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RESEARCH ARTICLE

Subdivision Strategies for Bone Models: A Comprehensive Analysis of Geometric and Visual Quality

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ABSTRACT Bone fracture modeling is a major challenge in medical image analysis and simulation, requiring accurate strategies to faithfully represent complex fracture patterns. This study conducts a comprehensive analysis of three subdivision strategies: approximation, triangulation, and a hybrid approach. The approximation method preserves mesh topology but exhibits visual inconsistencies with non-horizontal fractures. Triangulation accurately represents fractures but alters mesh topology. The hybrid approach balances geometric accuracy and visual fidelity by dynamically adjusting an approximation threshold. This minimizes deviations from the original fracture pattern and maintains visual quality. Using quality metrics, we evaluate these strategies for geometric accuracy, visual fidelity, and mesh topology. Our results indicate that the hybrid approach effectively balances accuracy and visual quality, making it a promising solution for bone fracture modeling. Expert validation and quantitative metrics underscore the importance of tailored approaches for different fracture patterns. This study significantly advances computational models for clinical and research applications, offering enhanced tools for improving the accuracy and realism of bone fracture simulations, ultimately benefiting surgical planning, prosthetic design, and medical training.

INDEX TERMS Bone fracture modeling, fracture pattern representation, geometric quality assessment, mesh subdivision strategies, quality metrics, triangulation techniques, visual realism.

I. INTRODUCTION

The research and practical application of bone models extend across diverse fields, including education, research, surgical planning, and medical device design [1]. In the biomedical and clinical field, bone models play a crucial role in various applications [2]. For research and education, they provide valuable resources for studying and teaching the anatomy, physiology, and biomechanics of bones [3]. They serve as effective visual learning tools, allowing for in-depth exploration of the skeletal system.

In surgical planning, bone models offer precise physical replicas of areas of interest, facilitating meticulous preoperative planning, enhancing surgical precision, and potentially reducing operation time and postoperative complications [4]. In medical device design and testing, such as for prosthetics and orthotics, bone models enable engineers to test functionality and durability in simulated environments that mimic the biomechanical properties of the human skeleton.

Bone models are also fundamental in biomechanical research, allowing for the simulation of mechanical loads, investigation of bone tissue properties, and examination of various skeletal disorders and injuries [5], [6]. The importance of bone models continues to grow, driven by advancements in modeling techniques, simulations, and applications [7], [8], [9]. High-fidelity and personalized digital bone models have significantly improved surgical planning,

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prosthetic design, and biomechanical research by enabling detailed modeling of individual patient anatomy [10], [11], [12].

The increasing use of 3D printing in medicine has further expanded the applications of bone models, enabling the production of physical bone replicas for surgical planning and medical education [13]. As we explore the significance of bone models, the challenges in creating high-quality triangle meshes become apparent. These challenges include balancing mesh accuracy and complexity, ensuring the quality of triangles, and addressing practical issues such as detecting and resolving triangle intersections, managing shape discontinuities, and adapting meshes to shape changes [14], [15].

Despite these challenges, continuous development and refinement of mesh generation techniques and algorithms ensure ongoing progress in this field. This study aims to analyze and propose strategies for subdividing bone models through the creation of high-quality triangle-based meshes. The specific goals include:

- Designing and comparing strategies for subdividing bone models: approximation, triangulation, and a hybrid approach.
- Improving the visual quality of triangle-based meshes, focusing on aesthetics and accuracy in representing bone structures.
- Evaluating the effectiveness and accuracy of different subdivision strategies regarding their impact on mesh quality and their ability to accurately represent bone geometry.
- Establishing an optimal balance between mesh quality and representation accuracy in the context of bone model subdivision.

The resulting models will be used for various applications, including medical simulations, detailed study of fracture morphology, fracture reduction simulations, surgical planning, and the creation of a database of fractured models. These applications will enhance diagnostic and treatment accuracy, facilitate patient education and informed consent through the use of 3D printed replicas, and provide detailed and precise data for research on various bone fractures.

The proposed strategies aim to achieve a subdivision of the original mesh based on various quality criteria. The approximation strategy minimizes distortion of the original mesh by moving cut points to the nearest vertices, which may lead to less accurate representation in complex features. A cut point occurs when there is an intersection between the fracture line and one side of a triangle of the triangle mesh representing the bone. The triangulation strategy maximizes geometric accuracy by incorporating cut points directly as new vertices and dividing existing triangles, creating a more accurate but complex mesh. The hybrid approach merges the benefits of both strategies, using a threshold to decide between approximation and triangulation, aiming to accurately represent geometry while maintaining reasonable mesh complexity.

The following section provides an overview of previous work related to bone fracture modeling and mesh subdivision strategies. Section III details the methodology developed in this study for simulating the subdivision of bone models represented by triangular meshes and section IV presents a thorough analysis of the proposed approximation and/or triangulation strategies applied to triangle meshes representing bone models with various fracture patterns. Finally, section V analyzes the results obtained from each subdivision strategy and the section VI summarize the findings of the study, highlighting the effectiveness of the hybrid approach and the identified optimal quality metrics.

II. PREVIOUS WORKS

There are different techniques to represent bone models in order to perform bone simulations, including point cloud representation, volumetric models, and triangular meshes [16].

Volumetric representation using voxels [16] is commonly employed in the medical domain, leveraging computed tomography images to create 3D representations of bones. Voxel-based models are advantageous as they provide a detailed and structured approach to representing complex anatomical structures, facilitating simulations and analyses that require high spatial accuracy. Additionally, voxel models are useful in finite element analysis (FEA) and computational fluid dynamics (CFD) simulations, where the discrete nature of voxels aids in precise calculations of stress, strain, and fluid flow within bone structures.

Wang et al. [17] introduce an algorithm combining voxel models with the finite cell method (FCM) to efficiently analyze workpiece deformation in five-axis milling. This approach reduces computation time by up to 19 times compared to traditional FEM methods, enabling fast and accurate deformation prediction and optimizing tool path design for thin-walled workpieces, addressing FEM limitations in rapidly changing physical domains.

Triangular mesh models are widely used for their balance between detail and computational efficiency. These models are often applied in surgical planning, prosthetic design, and virtual reality (VR) applications. These meshes can be generated from point clouds or voxel data through surface reconstruction algorithms, providing a continuous surface representation that is easier to manipulate and visualize. Each of these methods has unique advantages and limitations, making the choice of representation dependent on the specific requirements of the medical application.

Studies such as Sas et al. [18] emphasize that triangular mesh representation is considerably more accurate than using voxels, especially when considering superficial deformities in human bones. The process of obtaining a triangular mesh involves reconstructing the bone surface based on segmented computed tomography contours and converting identified vertices into a triangular topology mesh. Advanced segmentation and triangle mesh generation techniques are often employed to obtain representations of human bones from computed tomography scans.

Subsequently, a smoothing process [19] is applied to the bone model to mitigate 3D aliasing effects that might impact operations on the geometric model surface, contributing to generating continuity in regions and preventing abrupt artificial changes in surface curvature.

Fracturing a bone model involves determining the subdivision of the model representing a bone. Wu et al. [20] summarize various techniques that focus on different geometric and topological representations to determine the subdivision of a model. Some of these techniques use tetrahedral, hexahedral, and polyhedral meshes in combination with standard, polyhedral, compound, and finite element discretizations (Figure 1, a). These methods enable interactive subdivisions.

Mitani [21] present an algorithm that allows for the fracture of a model based on manually marked subdivision. It prioritizes simplicity and robustness over precision, with precision typically being the parameter prioritized by algorithms of this nature, despite having a higher implementation cost.

There are various approaches to cutting a model. Taking the cut points generated on the triangular mesh when intersecting the mesh with the fracture pattern as a starting point, several techniques can be employed to indicate how the triangles affected by that cut point will be configured. In some cases, the topology of the triangular mesh may change. Turkiyyah et al. [22] introduces a finite element-based method for obtaining cut points. This involves applying a fracture line on the triangular mesh and intersecting the line with the triangles. The result of this intersection is the fracture points, which will be located on the side of the affected triangle. The triangle should determine how it will be subdivided to incorporate the new vertex into the mesh topology without the subdivision line crossing the triangles but rather encircling them, respecting both the position of the line and the cut points (Figure 1, d).

In another study by Mitani [21], an algorithm is presented that cuts an object using fracture line simplification techniques to facilitate mesh subdivision (Figure 1, b). One technique for simplifying the fracture line involves shifting the generated cut point on the triangular mesh to the nearest vertex. Another technique involves shifting the subdivision segment to the nearest segment, displacing both the cut point and the segment. After applying these techniques, the algorithm subdivides the triangles affected by the subdivision line to ensure that the fracture line does not cross any triangles, encircling them (Figure 1, c).

Caligiana et al. [23] present a novel MR application to simulate surgical procedures with high precision and flexibility. This hands-free, real-time approach allows surgeons to interact with virtual bone models, enhancing preoperative planning and training while preserving geometry quality. This represents a significant advancement in MR for medical simulations.

Berrone et al. [24] propose refinement strategies for polygonal meshes in adaptive VEM discretization to enhance simulation accuracy and efficiency, especially in complex geometrical domains. Their methods improve the quality of mesh elements, applicable to various numerical methods and problems involving polygonal meshes.

Mesh quality metrics [25] are crucial for numerical simulations and scientific computations, helping to ensure accuracy and efficiency by evaluating geometrical properties and minimizing the number of elements. These metrics drive mesh generation and refinement processes, vital for reliable simulations in complex domains. In this context, Berrone and Auria [26] contribute a novel algorithm for quality-preserving polygonal mesh refinement, which enhances the adaptive refinement of high-quality meshes and improves initially low-quality meshes during the refinement process.

The limitations and challenges of previous works on bone fracture modeling are summarized in Table 1. This table provides a comparative analysis of different approaches, highlighting the specific limitations and challenges addressed by each study.

A. MAIN CONTRIBUTION REGARDING PREVIOUS WORKS

Our contribution focuses on the meticulous creation of bone models with precision and fidelity to real bone structures. We commence by employing high-resolution medical imaging techniques, such as computed tomography scans, to capture detailed and accurate representations of bone anatomy. These images undergo meticulous segmentation to isolate bone structures from surrounding tissues. Subsequently, we reconstruct the bone surface by converting segmented contours into a triangular mesh. This reconstruction process incorporates smoothing techniques to minimize aliasing effects and optimize mesh density, striving to strike a balance between detail precision and computational efficiency.

Moreover, we propose a novel method for incorporating cut points into the mesh topology, potentially altering the topology of its triangles. This is achieved through either the subdivision of affected triangles or the displacement of cut points, guided by a quality metric ensuring the optimal triangulation for that specific mesh. This approach enables dynamic adaptation of mesh topology, ensuring an accurate representation of cuts in the bone structure.

To incorporate fracture patterns, we project expertly validated two-dimensional patterns onto the three-dimensional bone model [27]. This validation process is further enhanced by forensic analysis [12], [28], [29]. In a subsequent study [30], advances were made in the projection of these patterns onto three-dimensional models for fracture simulations, increasing their use in medical applications. This iterative approach ensures the accuracy and authenticity of the projected fracture patterns, improving the realism and reliability of fracture simulations in the 3D bone model.

III. METHODS

The method developed in this study enables the simulation of the subdivision of a bone model represented by a triangular mesh. This is achieved by subdividing the model using a fracture pattern that indicates the location for the

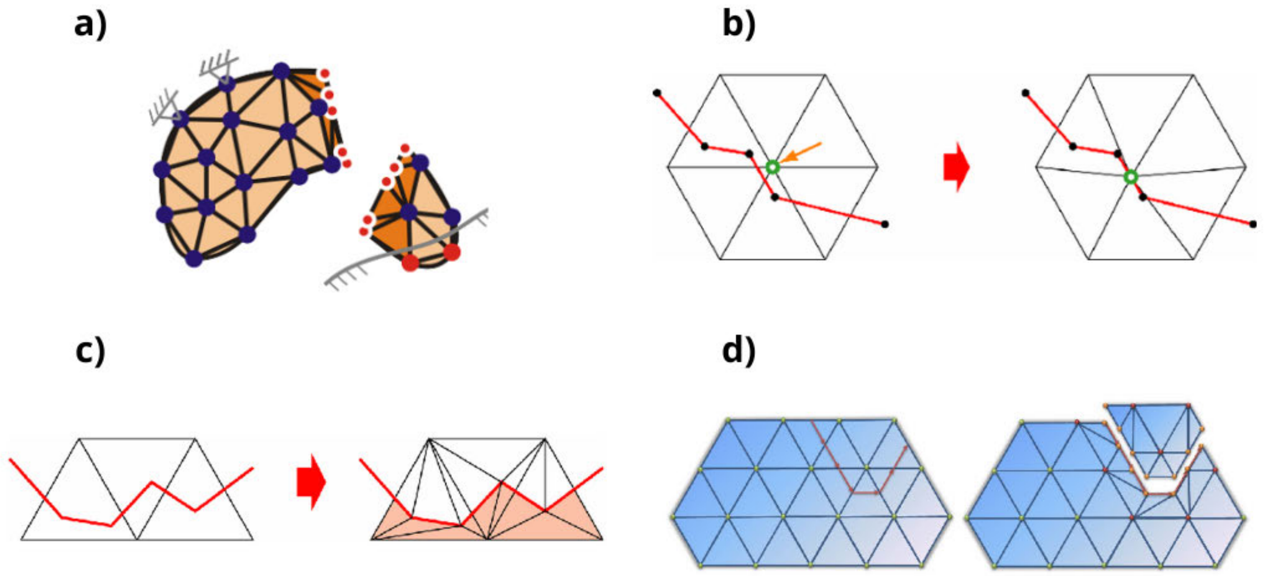

FIGURE 1. Representation of the various cutting strategies upon which we rely. a) Mesh subdivision process proposed by Wu et al. [20]; b) Cutting algorithm based on fracture line simplification by shifting the fracture line by Mitani [21]; c) Cutting algorithm with triangle subdivision by Mitani [21]; d) Method for obtaining cut points proposed by Turkiyyah et al. [22]. The figures have been extracted from the research of Wu et al. [20], Mitani [21] and Turkiyyah et al. [22].

TABLE 1. Summary of previous works on bone fracture modeling.

Reference	Approach	Limitations	Challenges
Xu et al. [16]	Voxel-based representation	Computationally intensive and less accurate for surface details compared to triangular meshes.	Optimizing voxel resolution for balance between detail and computational load.
Wang et al. [17]	Voxel model with finite cell method for deformation prediction	High computational demand for updating stiffness matrices.	Reducing computation time while maintaining accuracy in deformation prediction.
Sas et al. [18]	Triangular mesh representation	Limited in capturing detailed internal bone structures.	Ensuring high-quality mesh generation while preserving geometric details.
Wu et al. [20]	Finite element-based method	Computationally intensive and can introduce errors in cut point determination.	Balancing computational efficiency with accuracy in fracture representation.
Mitani et al. [21]	Manual marking and subdivision	Prone to human error and lacks precision in fracture line simplification.	Developing automated methods to enhance precision and reduce manual effort.
Turkiyyah [22]	Finite element method for cut points	Changes mesh topology, affecting subsequent analysis.	Maintaining mesh integrity while accurately representing complex fractures.
Caligiana et al. [23]	Mixed Reality (MR) techniques for real-time cutting	Requires specialized hardware and software integration; potential high cost.	Ensuring real-time performance and accuracy in complex medical simulations.
Berrone et al. [24]	Polygonal mesh refinement for VEM	Complex implementation and potential computational overhead.	Achieving optimal mesh quality and convergence rates in adaptive refinement processes.
Berrone et al. [26]	Quality-preserving polygonal mesh refinement	Refining good quality meshes while preserving quality; improving poor quality meshes during refinement.	Addressing convergence and optimality in adaptive methods for polygonal meshes.

subdivision, following the recommendations of the study conducted by Jimenez-Delgado et al. [27]. The fracture pattern provides a comprehensive representation of bone fractures, including details such as fracture lines, their orientations, and quantities, offering valuable insights into each part of the bone. Prior to subdivision, the fracture pattern is projected onto the geometric model. The model stores topological information about the location of lines belonging to the fracture pattern and the cut points produced by the intersection of the fracture pattern with the triangles in the triangular mesh.

The pattern projection method [30] used in this work is specifically designed for projecting patterns onto three-dimensional geometric bone models. This process ensures that the fracture patterns contain both geometric information (cut points) and topological information (distribution of lines belonging to the fracture) necessary for the subsequent subdivision of the triangular mesh. The goal is to represent real, fractured, and validated virtual bone models accurately.

The proposed method relies on two-dimensional fracture patterns extracted and validated by experts, along with dedicated tools. The projection procedure involves projecting these two-dimensional fracture patterns onto three-dimensional bone models in the diaphysis area, preserving the topological and geometric information contained in both the fracture pattern and the virtual bone model. This approach offers greater control over the fracture and its projection compared to other representations of fracture patterns, such as textures.

To project the two-dimensional pattern onto the model, a series of steps were followed based on previous work [30]. First, the height for the fracture pattern application is determined and scaled to match the bone thickness. The pattern is then transformed into three dimensions using a supporting cylinder, focusing on the central zone of long bones. The affected area of the bone model is unfolded over the cylinder, projecting the set of triangles forming the fracture area onto a plane. The fracture pattern is overlaid and adjusted to the actual length of the affected area. Finally, the fracture points, where the pattern intersects with the triangles, are determined and seamlessly incorporated into the three-dimensional bone model.

In the subdivision method proposed in this work, various bone models, with or without irregularities on their surface, were employed. To carry out the subdivision of the bone model, it is necessary to calculate cut points. The result produced is a set of cut points defining a contour equivalent to the fracture pattern on the model, which will either lead to new triangles or belong to the geometric location where the model is subdivided in the absence of triangulation.

Section C summarizes the whole process indicated in the method section.

A. STRATEGIES

The proposed method for subdividing triangular meshes consists of three alternatives for the triangulation and/or approximation of the triangles that constitute the mesh representing the geometric model. These strategies are referred to as approximation, triangulation, and hybrid approach.

- The approximation strategy involves bringing the fracture pattern's cut point closer to the nearest vertices of the triangles that constitute the mesh, without generating new triangles.
- The triangulation strategy generates new triangles within the triangles of the mesh, between the cut points and the vertices of the respective triangle.
- The hybrid approach combines the aforementioned strategies, allowing for the approximation of cut points to the vertices of the mesh triangles in some cases and, in others, generating new triangles using the implicated cut points and the vertices of the triangle on which the cut points are located. This results in the creation of new triangles in the triangular mesh, incorporating them into the mesh's topology.

The approximation strategy brings the cut point of the fracture pattern closer to the nearest vertex of the triangular mesh, resulting in no generation of new triangles but rather a displacement of the fracture line belonging to the pattern. The triangulation strategy creates new triangles in the triangular mesh using the cut points with a triangle and its vertices. This strategy ensures that the subdivision of the triangular mesh remains faithful to the original fracture line. Finally, the hybrid approach combines the previous approaches, allowing for the approximation of cut points to the vertices of the triangles in some cases and generating new triangles in other cases, based on a set of established thresholds.

1) APPROXIMATION STRATEGY

The approximation strategy allows determining, from a triangle affected by a cut point and the cut point itself, which vertex of the triangle this cut point should be approximated to, taking into account its proximity. Next, the method is detailed in greater depth. Given a triangle intersected by a fracture line and the cut point(s) on the side(s) of the affected triangle, the closest vertex to that cut point on the side of the triangle is determined, and once identified, the cut point is approximated to that vertex.

In all cases, the cut point is approximated to the closest vertex, where two different cut points on the same triangle can either approximate to the same vertex or different vertices of that triangle (Fig. 2). Given two cut points with the triangle, the considered cases are:

- Approximation of both cut points to the same vertex, as both cut points are close to the vertex.
- Approximation of both cut points to different vertices, as each cut point is close to a vertex. The fracture line adjusts to coincide with the side connecting both vertices.

With this approximation approach, each cut point is approximated to the nearest vertex. As the triangular mesh remains unchanged, there are no alterations in the quality of the fracture zone. This is because the resulting fracture line borders the triangles rather than traversing through them.

In the previous literature [21], the approximation of the fracture line generally involves displacing the entire line towards the nearest vertex, which can compromise geometric accuracy. Our strategy, however, only displaces the cut points located on the sides of the triangle towards the nearest vertex. This means that only a portion of the fracture line (the section between the cut points) is displaced, thereby maintaining the majority of the original fracture line intact and consequently improving the accuracy of the fracture representation.

2) TRIANGULATION STRATEGY

Given a triangle intersected by a fracture line and given the side(s) on which the cut points are located, the cut points are added to the mesh as new vertices, and the affected triangle is divided into two or more triangles, depending

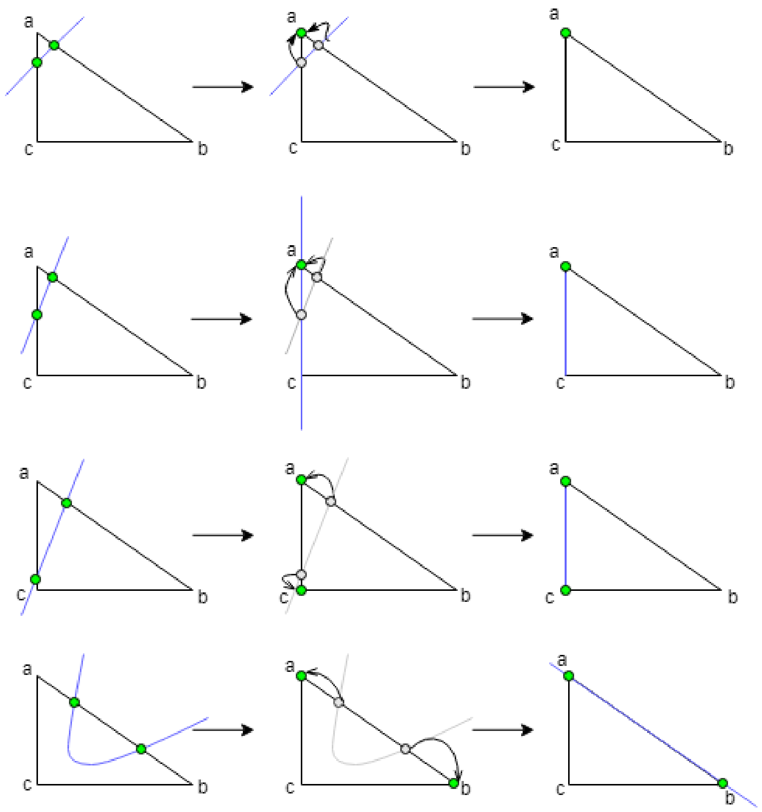

FIGURE 2. Situations considered in the approximation strategy. The first two examples represent the approximation of two cut points to the same vertex. The last two represent the approximation of two cut points to different vertices.

on the triangulation produced in that triangle (Fig. 3). The triangulation process involves creating new triangles from the cut points and the initial triangle.

- **Case 1:** Presents a single cut point due to a cutting line that terminates on one side of the affected triangle. In this case, no triangulation occurs since, even though a cut point appears on one of its sides, this cut will be taken into account in the adjacent triangle, with the subsequent triangulation. This case is not very common, as it is challenging for fracture lines to end on the side of the triangle. However, this can occur because an error bounded by an epsilon has been used in cases where the fracture line ends very close to the side of the triangle. This situation may arise when there are branches in the fracture pattern.
- **Case 2:** A single cut point, but the fracture line continues within the triangle, although it does not end on any other side of the triangle. In this case, a new auxiliary point is created, which will be treated as if it were a cut point, and it will be joined to the real cut point. Once this is done, the auxiliary cut point is connected to the remaining vertices, if possible due to area constraints, and four new triangles will be created to replace the initial one.
- **Case 3:** Both cut points are on the same side of the triangle. In this case, they are joined with the opposite vertex, forming three new triangles.
- **Case 4:** Two cut points on different sides of the triangle. In this case, both cut points are joined, and one of them is connected to the opposite vertex, so that the area of the resulting triangles falls within the limits defined by the user through a threshold. Thus, three new triangles are also obtained.

In the case of the triangulation strategy, the result is the original triangle, or between two and four triangles that replace the affected triangle. The triangles affected by triangulation undergo modifications in their shape, size, and neighborhood, so the quality of the individual triangle and the quality of the subdivision area will be altered. In general, the mesh quality would be compromised with the appearance of smaller and possibly less homogeneous triangles than the originals. Triangulation only occurs if the resulting triangle(s) are valid and non-degenerate. Otherwise, the approximation strategy is used for those cut points.

The previous version of this strategy [22] incorporates both the cut points and the fracture line vertices that are within the triangle. This method can result in high topological complexity and the creation of low-quality triangles. In contrast, our triangulation strategy subdivides the triangle by incorporating only the new cut points, thereby reducing complexity and improving the quality of the generated triangles.

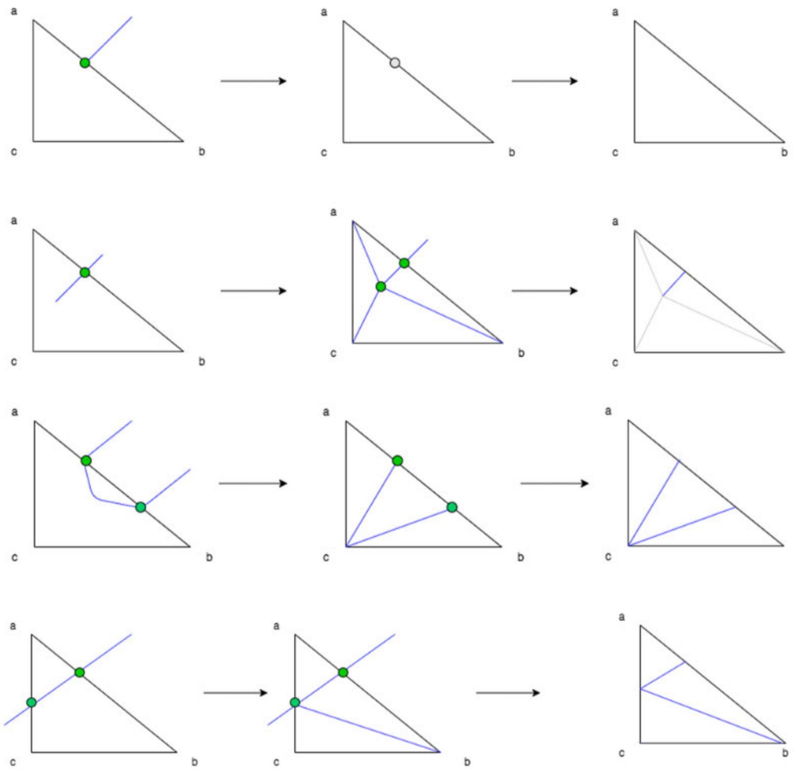

FIGURE 3. Different scenarios in the triangulation strategy from case 1(top) to case 4 (bottom).

3) HYBRID APPROACH

The hybrid approach has been proposed as an innovation in this work. This method dynamically combines approximation and triangulation strategies based on a predefined threshold. This approach distinguishes in which situation it is better to approximate to the nearest vertex, as is performed in the approximation strategy, and in which situations a triangulation strategy is more suitable. In fact, the same triangle can apply the approximation technique for one cut point and the triangulation technique for another.

This approach works with dynamically determined thresholds. These thresholds control, on one hand, the minimum distance between a cut point and its nearest vertex for the approximation situation, and, on the other hand, the minimum area value that a triangle must have to be triangulated. Based on both values, the method decides whether to approximate that cut point to the vertex or to triangulate the affected triangle. This threshold is set by the user at runtime through two parameters in the graphical interface of the application, although a study has been conducted to determine which threshold is more suitable for obtaining a better result.

The following is a description of how to perform the configuration of thresholds.

- **Configuration of thresholds:**
    1) **Minimum distance parameter.** The parameter refers to the minimum distance from the cut point to the nearest vertex. Determined empirically, this value is configured to be small enough to maintain geometric accuracy without unnecessarily increasing mesh complexity.
    2) **Minimum area parameter.** The parameter refers to the minimum area that is considered adequate for a triangle. Also determined empirically, this value is set to avoid creating excessively small triangles that could degrade mesh quality.

A study was conducted with various minimum distance values and minimum area values, evaluating the impact on mesh quality and geometric accuracy. The results of this study have been used to define default threshold values in the application.

B. QUALITY METRICS

The triangulation of an affected triangle, or equivalently, the generation of new triangles from the cut points and the vertices of the triangle, can be carried out in various ways for the same case. If we apply criteria that measure the quality of the resulting triangles to guide the generation of triangles, we can obtain a mesh that represents the resulting fragments with higher quality. A series of quality criteria, extracted from the work of Knupp et al. [31], have been selected to study how each of these quality criteria affects the subdivision of the mesh, trying to choose the best or most suitable criteria for this triangulation, given that this triangulation may differ depending on the order established in the cut points (Fig. 3). In this way, we will try to perform a triangulation that achieves the best average quality value according to one of these criteria.

The following describes the different metrics used to determine the quality of a triangle.

1) QUALITY BASED ON ASPECT RATIO

This metric implements the formula shown below (Eq. 1), where L_{max} is the longest side of the triangle under evaluation, and L_0, L_1 and L_2 are the different lengths of the triangle's sides, with A representing its area.

$$q = \frac{L_{max}\,(L_0 + L_1 + L_2)}{4\sqrt{3}A} \tag{1}$$

The quality values fall within a range of [1, DBL_MAX], where DBL_MAX is the maximum representable finite floating-point value. However, the acceptable range is within [1, 1.3], aiming to be as close as possible to 1, which is the value for an equilateral triangle. Refer to Table 2 for details.

TABLE 2. Quality ranges metrics for aspect ratio, edge ratio, and the ratio between the inradius and circumradius.

Dimension	Value
Acceptable range	[1, 1.3]
Normal range	[1, DBL_MAX]
Full range	[1, DBL_MAX]
q for an equilateral triangle	1

2) QUALITY BASED ON EDGE RATIO

This metric implements the formula provided below (Eq. 2), where L_{max} represents the longer side of the triangle under evaluation, and L_{min} is the shorter side.

$$\begin{aligned} L_{min} &= min(L_0, L_1, L_2) \\ L_{max} &= max(L_0, L_1, L_2) \\ q &= \frac{L_{max}}{L_{min}} \end{aligned} \tag{2}$$

The quality values fall within a range of [1, DBL_MAX], where DBL_MAX is the maximum representable finite floating-point value. However, the acceptable range is within [1, 1.3], aiming for proximity to 1, which is the value for an equilateral triangle. Refer to Table 2 for details.

3) QUALITY BASED ON THE RATIO BETWEEN THE INRADIUS AND CIRCUMRADIUS

This metric is implemented using the formula provided below (Eq.3), where R is the circumradius, and r is the inradius.

$$q = \frac{R}{2r} \tag{3}$$

The quality values fall within a range of [1, DBL_MAX], where DBL_MAX is the maximum representable finite floating-point value. However, the acceptable range is within [1, 3], aiming for proximity to 1, which is the value for an equilateral triangle. Refer to Table 2 for details.

TABLE 3. Quality ranges for the minimum angle of the triangle.

Dimension	Value
Acceptable range	[30º, 60º]
Normal range	[0º, 60º]
Full range	[0º, 360º]
q for an equilateral triangle	60º

TABLE 4. Quality ranges for the maximum angle of the triangle.

Dimension	Value
Acceptable range	[60º, 90º]
Normal range	[60º, 180º]
Full range	[0º, 180º]
q for an equilateral triangle	60º

To calculate the inradius (Eq. 4) and circumradius (Eq. 5), the following formulas are necessary, with A representing the area (Eq. 6) and $||\vec{L_i}||$ the Euclidean norms of the vector L_i to calculate the side lengths of the triangle:

$$r = \frac{2A}{\left\|\vec{L_0}\right\| + \left\|\vec{L_1}\right\| + \left\|\vec{L_2}\right\|} \tag{4}$$

$$R = \frac{\left\|\vec{L_0}\right\| \left\|\vec{L_1}\right\| \left\|\vec{L_2}\right\|}{2r\left(\left\|\vec{L_0}\right\| + \left\|\vec{L_1}\right\| + \left\|\vec{L_2}\right\|\right)} \tag{5}$$

$$A = \frac{1}{2}\left\|\vec{L_0} \times \vec{L_1}\right\| = \frac{1}{2}\left\|\vec{L_1} \times \vec{L_2}\right\| = \frac{1}{2}\left\|\vec{L_2} \times \vec{L_0}\right\| \tag{6}$$

4) QUALITY BASED ON THE MINIMUM ANGLE OF THE TRIANGLE

In order to use this quality metric, the arccosine of each angle of the triangle is calculated and multiplied by $180/\pi$ (Eq. 7). After this, the minimum value among the three resulting values is selected. The resulting range is between 0º and 360º, but the acceptable range is between 30º and 60º, with 60º being the value for an equilateral triangle. For details see Table 3.

$$q = \min_{n \in \{0,1,2\}} \left\{ \arccos\left(\frac{\vec{L_n} \cdot \vec{L_{n+1}}}{\left\|\vec{L_n}\right\| \left\|\vec{L_{n+1}}\right\|} \right) \left(\frac{180^\circ}{\pi} \right) \right\} \tag{7}$$

5) QUALITY BASED ON THE MAXIMUM ANGLE OF THE TRIANGLE

This metric is calculated in the same way as in the previous case, but instead of selecting the minimum resulting value, the maximum value is chosen (Eq. 8). The ranges are the same as in the previous metric (Table 4).

$$q = \max_{n \in \{0,1,2\}} \left\{ \arccos\left(\frac{\vec{L_n} \cdot \vec{L_{n+1}}}{\left\|\vec{L_n}\right\| \left\|\vec{L_{n+1}}\right\|} \right) \left(\frac{180^\circ}{\pi} \right) \right\} \tag{8}$$

C. ALGORITHM AND OUTCOMES FOR THE PROPOSED METHOD OF BONE MODEL SUBDIVISION

The following is an overview of the hybrid-based algorithm required to perform the subdivision method proposed in Section III.

1) **Input:**
    a) CT scan of the bone model.
    b) Previously validated fracture pattern [27].
    c) Minimum distance and minimum area parameters.
    d) Quality metric.
2) **Initialization:**
    a) Model reconstruction and segmentation.
    b) Projection of the fracture pattern on the model [30].
    c) Obtaining cut points by intersection between the fracture pattern and the triangulated grid [30].
3) **Steps:**
    a) **Algorithm steps:**
        i) For each cut point, calculate the distance to the nearest vertex.
        ii) If the distance is less than the minimum distance parameter:
            - Approximate the cut point to the nearest vertex. This ensures that small distances are managed without increasing the mesh complexity unnecessarily.
        iii) If the area of the affected triangle is greater than the minimum area parameter:
            - Perform triangulation by incorporating the cut point as a new vertex. Large triangles can be divided without compromising mesh quality, ensuring better geometric representation.
            - Calculate the deviation from the original geometry to ensure the new triangulation maintains fidelity and is the better of the two possible ones.
        iv) If the area of the affected triangle is less than the minimum area parameter:
            - Approximate the cut point to the nearest vertex. This avoids creating small triangles that could negatively impact the quality of the mesh.
    b) **Compare the metrics obtained with the initial values:**
        i) Quantitative comparison. Measure metrics are compared to the initial values to ensure minimal deviation.
        ii) Qualitative comparison. Visual inspection by experts to ensure the mesh visually aligns with expectations and maintains aesthetic quality.
        iii) Post-comparison actions. If significant deviations are found, adjustments are made either by re-running parts of the algorithm with adjusted thresholds or manually tweaking the mesh.
4) **Quality evaluation:**
    a) Evaluate the accuracy and fidelity of the generated mesh. Conducted continuously throughout the algorithm as described above.
    b) Expert evaluation:
        - Quantitative evaluation. Experts analyze the mesh using precise metrics and tools to measure geometric properties.
        - Qualitative evaluation. Experts visually inspect the mesh to assess aspects like smoothness, alignment, and overall appearance.
        - Combining results. Experts provide a combined assessment that includes both quantitative data and qualitative insights.
        - Adjustments. Based on the recommendations, the mesh is finalized or the necessary adjustments are made and re-evaluated.
5) **Output:**
    - Three-dimensional mesh with cut points and triangulated mesh triangles.
    - Detailed report on the quality metrics and parameters used in the subdivision process.

IV. RESULTS

A thorough investigation of the proposed approximation and/or triangulation strategies applied to triangle meshes representing bone models has been undertaken, employing various fracture patterns. The primary goal has been to scrutinize the arrangement of cutting points, the resulting approximations and triangulations, and to generate fractured models. To accomplish this, a preliminary selection of the most suitable quality metric and the thresholds ensuring a balance between the accuracy of the resulting model and its quality was necessary.

After executing the triangle mesh cutting and obtaining fragments through various strategies, a quality assessment of the subdivided mesh area (fracture area) has been conducted for each applied fracture pattern type. The obtained results have been compared with the initial quality of the mesh.

Upon obtaining these results, the discussion section will proceed to analyze the quality of the triangle mesh obtained in the fracture area, comparing different approaches and emphasizing the effectiveness of strategies guided by quality metrics in order to improve the quality of the fracture area.

To assess the quality in the fracture zone, it was imperative to select the type of fracture for examination. To accomplish this, six fracture patterns outlined in the AO/OTA classification [32] were chosen, which provides a comprehensive list of existing fracture patterns. This study specifically focused on those pertaining to long bones in the diaphyseal region. In particular, the investigation involves fracture patterns categorized as simple (A), wedge (B), and multifragmentary (C). The chosen fracture patterns to exemplify this research include spiral, oblique, transverse, wedge, segmentary, and fragmentary wedge fractures.

In the development of this work, the C++ programming language has been used for greater efficiency, together with OpenGL and OpenMesh, for the management of the meshes representing the models.

A. SELECTION OF QUALITY METRICS

The quality metric plays a crucial role in the triangulation process by influencing the generation of triangulation for an affected triangle to improve its quality. In this context, each triangle undergoes evaluation using the described metrics, determining which triangulation configuration among the available options will generate higher-quality triangles. The metric, through the calculation and comparison of various geometric parameters, guides the selection of the optimal triangulation configuration. Consequently, the final result of the triangulation process consists of triangles exhibiting the highest possible quality according to the selected quality metric. This approach aims to ensure that the subdivision of the mesh into triangles is optimal in terms of the specific geometric properties considered by the quality metric, thus guaranteeing a final result that meets the established criteria for triangulation quality.

Various quality metrics were analyzed in this study to select the one providing the best triangulation. The hybrid approach was employed, varying and evaluating the approximation thresholds incrementally. Triangles were generated using each of the studied quality metrics. The hybrid approach was chosen due to a perceived balance between mesh triangulations and approximations, resulting in a visually improved mesh.

After several tests and analyses, it was concluded that the quality metric based on the ratio between the inradius (the radius of the circle inscribed in the triangle) and the circumradius (the radius of the circle circumscribing the triangle) produced the best results (Table 5). This metric favored the generation of triangles that closely resembled equilateral triangles, generally considered high-quality triangles in other environments, such as in finite element analysis.

By selecting a quality metric for triangle subdivision, we contribute aspects to the resulting mesh that ensure the validity of the generated triangles, eliminating degenerate triangles and improving their overall arrangement. The choice of the indicated quality metric helped minimize the decline in mesh triangle quality after subdivision.

The quality metric based on the ratio between the inradius and the circumradius has the ability to assess the regularity and similarity of a geometric figure to a circular shape. With values close to 1, this metric reveals that the inscribed and circumscribed circles around the figure are in balanced proportions. This suggests a more regular and symmetrical shape, resulting in a geometric figure that more closely resembles a circle.

When compared to other metrics such as aspect ratio and edge ratio, which measure the relationship between length and width, as well as perimeter and circumference, respectively, the selected metric stands out by consistently showing values close to 1. Unlike these metrics, it specifically focuses on the relationship between the inscribed and circumscribed circles, providing a more direct assessment of geometric regularity. Although metrics related to maximum and minimum angles also yield significant improvements, their focus is different as they do not center on the overall shape of the figure.

B. THRESHOLD SELECTION FOR APPROXIMATION

The threshold stablished for the hybrid approach is essential. This parameter controls the transition between the approximation and subdivision strategy, determining when a cut-off point should be approximated and when the polygon containing it should be triangulated. Properly balancing these two approaches is fundamental to maintain model fidelity.

The threshold value must be less than 50% of the triangle side length. A threshold equal to or greater than 50% can lead to the inability to establish the vertex to approximate, limiting the method's effectiveness and reducing precision. High threshold values imply lower precision, highlighting the importance of avoiding excessively large thresholds.

Table 6 shows the results of several tests conducted using different threshold values to determine the optimal thresholds. Consistency was maintained by using a femur bone model with an oblique fracture pattern applied to the same region, ensuring identical initial subdivision regions and cut points across different thresholds. The best threshold was determined by varying its value and analyzing the sets of cut points that result in approximation versus those that lead to triangulation.

The results demonstrate that the hybrid approach effectively balances the two strategies, also balancing geometric accuracy and visual fidelity across different fracture patterns. The interval [10%, 30%] provides a suitable balance between approximations and subdivisions, offering a visually accurate subdivision without compromising the quality of the representation of the original fracture pattern. The results imply an inverse relationship between the threshold size and the number of triangulations performed on the mesh. A higher threshold leads to fewer modifications in the triangles, but this reduction in computational complexity must be carefully balanced with the requirement to maintain visual proximity between the fracture line and the original pattern.

A detailed analysis of the distribution of cut points based on the threshold provides a nuanced understanding of threshold selection. It is observed that the majority of cut points concentrate in the range of 15% to 30% of the side length, emphasizing the importance of considering these intervals to maintain visually adequate approximation.

In summary, while a higher threshold can reduce triangulations, avoiding significant displacements of the fracture line is critical to maintaining the visual quality of the model. Optimal threshold selection involves a careful balance

TABLE 5. Values for quality metrics of the fracture zone for each type of fracture pattern and for each type of quality metric using a hybrid approach for triangle subdivision.

Pattern	Metric				
	Aspect ratio	Edge ratio	Ratio between inradius and circumradius	Maximum angle	Minimum angle
Spiral	4.93	3.97	1.10	153.90	79.94
Oblique	4.86	3.83	1.01	154.38	80.10
Transverse	5.06	4.10	0.89	153.83	83.34
Wedge	5.89	4.21	0.96	154.08	80.63
Segmentary	5.56	4.80	1.24	157.55	80.73
Fragmentary wedge	5.18	3.57	1.13	154.42	77.93

TABLE 6. Number of approximations (a) and subdivisions (s) for each fracture pattern with varying approximation thresholds between 0% and 50%.

Pattern	Approximation Threshold																					
	0		5		10		15		20		25		30		35		40		45		50	
	a	s	a	s	a	s	a	s	a	s	a	s	a	s	a	s	a	s	a	s	a	s
Spiral	0	534	102	452	188	386	243	342	314	289	350	263	419	213	520	152	560	126	624	90	677	63
Oblique	0	576	44	539	171	439	229	395	295	344	365	292	429	248	498	199	569	154	641	115	709	80
Transverse	0	438	22	418	38	405	97	358	175	301	247	245	319	193	380	156	428	126	480	95	511	74
Wedge	0	903	195	743	386	517	394	591	588	356	639	414	694	238	823	82	831	78	824	30	720	7
Segmentary	0	870	5	867	138	738	495	477	997	152	1069	117	1103	100	1120	90	1127	85	1135	80	1140	77
Fragmentary wedge	0	1532	0	1532	467	1164	918	1013	774	924	1145	657	1237	298	1102	742	1476	416	1298	347	1717	260

between the quality of the resulting mesh structure and the visual quality of the model.

C. SUBDIVISION OF BONE MODELS ACCORDING TO STRATEGIES

The set of bone models chosen for bone subdivision and subsequent analysis comprises four different types of bones: femur, humerus, tibia, and fibula. This diversity of models allows the evaluation of strategies in various anatomical contexts. The fracture patterns used in this study include spiral, oblique, transverse, segmental, wedge, and fragmented wedge patterns, as indicated earlier. This selection aims to provide a comprehensive visualization of the flow of all methods developed in the bone model subdivision process. Given that the results obtained for different types of bones are similar, the test set applied to a specific type of bone, in this case, the femur model, is presented.

The strategies employed for the mentioned fracture patterns include the approximation strategy, the triangulation strategy, and the hybrid approach using different approximation thresholds (10% and 30%). The latter two strategies utilize the quality metric based on the ratio between the incenter and circumcenter, as described previously.

1) SUBDIVISION USING APPROXIMATION STRATEGY

Figure 4 illustrates the 2D projection of a transverse fracture pattern applied to the midpoint of the femur model. The cut points involved in the approximation strategy are depicted. In this approach, each time a cut point appears (highlighted in yellow), it is determined to which vertex of the affected triangle it should be approximated using a proximity criterion (marked in red).

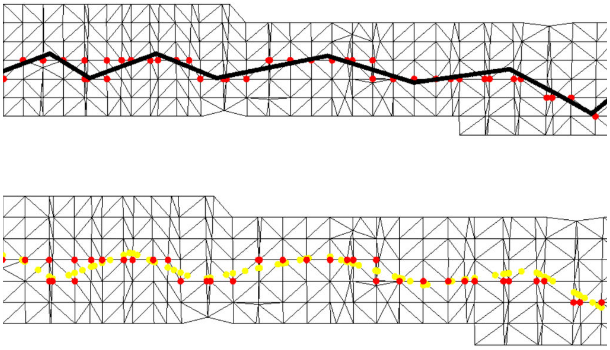

FIGURE 4. Approximation strategy applying a transverse pattern. Top: Subdivision area with the fracture line in black. Bottom: Subdivision area visualized with cutting points (yellow) and approximated points (red).

Figure 5 displays the mesh cut using this approximation strategy, through the generated cut points for each fracture type (spiral, oblique, transverse, wedge, segmentary and fragmentary wedge), presenting the resulting fragments.

2) SUBDIVISION USING TRIANGULATION STRATEGY

As shown in figure 6, the cut points coincide with those obtained for triangulation, as there is no form of approximation. These generated points are used to obtain different triangles using the corresponding quality metric.

Figure 7 illustrates the resulting triangle mesh after applying triangulation to it, generating new triangles that modify the structure of the original mesh.

Figure 8 depicts the mesh cut using the triangulation strategy, following the cut points generated for each type of fracture (spiral, oblique, transverse, wedge, segmentary, and fragmentary wedge), presenting the resulting fragments.

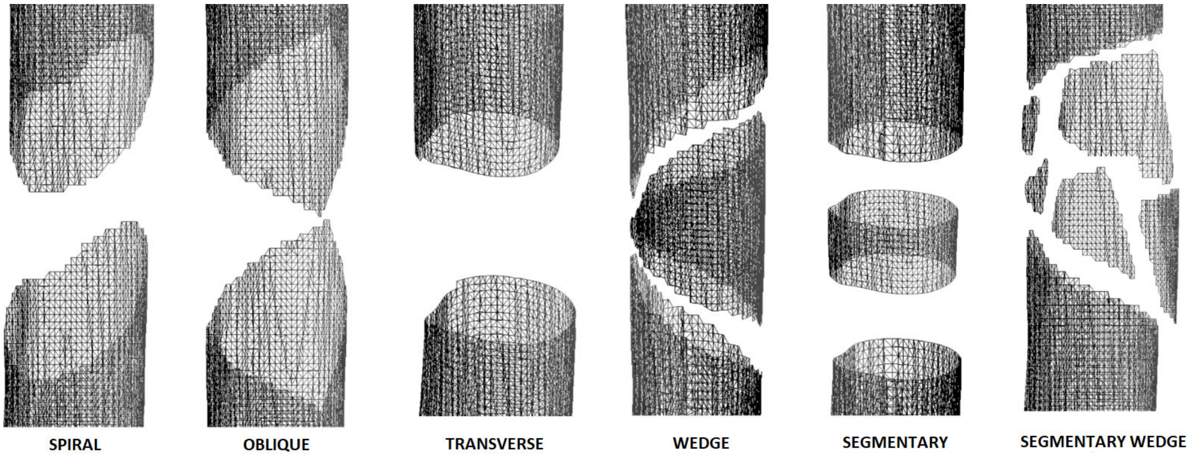

FIGURE 5. **Cut produced following the cutting points generated by the approximation strategy. For each type of fracture, its fragments are shown.**

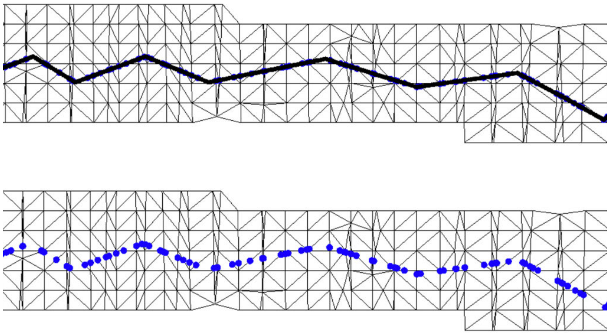

FIGURE 6. **Cutting points on triangles to be subdivided over the subdivision area in 2D.**

3) SUBDIVISION USING HYBRID APPROACH

The results of the hybrid approach in the subdivision of triangle meshes demonstrate a balanced combination of approximation and triangulation strategies. The goal was to improve the visual and geometric quality of the resulting mesh. This approach aims to keep the position of the cut points close to those of the fracture line, minimizing deviations.

The choice of the approximation threshold in the hybrid approach directly influences the number of approximations and triangulations performed. Among the selected thresholds, a 10% threshold favors a higher number of triangulations, fitting more precisely to the fracture line, while a 30% threshold involves more approximations and a slight displacement of the cut points.

Figure 9 illustrates the mesh cut using the hybrid approach, following the cut points generated for each type of fracture (spiral, oblique, transverse, wedge, segmentary and fragmentary wedge), presenting the resulting fragments.

D. QUALITY IN THE SUBDIVISION AREA

The quality of the subdivision area mesh has been measured before performing the cut and after subdivision with each strategy. The subdivision area involves the triangles affected by the fracture pattern lines, leaving the rest of the mesh triangles unchanged.

Table 7 presents the initial quality values of the subdivision area before applying different cutting strategies. The quality metrics indicated in the “Quality Metrics” section have been utilized. These values, corresponding to those obtained when applying the approximation strategy, reflect the mesh’s quality before topological changes according to each quality metric. It is evident that the approximation strategy does not introduce new points or re-triangulate the mesh.

Table 8 details the results of the subdivision area’s quality when applying the triangulation strategy with different quality metrics. Focusing on the quality metric based on the ratio between the inradius and circumradius, the resulting mesh’s quality is lower than the approximation strategy, but visually, it provides a more faithful representation of the fracture pattern.

Table 9 presents the quality values of the subdivision area when applying the hybrid approach with a 10% threshold. Compared to the triangulation strategy, the quality of the subdivision area improves with the hybrid approach. In the case of the metric based on the ratio between the inradius and circumradius, the closer the value of the ratio is to 1, the better the quality of the subdivision area. This approach proves effective in maintaining a balance between approximations and triangulations.

In Table 10, we observe the quality of the subdivision area when applying the hybrid approach with a 30% threshold. In this case, with a higher percentage of approximation than in other triangulation methods, the quality of the subdivision area is better, as fewer triangulations are performed.

Overall, comparing the results, it is concluded that the mesh quality is maintained with the approximation strategy, decreases with increasing triangulations, and the strategy guided by quality metrics offers a substantial improvement in the quality of the subdivision area compared to the triangulation strategy without using quality metrics to guide the process.

This quantitative analysis of the quality of the subdivision area provides crucial information to understand how each strategy affects the mesh structure and contributes to

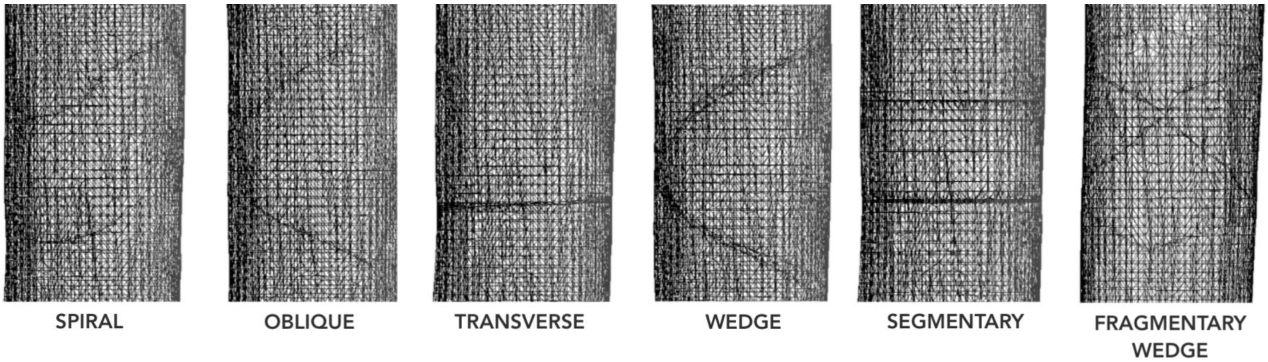

FIGURE 7. Femur with fracture patterns applied using the subdivision strategy. The model shows how the mesh topology has been modified, defining with its own geometry the location where the cutting line passes through and where the model will fracture.

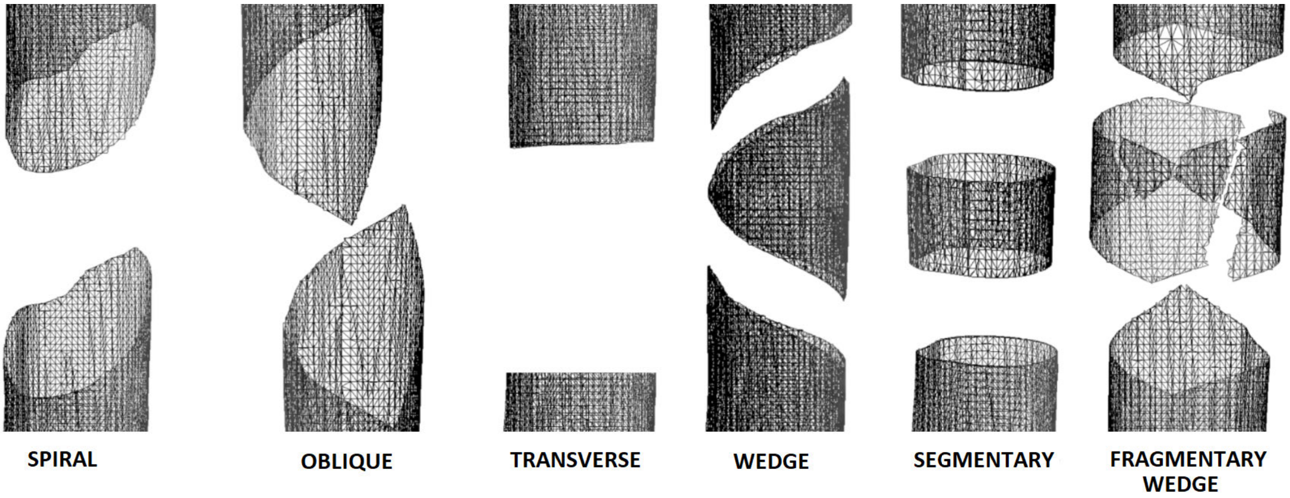

FIGURE 8. Cut produced following the cutting points generated by the triangulation strategy. For each type of fracture, its fragments are shown.

informed decision-making in choosing the most suitable subdivision strategy for specific bone models.

E. ASSESSMENT OF VISUAL QUALITY

To validate the quality of the resulting meshes from the applied triangulation strategies, an evaluation process was conducted by a committee of experts in 3D modeling and bone fracture. This analysis was performed through the review of subdivided models and the application of specific criteria to assess the suitability of each triangulation strategy used. The following details the procedure carried out in the validation and its results.

A selection of 5 experts was made, including two orthopedic surgeons, a radiologist, a fracture reduction specialist surgeon, and a 3D modeling expert with experience in evaluating meshes used in medical contexts. The next step involved providing the experts with the three-dimensional subdivided bone models using approximation, triangulation, and a hybrid approach.

Regarding the evaluation criteria used to determine the visual quality of the mesh, specific criteria based on anatomy, visual continuity, and structural coherence of the resulting meshes were established. Factors considered included the accuracy of the cut, the smoothness of the transition between fragments, and the preservation of the initial geometry.

As for the results of the visual quality validation (Table 11), the experts observed that in the case of the approximation strategy, there was adequate preservation of the original geometry and a smooth transition between fragments, highlighting visual coherence in the case of horizontal fractures. However, they also noted the presence of unusual ‘‘peaks’’ in the rest of the fractures. Focusing on the triangulation strategy, the experts determined that the preservation of visual and structural mesh quality was very good, faithfully representing the fracture pattern. They found that the triangles of the mesh were perfectly altered, adapting to the distribution of fracture points. Finally, the experts determined that the hybrid approach presented an adequate balance in its fracture line, likely due to the combination of approximations and subdivisions. Additionally, they highlighted the ability of a hybrid approach to maintain the position of fracture line points, minimizing deviations.

In conclusion, the expert validation determined that there were acceptable visual results in the approximation strategy but with some issues in the case of non-horizontal fractures. They also emphasized the visual and structural consistency of the triangulation strategy. The hybrid approach was positively received for its ability to balance approximations and triangulations, adapting well to different fracture patterns.

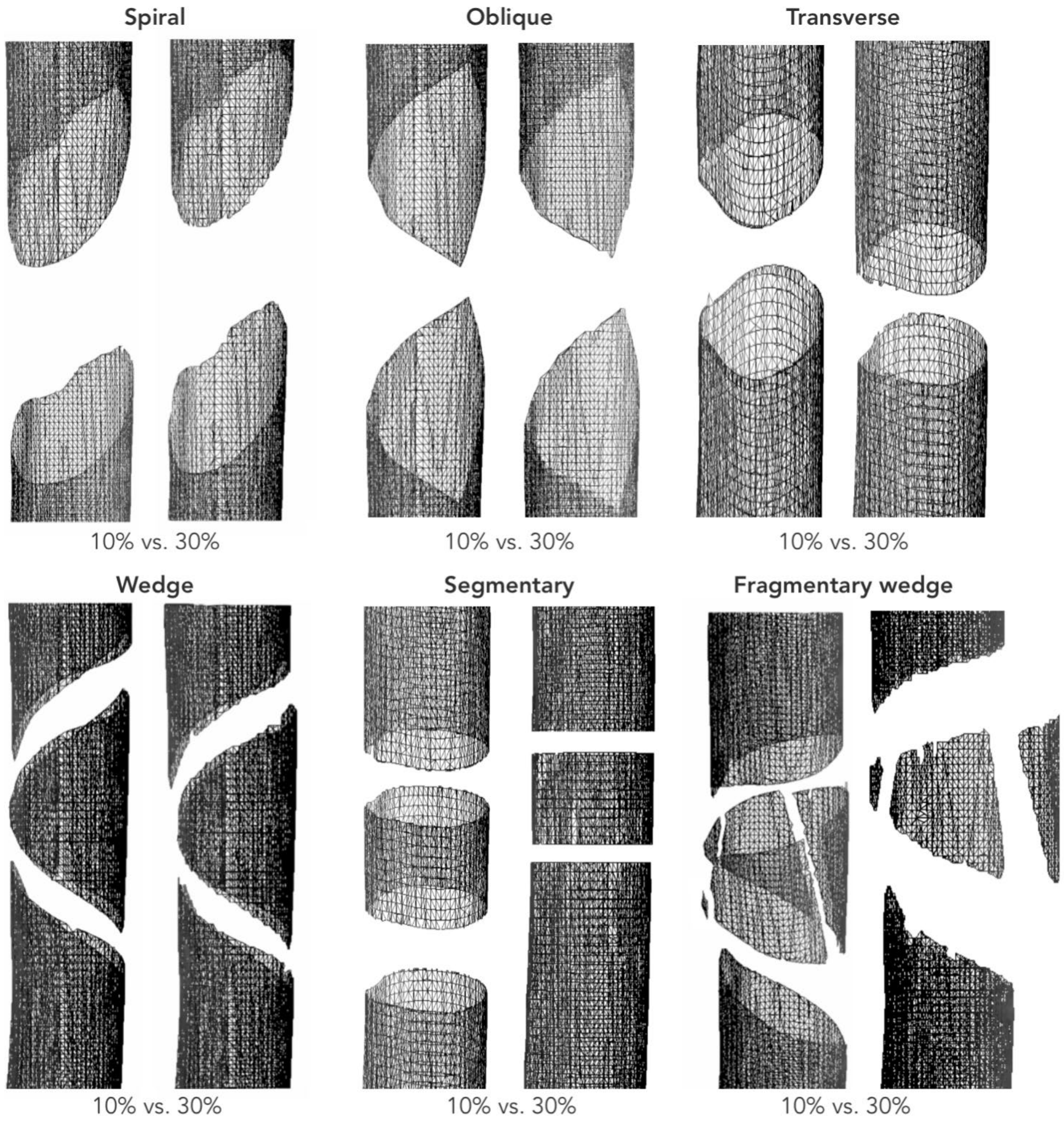

FIGURE 9. Cut produced following the cutting points generated by the hybrid approach. The fragments are shown for each type of fracture (with a 10% threshold and with a 30% threshold).

This expert validation supports the choice of subdivision strategies, providing an external evaluation that considers aesthetic and structural aspects specific to bone structures, crucial in medical and three-dimensional modeling applications.

V. DISCUSSION

This section analyzes the results obtained with each subdivision strategy and the quality of the triangles obtained, addressing both the perspective of quality metrics and the visualization of the resulting models.

A. ANALYSIS OF SUBDIVISION VIA APPROXIMATION

Subdivision via approximation is a strategy used in the processing of three-dimensional models to generate a fractured or subdivided representation of a triangle mesh. This approach is based on identifying cut points along a fracture line and subsequently approximating these points towards the nearest vertices in the original mesh. Through this process, the goal is to create a realistic three-dimensional representation of the subdivision, preserving the topology of the original mesh as much as possible.

One key advantage of subdivision via approximation is its ability to retain the original mesh's topology (Fig. 4), as no new vertices or triangles are introduced. However, this strategy may present visual drawbacks, especially when the fracture line is not horizontal. In such cases, visual issues such as the appearance of "peaks" in the obtained models were observed.

The approximation strategy proves particularly effective in situations where the fracture is nearly horizontal, such as in transverse or segmental fractures. In these cases, visual consistency improves, as all triangles follow the same approximation criterion. Furthermore, subdivision via approximation does not negatively impact the mesh's metric quality, suggesting that this strategy can be a suitable choice

TABLE 7. Initial quality values with different metrics applied to the triangles in the fracture zone. These values coincide with those obtained using the approximation strategy.

Pattern	Metric				
	Aspect ratio	Edge ratio	Ratio between inradius and circumradius	Maximum angle	Minimum angle
Spiral	1.62	2.50	0.44	145.71	91.55
Oblique	1.82	2.91	0.46	144.97	91.18
Transverse	2.11	2.90	0.49	146.35	91.52
Wedge	1.84	2.74	0.47	145.65	91.21
Segmentary	2.04	3.09	0.40	150.40	91.22
Fragmentary wedge	2.47	3.92	0.44	145.92	91.48

TABLE 8. Quality values with different metrics applied to the triangles in the fracture zone using a triangulation strategy.

Pattern	Metric				
	Aspect ratio	Edge ratio	Ratio between inradius and circumradius	Maximum angle	Minimum angle
Spiral	6.42	5.56	1.71	157.66	81.68
Oblique	39.66	14.09	5.03	156.84	76.08
Transverse	7.70	4.91	1.80	154.96	82.61
Wedge	19.40	12.42	1.86	156.76	77.39
Segmentary	7.82	6.31	1.84	158.95	83.14
Fragmentary wedge	48.76	7.95	2.80	157.26	75.27

TABLE 9. Quality values with different metrics applied to the triangles in the fracture zone using an hybrid approach with a 10% threshold.

Pattern	Metric				
	Aspect ratio	Edge ratio	Ratio between inradius and circumradius	Maximum angle	Minimum angle
Spiral	3.95	3.02	1.24	153.20	79.61
Oblique	4.11	3.29	0.96	152.65	79.87
Transverse	4.37	3.10	0.98	152.52	82.12
Wedge	3.28	2.79	1.01	152.40	81.54
Segmentary	5.61	3.72	1.18	158.12	74.43
Fragmentary wedge	3.64	2.80	0.97	152.69	79.92

TABLE 10. Quality values with different metrics applied to the triangles in the fracture zone using an hybrid approach with a 30% threshold.

Pattern	Metric				
	Aspect ratio	Edge ratio	Ratio between inradius and circumradius	Maximum angle	Minimum angle
Spiral	5.35	4.11	1.29	155.33	78.38
Oblique	5.30	3.91	1.15	155.30	78.19
Transverse	5.20	4.29	1.23	154.56	82.76
Wedge	7.58	5.20	1.12	155.63	78.15
Segmentary	5.86	5.12	1.76	157.73	80.87
Fragmentary wedge	4.80	4.04	1.47	155.57	77.51

when preserving the geometric quality of the original mesh is valued. However, it is essential to consider the associated visual limitations, especially in fractures with non-horizontal lines.

TABLE 11. Visual quality validation conducted by experts, utilizing the 5-point Likert scale.

Expert	Model	Strategy												Average
		Approximation				Subdivision				Hybrid 10%				
		Cutting precision	Smoothness of transition	Conservation of geometry	Adaptation to the pattern	Cutting precision	Smoothness of transition	Conservation of geometry	Adaptation to the pattern	Cutting precision	Smoothness of transition	Conservation of geometry	Adaptation to the pattern	
1	Spiral	3.00	4.00	5.00	3.00	5.00	5.00	1.00	5.00	5.00	5.00	3.00	5.00	4.08
	Oblique	3.00	4.00	5.00	3.00	5.00	5.00	0.00	5.00	5.00	5.00	4.00	5.00	4.08
	Transverse	5.00	5.00	5.00	5.00	5.00	5.00	2.00	5.00	5.00	5.00	4.00	5.00	4.67
	Wedge	3.00	4.00	5.00	3.00	5.00	5.00	1.00	5.00	5.00	5.00	3.00	5.00	4.08
	Segmentary	5.00	5.00	5.00	5.00	5.00	5.00	1.00	5.00	5.00	5.00	3.00	5.00	4.50
	Fragmentary wedge	3.00	4.00	5.00	2.00	5.00	4.00	2.00	5.00	5.00	5.00	3.00	4.00	3.92
	Average	3.67	4.33	5.00	3.50	5.00	4.83	1.17	5.00	5.00	5.00	3.33	4.83	4.22
2	Spiral	3.00	4.00	5.00	4.00	5.00	4.00	2.00	5.00	5.00	5.00	3.00	5.00	4.17
	Oblique	3.00	4.00	5.00	4.00	5.00	5.00	2.00	5.00	5.00	5.00	3.00	5.00	4.25
	Transverse	5.00	5.00	5.00	5.00	5.00	5.00	3.00	5.00	5.00	5.00	5.00	5.00	4.83
	Wedge	2.00	3.00	5.00	4.00	5.00	4.00	3.00	5.00	5.00	5.00	4.00	5.00	4.17
	Segmentary	5.00	5.00	5.00	5.00	5.00	5.00	3.00	5.00	5.00	5.00	4.00	5.00	4.75
	Fragmentary wedge	2.00	3.00	5.00	3.00	5.00	4.00	2.00	5.00	5.00	5.00	4.00	5.00	4.00
	Average	3.33	4.00	5.00	4.17	5.00	4.50	2.50	5.00	5.00	5.00	3.83	5.00	4.36
3	Spiral	4.00	4.00	5.00	4.00	5.00	4.00	1.00	5.00	5.00	5.00	4.00	5.00	4.25
	Oblique	4.00	4.00	5.00	4.00	5.00	4.00	1.00	5.00	5.00	5.00	4.00	5.00	4.25
	Transverse	5.00	5.00	5.00	5.00	5.00	5.00	2.00	5.00	5.00	5.00	5.00	5.00	4.75
	Wedge	2.00	4.00	5.00	3.00	5.00	5.00	1.00	5.00	5.00	5.00	4.00	5.00	4.08
	Segmentary	5.00	5.00	5.00	5.00	5.00	5.00	1.00	5.00	5.00	5.00	4.00	5.00	4.58
	Fragmentary wedge	3.00	3.00	5.00	3.00	5.00	4.00	0.00	5.00	5.00	5.00	3.00	4.00	3.75
	Average	3.83	4.17	5.00	4.00	5.00	4.50	1.00	5.00	5.00	5.00	4.00	4.83	4.28
4	Spiral	3.00	3.00	4.00	3.00	5.00	4.00	2.00	5.00	5.00	5.00	3.00	5.00	3.92
	Oblique	4.00	4.00	5.00	3.00	5.00	5.00	2.00	5.00	5.00	5.00	3.00	5.00	4.25
	Transverse	5.00	5.00	5.00	5.00	5.00	5.00	3.00	5.00	5.00	5.00	4.00	5.00	4.75
	Wedge	2.00	2.00	4.00	2.00	5.00	5.00	2.00	5.00	5.00	5.00	4.00	5.00	3.83
	Segmentary	5.00	5.00	5.00	5.00	5.00	5.00	2.00	5.00	5.00	5.00	4.00	5.00	4.67
	Fragmentary wedge	2.00	2.00	5.00	1.00	5.00	4.00	1.00	5.00	5.00	5.00	3.00	5.00	3.58
	Average	3.50	3.50	4.67	3.17	5.00	4.67	2.00	5.00	5.00	5.00	3.50	5.00	4.17
5	Spiral	3.00	4.00	5.00	3.00	5.00	5.00	1.00	5.00	5.00	5.00	3.00	5.00	4.08
	Oblique	3.00	4.00	5.00	4.00	5.00	5.00	1.00	5.00	5.00	5.00	3.00	5.00	4.17
	Transverse	5.00	5.00	5.00	5.00	5.00	5.00	1.00	5.00	5.00	5.00	4.00	5.00	4.58
	Wedge	3.00	3.00	5.00	2.00	5.00	5.00	1.00	5.00	5.00	5.00	3.00	4.50	3.88
	Segmentary	5.00	5.00	5.00	5.00	5.00	5.00	1.00	5.00	5.00	5.00	3.00	5.00	4.50
	Fragmentary wedge	3.00	3.00	5.00	1.00	5.00	4.00	0.00	5.00	5.00	5.00	3.00	4.00	3.58
	Average	3.67	4.00	5.00	3.33	5.00	4.83	0.83	5.00	5.00	5.00	3.17	4.75	4.07
Average		3.60	4.00	4.93	3.63	5.00	4.67	1.50	5.00	5.00	5.00	3.57	4.88	4.22

B. ANALYSIS OF SUBDIVISION VIA TRIANGULATION

The triangulation strategy is based on generating new triangles by integrating cut points into the mesh's topology. Visual coherence with the fracture pattern is a prominent feature of the triangulation strategy, where the mesh's topology is modified, creating new triangles by combining cut points with the original vertices. This process ensures direct visual coherence with the applied fracture pattern, providing a more accurate and detailed representation.

Despite the visual improvement, it is essential to consider the changes in topology that triangulation involves. Unlike the approximation strategy, this technique introduces new triangles and vertices, altering the original structure and generating a more detailed representation adapted to the fracture. This impact on topology can influence the metric quality of the resulting mesh.

Computational cost is another relevant aspect to consider. Although the triangulation strategy provides superior visual results, its implementation often involves higher computational resource consumption compared to approaches that simply adjust the position of existing points.

The subdivision via triangulation strategy stands out for its ability to generate visually accurate meshes that faithfully reflect fracture patterns. However, a meticulous evaluation of metric quality and an analysis of computational costs are essential to determine its suitability for specific bone models.

C. ANALYSIS OF SUBDIVISION WITH HYBRID APPROACH

The subdivision approach using the hybrid strategy represents a strategic synthesis that integrates both approximation and triangulation approaches in the subdivision of triangle meshes, aiming to enhance both the visual and geometric quality of the resulting mesh. The applicability of this approach has been thoroughly explored in the context of fractured bone models.

The hybrid approach more effectively maintains the position of points along the fracture line, minimizing deviations and preserving visual coherence with the fracture pattern. This feature is crucial to ensure that the geometric representation of the fractured model remains faithful to the original fracture.

The choice of a 10% or 30% threshold implies specific trade-offs. A 10% threshold, by favoring more triangulations, achieves a more precise fit to the fracture line and a smoothed edge. In contrast, a 30% threshold produces more approximations, leading to a slight displacement of cut points but maintaining coherence with the fracture line. Decision-making in this aspect becomes a critical component of the fragment acquisition process.

The hybrid approach emerges as an effective strategy in the subdivision of triangle meshes for fractured bone models. Its ability to adapt to different contexts, coupled with the option to adjust the approximation threshold, positions it as a versatile tool. The combination of the strengths of approximation and triangulation in a single conceptual framework provides an optimal balance between visual quality and metrics.

D. ANALYSIS OF THE QUALITY OF RESULTING TRIANGLES

Analyzing the quality of resulting triangles in the subdivision of triangle meshes is an essential component for evaluating the effectiveness and suitability of the applied strategies. This study addresses triangle quality from two key perspectives: quality metrics and the visualization of the resulting model.

In the case of subdivision through approximation, it is observed that the mesh quality is not negatively affected in terms of quality metrics. However, visually, in most cases, the quality of the resulting mesh is not optimal. The approximation strategy presents visual drawbacks, especially in non-horizontal fractures. These issues become more evident when the fracture line shifts toward the nearest vertex each time it intersects a triangle in the mesh. In practically horizontal fractures, the approximation strategy may offer better visual consistency since all triangles apply the same approximation criterion. It is important to note that the mesh's topology is not modified due to approximations, as no new vertices are added, and no new triangles are generated.

On the other hand, in the case of subdivision through triangulation, an interesting phenomenon is evident. Although the visual quality of the subdivision area is significantly superior compared to the approximation strategy, the metric quality of the resulting mesh is inferior. Triangulation generates a mesh composed of smaller triangles and generally of lower quality than in the original case. This aspect highlights the dichotomy between visual and metric quality, where triangulation can offer a more faithful representation of the fracture pattern but at the expense of the mesh's metric quality.

The hybrid approach stands out for maintaining the position of points along the fracture line, achieving a precise balance between visual and metric quality, with minimal deviations limited to the action range of the approximation strategy. The choice of the threshold in the hybrid approach provides flexibility to adjust the visual and metric quality of the resulting mesh according to specific requirements.

VI. CONCLUSION

This study aimed to determine a suitable cutting strategy for the subdivision of bone models, considering both geometric and visual quality aspects in the resulting mesh.

Significant differences in results were observed depending on the strategy used. The approximation strategy did not negatively affect the metric quality of the mesh, but visually, in some cases, it was unsatisfactory due to the irregular distribution of cut points. Moreover, the triangulation strategy altered the mesh topology but produced a more precise and consistent fracture line with the original pattern.

The explored hybrid approach proved useful in balancing mesh quality and fracture line accuracy. This approach uses a specific approximation threshold and a metric based on the ratio between the inradius and circumcenter, being particularly valuable when a balance between precision and visual quality is needed.

Results indicate that subdivision strategies can significantly impact the quality and precision of the resulting mesh, making the choice of approach critical in different applications. The metric based on the ratio between the inradius and circumcenter was identified as optimal in most studied cases. The analysis underscores the importance of precision in the subdivision process and suggests that using quality metrics to guide triangulation can enhance the overall quality of the resulting mesh, providing more accurate and visually appealing representations of the original model.

These findings highlight the importance of carefully choosing and adjusting subdivision strategies and metrics, impacting the quality and precision of results.

Future research directions are proposed, such as the use of dynamic quality metrics based on triangle types, the incorporation of intermediate points in triangulations for improved quality, and the automation and dynamic adaptation of thresholds based on the specific geometry of the mesh and its triangles.

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