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RESEARCH ARTICLE

Nonfragile Observer-Based Control With Passivity and H_∞ Performance for a Class of Power-Line Inspection Robots With Input Time-Delay

BINGSHAN LIU[1] **AND CHEN ZHANG**[2]
[1]Engineering Research Center of Digital Forensics, Ministry of Education, Nanjing University of Information Science and Technology, Nanjing 210044, China
[2]Faculty of Automation, Huaiyin Institute of Technology, Huaian 223003, China
Corresponding author: Bingshan Liu (sanderlbs@126.com)

ABSTRACT This study introduces a nonfragile observer-based control strategy for power-line inspection robots (PILRs) that adeptly tackles input time delays while integrating passivity and H_∞ performance. The research commences with the development of a nonlinear system model for PILRs, where the equilibrium manifold linearization method is employed to transform these underactuated systems into a linear framework using scheduling variables. Given the challenges posed by input delays, parameter disturbances, and uncertainties in real-world applications, this study proposes a robust observer-based control framework. Utilizing Lyapunov-Krasovskii functions and linear matrix inequalities (LMIs), the strategy confirms the system's robustness in maintaining mixed passivity and H_∞ performance. Simulation assessments substantiate the effectiveness of the proposed control approach, demonstrating its capacity to uphold system stability under diverse operational conditions. The strategy significantly enhances the reliability and safety of PILRs operating in complex environments, marking a pivotal advancement in robotic technologies for maintaining and inspecting utility infrastructures. This contribution not only strengthens the operational capabilities of PILRs but also provides a scalable approach to handling uncertainties and disturbances in robotic control systems.

INDEX TERMS Power-line inspection robots (PILRs), passivity, H_∞ performance, input delays, nonfragile control.

I. INTRODUCTION

Robots have been deployed across a myriad of sectors, as documented extensively in the literatures [1], [2], and [3] and so on. In the system of electricity, for example, the Power Line Inspection robot(PLIR) is capable of automatic or remote inspection of the external high voltage equipment(HVE) in substations, examine all kinds of problems in the electrical equipment, and then supply related data to the operator so that they can diagnose potential accidents, see references [4], [5], and [6]. Hence, to guarantee the safety and stability of the transmission line, it is necessary to inspect the transmission line periodically. Recently, research on Power Line Inspection robots(PLIRs) has become a popular topic, and many researchers at home and abroad have done a great deal of research on detecting robots, for example, testing, maintenance, nondestructive testing, etc., in [7], [8], [9], and [10].

In practice, there are a number of PLIRs that operate in the nonlinear system with underactuation [11], [12]. Since actuators in the PLIRs system with underactuation are smaller than the freedom degrees, the system operating cost is reduced, thus in high voltage (HV) line detection it is wide to be applied. Nonetheless, controlling an underactuated system presents a significantly greater challenge compared to managing a fully actuated system. In particular, when

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the robot operates on HV, the underactuated system will be influenced by any outside interference, which will cause the system performance to deteriorate [13], [14], [15]. Additionally, the effect of input delay is also taken into account. When PLIR is concerned with HVE, there exists the delay in the date and in fact in other cases [16], [17]. In a lot of practical problems, the state feedback control can not ensure the desired stability when some system states cannot be measured. In this situation, a controller based on observation will assist in reconstructing the system's status and providing the system with the desired feedback, such as [18]. The main advantage of an observer based control design is that it can stabilize the plant with an output feedback control unit even in the presence of external disturbances. Therefore, it remains a challenge to enhance the capability of processing controller in aspect of the uncertain PLIR system based on observer with input delay. That is one of the motives for the study of this thesis.

Among the above results, there have been many papers on H_∞ control problems in PILR systems in [19], [20], and [21]. H_∞ robust control is a useful tool for dealing with uncertainty, and there has been lots of research on it. In some nonlinear system time-delay, the H_∞ control under discrete-time and continuous time has been studied. But there has not been a novel approach for the problem of the nonlinear PLIRs system based on observation. Passive control approaches have garnered considerable interest due to their direct and seamless applicability to power converters. Furthermore, they have become a focal point of attention. The fundamental concept behind a passive controller involves acquiring the energy and its fluctuations within the closed-loop system in alignment with a achieved state trajectory. This is achieved in virtue of incorporating the desired damping and effectively channeling the force that lacks work into the function about energy dissipation. Muhammad and Cerezo [22] proposed passive controllers were used in the dynamic ship model. Passivity-based fault-tolerant synchronization has been proposed by [23] to address the problem of failure in neural networks. The passive sliding mode control problem for uncertain, non-linear and input time-delay PLIR systems are presented by [24].

Concurrently, the nonfragile control problem has garnered significant attention in both theoretical and practical domains [25]. Considerable focus has been directed towards the development of robust feedback controllers designed to substantially diminish the impact of disturbances on the plant. Furthermore, a nonfragile controller possesses the ability to withstand a bounded alteration in controller gain. Then, a number of researchers have studied the problem of nonfragile control methods based on the observer. Likewise, nonfragile H_∞ control has been explored in the context of numerous singular systems with discrete-time featuring uncertainties and input delay in parameters [26], [27]. But, as far as we are concerned, nonfragile observer-based control, incorporating passivity and H_∞ performance, for a specific category of power-line inspection robots with input time-delay has not been extensively examined.

Generally speaking, in this paper, we consider the topic of nonfragile observer-based control with passivity and H_∞ performance for a class of power-line inspection robots with input time-delay. Within the framework of the nonfragile controller, LKF are devised to fully exploit the information inherent in the PLIRs system. Linear Matrix Inequalities (LMIs) are then presented to establish that the closed-loop PLIR system achieves exponential asymptotic stability, satisfying the essential conditions for system stability. The key developments in this paper can be encapsulated as follows:

(1)Motivated by the nonlinear PLIR system is constructed, and the equilibrium manifold linearization method is utilized to convert the underactuated PLIR into a linear model by incorporating scheduling variables.

(2)For dynamic PLIR system, comparing with previous researches, considering input delay, parameter disturbances, uncertainties in practical situations, nonfragile observer-based control is presented to demonstrate the stability of the system above described.

(3)Lyapunov-Krasovskii functions and linear matrix inequalities are introduced to demonstrate the passivity and H_∞ performance of the established system. Subsequently, simulation verifications are performed to confirm the robustness for discussed system.

This paper is structured in the following manner: Section "Preliminary results" considers the problems in this paper, introducing "Dynamic Model for PLIR", "Construction of equilibrium manifold linearization model for PLIR" and "Mathematical Preliminary Findings". Section "Principal findings and demonstrations" describes main results in which two theorems clarify the stability of the system. Section "Simulations" includes a numerical simulation example aimed at validating the reliability of the suggested controller. The conclusion is outlined in Section "Conclusion".

II. PRELIMINARY RESULTS

A. DYNAMIC MODEL FOR PLIR

The PLIR system is a two-degree-of-freedom underactuated system subjected to outside interference and input delay. Figure 1-3 illustrates the balance adjusting parameters of the PLIR. [28] Let α_1 be the angle of inclination of the robot's body to the X_1axis, and α_2 represents the angle measured from the initial position of the driven rod to its engaged orientation. Let r_1 denote the horizontal distance from the cable to the body centroid and r_2 denote the horizontal distance between the counterweight boxes, m_a and m_b represent the mass of the robot and the weight of the counter [29]. Let u denote a system control moment, and τ_σ is an external perturbation, then τ_δ represents the system input delay. The input torque is represented as $u(t-\tau_\delta)$ when the

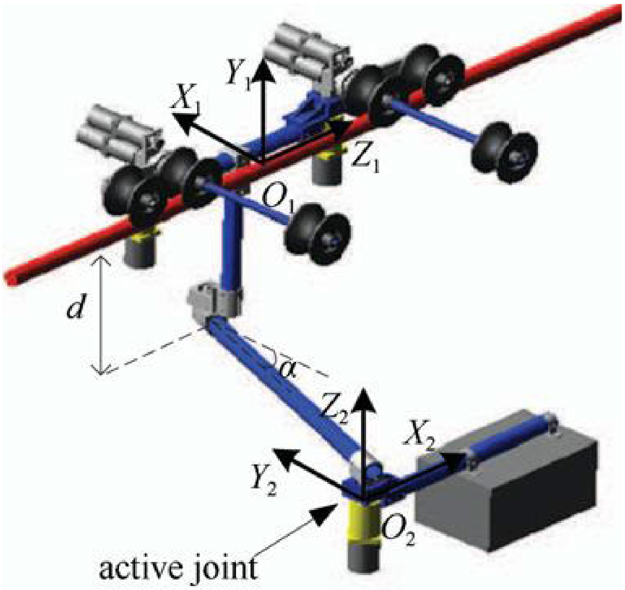

FIGURE 1. 3D model and framework illustration of PLIR.

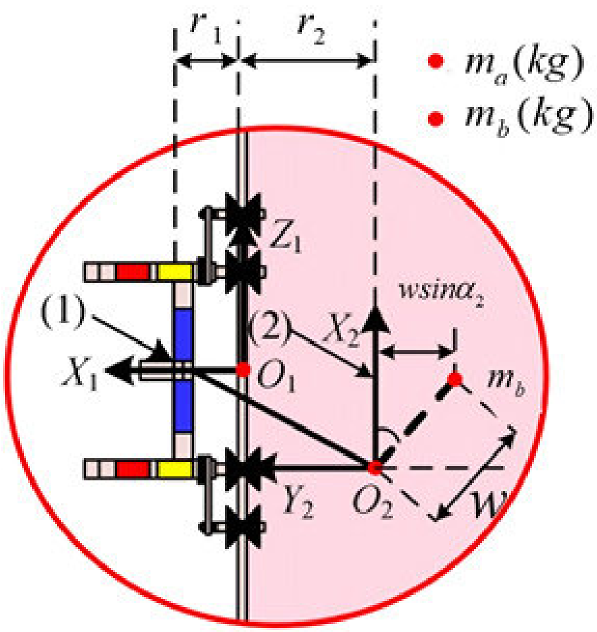

FIGURE 2. Illustration of the frame in the $X_1O_1Z_1$ plane.

system is influenced by an input delay, which can also be abbreviated as τ_δ.

Utilizing the Euler-Lagrange equation, we derive the motion equation governing the balance adjustment process of the PLI robot under the influence of parameter disturbances and input delays. The equation can be described as:([28])

$$u_{i\tau_\delta} = \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\alpha}_i}\right) - \frac{\partial \mathscr{L}}{\partial \alpha_i},$$

where, the external moment acting on the ith ($i = 1, \ldots n$) generalized coordinate with a time delay τ_δ is represented by

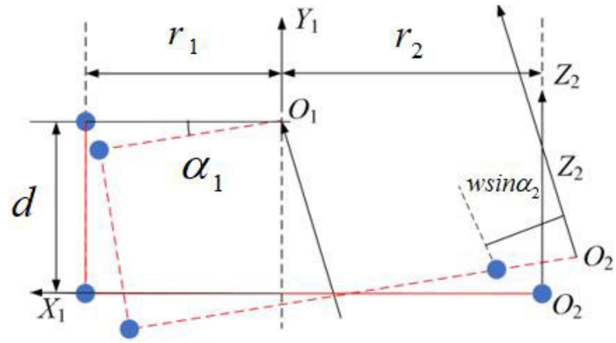

FIGURE 3. Illustration of the equilibrium adjustment for the PLIR.

$u(t - \tau_\delta)$, and the equation $\mathscr{I} = \mathscr{X} - \mathscr{Y}$ represents the balance regulation of motion for a PLI robot, where $\mathscr{X}$ and $\mathscr{Y}$ denote the kinetic and potential energy, respectively.

$$\mathscr{X} = \frac{m_a r_1^2 \dot{\alpha}_1^2}{2} + \frac{m_b w^2 \dot{\alpha}_2^2}{2} + \frac{m_b\left[d^2 + (-r_2 + w\sin\alpha_2)^2\right]\dot{\alpha}_1^2}{2},$$
$$\mathscr{Y} = -m_a g r_1 \sin\alpha_1 + m_b g\left[(r_2 + w\sin\alpha_2)\sin\alpha_1 - d\cos\alpha_1\right],$$

where g is the acceleration of gravity, w and d denote the length of the actuator rod and the height of the T-shaped base, respectively. We can choose $m_a r_1 = m_b r_2$, then $\mathscr{P}$ can be rewritten as

$$\mathscr{P} = -m_a g\left(-d\cos\alpha_1 + w\sin\alpha_2\sin\alpha_1\right).$$

Based on the previous research on the PLIR's dynamic model, the PLIR state equation is chosen as:

$$x = \left[x_1\ x_2\ x_3\ x_4\right]^\top = \left[\alpha_1\ \dot{\alpha}_1\ \alpha_2\ \dot{\alpha}_2\right]^\top,$$

where x is a continuous function, define $\dot{x} = \frac{dx}{dt}$, then the state equations are able to be formulated as:

$$\dot{x} = f\left[x, u\left(t - \tau_\delta\right)\right],$$
$$\begin{cases} \dot{x_1} = x_2, \\ \dot{x_2} = \frac{-m_b g d\sin x_1 - m_b g w\sin x_3\cos x_1}{m_a l_1^2 + m_b\left[d^2 + (-r_2 + w\sin x_3)^2\right]} + \tau_\sigma, \\ \dot{x_3} = x_4, \\ \dot{x_4} = \frac{u_{\tau_n} + m_b d(-r_2 + w\sin x_3)(\cos x_3)x_2^2 - m_b g d\sin x_1\cos x_3}{m_b d^2}. \end{cases} \tag{1}$$

From the equation above, it is evident that all n-order derivatives of the state function remain continuous and bounded, which qualifies the state function as the C^n functions. The state space equation (1) for a PLIR has an equilibrium point $x = 0$ when the control input satisfies $u_0 = 0$, i.e.

$$x = \left[x_1\ x_2\ x_3\ x_4\right]^\top = \left[0\ 0\ 0\ 0\right]^\top.$$

In such case, the state space equation of PLIR is as follows:

$$\begin{cases} 0 = x_2, \\ 0 = \frac{-m_b g d\sin x_1 - m_b g w\sin x_3\cos x_1}{m_a r_1^2 + m_b\left[d^2 + (-r_2 + w\sin x_3)^2\right]} + \tau_\sigma, \\ 0 = x_4, \\ 0 = \frac{u_{\tau_\delta} + m_b d(-r_2 + w\sin x_3)(\cos x_3)x_2^2 - m_b g d\sin x_1\cos x_3}{m_b d^2}, \end{cases} \tag{2}$$

then, the derivation of equation (2) leads to:

$$\begin{cases} x_2 = 0, \\ x_3 = \arcsin\left(-dw^{-1}\tan x_1\right), \\ x_4 = 0, \\ u_o = u(0) = m_b gw \sin x_1 \cos\left[\arcsin\left(-dw^{-1}\tan x_1\right)\right], \end{cases} \tag{3}$$

and from eq. (3) we can obtain: for every x_1, $-1 \leq -dw^{-1}\tan x_1 \leq 1$, (3) has the equivalent x_3 and u_0. Given that the equilibrium family encompasses every equilibrium point within a nonlinear system, it is evident that system (1) also contains at least one equilibrium state.

B. CONSTRUCTION OF EQUILIBRIUM MANIFOLD LINEARIZATION MODEL FOR PLIR

The PLIR method involves approximating a nonlinear system's behavior by linearizing it about an equilibrium point. This process transforms the original nonlinear equations into their linear counterparts with respect to this specific point $x = \begin{bmatrix} x_1 \ x_2 \ x_3 \ x_4 \end{bmatrix}^\top = \begin{bmatrix} 0 \ 0 \ 0 \ 0 \end{bmatrix}^\top$, is transformed into the following eq.(3),

$$\begin{aligned} \dot{x} = f\left(x, u_{\tau_\delta}\right) &= \left.\frac{\partial f}{\partial x}\right|_{(x=0,u=0)} x \\ &+ \left.\frac{\partial f}{\partial u}\right|_{(x=0,u=0)} u_{\tau_\delta} + o\left(x, u_{\tau_\delta}\right) \\ &= \mathscr{A}x + \mathscr{B}u_{\tau_\delta} + o\left(x, u_{\tau_\delta}\right), \end{aligned}$$

where $o\left(x, u_{\tau_\delta}\right)$ is a higher order approximation term that is often omitted,

$$\begin{aligned} \mathscr{A} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-m_b gd}{m_a r_1^2 + m_b(d^2+r_2^2)} & 0 & \frac{-m_b gw}{m_a r_1^2 + m_b(d^2+r_2^2)} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-g}{w} & 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & 0 & a_{23} & 0 \\ 0 & 1 & 0 & 0 \\ a_{41} & 0 & 0 & 0 \end{bmatrix}, \\ \mathscr{B} &= \left.\frac{\partial f}{\partial u}\right|_{u_0=0} = \begin{bmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_2}{\partial u} & \frac{\partial f_3}{\partial u} & \frac{\partial f_4}{\partial u} \end{bmatrix}^\top \\ &= \begin{bmatrix} 0 \ 0 \ 0 \ \frac{1}{m_b w^2} \end{bmatrix}^\top = \begin{bmatrix} 0 \ 0 \ 0 \ b_4 \end{bmatrix}^\top, \end{aligned}$$

where, we can think of $o\left(x, u_{\tau_\delta}\right)$ as nonlinear function $\mathscr{H}(x)$ and external interference $\varpi(t)$, then let $y = \begin{bmatrix} \alpha_2 \ \dot{\alpha}_2 \end{bmatrix}^T = \begin{bmatrix} x_3 \ x_4 \end{bmatrix}^T$, so that the state space equation can be expressed as:

$$\begin{cases} \dot{x}(t) = \mathscr{A}x(t) + \mathscr{B}u(t-\tau_\delta) + \mathscr{D}\mathscr{H}(g) + \mathscr{F}_1\varpi(t) \\ y(t) = \mathscr{C}x(t) \\ z(t) = \mathscr{C}_1 x(t) + \mathscr{F}_2\varpi(t) \end{cases}, \tag{4}$$

where, $x(t)$ represents the state of a dynamic system at time t, which is the vector of n dimension and $u(t-\tau_\delta) \in R_n$ denote the vector for control input in which τ_δ is a time delay. The function $\mathscr{H}(x) \in R_n$ is a vector-valued function with nonlinearity that fulfills the condition that is quadratic bounded with respect to increments. Additionally, $\varpi(t)$ belongs to the space R^q and represents an external disturbance. The measured output is denoted as $y(t) \in R^l$, while $z(t) \in R^q$ represents an output subject to regulation. $\mathscr{A}$, $\mathscr{B}$, $\mathscr{C}$, $\mathscr{C}$, $\mathscr{D}$, $\mathscr{F}_1$, $\mathscr{F}_2$ are matrices with proper dimensions.

In order to ensure that there is no instability in the control process, A non-fragile robust controller that incorporates an observer can be developed using the following approach:

$$\begin{cases} \dot{\hat{x}}(t) = \mathscr{A}\hat{x}(t) + \mathscr{B}u(t-\tau_\delta) + \mathscr{D}\mathscr{H}(\hat{f}) \\ +(\mathscr{L} + \Delta\mathscr{L}(t))[y(t) - \hat{y}(t)] \\ \hat{y}(t) = \mathscr{C}\hat{x}(t) \\ u(t) = -(\mathscr{K} + \Delta\mathscr{K}(t))\hat{x}(t) \\ \hat{f} = \mathscr{E}\hat{x}(t), \end{cases} \tag{5}$$

here, $\hat{x}(t)$ represents the state of a dynamic system at time t and it is the vector of n dimension with estimation. There is a control gain matrix denoted by $\mathscr{K}$, which is of dimension $m\times n$. An observer gain matrix, represented as$\mathscr{L}$ has dimensions of $n\times p$. $\Delta\mathscr{K}(t)$ stands for a time-varying disturbance matrix within $R^{m\times n}$, and $\Delta\mathscr{L}(t)$ represents another time-varying disturbance matrix within $R^{n\times p}$. It is important to note that these perturbations are as follows:

$$\begin{bmatrix} \Delta\mathscr{K}(t) \ \Delta\mathscr{L}(t) \end{bmatrix}^T = \begin{bmatrix} \mathscr{M}_1\Delta_1(t)\mathscr{N}_1 \ \mathscr{M}_2\Delta_2(t)\mathscr{N}_2 \end{bmatrix}^T, \tag{6}$$

where, $\mathscr{M}_1$, $\mathscr{M}_2$, $\mathscr{N}_1$, $\mathscr{N}_2$ consist of some matrices with suitable dimensions. $\Delta_1(t)$ and $\Delta_2(t)$ represent continuous unknown functions, subject to the conditions $\Delta_1^T(t)\Delta_1(t) \leq I$ and $\Delta_2^T(t)\Delta_2(t) \leq I$, where I is the identity matrix.

Combining the aforementioned equation (4) with equation (5), we derive the following expression:

$$\begin{cases} \dot{x}(t) = \mathscr{A}x(t) - \mathscr{B}(\mathscr{K} + \Delta\mathscr{K}(t))x(t-\tau_\delta) \\ \qquad +\mathscr{B}(\mathscr{K} + \Delta\mathscr{K}(t))\sigma(t-\tau_\delta) + \mathscr{D}\mathscr{H}(f) + \mathscr{F}_1\varpi(t) \\ \dot{\sigma}(t) = (\mathscr{A} - \mathscr{L}\mathscr{C} - \Delta\mathscr{L}(t)\mathscr{C})\sigma(t) + \mathscr{D}\tilde{\mathscr{H}}(f) \\ +\mathscr{F}_1\varpi(t) \\ z(t) = \mathscr{C}_1 x(t) + \mathscr{F}_2\varpi(t), \end{cases} \tag{7}$$

where $\tilde{\mathscr{H}}(f) = \mathscr{H}(f) - \mathscr{H}(\hat{f})$.

Remark 1: We propose an innovative enhancement to the non-fragile robust controller by harnessing an iterative estimation technique. This technique, 'iterative proportional-integral interval estimation,' is traditionally utilized in linear discrete-time systems to incrementally perfect the state estimation. It operates by fine-tuning the proportional and integral components of the controller within a defined range, thereby ensuring that the discrepancy between the estimated and actual values is minimized to a preset margin. The integration of this iterative mechanism could endow the controller with superior precision and dependability, especially in fluctuating operational contexts where the system's parameters or external perturbations are prone to fluctuation. For power-line inspection robots (PILRs) that are frequently subjected to varying conditions and demand acute accuracy in monitoring and manipulative tasks, this method may offer distinct advantages.

C. MATHEMATICAL PRELIMINARY FINDINGS

For the investigation of the study outlined above, we introduce the following assumptions, definitions, and lemmas.

Assumption 1 ([24]): The pair $(\mathscr{A}, \mathscr{B})$ exhibits full controllability, and the pair $(\mathscr{A}, \mathscr{C})$ demonstrates full observability.

Assumption 2 ([25]): If the nonlinear function $\mathscr{H}(q)$ is defined for all values of q, then there must exist a symmetry matrix $\mathscr{T}_\rho$ which fulfills the condition expressed by the inequality

$$\begin{bmatrix} q-\hat{q} \\ \mathscr{H}(q)-\mathscr{H}(\hat{q}) \end{bmatrix}^T \mathscr{T}_\rho \begin{bmatrix} q-\hat{q} \\ \mathscr{H}(q)-\mathscr{H}(\hat{q}) \end{bmatrix} \geq 0$$

is valid, in that case, the non-linear function $\mathscr{H}(q)$ satisfies the condition of being increasing and quadratically bounded, while the symmetric matrix $\mathscr{T}_\rho$ is denoted as the incremental multiplier matrix.

Definition 1 ([30]): The composite system (7) exhibits passivity if

$$\int_0^t \varpi^T(t)z(t)dt \geq \eta, \forall \varpi(t) \in L_2[0,\infty),$$

where η is a constant that might vary according to the initial conditions.

Definition 2 ([31]): That composite system (7) achieves H_∞ performance when the following inequality holds true:

$$\int_0^t z^T(s)z(s)ds \leq \gamma^2 \int_0^t \varpi^T(s)\varpi(s)ds,$$

where γ being a constant that may be influenced by the system's initial conditions.

Lemma 1 ([32]): For a given matrix $\mathscr{S} = \begin{bmatrix} \mathscr{S}_{11} & \mathscr{S}_{12} \\ \mathscr{S}_{12}^T & \mathscr{S}_{22} \end{bmatrix}$ with $\mathscr{S}_{11} = \mathscr{S}_{11}^T, \mathscr{S}_{22} = \mathscr{S}_{22}^T$, the equivalent conditions are as follows:

(1) $\mathscr{S} < 0$;
(2) $\mathscr{S}_{11} < 0, \quad \mathscr{S}_{22} - \mathscr{S}_{12}^{\mathrm{T}}\mathscr{S}_{11}^{-1}\mathscr{S}_{12} < 0$;
(3) $\mathscr{S}_{22} < 0, \quad \mathscr{S}_{11} - \mathscr{S}_{12}\mathscr{S}_{22}^{-1}\mathscr{S}_{12}^{\mathrm{T}} < 0$.

Lemma 2 ([33]): Given arbitrary real matrices $\mathscr{P}$, $\mathscr{Q}$, and a matrix function $F(t)$ of appropriate dimensions, where $F(t)$ satisfies $F^{\mathrm{T}}(t)F(t) \leq I$ for all $t \in R$. Then for every number $\varepsilon > 0$, the following inequality holds true:

$$\mathscr{P}F(t)\mathscr{Q} + \mathscr{Q}^T F^T(t)\mathscr{P}^T \leq \lambda\mathscr{P}\mathscr{P}^T + \lambda^{-1}\mathscr{Q}^T\mathscr{Q}.$$

Lemma 3 ([34]): There exists an inequality $\mathscr{W} + \mathscr{Q}^T\mathscr{Y}^T + \mathscr{Y}\mathscr{Q} < 0$ for proper dimensions matrices $\mathscr{W}$, $\mathscr{Y}$, $\mathscr{Q}$ and $\mathscr{U}$, if $\mu \neq 0$ is a constant, which is equal to

$$\begin{bmatrix} \mathscr{W} & \mu\mathscr{Y} + \mathscr{Q}^T\mathscr{U}^T \\ * & -\mu\mathscr{U} - \mu\mathscr{U}^T \end{bmatrix} < 0.$$

In order to verify the stability of the system (4), the LMI method is used to assess whether the integrated system, as described by equation (7), fulfills the specified criteria for passivity and H_∞ robustness.

III. PRINCIPAL FINDINGS AND DEMONSTRATIONS

In this segment, a condition that proves sufficient is explored. The system in question is inherently reactive, maintaining equilibrium without the influence of external inputs, and demonstrates stability that gradually approaches an asymptotic state. To commence, Theorem 1 is formulated as follows.

Theorem 1: Considering the amalgamated system (7), assume the existence of positive constants λ_1, λ_2, λ_3, u, v, along with normal η and non-zero constants μ. Additionally, consider positive definite matrix, denoted as $\mathscr{P}$, $\mathscr{Q}_1$, $\mathscr{Q}_2$, and $\mathscr{R} \in \mathbb{R}^{n\times n}$, as well as the matrices $\mathscr{U} \in \mathbb{R}^{m\times m}$, $\mathscr{K} \in \mathbb{R}^{m\times n}$, and $\mathscr{L} \in \mathbb{R}^{n\times q}$. Subsequently, the integration of the disturbance specified in (6) into the system denoted as (4) results in a configuration that demonstrates both passive stability and asymptotic stability when subjected to control by the controller based on observer (5). The gain of controller is determined by $\mathscr{K} = \mathscr{U}^{-1}\mathscr{K}$, and the gain of observer is given by $\mathscr{L} = \mathscr{R}^{-1}\mathscr{L}$. These conditions are satisfied when the following Linear Matrix Inequality (LMI) is valid:

$$\begin{bmatrix} U_1 & -\mathscr{B}\hat{\mathscr{K}} & 0 & \mathscr{B}\hat{\mathscr{K}} & U_2 & 0 & \mathscr{P}\mathscr{F}_1 - \mathscr{C}_1^T & U_3 & \mho_1 \\ * & \varphi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & U_4 & 0 & 0 & U_5 & \mathscr{R}\mathscr{F}_1 & \hat{\mathscr{K}} & \mho_2 \\ * & * & * & \varphi_4 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & u\mathscr{T}_{22} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & v\mathscr{T}_{22} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\mathscr{F}_2^T - \mathscr{F}_2 & 0 & 0 \\ * & * & * & * & * & * & * & U_6 & 0 \\ * & * & * & * & * & * & * & * & \mho_3 \end{bmatrix} < 0, \tag{8}$$

where

$$U_1 = \mathscr{A}^T\mathscr{P} + \mathscr{P}\mathscr{A} + u\hat{\mathscr{T}}_{11} + \mathscr{Q}_1, U_2 = \mathscr{P}\mathscr{D} + u\hat{\mathscr{T}}_{12},$$
$$U_3 = \mu(\mathscr{P}\mathscr{B} - \mathscr{B}\mathscr{U}) - \hat{\mathscr{K}},$$
$$U_4 = \mathscr{A}^T\mathscr{R} + \mathscr{R}\mathscr{A} - \mathscr{C}^T\hat{\mathscr{L}}^T - \hat{\mathscr{L}}\mathscr{C}$$
$$+ \lambda_3\mathscr{C}^T N_1^T N_1\mathscr{C} + \mathscr{Q}_2 + v\hat{\mathscr{T}}_{11},$$
$$U_5 = \mathscr{R}\mathscr{D} + v\hat{\mathscr{T}}_{12}, U_6 = -\mu\mathscr{U} - \mu\mathscr{U}^T,$$
$$\varphi_2 = -\mathscr{Q}_1 + \lambda_1\mathscr{N}_2^T\mathscr{N}_2, \varphi_4 = -\mathscr{Q}_2 + \lambda_2\mathscr{N}_2^T\mathscr{N}_2,$$
$$\mho_1 = [\mathscr{R}\mathscr{M}_1, \mathscr{P}\mathscr{B}\mathscr{M}_2, 0], \mho_2 = [0, 0, \mathscr{R}\mathscr{M}_1],$$
$$\mho_3 = \mathrm{diag}\{-\lambda_1 I, -\lambda_2 I, -\lambda_3 I\}.$$

Proof: For controllers based on system (4) and observers (5), contemplate the subsequent Lyapunov-Krasovskii functional candidate:

$$\mathscr{V}(t) = \mathscr{V}_1(t) + \mathscr{V}_2(t) + \mathscr{V}_3(t) + \mathscr{V}_4(t),$$
$$\mathscr{V}_1(t) = x^T(t)\mathscr{P}x(t), \mathscr{V}_2(t) = \int_{t-\sigma}^t x^T(t)\mathscr{Q}_1 x(t)\mathrm{d}t,$$
$$\mathscr{V}_3(t) = \sigma^T(t)\mathscr{R}\sigma(t), \mathscr{V}_4(t) = \int_{t-\sigma}^t \sigma^T(t)\mathscr{Q}_2\sigma(t)\mathrm{d}t,$$

thus, the rate of change of the potential energy function $V(t)$ with respect to time can be described by the sum of the time derivatives of its individual components. This is mathematically expressed as:

$$\dot{\mathscr{V}}(t) = \dot{\mathscr{V}_1}(t) + \dot{\mathscr{V}_2}(t) + \dot{\mathscr{V}_3}(t) + \dot{\mathscr{V}_4}(t),$$

where,

$$\begin{aligned}
\dot{\mathscr{V}_1}(t) &= \dot{x}^T(t)\mathscr{P}x(t) + x^T(t)\mathscr{P}\dot{x}(t) \\
&= [\mathscr{A}x(t) - \mathscr{B}(\mathscr{K} + \Delta\mathscr{K})x(t-\tau_\delta) \\
&\quad + \mathscr{B}(\mathscr{K} + \Delta\mathscr{K})\sigma(t-\tau_\delta) \\
&\quad + \mathscr{D}\mathscr{H}(f) + \mathscr{F}_1\varpi(t)]^T\mathscr{P}x(t) + x^T(t)\mathscr{P}[\mathscr{A}x(t) \\
&\quad - \mathscr{B}(\mathscr{K} \\
&\quad + \Delta\mathscr{K})x(t-\tau_\delta) + \mathscr{B}(\mathscr{K} + \Delta\mathscr{K})\sigma(t-\tau_\delta) \\
&\quad + \mathscr{D}\mathscr{H}(f) + \mathscr{F}_1\varpi(t)], \\
\dot{\mathscr{V}_2}(t) &= x^T(t)\mathscr{Q}_1x(t) - x^T(t-\tau_\delta)\mathscr{Q}_1x(t-\tau_\delta), \\
\dot{\mathscr{V}_3}(t) &= \dot{\sigma}^T(t)\mathscr{R}\sigma(t) + \sigma^T(t)\mathscr{R}\dot{\sigma(t)} \\
&= \left[(\mathscr{A} - \mathscr{L}\mathscr{C} - \Delta\mathscr{L}\mathscr{C})\sigma(t) + \mathscr{D}\tilde{\mathscr{H}} + \mathscr{F}_1\varpi(t)\right]^T \\
&\quad \times \mathscr{R}\sigma(t) \\
&\quad + \sigma^T(t)\mathscr{R}\left[(\mathscr{A} - \mathscr{L}\mathscr{C} - \Delta\mathscr{L}\mathscr{C})\sigma(t) + \mathscr{D}\tilde{\mathscr{H}(f)}\right. \\
&\quad \left. + \mathscr{F}_1\varpi(t)\mathscr{F}_1\varpi(t)\right], \\
\dot{\mathscr{V}_4}(t) &= \sigma^T(t)\mathscr{Q}_2\sigma(t) - \sigma^T(t-\tau_\delta)\mathscr{Q}_2\sigma(t-\tau_\delta).
\end{aligned}$$

So that

$$\begin{aligned}
&\dot{\mathscr{V}}(t) \\
&= \dot{\mathscr{V}_1}(t) + \dot{\mathscr{V}_2}(t) + \dot{\mathscr{V}_3}(t) + \dot{\mathscr{V}_4}(t) \\
&= x^T(t)(\mathscr{A}^T\mathscr{P} + \mathscr{P}\mathscr{A} + \mathscr{Q}_1)x(t) + x^T(t)(-\mathscr{P}\mathscr{B}\mathscr{K} \\
&\quad - \mathscr{P}\mathscr{B}\Delta\mathscr{K})x(t-\tau_\delta) + x^T(t)(\mathscr{P}\mathscr{B}\mathscr{K} \\
&\quad + \mathscr{P}\mathscr{B}\Delta\mathscr{K})\sigma(t-\tau_\delta) \\
&\quad + x^T(t)\mathscr{P}\mathscr{D}\mathscr{H}(f) + x^T(t)\mathscr{P}\mathscr{F}_1\omega(t) \\
&\quad + x^T(t-\tau_\delta)(-\mathscr{Q}_1)x(t \\
&\quad - \tau_\delta) + x^T(t-\tau_\delta)(-\mathscr{K}^T\mathscr{B}^T\mathscr{P} - \Delta\mathscr{K}^T\mathscr{B}\mathscr{P})x(t) \\
&\quad + \sigma^T(t)(\mathscr{R}\mathscr{A} + \mathscr{A}^T\mathscr{R} - \mathscr{C}^T\mathscr{L}^T\mathscr{R} \\
&\quad - \mathscr{R}\mathscr{L}\mathscr{C} - \mathscr{C}^T\Delta\mathscr{L}^T\mathscr{R} \\
&\quad - \mathscr{R}\Delta\mathscr{L}\mathscr{C} + \mathscr{Q}_2)\sigma(t) + \sigma^T(t)\mathscr{R}\mathscr{D}\tilde{\mathscr{H}(f)} \\
&\quad + \sigma^T(t)\mathscr{R}\mathscr{F}_1\omega(t) \\
&\quad + \sigma^T(t-\tau_\delta)(-\mathscr{Q}_2)e(t-\tau_\delta) + \varpi^T(t)\mathscr{F}_1^T\mathscr{R}\sigma(t) \\
&\quad + \sigma^T(t-\tau_\delta)(\mathscr{K}^T\mathscr{B}^T\mathscr{P} + \Delta\mathscr{K}^T\mathscr{B}^T\mathscr{P})x(t) \\
&\quad + \mathscr{H}(f)^T(f)\mathscr{D}^T\mathscr{P}x(t + \varpi^T(t)\mathscr{F}_1^T\mathscr{P}x(t) \\
&\quad + \tilde{\mathscr{H}}(f)^T\mathscr{D}^T\mathscr{R}\sigma(t).
\end{aligned} \tag{9}$$

Utilizing *Lemma*2, we obtain

$$\begin{aligned}
\varrho_1 &= x^T(t)(-\mathscr{P}\mathscr{B}\Delta\mathscr{K})x^T(t-\tau_\delta) \\
&\quad + x^T(t-\tau_\delta)(-\Delta\mathscr{K}^T\mathscr{B}^T\mathscr{P})x^T(t) \\
&= x^T(t)(-\mathscr{P}\mathscr{B}\mathscr{M}_2\Delta_2(t)\mathscr{N}_2)x(t-\tau_\delta) + x^T(t \\
&\quad - \tau_\delta)(-\mathscr{N}_2^T\Delta_2^T(t)\mathscr{M}_2^T\mathscr{B}^T\mathscr{P})x(t) \\
&\le \lambda_1^{-1}x^T(t)\mathscr{P}\mathscr{B}\mathscr{M}_2\mathscr{M}_2^T\mathscr{B}^T\mathscr{P}x(t) \\
&\quad + \lambda_1x^T(t-\tau_\delta)\mathscr{N}_2^T\mathscr{N}_2x(t-\tau_\delta), \\
\varrho_2 &= x^T(t)\mathscr{P}\mathscr{B}\Delta\mathscr{K}\sigma(t-\tau_\delta) + \sigma^T(t-\tau_\delta)\Delta\mathscr{K}^T\mathscr{B}^T\mathscr{P}x(t) \\
&= x^T(t)\mathscr{P}\mathscr{B}\mathscr{M}_2\Delta_2(t)\mathscr{N}_2\sigma(t-\tau_\delta) \\
&\quad + \sigma^T(t-\tau_\delta)\mathscr{N}_2^T\Delta_2^T(t)\mathscr{M}_2^T\mathscr{B}^T\mathscr{P}x(t) \\
&\le \lambda_2^{-1}x^T(t)\mathscr{P}\mathscr{B}\mathscr{M}_2\mathscr{M}_2^T\mathscr{B}^T\mathscr{P}x(t) \\
&\quad + \lambda_2\sigma^T(t-\tau_\delta)\mathscr{N}_2^T\mathscr{N}_2\sigma(t-\tau_\delta), \\
\varrho_3 &= -\mathscr{C}^T\Delta\mathscr{L}^T\mathscr{R} - \mathscr{R}\Delta\mathscr{L}\mathscr{C} \\
&= -\mathscr{C}^T\mathscr{N}_1^T\Delta_1^T(t)\mathscr{M}_1^T\mathscr{R} - \mathscr{R}\mathscr{M}_1\Delta_1(t)\mathscr{N}_1\mathscr{C} \\
&\le \lambda_3^{-1}\mathscr{R}\mathscr{M}_1\mathscr{M}_1^T\mathscr{R} + \lambda_3\mathscr{C}^T\mathscr{N}_1^T\mathscr{N}_1\mathscr{C}.
\end{aligned} \tag{10}$$

Replacing the inequalities from (10) into (9) leads to

$$\begin{aligned}
&\dot{\mathscr{V}}(t) \\
&= \dot{\mathscr{V}_1}(t) + \dot{\mathscr{V}_2}(t) + \dot{\mathscr{V}_3}(t) + \dot{\mathscr{V}_4}(t) \\
&\le x^T(t)(\mathscr{A}^T\mathscr{P} + \mathscr{P}\mathscr{A} + \lambda_1^{-1}\mathscr{P}\mathscr{B}\mathscr{M}_2\mathscr{M}_2^T\mathscr{B}^T\mathscr{P} \\
&\quad + \lambda_2^{-1}\mathscr{P}\mathscr{B}\mathscr{M}_2\mathscr{M}_2^T\mathscr{B}^T\mathscr{P} + \mathscr{Q}_1)x(t) \\
&\quad + x^T(t)(-\mathscr{P}\mathscr{B}\mathscr{K})x(t \\
&\quad - \tau_\delta) + x^T(t)(\mathscr{P}\mathscr{B}\mathscr{K})\sigma(t-\tau_\delta) + x^T(t)\mathscr{P}\mathscr{D}\mathscr{H}(g) \\
&\quad + x^T(t)\mathscr{P}\mathscr{F}_1\varpi(t) + x^T(t-\tau_\delta)(-\mathscr{Q}_1 + \lambda_1\mathscr{N}_2^T\mathscr{N}_2) \\
&\quad \times x(t-\tau_\delta) \\
&\quad + x^T(t-\tau_\delta)(-\mathscr{K}^T\mathscr{B}^T\mathscr{P})x(t) + \sigma^T(t)(\mathscr{A}^T\mathscr{R} + \mathscr{R}\mathscr{A} \\
&\quad - \mathscr{C}^T\mathscr{L}^T\mathscr{R} - \mathscr{R}\mathscr{L}\mathscr{C} + \lambda_3^{-1}\mathscr{R}\mathscr{M}_1\mathscr{M}_1^T\mathscr{R} \\
&\quad + \lambda_3\mathscr{C}^T\mathscr{N}_1^T\mathscr{N}_1\mathscr{C} \\
&\quad + \lambda_1\mathscr{N}_2^T\mathscr{N}_2 + \mathscr{Q}_2)\sigma(t) + \sigma^T(t)\mathscr{R}\mathscr{G}\tilde{\mathscr{H}} \\
&\quad + \sigma^T(t)\mathscr{R}\mathscr{F}_1\varpi(t) \\
&\quad + \sigma^T(t-\tau_\delta)(-\mathscr{Q}_2 + \lambda_2\mathscr{N}_3^T\mathscr{N}_3)\sigma(t-\tau_\delta) + \sigma^T(t \\
&\quad - \tau_\delta)(\mathscr{K}^T\mathscr{B}^T\mathscr{P})x(t) + \mathscr{H}^T(f)\mathscr{G}^T\mathscr{P}x(t) \\
&\quad + \varpi^T(t)\mathscr{F}_1^T\mathscr{P}x(t) \\
&\quad + \tilde{\mathscr{H}}^T\mathscr{G}^T\mathscr{R}\sigma(t) + \varpi(t)^T(t)\mathscr{F}_1^T\mathscr{R}\varpi(t),
\end{aligned}$$

then,

$$\dot{\mathscr{V}}(t) - 2\varpi(t)z(t) \le \chi^T(t)\Upsilon_1\chi(t),$$

in which,

$$\begin{aligned}
&\chi^T(t) \\
&= \left[x^T(t)\; x^T(t-\tau_\delta)\; \sigma^T(t)\; \sigma^T(t-\tau_\delta)\; \mathscr{H}^T(f)\; \tilde{\mathscr{H}}^T\; \varpi^T(t)\right]^T.
\end{aligned}$$

If Υ_1 is less than zero, then the expression $\dot{\mathscr{V}}(t) - 2\varpi(t)z(t)$ is negative. Consequently,

$$\int_0^t \varpi^T(t)z(t)dt > \frac{1}{2}\int_0^t V(t)dt = \eta,$$

subsequently, the above can prove that $\Upsilon_1 < 0$.

$$\Upsilon_1 = \begin{bmatrix} \varphi_1 & -\mathscr{P}\mathscr{B}\mathscr{K} & 0 & \mathscr{P}\mathscr{B}\mathscr{K} & \mathscr{P}\mathscr{D} & 0 & \mathscr{P}\mathscr{F}_1 - \mathscr{C}_1^T \\ * & \varphi_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & \varphi_3 & 0 & 0 & \mathscr{R}\mathscr{D} & \mathscr{R}\mathscr{F}_1 \\ * & * & * & \varphi_4 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & -\mathscr{F}_2^T - \mathscr{F}_2 \end{bmatrix},$$

where

$$\begin{aligned} \varphi_1 &= \mathscr{A}^T\mathscr{P} + \mathscr{P}\mathscr{A} + \lambda_1^{-1}\mathscr{P}\mathscr{B}\mathscr{M}_2\mathscr{M}_2^T\mathscr{B}^T\mathscr{P} \\ &\quad + \lambda_2^{-1}\mathscr{P}\mathscr{B}\mathscr{M}_2\mathscr{M}_2^T\mathscr{B}^T\mathscr{P} + \mathscr{Q}_1, \\ \varphi_2 &= -\mathscr{Q}_1 + \lambda_1\mathscr{N}_2^T\mathscr{N}_2, \varphi_4 = -\mathscr{Q}_2 + \lambda_2\mathscr{N}_3^T\mathscr{N}_3 \\ \varphi_3 &= \mathscr{R}\mathscr{A} - \mathscr{C}^T\mathscr{L}^T\mathscr{R} + \mathscr{A}^T\mathscr{R} - \mathscr{R}\mathscr{L}\mathscr{C} \\ &\quad + \lambda_3^{-1}\mathscr{R}\mathscr{M}_1\mathscr{M}_1^T\mathscr{R} \\ &\quad + \lambda_3\mathscr{C}^T\mathscr{N}_1^T\mathscr{N}_1\mathscr{C} + \lambda_1\mathscr{N}_2^T\mathscr{N}_2 + \mathscr{Q}_2. \end{aligned}$$

Given the presence of the nonlinear term $\mathscr{P}\mathscr{B}\mathscr{K}$ in Υ_1, we proceed to address the nonlinear term as follows: since the function $\mathscr{H}(f)$ meets the bounded condition for quadratic increment, the following inequality holds true:

$$\begin{bmatrix} x(t) \\ \mathscr{H}(f) \end{bmatrix}^T \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix}^T \mathscr{T}_\rho \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x(t) \\ \mathscr{H}(f) \end{bmatrix} \geqslant 0$$

$$\begin{bmatrix} \sigma(t) \\ \tilde{\mathscr{H}} \end{bmatrix}^T \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix}^T \mathscr{T}_\rho \begin{bmatrix} G & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \sigma(t) \\ \tilde{\mathscr{H}} \end{bmatrix} \geqslant 0,$$

Set

$$\mathscr{T}_\rho = \begin{bmatrix} \mathscr{T}_{11} & \mathscr{T}_{12} \\ \mathscr{T}_{21} & \mathscr{T}_{22} \end{bmatrix}, \quad \varphi = \begin{bmatrix} G & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix},$$

then,

$$\chi^T(t)\varphi^T \begin{bmatrix} u\mathscr{T}_{11} & 0 & 0 & 0 & u\mathscr{T}_{12} & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & u\mathscr{T}_{22} & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix} \varphi\chi(t) \geqslant 0,$$

$$\chi^T(t)\varphi^T \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & v\mathscr{T}_{11} & 0 & 0 & v\mathscr{T}_{12} & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & v\mathscr{T}_{22} & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix} \varphi\chi(t) \geqslant 0,$$

where

$$\dot{\mathscr{V}}(t) - 2\varpi(t)z(t) \leq \chi^T(t)\Upsilon_1\chi(t),$$

$$\Upsilon = \begin{bmatrix} \psi_1 & -\mathscr{P}\mathscr{B}\mathscr{K} & 0 & \mathscr{P}\mathscr{B}\mathscr{K} & \psi_2 & 0 & \mathscr{P}\mathscr{F}_1 - \mathscr{C}_1^T \\ * & \varphi_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & \psi_3 & 0 & 0 & \psi_4 & \mathscr{R}\mathscr{F}_1 \\ * & * & * & \varphi_4 & 0 & 0 & 0 \\ * & * & * & * & u\mathscr{T}_{22} & 0 & 0 \\ * & * & * & * & * & v\mathscr{T}_{22} & 0 \\ * & * & * & * & * & * & -\mathscr{F}_2^T - \mathscr{F}_2 \end{bmatrix},$$

and

$$\psi_1 = \varphi_1 + u\hat{\mathscr{T}}_{11}, \psi_2 = \mathscr{P}\mathscr{D} + u\hat{\mathscr{T}}_{12}, \psi_3 = \varphi_3 + v\hat{\mathscr{T}}_{11},$$
$$\psi_4 = \mathscr{R}\mathscr{D} + v\hat{\mathscr{T}}_{12}\hat{\mathscr{T}}_{11} = G^T\mathscr{T}_{11}G, \hat{\mathscr{T}}_{12} = G^T\mathscr{T}_{12}G.$$

Imagine that a nonlinear event can be represented as

$$\mathscr{P}\mathscr{B}\mathscr{K} = (\mathscr{P}\mathscr{B} - \mathscr{B}\mathscr{U})\mathscr{U}^{-1}\hat{\mathscr{K}} + \mathscr{B}\hat{\mathscr{K}}, \quad \mathscr{K} = \mathscr{U}^{-1}\hat{\mathscr{K}},$$

at this juncture, Υ has the potential to undergo transformation into

$$\bar{\Upsilon} = \begin{bmatrix} \psi_1 & -\mathscr{B}\hat{\mathscr{K}} & 0 & \mathscr{B}\hat{\mathscr{K}} & \psi_2 & 0 & \mathscr{P}\mathscr{F}_1 - \mathscr{C}_1^T \\ * & \varphi_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & \psi_3 & 0 & 0 & \psi_4 & \mathscr{R}\mathscr{F}_1 \\ * & * & * & \varphi_4 & 0 & 0 & 0 \\ * & * & * & * & u\mathscr{T}_{22} & 0 & 0 \\ * & * & * & * & * & v\mathscr{T}_{22} & 0 \\ * & * & * & * & * & * & -\mathscr{F}_2^T - \mathscr{F}_2 \end{bmatrix} + \begin{bmatrix} 0 & \psi_5 & 0 & \psi_6 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

where,

$$\psi_1 = \varphi_1 + u\hat{\mathscr{T}}_{11}, \psi_2 = \mathscr{P}\mathscr{D} + u\hat{\mathscr{T}}_{12}, \psi_3 = \varphi_3 + v\hat{\mathscr{T}}_{11},$$
$$\psi_4 = \mathscr{R}\mathscr{D} + v\hat{\mathscr{T}}_{12}, \psi_5 = -(\mathscr{P}\mathscr{B} - \mathscr{B}\mathscr{U})\mathscr{U}^{-1}\hat{\mathscr{K}},$$
$$\psi_6 = (\mathscr{P}\mathscr{B} - \mathscr{B}\mathscr{U})\mathscr{U}^{-1}\hat{\mathscr{K}},$$
$$\hat{\mathscr{T}}_{11} = G^T\mathscr{T}_{11}G, \hat{\mathscr{T}}_{12} = G^T\mathscr{T}_{12}G.$$

If

$$\mathscr{Y} = \begin{bmatrix} \mathscr{P}\mathscr{B} - \mathscr{B}\mathscr{U} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathscr{Z} = \mathscr{U}^{-1} \begin{bmatrix} -\hat{\mathscr{K}}^T \\ 0 \\ \hat{\mathscr{K}}^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T,$$

subsequently, based on the Lemma 3, we obtain

$$\bar{\Upsilon}' = \begin{bmatrix} \psi_1 & -\mathscr{B}\hat{\mathscr{K}} & 0 & \mathscr{B}\hat{\mathscr{K}} & \psi_2 & 0 & \mathscr{P}\mathscr{F}_1 - \mathscr{C}_1^T & \Omega_1 \\ * & \varphi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \psi_3 & 0 & 0 & \psi_4 & \mathscr{R}\mathscr{F}_1 & \hat{\mathscr{K}} \\ * & * & * & \varphi_4 & 0 & 0 & 0 & 0 \\ * & * & * & * & u\mathscr{T}_{22} & 0 & 0 & 0 \\ * & * & * & * & * & v\mathscr{T}_{22} & 0 & 0 \\ * & * & * & * & * & * & -\mathscr{F}_2^T - \mathscr{F}_2 & 0 \\ * & * & * & * & * & * & * & \Omega_2 \end{bmatrix}.$$

where, $\Omega_1 = \mu(\mathscr{P}\mathscr{B} - \mathscr{B}\mathscr{U}) - \hat{\mathscr{K}}$, $\Omega_2 = -\mu\mathscr{U} - \mu\mathscr{U}^T$.

Applying the Schur complement, if $\bar{\Upsilon}' < 0$, it implies that $\Upsilon < 0$. Consequently, the validity of LMI (8) is established, and this indicates that $\Upsilon_1 < 0$, signifying the passive and asymptotically stable nature of the system (7).

Indeed, when $\omega(t) = 0$, similarly, we readily derive

$$\dot{\mathscr{V}}(t) \leq \xi^T(t)\Lambda\xi(t),$$

where

$$\xi(t) = \left[x^T(t)\ x^T(t-\tau_\delta)\ \sigma^T(t)\ \sigma^T(t-\tau_\delta)\ \mathscr{H}^T(g)\ \tilde{\mathscr{H}}\right]^T,$$

and

$$\Lambda = \begin{bmatrix} \varphi_1 & -\mathscr{P}\mathscr{B}\mathscr{K} & 0 & \mathscr{P}\mathscr{B}\mathscr{K} & \mathscr{P}\mathscr{D} & 0 \\ * & \varphi_2 & 0 & 0 & 0 & 0 \\ * & * & \varphi_3 & 0 & 0 & RG \\ * & * & * & \varphi_4 & 0 & 0 \\ * & * & * & * & u\mathscr{T}_{22} & 0 \\ * & * & * & * & * & v\mathscr{T}_{22} \end{bmatrix} < 0.$$

If $\Upsilon_1 < 0$, Schur Complement Lemma implies that $\Lambda < 0$. Additionally, $\dot{V}_1(t) \leq \xi^T(t)\Lambda\xi(t)$ holds for $\xi(t) \neq 0$; indicating the asymptotic stability of system (7) when $\xi(t) = 0$. This concludes the proof.

Building upon the investigation into the examination of the passive and stable characteristics for closed-loop systems, in above system we delve into the H_∞ optimal properties, as established in Theorem 1. Consequently, Theorem 2 unfolds as follows. □

Theorem 2: Assuming γ remains constant, consider the external disturbance $\varpi(t)$ to be not zero. Assume the existence of positive constants $\lambda_1, \lambda_2, \lambda_3, u, v$, normal η, non-zero constants μ, positive definite matrices $\mathscr{P}$, $\mathscr{Q}1$, $\mathscr{Q}2$, $\mathscr{R} \in \mathbb{R}^{n\times n}$, matrices $\mathscr{U} \in \mathbb{R}^{m\times m}$, $\hat{\mathscr{K}} \in \mathbb{R}^{m\times n}$, and $\hat{\mathscr{L}} \in \mathbb{R}^{n\times q}$. The closed-loop system exhibits passivity, asymptotic stability and H_∞ performance when it meets the conditions of the LMI (11) stated below:

$$\begin{bmatrix} \bar{U}_1 & -\mathscr{B}\hat{\mathscr{K}} & 0 & -\mathscr{B}\hat{\mathscr{K}} & U_2 & 0 & \Xi_1 & U_3 & \mho_1 \\ * & \varphi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & U_4 & 0 & 0 & U_5 & \mathscr{R}\mathscr{F}_1 & \hat{\mathscr{K}} & \mho_2 \\ * & * & * & \varphi_4 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & u\mathscr{T}_{22} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & v\mathscr{T}_{22} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_2 & 0 & 0 \\ * & * & * & * & * & * & * & U_6 & 0 \\ * & * & * & * & * & * & * & * & \mho_3 \end{bmatrix} < 0, \tag{11}$$

where

$$\bar{U}_1 = U_1 + I,$$
$$\Xi_1 = \mathscr{P}\mathscr{F}_1 - \mathscr{C}_1^T + \mathscr{C}_2^T\mathscr{F}_2$$
$$\Xi_2 = -\mathscr{F}_2^T - \mathscr{F}_2 + I - \gamma^2.$$

TABLE 1. Specifications for a single-link flexible manipulator's parameters.

parameter	unit	numeric value
m_a	kg	63
m_b	kg	27
r_1	m	0.18
r_2	m	0.42
d	m	0.5
w	m	0.5

Proof: For any perturbation $\omega(t)$ that is non-zero, regardless of its specific values, we obtain:

$$\dot{\mathscr{V}}(t) + z^T(t)z(t) - \gamma^2\varpi^T(t)\varpi(t) < 0,$$

subsequently,

$$\int_0^t z^T(t)z(t)dt < \gamma^2\int_0^t \varpi^T(t)\varpi(t)dt - \mathscr{V}_1(0) + \mathscr{V}_1(t).$$

As stated in Theorem 1, the discussed system exhibits asymptotic stability. Therefore, as time approaches infinity ($t \to \infty$), we can derive:

$$\int_0^t z^T(t)z(t)dt < \gamma^2\int_0^t \varpi^T(t)\varpi(t)dt.$$

If so, it is asserted that the H_∞ performance criteria are satisfied. Given that $\dot{\mathscr{V}}(t) + z^T(t)z(t) - \gamma^2\varpi^T(t)\varpi(t) \leqslant \chi^T(t)\Upsilon_4\chi(t)$, the verification for the matrix inequality (11) is readily attainable, where, as shown in the equation at the bottom of the next page.

Hence, $\chi^T(t)\Upsilon_4\chi(t) < 0$ is synonymous with $\int_0^t z^T(t)z(t)dt < \gamma^2\int_0^t \varpi^T(t)\varpi(t)dt$, fulfilling the $H\infty$ performance criterion.

IV. SIMULATIONS

To assess the efficacy of the suggested nonfragile observer-based controller, this section will involve conducting MATLAB simulations on a PLIR system.

The precise parameters of the PILR are outlined in Table 1. For a more comprehensive set of physical parameters regarding the system, please refer to citation [29].

The simulation design incorporates the following model parameters:

$$\mathscr{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix},$$

$$\mathscr{B} = \begin{bmatrix} 0 & 21.6 & 0 & 0 \end{bmatrix}^T, \mathscr{H}(f) = \sin(f),$$

$$\mathscr{D} = \begin{bmatrix} 0 \\ 0 \\ -2.44 \\ 0 \end{bmatrix}, \mathscr{C} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}^T,$$

$$\mathscr{E} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \mathscr{F} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

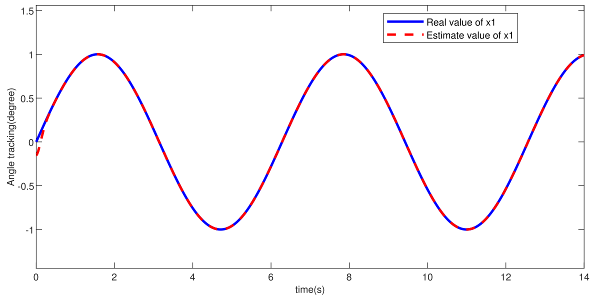

FIGURE 4. Actual value and estimate value of x_1.

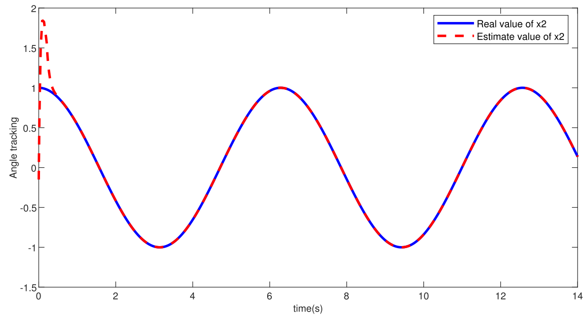

FIGURE 5. Actual value and estimate value of x_1.

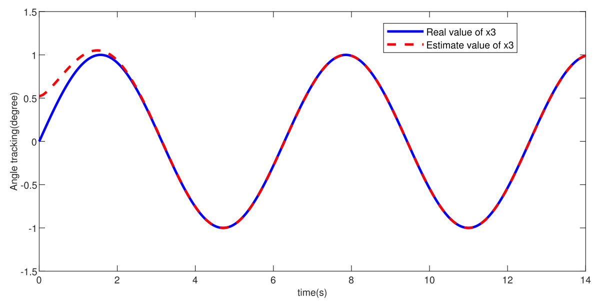

FIGURE 6. Actual value and estimate value of x_3.

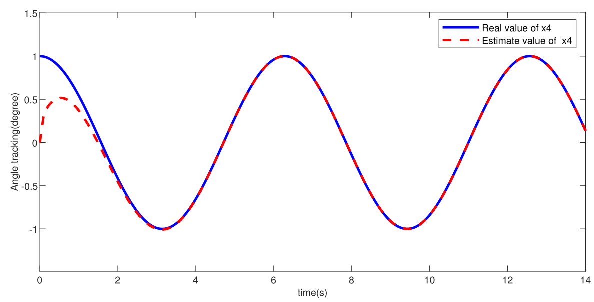

FIGURE 7. Actual value and estimate value of x_4.

$$\varpi(t) = \begin{bmatrix} eps(0) \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The matrix representing the perturbation (6) can be deduced from the given data as follows:

$$\mathscr{M}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathscr{M}_2 = 1, \mathscr{N}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathscr{N}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

$$\Delta_2(t) = e, \Delta_3(t) = \begin{bmatrix} 0 & p \\ h & 0 \end{bmatrix}, \tau_\delta = eps(0),$$

where,

$$e = \cos(4t), p = \cos(3t), h = -0.6\cos(2t).$$

Given that Assumption 2 is valid, it implies that the nonlinear function $\mathscr{H}(f)$ satisfies the quadratic bound constraint, implying that as f increases, so does the growth of $\mathscr{H}(f)$, so the incrementing matrix can be expressed in the following manner:

$$\mathscr{T}_\rho = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

To guarantee the system stability, one can solve the LMIs of Theorem 1. Subsequently, the gain of controller and the gain of observer can be determined, respectively:

$$\mathscr{K} = \begin{bmatrix} 4.58 & 5.24 & -0.43 & -0.83 \end{bmatrix},$$

$$\mathscr{L} = \begin{bmatrix} 13.13 & -12.00 & 14.59 & 15.81 \\ 14.33 & -26.98 & -26.68 & 14.73 \end{bmatrix}^T.$$

Upon substituting the gain of controller and the gain of observer into the non-fragile robust controller derived from observer (5), associated with the original system (4), and then simulating the system using MATLAB software, Figures 4 to 11 can be generated.

$$\Upsilon_4 = \begin{bmatrix} U_1 + I & -\mathscr{B}\hat{\mathscr{K}} & 0 & -\mathscr{B}\hat{\mathscr{K}} & U_2 & 0 & \mathscr{P}\mathscr{F}_1 - \mathscr{C}_1^T & U_3 & \mho_1 \\ * & \varphi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & U_4 & 0 & 0 & U_5 & \mathscr{R}\mathscr{F}_1 & \hat{\mathscr{K}} & \mho_2 \\ * & * & * & \varphi_4 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & u\mathscr{T}_{22} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & v\mathscr{T}_{22} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\mathscr{F}_2^T - \mathscr{F}_2 & 0 & 0 \\ * & * & * & * & * & * & * & U_6 & 0 \\ * & * & * & * & * & * & * & * & \mho_3 \end{bmatrix}.$$

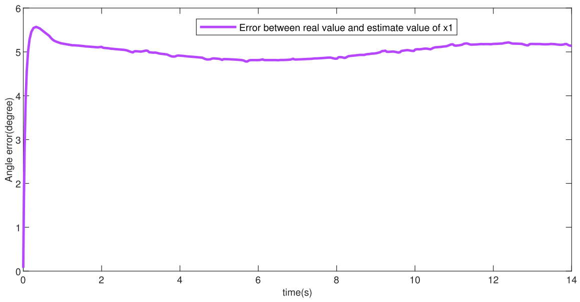

FIGURE 8. Error between real value and estimate value of x_1.

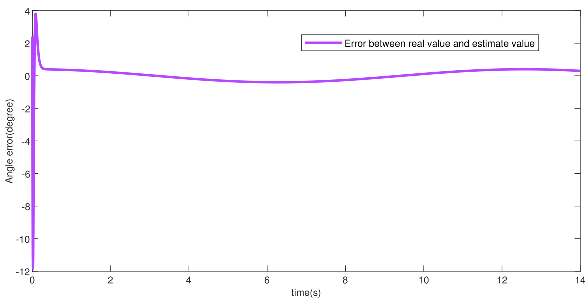

FIGURE 9. Error between real value and estimate value of x_2.

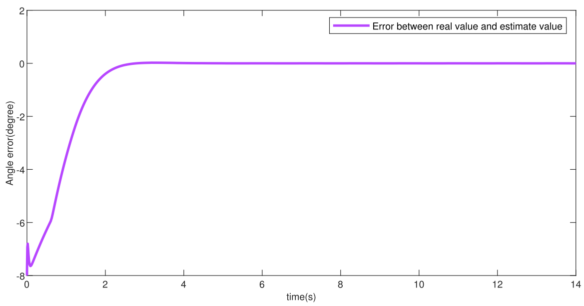

FIGURE 10. Error between real value and estimate value of x_3.

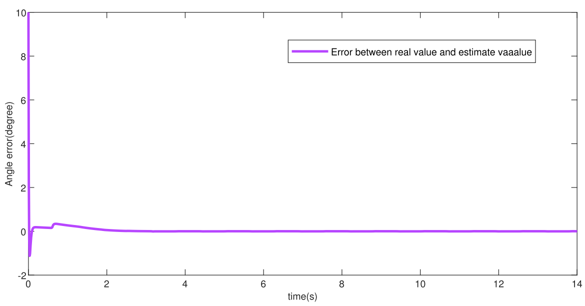

FIGURE 11. Error between real value and estimate value of x_4.

Figures 4 to 7 illustrate that, when the influence of the observer-based controller (5) works, the state for above system converges toward stability, and the estimated system state progressively approaches its actual value. Figures 8 to 11 reveal a diminishing trend in the estimation error of the system, ultimately approaching zero.

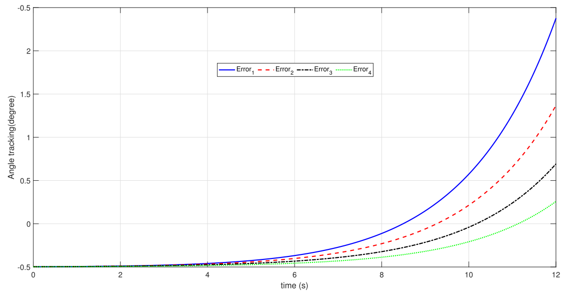

FIGURE 12. Angle tracking error.

When the novel controller is not used, the following contrast graph present the angle tracking error:

V. CONCLUSION

This investigation has comprehensively analyzed perturbations affecting both the controller gain and the observer gain within a non-fragile observer-based control framework for Power-Line Inspection Robots (PILRs). Unlike previous methodologies, the primary goal of this study is to develop an effective non-fragile state feedback controller grounded in observational data. By employing Lyapunov-Krasovskii Functional (LKF) methods and Linear Matrix Inequalities (LMIs), we demonstrate that the states of the closed-loop system achieve passivity and are stabilized with H_∞ performance, even within a continuous-time context. The efficacy and robustness of the proposed approach are rigorously validated through theoretical derivations and a series of simulation experiments, confirming its superior performance and reliability in enhancing the operational stability of PILRs. Nevertheless, the appliation and sustainability of PLIR remain improve and develop.

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BINGSHAN LIU was born in Jiangsu, China, in 2003. He is currently pursuing the bachelor's degree in computer science and technology with Nanjing University of Information and Science Technology. From 2021 to 2024, he was a Research Associate with the Engineering Research Center, Ministry of Digital Forensics. He owns two computer software copyrights and one Chinese invention patent. His research interests include robot control, UAV control, and data mining. He won the National Gold Medal from China International "Internet+" College Student Innovation and Entrepreneurship Competition and the Second Prize from the National College Student Information Security Competition.

CHEN ZHANG received the B.S. degree from Southeast University, Nanjing, China, in 2008, and the M.S. degree from Jiangsu University, Zhenjiang, China, in 2017. He is currently an Experimenter with Huaiyin Institute of Technology. His research interests include robot control and nonlinear intelligent control of motors.