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RESEARCH ARTICLE

Traffic Management System With Symbolic Discrete Controller Synthesis Technique

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ABSTRACT In contemporary urban environments, the growing utilization of vehicles has emerged sophisticated challenges. Notably, degradation in air quality and amplified fuel consumption are the most issues that are being faced in urban cities due to traffic-related disruptions. Addressing these challenges is necessary to concern the adverse impacts on public health and economic aspects. This paper presents a comprehensive Traffic Management System employing the Symbolic Discrete Controller Synthesis Technique, offering a paradigm shift in urban traffic control. Our approach synthesizes effective controllers to reduce congestion and enhance system reliability by benefitting formal control frameworks and advanced modeling techniques. The key features of the approach include a symbolic safety algorithm to ensure compliance with regulations, an optimization algorithm to minimize congestion costs. Simulations across various scenarios validate the efficacy and robustness of our framework and also it suggests its potential to significantly improve urban mobility. The research directions may explore scalability and real-time data integration for broader applicability, however, our work lays a foundation for integrated traffic management systems combining formal control techniques, safety algorithms, and optimization strategies.

INDEX TERMS Discrete event systems, discrete controller synthesis, safety algorithm, optimization algorithm, traffic management systems.

I. INTRODUCTION

In contemporary urban settings, the escalating utilization of vehicles within transportation systems has given rise to multifaceted challenges, including amplified fuel consumption and a consequential degradation in the quality of ambient air due to traffic-related disruptions [1], [2].

A preeminent concern among these challenges is the adverse impact on air quality, contributing significantly to environmental pollution. Furthermore, the global annual mortality rate, accounting for %2.2 of the world population, is notably influenced by fatalities resulting from vehicular accidents [3]. The accessibility of emergency response teams to incidents such as traffic accidents, fires, and medical emergencies is hindered by transportation-related delays, underscoring the critical importance of timely intervention to mitigate potential adverse outcomes [4].

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Moreover, disruptions in transportation not only result in tangible time losses but also induce psychological stress among individuals [5]. The burgeoning population and corresponding surge in vehicular traffic in contemporary urban conglomerates, often referred to as modern cities, exert unprecedented strain on the existing transportation infrastructure. Consequently, there is a perceptible inclination towards the expedient resolution of this quandary through the construction of novel thoroughfares [6].

The escalation in vehicular volume on roadways leading to traffic congestion manifests repercussions both in terms of tangible and intangible losses. Traditional interventions involving the construction of new urban structures, encompassing roads, bridges, and intersections, while essential, may not singularly suffice for the amelioration of the transportation system. Scholarly discourse posits that the integration of sustainable transportation systems, coupled with advancements in technology, holds promise for system enhancement [7].

Furthermore, inherent inefficiencies in conventional traffic control mechanisms, exemplified by reliance on traffic police and consequential signaling discrepancies at traffic lights leading to delays, contribute significantly to the genesis of traffic congestion [8]. Consequently, there exists a compelling rationale for a paradigm shift in traffic management paradigms, advocating for the incorporation of emerging technologies to enable a more intelligent and responsive control framework.

In brief, occurrences leading to traffic congestion, such as signal malfunctions at traffic lights and traffic accidents, contribute to an escalation in vehicular density on roadways. The ramifications of this escalation extend to both economic facets, notably fuel consumption, and public health concerns, including the deterioration of air quality with associated psychological effects. Beyond the conventional approach of infrastructural expansion involving the construction of new roads, bridges, and interconnections, addressing these challenges necessitates a dedicated pursuit of solutions through modeling and the development of sophisticated traffic control strategies. Various methodologies have been explored in pertinent research endeavors, encompassing machine learning, artificial intelligence, image processing.

A. RELATED WORK

Highway, roadway and urban traffic control systems have lately been developed utilising a variety of strategies in an effort to enhance faith in modern technologies as a solution to traffic problems. Ensuring the safety and efficacy of these techniques in the realm of traffic control necessitates comprehensive studies aimed at mitigating challenges and issues. These endeavors encompass the utilization of mathematical modeling, machine learning-based Internet of Things (IoT), or, more specifically, machine learning-based approaches.

The research efforts aimed at improving traffic safety have also focused on proactive approaches. This strategy optimises traffic flow and ensures a safer environment by strategically placing traffic signals, roundabouts, and traffic signs. In addition, sophisticated traffic signal systems that are furnished with sensors and algorithms have been utilised to significantly reduce the frequency of traffic incidents. Notably, deep learning models are being used more and more in this field of study as academics realise how effective they are at producing performance results that are impressive. The research carried out by [9] showed that using simpler models can lead to better performance. The results of [10] show that the LSTM-CNN hybrid model performs better than other models, especially when it comes to sensitivity, as evidenced by its area under the curve (AUC) value. A hybrid model was used in another experiment, as reported by [11]. Interestingly, this hybrid model was created with a streamlined feature set, making it more suited for attaining greater true-positive rates and enabling proactive real-time actions in the context of traffic security. Convolutional Neural Networks (CNN) were utilised in the research by [12] in order to efficiently capture the unique characteristics of the traffic flow environment. Reference [13] used Gated Convolution Network (G-CNN) to highlight the importance of traffic density in the field of security and evaluate the possible risk of accidents related to traffic density. Notably, recurrent neural networks (RNN) served as an influence for this network type. A Reinforcement Learning (RL) model was developed by [14] by adding domain-specific rules into their research. The goal of this integration was to reduce any security flaws in traffic control systems that might occur at crossings. Using RL technique, [15]'s research developed a dual agent-based strategy to improve traffic utilization's comfort, safety, and efficiency. Using traffic data, this technology allows for real-time modifications to the model parameters. The incorporation of autonomous driving functionalities into newly released electric cars represents a steady but noteworthy progression. In the future, these cars' data combined with increased inter-vehicle connectivity will likely bring a new level of detail to traffic control systems. A notable addition was made by [16], who presented an improved Reinforcement Learning (RL)-based model with rule-based algorithms integrated in a synergistic way for autonomous cars.

The increasing development of technology has brought up the popular discussion of smart cities. The amount of data that is generated, especially when it comes to traffic density, as a result of the interaction between sensors, mobile devices, and autonomous cars, is going to increase in the future [17]. Notably, researchers' attention has been drawn to Internet of Things (IoT) traffic management systems. The goal of controlling traffic density in smart cities and guaranteeing the safe transfer of necessary data is what is attracting this focus. A machine learning-based cloud-based transmission control system has been developed, as demonstrated by [18]. This technology takes pictures at intersections, stores them, and sends them to the next traffic light all at once. By means of this method, the traffic lights at successive crossings participate in a mutual status monitoring process, which enables them to function in compliance with the feedback that is observed.

The control synthesis is also used in different areas such as schedule processes. The schedule processes need a controller to check the system faults. For instance [19] presents a supervisory control-based fault-tolerant scheduler for real-time tasks over on multi-core platforms. It performs a robust scheduling process and efficient resource under transient faults. Another work [20] introduces a formal scheduler synthesis framework for safety-critical systems on multi-core platforms. It provides a supervisory control of timed discrete event systems and binary decision diagrams to manage complexity. Reference [21] proposes a methodology to synthesize ramp metering control strategies for freeway networks using linear temporal logic specifications. The methodology uses advantages of both the cell transmission model and recent advances in control theory to achieve high correctness. The methodology of [22] addresses scalability challenges in synthesizing controllers for large signalized vehicular traffic networks using a compositional approach with contracts between sub-networks.

This approach is employed to systematically address diverse parameters collectively contributing to traffic congestion by using discrete control synthesis.

Upon a comprehensive analysis of the overarching factors contributing to urban traffic congestion, our focus shifts to conceptualizing traffic lights as a model interlinked in a network configuration, encompassing connections, roundabouts, side roads, and intersections. Previous investigations either limited their scope to freeway scenarios [23], [24], [25] or concentrated solely on accident prevention [26], [27], [28]. In this study, our endeavor involves the development of a model that not only facilitates economic efficiencies but also prioritizes traffic safety. Furthermore, our model allows for seamless integration of various connections, including junctions, main arteries, ground passages, and connecting roads, ensuring their effortless incorporation into the system without disrupting its operational integrity.

The traffic light control system demonstrates as a discrete time system. The construction of a such system consists lights that can be described as a state in a formal language, traffic density that is a value where it is measured by sensors can be shown as a variable and junctions that are main states of the discrete system. This system is compatible with discrete control synthesis essentially. In order to avoid any gap in the system, all possible situations regarding to the system are considered in this study by down to the last detail. To provide this system without error, we proposed strict rules. According to these rules, all possible situations that may occur in traffic system are performs with an minimum error. Besides, a new rule can be adapted to the system easily if necessary.

In our comprehensive literature review, it has been observed that traffic management systems represent a feedback control problem. Our research idea revolves around the notion that traffic management systems with complex structures can be decomposed into smaller components and modeled in a simple manner with formal correctness, along with safety and optimization control objectives.

The main contribution of this study is

- The systematic management of traffic management systems, whereby even sophisticated systems are designed and modeled as discrete event systems with smaller structures, enabling formal correctness, including optimization processes, to achieve desired system behaviors.
- It is crucial to address safety-critical systems deterministically with formal correctness, as in traffic management systems, rather than relying on meta-heuristic approaches.
- Our proposed method facilitates this through model checking. In optimization processes, the focus is on minimizing traffic congestion through event controls rather than time considerations. Since all possible future scenarios are explored, the resulting controller always yields the best outcome.
- The presented optimization algorithm is pessimistic, aiming to avoid worst-case scenarios. Although the tooling performance during compilation may be costly, this process is repeated only once, and once the controller is obtained, it dynamically responds to system and environmental factors.
- During the modeling phase, traffic flows at intersecting points are modeled as interdependent while being independent of situations at other intersections, facilitating straightforward modeling. However, due to the established queuing model, global control objectives in the optimization process naturally influence each other.

The rest of the paper is designed as follow: In Section-II, a brief simple background of Symbolic discrete controller synthesis technique is defined by all necessary including. Traffic management system is handled with all main sections and strict rules in Section-III. Important results regarding to rules are shown in Section-IV, and in the last section conclusion is touched on with future work.

II. BACKGROUND OF SYMBOLIC DISCRETE CONTROLLER SYNTHESIS TECHNIQUE

The control of discrete-event systems was initially formally introduced by [29] and [30] as a language theory. This theory has primarily been utilized for the synthesis of controllers, making it the pioneering work in the realm of discrete control synthesis [31], [32], [33]. Subsequently, [34] and [35] made significant advancements, particularly in the context of infinite systems. Discrete control synthesis finds applications across a wide spectrum of domains, as evidenced by recent studies, including [36], [37], and [38]'s exploration of energy efficiency in hardware circuits. Those systems are in common domain of daily life such as smartphones, transport, automotive, heating-ventilating systems etc [39].

The important spot of discrete controller synthesis theory is that it discriminates the feedback loops from the loops of discrete event systems. This allows discrete event systems to be autonomously analysed and controlled in predetermined control objectives. In order to operate a discrete controller synthesis, a labeled transition systems is applied on a discrete event system to allow a controller to operate the system. This controller manages some events in order to affect how the discrete events behaves.

These theory designs are helpful in real-world issues when there are several ways to perform a specific computing operation, each with varying amounts of resource consumption, service quality, and other factors. Nevertheless, determining the suitable configurations of a network, for instance traffic optimal control, at any time sequence is a difficult challenge. In order to find an optimal model to design for handling high accuracy traffic control system by minimizing the overall energy consumption in traffic intensity, while controlling all traffic lights, we proposed an abstract symbolic system model. Thus, the structure of this study correspond to an abstract symbolic system model to exhibit much more effective results. The determined issue in the traffic control with optimal design can be attained by concentrating on runs that achieve a certain set of goal states. These goal states are determined in the next section.

We indicate the system to manage traffic control with a set of symbols accepted by the symbolic system with the letter $\mathcal{S}$ to adapt it to the proposed model. These symbol sets are associated with certain domains $\mathcal{D}$. For each domain, it is expressed as $\mathcal{S} \rightarrow \mathcal{D} \times \mathcal{D}$. All domains that are used in the system are defined as follows:

- **Boolean domain**: In this domain, operations are performed according to True-True and False-False conditions. It is defined as $\mathcal{B} = \{\text{TT, FF}\}$.
- **Numerical domanins**: This domain represents the set of symbols used as numerically where $\mathcal{N} \in \mathbb{Z}$ and $\mathcal{N} \in \mathbb{Q}$.
- **Constant numbers**: This domain keeps the set of symbols where it contains constant values. These values can be integers, rational numbers or boolean values. $c \in \mathbb{Z}|c \in \mathbb{Q}|c \in \mathcal{B}$

A given domain is represented as $\delta \in \mathcal{D}$, and all are held as a set in X_δ. Each domain must satisfy the grammar accepted by the symbolic system. These grammar structures are represented by the formulas ξ_δ where

- $\xi_\delta :=$ if $\xi_B =$ True then s else ξ_δ, where s $= \xi_\delta$
- $\xi_B := \neg\xi_B|\xi_B\alpha\xi_B|\xi_B \rightarrow \xi_B|\xi_\delta = \xi_\delta|\xi_N\beta\xi_N$ where $\alpha \in \{and, or\}$, $\beta \in \{<, \leq\}$
- $\xi_\mathcal{N} := k.\xi_\mathcal{N}|\xi_\mathcal{N}\theta\xi_\mathcal{N}$ where $\mathcal{N} \in \mathbb{Z}|\mathbb{Q}$, $k \in \mathcal{N}$,and $\theta \in \{+, -\}$

Here, s is a symbol in $\mathcal{S}$. Also the rules ξ_δ has a generic form where they are similar conditional structure in the domains. The grammar also contains conditional logical structures as shown $\neg$, *and*, and *or* that are in syntactic type to make ease the readability for boolean rules. The numerical rules are shown with some mathematical expression to be specified the relationship between them. For instance, two different numerical values can be equal or magnitude between them by showing with $(<, \leq)$ or they can be added or subtract between each other $(+, -)$.

In the symbolic systems, the input data (I) and system states (S) are shown as a discrete sets. In initial step, all states are assigned with a fixed value. Transitions between states are similar with discrete events and they are evolved by using transition rules (T) which contain one assignment for per states in system. Each transition (t_s) has value that has symbolic expression that depends on the current state and input values. This kind of system performs as a finite state machine where it contains a variable for each states and transitions for each states at discrete time.

III. TRAFFIC MANAGEMENT SYSTEM

The current traffic control system is being performed by two construction: traffic lights and traffic polices. However, some important issues occur due to performance of this system. For instance, the time interval for the signal of green and red lights are commonly set as a fixed time. Also, there is no any communication between lights at each intersections. If we consider these both condition traffic congestion always increase in city urban. The other issue is the use of traffic officers to control the traffic flow. The discretion of the officer is the only decision in this system. Thus, usually it might not be a best and accurate control in traffic flow due to reduce some roads to one lane, open the road to finish the busy section, but cause traffic congestion on the road they close. In such cases, there is usually no logical operation. A more sophisticated and effective traffic management system is required, one that can adjust dynamically to shifting traffic patterns and maximise traffic flow. In order to provide an intelligent system that helps to control traffic congestion, reduce pollution, and transportation costs, we proposed a novel model.

The proposed model is designed by integrating certain strict rules which take into account all interactions that may cause traffic congestion. The proposed method can be likened to symbols stored within a stack, where empty stack, full stack, or partial full stack can be examined. This situation suggests the development of a traffic management system that alters traffic flow at specific points by considering the occupancy rate of the stack. Thus, the system potentially prevents congestion at those locations. The common events that may arise in traffic flow, the processing of traffic congestion, and the response of the model are proceeded through the following steps:

- Traffic stops when the traffic light turns red and flows when it turns green.
- The permissions for the routes that vehicles at the same intersection can take must be determined. In this manner, the lights at the same intersection are processed as a whole within a single node to determine permissions as shown in FIGURE 2.
- Be noted that the traffic lights at each intersection are interconnected each other. The system re-designs the operating times of all traffic lights in the system to prevent or minimize traffic congestion that may occur at one point from affecting other sections. In order to do this purpose, a scheduling policy should be prepared. According to this policy:
  - The implementation of a fairness policy is required to adjust the time of signal at an intersection. According to this policy, processing extend time for green light at a specific location in order to reduce traffic congestion will lead to the occurrence of new traffic congestion at other locations. To prevent this, a fair policy adjustment is necessary. By implementing this policy in the proposed model, we are addressing the traffic system in a fair manner.
  - The other policy that provides a fair traffic flow is the condition of roads being completely empty. According to this policy, at least one traffic light in the system must be green to ensure uninterrupted transition between states within the system. Thus, in the scenario where the roads are empty, the system will continue to operate without major errors.

– Another important policy aims to prevent traffic lights from never transitioning to a red signal. The term 'starvation' is used toe determine this condition. In order to facilitate transitions in this condition, a concurrency model has been defined.

In summary, the proposed model consists all traffic lights at each intersection and suggests the development of a traffic management system that modifies traffic congestion by considering every possible scenario. The traffic flow ensured by specific strict rules also eases potential issues that may arise with important policies.

A. OVERVIEW

The initial construction of the traffic management system model follows our proposed symbolic approach, organized as a synchronous parallel composition. This composition encompasses various components: i) strict rules, which handle scenarios like coming to a complete stop at a red light or ensuring that intersecting roads do not have a green signal simultaneously; ii) a queuing model, representing different states of traffic congestion; iii) scheduling policies governing the coordination between traffic lights at a junction node; and iv) a cost function integral to the optimization process.

Subsequently, a safety objective is formulated as a conjunction of propositional formulas involving the state variables of the previously mentioned models. These formulas serve various purposes, ranging from ensuring compliance with red light signals to expressing mutual exclusion constraints among process configurations that share resources, such as intersecting roads. At this juncture, a symbolic safety control algorithm proves instrumental in calculating a strategy for the process configuration inputs (i.e., the controllable input variables of our model). This strategy is designed to guarantee the satisfaction of the safety objective. The resultant safety objective, combined with our model, coalesces to form a manager. This manager generates configuration choices for each traffic light, dictating when it should be active or inactive. Importantly, the design resulting from this manager's outputs is inherently unable to violate the previously mentioned mutual exclusion constraints.

Alternatively, for further refinement, our approach involves the application of a symbolic optimal control algorithm to achieve an additional optimization objective. In this context, the cost function defining the new strategy primarily focuses on the minimization of traffic congestion. The refined strategy obtained through this process can be applied similarly, dynamically selecting configurations for each traffic light. This integrated approach ensures both safety and optimization in the traffic management system.

Our approach involves a generalized technique. Regardless of how sophisticated the given system as in FIGURE 2 is, it is systematically decomposed into parts referred to as "nodes" and then modeled as discrete-event systems through synchronous parallel compositions. Our control objectives, as mentioned above, are evaluated globally and compiled to produce a controller. A "node" in our context refers to a sub-traffic system consisting of interdependent roadways and traffic signals, while being independent of the roadways and signals within another node. This is formulated below. It's worth noting that our discrete-event systems operate globally on control objectives, allowing for the modeling and management of sophisticated systems. For instance, dependencies or indirect effects among queues formed by vehicles within traffic flow are managed through global control objectives.

$\forall Comp \in \mathcal{N}_{\mathtt{i}} \perp Comp \in \mathcal{N}_{\mathtt{i+1}}$ where $i \in 1, 2, \ldots, n$ Here, *Comp* is all components (i.e. lights, conjunctions, etc.) where they belong to $\mathcal{N}_{\mathtt{i}}$.

The other strict rule is the control of traffic lights giving permission to use the same sharing resources, where the set of lights is called as Node $\mathcal{N}$ and the number of Nodes in a given system is denoted by $|\mathcal{N}|$. An example of a Node is a set composed of 4 traffic lights in FIGURE 2 denoted as ($\mathcal{N}_{\mathtt{i}} = \{S_1, S_2, S_3, S_4\}$), where $i \in \{1, \ldots, |\mathcal{N}|\}$. The generic concept of Node is given by means of a crossroad example; however, traffic lights relevant to any kind of the same sharing resources (such as roundabout) can be modeled as Node.

B. STRICT RULES

The strict rules make a control scheme, on one traffic light as well as cooperative problem formulation. The safety objective enforces that the traffic light activities and the cooperation of the relevant lights must be reliable.

1) PERMISSION MODEL

FIGURE 1 represents the permission model which is the basic rule for traffic lights: stopping at a red light and going through a green light. A unique traffic light is denoted by a state symbol $S_{\mathtt{i}}$ that takes its values in $\{\mathtt{R}, \mathtt{Y}, \mathtt{G}\} \in \mathcal{D}_{\mathtt{fin}}$, where the number of total lights in a given traffic network is denoted by $|S|$ and $i \in \{1, \ldots, |S|\}$. Each state symbol $S_{\mathtt{i}}$ is accompanied with a controllable signal $c_{\mathtt{i}}$ that gives a permission in order to leave from the states R and G. The state Y is considered as a small waiting for the transitions between the states R and G; and $u_{\mathtt{i}}$ is an uncontrollable input for $S_{\mathtt{i}}$ and indicates that the waiting time is completed by means of a signal from the given system. As a note that the lights are considered to give general permission for going through *i.e.,* the permission of separate left/right turns is not considered in this research.

2) MUTUAL EXCLUSION CONSTRAINTS

The other strict rule is the control of traffic lights giving permission to use the same sharing resources, where the set of lights is called as Node $\mathcal{N}$ and the number of Nodes in a given system is denoted by $|\mathcal{N}|$. An example of a Node is a set composed of 4 traffic lights in FIGURE 2 denoted as ($\mathcal{N}_{\mathtt{i}} = \{S_1, S_2, S_3, S_4\}$), where $i \in \{1, \ldots, |\mathcal{N}|\}$. The generic concept of Node is given by means of a crossroad example; however, traffic lights relevant to any kind of the same sharing resources (such as roundabout) can be modeled as Node.

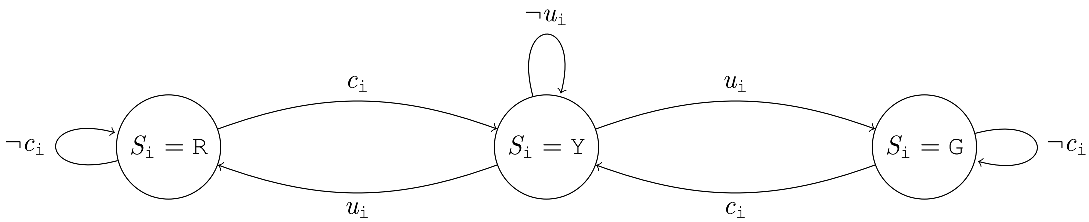

FIGURE 1. Representation of permission model encoded as S_i.

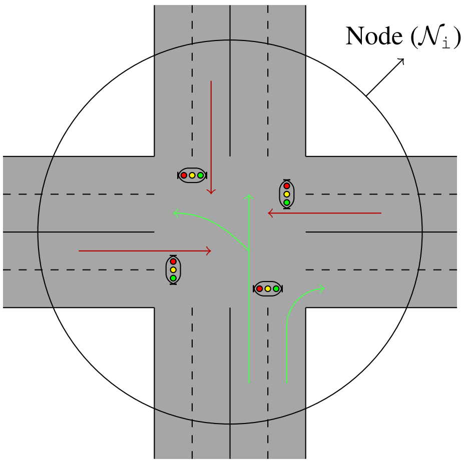

FIGURE 2. The representation of a node for a crossroad that includes 4 traffic lights (S_1,S_2,S_3,S_4).

As it is shown in FIGURE 2, there are three possible ways, represented with green lines, for a vehicle to pass through the light at the crossroad, and all these ways cross the other transit routes, represented with red lines. As can be seen clearly, only one traffic light must turn green at the same time. To this end, mutual exclusion constraints on shared resources are enforced in order to meet the objective. A symbol of mutual exclusion constraints $\mathcal{M}_i$ for a Node $\mathcal{N}_i$ is expressed as the EXNOR (exclusive not or) operation of states in $\mathcal{N}_i$ excluding the AND operation of them, and encoded as:

$$\mathcal{M}_i \triangleq \neg \bigoplus S_j \setminus \bigwedge S_j, \tag{1}$$

where $i \in |\mathcal{N}|$, $S_j \in \mathcal{N}_i$, and $\bigoplus$ denotes the operator EXOR.

C. QUEUEING MODEL

When representing traffic flow in various scenarios or modeling it, there are numerous methods at our disposal. For instance, we can consider of it in terms of a stack, much like symbols stacked upon each other. The choice of the modeling mechanism ultimately depends on whether this stack is empty, filled, or contains a certain number of stack symbols, or overflow checking for bounded stack when we examine the traffic management system from a holistic perspective.

In our case, the development of an effective traffic management system revolves around altering the flow speed of traffic at specific points, thus preventing the overflow of an unbounded stack, ultimately mitigating traffic congestion. Furthermore, if we envision such a manager in this context as consider to playing a game, the system manager should realize the stack's state, whether it's empty or near-empty. By strategically deciding on moves based on potential traffic flow scenarios, they can effectively reduce traffic congestion. In our efforts to address the issue of space explosion in tackling the traffic flow problem, we abstracted our traffic flow model by constraining it to a limited domain, ensuring that it takes values from a finite set ($\{Light, Moderate, Heavy, Stop\ \ and\ \ Go, Gridlock\}$). Apart from the standard input and output processes that can occur in a traffic channel, there may be known or unknown factors, such as vehicles exiting or entering from an alternative route. We define this non-deterministic process with the symbol ξ and offer a deterministic modeling approach.

Our traffic model which represents the abstracted behaviors of traffic as described, operates within a finite domain. The transitions within this domain are facilitated by ξ_i, in conjunction with $prod_i$ and $cons_i$ pairs. Additionally, it's worth noting that multiple processes can enter or leave the channel simultaneously. These aspects will be further modeled in the next section as queuing discipline and admission policy.

The pair $prod_i$ and $cons_i$ represents the vehicles entering and exiting traffic. S_i indicates the current state of a traffic signal. If S_i is not green and at the same time the previous connected signals ($\widetilde{S_i}$) that can access it, are green, then $prod_i$ will be generated. For a $cons_i$ operation to occur, it is sufficient for the current traffic signal to be in a green state. The relevant dataflow equation is provided below:

$$prod_i \triangleq (S_i != G) \wedge (\widetilde{S_i} = G) \tag{2}$$

$$cons_i \triangleq (S_i = G) \tag{3}$$

D. SCHEDULING POLICIES

Taking necessary precautions for exceptional situations within traffic is crucial. At an intersection where roads intersect, scheduling is essential for the traffic signals since a road dependent on a traffic signal can either be completely empty or perpetually congested. To address these scenarios in a fair manner, the following modeling policy is being followed. First and foremost, as part of your safety objectives, we introduce a scheduling constraint in the form of a virtual unbounded counter (ϱ_i), to be used as a time quadrant.

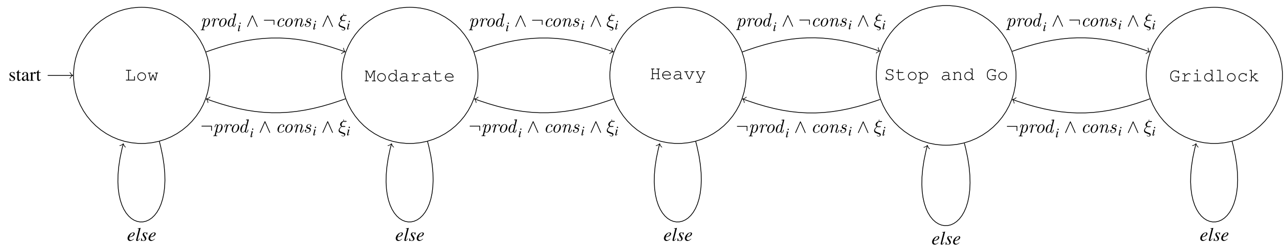

FIGURE 3. Queueing model for traffic flow.

1) FAIRNESS

The virtual queue we have constructed for the fairness policy is presented below. Whenever the pertinent signal is activated, our virtual queue is reset, and when traffic signals linked to this signal ($\widetilde{S_{\mathtt{i}}}$) are triggered by their associated control variable ($\widetilde{c_{\mathtt{i}}}$), they generate some data token. This process results in the establishment of a time quadrant, which is formally modeled through the dataflow equation provided below:

$$\varrho_i := \begin{cases} 0 & \text{if } S_{\mathtt{i}} = \mathtt{G} \\ \varrho_i + 1 & ((\widetilde{S_{\mathtt{i}}} = \mathtt{G}) \wedge \widetilde{c_{\mathtt{i}}}) \vee ((\widetilde{S_{\mathtt{i}}} = \mathtt{R}) \wedge \widetilde{c_{\mathtt{i}}}) \\ \varrho_i & \text{otherwise,} \end{cases} \tag{4}$$

In addition, to create a priority list, we introduce a new symbol, denoted as $\mathfrak{p}$, for sorting and valuation, which is constrained by the invariants of Ψ^{sort}_{fair}. Furthermore, in this context, we solely consider traffic signals within the node ($\mathcal{N}_{\mathtt{x}}$, where $x \in |\mathcal{N}|$), and the total count here is indicated using the cardinality symbol. Ψ^{sort}_{fair} is formally defined as:

$$\Psi^{sort}_{fair} = \bigwedge_{i \in \{1,\ldots,|S_{\mathcal{N}_{\mathtt{x}}}|-1\}} \mathfrak{p}_i \geq \mathfrak{p}_{i+1} \tag{5}$$

Following the ordering process conducted above, the valuation process has been executed. Ψ^{val}_{fair} is formally defined as:

$$\begin{aligned} \Psi^{val}_{fair} &= \bigwedge_{i \in \{1,\ldots,|S_{\mathcal{N}_{\mathtt{x}}}|\}} \bigvee_{i \in |S_{\mathcal{N}_{\mathtt{x}}}|} \mathfrak{p}_i = \varrho_i \wedge \sum_{i \in \{1,\ldots,|S_{\mathcal{N}_{\mathtt{x}}}|\}} \mathfrak{p}_i \\ &= \sum_{i \in |S_{\mathcal{N}_{\mathtt{x}}}|} \varrho_i \end{aligned} \tag{6}$$

If our fairness-preserving invariant is represented as Ψ_{fair}, combining the sorting and valuation processes, we manage our control variables in a fair manner. When a light turns green, we assign a value from the priority list to the virtual queue values. The formal representation of this process is provided in the form of a dataflow equation below:

$$\begin{aligned} \Psi_{fair} &= \Psi^{sort}_{fair} \wedge \Psi^{val}_{fair} \wedge \\ &\bigwedge_{j \in |S_{\mathcal{N}_{\mathtt{x}}}|} (S_{\mathtt{j}} = \mathtt{G}) \Rightarrow \varrho_j \in \{\mathfrak{p}_1, \ldots, \mathfrak{p}_{|S_{\mathcal{N}_{\mathtt{x}}}|}\} \end{aligned} \tag{7}$$

2) EMPTINESS

The dataflow equation provided above would pose a problem in the scenario where all of our paths are completely empty. Based on the current situation, if all our paths are entirely empty, the traffic lights will be unable to initiate a transition to green. This is because this action is based on a fairness policy determined by the evaluation of the queue. In the absence of a green light within a node, the virtual queue counter will not increase.

To break this deadlock, even in the situation where all paths in a node are completely empty, we are enforcing at least one traffic light to be green with the following formulation:

$$\Psi_{empt} = \mathcal{T}_{clk} \Rightarrow c_{\mathtt{i}}, \tag{8}$$

where $i \in |S|$ and the deadlock is resolved when triggered within the specified time interval $\mathcal{T}_{clk}$.

3) STARVATION

Ensuring the absence of starvation is crucial in cases where a process never undergoes a transition to the red light. In our applications, we have encountered scenarios where this situation occurs. Therefore, a concurrency model has been defined, specifying that processes are compelled to facilitate this transition under certain conditions.

$$\Psi_{star} = \mathcal{T}_{clk} \Rightarrow c_{\mathtt{i}}, \tag{9}$$

where $i \in |S|$ and triggering mechanism is activated at each specified time interval $\mathcal{T}_{clk}$ to prevent starvation.

E. CONTROL

In this context, our control objectives consist of two components: safety and optimization objectives.

The safety objective, ensuring the invariance, is defined by evaluating controlled variables to be always equal to alway "true", achieved through the convergence of all invariants as described below:

$$\Psi = \Psi_{fair} \wedge \Psi_{empt} \wedge \Psi_{star} \wedge \mathcal{M}_{\mathtt{i}} \tag{10}$$

Our optimization objective is defined as an effort to minimize the cost of a cost function modeled through the traffic flow model, as outlined below:

$$\begin{aligned} \mathcal{O}ptim. = \sum_{i \in |S|} S_{\mathtt{i}} \perp \{&Light, Moderate, Heavy, \\ &Stop\ and\ Go, Gridlock\}, \end{aligned} \tag{11}$$

where $\perp$ indicates the density of traffic depending on the traffic light.

IV. EXPERIMENTAL EVALUATION

Our experimental evaluation is conducted on two different platforms, namely MATLAB and ReaX. All the algorithms applied in the simulation environments on both platforms were executed on a computing device equipped with an Intel i7 3.3GHz CPU and 16GB RAM.

Initially, we developed a script in MATLAB to generate a random traffic simulation design (plant). Subsequently, we constrained the traffic flow in this simulation to adhere strictly to solid safety principles, such as avoiding the simultaneous green signal for intersecting roads. In other words, we considered only rigid safety objectives.

To compare with our approach, we activated three distinct control mechanisms. These included: random alterations in the states of lights with upper and lower bound durations; lights consistently changing at fixed intervals; and a manual-manager (user) intervention mechanism, which involved adjusting the durations of lights based on observed traffic patterns, especially in scenarios with high potential traffic density.

To implement our proposed approach, we generated a random traffic simulation design in MATLAB and concurrently modeled the same environment (i.e., the initially uncontrolled system behaviors, i.e., plant) in the ReaX environment according to the systematic modeling framework we presented. In our ReaX model, system objectives and observers such as strict rules, queuing models, and scheduling policies were coded within the ReaX environment. Subsequently, the safety and optimization algorithms proposed in our framework were applied to this traffic model, and the resulting controller C code was translated into MATLAB for controller implementation. The outcomes obtained from our approach are comparatively reported in TABLE 1.

TABLE 1. Comparative performance criteria results (each metric is given in a range of [min,max] for the 30 different scenarios).

Method	$\lambda_{dens.}$ (%)	$\lambda_{avg.}$ (t)	λ_{energy} (%)
$\mathcal{A}_{DCS}$	[-21,-16]	[76,85]	[-27,-14]
$\mathcal{A}_{rand.}$	[-3,3]	[105,107]	[-3,2]
$\mathcal{A}_{fixed}$	[0,0]	[100,100]	[0,0]
$\mathcal{A}_{manu.}$	[-7,-4]	[83,88]	[-11,-9]

In TABLE 1, the presented approaches are $\mathcal{A}_{DCS}$ (our proposed approach), $\mathcal{A}_{rand.}$ (randomly changing the states of traffic lights within upper and lower bound durations), $\mathcal{A}_{fixed}$ (keeping the states of lights fixed at initially specified durations), and $\mathcal{A}_{manu.}$ (intuitively allowing the user to adjust durations at the beginning). Our performance criteria are as follows: $\lambda_{dens.}$ (traffic density), $\lambda_{avg.}$ (the average time a vehicle spends in traffic), and λ_{energy} (percentage of average energy consumption per vehicle).

We conducted coding to explain the performance criteria and how we calculated them. This coding shows that the values of $\mathcal{A}_{fixed}$ are determined as a fixed system, and we compared them with other approaches. The obtained results were calculated by considering the performance criteria as 0 percent for $\mathcal{A}_{fixed}$ and comparing other approaches ($\mathcal{A}_{DCS}$, $\mathcal{A}_{rand.}$, $\mathcal{A}_{manu.}$) to the values of $\mathcal{A}_{fixed}$. The time unit was accepted as 100 units and used as a counter for $\mathcal{A}_{fixed}$.

TABLE 2. Synthesis time and memory occupation.

State (amount)	Time (s)	Memory (KB)
10	0.19	56736
15	13.46	217104
20	63.72	591468
25	147.96	1759738
30	232.87	2793726
35	265.91	3143764
40	342.73	3334942

Additionally, the system's ability to generate a controller and the memory space it occupies are provided in TABLE 2, along with the performance criteria of the presented tool. The most crucial criterion here is the number of states. As observed in TABLE 2, synthesis time and memory usage increase with the growing complexity, particularly in relation to the number of states. It is important to note that the states in the table represent not all observers in the model but solely the representation of traffic lights.

In evaluating our approach, we compare it with related works in traffic management systems. While [9] suggests the superiority of simpler models for better performance, [13] emphasizes the importance of considering traffic density using Gated Convolutional Networks (G-CNN). Additionally, [18] explores IoT-based traffic management systems, [14] focuses on reducing security vulnerabilities through domain-specific rules in Reinforcement Learning (RL) models, and [23] proposes a comprehensive model prioritizing economic efficiencies and traffic safety.

When comparing our approach with the meta-heuristic methods mentioned above, it not only outperforms in terms of performance but also ensures formal correctness. Additionally, such meta-heuristic approaches typically require large datasets and entail lengthy training periods. In contrast, our proposed approach can deterministically fulfill safety objectives and manage optimization processes without the need for any data or training. Although the compilation process of DCS technique may take some time, it exhibits significantly superior performance compared to these meta-heuristic algorithms, as shown in TABLE 2. From a modeling perspective, meta-heuristic algorithms are error-prone and inherently possess complex structures. Conversely, our proposed modeling technique allows for the systematic construction of discrete event systems with small structures, and a design tool could be easily devised to automatically facilitate this modeling process. Our approach effectively addresses the complexities of urban traffic congestion, offering simplicity, adaptability, and enhanced reliability. Overall, our work contributes to advancing traffic management systems by offering novel perspectives and methodologies for improved efficacy and reliability in diverse urban settings.

As a result, as demonstrated in TABLE 1 through our comparative evaluation with standard approaches, the proposed approach has exhibited notably high performance. When considering the synthesis times presented in TABLE 2, the scalability of the application appears to be quite reasonable. Moreover, a review of existing literature underscores the significance of employing an approach that ensures formal correctness in such systems, thereby highlighting the prominence of our approach. In control experiments, our symbolic approach stands out not only for reducing complexity but also for its ability to model optimization goals without the need for a final target state, as compared to studies in the literature. Finally, through both our own evaluation and comparisons with other studies in the literature, our method proves the appropriateness of using it as a traffic manager. This underscores the inspirational nature of our approach for adoption in similar systems.

The effectiveness of the proposed approach in addressing 30 different scenarios related to traffic management systems has been validated, as depicted in TABLE 1, employing various methodologies within minimum and maximum intervals. The foundation of verification processes lies in the discrete-event symbolic system's adherence to model checking techniques. Model checking serves as a verification method widely employed in computer science to validate system accuracy and identify errors. It verifies whether system behaviors conform to specified specifications.

V. CONCLUSION

In conclusion, our Traffic Management System, utilizing the Symbolic Discrete Controller Synthesis Technique, represents a significant advancement in addressing urban traffic complexities. By employing parallel synchronous languages within a formal control framework, our approach facilitates the synthesis of effective controllers, modeling traffic dynamics comprehensively. The symbolic safety algorithm ensures compliance with strict rules and limitations, enhancing system reliability, while the optimization algorithm minimizes a cost function to reduce congestion. Simulations across diverse scenarios validate the robustness of our framework, highlighting its potential to improve urban mobility. Future research may explore scalability and real-time data integration for broader applicability, but our work demonstrates a comprehensive solution combining formal control techniques, safety algorithms, and optimization strategies to enhance traffic system efficiency and safety.

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