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RESEARCH ARTICLE

Optimizing Urban Parking Utility: Strategic and Operational Planning of Fixed and Mobile EV Charging Services in Hybrid Parking Systems

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ABSTRACT This paper introduces a novel hybrid parking system that integrates stationary charging piles (SCPs) and mobile charging robots (MCRs) to optimize urban parking utility. The model categorizes parking spaces based on the presence of SCPs, considering customer behavior including improper parking. It also introduces an operational algorithm—Earliest Available Device First (EADF)—to manage real-time scheduling of MCRs efficiently. Through strategic planning and operational management, the system aims to enhance social welfare by balancing cost-efficiency with flexible charging solutions. We evaluate our approach based on real-world data, demonstrating how MCRs significantly improve both the strategic and accumulated operational aspects of urban parking facilities. The results showcase the potential of hybrid systems in urban environments, promoting higher utility and cost-effective management.

INDEX TERMS Mobile charging robots, stationary charging piles, urban parking utility, hybrid charging systems, operational optimization, strategic planning.

I. INTRODUCTION

The rising adoption of electric vehicles (EVs), encompassing Battery Electric Vehicles (BEVs) and Plug-in Hybrid Electric Vehicles (PHEVs), reflects their attributes of efficiency, eco-friendliness, and cost-effective maintenance. In 2023, the global count of these vehicles ascended to 14.2 million, a significant 35% climb from the previous year, indicating a robust and growing market [1].

The advent of electric vehicles has spurred a surge in research within this domain, encompassing areas such as charging frameworks [2], [3], [4], wireless charging [5], [6], [7], [8], [9], [10], and charging safty or state monitoring [11], [12]. Conventional charging infrastructures, predominantly Fixed Charging Stations (FCS), are extensively studied and integrated into urban frameworks, aiming to enhance charging efficacy [13], [14], [15], [16], [17], [18]. Sadeghi-Barzani et al. advanced an optimization strategy for strategic placement and sizing of fast charging stations [13]. Luo et al. tailored an optimization model for the optimal allocation of charging stations comprising multi-types of charging facilities [15]. Further, Dong et al. introduced an EV charging pricing strategy to leverage the load flexibility [16]. Arias et al. developed a time-spatial predictive model for EV charging-power demand in urban landscapes [14].

Although fixed charging stations are instrumental and have been widely deployed, they present limitations in terms of location dependency and user convenience. To address these constraints, the concept of "Mobile Charging Stations (MCS)" has been introduced, promising to improve accessibility and operational adaptability for EV charging [19], [20], [21]. This model leverages the mobility of charging

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units, enabling on-demand service to EVs regardless of their location and obviating the need for EV owners to search for fixed charging stations. Important studies in this domain include Huang et al., who introduced a service model for mobile charging based on the nearest-work-next principle, enhancing scheduling efficiency [22]. The dispatching problem of mobile chargers is modeled as a static vehicle routing problem with time windows and is solved by CPLEX in [23]. Cui et al. formulated a location-routing problem for static mobile charging operations, analyzing sensitivity to battery capacity and recharging rates [24]. Raeesi and Zografos's application of vehicle routing problem frameworks to mobile charging [25], and Tang et al.'s simulation-based optimization for mobile charging systems design and evaluation [26].

The introduction of mobile charging stations has indeed augmented the user experience and operational agility in urban parking ecosystems, but these conveniences come at a cost, particularly when they rely on auxiliary vehicles for the deployment of charging devices. This has spurred a new wave of innovation focused on Mobile Charging Robots (MCRs), which offer an even more adaptable approach to charging services within parking facilities [27], [28], [29], [30], [31], [32]. Autonomous by design, equipped with advanced sensors and cameras, MCRs are capable of independently navigating parking structures to connect with and charge EVs, substantially enriching both the user experience and the degree of automation in charging services.

Nevertheless, the discourse on MCRs has predominantly revolved around their technological development, with research concentrating on their charging mechanisms, such as the automated detection of charging ports [27], precise connection capabilities [28], and their mechanical and kinematic designs [29], [30], [32]. Conversely, the operational and strategic frameworks for the deployment of MCRs are areas yet to be fully explored. A holistic review of MCR technology can be found in [33], offering a wide-ranging overview of current advancements.

In the realm of urban planning, the planning and scheduling of charging device allocation are critical components of enhancing parking utility and societal benefits. Studies like [34] have introduced EV charging station planning with realistic mobility constraints, while [21] proposed a charging planning method for shared electric vehicles. Reference [35] discusses planning scheduling policy for electric buses, aimed at reducing costs and meeting bus route constraints. Work by Chen et al. [36] tackles MCS power management within the Internet of Things (IoT) framework to maximize long-term profit, and [37] delves into MCS dispatching algorithms designed to minimize customer wait times. Research by Wang et al. [19] presents an equilibrium model that delineates customer behavior within systems incorporating both fixed and mobile charging solutions. Adak et al. integrated renewable energy sources and EVs into a micro-grid system and analyzed the impact of the stochastic charging/discharging of EVs on the secure and stable operation of the microgrids [38]. They demonstrated that the coordinated control between electric vehicles and renewable energy sources is essential and should be considered in the design of microgrid systems. Additionally, Chauhan and Gupta [39] have devised heuristic scheduling algorithms for mobile charging stations to complement the capacity of fixed stations.

Although insightful for mobile charging station strategies, these studies do not directly translate to the context of MCRs due to their distinctive operational and strategic nuances. One key differential is the co-location of MCRs and Stationary Charging Piles (SCP) within the same parking facility, a factor that significantly influences their joint operation, which is neglected in existing literature. Moreover, while cost-minimization is often a focal point, the overall enhancement of parking utility or social welfare is sometimes overlooked. Presently, there exists a gap in the systematic study of the strategic and operational integration of MCRs to augment societal benefit comprehensively.

This paper introduces an innovative hybrid parking system combining SCPs and MCRs to elevate urban parking utility, as shown in Figure 1. Within this system, parking spaces are bifurcated into two categories based on the presence of SCPs, with customers selecting space according to their specific characteristics. The parking system is depicted as a graph-based model, inclusive of the MCR travel network, and a comprehensive state transformation model to facilitate the optimization of social welfare.

We delve into the optimization challenges pertinent to MCR scheduling during strategic and operational configuration planning, with an overarching aim to amplify overall social welfare while concurrently mitigating investment and operational expenditures. Figure 2 provides an overview of the optimization process.

In the Strategic Planning layer, the objective is to determine the optimal configuration of the hybrid parking system given predefined external factors. The process evaluates all combinations of key factors—such as the number of rows and columns in the parking lot, the number of vertical tracks, and the quantities of stationary charging piles and charging robots—and selects the configuration that maximizes overall social welfare. This optimal configuration is identified through simulation, which incorporates customer characteristics and parking demand data.

During the simulation, the system must assign a charging device to each charging request based on the current system state to maximize expected operational utility. This requirement brings us to the Operational Optimization layer. To address this challenge, we propose the Earliest Available Device First (EADF) algorithm, which aims to enhance operational utility by efficiently allocating charging devices.

The main contributions of this paper are as follows:

- Hybrid Parking System Model: We introduce an innovative model for parking facilities that synergizes stationary charging piles (SCPs) with mobile charging

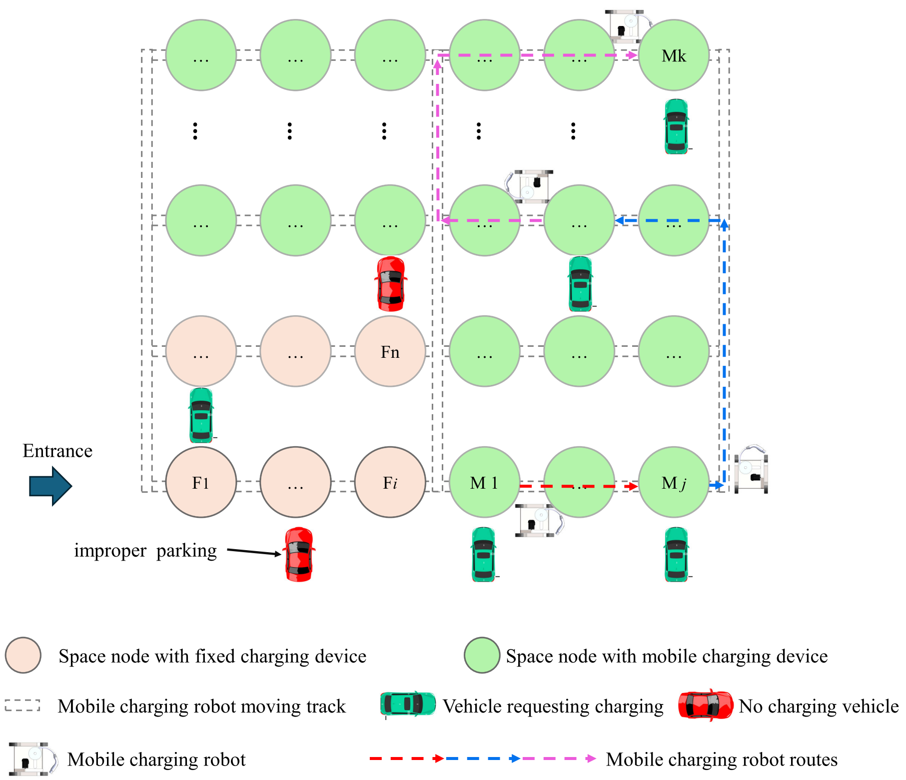


FIGURE 1. **Diagrammatic Representation of the hybrid Parking System. Note that each vehicle should be aligned with the identical node representing the parking space it occupies. To clarify the illustration, vehicle icons are positioned directly beneath their respective parking space nodes. Similarly, mobile charging robot icons are placed above the respective parking space nodes where they are operational or stationed.**

robots (MCRs), targeting both cost-efficiency and adaptability to diverse charging demands within the parking infrastructure.

- Improper Parking Behavior: Acknowledging the real-world complexity, our model integrates the often-ignored aspect of improper parking by non-charging vehicles. We examine how this behavior influences system state transformations and the corresponding impact on optimization procedures.
- Earliest Available Device First (EADF) Algorithm. To solve the operational optimization problem in real-time, we propose the EADF algorithm, an solution that is both efficient and robust, facilitating the prioritization of charging assignments based on device availability. According to the evaluation results, the EADF algorithm can improve the operational utility by up to 1.9% compared to existing algorithms, with its time expense three to five orders of magnitude lower.
- Comprehensive Evaluation of the Framework: We evaluate the EADF algorithm and strategic planning of the proposed framework based on customers characteristiced from a real-world dataset. The evaluation results demonstrate that the proposed EADF algorithm can better optimize the utility while ensuring real-time scheduling. Moreover, the results also affirm the value added by MCRs in elevating the operational and strategic utility of urban parking facilities, with a balanced consideration of associated costs.

The remainder of this paper is organized as follows. Section II presents the hybrid system model and mathematical formulation of the optimization problem. Section III describes the operational dynamics of the system and Section IV presents the algorithm for the operational optimization problem. Section V discusses the strategic planning of hybrid parking system configuration. Section VI presents the evaluation results. Finally, Section VII concludes the paper and outlines directions for future work.

II. SYSTEM MODEL AND PROBLEM DEFINITION

A. MOTIVATION OF HYBRID PARKING SYSTEM

The proposed hybrid parking system combines two distinct types of parking spaces: fixed pile parking space and flexible parking space. The fixed pile parking space (FPPS), as its name suggests, is distinguished by the presence of a stationary charging pile (SCP) situated adjacent to it. The flexible parking space (FPS) is essentially a normal parking space, but incorporating a specialized overhead track network. Each node of this network corresponds to a flexible parking space, intricately connected through a series of tracks. A fleet of mobile charging robots (MCR) is deployed in the network to deliver charging services, each of which can

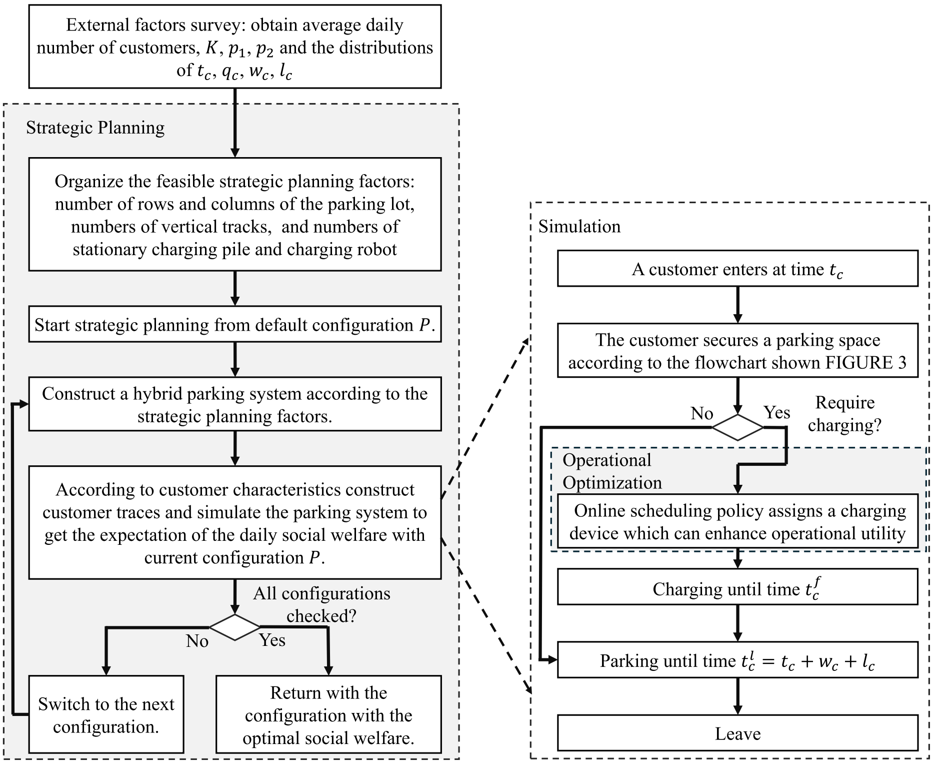


FIGURE 2. Framework for optimizing urban parking utility: strategic and operational planning of stationary and mobile EV charging services in hybrid parking systems.

navigate effortlessly from any one node to another through tracks.

A distinctive innovation within the hybrid parking system is the integration of the vehicle recharging power grid with the track network. This integration allows MCRs to continuously draw power from the grid for both locomotion and charging EVs, eliminating the need for onboard batteries. This design yields three primary benefits: firstly, it allows for a reduction in MCR hardware costs; secondly, it significantly decreases the weight of MCRs, thereby reducing operational costs; and thirdly, it negates the need for MCRs to detach for battery recharging, thus enhancing overall system utilization.

The fixed pile parking space represents a cost-effective solution because of its implicity and lower installation cost relative to more complex mobile charging system. However, its rigidity in spatial configuration may not accommodate irregular parking patterns. As shown in Figure 1, a vehicle without charging demand obstructs a FPPS, rendering the charging pile unusable until the obstructing vehicle departs. Thus, a singular reliance on FPS may not achieve the optimal facility utilization.

On the other hand, flexible parking spaces, equipped with an overhead track network for MCRs, offer a dynamic and adaptable charging alternative. This design ensures that the charging service's availability or efficiency won't be disrupted by improperly parked vehicles, thereby boosting the utilization of facilities and, by extension, the potential for increased revenue from more frequent parking and charging sessions.

However, the advanced nature of FPS entails higher initial and operational costs due to the complexity of the mobile charging infrastructure. Given the unpredictable patterns of improper parking and the variable demand for electric vehicle (EV) charging based on the facility's location, an exclusive reliance on FPS and MCR could result in excessive construction and operational costs, reducing overall efficiency.

Therefore, in alignment with the objective of optimizing urban parking utility, we propose the hybrid parking system that blends FPPS and FPS which supported by a network of track and power grid, and a fleet of MCRs. This composite approach allows the system to cater to the diverse parking and charging requirements of both conventional vehicles and EVs, ensuring effective utilization across various environments. This strategic blend aligns with the goals of enhancing urban parking efficiency and accommodating fluctuating charging demands within diverse urban landscapes.

B. SYSTEM MODEL

1) TRACK NETWORK AND MCR

Notation: in this paper, scalars are represented by letters, e.g., v or V, and vectors are denoted by bold lowercase letters, e.g., $\mathbf{v}$. Sets and matrices are denoted by bold uppercase letters, e.g., $\mathbf{V}$.

We consider a hybrid parking system composed of n^x fixed pile parking spaces each paired with a charging pile, n^y flexible parking spaces, n^z mobile charging robots, and a track network. The parking spaces and the track network are modeled as a directed graph G ($\mathbf{N}$, $\mathbf{A}$), where $\mathbf{N}$ is the set of nodes and $\mathbf{A}$ is the set of links. Each node N_i corresponds to the ith parking space, which can be either an FPPS or an FPS. For any two nodes N_i and N_j, the link A_{ij} represents the shortest track path connecting them, with $d(N_i, N_j)$ symbolizing the travel distance between these nodes. Then, a parking system can be configured by a tuple $P = (G, n^x, n^y, n^z)$.

This system architecture comprises two categories of charging devices: n^x SCPs and n^z MCRs, collectively represented by $\mathbf{E}$. The charging period for both device types, SCP or MCR, correlates directly with the requisite energy, as characterized by the charging rate r.

Parking spaces in G are organized by proximity to the entrance, with FPPSs prioritized over FPSs. Likewise, SCPs precede MCRs within $\mathbf{E}$, each SCP indexed to its respective FPPS for streamlined consistency.

An MCR stationed for charging could potentially obstruct the pathway for others. To tackle such complexity, it is assumed that each MCR is outfitted with a set of auxiliary tracks such that it does not hinder the movement of other MCRs, thus allowing for seamless bypassing with minimal time delay. Additionally, all MCRs are assumed to travel at a constant speed V along the track, thus the travel time t_{ij} for an MCR moving from N_i to N_j is computed as:

$$t_{ij} = d(N_i, N_j)/V \tag{1}$$

2) CUSTOMER AND CHARGING REQUEST

Operational dynamics involve the generation of potential customers $\mathbf{C} = \{\cdots, c, \cdots\}$, who arrive at the facility entrance according to a specific temporal stochastic process. The generation of customers is strongly influenced by external factors related to the facility, such as its geographic location, the penetration rate of electric vehicles, and local traffic conditions. For example, parking facilities near large commercial centers often see peak customer inflow during evening hours. Additionally, areas with higher electric vehicle adoption rates tend to have a larger share of customers needing charging services. Let K denote the fraction of customers who require charging services. For convenience, we will refer to customers requiring charging as DC-customers and those who do not as NC-customers.

Each customer c is characterized by a tuple (t_c, q_c, w_c, l_c, p_1, p_2) reflecting individual characteristics: t_c represents the entering time, q_c denotes the required charging energy amount, w_c specifies the acceptable waiting time window for avoiding additional charging waiting costs, l_c signifies the maximum allowable waiting time, representing the customer's opportunity cost for seeking alternate charging options, p_1 and p_2 are the probabilities of improper parking in two scenarios: (1) at least one FPS is available; (2) no FPS but one FPPS is available, respectively. It is reasonable to assume that $p_1 < p_2$. Note that p_1 and p_2 are meaningful only for NC-customers; q_c and l_c are meaningful only for DC-customers. Therefore, for DC-customers, p_1 and p_2 are set to 0. For NC-customers, q_c and l_c are set to 0 and their w_c denotes the desired parking duration. Different values of w_c and l_c can capture distinct characteristics of DC-customers. For instance, a customer intending to charge during a one-hour dinner may have $w_c = 60$min, with a potential $l_c = 30$min. Conversely, a vehicle nearing battery depletion may corresponds to a small w_c, such as 5 minutes, and a large l_c, due to the difficulty of locating an alternative charging station. We stipulate that the completion time for immediate charging must not surpass the customer's maximum waiting threshold, defined as $t_c^l = t_c + w_c + l_c$, i.e.,

$$\frac{q_c}{r} \leq w_c + l_c, \forall c \in \mathbf{C} \tag{2}$$

Although customer generation is stochastic and varies every day, one can assume the process follows certain distributions. The stochastic process of customer generation is now defined by a distribution tuple of above six factors: F, with the assumption that F is established through extensive market research.

A DC-customer would opt to park at the FPPS closest to the entrance for the quickest possible charging. Should no FPPS be available, the vehicle is directed to the FPS closest to the entrance, followed by the submission of a charging request $R_c = (t_c, q_c, w_c, l_c, N_c)$, where N_c is c's parking space. Next, the hybrid parking system tries to allocate an MCR to serve the customer. The charging finish time t_c^f for R_c should not surpass the waiting limit t_c^l:

$$t_c^f \leq t_c^l \tag{3}$$

If no MCR can meet this timing requirement, the request R_c is declined, and the customer c exits the parking system. Conversely, if the timing constraint is manageable, an appropriate MCR is designated to R_c. However, this MCR might not proceed to N_c instantly, as it needs to fulfill any prior charging requests first.

C. PROBLEM DEFINITION

The goal of this paper is to optimize urban parking utility through the proposed hybrid parking system. Now, we first introduce the formal mathematical definition of utility.

For customers not receiving charging services—either NC-customers or those with rejected requests—the utility u_c is 0. If the request R_c is served, the utility u_c stems from the customer's opportunity waiting cost. If charging completes within the acceptable waiting time ($t_c < t_c^f \leq t_c + w_c$), u_c is a constant l_c. Otherwise, u_c is determined by the reduction in waiting time. Finally, we have:

$$u_c = \begin{cases} 0, & \text{if } q_c = 0 \text{ or } t_c = t_c^f \\ l_c, & \text{if } t_c < t_c^f \leq t_c + w_c \\ t_c^l - t_c^f, & \text{if } t_c + w_c < t_c^f \leq t_c^l \end{cases} \tag{4}$$

Here, $q_c = 0$ identifies a NC-customer, and $t_c = t_c^f$ signifies a rejected R_c.

The optimization of our hybrid parking systems is divided into two layers: operational scheduling and strategic planning. In the operational layer, given the system configuration P and a customer trace $\mathbf{C}$, the goal is scheduling the MCR fleet to fulfill the charging request, maximizing the operational utility as the customers's utility minus operational costs:

$$\max\ \theta \sum_{c\in\mathbf{C}} u_c - \gamma \cdot D^{mcr} \tag{5}$$

where *theta* is the value of time, D^{mcr} is the total mileage accumulated by all MCRs and γ denotes the cost per unit of meter. Consequently, $\gamma \cdot D^{mcr}$ embodies the operational cost. In the strategic layer, given the customer generation distribution tuple F, the aim is to find the optimal system configuration P which maximizes the average daily social welfare:

$$\max\ \underset{\mathbf{C}\sim F}{\mathbf{Exp}}\{\frac{\theta}{L}\sum_{c\in\mathbf{C}} u_c - \gamma \cdot D^{mcr}\} - C^f \tag{6}$$

where C^f represents the daily facility capital cost covering the purchase of MCRs and infrastructure development, and L is the duration in days over which the average utility is calculated. The expectation operator $\underset{\mathbf{C}\sim F}{\mathbf{Exp}}(\cdot)$ calculates the expectation for customers $\mathbf{C}$ following F.

III. SYSTEM OPERATION PROCESS

To solve the operational optimization problem (5), an in-depth examination of the operational dynamics is necessary.

A. OPERATIONAL PRINCIPLES

The online operation is driven by customer and MCR events in a dynamic manner. To facilitate the development of an efficient online scheduling algorithm targeting (5), we adopt the following principles to simply the process.

- **Immediate System Response**: The scheduling policy ensures immediate determination of a charging request's viability, including acceptance or rejection based on the availability of an MCR to meet the request's need (3), and, if accepted, the scheduling of start and end times for charging.
- **First-In-First-Out Queue Management**: Accepted requests are queued in a strict First-In-First-Out (FIFO) manner specific to the assigned MCR. This strict queuing discipline prohibits any cancellations or reordering of requests.
- **Temporal Aspect Simplifications**: For analytical simplicity, the model abstracts from the temporal dimensions related to parking space searching, vehicle parking, submission and processing of charging requests, and the preparing and terminating of charging cycles. This simplification allows for a focused examination of the system's scheduling policy and operational dynamics.
- **Dedicated Use of Stationary Charging Piles**: Stationary charging piles are exclusively allocated for the recharging of vehicles positioned in the corresponding, designated parking space. This exclusivity principle restricts the use of these stations to vehicles within their assigned spaces, thereby preventing utilization by vehicles in adjacent or nearby spaces.

Adopting these principles allows for a precise calculation of charging completion time t_c^f, streamlining scheduling and optimizing charging operations in hybrid parking systems.

B. OPERATIONAL STATE

The operation of hybrid parking systems involves the management of two crucial resources: parking spaces and charging devices. System dynamics are propelled by customer actions and MCR activities, which in turn affect the availability of these resources. Online operational state is therefore depicted by resource availability.

Mathematically, the availability of a parking space N_i at any given time t is binary, either be vacant or occupied by a customer. Without loss of generality, we define the parking space available time $\tau_i^p(t)$.

$$\tau_i^p(t) = \begin{cases} t, & \text{if } N_i \text{ is vacant at time } t, \\ t^e, & \text{if } N_i \text{ is occupied by c at time } t \end{cases} \tag{7}$$

Here, t^e signifies the anticipated exit time for customer c.

Analogously, the availability of a charging device E_i is determined by its queue status, expressed by $\tau_i(t)$. This is set to the current time t if the device's queue is empty, or the charging finish time $t^f_{c^i_{-1}}$ of the last customer c^i_{-1} in the queue if it is not empty:

$$\tau_i(t) = \begin{cases} t, & \text{if the queue of } E_i \text{ is empty at } t, \\ t^f_{c^i_{-1}}, & \text{if the queue of } E_i \text{ is not empty at } t \end{cases} \tag{8}$$

The state of hybrid parking systems should encapsulates the dynamic availability of the system's resources and thus is defined by the tuple $(\mathbf{Z}^{fpps}(t), \mathbf{Z}^{fps}(t), \mathbf{Z}^{dev}(t))$, where $\mathbf{Z}^{fpps}(t)$, $\mathbf{Z}^{fps}(t)$, and $\mathbf{Z}^{dev}(t))$ are vectors that contain the available times of the n^x fixed pile parking spaces, n^y flexible parking space and $n^x + n^z$ charging devices, respectively. Each set provides a snapshot of the current occupancy and expected turnover of parking or charging units, with the ordering within these sets reflecting the corresponding arrangement of parking and charging units in $\mathbf{N}$ and $\mathbf{E}$, ensuring consistent system representation.

Denote the system operational state before an event occuring at time t as $S(t) = (\mathbf{Z}^{fpps}(t), \mathbf{Z}^{fps}(t), \mathbf{Z}^{dev}(t))$, then, after the event, the system state should be updated to $S^+(t) = (\mathbf{Z}^{fpps+}(t), \mathbf{Z}^{fps+}(t), \mathbf{Z}^{dev+}(t))$, which is discussed in the next section.

C. OPERATIONAL STATE TRANSFORMATION

Customer events fall into categories such as entering, parking, requesting charging, waiting for charging devices, charging and departure. Owing to temporal simplification, entering, parking, and requesting charging are considered

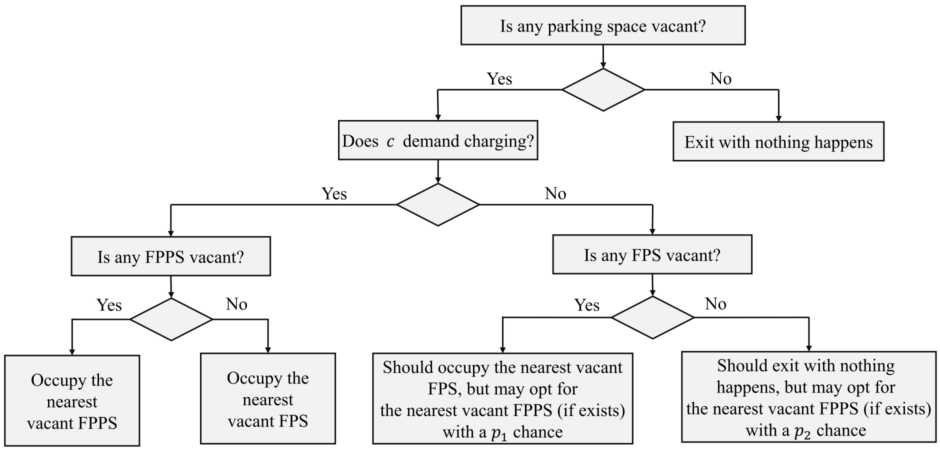


FIGURE 3. Decision-Making flowchart for vehicle parking in urban hybrid parking systems.

simultaneous. Following a scheduling decision, immediate system response allows for swift determination of waiting, charging, and departure activities. Consequently, the system state could be recalculated instantly after the customer enters. The focus, therefore, is on the state transition triggered by the arrival event, synthesizing a unified entry event within the operational framework.

As detailed in the system model, a customer entering at time t_c may either secure a FPPS, or a FPS, or exit without parking. The decision flowchart is outlined in Figure 3. If a customer exits without parking, the system state $S(t_c)$ remains unchanged; otherwise, it updates to incorporate the new parking and charging allocations.

1) CUSTOMER WITHOUT CHARGING REQUEST

In this case, the parking duration of customer c is w_c.

Case 1: If an FPPS is selected, it is clear that set $\mathbf{Z}^{fps}$ is unchanged, i.e., $\mathbf{Z}^{fps+}(t_c) = \mathbf{Z}^{fps}(t_c)$. The selected FPPS must be unoccupied and nearest to the entrance. The vacant spaces at time t_c are depicted by the set $\mathbf{N}^v(t_c) = \{N_i|\mathbf{Z}_i^{fpps}(t_c) \le t_c\}$. As previously noted, the order of nodes in $\mathbf{Z}^{fpps}$ are consistent with the arrangement of parking spaces in $\mathbf{N}$, i.e., sorted by their proximity to the entrance. Consequently, $\mathbf{N}^v(t_c)$ adheres to this order, presenting the nearest available FPPS as $\mathbf{N}^v(t_c)[0]$. The index of $\mathbf{N}^v(t_c)[0]$ in $\mathbf{Z}^{fpps}$ and $\mathbf{N}$ are the same, denoted as $I^\dagger$. Then we have the updated availability set for FPPSs:

$$\mathbf{Z}^{fpps+}(t_c) = Modify(\mathbf{Z}^{fpps}(t_c), I^\dagger, t_c + w_c). \tag{9}$$

Here, $Modify(Z, i, a)$ is a operator that modifies the value of the ith element in Z to a while other elements unchanged, i.e., $Modify(Z, i, a) = a \times \mathbf{e}_i^{|Z|} + Z \odot (\mathbf{1} - \mathbf{e}_1^{|Z|})$, where $\mathbf{e}_i^{|Z|}$ is a $|Z|$-dimension base vector whose elements are all zeros except that the ith element is 1 and $\odot$ denotes the element-wise product of two vectors.

Since a SCP is associated with each FPPS, its available time also needs updating. Recall that charging piles in $\mathbf{E}$ are sorted identically to parking spaces in $\mathbf{N}$, thus $E_{I^\dagger}$ is the associated stationary charging pile. Then,

$$\mathbf{Z}^{dev+}(t_c) = Modify(\mathbf{Z}^{dev}(t_c), I^\dagger, t_c + w_c) \tag{10}$$

Case 2: If an FPS is chosen, the state of all FPPSs and charging devices are unchanged, i.e., $\mathbf{Z}^{fpps+}(t_c) = \mathbf{Z}^{fpps}(t_c)$, $\mathbf{Z}^{dev+}(t_c) = \mathbf{Z}^{dev}(t_c)$. In this case, the vacant space set is $\mathbf{N}^v(t_c) = \{N_i|\mathbf{Z}_i^{fps}(t_c) \le t_c\}$. Following the same logic, the selected FPS is $\mathbf{N}^v(t_c)[0]$ and the corresponding node index in $\mathbf{Z}^{fps}$ is also denoted as $I^\dagger$. Similarly, we have:

$$\mathbf{Z}^{fps+}(t_c) = Modify(\mathbf{Z}^{fps}(t_c), I^\dagger, t_c + w_c) \tag{11}$$

2) CUSTOMER WITH CHARGING REQUEST

In this case, we first calculate the charging completion time, as it may effect the departure time of the customer. Same notations in previous analysis are adopted here.

Case 3: For the FPPS case, $\mathbf{Z}^{fps+}(t_c) = \mathbf{Z}^{fps}(t_c)$, and $E_{I^\dagger}$ is the associated stationary charging pile. Under the temporal simplification assumption and dedicated use principles, vehicles begin charging immediately. The charging finish time of customer c then yields:

$$t_c^f = t_c + \frac{q_c}{r} \tag{12}$$

Then, consider the expected parking time w_c, we can predict the departure time and update $\mathbf{Z}^{fpps}$ as:

$$\mathbf{Z}^{fpps+}(t_c) = Modify(\mathbf{Z}^{fpps}(t_c), I^\dagger, t_c + max(\frac{q_c}{r}, w_c)) \tag{13}$$

Consequently, the available time of $E_{I^\dagger}$ is updated:

$$\mathbf{Z}^{dev+}(t_c) = Modify(\mathbf{Z}^{dev}(t_c), I^\dagger, t_c + max(\frac{q_c}{r}, w_c)) \tag{14}$$

Case 4: For the FPS cases, $\mathbf{Z}^{fpps+}(t_c) = \mathbf{Z}^{fpps}(t_c)$, and customer c must be assigned an MCR E_k, with $n^x < k \le n^x + n^z$. Following FIFO scheduling, each MCR maintains a queue for charging tasks. Let N_i^k and q_i^k denote the target node and energy requirement of ith request in E_k's queue before the scheduling event. Post-scheduling, a new request R_c is appended to E_k's queue, extending E_k's availability by the travel and charging times, resulting in:

$$\mathbf{Z}^{dev+}(t_c) = \mathbf{Z}^{dev}(t_c) + (d(N_{-1}^{k}, N_c)/V + q_c/r)\mathbf{e}_k^{n^x+n^z} \quad (15)$$

where index -1 means the last charging request before scheduling. If the queue is empty before the scheduling event, N_{-1}^{k} denotes the parking space where E_k stationed at t_c. After updating $\mathbf{Z}^{dev+}(t_c)$, we can directly obtain the charging finish time for customer c.

$$t_c^f = Z_k^{dev+}(t_c). \quad (16)$$

Finally, the $\mathbf{Z}^{fps}$ is updated as:

$$\mathbf{Z}^{fps+}(t_c) = Modify(\mathbf{Z}^{fps}(t_c), I^{\dagger}, max(t_c^f, t_c + w_c)) \quad (17)$$

Finally, to summarize the above 4 cases, for customer c entering at time t_c, the parking system state $S = (\mathbf{Z}^{fpps}, \mathbf{Z}^{fps}, \mathbf{Z}^{dev})$ before the event can be updated to $(\mathbf{Z}^{fpps+}, \mathbf{Z}^{fps+}, \mathbf{Z}^{dev+})$ after the event.

$$\mathbf{Z}^{fpps+} = \begin{cases} Modify(\mathbf{Z}^{fpps}, I^{\dagger}, t_c + w_c), & \text{Case 1} \\ \mathbf{Z}^{fpps}, & \text{Case 2 and 4} \\ Modify(\mathbf{Z}^{fpps}, I^{\dagger}, t_c + max(\frac{q_c}{r}, w_c)), & \text{Case 3} \end{cases}$$

$$\mathbf{Z}^{fps+} = \begin{cases} \mathbf{Z}^{fps}, & \text{Case 1 and 3} \\ Modify(\mathbf{Z}^{fps}, I^{\dagger}, t_c + w_c), & \text{Case 2} \\ Modify(\mathbf{Z}^{fps}, I^{\dagger}, max(t_c^f, t_c + w_c)), & \text{Case 4} \end{cases}$$

$$\mathbf{Z}^{dev+} = \begin{cases} Modify(\mathbf{Z}^{dev}, I^{\dagger}, t_c + w_c), & \text{Case 1} \\ \mathbf{Z}^{dev}, & \text{Case 2} \\ Modify(\mathbf{Z}^{dev}, I^{\dagger}, t_c + max(\frac{q_c}{r}, w_c)), & \text{Case 3} \\ \mathbf{Z}^{dev} + (d(N_{-1}^{k}, N_c)/V + q_c/r)\mathbf{e}_k^{n^x+n^z}, & \text{Case 4} \end{cases}$$

IV. ONLINE SCHEDULING POLICY OF MCRS

A. REQUIREMENTS OF THE SCHEDULING POLICY

When a DC-customer initiates a charging request, the online scheduling policy should designate an appropriate charging device. Theoretically, an optimal scheduling policy would enhance operational utility by considering both immediate and future customer demands, as shown in the following optimization problem:

$$\max_{\mathbf{C}^{fu} \sim F} \ \mathbf{Exp}\,\{U(S(t_c), \mathbf{C}^{fu}, \mathbf{b})\} \quad (18)$$

$$\text{s. t. } U(S(t_c), \mathbf{C}^{fu}, \mathbf{b}) = \sum_{i=0}^{W} u(S(t_i), c_i^{fu}, b_i) \quad (19)$$

where $U(S(t_c), \mathbf{C}^{fu}, \mathbf{b})$ is the cumulative operational utility of sampled future traces $\mathbf{C}^{fu}$ and $u(S(t_i), c_i^{fu}, b_i)$ represents the incremental utility achieved when a charging device E_{b_i} is allocated to c_i^{fu} under the system state $S(t_i)$.

Although this approach is theoretically optimal, the indeterminate and variability in future customers generation pose a serious challenge to it. Predicting the influence of current allocations on future demands typically involves simulating numerous potential customer traces, a method impractical for real-world application due to:

- The necessity to solve complex sub-problems for each sampled customer trace, constituting a computational challenge comparable to NP-hard problems.
- The requirement to process extensive simulations for an array of potential customer traces, which is computationally demanding.
- Observations from systematic trials that reveal the utility for future customers is sensitive to the variation in customer traces, leading to unreliable future utility estimates in certain instances. This constraint compromises the robustness since the actual future customer generation may differ from those forecasted in simulation.

For practical application, the scheduling policy must prioritize low computational complexity to ensure immediate system response. Additionally, the assignment process should also be aimed at boosting the operational utility of the hybrid parking system and maintain robustness across various system scenarios. Considering the real-time and robustness requirements of the scheduling policy, we propose an online scheduling policy selecting the mobile charging robot which can start charging the customer the earliest.

Algorithm 1 Earliest Available Device First Policy

Input: $G(\mathbf{N}, \mathbf{A})$, $S(t_c)$, c, N_c
Output: b_c

if c is a NC-customer **then**
  return $\emptyset$
end if
if N_c is a FPPS **then**
  return the index of N_c's charging pile
end if
$b_c \leftarrow \emptyset$, $t_{best}^{start} \leftarrow$ inf
for i in $\{1, \cdots, |\mathbf{E}|\}$ **do**
  if E_i is an MCR **then**
    if $t_c^{start}(k) + q_c/r > t_c^l$ **then**
      continue
    end if
    if $t_c^{start}(k) < t_{best}^{start}$ **then**
      $t_{best}^{start} \leftarrow t_c^{start}(k)$
      $b_c \leftarrow i$
    end if
  end if
end for
return b_c

B. EARLIEST AVAILABLE DEVICE FIRST POLICY

The idea behind the Earliest Available Device First (EADF) policy is trying to balance the utilization of all mobile charging robots. An MCR with an earier available time is more likely to be less utilized than the others. Meanwhile,

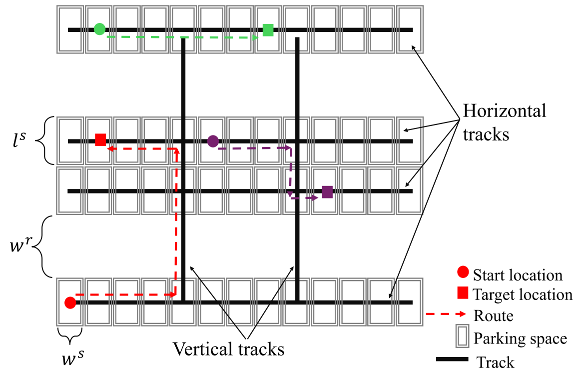


FIGURE 4. The adopted type of track network layout integrated in hybrid parking systems for mobile charging robots.

serving the customer as soon as possible is also beneficial for optimizing the operational utility. By adopting this approach, the policy seeks a near-optimal solution while balancing service promptness and MCRs utilization.

Given the current state of the system and the characteristics of c, the start time that a charging device E_k can serve c is calculated as below, which is a variation of (15):

$$t_c^{start}(k) = Z_k^{dev+}(t_c) + d(N_{-1}^k, N_c)/V \tag{20}$$

Its calculation is based on $Z_k^{dev+}(t_c)$, the anticipated charging finish time of the last request in E_k. The policy selects the charging device that can start charging c the earliest. The pseudo code of the policy is shown in Algo. 1.

The algorithm first checks if the customer is a NC-customer. If so, the algorithm returns an empty set. If the customer parks at a FPPS, the algorithm returns the index of the charging pile at the FPPS. Otherwise, the algorithm iterates through all the charging devices and selects the charging device with earliest available time $t_c^{start}(k)$ to start charging the customer. The algorithm returns the index of the selected charging device. If no charging device can meet the customer's waiting limit t_c^l, the algorithm returns an empty set.

Lem. 1: The time complexity of the Earliest Available Device First policy is $O(n)$.

Proof:

The algorithm iterates through all the $|\mathbf{E}|$ charging devices to find the one with the earliest available time. Then, the scale is related to the number of charging devices and it is clear that the time complexity is $O(n)$. ■

V. STRATEGIC PLANNING OF HYBRID PARKING SYSTEMS

Given a customer distribution F, strategic optimization entails determining the optimal configuration of a hybrid parking system. This includes establishing the total count and arrangement of parking spaces, quantifying the stationary charging pile number n^x, specifying the fleet size n^z of mobile charging robots, and the layout of track network.

A. LAND INVESTMENT

This paper addresses a conventional parking lot arrangement comprising parallel rows of parking spaces, as shown in Figure 4. Denote as n^r and n^c for the number of rows and columns, the aggregate count of parking spaces is $n^r \times n^c$. A road with a width of w^r is posited between alternate rows. A primary component of infrastructural expenditure includes land leasing or acquisition expenses C_{land}^f directly correlating with the parking structure's footprint:

$$C_{land}^f = \alpha(n^r l^s + \lfloor \frac{n^r}{2} \rfloor w^r) n^c w^s. \tag{21}$$

Here l^s and w^s represent the parking space's length and breadth, while α is the daily cost per unit of land.

B. TRACK NETWORK INVESTMENT

Complementary to the above arrangement of parking spaces, a track network is designed, placing a horizontal track above each row, enabling connectivity among spaces within the same row, as illustrated in Figure 4. To bridges the horizontal tracks, n^v vertical tracks are installed, allowing the MCRs to move between rows. The track network investment C_{track}^f is proportional to the track network's aggregate length:

$$C_{track}^f = \beta[n^r * (n^c - 1)w^s + n^v * ((n^r - 1)l^s + \lfloor n^r/2 \rfloor w^r)] \tag{22}$$

Here β symbolizes the daily financial cost per track unit length.

The track configuration suggests that n^v can potentially influence the MCRs' transit efficacy and thus plays a role in strategic framework deliberation. Although additional vertical tracks can diminish MCR transit time and length, they also elevate the C_{track}^f and potentially impact the system's strategic utility adversely. Thus, n^v is retained as a factor in the strategic optimization process.

C. INVESTMENT OF CHARGING DEVICES

Identifying the most advantageous mix of n^x and n^z is key to optimizing the strategic utility of hybrid parking systems, considering the balance between cost-efficiency and adaptability. The daily capital costs for charging devices, represented by C_{dev}^f, is calculated by:

$$C_{dev}^f = \delta n^x + \eta n^z, \tag{23}$$

where δ and η are the daily costs per unit of stationary charging pile and mobile charging robot, respectively.

D. FORMULATION OF THE STRATEGIC PLANNING

Strategic planning aims to maximize the expected daily social welfare for traces shaped by F, denoted as $Q(\mathbf{C}, n^r, n^c, n^x, n^z, n^v)$, which is the accumulated operational utility minus the investment costs for described aspects:.

$$Q = \max_{\mathbf{b}} U(S_{init}, \mathbf{C}, \mathbf{b}) - C_{land}^f - C_{track}^f - C_{dev}^f \tag{24}$$

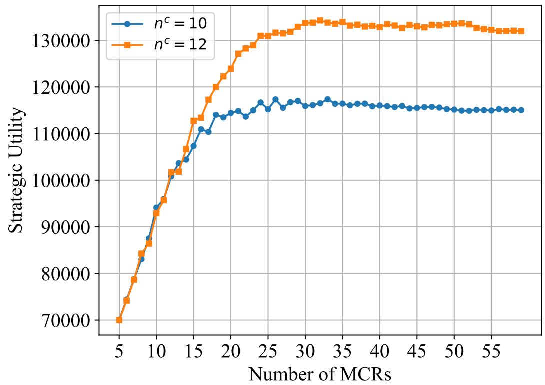


FIGURE 5. **The strategic utility for different n^z and n^c. Note the local peaks in both curves.**

The strategic planning problem is formulated with the following constraints:

$$\max_{n^r, n^c, n^x, n^z, n^v} \quad \underset{\mathbf{C}\sim F}{\mathbf{Exp}}\{Q(\mathbf{C}, n^r, n^c, n^x, n^z, n^v)\} \tag{25}$$

$$\text{subject to} \quad 1 \le n^r \le n^r_{\max}, \; 1 \le n^c \le n^c_{\max}, \tag{26}$$

$$0 \le n^x \le n^x_{\max}, \; 0 \le n^z \le n^z_{\max}, \tag{27}$$

$$1 \le n^v \le n^v_{\max}, \tag{28}$$

$$1 \le n^r * n^c \le n^{total}_{max}, \tag{29}$$

$$1 \le n^x + n^z \le n^r * n^c, \tag{30}$$

$$n^r, n^c, n^x, n^z, n^v \in \mathbb{N}^+. \tag{31}$$

Constraints (26), (27) and (28) ensure that the numbers of parking spaces, charging piles, and tracks are within feasible limits (29) limits the total number of parking spaces. Linear constraint (30) guarantees the number of charging devices does not exceed the parking capacity. All parameters are constrained to be non-negative integers, ensuring a practical and implementable solution.

E. OPTIMIZATION OF STRATEGIC PLANNING

We use simulation-based method to evaluate the expected strategic utility $\underset{\mathbf{C}\sim F}{\mathbf{Exp}}(Q)$ for different configurations of n^r, n^c, n^x, n^z and n^v. The time expense for calculating Q primarily hinges on $|\mathbf{C}|$, the length of the simulated customer trace. Note that although $|\mathbf{E}|$ is also a factor could theoretically influence the time complexity, as noted by Lem. 1, the time expense of EADF algorithm is generally negligible compared to the simulation due to its high efficiency. Therefore, with $|\mathbf{C}|$ held constant, we can assume the time complexity of calculating strategic utility is $\mathcal{O}(1)$. Then, given the search space is a 5-dimensional grid, it is clear that the time complexity of the strategic utility optimization is $\mathcal{O}(n^5)$.

Strategic utility Q is a nonlinear and multimodal function of n^r, n^c, n^x, n^z and n^v, as empirical data suggests. Visual evidence, such as that provided in Figure 5, demonstrates that strategic utility fluctuates with different combinations of n^z and n^c. The existence of local optima in the strategic utility curve renders existing convex optimization algorithms inapplicable.

The strategic planning is an integer linear programming (ILP) problem. Since strategic planning is usually performed offline, the computational complexity is not a major concern. Heuristic algorithms like Genetic Algorithms (GA) or Simulated Annealing are appropriate to obtain near-optimal solutions. Alternatively, given the polynomially bounded search space, a brute-force approach is feasible and guarantees finding the global optimum. Moreover, in brute-force searching, parallelization is practical due to the separability of the configuration evaluations, facilitating a more efficient optimization procedure.

VI. EVALUATION

In this section, we evaluate and validate the proposed operational scheduling optimization algorithms and the strategic-level planning algorithm.

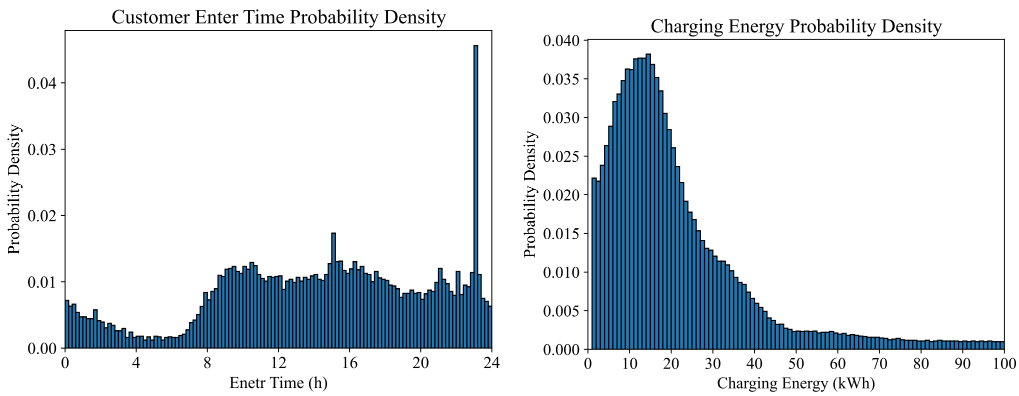
First, we compare the proposed Earliest Available Device First (EADF) policy with two other scheduling algorithms in terms of the accumulated operational utility: (1) the Greedy Scheduling Algorithm (GSA); and (2) the Recursive Approximation Algorithm (RAA) proposed in [26]. Then we verify that the joining of mobile charging robots will improve the strategic utility. Afterward, we explore the sensitivity of this utility enhancement to customer parameters under different settings. Finally, we report the time consumption and scalability of the proposed operational scheduling algorithms.

A. EXPERIMENTAL SETUP

We adopted a real-world dataset from public charging stations in the Beijing area to model the customer generation [26]. The dataset contains 396,077 transactions between January and March 2018. The majority of charging requests, notably peaking at 11 p.m. due to lower electricity rates. Orders are uniformly distributed from 8 a.m. to midnight and drops significantly after 1 a.m. The average charging quantity is 14 kWh and the standard deviation is 10 kWh. Both distributions are shown in Figure 6. The other default parameters adopted in the evaluation are listed in Table 1.

B. COMPARISON OF SCHEDULING ALGORITHMS

We compare the performance of the EADF, GSA and RAA algorithms in terms of the accumulated operational utility. The GSA algorithm, as the name suggests, selects the charging robot that maximizes the immediate utility $u(S(t_c), c, k)$ for the customer at each scheduling instance. The RAA algorithm, introduced by [26], addresses the online scheduling of mobile charging stations. At each scheduling instance, the algorithm selects the optimal charging station by considering both immediate and a limited number ($tracelen^f$) of future customer demands. To reduce computational complexity, the RAA algorithm retains only the best B assignment schemes at each step, narrowing the search space. Consequently, the time complexity of the RAA algorithm is influenced by $tracelen^f$ and B. In our evaluation, we set $tracelen^f = 5$ and $B = 2$.



(a) Distribution of customer enter times (b) Distribution of charging energy quantity

FIGURE 6. Distribution of custoer enter times and charging energy quantity.

TABLE 1. Default configuration for simulation.

Parameter	Value
Parking Lot Configuration	
Parking Spaces Row Number n^r	8
Parking Spaces Column Number n^c	40
Parking Space Breadth w^s	2.5 meters
Parking Space Length l^s	5.5 meters
Road Width w^r	5 meters
Number of Mobile Charging Robots n^z	10
Number of Stationary Charging Piles n^x	10
Number of Vertical Tracks n^v	2
MCR Moving Speed V	2 m/s
Charging Rate r	12 kWh/h
Simulation Parameters	
Customer Number Per Day	800
Simulation Time	4 days
NC-customer Settings	
Mean Parking Time	2 hours
Standard Deviation of Parking Time	1 hour
Minimum Parking Time	1 minute
Maximum Parking Time	12 hours
Improper Parking Probability p_1 (With Vacant FPS)	0.3
Improper Parking Probability p_2 (No Vacant FPS)	0.7
DC-customer Settings	
Proportion of Vehicles Demanding Charging K	45%
w_c and l_c Choices	[[0, 120], [30, 90], [60, 90]] minutes
Probability of w_c and l_c Choices	[0.2, 0.5, 0.3]
Utility Calculation Parameters	
Value of Customer Time θ	60 yuan/hour
Operational Cost Per MCR Unit Distance γ	0.005 yuan/m
Daily Capital Cost Per Unit Of Land α	1.1 yuan/m^2
Daily Capital Cost Per Track Unit Length β	0.2 yuan/m
Daily Capital Cost Per Stationary Charging Pile δ	20 yuan
Daily Capital Cost Per Mobile Charging Device η	40 yuan

Figure 7 shows the accumulated operational utilities obtained by EADF, GSA and RAA at different settings. We can make the following observations: (1) In general, EADF could give higher accumulated operational utility than GSA and RAA in most cases. This is because GSA may overutilize charging robot resources due to the immediate utility maximization. As shown in (4), the customer utility is a constant l_c if the customer's charging finishes before the acceptable waiting time. The EADF adopts the charging start time as the criterion and thus can better balance the utilization of all MCRs. (2) The accumulated operational utility of EADF is strictly negative linear with the moving cost coefficient γ. This is because EADF selects MCR just based on the available time, and is unrelated to the

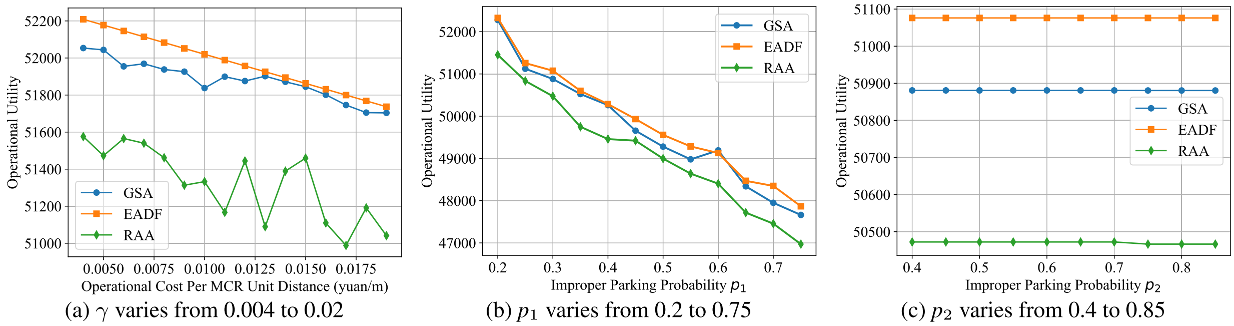


FIGURE 7. Accumulated operational utilities obtained by EADF, GSA and RAA at different settings. Parameters not mentioned are defaults.

moving cost. Therefore the customer utility of EADF is not affected by the moving cost, while the operational utility is decreased linearly as the moving cost increases. (3) The RAA algorithm yields the lowest utilities in most scenarios due to its inability to effectively capture the stochastic nature of customer generation with a limited number of future customers and simulation repetitions. As analyzed in Section IV-A, slight deviations in real future customer can degrade performance. Additionally, the utility fluctuation of RAA is more pronounced compared to EADF and GSA, supporting this analysis. Despite the current settings of $tracelen^f = 5$ and $B = 2$, the RAA algorithm already incurs a significant time cost of approximately 0.5 seconds, as shown in the subsequent subsection. Increasing $tracelen^f$ and B could potentially improve performance, but this would result in much longer processing times, making RAA unsuitable for real-time scheduling applications. (4) The improper parking probability p_1 has a significant impact on the accumulated operational utility while p_2 poses negligible impact on the utility. This is because the scenarios for p_2 is that all FPSs are occupied while at least one FPPS is vacant. As we set that FPPSs are more preferred than FPSs, these scenarios are seldom encountered.

For a clear comparison, Table 2 presents the quantitative differences among the three algorithms. The table reports the average and maximal improvements of EADF over GSA and RAA across three scenarios. Results indicate that EADF achieves an average improvement around 0.3% over GSA and around 1.3% over RAA. The maximal improvements are 0.83% over GSA and 1.9% over RAA. These results further confirm the effectiveness of the EADF algorithm in enhancing the operational utility of the hybrid parking system.

C. STRATEGIC UTILITY IMPROVEMENT

In this section, we verify that adding mobile charging robots in our hybrid parking system will improve the strategic utility. For a fair comparison, we set the number of total charging devices, $n^x + n^z$, to be constant, and the operational cost of MCR is twice that of SCP. For a clear presentation, other parameters are constant, thus in this experiment the strategic utility is a function of n^x and n^z, $Q(n^x, n^z)$. The strategic utility improvement between configurations (n_0^x, n_0^z)

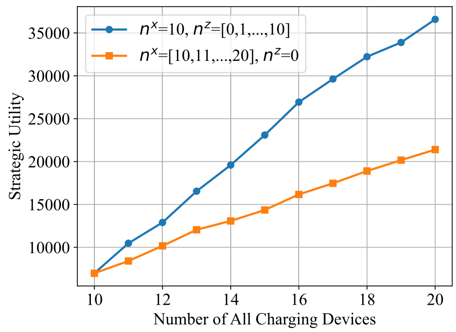


FIGURE 8. The strategic utility improvement when part of the SCPs are replaced by MCRs. Parameters not mentioned are defaults.

and (n_1^x, n_1^z) is defined as

$$\Delta Q(n_1^x, n_1^z, n_0^x, n_0^z) = Q(n_1^x, n_1^z) - Q(n_0^x, n_0^z) \quad (32)$$

Figure 8 shows the strategic utility improvement when the charging devices number $n^x + n^z$ varies from 10 to 20 in two cases.

- Case 1: $n^x = 10$, while n^z varies from 0 to 10.
- Case 2: No MCR, i.e., $n^z = 0$, while n^x varies from 10 to 20. For fair comparison, the daily cost of track network investment C_{track}^f is 0 in this case.

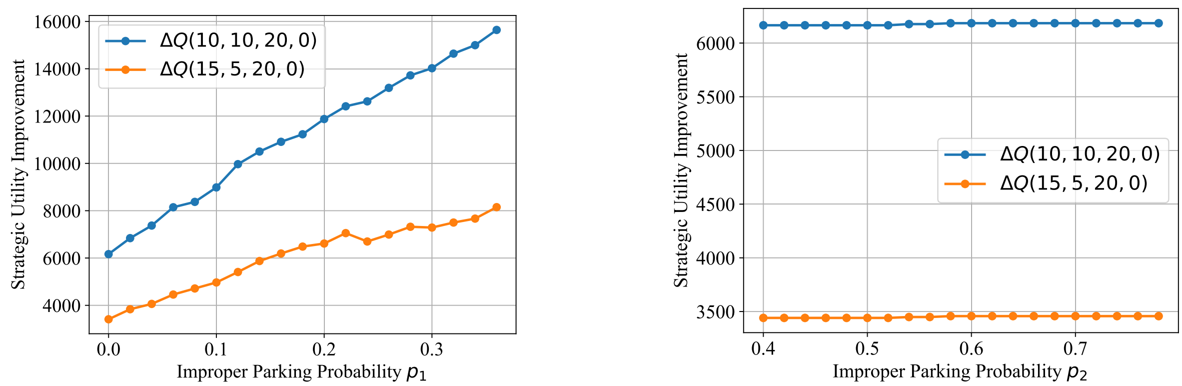
The strategic utility experiences a marked enhancement when mobile charging robots are added to the system. This enhancement amplifies with the addition of more MCRs. The adaptability of MCRs to accommodate improperly parked vehicles and their capacity to serve customers beyond the reach of FPPSs substantially contributes to this improvement.

To further investigate the impact of customer parameters on the strategic utility improvement, we report the strategic utility improvements $\Delta Q(10, 10, 20, 0)$ and $\Delta Q(15, 5, 20, 0)$ when the improper parking probability p_1 varies from 0 to 0.36 and and p_2 from 0.4 to 0.78, respectively. For control, only one of the probabilities is varied while the other is held constant at 0.

The results shown in Figure 9a reveal a direct correlation between strategic utility improvement and p_1: the higher the probability of improper parking probability p_1, the more significant the contribution of MCRs to the strategic utility. On the other hand, Figure 9b demonstrates the strategic utility improvement is almost non-related to p_2, which is consistent

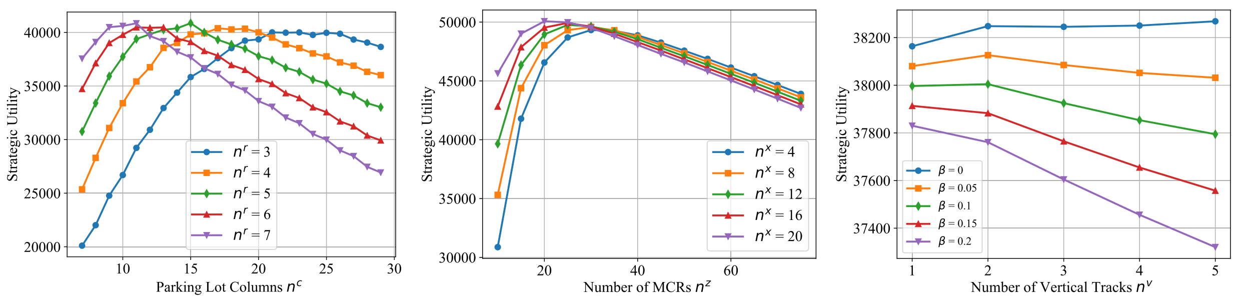
TABLE 2. The improvement of EADF over GSA and RAA.

Scenario	Average Improvement over GSA	Maximal Improvement over GSA	Average Improvement over RAA	Maximal Improvement over RAA
varying γ	0.18%	0.37%	1.27%	1.64%
varying p_1	0.34%	0.83%	1.45%	1.90%
varying p_2	0.38%	0.38%	1.20%	1.21%



(a) p_1 varies in range $[0, 0.36]$ with step 0.02 and $p_2 = 0$ (b) $p_1 = 0$ and p_2 varies in range $[0.4, 0.78]$ with step 0.02

FIGURE 9. The strategic utility improvement of configurations ($n^x = 10, n^z = 10$) and ($n^x = 15, n^z = 5$) to configuration ($n^x = 20, n^z = 0$). Parameters not mentioned are defaults.



(a) Rows and Columns Numbers (b) SCP and MCR Numbers (c) Vertial Track Number and Daily Cost β

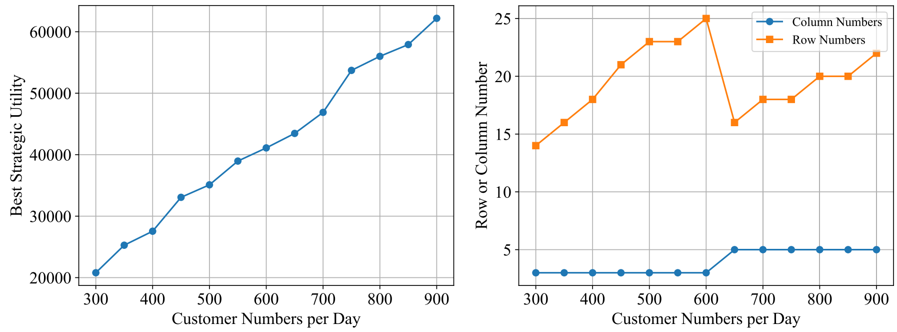
FIGURE 10. The strategic utility for different n^r, n^c, n^x, n^z, n^v. Parameters not mentioned are defaults.

with the previous observation. Another observation is that the configuration with a higher proportion of MCRs, ($n^x = 10, n^z = 10$), exhibits a more pronounced improvement than the one with less MCRs, ($n^x = 15, n^z = 5$), which further strengthens the argument that MCRs play a crucial role in enhancing the strategic utility of hybrid parking systems. Remarkably, even in scenarios where p_1 is 0—indicating no improper parking—the strategic utility still sees an upturn. This improvement arises because MCRs can swiftly handle other charging requests once queued tasks are finished, unlike stationary charging piles that remain occupied until the current customer vacates. Consequently, the inclusion of MCRs contributes to a more dynamic and efficient system, demonstrating their significant role in enhancing the overall strategic utility of parking systems.

D. STRATEGIC UTILITY SENSITIVITY

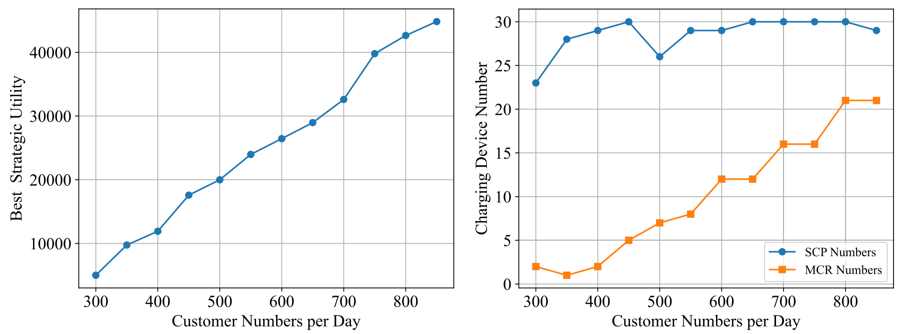
In this section, we investigate how the strategic utility changes when the parameters n^r, n^c, n^x, n^z, n^v vary and analyze the sensitivity of the strategic utility to these parameters. For clearity, in each experiment, we vary one or two parameters while keeping the others constant. The results are shown in Figure 10.

The following observations can be made: (1) larger values of these parameters do not necessarily lead to higher strategic utility. For example, the strategic utility first increases and then decreases with the increase of n^r and n^c. This is because the investment cost of land and track network increases with the increase of n^r and n^c. (2) As Figure 10 shows, the numbers of rows and columns, i.e., n^x and n^z, exhibit a mutual influence on the strategic utility. Actually, they both can be categorized as the parking space resource. A larger n^x results in a smaller optimal n^z where the strategic utility peak is achieved, and vice versa. One can also observe a similar phenomenon in the case studies regarding the number of SCPs and MCRs, since they can be categorized as the charging device resource. (3) For the cases that the daily cost per track unit $\beta \geq 0.1$, the track number n^v has a negative impact on the strategic utility. One reason is that the installing more vertical tracks just reduces the travelling cost of MCRs in part scenarios. Such reduction is trivial compared to the additional investment cost. For the case $\beta \leq 0.5$,



(a) Strategic Utility (b) Optimal Columns and Rows Numbers

FIGURE 11. **The performance of Brutal Searching Strategic Optimization for different customer numbers per day in the search space of $n^c \in [1, 25]$ and $n^r \in [1, 25]$. Parameters not mentioned are defaults.**



(a) Strategic utility (b) SCP and MCR Numbers

FIGURE 12. **The performance of Brutal Searching Strategic Optimization for different customer numbers per day in the search space of $n^x \in [1, 30]$ and $n^z \in [1, 30]$. Parameters not mentioned are defaults.**

we can observe that the strategic utility first increases and then decreases with the increase of n^v. If $\beta = 0$, one can observe a positive correlation between n^v and the strategic utility. Thus, we can conclude that whether adding more vertial tracks will lead to a higher strategic utility depends on the value of β as well as the parking lot size.

E. STRATEGIC OPTIMIZATION

Now, we evaluate the performance of the brutal searching method for strategic optimization. As aforementioned, we parallelize the algorithm by 16 processes to accelerate the searching process. The optimized strategic utility and the best solution found by the algorithm for different number of customer per day are shown in Figure 11 and Figure 12. For clear comparison, in the first case study, the numbers of SCP and MCR are fixed to 20 and 20, respectively and the track number is also set as 1. The method searches parking lot column number n^c and row number n^r in the same region [1, 25]. Similarly, the search space in the second case study is based on SCP number $n^x \in [1, 30]$ and MCR number $n^z \in [1, 30]$, while the row and column numbers are both 19, and the track number is 1.

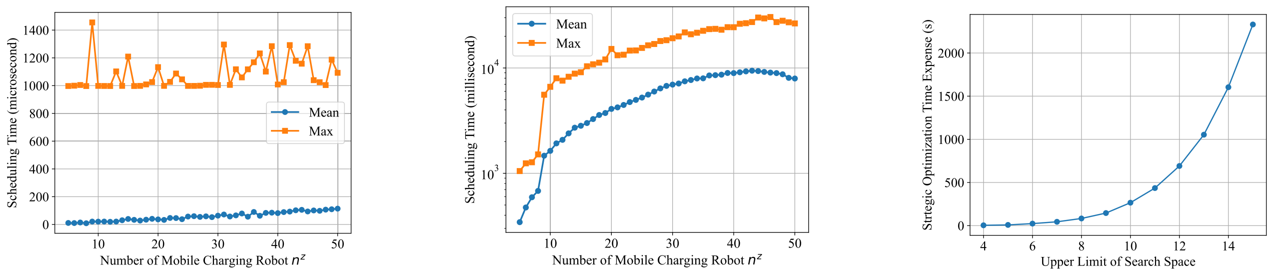
Observe that the optimal strategic utility is generally linear with the number of customers per day in both figures. This is reasonable since more customers mean more charging demands and thus more opportunities for gain utility. In Figure 11b, one can find that the parking row number n^r is preferred to the column number n^c for increasing customer numbers. Such pattern can better match the process of customer finding parking space and charging. In Figure 12b, the searching method prefers SCP to MCR when the number of customers gets larger. This is because SCP is more cost-effective than MCR. The results demonstrate the effectiveness of the proposed strategic optimization method in enhancing the strategic utility of the hybrid parking system.

F. TIME EXPENSE OF ALGORITHMS

In this section, all the simulations are conducted on a HP ZBOOK-G10 laptop with an Intel Core i7-13700H CPU and 32GB RAM. The simulation is implemented in Python 3.10.13 and the time consumption is measured by the Python time module.

First, we report the time consumption of EADF and RAA scheduling algorithm. We vary the number of MCRs from 5 to 50 and report the mean and max time consumption of each scheduling instance. Note that the simulation length is extended to 30 days to obtain more accurate results.

As Figure 13a demonstrates, the mean scheduling time consumption of EADF is roughly linear with the number of MCRs and just around 200 microseconds or shorter. The max scheduling time consumptions are around 1200 microseconds and is still significantly short for real-time scheduling. From Figure 13b, the mean scheduling times of RAA are



(a) The time consumption of EADF for different MCR numbers.

(b) The time consumption of RAA for different MCR numbers.

(c) The searching time of brutal strategic planning v.s. space size.

FIGURE 13. The time consumption of EADF, RAA during online scheduling and brutal searching method for strategic planning.

approximately 500 milliseconds when $n^z \leq 8$, which is about 2500 times longer than EADF. For scenarios with larger n^z, the mean scheduling time of RAA varies from 1 to 10 seconds and the max scheduling time is around 30 seconds. This extended duration is due to the RAA algorithm's need to simulate future customer demands and solve a combinatorial optimization problem at each scheduling instance. In general, the scheduling time of RAA is about three to five orders of magnitude longer than EADF and it is challenging to implement RAA in real-time MCR scheduling.

Finally, we check the time consumption of the brutal searching strategic optimization algorithm with EADF integrated and evaluate if the algorithm is practical. In the case studies, the number of candidates for each parameter n^c, n^r, n^x, and n^z are set to be same and vary from 4 to 15, to represent the growing of search space. The track number is set to 1, as the strategic utility is not sensitive to the track number. The worst case happens when the candidate number is 15 for each parameter, i.e., $15^4 = 50625$ configurations to be simulated in total. The results are shown in Figure 13c.

We can observe that the time expense for the strategic optimization increases rapidly as the search space growing, following the previous analysis that the time complexity of brutal searching is $\mathcal{O}(n^5)$. Nevertheless, the time consumption is still acceptable for strategic optimization. In the worst case, the time consumption is around 40 minutes. Consider that the algorithm is implementation in Python with no optimization and executed on a laptop, we believe that the algorithm is practical after further optimization and branch cutting in a real-world scenario.

VII. CONCLUSION AND FUTURE WORK

This study confirms that the integration of SCPs and MCRs within a hybrid parking system significantly enhances urban parking utility. The Earliest Available Device First (EADF) algorithm proves effective in managing the dynamic operational challenges posed by varying customer demands and parking behaviors. Our evaluations indicate that the proposed hybrid model not only optimizes facility utilization but also aligns with economic efficiency and flexibility in charging services. The results emphasize the critical role of adaptive strategies in future urban infrastructure development.

Future research will focus on new optimization algorithms for scheduling with direct consideration of future customer utility. Moreover, strategic planning for larger-scale scenarios and more diverse urban layouts is also a key area of interest. Additionally, exploring the integration of renewable energy sources into the charging network could further enhance the sustainability and cost-effectiveness of the hybrid parking system.

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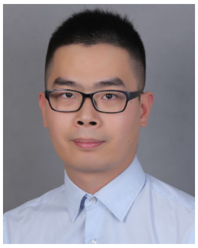

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