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RESEARCH ARTICLE

An Insight on the Microwave Circulator Theory

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ABSTRACT This contribution is devoted to clarifying the analytical derivation of the microwave circulator matrixes. Although this topic seems well-acknowledged, the approach of many books is affected by an initial error that, in turn, affects the final result. It is worth noting that the error is not typographical, but a direct consequence of an incorrect assumption reported in many books. Starting from the knowledge that a three-port network cannot be simultaneously lossless, matched and reciprocal, the solutions of the system are usually calculated by verifying the unitary conditions on a matched matrix, which is the point where the error arises in many books. As a matter of fact, it should not be underestimated because microwave and millimeter-wave circulators are unique and basic components used in a huge number of applications due to their great ability to redirect microwave signals, and the related theory is covered within most of the microwave engineering courses. In view of that, it is of utmost importance clarifying the related aspects by providing all the mathematical steps to obtain the final matrixes of circulator.

INDEX TERMS Microwave circulators, non-reciprocal components, scattering matrix, three-port networks, unitary condition.

I. INTRODUCTION

Circulators are key components, widely exploited to manage the flow of microwave signals within circuits and systems. They are passive three-port devices, whereby each port can be either considered an input or output. In a clockwise circulator, a signal applied to the port 1 will be routed to port 2, whereas a signal applied to port 2 or 3 will be routed to port 3 or 1, respectively. On the other hand, in counterclockwise circulators the signal direction is the opposite. Circulators can be also exploited to obtain other useful components, e.g., by terminating one of the three ports with a matched load, a circulator will reproduce the behavior of an isolator [1], [2].

Reference [3] is one of the earliest contributions where the conditions to physically design non-reciprocal networks are described. It demonstrates that any lossless, matched and nonreciprocal three-port microwave network is a perfect three port circulator, and this has been remarked in [4]. On the other hand, it is possible to demonstrate that the three conditions of being lossless, matched and reciprocal cannot be simultaneously obtained in a three-port network [5]. The possibility to have a non-reciprocal network which is simultaneously matched and lossless is very beneficial for many microwave circuits. As an example, the feature of circulators to route the signal towards the transmitting or receiving stages, can be exploited to obtain a transceiver with a single transmitting/receiving antenna. Such a component, commonly referred as duplexer, prevents damage to the sensitive receiver circuits from the high transmitter power levels.

The scientific literature reports many recent studies concerning circulators, thus highlighting that the topic is up-to-date and of utmost importance [6], [7], [8].

In [9], a wideband gyrator has been designed with the aim to obtain an on-chip circulator or isolator. The contribution proposes a novel topology exploiting mixers in superconductor–insulator–superconductor technology and phase-delay circuits. In [10] multiband circulators and isolators are designed in a magnet-less configuration. The design exploits transversal frequency-selective signal paths based on the frequency-tunable and spatiotemporally modulated resonators. Some samples have been implemented and tested at very high frequency (VHF) band. Similarly, in [11], the spatiotemporal modulation of microstrip filtering delay networks has been exploited to obtain an ultra-high frequency

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(UHF) filtering nonmagnetic circulator. At the same time circulators can be also used to design other components. As an example, in [12] the characteristic of the circulator of being a non-reciprocal network is exploited to obtain a multifunctional component in S-band. In detail, both non-reciprocal single-ended and balanced bandpass filters based on two circulators are proposed. Moreover, depending on the ports used as inputs, the nonreciprocal balanced bandpass filter can be viewed as filter or power divider. In a similar fashion, [13] reports on the design of multiport circulators with bandpass filtering functionality. In detail, the filtering transfer function is obtained in one direction of propagation, while a high signal suppression can be observed in the reverse direction.

Recently, researchers proposed a novel three-terminal circulator based on the skyrmion Hall effect [14]. Compared to classic microwave circulators, the circulating direction can be changed to either clockwise or counterclockwise by changing the sign of the topological charge. Finally, they are used in a great variety of microwave transceiver with applications concerning different sectors, e.g., microwave space components and radars [15], [16], [17], [18], [19], [20], [21], [22]. As an example, in [23], a Ku-band high-power Y-junction circulator is proposed for radar applications, whereby two circulators are combined using power splitters to obtain an isolation level and return loss equal to 20 dB. The theoretical description of circulators has been proposed by different and very famous books for microwave engineers as [4], [5], and [24]. Reference [5], in detail, is probably the reference book for each microwave engineer. These books represent a precious guide both for researchers working in the field and for professors and students interested in studying circulators in the microwave engineering courses and it is therefore of utmost importance that every information, particularly the theoretical analysis is rigorously described and correct.

However, after a careful analysis, it is possible to note that the description of three-port circulators reported in [5] and [24], and consequently in many books based on these famous references, includes some wrong steps which may lead the readers to mistake. The wrong steps, related to the system of equations describing the linear behavior of a circulator network, are difficult to uncover because they concern a system of equations with many solutions. The known circulator matrixes are correctly reported in the main reference books only because many solutions of the wrongly derived matrixes are omitted. Otherwise, the additional meaningless matrixes would make the initial error evident; indeed, in this contribution, some related examples are reported to clarify the source of mistake. The solution of the problem lies in the proper verification of the lossless condition, as it will be described throughout this contribution. The wrong description may lead to wrong conclusions, that of course will be very detrimental for the researchers, professors, and students in the field, whereby having a correct reference book is of utmost importance.

Throughout the text, the initial error affecting many reference books is pointed out, and the correct derivation of the circulator matrixes is described and demonstrated.

The purpose of this contribution is to point out the wrong steps and provide the correct solution after a comprehensive and straightforward analysis. Of course, the aim of this contribution is not to question the always clear and in-depth dissertation of the reported previous books concerning the three port networks but to make the reader aware of some wrong mathematical steps that are, however, fundamental for a deep understanding of the circulator theory.

The paper is organized as follows. The classic theory of circulators, including that discussed in [4], [5], and [24], is reported in Section II. Section III shows in detail the passages to obtain the correct three-port circulator clockwise and counterclockwise matrixes. Finally, the conclusions are drawn in Section IV.

II. CLASSIC THEORY OF THREE-PORT CIRCULATOR

The analysis of the circulator theory, as reported for example in [4], [5], and [24], starts with the description of the matrix related to a three-port component reported in (1).

$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \tag{1}$$

The zero diagonal matrix is a direct consequence of the matching hypothesis, i.e., the reflection coefficient is equal to zero for all the three ports. Indeed, the purpose is to verify the solutions for a three-port that is simultaneously matched, lossless and non-reciprocal as remarked in [3] and [4].

If i and j are the row and column indexes of the scattering matrix, the lossless condition can be written as in (2) for each column, if $i = j$.

$$\sum_{k=1}^{N} S_{ki}^{*} S_{ki} = 1 \tag{2}$$

whereas it can be written as in (3) for each column, if $i \neq j$.

$$\sum_{k=1}^{N} S_{ki}^{*} S_{kj} = 0 \tag{3}$$

From (2)-(3), it is possible to verify the unitary condition and thus that the component is lossless by checking that the dot product of any column of [S] with the conjugate of the same column gives unity and the dot product of any column with the conjugate of a different column gives zero, i.e., the columns are orthonormal. In other words, the component is lossless whether the identity matrix is the result of the dot product between the matrix and its conjugate transpose. The classic analysis of the circulator theory fails in many books, as reported for example in [5] and [24], to extract the following conditions, reported in (4)-(9), from (2) and (3), respectively.

$$|S_{12}|^2 + |S_{13}|^2 = 1 \tag{4}$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \tag{5}$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \tag{6}$$

$$S_{31}^* S_{32} = 0 \tag{7}$$

$$S_{21}^* S_{23} = 0 \tag{8}$$

$$S_{12}^* S_{13} = 0 \tag{9}$$

Equations (4)-(9) are the commonly reported solutions of (2)-(3), however the (4)-(6) are not correct. Indeed, they are derived for a symmetric matrix, by applying the unitary condition on the rows instead of columns. Of course, this is not the case of the circulator which, ideally, is non-reciprocal, i.e., the matrix is asymmetric by definition in [3]. Since the circulator is asymmetric, then applying the unitary condition on the rows is not allowed as, instead erroneously applied in [5] and [24]. Accidentally, two solutions of (4)-(9) are the correct ones, i.e., $S_{12} = S_{23} = S_{31} = 0$, $|S_{21}| = |S_{32}| = |S_{13}| = 1$ and $S_{21} = S_{32} = S_{13} = 0$, $|S_{12}| = |S_{23}| = |S_{31}| = 1$. However, these results can be considered neither satisfactory nor correct, because they don't exclude the presence of other solutions. Indeed, it is worth noting that additional solutions are totally ignored in the main reference books, although they exist. As an example, it is possible to observe that $S_{32} = S_{23} = S_{13} = 0$, $|S_{21}| = |S_{31}| = |S_{12}| = 1$ and $S_{32} = S_{23} = S_{12} = 0$, $|S_{21}| = |S_{31}| = |S_{13}| = 1$ solve (4)-(9). These solutions result in the matrixes reported in (10) and (11), in turns corresponding to the networks shown in Fig. 1a-b respectively, which are meaningless. Indeed, a bidirectional component for the same ports is senseless in the microwave field due to the aforementioned reasons, thus, Fig 1a-b does not represent a circulator. For the sake of conciseness, the other solutions are not reported.

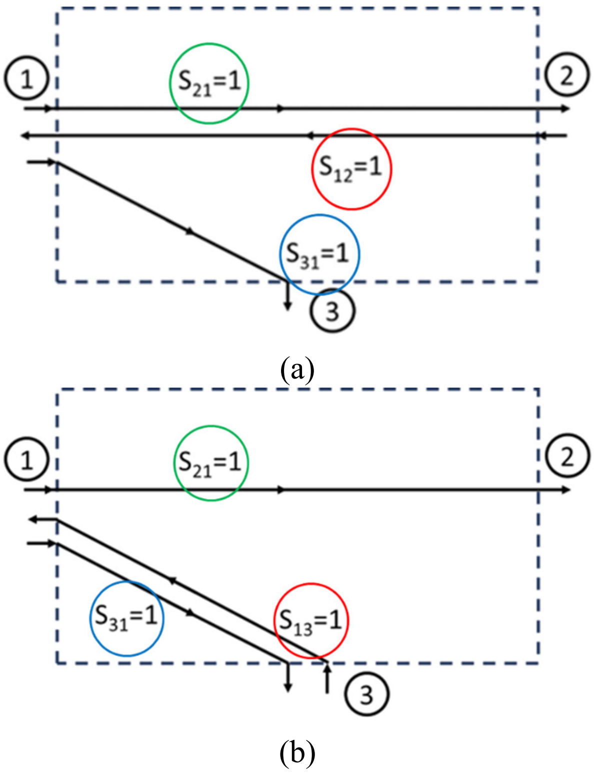

FIGURE 1. Networks corresponding to the wrong matrixes reported in (10) and (11).

$$\begin{bmatrix} 0 & \textcircled{1} & 0 \\ \textcircled{1} & 0 & 0 \\ \textcircled{1} & 0 & 0 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} 0 & 0 & \textcircled{1} \\ \textcircled{1} & 0 & 0 \\ \textcircled{1} & 0 & 0 \end{bmatrix} \tag{11}$$

Clearly, the additional wrong matrixes are direct consequences of the initial error that will be solved in the description proposed in the next Section.

III. PROPOSED THEORY OF THREE-PORT CIRCULATOR

To solve the issue highlighted in Section II, recall that a matrix is lossless if it the unitary condition, in (12), can be verified.

$$[S]^{*t}\,[S] = [U] \tag{12}$$

where $[\square]^t$ and $[\square]^*$ are the transpose and conjugate operators respectively, and $[U]$ is the identity matrix, that is a matrix with ones on the main diagonal and zeroes elsewhere. By applying (12) to a 3 × 3 matched matrix, it is possible to obtain:

$$\begin{bmatrix} 0 & S_{21}^* & S_{31}^* \\ S_{12}^* & 0 & S_{32}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix} \times \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$
$$= \begin{bmatrix} S_{21}^* S_{21} + S_{31}^* S_{31} & S_{31}^* S_{32} & S_{21}^* S_{23} \\ S_{32}^* S_{31} & S_{12}^* S_{12} + S_{32}^* S_{32} & S_{12}^* S_{13} \\ S_{23}^* S_{21} & S_{13}^* S_{12} & S_{13}^* S_{13} + S_{23}^* S_{23} \end{bmatrix}$$
$$= [U] \tag{13}$$

whereas, from (13) and by considering the unitary condition, new equations are extracted:

$$S_{21}^* S_{21} + S_{31}^* S_{31} = 1 \tag{14}$$

$$S_{12}^* S_{12} + S_{32}^* S_{32} = 1 \tag{15}$$

$$S_{13}^* S_{13} + S_{23}^* S_{23} = 1 \tag{16}$$

$$S_{31}^* S_{32} = 0 \tag{17}$$

$$S_{21}^* S_{23} = 0 \tag{18}$$

$$S_{12}^* S_{13} = 0 \tag{19}$$

It is worth noting that (17)-(19) are equal to (7)-(9), whereas (14)-(16) are new equations that, contrary to (4)-(6), are correct. They have two and only two solutions, namely:

$$\begin{bmatrix} 0 & 0 & \textcircled{1} \\ \textcircled{1} & 0 & 0 \\ 0 & \textcircled{1} & 0 \end{bmatrix} \tag{20}$$

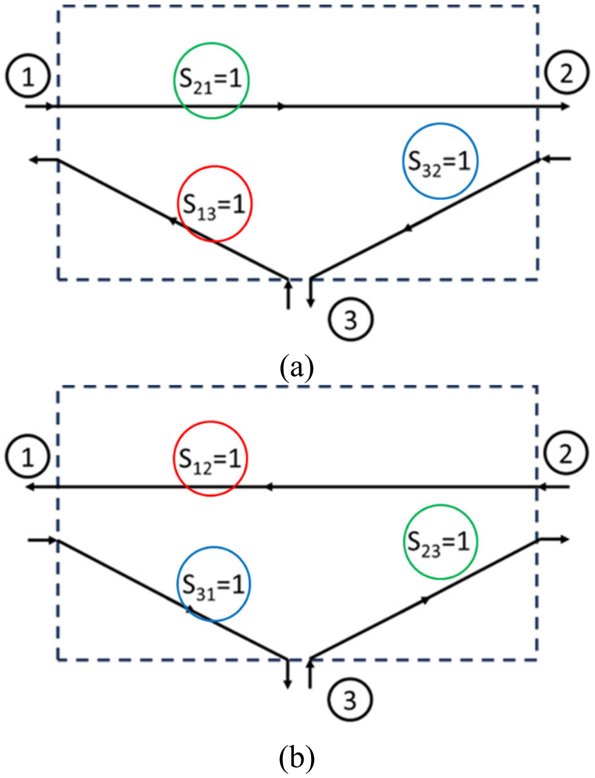

FIGURE 2. Networks corresponding to the (a) clockwise and (b) counterclockwise circulator matrixes of (20) and (21), respectively.

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{21}$$

The matrixes in (20) and (21) in turns correspond to the networks shown in Fig. 2, respectively, which correspond to the classic clockwise and counterclockwise circulator. Indeed, in contrast to the examples shown in Fig. 1a-b, the networks in Fig. 2a-b are characterized by the correct power flow where also the isolation requirement is respected. A circulator can be employed in a marine radar system to manage the power flow among the antenna, the magnetron and the front end. Although the two final matrices are correct in every circulator reference book, the main problem of the classic description of the circulator theory relies on the application of the lossless condition on the circulator matrix, resulting in more solutions than expected. On the other hand, starting from the unitary condition, it is possible to obtain the only two solutions to solve the system of equations.

IV. CONCLUSION

In this paper, the mathematical derivation of the circulator matrix has been described. The interest of the authors was fueled by some inaccuracies in the classic description of the circulator theory reported in many reference books. Due to the great importance of microwave and millimeter-wave circulators both in the industrial and academic fields, the present contribution might represent a suitable starting reference point for those engineers interested in the field.

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