toss" decision to choose a specific road section for travel and finally achieving success.
p_i	The probability of using past traffic flow and travel situation information to choose a specific road section for travel and achieving success ($i = 1, 2, \ldots, M$).
p'_i	A probability that equals to $(p + p_i)/2$.
$\varepsilon[t]$	A Gaussian white noise process with zero mean value and unit variance.
$L[t]$	A reference value of the traffic flow change on a specific road section.
R	Rating threshold that travelers should reach when making certain micro route choice decisions.
R^*	Successful travel decision-making strategy.
$\underline{R}^*$	Anti-correlation travel decision-making strategy of R^*.
P	The total number of local day-to-day travelers.
s	The number of decision making strategies that each traveler has.
m	The mastery of the past information length of travelers, usually $m = n - 1$, and $M = 2^m$.

REFERENCES

[1] M. A. van Essen, "One route or the other?" Ph.D. dissertation, Fac. Eng., Univ. Twente, Enschede, The Nethelands, 2014.

[2] W. Nai, Z. Yang, D. Lin, D. Li, and Y. Xing, "A vehicle path planning algorithm based on mixed policy gradient actor-critic model with random escape term and filter optimization," *J. Math.*, vol. 2022, no. 1, Jan. 2022, Art. no. 3679145, doi: 10.1155/2022/3679145.

[3] P. D. Site, "On the equivalence between SUE and fixed-point states of day-to-day assignment processes with serially-correlated route choice," *Netw. Spatial Econ.*, vol. 17, no. 3, pp. 935–962, Sep. 2017, doi: 10.1007/s11067-017-9353-2.

[4] W. Nai, Z. Yang, D. Li, L. Liu, Y. Fu, and Y. Guo, "Urban day-to-day travel and its development in an information environment: A review," *Sustainability*, vol. 16, no. 6, p. 2572, Mar. 2024, doi: 10.3390/su16062572.

[5] W. Li, S. Ma, N. Jia, and Z. He, "An analyzable agent-based framework for modeling day-to-day route choice," *Transportmetrica A, Transp. Sci.*, vol. 18, no. 3, pp. 1517–1543, Dec. 2022, doi: 10.1080/23249935.2021.1952336.

[6] X. Xu, K. Qu, A. Chen, and C. Yang, "A new day-to-day dynamic network vulnerability analysis approach with weibit-based route adjustment process," *Transp. Res. Part E, Logistics Transp. Rev.*, vol. 153, Sep. 2021, Art. no. 102421, doi: 10.1016/j.tre.2021.102421.

[7] Z. Zhu, A. Mardan, S. Zhu, and H. Yang, "Capturing the interaction between travel time reliability and route choice behavior based on the generalized Bayesian traffic model," *Transp. Res. Part B, Methodol.*, vol. 143, pp. 48–64, Jan. 2021, doi: 10.1016/j.trb.2020.11.005.

[8] C. Meneguzzer, "Modeling the combined effect of travelers' contrarian behavior, learning and inertia on the day-to-day dynamics of route choice," *Appl. Sci.*, vol. 13, no. 5, p. 3294, Mar. 2023, doi: 10.3390/app13053294.

[9] R. Zhong, J. Xiong, Y. Huang, A. Sumalee, A. H. F. Chow, and T. Pan, "Dynamic system optimum analysis of multi-region macroscopic fundamental diagram systems with state-dependent time-varying delays," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 9, pp. 4000–4016, Sep. 2020, doi: 10.1109/TITS.2020.2994347.

[10] R. Kazhamiakin, E. Loria, A. Marconi, and M. Scanagatta, "A gamification platform to analyze and influence citizens' daily transportation choices," *IEEE Trans. Intell. Transp. Syst.*, vol. 22, no. 4, pp. 2153–2167, Apr. 2021, doi: 10.1109/TITS.2021.3049792.

[11] P. Zhang, F. Lei, Y. Ma, L. Lu, and G. Zhu, "The general system model of vehicle TDM effect on urban road traffic," *IEEE Intell. Transp. Syst. Mag.*, vol. 13, no. 1, pp. 160–168, Sep. 2021, doi: 10.1109/MITS.2018.2889689.

[12] F. Yang and D. Zhang, "Day-to-day stationary link flow pattern," *Transp. Res. Part B, Methodol.*, vol. 43, no. 1, pp. 119–126, Jan. 2009, doi: 10.1016/j.trb.2008.05.005.

[13] M. J. Smith, "The existence and calculation of traffic equilibria," *Transp. Res. Part B, Methodol.*, vol. 17, no. 4, pp. 291–303, Aug. 1983.

[14] M. J. Smith, "The stability of a dynamic model of traffic assignment—An application of a method of Lyapunov," *Transp. Sci.*, vol. 18, no. 3, pp. 245–252, Aug. 1984, doi: 10.1287/trsc.18.3.245.

[15] W. Zhu, S. Ma, J. Tian, and G. Li, "Nonlinear relative-proportion-based route adjustment process for day-to-day traffic dynamics: Modeling, equilibrium and stability analysis," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 40, pp. 129–137, Nov. 2016, doi: 10.1016/j.cnsns.2016.04.017.

[16] H. Alibabai and H. S. Mahmassani, "Foxes and sheep: Effect of predictive logic in day-to-day dynamics of route choice behavior," *EURO J. Transp. Logistics*, vol. 5, no. 1, pp. 53–67, Mar. 2016, doi: 10.1007/s13676-015-0088-2.

[17] T. L. Friesz, D. Bernstein, N. J. Mehta, R. L. Tobin, and S. Ganjalizadeh, "Day-to-day dynamic network disequilibria and idealized traveler information systems," *Oper. Res.*, vol. 42, no. 6, pp. 1120–1136, Dec. 1994, doi: 10.1287/opre.42.6.1120.

[18] W.-L. Jin, "A dynamical system model of the traffic assignment problem," *Transp. Res. Part B, Methodol.*, vol. 41, no. 1, pp. 32–48, Jan. 2007, doi: 10.1016/j.trb.2006.02.010.

[19] R.-Y. Guo and H.-J. Huang, "Chaos and bifurcation in dynamical evolution process of traffic assignment with flow 'mutatio,'" *Chaos, Solitons Fractals*, vol. 41, no. 3, pp. 1150–1157, Aug. 2009, doi: 10.1016/j.chaos.2008.04.046.

[20] A. Nagurney and D. Zhang, "Projected dynamical systems in the formulation, stability analysis, and computation of fixed-demand traffic network equilibria," *Transp. Sci.*, vol. 31, no. 2, pp. 147–158, May 1997, doi: 10.1287/trsc.31.2.147.

[21] D. P. Bertsekas, and E. M. Gafni, "Projection methods for variational inequalities with application to the traffic assignment problem," *Math. Program. Study*, vol. 17, pp. 139–159, Aug. 1982.

[22] W. H. Sandholm, "Potential games with continuous player sets," *J. Econ. Theory*, vol. 97, no. 1, pp. 81–108, Mar. 2001.

[23] H. Ye, F. Xiao, and H. Yang, "Exploration of day-to-day route choice models by a virtual experiment," *Transp. Res. Part C, Emerg. Technol.*, vol. 94, pp. 220–235, Sep. 2018, doi: 10.1016/j.trc.2017.08.020.

[24] M. L. Hazelton, "Day-to-day variation in Markovian traffic assignment models," *Transp. Res. Part B, Methodol.*, vol. 36, no. 7, pp. 637–648, Aug. 2002.

[25] M. L. Hazelton and D. P. Watling, "Computation of equilibrium distributions of Markov traffic-assignment models," *Transp. Sci.*, vol. 38, no. 3, pp. 331–342, Aug. 2004, doi: 10.1287/trsc.1030.0052.

[26] M. Wahba and A. Shalaby, "Large-scale application of MILATRAS: Case study of the Toronto transit network," *Transportation*, vol. 38, no. 6, pp. 889–908, Nov. 2011, doi: 10.1007/s11116-011-9358-5.

[27] M. Wahba and A. Shalaby, "Learning-based framework for transit assignment modeling under information provision," *Transportation*, vol. 41, no. 2, pp. 397–417, Mar. 2014, doi: 10.1007/s11116-013-9510-5.

[28] Y. Huang, Z. Xiao, D. Wang, H. Jiang, and D. Wu, "Exploring individual travel patterns across private car trajectory data," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 12, pp. 5036–5050, Dec. 2020, doi: 10.1109/TITS.2019.2948188.

[29] M. A. Abdel-Aty, R. Kitamura, and P. P. Jovanis, "Investigating effect of travel time variability on route choice using repeated measurement stated preference data," *Transp. Res. Rec.*, vol. 1493, pp. 39–45, Jun. 1995.

[30] Q. Cheng, W. Deng, and M. Ammar Raza, "Analysis of the departure time choices of metro passengers during peak hours," *IET Intell. Transp. Syst.*, vol. 14, no. 8, pp. 866–872, Aug. 2020, doi: 10.1049/iet-its.2019.0442.

[31] R. Lahkar and R. M. Seymour, "Reinforcement learning in population games," *Games Econ. Behav.*, vol. 80, pp. 10–38, Jul. 2013, doi: 10.1016/j.geb.2013.02.006.

[32] F. Wei, S. Ma, and N. Jia, "A day-to-day route choice model based on reinforcement learning," *Math. Problems Eng.*, vol. 2014, pp. 1–19, Aug. 2014, doi: 10.1155/2014/646548.

[33] Z. Cui and Y. Long, "Perspectives on stability and mobility of transit passenger's travel behaviour through smart card data," *IET Intell. Transp. Syst.*, vol. 13, no. 12, pp. 1761–1769, Dec. 2019, doi: 10.1049/iet-its.2019.0212.

[34] K. Kim, "Effects of weather and calendar events on mode-choice behaviors for public transportation," *J. Transp. Eng., Part A, Syst.*, vol. 146, no. 7, Jul. 2020, Art. no. 04020056, doi: 10.1061/jtepbs.0000371.

[35] T. L. Friesz and S. Shah, "An overview of nontraditional formulations of static and dynamic equilibrium network design," *Transp. Res. Part B, Methodol.*, vol. 35, no. 1, pp. 5–21, Jan. 2001.

[36] K. Soltani Naveh and J. Kim, "Urban trajectory analytics: Day-of-week movement pattern mining using tensor factorization," *IEEE Trans. Intell. Transp. Syst.*, vol. 20, no. 7, pp. 2540–2549, Jul. 2019, doi: 10.1109/TITS.2018.2868122.

[37] I. I. Sirmatel and N. Geroliminis, "Nonlinear moving horizon estimation for large-scale urban road networks," *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 12, pp. 4983–4994, Dec. 2020, doi: 10.1109/TITS.2019.2946324.

[38] D. Li, M. Yang, C.-J. Jin, G. Ren, X. Liu, and H. Liu, "Multi-modal combined route choice modeling in the MaaS age considering generalized path overlapping problem," *IEEE Trans. Intell. Transp. Syst.*, vol. 22, no. 4, pp. 2430–2441, Apr. 2021, doi: 10.1109/TITS.2020.3030707.

[39] L. Ma, F. Yuan, X. Yan, and J. Zhang, "Disentangling city-level macroscopic traffic performance patterns through a trigonometric multiseasonal filtering algorithm: Inspiration from big data of ride-sourcing trips," *J. Transp. Eng., Part A, Syst.*, vol. 148, no. 3, Mar. 2022, Art. no. 04021120, doi: 10.1061/jtepbs.0000628.

[40] J. G. Wardrop and J. I. Whitehead, "Correspondence. Some theoretical aspects of road traffic research," *Proc. Inst. Civil Eng.*, vol. 1, no. 5, pp. 767–768, Oct. 1952, doi: 10.1680/ipeds.1952.11362.

[41] X. Guo and H. X. Liu, "Bounded rationality and irreversible network change," *Transp. Res. Part B: Methodol.*, vol. 45, no. 10, pp. 1606–1618, Dec. 2011, doi: 10.1016/j.trb.2011.05.026.

[42] X. Guo, "Toll sequence operation to realize target flow pattern under bounded rationality," *Transp. Res. Part B, Methodol.*, vol. 56, pp. 203–216, Oct. 2013, doi: 10.1016/j.trb.2013.08.006.

[43] P. D. Site, "Fixed-point states of day-to-day assignment processes with state-dependent route choice," *Transp. Res. Proc.*, vol. 27, pp. 1009–1016, Sep. 2017, doi: 10.1016/j.trpro.2017.12.068.

[44] M. Torkjazi, P. S. Mirjafari, and H. Poorzahedy, "Reliability-based network flow estimation with day-to-day variation: A model validation on real large-scale urban networks," *J. Intell. Transp. Syst.*, vol. 22, no. 2, pp. 121–143, Mar. 2018, doi: 10.1080/15472450.2017.1413555.

[45] X. Chen, J. Yin, G. Qin, K. Tang, Y. Wang, and J. Sun, "Integrated macro-micro modelling for individual vehicle trajectory reconstruction using fixed and mobile sensor data," *Transp. Res. Part C, Emerg. Technol.*, vol. 145, Dec. 2022, Art. no. 103929, doi: 10.1016/j.trc.2022.103929.

[46] O. Nave, "Modification of semi-analytical method applied system of ODE," *Modern Appl. Sci.*, vol. 14, no. 6, p. 75, May 2020, doi: 10.5539/mas.v14n6p75.

[47] H. S. Mahmassani and Y.-H. Liu, "Dynamics of commuting decision behaviour under advanced traveller information systems," *Transp. Res. Part C, Emerg. Technol.*, vol. 7, nos. 2–3, pp. 91–107, Apr. 1999.

[48] C. Ratti, D. Frenchman, R. M. Pulselli, and S. Williams, "Mobile landscapes: Using location data from cell phones for urban analysis," *Environ. Planning B, Planning Design*, vol. 33, no. 5, pp. 727–748, Oct. 2006, doi: 10.1068/b32047.

[49] E. Ben-Elia and Y. Shiftan, "Which road do I take? A learning-based model of route-choice behavior with real-time information," *Transp. Res. Part A, Policy Pract.*, vol. 44, no. 4, pp. 249–264, May 2010, doi: 10.1016/j.tra.2010.01.007.

[50] A. Tsirimpa, "Modeling the impact of traffic information acquisition from mobile devices during the primary tour of the day," *J. Intell. Transp. Syst.*, vol. 19, no. 2, pp. 125–133, Apr. 2015, doi: 10.1080/15472450.2015.1012865.

[51] P. C. Lila and M. V. L. R. Anjaneyulu, "Networkwide impact of telework in urban areas: Case study of Bangalore, India," *J. Transp. Eng., Part A, Syst.*, vol. 143, no. 8, Aug. 2017, Art. no. 05017004, doi: 10.1061/jtepbs.0000061.

[52] W.-X. Wu and H.-J. Huang, "A study of modeling travelers' route choice behavior and information release strategy," in *Proc. Int. Conf. Intell. Comput. Technol. Autom. (ICICTA)*, Changsha, China, Oct. 2008, pp. 311–315, doi: 10.1109/icicta.2008.186.

[53] G. Fusco, A. Bracci, T. Caligiuri, C. Colombaroni, and N. Isaenko, "Experimental analyses and clustering of travel choice behaviours by floating car big data in a large urban area," *IET Intell. Transp. Syst.*, vol. 12, no. 4, pp. 270–278, May 2018, doi: 10.1049/iet-its.2018.0015.

[54] G. Jiang, Z. Zheng, Z. Bai, L. Dai, and J. Zhao, "Simulation-based assessment of variable message signs route guidance information under congestion condition," *J. Jilin Univ. Engineering Technol. Edition*, vol. 36, no. 2, pp. 183–187, 2006.

[55] H. Rahimi-Farahani, A. A. Rassafi, and B. Mirbaha, "Forced-node route guidance system: Incorporating both user equilibrium and system optimal benefits," *IET Intell. Transp. Syst.*, vol. 13, no. 12, pp. 1851–1859, Dec. 2019, doi: 10.1049/iet-its.2018.5457.

[56] E. B. Diop, S. Zhao, and V. D. Tran, "Modeling travelers' acceptance of variable message signs: A hierarchical hybrid choice model," *J. Transp. Eng., Part A, Syst.*, vol. 146, no. 12, Dec. 2020, Art. no. 04020134, doi: 10.1061/jtepbs.0000456.

[57] B. Zhou, M. Xu, Q. Meng, and Z. Huang, "A day-to-day route flow evolution process towards the mixed equilibria," *Transp. Res. Part C, Emerg. Technol.*, vol. 82, pp. 210–228, Sep. 2017, doi: 10.1016/j.trc.2017.06.018.

[58] C. Meneguzzer, "Contrarians do better: Testing participants' response to information in a simulated day-to-day route choice experiment," *Travel Behaviour Soc.*, vol. 15, pp. 146–156, Apr. 2019, doi: 10.1016/j.tbs.2019.02.003.

[59] Q. Ye and S. V. Ukkusuri, "Resilience as an objective in the optimal reconstruction sequence for transportation networks," *J. Transp. Saf. Secur.*, vol. 7, no. 1, pp. 91–105, Jan. 2015, doi: 10.1080/19439962.2014.907384.

[60] F. Wei, N. Jia, and S. Ma, "Day-to-day traffic dynamics considering social interaction: From individual route choice behavior to a network flow model," *Transp. Res. Part B, Methodol.*, vol. 94, pp. 335–354, Dec. 2016, doi: 10.1016/j.trb.2016.10.002.

[61] C. Zhang, T.-L. Liu, H.-J. Huang, and J. Chen, "A cumulative prospect theory approach to commuters' day-to-day route-choice modeling with friends' travel information," *Transp. Res. Part C, Emerg. Technol.*, vol. 86, pp. 527–548, Jan. 2018, doi: 10.1016/j.trc.2017.12.005.

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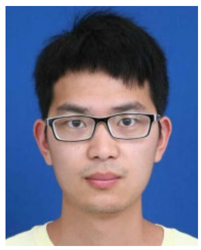

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