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RESEARCH ARTICLE

Joint Interference and Power Minimization for Fault-Tolerant Topology in Sensor Networks

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ABSTRACT Energy conservation is crucial in wireless ad hoc sensor network design to increase network lifetime. Since communication consumes a major part of the energy used by a sensor node, efficient communication is important. Topology control aims at achieving more efficient communication by dropping links and reducing interference among simultaneous transmissions by adjusting the nodes' transmission power. Since dropping links make a network more susceptible to node failure, a fundamental problem in wireless sensor networks is to find a communication graph with minimum interference and minimum power assignment aiming at an induced topology that can satisfy fault-tolerant properties. In this paper, we examine and propose linear integer programming formulations and a hybrid meta-heuristic GRASP/VNS (Greedy Randomized Adaptive Search Procedure/Variable Neighborhood Search) to determine the transmission power of each node while maintaining a fault-tolerant network and simultaneously minimize the interference and the total power consumption. Optimal biconnected topologies for moderately sized networks with minimum interference and minimum power are obtained using a commercial solver. We report computational simulations comparing the integer programming formulations and the GRASP/VNS, and evaluate the effectiveness of three meta-heuristics in terms of the tradeoffs between computation time and solution quality. We show that the proposed meta-heuristics are able to find good solutions for sensor networks with up to 400 nodes and that the GRASP/VNS was able to systematically find the best lower bounds and optimal solutions.

INDEX TERMS Wireless sensor networks, network design, fault-tolerant topology control, wireless interference, power control, integer programming, meta-heuristics.

ACRONYMS

BVNS	Basic variant of VNS.
CPLEX	IBM ILOG CPLEX Optimization Studio.
D	Relative degradation.
fea	Feasibility.
FIP	Incremental Power formulation.
FMMI	Mathematical formulation to solve the MMI problem.
FMMITP	Mathematical formulation to solve the MMITP problem .
FMTI	Mathematical formulation to solve the MTI problem.
FMTITP	Mathematical formulation to solve the MTITP problem.
GRASP	Greedy Randomized Adaptive Search Procedure.
L	Aaverage relative linear relaxation gap.
M	Average relative MIP gap.
MaxI	Average maximum interference.

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MInt	Average minimum maximum interference.
MS	Minimum Spanning Tree.
MMI	Minimum Maximum Interference.
MMITP	Minimum Maximum Interference with Minimum Total Power.
MTI	Minimum Total Interference.
MTITP	Minimum Total Interference with Minimum Total Power.
opt	Optimality.
TInt	Average minimum total interference.
TotalI	Average total interference.
VNS	Variable Neighborhood Search.
WSN	Wireless Sensor Networks.

I. INTRODUCTION

A wireless sensor node is a micro-computer featuring functionalities in combination with a wireless communication device used to connect with other wireless sensor nodes. Wireless networks of such sensor nodes have many potential applications such as surveillance, environmental monitoring, and biological detection [1].

There are various constraints in the design of wireless sensor nodes. For instance, nodes are most of the time not connected to the communication infrastructure, and they should be small, low-priced and last as long as possible without recharging. Therefore, setting wireless sensor networks (WSN) must aim to conserve as much energy as possible. Since communication consumes a major part of the energy used by a sensor node, efficient communication is important. The aim of topology control is to determine the transmission power of each node to compute a subset of all possible communication links that allow efficient communication and energy conservation [2], [3].

Topology control in sensor networks is important to maintain reliable communication links between base stations and nodes and to maximize the battery life. Instead of transmitting with maximal power, each node adjusts its transmission power to define the proper network topology. The power settings are given by the topology control algorithm. When using lower transmission power, topology control algorithms decrease the number of links in the network, reducing the number of possible routing paths. A link between two nodes is established when the received powers are equal to or greater than their sensitivities. Dropping links in the network makes it more susceptible to failure. A large number of link failures, provided by critical application domains, fading, or obstructions [4], may be disastrous, especially in the case of sensor networks. This problem can be mitigated if an adequate level of routing redundancy can be properly figured into the topology [5], [6], [7], [8], [9], [10], [11], [12], [13].

Topology control can avoid extra resource consumption in data transmission, which can maintain network connectivity while reducing energy consumption and minimizing radio interference [14]. A message packet transmitted by a sensor device is often received by many nodes in the vicinity of the receiver node. This property is known as wireless multicast advantage [15]. The wireless multicast advantage generates interference, reducing communication efficiency since it causes message collisions and retransmissions. Minimizing interference through topology control in wireless sensor networks is a well-known open algorithmic problem [16], [17]. Reducing interference may increase throughput, reduce energy consumption, and increase network lifetime. Thus, developing interference-aware topology control algorithms is a necessity for such networks.

Several graph-based topology control models for interference have been proposed in the literature. Topology control under graph-based interference models can employ classic graph-theoretical tools, such as graph coloring, for algorithm design and analysis. Two different graph-based interference models exist, named sender-centric and receiver-centric [18]. In the sender-centric model, the interference is considered to be an issue on the sender side, where interference is based on the number of nodes affected by communication over a given link. On the other hand, in the receiver-centric model, the interference is considered at the receiver side, where message collisions prevent proper reception. In addition, there are two different approaches for computing the interference of a network, known as maximum and total (or average) interference. The maximum interference approach computes the interference of a network considering the maximum interference of a link or a node in a network [19], [20], [21], whereas the total interference approach computes the interference of a network considering total interference of whole links or nodes in a network [22]. In recent years, there was a substantial amount of research on interference-aware topology control for wireless ad hoc networks using graph-based interference models [11], [16], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35]. Works in [36] and [37] also consider topology control algorithms to minimizing both power consumption as well as interference.

A. CONTRIBUTIONS

It is evident from the aforementioned literature that the main goals of topology control include reductions in power usage and network interference. While many solutions focus on minimizing energy or interference individually, few address both drawbacks simultaneously. Thus, the algorithms proposed in this paper include:

- problem formulations taking into account only the interference issue;
- new problem formulations in which both interference and total power consumption are concurrently considered.

To this aim, we use mathematical programming, meta-heuristics and a hybrid meta-heuristic to solve the problems. Hence, we propose integer programming formulations and algorithms to solve four interference minimization problems named Minimum Maximum Interference (MMI), Minimum Total Interference (MTI), Minimum Maximum Interference with Minimum Total Power (MMITP), and Minimum Total Interference with Minimum Total Power (MTITP). The first two problems, MMI and MTI, minimize the interference in a sensor network such that the network topology becomes fault-tolerant. The MMITP and MTITP problems also create a fault-tolerant network topology. However, they simultaneously minimize the interference and the total power consumption.

Moreover, in this paper, we compare the hardness of the proposed formulations to obtain a biconnected optimal solution using CPLEX for moderately sized networks with up to 50 sensors for the four studied problems. We show that the simultaneous minimization approach, problems MMITP and MTITP, can find the same interference optimal solution value with smaller total power value when compared with the optimal solution values found for problems MMI and MTI. We also implement and compare three meta-heuristics: one GRASP, one VNS, and a hybrid GRASP/VNS. We show that the hybrid GRASP/VNS can find all known optimal solutions for all problems with biconnectivity requirements. It also provides the best solutions for real-sized networks with up to 400 sensors.

The remainder of the paper is organized as follows. We first introduce the related works in Section II. The WSN model and the optimization problems are presented in Section III. In Section IV, we define the k-connected minimum wireless ad hoc interference problem, its variants, as well as the integer programming formulations used to solve problems in moderately sized networks. A GRASP/VNS heuristic used to approximately solve large problem instances with biconnectivity requirements is proposed in Section V. Computational results are reported and discussed in Section VI, and concluding remarks are made in Section VII.

II. RELATED WORKS

Reference [19] first raised a fundamental question "Does topology control reduce interference?". They showed that traditional topology control methods with implicit interference reduction can fail to effectively achieve interference minimization. Since then, several explicit definitions of interference were proposed in literature. Researchers often use graph-based models in which the transmission range of each device is modeled as a circle with a certain radius. Two devices can exchange messages only if they are within each other's transmission range. In this case, the two most relevant models, among those measuring interference at devices, are the sender interference model and the receiver interference model. In the first model, the interference is viewed as an issue of a transmitting device and is measured as the cardinality of the set of devices to whom it can send messages in one-hop. In the second model, the interference is viewed as an issue of a receiving node and is measured as the cardinality of the set of devices from which it can receive messages directly. Graphs can also be used to represent interference models considering communication irregularities [38]. Reference [39] show how a graph-based problem instance can be built to solve multi-rate and variable-rate scheduling problems in wireless ad-hoc networks where signal-to-interference-plus-noise ratios are kept above a certain threshold.

In this paper, we focus on the receiver interference model since it seems to better reflect the intuition of the real world [21], [25], when compared with the sender interference model. Further, polynomial time algorithms for computing an optimal solution for many connectivity predicates, like strong connectivity and spanner, are known for the sender interference model [19], [25], [37]. On the other hand, considering the receiver interference model, [40] proved that minimizing the maximum node interference is hard to approximate, while [32] showed that the problem of assigning powers to get a connected graph minimizing the maximum node interference is NP-complete for the 2-dimensional case.

Researchers studied the graph-based interference problem in two widely accepted models: minimizing the maximum interference and minimizing the total (or average) interference. Considering the minimization of the maximum node interference, [19] proposed several methods to build topologies whose maximum link interference is minimized while the topology is connected or is a spanner for Euclidean length. Reference [21] described an $\sqrt[4]{\delta}$-approximation of the optimal connectivity-preserving topology in the general highway model, where δ is the maximum node degree. In [27], the authors generalized the highway one-dimensional (1D) case from [21] to the two-dimensional (2D) one.

When considering the problem of computing the minimum total interference [31] developed an asymptotically optimal algorithm with an approximation ratio of $O(\log n)$ for minimizing total interference in 2D networks. References [22] and [41] provided central and local algorithms to minimize the maximum and total interference of a network by constructing interference-based single-hop local Minimum Spanning Tree. Reference [17] studied the minimization of the total and the maximum receiver-centric interference for the highway model. Among other results, they proposed a polynomial-time exact algorithm that can construct a connected topology with minimum total interference. Reference [16] improved the results given in [17] for the minimization of total interference problem. Other topology control algorithms using the graph-based interference model can be found in [20], [24], [26], [28], [34], [35], [37], [42], and [43].

Fault-tolerant topology control is also a critical problem in wireless sensor networks [6]. Several fault-tolerant topology control algorithms have been proposed to create a power-efficient network topology in WSN [6], [10], [12], [44], [45], [46]. However, most of the existing proposed fault-tolerant topology control algorithms do not take the interference issues into consideration. Some fault-tolerant interference-aware topology control algorithms were presented in [7], [11], [13], and [47]. Reference [7] formulated the problem of constructing minimum interference path preserving and fault-tolerant wireless ad hoc networks and then provide centralized and distributed algorithms to solve the problem. Reference [47] introduced the fault-tolerant multi-constrained spanning tree problem and proposed a $2K$-approximation algorithm to solve it, where K is the number of non-negative real-value edge weights. Reference [13] proposed and proved that their algorithms could induce a k-fault resistant energy spanner and furthermore the interference is minimized. Reference [11] proposed four integer programming formulations for the k-connected minimum wireless ad hoc interference problem, which consists in a topology control technique to find a power assignment to the nodes of an ad hoc wireless network such that the resulting network topology is k-vertex connected and the radio interference is minimum. They showed that the proposed formulations give solutions in reasonable computational time with low transmission powers and minimum interference, considering continuous and noncontinuous interference models.

Some works deal with mixed integer programming models, optimization, and management of wireless (sensor) networks. In [48], the authors present a novel approach to optimize the number of simultaneous transmissions in wireless networks, focusing on satisfying the signal-to-noise-and-interference ratio (SINR) at the receivers. Traditionally, this task is tackled using an integer programming model with explicit SINR constraints. However, [48] presents a new exact algorithm for maximum link activation: an integer programming algorithm that provides a more efficient representation of the SINR constraints. To evaluate the performance of their proposed algorithm, the authors used six network groups with 50 to 100 nodes transmitting with uniformly set power. The time required to reach optimality is compared, and results show that their proposed algorithm based on inequalities from SINR cover outperforms the conventional approach in proving and approaching the global optimum. Such improvement is more significant for smaller-sized networks.

A study on minimization of the average and the maximum interference for the highway model, where all the nodes are stationary and arbitrarily distributed along a line, is presented in [17]. The authors prove that there is always an optimal planar topology with minimum interference, and present two exact algorithms: one to minimize the average interference in polynomial time, and another to minimize the maximum interference in sub-exponential time. The optimal topologies constructed by their methods are planar. The authors also state that the question of whether it is NP-hard to minimize the maximum interference for the highway model is still open. By its turn, [49] focuses on the design of body area networks, a topic related to healthcare applications of wireless sensor networks. They propose a robust optimization graph-based model for jointly optimizing the topology and the routing in (body area) wireless sensor networks under traffic uncertainty. The authors propose a fast hybrid exact-heuristic optimization algorithm that exploits suitable linear relaxations and present experiments that show that their algorithm performs better than a solver like CPLEX in the vast majority of cases.

The two main objectives of a topology control problem are to minimize power consumption and the network's interference. Several works have been proposed for minimizing the energy and interference separately, and only some algorithms are available in literature that address these two problems simultaneously. Reference [36] present an interference and energy-aware topology control protocol based on Minimum Spanning Tree (MST). Reference [37] proposed a local search-based heuristic for the problem of assigning transmission power to each network sensor, such that the total power consumption and interference are minimum along with the constraint that the resulting topology consisting of bidirectional links is strongly connected. Reference [50] studied a localized construction protocol that simultaneously satisfies many design goals, among them power and interference. Reference [47] studied the interference and power-constrained broadcast/multicast and the delay-bounded interference and power-constrained broadcast/multicast routing problems in wireless ad hoc networks using directional antennas. They proposed approximation and heuristic algorithms for the two problems. Reference [51] described a localized topology control algorithm to design a strongly connected topology that is very efficient in terms of interference while minimizing energy.

The authors of [52] proposed a distributed optimization model using Game Theory to maximize energy efficiency and connectivity in multi-radio WSN. An allocation algorithm was developed to optimize power and channels, achieving a Nash Equilibrium with low complexity. The simulation results demonstrate that the proposed approach reduces network interference and transmission errors, enhancing energy efficiency and data transmission. A dual interference model optimization algorithm was formulated in [53] for efficient routing in WSN, minimizing total link interference while maximizing interference at the receiver node. The optimized results were obtained with minimal interference, transmission cost, and hop counts.

Related to the energy-aware cluster-based routing problem, [54] and [55] addressed the problem of energy-efficient data collection in robotic WSN using an Unmanned Aerial Vehicle (UAV). The study introduces the role of Cluster Head (CH) robots responsible for task assignment and data collection within their respective clusters. The UAV, which has a limited battery capacity, visits these CH robots in an optimal manner to minimize battery energy consumption. Considering its battery constraints, the paper presents an analytical approach to determine the optimal subset of CH robots for the UAV to visit. The work considers the UAV's constant battery capacity, contrary to previous studies that assumed variable or sufficient energy availability.

In this paper, we develop integer programming formulations and heuristics to solve the fault-tolerant topology control variants of the MMI, MTI, MMITP and MTITP problems, considering the receiver interference model. These optimization problems are able to minimize only the interference, as well as to minimize the interference and the total power consumption simultaneously. Figure 1 shows a flowchart of the general topology control method used to solve the biconnected variant.

III. WSN MODEL AND OPTIMIZATION PROBLEMS

This section describes the WSN model and the set of minimization problems we focus on. We consider stationary WSN located in the Euclidean plane with omnidirectional transceivers. The parameters used in the modeling are described in Table 1.

TABLE 1. Mathematical notation defined in the system model.

Notation	Description
k	the k-connectivity constraint
V	set of transceivers (nodes), numbered $0, 1, \ldots, \lvert V\rvert - 1$
p_u	transmission power associated with each node $u \in V$
p	set of all transmission powers
ε	path loss exponent
d_{uv}	distance from node $u \in V$ and $v \in V$
$G(p)$	communication graph set up by transmission powers p
$A(p)$	set of direct arcs (u, v) set up by transmission powers p
$E(p)$	set of undirect edges $[u, v]$ set up by transmission powers p
$\mathrm{I}(v)$	interference value of a single node v in the receiver model

A WSN can be modeled by a set V of transceivers (nodes), numbered $0, 1, \ldots, |V| - 1$, together with their locations or distances between them. A transmission power p_u is associated with each node $u \in V$. A transmitter-receiver pair (or link) is defined as the ordered pair $(u, v) : u, v \in V$, where u and v denote a transmitting terminal and its corresponding receiving terminal, respectively. We assume that each node can adjust its transmission power depending on the position of the immediate receiver up to a maximum fixed for every node.

In the most common power attenuation model [56], the signal power falls with $1/d^{\varepsilon}$, where d is the distance from the transmitter and ε is the path loss exponent (e.g. $\varepsilon = 2$ for the free space model and $\varepsilon = 4$ for the two-ray ground model). Under this model, the power requirement at node u for supporting the transmission through a link from u to v (i.e., node u can send packets to node v that is d_{uv} away, without consideration of interference) is given by

$$p_u \geq d_{uv}^{\varepsilon}.q_v, \tag{1}$$

where q_v is the receiver's power threshold for signal detection, which is usually normalized to 1. Assuming a deterministic path loss model and $q_v = 1$, we have $p_u \geq d_{uv}^{\varepsilon}$.

For each ordered pair (u, v) of transceivers, with $u, v \in V$, we are given a non-negative arc weight equal to d_{uv}^{ε} such that a signal transmitted by the transceiver u can be received at node v, if and only if, the transmission power of u is at least equal to d_{uv}^{ε}, i.e. if $p_u \geq d_{uv}^{\varepsilon}$. Therefore, the directed communication graph of a network is represented by a graph $G(p) = (V, A(p))$, where $A(p) = \{(u, v) : u \in V, v \in V, p_u \geq d_{uv}^{\varepsilon}\}$. However, in order to simplify routing protocols, it is desirable to have bidirectional links, thus only the set of undirected edges $E(p) = \{[u, v] : u \in V, v \in V, p_u \geq d_{uv}^{\varepsilon}, p_v \geq d_{vu}^{\varepsilon}\}$, such that $E(p) \subseteq A(p)$, is considered to enforce the k-connectivity constraint. Nevertheless, a direct arc $(u, v) \in A(p)$ can cause interference.

Given a communication graph $G(p)$, a signal can be successfully received and decoded only if the interference constraints are satisfied. We define the receiver interference model [21] of a node v as the number of terminals whose transmissions disturb reception at v (see Figure 2). Formally, given a directed communication graph $G(p) = (V, A(p))$, the interference value of a single node is formally defined as

$$\mathrm{I}(v) = |\{u \in V \setminus \{v\} : (u, v) \in A(p)\}|. \tag{2}$$

Adopting the receiver interference model, we formally define the set of minimization problems for the sensor networks we focus on. The common input for all includes the node set V, the arc weights d_{uv}^{ε} for any $u, v \in V$, and the k-connectivity property. The output is a transmission power assignment $p_u : u \in V$, such that the set of undirected edges $E(p) \subseteq A(p)$ established from the directed communication graph $G(p) = (V, A(p))$ satisfies the given k-connectivity property. The interference minimization problems are listed below.

1) The Minimum Maximum Interference (MMI): the objective function is the maximum interference $[\max_{v\in V} \mathrm{I}(v)]$ experienced by any node.
2) The Minimum Total Interference (MTI): the objective function is the total interference experienced by any of the nodes, which is mathematically defined as $\sum_{v\in V} \mathrm{I}(v)$.
3) The Minimum Maximum Interference with Minimum Total Power (MMITP): the objective function is the maximum interference $[\max_{v\in V} \mathrm{I}(v)]$ experienced by any of the nodes, and the total power defined as $\sum_{u\in V} p_u$.
4) The Minimum Total Interference with Minimum Total Power (MTITP): the objective function is the total interference $[\sum_{v\in V} \mathrm{I}(v)]$ experienced by any of the nodes, and the total power denoted as $\sum_{u\in V} p_u$.

Similarly to widely studied interference-related problems, the nodes considered in the problem proposed here are located at fixed positions in the 2D plane, and the main task is to select the transmission range of each node to achieve minimum interference. However, unlike our proposal, the theoretic works regarding receiver interference minimization require a k-connected resulting network where $k = 1$, while in our problem, the resulting communication graph must satisfy a k-connectivity property such that $k \geq 2$.

Considering the 1-connected property in both the symmetric ($d_{uv}^{\varepsilon} = d_{vu}^{\varepsilon}$) and asymmetric ($d_{uv}^{\varepsilon} \neq d_{vu}^{\varepsilon}$) models, minimizing the maximum receiver interference (MMI problem) is NP-hard [40], [57]. Taking into account the minimization of the total interference (MTI problem) to obtain a 1-connected communication graph, Lam et al. [58]

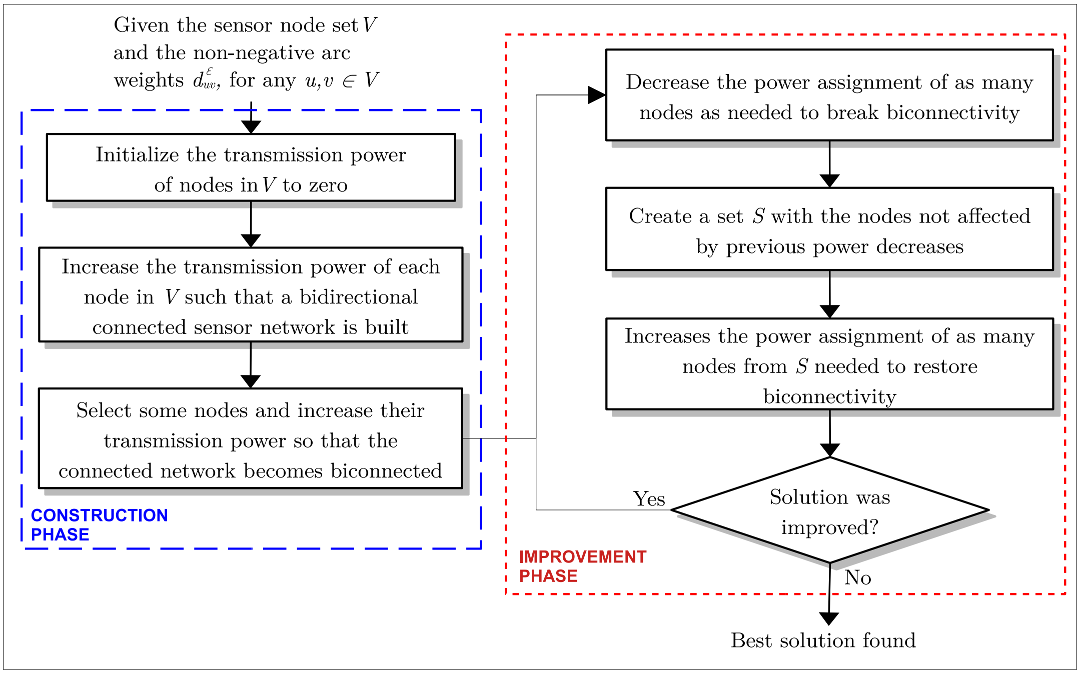


FIGURE 1. General flowchart of the proposed topology control method.

prove that the symmetric network model case is NP-hard and Abu-Affash et al. [59] prove that the asymmetric network model case is also NP-hard. Although the complexity of the k-connectivity property such that $k \geq 2$ is under investigation, we conjecture that, for any positive integer k, the k-connected interference problems addressed in this work are NP-hard as well. Thus, polynomial-time approximation algorithms are expected, as in, for example, the proposal described in [60], in which approximation algorithms are employed to the k-connected interference minimization problem such the maximum interference at any node is minimized.

Different transmission power values can be used without changing the interference minimization value in the aforementioned interference context. Moreover, it should be stressed that a node can use a higher power without increasing the number of receiving nodes and, consequently, without affecting the maximum or total interference value. The proposed solutions (see Section IV) for the problems that jointly address interference minimization and power minimization seek the lowest possible power allocation without changing the original interference problem. In other words, it is a specific (lowest) power assignment among several existing ones for the same minimum interference solution.Hence, it is worth commenting that all formulated problems have the same computational complexity. Furthermore, the topology control problem of minimizing total transmission power is shown to be NP-complete [10] as well.

IV. INTEGER PROGRAMMING FORMULATIONS

An efficient way to formulate the k-connectivity constraints consists in defining a set C of $\lceil k|V|/2 \rceil$ commodities with

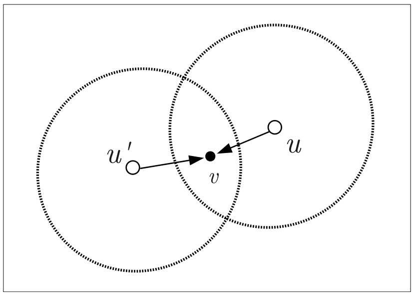


FIGURE 2. Receiver interference model. Node v suffers interference from u and u', $I(v) = 2$.

a demand of one unit [10]. For each commodity $c \in C$, we represent by $o(c)$ its origin and by $d(c)$ its destination. For any node $i \in V$ and any commodity $c \in C$, let $D_c(i) = -k$ if $i = o(c)$, $D_c(i) = +k$ if $i = d(c)$ and $D_c(i) = 0$ otherwise. The discrete variable f^c_{ij} represents the flow of commodity c through arc (i, j). The binary variable f^c_{ij} is equal to one if the arc (i, j) is used by commodity c for communication from node i to j. Otherwise, it is equal to zero. Table 2 shows the notation and description of the parameters used in the formulations.

Let $Q_i = [q^1_i, \ldots, q^{\phi(i)}_i]$ be a finite list of successive cumulative increments in the power setting that can be assigned to node i, for any $i \in V$. Furthermore, $\phi(i) \leq |V| - 1$ and for any $\ell = 1, \ldots, \phi(i)$ we define T^ℓ_i as the set of new nodes reachable from node i if an additional increment q^ℓ_i is added to its current power assignment, as illustrated in Figure 3.

The binary variable x^ℓ_i takes the value one if there is a node $j \in T^\ell_i$ such that a directional arc (i, j) is used

TABLE 2. Mathematical notation defined in the integer programming formulations.

Notation	Description
C	set of $\lceil k\|V\|/2 \rceil$ commodities with a demand of one unit
$o(c)$	the origin of commodity $c \in C$
$d(c)$	the destination of commodity $c \in C$
$D_c(i)$	flow conservation of node $i \in V$
f_{ij}^c	discrete variable representing the flow of commodity c through arc (i,j)
$\phi(i)$	the number of successive cumulative increments in the power setting that can be assigned to node i
Q_i	the finite list of successive cumulative increments $[q_i^1, \ldots, q_i^{\phi(i)}]$ in the power setting that can be assigned to node i
T_i^ℓ	the set of new nodes reachable from node i if an additional increment q_i^ℓ is added to its current power assignment, where $\ell = 1, \ldots, \phi(i)$
x_i^ℓ	binary decision variable, taking the value one if there is a node $j \in T_i^\ell$ such that a directional arc (i,j) is used for communication from i to j
$\bar{\ell}(i)$	the smallest cumulative increment needed to establish the k-connectivity requirement from node i
P_{max}	the total power consumption of the network when all nodes are transmitting with its maximum power
P_{norm}	the network total power consumption normalized in the interval $[0.0, 1.0)$
$I(i)$	the receiver interference of each node i
INT	network maximum interference
Z_1	objective function of the FMMI formulation
Z_2	objective function of the FMMITP formulation
Z_3	objective function of the FMTI formulation
Z_4	objective function of the FMTITP formulation

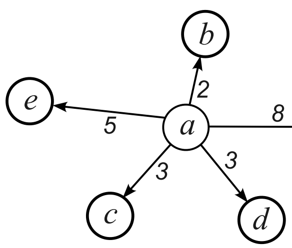


FIGURE 3. List of successive cumulative increments: $Q_a = [2, 1, 2, 3]$ and $T_a^1 = \{b\}$, $T_a^2 = \{c, d\}$, $T_a^3 = \{e\}$, $T_a^4 = \{f\}$.

for communication from i to j, otherwise it is equal to zero. We also define $\bar{\ell}(i) \in \{1, \ldots, \phi(i)\}$ considering that $\sum_{\ell=1}^{\bar{\ell}(i)-1} |T_i^\ell| < k$ and $\sum_{\ell=1}^{\bar{\ell}(i)} |T_i^\ell| \geq k$. Then, for any node i, $\sum_{\ell=1}^{\bar{\ell}(i)} |T_i^\ell|$ gives the minimum number of nodes needed to establish the k-connectivity requirement from node i.

The integer program formulation FMMI defined by the objective function (3) and constraints (4)–(13), presented in Figure 4, is a valid formulation for the k-connected MMI problem using the receiver interference model.

Constraints (5) and (4) give, respectively, the receiver interference $I(i)$ of each node i, and the maximum interference INT which is minimized by the objective function (3). Constraint (6) represent the flow conservation equations, which means that for $k = 2$, $D_c(i) = -2$ if node i is origin of the flow c, $D_c(j) = 2$ if node j is the destination of the flow c, and for all node $u \in V \setminus \{i, j\}$, $D_c(u) = 0$. These constraints algebraically state that the sum of the flow through arcs directed toward a node plus that node's supply (if any) equals the sum of the flow through arcs directed away from that node plus that node's demand (if any).

minimize $\quad Z_1 = INT \qquad (3)$

subject to

$$INT \geq \mathrm{I}(i) \qquad \forall i \in V \qquad (4)$$

$$\mathrm{I}(i) \geq \sum_{j \in V \setminus \{i\}} x_j^\ell \qquad \forall i \in V, \ell : i \in T_j^m \qquad (5)$$

$$\sum_{j \in V} f_{ji}^c - \sum_{l \in V} f_{il}^c = D_c(i), \qquad \forall c \in C, \forall i \in V \qquad (6)$$

$$\sum_{j \in V} f_{ij}^c \leq 1, \qquad \forall c \in C, \forall i \in V : i \neq o(c), i \neq d(c) \qquad (7)$$

$$x_i^\ell \geq f_{ij}^c + f_{ji}^c, \qquad \forall i \in V, \forall c \in C, \forall j \in T_i^\ell, \ell = 1, \ldots, \phi(i) \qquad (8)$$

$$x_i^{\ell+1} \leq x_i^\ell, \qquad \forall i \in V, \ell = 1, \ldots, \phi(i) - 1 \qquad (9)$$

$$x_i^\ell = 1, \qquad \forall i \in V, \ell = 1, \ldots, \bar{\ell}(i) \qquad (10)$$

$$f_{ij}^c \in \{0, 1\}, \qquad \forall i, j \in V, \forall c \in C \qquad (11)$$

$$x_i^\ell \in \{0, 1\}, \qquad \forall i \in V, \ell = 1, \ldots, \phi(i) \qquad (12)$$

$$0 \leq INT \leq |V| \qquad (13)$$

FIGURE 4. Integer program formulation FMMI to minimize the maximum interference with k-connectivity constraints.

Inequality (7) ensures node-disjointness and, to illustrate its benefit, suppose a scenario in which $k = 2$ units of flow are transmitted from node u to node v. In this case, the formulation forces the flow transmission on 2 node-disjoint paths between these nodes. Then, inequality (7) ensures that at most one unit of a commodity flows into [and out of due to flow balance constraint (6)] any node that is not the source or destination of the commodity. This strategy assures the required number of node-disjoint paths in the network.

Inequality (8) states that x_i^ℓ must be set to one if there is a node $j \in T_i^\ell$ such that arc (i, j) or arc (j, i) is used for communication from node i to j by commodity c. This restriction forces the setting of the transmission range of node i, if there is a flow through the directed arc (i, j), or through the directed arc (j, i). That is, Inequality (8) ensures that a bidirectional edge $[i, j]$ is used if there is flow from i to j or from j to i. For the inequality (8) to be valid, we also have to prove that both f_{ij}^c and f_{ji}^c cannot be simultaneously equal to one. This cannot be true because otherwise, there would be a cycle of commodity c through nodes i and j.

Constraint (9) enforces $x_i^{\ell+1}$ to be equal to zero if the previous increment was not used, i.e., if $x_i^\ell = 0$. It imposes the successive cumulative increments in the power setting. Constraint (10) sets to one the incremental powers necessary to reach at least the k closest nodes of each node i, because, in order to be k-connected, any solution requires at least k incident edges to any node. Constraints (11), (12), and (13) express the integrality and bounds requirements on the variables.

The objective function (3) gives an integer value *INT* corresponding to the minimum maximum interference, without considering power values nor the number of nodes with maximum interference. Therefore, getting different power settings that does not affect the maximum interference is possible. To find the lowest power setting simultaneously with the interference minimization given by the objective function (3), we propose the addition of a normalized total power consumption in the interval [0.0, 1.0) to the integer value corresponding to the minimum maximum interference.

Let $P_{max} = \sum_{i \in V} \max(p_i)$ be the maximum total power consumption of the network when all nodes are transmitting with its maximum power. It can be verified from the FMMI formulation that the power setting assigned to each node i can be calculated as $\sum_{\ell=1}^{\phi(i)} q_i^\ell \cdot x_i^\ell$ and, consequently, the total power consumption can be normalized in the interval [0.0, 1.0) by Equation (14):

$$P_{norm} = \frac{\sum_{i \in V} \sum_{\ell=1}^{\phi(i)} q_i^\ell \cdot x_i^\ell}{P_{max}}. \tag{14}$$

Since the objective function (4) gives an integer value, we can add the normalized total power consumption to it without affecting the minimum interference calculation. The normalized total power penalizes the objective function with high power settings. Thus, if we replace Equation (4) with Equation (15) we have the integer program formulation FMMITP for the k-connected MMITP problem using the receiver interference model

$$\textbf{minimize} \quad Z_2 = INT + P_{norm}. \tag{15}$$

The receiver interference of each node i is given by $I(i)$ calculated in Equation (5), and the network total interference can be calculated as $\sum_{i \in V} \mathrm{I}(i)$. As a result, to solve the k-connected MTI problem using the receiver interference model, we propose the FMTI integer program formulation defined by the objective function (16), and by the constraints (5)–(13) presented in Figure 4

$$\textbf{minimize} \quad Z_3 = \sum_{i \in V} \mathrm{I}(i). \tag{16}$$

Similar to the FMMI formulation, different power settings can result in the same value of the objective function in the FMTI formulation. Again, we propose the addition of the normalized total power consumption to find the objective function with the lowest power settings. Since the objective function (16) gives an integer value, the normalized total power consumption do not affect its calculation. Therefore, if we replace Equation (16) with Equation (17) we have the integer program formulation FMTITP for the k-connected MTITP problem using the receiver interference model

$$\textbf{minimize}\ Z_4 = \sum_{i \in V} \mathrm{I}(i) + P_{norm}. \tag{17}$$

In Section VI-A, we compare the proposed formulations using results obtained by a commercial solver. We focus our analysis on the biconnected case ($k = 2$) since it gives the most useful fault-tolerant property. With biconnectivity requirements, at least two node-disjoint paths exist between any pair of nodes. Biconnected communication graphs are important to ensure fault tolerance since ad hoc networks are used in critical application domains where failures are likely to occur.

V. GRASP/VNS HEURISTIC

A Greedy Randomized Adaptive Search Procedure (GRASP) [61], [62] is a multi-start meta-heuristic. Each of its iterations consists of two phases: a construction phase, in which a feasible solution is built, and a local search phase, in which a local optimum in the neighborhood of the current solution is sought. These two phases are then repeated until a certain termination criterion is met and the best overall solution is returned. The greedy randomized solutions are generated by adding elements to the problem solution set from a list of elements ranked by a greedy function, according to the quality of the solution they will achieve. The combination of greediness and randomness in the generation of solutions is achieved by means of restricted candidate lists (RCL) of variable sizes that control both features. Since GRASP was first introduced in [63] it has been successfully applied to a variety of optimization problems [61], [62].

A Variable Neighborhood Search (VNS) [64], [65] is another meta-heuristic for solving combinatorial optimization problems. VNS applies a strategy addressed to searching for optimal solutions by using a finite set of different neighborhood structures. The basic VNS (BVNS) variant explores the search space by doing systematic changes between the given neighborhood structures. This strategy makes the search more flexible when compared to single-neighborhood-based local search algorithms.

The BVNS consists of three phases: shake, local search, and neighborhood change. During the shake phase, a new solution is randomly selected from the neighborhood of the current solution, helping the algorithm to escape from the local optimum. Local search phase tries to improve the current solution by exploring one or more neighborhood structures. In the neighborhood change phase, the new solution given by local search phase is compared. If the new solution performs better than the previous best, it is chosen as the new optimum, and the current neighborhood structure is updated. These three phases are repeated until a termination criterion is satisfied. The adaptability of the methodology has resulted in several variants in recent years (see [64] for a recent survey on the methodology).

This paper proposes the implementation of a hybrid GRASP/VNS algorithm to provide solutions to the four interference minimization problems. The proposed hybrid algorithm incorporates two powerful features, the effective constructive and improving ability of GRASP and the flexibility of VNS to explore different search spaces for the problem. The hybridization consists of generating solutions using four phases: construction, shake, local search, and neighborhood change. In the remainder of this section, we customize the four hybrid heuristic phases for the interference minimization problems (MMI, MMITP, MTI, MTITP) with asymmetric input and bidirectional biconnected communication graph.

In Section VI, we compare the results of our proposed strategy with the optimal solutions provided by the mathematical formulations described in Section IV for moderately sized networks.

A. CONSTRUCTION PHASE

In the construction phase, a feasible solution is iteratively constructed, one element at a time. At each construction iteration, the choice of the next element to be added is determined by ordering all candidate elements (i.e. those that can be added to the solution) in a candidate list L with respect to a real-valued greedy function $g(.)$. This function measures the benefit of selecting each element. In a purely greedy implementation, the top candidate is always selected. The probabilistic component of a GRASP is characterized by randomly choosing one of the best candidates in the list, but not necessarily the top candidate. The list of best candidates is called the restricted candidate list.

We propose a construction phase algorithm based on the constructive procedure for the bidirectional biconnected minimum power consumption problem developed by [66]. The mathematical notation, as well as important descriptions, are shown in Table 3.

TABLE 3. Mathematical notation defined in Heuristics.

Notation	Description
L	candidate list
$g(.)$	real-valued greedy function
d^{ε}_{uv}	non-negative arc weights
α	greediness constructive procedure control parameter
RCL	restricted candidate list
$\overline{g}$	maximum computed greedy cost over all candidate nodes
$\underline{g}$	minimum computed greedy cost over all candidate nodes
$H(p)$	construction phase working graph
r	construction phase randomly selected initial node
$f(.)$	objective function value depending on the problem being solved
w_{max}	parameter that indicates the maximum neighborhood to be explored in the VNS heuristic

Our proposed algorithm for solving minimum interference problems requires the same inputs as [66]: the node set V, the non-negative arc weights d^{ε}_{uv} for any $u, v \in V$ and α, which controls the greediness/randomness of the constructive procedure and the same constructive steps. The only difference between both algorithms is the greedy cost function defined in our procedure as $g(u, v) = \max\{d^{\varepsilon}_{uv}, d^{\varepsilon}_{vu}\}$, for any $u, v \in V$. Since we have asymmetric inputs, then $d^{\varepsilon}_{uv} \neq d^{\varepsilon}_{vu}$.

The algorithm begins by setting up $p_u = 0$ for all $u \in V$, and initializing a working graph $H(p) = (V', E(p))$ with $V' = \{r\}$ and $E(p) = \{[u, v] : u \in V', v \in V', p_u \geq d^{\varepsilon}_{uv}, p_v \geq d^{\varepsilon}_{vu}\} = \emptyset$, where $r \in V$ is any randomly selected initial node.

The first stage of our construction phase builds a bidirectional connected graph, one node at a time. For every node $u \notin V'$, let $g(u) = \min_{v \in V'}\{g(u, v)\}$ be the minimum computed greedy cost. Let $\underline{g} = \min_{u \in V \setminus V'}\{g(u)\}$ and $\overline{g} = \max_{u \in V \setminus V'}\{g(u)\}$ be, respectively, the minimum and maximum computed greedy cost over all candidate nodes (i.e., those not in the current solution). The RCL is formed by all nodes $u \in V \setminus V'$ such that $g(u) \leq \underline{g} + \alpha(\overline{g} - \underline{g})$, with $0.0 \leq \alpha \leq 1.0$. The case $\alpha = 0.0$ corresponds to a pure greedy algorithm, while $\alpha = 1.0$ is equivalent to a completely random construction. A node u is randomly selected from RCL and inserted into V'. The power assignments of the nodes $u \in V \setminus V'$ and $v \in V'$ such that $g(u) = g(u, v)$ are updated to $\max\{p_u, d^{\varepsilon}_{uv}\}$ and $\max\{p_v, d^{\varepsilon}_{vu}\}$, respectively, for example, if $p_u \geq d^{\varepsilon}_{uv}$, then the unidirectional communication between u and v is already set up. Consequently, the bidirectional edge $[u, v]$ is inserted into $E(p)$. This stage finishes when $V' = V$, ensuring that a connected graph $H(p) = (V, E(p))$ is obtained.

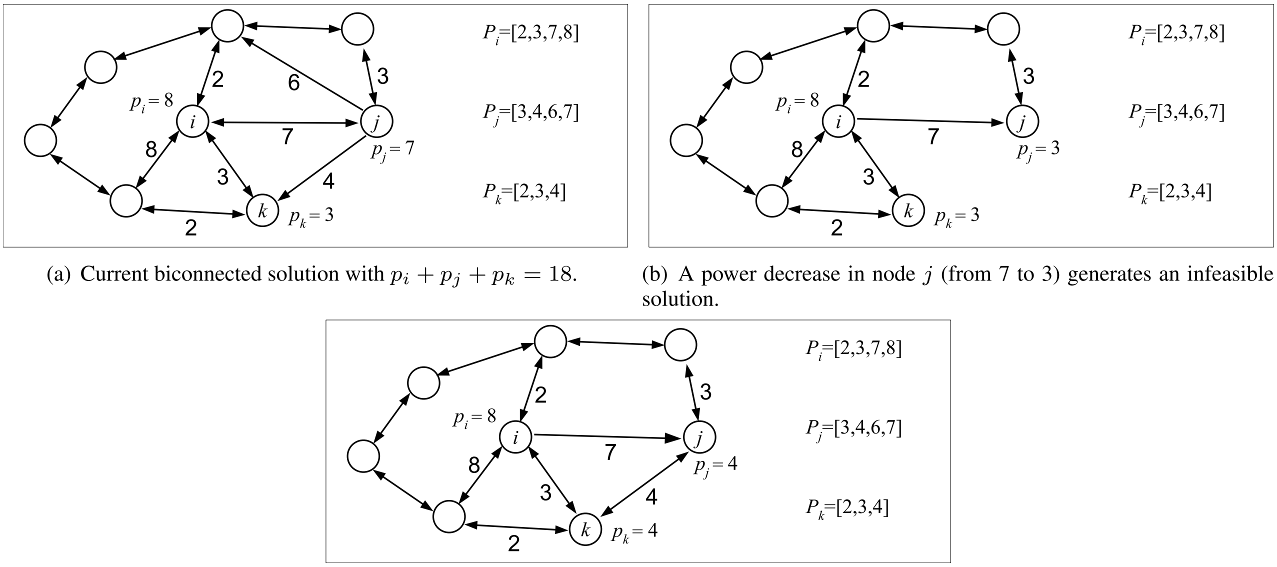
The second construction stage produces a biconnected graph $G(p) = (V, B(p))$ using the Tarjan's algorithm [67] to compute the biconnected components and the articulation points of the current solution and, in the following, connecting two nodes which are not articulation points. Connecting nodes that are not articulation points of the current solution progressively reduces the number of biconnected components until a biconnected graph is obtained [66]. The algorithm stops when a biconnected graph is built.

B. SHAKE PHASE

In the shake phase, a solution is randomly generated by applying the corresponding neighborhood structure, i.e., the w-th neighborhood structure, with w_{max} representing the total number of neighborhood structures. The sequence of neighborhood structures has been chosen following the ideas described by [66] where the definition of the neighborhoods makes use of two basic operations for decreasing and increasing the power assignments.

Applied to a node $i \in V$ (see Figure 5(a) and 5(b)), the first operation decreases its current power assignment $p_i = p^{\ell}_i$ (with $\ell \geq 2$) to $p_i = p^{\ell'}_i$, where ℓ' is the highest level that supports a bidirectional edge. Consequently, the decreasing operation removes links (arcs and edges) between nodes and reduces total power assignment.

Applied to a node $i \in V$ (see Figure 5(c)), the second operation increases its current power $p_i = p^{\ell}_i$ (with $\ell \leq \phi(i) - 1$) to $p_i = p^{\ell+1}_i$. The power increase operation must ensure the insertion of the bidirectional edge $[i, j]$. Thus, the operation can also increase the current power p_j to p^{l}_j, $l = 1, \ldots, \phi(j)$, such that $p_j < p^{l}_j$ and $i \in T^{l}_j$ and the objective function is increased by $(p^{\ell+1}_i - p^{\ell}_i) + (p^{l}_j - p_j)$. Otherwise, if there exists a node $j \in T^{\ell+1}_i$ and a power level



(a) Current biconnected solution with $p_i + p_j + p_k = 18$.

(b) A power decrease in node j (from 7 to 3) generates an infeasible solution.

(c) A power increase in nodes j and k (from 3 to 4) restores feasibility and creates a better solution with $p_i + p_j + p_k = 16$.

FIGURE 5. Example of a complete local search move [66].

$l = 1, \ldots, \phi(j)$ so that $i \in T_j^l$ and $p_j \geq p_j^l$, then the objective function is increased only by $(p_i^{\ell+1} - p_i^{\ell})$.

The `Shake(p,w)` procedure used in line 5 of Algorithm 3 and line 7 of Algorithm 4 (explained in Section V-F) generates a solution p' at random from the w-th neighborhood of the current solution p in which neighborhood $w = 1$ means apply a decrease operation in one node at random assuring that biconnectivitiy of current solution p is not broken. For neighborhood $w = 2$, an increase operation is applied in one node at random of the current solution p.

C. LOCAL SEARCH PHASE

The local search phase explores the neighborhood of the current solution, attempting to reduce the total power consumption, and it is based on the local search procedure proposed by [66]. The proposed local search method follows a first-improvement strategy. In particular, the method visits neighbor solutions by following a non-increasing order of power decrease (i.e., start by largest power decrease), and replaces the incumbent solution with the first neighbor solution that has a better objective function value; the search is subsequently restarted in the neighborhood of the updated solution. The search stops when a local optimum is reached, i.e., no better solution can be found in the neighborhood.

A move starts (see Figure 5(b)) by decreasing the power assignment of as many nodes as needed to break biconnectivity, followed by a sequence of as many power increases as necessary to restore biconnectivity. Decrease operations are performed in non-increasing order of power decrease (i.e., start by largest power decrease). Increase operations are performed in a non-decreasing order of power increase (i.e., start by smallest power increase). To get more effective moves, increase operations are applied only to nodes not affected by previous power decrease operations. The first improvement move is accepted, and the search moves to the new neighbor. The procedure continues until no further improvement move exists. An improvement is achieved if $f(p') < f(p)$, where p is the current solution, p' its neighbor and $f(.)$ the function which gives the objective function value depending on the problem being solved (Equations (3), (15), (16) and (17)). The proposed local search method, named `Local_Search(p)`, is used in line 4 of Algorithm 2 (see Section V-E), in line 6 of Algorithm 3, and line 8 of Algorithm 4.

D. NEIGHBORHOOD CHANGE PHASE

The neighborhood change phase is executed in order to evaluate the new solution given by local search phase and to select the subsequent neighborhood to be explored. Algorithm 1 (see Algorithm 5 for further details) implements neighborhood change phase as a function named `Neighborhood_Change(p,p'',w)`. This function requires three inputs: the incumbent solution p, the solution given by local search phase p'', and the current neighborhood w. Function `Neighborhood_Change(p,p'',w)` compares the incumbent value $f(p)$ with the new value $f(p'')$ obtained from the w-th neighborhood (line 1). The function $f(.)$ gives the objective function value depending on the problem being solved (Equations (3), (15), (16) and (17)). If an improvement is obtained, the incumbent is updated (line 2), and w is returned to its initial value (line 3). Otherwise, the next neighborhood is considered (line 5). The current incumbent solution p and the updated current neighborhood w are returned in line 7.

Algorithm 1 Function `Neighborhood_Change` (p, p'', w)

Require: Incumbent solution p, local search solution p'' and the current neighborhood w.
Ensure: Updated incumbent solution p and current neighborhood w.

1: **if** $f(p'') < f(p)$ **then**
2: $\quad p \leftarrow p''$;
3: $\quad w \leftarrow 1$;
4: **else**
5: $\quad w \leftarrow w + 1$;
6: **end if**
7: **return** p, w;

E. GRASP HEURISTIC

Algorithm 2 (see Algorithm 6 for further details) presents the pseudo-code for the GRASP algorithm proposed in this work to solve the four interference minimization problems.

Algorithm 2 Pseudo-Code of GRASP

Require: Node set V, arc weights d^{ε}_{uv} for any $u, v \in V$, *Stop_Condition* and parameter α.
Ensure: Best known solution p^*

1: $p^* \leftarrow \emptyset$;$f(p^*) \leftarrow \infty$;
2: **while** *Stop_Condition* is not met **do**
3: $\quad p \leftarrow$ `Greedy_Randomized_Construction` $(V, d^{\varepsilon}_{uv}, \alpha)$;
4: $\quad p' \leftarrow$ `Local_Search` (p);
5: $\quad$ **if** $f(p') < f(p^*)$ **then**
6: $\quad\quad p^* \leftarrow p'$;
7: $\quad$ **end if**
8: **end while**
9: **return** p^*;

Algorithm 2 requires the node set V and arc weights d^{ε}_{uv} for any $u, v \in V$, and two parameters: *Stop_Condition*, that indicates the stopping condition of the algorithm, and α, that controls the greediness/randomness of the constructive procedure. In line 1, the best-known solution and its objective function value are initialized. The function $f(.)$ gives the objective function value depending on the problem being solved (Equations (3), (15), (16) and (17)). Each iteration of the loop in lines 2 to 8 finds a new solution to the problem until the stopping condition is reached. The procedure in line 3 finds a greedy randomized solution, as discussed in Section V-A, which is submitted to the local search procedure in line 4 as discussed in Section V-C. If the solution found by local search improves upon the best previously found solution, then the best solution is updated in line 6. The best power assignment p^* is returned in line 9.

F. VNS HEURISTIC

Algorithm 3 (see Algorithm 7 for further details) depicts the pseudo-code of the BVNS algorithm to solve the four interference minimization problems.

Algorithm 3 Pseudo-Code of VNS

Require: Node set V, arc weights d^{ε}_{uv} for any $u, v \in V$, *Stop_Condition*, maximum neighborhood w_{max} to be explored during the search.
Ensure: Best known solution p

1: $p \leftarrow$ `Greedy_Randomized_Construction` $(V, d^{\varepsilon}_{uv}, \alpha = 0.0)$;
2: **while** *Stop_Condition* is not met **do**
3: $\quad w \leftarrow 1$;
4: $\quad$ **repeat**
5: $\quad\quad p' \leftarrow$ `Shake` (p, w);
6: $\quad\quad p'' \leftarrow$ `Local_Search` (p');
7: $\quad\quad p, w \leftarrow$ `Neighborhood_Change` (p, p'', w);
8: $\quad$ **until** $w = w_{max}$
9: **end while**
10: **return** p;

Algorithm 3 requires the node set V and arc weights d^{ε}_{uv} for any $u, v \in V$, as well as two parameters: *Stop_Condition*, that indicates the stopping condition of the algorithm and w_{max}, that indicates the maximum neighborhood to be explored during the search. An initial solution p is created in line 1 by applying the construction phase, as described in Section V-A, with parameter α fixed in 0.0. Thus, the function `Greedy_Randomized_Construction` $(\alpha = 0.0)$ returns the pure greedy solution without randomness.

The neighborhood size parameter w is initialized to 1 (line 3). The search explores a sequence of increasing-order neighborhoods. Starting from the initial solution p, the loop (lines 4 to 8) is repeated until the neighborhood size parameter exceeds the maximum size w_{max}, in that case, all neighborhoods have been explored without finding any improving solution. To do this, a solution p' belonging to the w-th order neighborhood of p is randomly generated in line 5 by procedure `Shake` (p, w), as described in Section V-B. In line 6, the `Local_Search` (p') procedure is applied to p', and a local optimum p'' is found, as described in Section V-C. In line 7, the function `Neighborhood_Change` (p, p'', w) executes the neighborhood change stage, as described in Section V-D, in order to select the subsequent neighborhood to be explored and to update the best-known solution p. The best power assignment p is returned in line 10.

G. HYBRID GRASP/VNS HEURISTIC

The proposed hybrid algorithm incorporates the powerful features of the algorithms: the effective constructive and the improving ability of GRASP, and the flexibility of VNS to explore different search spaces for the problem. Algorithm 4 (see Algorithm 8 for further details) shows the pseudo-code of the hybrid GRASP/VNS. It solves the four interference minimization problems (MMI, MMITP, MTI, MTITP) for

Algorithm 4 Pseudo-Code of Hybrid GRASP/VNS

Require: Node set V, arc weights d_{uv}^{ε} for any $u, v \in V$, *Stop_Condition*, maximum neighborhood w_{max} to be explored during the search and parameter α.
Ensure: Best known solution p^*

1: $p^* \leftarrow \emptyset$;$f(p^*) \leftarrow \infty$;
2: **while** *Stop_Condition* is not met **do**
3:   $p \leftarrow$ `Greedy_Randomized_Construction` $(V, d_{uv}^{\varepsilon}, \alpha)$;
4:   **repeat**
5:     $w \leftarrow 1$;
6:     **repeat**
7:       $p' \leftarrow$ `Shake` (p, w);
8:       $p'' \leftarrow$ `Local_Search` (p');
9:       $p, w \leftarrow$ `Neighborhood_Change` (p, p'', w);
10:     **until** $w = w_{max}$
11:   **until** no improvement
12:   **if** $f(p) < f(p^*)$ **then**
13:     $p^* \leftarrow p$;
14:   **end if**
15: **end while**
16: **return** p^*;

asymmetric input graphs and bidirectional biconnected communication graphs. It requires the node set V and arc weights d_{uv}^{ε} for any $u, v \in V$, and three parameters: *Stop_Condition*, w_{max} and α. In line 1, the best-known solution and its objective function value are initialized. Each iteration of the loop in lines 2 to 15 finds a new solution until the stopping condition is reached. In line 3, the procedure `Greedy_Randomized_Construction` $(V, d_{uv}^{\varepsilon}, \alpha)$ finds a greedy randomized solution that is submitted to a VNS procedure in lines 4 to 11 until no improvement is achieved, as described in Section V-F. If the solution found by the VNS procedure improves upon the best previously found solution, then the best solution is updated in line 13. The best power assignment p^* is returned in line 16.

VI. RESULTS AND DISCUSSION

The analysis of the obtained results is divided into two parts: optimal solutions and heuristics solutions. The former is devoted to exactly solving a set of very moderately sized problems using a commercial solver and the integer programming formulations proposed in Section IV, while the latter was generated by the heuristics used to solve real-life sized problems. We also report computational simulations in which we compare the formulations and the interference models. Moreover, we evaluate the effectiveness of the heuristics variants in terms of the trade-offs between computation time and solution quality. We focus our analysis on the biconnected case ($k = 2$), since it gives the most useful fault tolerant property.

A. OPTIMAL SOLUTIONS

Numerical simulations, aiming for optimal solutions, have been carried out on a set of random moderately sized asymmetric Euclidean Instances with $|V| \in [10, 50]$ nodes uniformly distributed in the unit square grid. The asymmetric weight of the arc between nodes u and v is set as $Fd_{u,v}^{\varepsilon}$, where $d_{u,v}$ is the Euclidean distance between nodes u and v, the path loss exponent ε is set at 2, and $F \in [0.8, 1.2]$ is a random uniform perturbation. The set of instances used in this work is the same set generated by [11] and [66], consisting of fifteen random instances for each considered size (problem dimension).

A computational cluster with an Intel Xeon with a 2.40 GHz clock and 8 Gbytes of RAM memory running under GNU/Linux 2.6.24 was used in the simulations. ILOG CPLEX 12.4 was used as both a linear and integer programming solver with parallel features disabled. We have created fifteen random instances for each problem dimension. The simulations consist of one run of each formulation in CPLEX, over the fifteen randomly generated Euclidean Instances of the same size.

We also used the optimal minimum power in the comparisons. The optimal minimum power is calculated by the Incremental Power formulation (FIP) proposed by [10] to solve the k-connected minimum power consumption problem, which consists of finding a power assignment to the nodes of a wireless network such that the resulting network topology is k-vertex connected and the total power consumption is minimum.

For each problem dimension $|V| \in \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$ and each formulation, Table 4 shows the total number of instances solved to optimality and feasibility, i.e., a feasible solution for the model has been proven to exist, but the model has been not solved to optimality by CPLEX in less than six hours of CPU time. All formulations found the fifteen optimal solutions for $|V| \leq 25$ and feasible solutions for $|V| \leq 50$, within six hours. Although the number of constraints and variables is given by $O(|V|^3)$ in all formulations, model FIP found the most optimal solutions since it has no interference constraints, i.e., the addition of interference constraints makes the models harder to solve. Although the interference models require a greater computational effort, all interference formulations had similar performance, independently of the objective function.

Table 5 shows, for each problem dimension $|V| \in \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$ and each formulation, the average time results in seconds over the optimal solutions found by all formulations within six hours of computations. The instances that did not find an optimal solution by a given formulation within six hours of computations were discarded. The first column of Table 5 displays, in parentheses, the number of instances used to calculate the averages. Cells in blank correspond to formulations with the lower number of instances solved to optimality, as shown in Table 4.

TABLE 4. Number of instances solved to optimality and feasibility in six hours of running time for all formulations.

	FIP		FMMI		FMMITP		FMTI		FMTITP	
$\|V\|$	opt	fea	opt	fea	opt	fea	opt	fea	opt	fea
10	15	15	15	15	15	15	15	15	15	15
15	15	15	15	15	15	15	15	15	15	15
20	15	15	15	15	15	15	15	15	15	15
25	15	15	15	15	15	15	15	15	15	15
30	15	15	14	15	13	15	14	15	14	15
35	14	15	9	12	7	14	7	15	12	15
40	12	15	5	5	4	8	5	13	4	14
45	8	15	2	2	2	3	1	6	1	5
50	4	8	2	2	3	3	0	3	0	1

The results presented in Table 5 also show that the model FIP has the lowest computational time, which reflects the same behavior described by the results shown in Table 4. Table 5 also shows that, as the size of the problem dimension increases, the FMMITP and FMTITP become faster than the models with no power minimization (FMMI and FMTI).

TABLE 5. Average running time (in seconds) to find optimal solutions for all formulations. The first column displays, in parentheses, the number of instances used to calculate the averages.

$\|V\|$	FIP	FMMI	FMMITP	FMTI	FMTITP
10 (15)	0.32	0.25	0.79	0.96	1.59
15 (15)	1.89	5.06	5.34	7.27	9.63
20 (15)	17.02	70.76	39.12	75.27	28.98
25 (15)	109.51	1141.99	732.92	600.66	342.73
30 (12)	329.90	2471.91	2988.87	2735.39	2914.51
35 (5)	332.16	5493.89	2012.23	6035.40	2649.74
40 (12)	7134.33	–	–	–	–
45 (8)	8928.67	–	–	–	–
50 (4)	13790.09	–	–	–	–

In Tables 6 and 7, we consider the instances solved to feasibility (instead of optimality) by CPLEX within six hours of computation time. Since the interference formulations give a few feasible solutions for $|V| \geq 45$, we consider the problem dimension $|V| \in \{10, 15, 20, 25, 30, 35, 40\}$. Table 6 shows the average relative primal-dual gap D in percentage between the primal and the dual feasible solutions found by all formulations within six hours of computations. The optimal solution was found in all instances when $D = 0.00\%$. All values are averaged over at least 8 integer feasible solutions for each problem dimension and each formulation. For large instance size, results show that, when formulations do not reach the optimality, the dual bound can be very large when the total interference is minimized in the FMTI and FMTITP models. The largest average gap remains 80.98% after six hours of computation.

Table 7 presents the average relative linear relaxation gap (column L) in percentage between the primal solution and that of the linear relaxation bound, as well as the average relative MIP gap (column M) in percentage between the first integer solutions found, and the linear relaxation bound at that time. Regardless of the instance sizes, Table 7 shows that the FIP, FMMI and FMMITP formulations keep the optimum linear relaxation solutions close to the optimum

TABLE 6. Average relative primal-dual gap D(%) for all formulations.

$\|V\|$	FIP	FMMI	FMMITP	FMTI	FMTITP
10 (15)	0.00	0.00	0.00	0.00	0.00
15 (15)	0.00	0.00	0.00	0.00	0.00
20 (15)	0.00	0.00	0.00	0.00	0.00
25 (15)	0.00	0.00	0.00	0.00	0.00
30 (15)	0.00	1.33	0.01	1.94	0.52
35 (11)	0.15	1.79	1.06	47.20	0.37
40 (8)	2.83	–	6.51	73.80	80.98

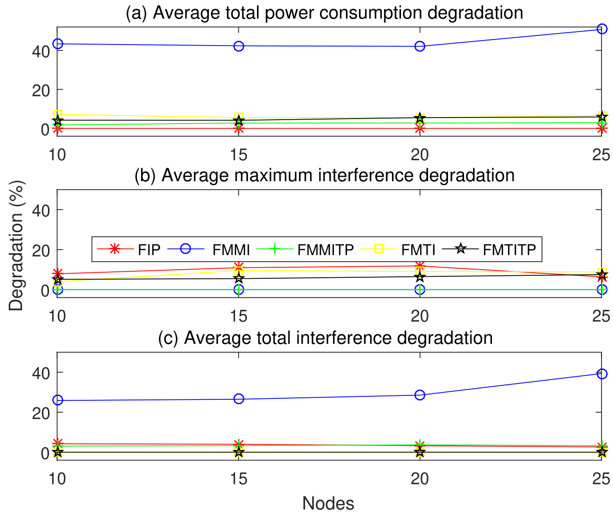


FIGURE 6. Average relative degradation for power and interference values.

integer solutions and also keep a constant MIP gap for the tested problems. For the FMTI and FMTITP formulations, Table 7 shows that the total interference minimization leads to an increase in the optimum linear relaxation solutions and in the objective function value of the first integral solution found and, consequently, to an increase in the relative gap L and M, respectively. These results, together with Tables 4 and 6, allow us to conclude that the interference formulations FMTI and FMTITP are, on average, the best methods at finding feasible integer solutions. However, these solutions can be far from the optimal that requires larger computing effort. Formulations FMMI and FMMITP have greater difficulty in finding feasible solutions, but when they find them, the feasible solutions are closer to the optimum.

The results presented in Table 8 show the average solution quality given by all formulations. Table 8 and Figure 6 show the average power and interference values and its percentage degradation with respect to the best average (in bold), for each formulation. The average values are shown for the instances with $|V| \leq 25$ since all formulations find the fifteen optimal solutions for $|V| \leq 25$.

Table 8 and Figure 6 show that the minimum total interference formulation FMTI has its total power values close to the optimal with at most 7.23% from the optimal. On the other hand, the FMMI formulation minimizes the maximum interference and gives solutions with total power

TABLE 7. Average relative linear gap $L(\%)$ and MIP gap $M(\%)$ for all formulations.

	FIP		FMMI		FMMITP		FMTI		FMTITP	
$\|V\|$	L(%)	M(%)	L(%)	M(%)	L(%)	M(%)	L(%)	M(%)	L(%)	M(%)
10 (15)	9.49	22.92	11.54	8.25	17.79	8.15	16.81	12.12	10.70	13.23
15 (15)	7.87	13.88	15.81	3.83	15.42	9.02	15.40	13.45	15.38	13.60
20 (15)	12.90	15.53	11.46	11.44	16.52	8.22	15.59	29.59	15.58	18.48
25 (15)	5.95	19.17	13.77	10.32	13.57	11.93	9.53	51.13	9.52	36.54
30 (15)	6.56	22.26	13.88	11.09	13.71	10.15	11.48	65.82	10.25	59.71
35 (11)	5.43	16.30	7.82	4.33	6.13	3.39	51.13	36.40	8.61	44.90
40 (8)	8.16	18.83	–	–	12.60	7.83	75.67	70.17	82.35	69.82

TABLE 8. Average power and interference values and relative degradation for each formulation.

Average minimum power for each formulation										
	FIP		FMMI		FMMITP		FMTI		FMTITP	
$\|V\|$	power	D(%)	power	D(%)	power	D(%)	power	D(%)	power	D(%)
10	**1.66**	**0.00**	2.38	43.37	1.69	1.80	1.78	7.23	1.73	4.22
15	**1.44**	**0.00**	2.05	42.36	1.48	2.77	1.52	5.55	1.50	4.17
20	**1.45**	**0.00**	2.06	42.07	1.49	2.76	1.53	5.52	1.53	5.52
25	**1.36**	**0.00**	2.05	50.73	1.40	2.94	1.45	6.62	1.44	5.88
Average minimum maximum interference (MInt) for each formulation										
	FIP		FMMI		FMMITP		FMTI		FMTITP	
$\|V\|$	MInt	D(%)	MInt	D(%)	MInt	D(%)	MInt	D(%)	MInt	D(%)
10	5.47	7.89	**5.07**	**0.00**	**5.07**	**0.00**	5.27	3.94	5.33	5.13
15	5.47	10.95	**4.93**	**0.00**	**4.93**	**0.00**	5.40	9.53	5.20	5.48
20	5.67	11.83	**5.07**	**0.00**	**5.07**	**0.00**	5.53	9.07	5.40	6.50
25	5.67	6.38	**5.33**	**0.00**	**5.33**	**0.00**	5.80	8.82	5.73	7.50
Average minimum total interference (TInt) for each formulation										
	FIP		FMMI		FMMITP		FMTI		FMTITP	
$\|V\|$	TInt	D(%)	TInt	D(%)	TInt	D(%)	TInt	D(%)	TInt	D(%)
10	34.40	4.24	41.53	25.85	34.00	3.03	**33.00**	**0.00**	**33.00**	**0.00**
15	50.53	3.97	61.47	26.48	50.20	3.29	**48.60**	**0.00**	**48.60**	**0.00**
20	67.73	3.25	84.33	28.55	68.00	3.66	**65.60**	**0.00**	**65.60**	**0.00**
25	83.00	2.63	112.67	39.32	83.53	3.29	**80.87**	**0.00**	**80.87**	**0.00**

up to 50.73% greater than the optimum. Both FMMITP and FMTITP formulations gave low power settings with, at most, 5.88% from the optimal. They found the minimum power for the optimal interference.

Table 8 and Figure 6 also show that the FMMI and FMMITP models found the same optimal interference values. They show that the other models gave solutions close to the optimal, even not solving the minimum maximum interference explicitly. These results also show that the FMTI and FMTITP models found the same optimal interference values and show that the other models gave solutions close to the optimal (except for the FMMI), even not solving the minimum maximum interference explicitly.

These results allow us to conclude that the FIP formulation, even not explicitly solving the interference, gives interference values close to the optimum, regardless of the adopted interference objective value. We also conclude that FMMI formulation minimizes the maximum interference but gives solutions with total power and total interference above 25.85% greater than the optimum. Thus, when considering the minimization of the maximum interference, it is important to minimize the total power simultaneously, as demonstrated by the results obtained by the FMMITP formulation. In the case of total interference minimization, FMTI formulation finds solutions with low power settings, even not considering the power reduction explicitly. Minimizing the total interference gets optimal solutions with lower power settings when compared with the objective of minimizing the maximum interference. This happens because, when minimizing the maximum interference, it is possible to increase the node power to provide more links and, consequently, more connectivity without an increase in the maximum interference. On the other hand, when minimizing the total interference, increasing node power always causes increases in the total interference.

TABLE 9. The percentage difference between all nodes and the interference-free nodes, i.e., $I(v) = k$.

$\|V\|$	FIP (%)	FMMI (%)	FMMITP (%)	FMTI (%)	FMTITP (%)
10 (15)	31.33	8.67	32.00	**34.00**	32.00
15 (15)	29.78	12.00	29.33	**33.78**	33.33
20 (15)	29.33	7.67	26.00	**34.00**	33.33
25 (15)	28.27	5.33	27.20	**30.93**	**30.93**
30 (12)	30.83	8.89	30.00	35.56	**36.11**
35 (5)	40.00	6.86	38.29	**42.29**	41.14
40 (12)	**28.54**	–	–	–	–

Table 9 presents the average relative difference between all nodes and the interference-free nodes. The first column displays, in parentheses, the number of instances used to build the averages. We consider $I(v) = k$ as the interference-free condition to receiver v, i.e., receiver v is covered by the minimum necessary communication links needed to achieve the k-connectivity property. The biggest difference for each instance size is in bold. Table 9 shows that FMTI and FMTITP formulations have the largest relative amount of nodes with no interference. For instance, with $|V| = 35$, solutions given by FMTI formulation have, on average, 42.29% of nodes with no interference. Table 9 also shows that FMMI formulation, although minimizing the maximum interference, gives solutions that keep many nodes with high interference. A comparison with results obtained by formulations FMMI and FMMITP, confirms that the minimization of the maximum interference, without consider the power reduction simultaneously, can lead to networks with high interference and power settings. The importance of power reduction is also supported by the results from the FIP formulation, which builds low interference networks while minimizing the total power without explicitly considering interference.

The difficulties faced by CPLEX in solving large instances support the need for efficient heuristics capable of finding good approximate solutions for large-size problems in reasonable computation times.

B. HEURISTIC SOLUTIONS

The heuristics described in Section V were coded in Matlab, version 4.1 2018a. An Intel Core i5-8265 machine with a 1.80 GHz clock and 8 Gbytes of RAM memory running under Windows 10 Home was used in all heuristic simulations. The value for the α parameter was set to 0.2 because it often leads to good solutions in the presence of a relatively large variance [62]. The *Stop_Condition* parameter varied according to the simulations being performed.The w_{max} parameter was set to 2 due to the two different neighborhood structures, as described in Section V-B. It has been observed that the best value for the parameter w_{max} is often 2 or 3 [64]. Computational simulations with optimal solutions have been carried out on the same set of random moderately sized asymmetric Euclidean Instances with $|V| \in [10, 50]$ extended to up to 400 nodes ($|V| \in [10, 400]$), also consisting in fifteen randomly instances for each problem dimension. We focus our analysis on the biconnected case ($k = 2$) of the four interference minimization problems.

1) COMPARISON BETWEEN HEURISTICS AND THE OPTIMAL VALUES

In the first simulations, each instance was solved once by each heuristic, and the heuristics stopped when the optimal solution was found or when the time limit of 30 seconds was reached since the GRASP/VNS (G^V) heuristic found all known optimal solutions in less than 30 seconds.

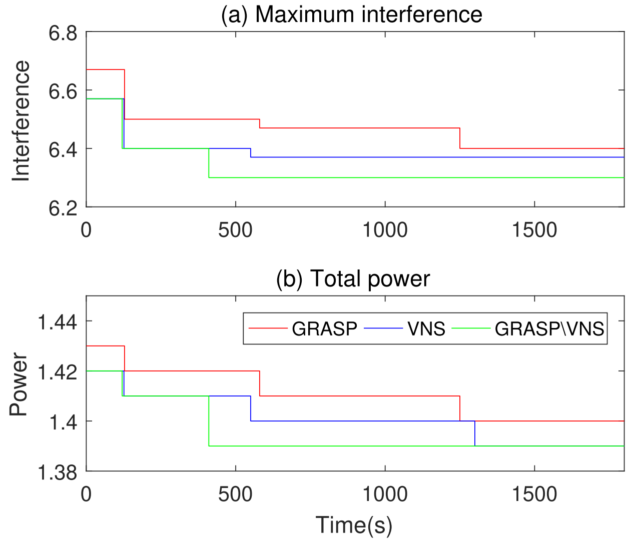


FIGURE 7. **Progressive improvement in solution values along the running time for all heuristics on instance with $|V| = 100$ nodes and problem MMITP.**

Table 10 shows, for each heuristic and for each moderately sized problem dimension $|V| \leq 50$, the average interference values over the optimal solutions found by the formulations FMMI, FMMITP, FMTI and FMTITP. This means that the instances that do not reach an optimal solution by any formulation within six hours of computations were discarded. Columns "opt" of Table 10 display the number of instances with the optimal solution found by each formulation. The results in Table 10 show that the hybrid G^V algorithm is the most effective heuristic when compared to the standard variants of GRASP (G) and VNS (V) algorithms since G^V was the only one able to find all optimal solutions in less than 30 seconds.

2) COMPARISON BETWEEN HEURISTICS ON LARGE INSTANCES

The next set of simulation results presents the comparison between heuristics given real-sized instances with up to 400 sensors for the MMITP and MTITP problems. As described in previous Sections, the solutions of MMITP and MTITP also guarantee power consumption reduction, besides reducing the interference. On the other hand, Table 8 and Figure 6 show that the solutions of the variants MMI and MTI reduce the interference but can give solutions with total power up to 50.73% greater than the optimum.

For the instances with 100, 200, and 400 nodes, Tables 11 and 12 display the power and interference values of the best solution found over five runs for one instance of each type, as the running time limit increases from 30 to 1800 seconds for the MMITP and MTITP problems, respectively. All heuristics continue to improve their solutions as the time limit increases. Algorithm G^V found the best average solution values in most of the situations, as depicted in bold in

TABLE 10. Number of optimal values found by each heuristic within 30 seconds.

	MMI				MMITP				MTI				MTITP			
$\|V\|$	opt	G	V	G^V	opt	G	V	G^V	opt	G	V	G^V	opt	G	V	G^V
10	15	**15**	**15**	**15**	15	**15**	**15**	**15**	15	**15**	**15**	**15**	15	**15**	**15**	**15**
15	15	**15**	**15**	**15**	15	**15**	**15**	**15**	15	**15**	**15**	**15**	15	**15**	**15**	**15**
20	15	**15**	**15**	**15**	15	**15**	**15**	**15**	15	**15**	**15**	**15**	15	**15**	**15**	**15**
25	15	**15**	**15**	**15**	15	**15**	**15**	**15**	15	**15**	**15**	**15**	15	**15**	**15**	**15**
30	14	**14**	**14**	**14**	13	**13**	**13**	**13**	14	**14**	**14**	**14**	14	**14**	**14**	**14**
35	9	**9**	**9**	**9**	7	**7**	**7**	**7**	7	**7**	**7**	**7**	12	11	**12**	**12**
40	5	**5**	**5**	**5**	4	**4**	**4**	**4**	5	**5**	**5**	**5**	4	2	2	**4**
45	2	0	0	**2**	2	1	1	**2**	1	0	0	**1**	1	0	0	**1**
50	2	0	0	**2**	3	0	0	**3**	0	–	–	–	0	–	–	–
total	92	88	88	**92**	89	85	85	**89**	87	86	86	**87**	91	88	88	**91**

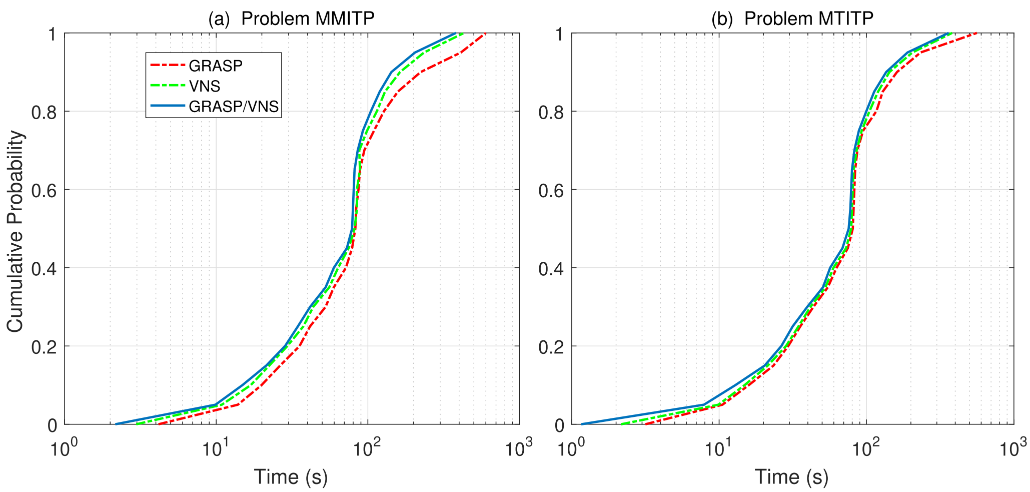


FIGURE 8. Empirical distributions of the time to target-solution-value for heuristics on instances with 200 nodes.

TABLE 11. Average power and interference values for instances with 100, 200 and 400 nodes for problem MMITP.

		Max. interferece			Total power		
$\|V\|$	time (s)	G	V	G^V	G	V	G^V
100	30	6.67	**6.57**	**6.57**	1.43	**1.42**	**1.42**
	120	6.50	**6.40**	**6.40**	1.42	**1.41**	**1.41**
	600	6.47	6.37	**6.30**	1.41	1.40	**1.39**
	1800	6.40	6.37	**6.30**	1.40	**1.39**	**1.39**
200	30	7.50	**7.40**	**7.40**	1.82	**1.80**	**1.80**
	120	7.40	7.37	**7.33**	1.79	1.79	**1.78**
	600	7.40	7.33	**7.27**	1.78	1.77	**1.75**
	1800	7.37	7,27	**7.07**	1.78	1.76	**1.75**
400	30	7.57	7.53	**7.47**	2.88	2.88	**2.86**
	120	7.47	7.47	**7.40**	2.87	2.86	**2.83**
	600	7.43	7.40	**7.33**	2.86	2.85	**2.83**
	1800	7.40	7.33	**7.27**	2.84	2.83	**2.82**

TABLE 12. Average power and interference values for instances with 100, 200 and 400 nodes for problem MTITP.

		Total interferece			Total power		
$\|V\|$	time (s)	G	V	G^V	G	V	G^V
100	30	203.04	**202.73**	**202.73**	1.47	**1.45**	**1.45**
	120	202.25	**202.16**	**202.16**	1.45	**1.44**	**1.44**
	600	201.57	200.75	**200.23**	1.44	1.43	**1,42**
	1800	200.67	200,26	**200.03**	1.43	**1.42**	**1.42**
200	30	214.29	**213.45**	**213.45**	1.82	1.81	**1,80**
	120	212.89	**212.12**	**212.12**	1.80	1.80	**1.79**
	600	212.65	212.07	**211.78**	1.80	1.78	**1.77**
	1800	212.38	211.64	**211.56**	1.79	1.78	**1.77**
400	30	231.25	230.82	**230.79**	2.91	**2.87**	**2.87**
	120	230.53	229.72	**229.44**	2.89	2.88	**2.87**
	600	229,37	228.93	**228.51**	2.88	2.86	**2.85**
	1800	229.17	228.88	**228.46**	2.87	2.85	**2.84**

Tables 11 and 12. Figure 7 illustrates, using MMITP problem, the behavior of each algorithm for one run and one instance with $|V| = 100$ nodes. It shows that better locally optimal solutions are continuously found as the running time increases up to 1800 seconds.

We also compared the three heuristics on one selected instance with $|V| = 200$ using the methodology proposed by [68] and [69]. Two hundred independent runs have been performed for each algorithm and for each instance. Each run was terminated when a solution with a value less than or equal to a given target was found. We use a sub-optimal value chosen such that at least one run of the slowest variant could find it in less than 10 min (600 seconds) of computation time. The empirical probability distributions of the time observed to find a solution value less than or equal to the target are plotted in Figure 8. To plot the empirical distribution for each

TABLE 13. Average solution values when considering power consumption, maximum interference, and total interference for all problems (IP, MMI, MMITP, MTI and MTITP) solved by GRASP/VNS heuristic.

Average power consumption for each problem										
	IP		MMI		MMITP		MTI		MTITP	
$\|V\|$	power	D(%)	power	D(%)	power	D(%)	power	D(%)	power	D(%)
25	**1.36**	**0.00**	2.05	50.73	1.40	2.94	1.45	6.61	1.44	5.88
50	**1.27**	**0.00**	1.78	40.16	1.32	3.94	1.38	8.66	1.35	6.30
100	**1.35**	**0.00**	1.95	44.44	1.39	2.96	1.47	8.88	1.42	5.18
200	**1.69**	**0.00**	2.17	28.40	1.75	3.55	1.80	6.50	1.77	4.73
400	**2.81**	**0.00**	3.02	7.47	2.82	0.35	2.87	2.13	2.84	1.06
Average maximum interference for each problem										
	IP		MMI		MMITP		MTI		MTITP	
$\|V\|$	MaxI	D(%)	MaxI	D(%)	MaxI	D(%)	MaxI	D(%)	MaxI	D(%)
25	5.67	6.38	**5.33**	**0.00**	**5.33**	**0.00**	5.80	8.82	5.73	7.50
50	6.25	25.00	**5.00**	**0.00**	**5.00**	**0.00**	6.07	21.40	6.00	20.00
100	7.27	14.85	**6.33**	**0.00**	**6.33**	**0.00**	6.47	2.21	6.47	2.21
200	7.33	3.67	**7.07**	**0.00**	**7.07**	**0.00**	7.27	2.83	7.27	2.83
400	7.83	6.82	**7.27**	**0.00**	**7.27**	**0.00**	7.33	0.82	7.33	0.82
Average total interference for each problem										
	IP		MMI		MMITP		MTI		MTITP	
$\|V\|$	TotalI	D(%)	TotalI	D(%)	TotalI	D(%)	TotalI	D(%)	TotalI	D(%)
25	83.00	2.63	112.67	39.32	83.53	3.29	**80.87**	**0.00**	**80.87**	**0.00**
50	153.75	1.97	223.50	48.23	152.30	1.00	182.50	21.03	**150.78**	**0.00**
100	207.36	3.66	214.39	7.18	213.58	6.77	201.35	0.66	**200.03**	**0.00**
200	216.35	2.26	223.64	5.70	221.82	4.85	214.69	1.48	**211.56**	**0.00**
400	232.68	1.85	239.35	4.76	233.68	2.28	**227.05**	**0.00**	228.46	0.62

Algorithm 5 High Level Pseudo-Code of Function `Neighborhood_Change` (p, p'', w)

Require: Incumbent solution p, local search solution p'' and the current neighborhood w.
Ensure: Updated incumbent solution p and current neighborhood w.

```
if an improvement in objective function value f(.) is obtained, f(p'') < f(p) then
    the incumbent solution is updated, p ← p'';
    w is returned to its initial value, w ← 1;
else
    w is updated to the next neighborhood, w ← w + 1;
end if
return the current incumbent solution p and the updated current neighborhood w are returned, p, w;
```

Algorithm 6 High Level Pseudo-Code of GRASP

Require: Node set V, arc weights d^{ε}_{uv} for any $u, v \in V$, *Stop_Condition* and parameter α.
Ensure: Best known solution p^*

```
the set of transmission powers which stores the best known solution is initialized empty, p* ← Ø;
the objective function value of an empty set of transmission powers is initialized with a very large value, f(p*) ← ∞;
while Stop_Condition is not met do
    finds a greedy randomized solution p, p ← Greedy_Randomized_Construction (V, d^ε_uv, α);
    explores the neighborhood of the current solution p attempting to find a local optimum p' with reduced total power
    consumption, p' ← Local_Search(p);
    if the solution found by local search p' improves upon the best previously found solution p*, f(p') < f(p*) then
        the best solution is updated, p* ← p';
    end if
end while
return best power assignment p* is returned;
```

algorithm, we associate a probability $p_i = (i - \frac{1}{2})/200$ with the i-th smallest running time t_i, and we plot the points $z_i = (t_i, p_i)$, for $i = 1, \ldots, 200$.

Figure 8 shows that G^V heuristic is the fastest variant for $|V| = 200$ with the given target solution value. The combination of the GRASP and VNS strategies gives more

Algorithm 7 High Level Pseudo-Code of VNS

Require: Node set V, arc weights d_{uv}^{ε} for any $u, v \in V$, *Stop_Condition*, maximum neighborhood w_{max} to be explored during the search.
Ensure: Best known solution p

finds the greedy solution p, $p \leftarrow$ `Greedy_Randomized_Construction` $(V, d_{uv}^{\varepsilon}, \alpha = 0.0)$;
while *Stop_Condition* is not met **do**
  the neighborhood size parameter w is initialized to 1, $w \leftarrow 1$;
  repeat
    generates a solution p' at random from the w-th neighborhood of the current solution p, $p' \leftarrow$ `Shake` (p, w);
    explores the neighborhood of the current solution p' attempting to find a local optimum p'' with reduced total power consumption, $p'' \leftarrow$ `Local_Search` (p');
    evaluates the new solution p'' given by local search and selects the subsequent neighborhood to be explored or returns w to its initial value if an improvement was obtained, $p, w \leftarrow$ `Neighborhood_Change` (p, p'', w);
  until eighborhood size parameter exceeds the maximum size, $w = w_{max}$
end while
return best power assignment p is returned;

Algorithm 8 High Level Pseudo-Code of Hybrid GRASP/VNS

Require: Node set V, arc weights d_{uv}^{ε} for any $u, v \in V$, *Stop_Condition*, maximum neighborhood w_{max} to be explored during the search and parameter α.
Ensure: Best known solution p^*

the set of transmission powers which stores the best known solution is initialized empty, $p^* \leftarrow \emptyset$;
the objective function value of an empty set of transmission powers is initialized with a very large value, $f(p^*) \leftarrow \infty$;
while *Stop_Condition* is not met **do**
  randomly builds a biconnected graph $G(p) = (V, B(p))$, $p \leftarrow$ `Greedy_Randomized_Construction` $(V, d_{uv}^{\varepsilon}, \alpha)$;
  repeat
    the neighborhood size parameter w is initialized to 1, $w \leftarrow 1$;
    repeat
      generates a solution p' at random from the w-th neighborhood of the current solution p, $p' \leftarrow$ `Shake` (p, w);
      explores the neighborhood of the current solution p' attempting to find a local optimum p'' with reduced total power consumption, $p'' \leftarrow$ `Local_Search` (p');
      evaluates the new solution p'' given by local search and selects the subsequent neighborhood to be explored or returns w to its initial value if an improvement was obtained, $p, w \leftarrow$ `Neighborhood_Change` (p, p'', w);
    until neighborhood size parameter exceeds the maximum size w_{max}, $w = w_{max}$
  until submitted the VNS procedure to the current solution p until no improvement is achieved
  if solution found by the VNS procedure improves upon the best previously found solution, $f(p) < f(p^*)$ **then**
    updated the best solution, $p^* \leftarrow p$;
  end if
end while
return best power assignment p^* is returned;

diversity and intensification. Diversity and intensification improve the probability of finding good solutions in less time. Once the best heuristic was identified, it was applied in the next evaluations.

3) PROBLEM COMPARISON USING THE GRASP/VNS HEURISTIC

In this simulation, we assess the quality of the solutions in terms of their power consumption, maximum interference, and total interference produced by GRASP/VNS heuristic when solving the five different problems (IP, MMI, MMITP, MTI and MTITP). The set of instances with $25 \leq |V| \leq 400$ considered in Table 13 was solved with GRASP/VNS using a fixed amount of time. As the GRASP/VNS heuristic (see Tables 11 and 12) finds better values as more time is given, the time of 30 minutes was used to deal with the trade-off between quality solution and the total test running time (15 instances of each size for 5 different problems; and for 30 minutes there are up to 187.5 hours of computation time).

Table 13 shows the average power consumption, maximum interference, and total interference degradation obtained by each problem solution, with respect to the best average power consumption, maximum interference, and total interference, respectively. The best average values are depicted in bold in Table 13.

The approximate results given by the GRASP/VNS presented in Table 13 behave similarly to the comparison made with optimal solutions shown in Table 8. The solutions of problem IP give minimum power consumption, while solutions of the MMITP and MTITP problems give minimum interference values with lower power consumption. This is a strong indication that the GRASP/VNS heuristic is robust and can be considered a good strategy to find approximate solutions for large problems that cannot be tackled by CPLEX.

Regarding the interference problems, we can observe that the solution of the MMI problem leads to the highest values of power consumption and total interference. In contrast, it is sufficient to insert the power minimization (MMITP problem) to achieve lower power and total interference than MMI. The solution of the MTI and MTITP problems gives, in most cases, low degradation in both power consumption and maximum interference. When GRASP/VNS heuristic solves the IP problem, where interference is not considered explicitly in the objective function, solutions present low interference degradations, regardless of how interference is calculated. Therefore, solutions with minimum total power consumption give low interference values.

VII. CONCLUDING REMARKS

In this paper, we studied the fundamental problem of joint minimization of interference and power in wireless sensor networks. We formulated it as a graph-based topology control problem that aims to minimize a function of receiver interference and total power subject to k-connectivity constraints. Four interference-aware formulations and three metaheuristics were proposed and compared through comprehensive and extensive computational simulations.

We showed that CPLEX expends similar computational effort to solve all formulations. We also found that the joint minimum interference (maximum or total) with minimum power formulations provided solutions with minimum interference values and low power consumption. When considering only minimum interference, the formulation that minimizes the maximum interference without power minimization gives solutions with the highest node power.

The difficulties faced by CPLEX in solving large instances support the need for efficient heuristics capable of finding good approximate solutions for large-size problems in reasonable computation times. Therefore, we evaluated the effectiveness of three meta-heuristics in terms of the tradeoffs between computation time and solution quality. They were able to find good solutions for very large problem sizes with up to 400 nodes. The combination of both GRASP and VNS heuristics in a hybrid GRASP/VNS gives more diversity and intensification, which increases the probability of finding good solutions in less time. The GRASP/VNS systematically found the best lower bounds and optimal solutions.

All optimal and approximate results show the importance of considering power reduction along with interference when minimizing interference. Thus, considering the application of the type of WSN evaluated in this work in scenarios with limited energy resources (the sensors are battery-based), we are preparing an experimental proof-of-concept. Thus, we are designing a network with connectivity based on the Thread protocol, a promising low-rate and low-power wireless protocol that allows the deployment of mesh networks. With a leader router, this open-source technology permits route configuration for communication between end-device nodes, facilitating the deployment of certain scenarios in a specific topology before tests with the optimization provided by our proposal. It is worth mentioning that the proposed strategy is off-line applicable, i.e., knowing the environmental area, the network is optimally designed, configured, and then applied. As future works, we also envision the development of adaptive algorithms aiming at dynamic network configuration adjustments, extending optimization frameworks to incorporate additional constraints such as fault tolerance and energy harvesting. The integration of machine learning for predictive analysis and performance optimization can be an added value.

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