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RESEARCH ARTICLE

STEM-Based Bayesian Computational Learning Model-BCLM for Effective Learning of Bayesian Statistics

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ABSTRACT This work contributes to the comprehension of Bayes' theorem inclusive Bayesian probabilities and Bayesian inferencing within the framework of STEM (Science, Technology, Engineering, Arts, and Mathematics) and cognitive learning w.r.t Bloom's taxonomy (BT). Bayes' theorem is taken as a crucial statistical instrument employed in the development of intelligent systems and the management of risks, commonly utilized by engineers for tasks in machine learning and managerial decision-making. The fundamental concept behind Bayes' theorem revolves around comprehending the degree of truth within the confines of an explicit perspective. This involves partitioning the entire sample space of possible evidence and utilizing the subset containing the relevant perspective to estimate the uncertainty of an event or the reliability of a model. However, it is often found difficult for students to understand Bayes' theorem to the level of applying it to real-world problems. Considering this, the proposed learning method in this paper elucidated the acquisition of Bayes' mathematical formulation by leveraging computational thinking, leading to the development of a computational model. The proposed model is named the Bayesian Computational Learning Model (BCLM). Subsequently, we have probed the utility of BCLM in the design and plan of learning activities, coherent to the STEM paradigm and BT cognitive learning hierarchy.

INDEX TERMS Bloom's taxonomy, Bayes' theorem, computational thinking, computer simulations, decision making, engineering education, frequentist, intelligent systems design, machine learning, project management, risk analysis, STEM.

I. INTRODUCTION

Bayes' theorem is a fundamental principle applied in probability and statistics to determine the likelihood of an event happening, given that another event has already occurred. This theorem enables the assessment of the influence of one event on another, highlighting the interdependence between them. This interdependence can also be on a series of events that have occurred independently already, also called naive Bayes'. Mathematically if a and b are two dependent events such that b has occurred already and then the occurrence of a is to be observed, then the subsequent occurrence of a, i.e. $a|b$, can be ascertained by using Bayes' theorem as introduced in Eq. (1).

$$P(a|b) = \frac{P(a)\,P(b|a)}{P(b)} \tag{1}$$

This is the famous Bayes' equation, credited to Thomas Bayes but published posthumously in 1763 [1]. In Eq. (1) the components making up the Bayes' theorem recipe are as follows:

$P(a)$ is the prior probability of the event a.

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$P(b|a)$ is the likelihood.

$P(b)$ is the total marginal probability of the event b or evidence.

$P(a|b)$ is the posterior probability of a given b has occurred.

Prior probability refers to the initial belief of the variable under consideration before incorporating any evidence. Following the receipt and utilization of the evidence's likelihood, the prior probability undergoes an update, and a modified version of this probability is obtained as posterior. Both posterior and marginal likelihood represent conditional probabilities. Marginal likelihood in Eq. (1), does not alter the configuration of the posterior probability but merely adjusts its scale.

The concept of prior is synonymous with the idea of perspective. Therefore, any alteration in the prior or perspective will consequently impact the posterior. It is imperative to acknowledge this aspect when addressing problems involving Bayes' theorem. To comprehend the impact of prior on posterior and then properly apply (1), different learning techniques can be used. However, when conveying information to engineering students, it is beneficial if they can grasp the subject through a computational approach. Computational thinking (CT) enhances problem-solving abilities and develops technology-driven solutions [2]. This capacity to comprehend information and think in a manner that aligns with technological procedures is not only vital for an engineering student's professional preparedness but also helps in achieving the learning goals of solving complex engineering problems (CEP) and problem-based learning (PBL) in a more strategic way as required by STEM learning architype and Washington Accord which globally acknowledges tertiary level engineering qualification [3].

Computational thinking (CT) requires the identification of a precise, well-defined, step-by-step resolution to a problem. It involves breaking down the problem into smaller parts, recognizing patterns, and eliminating extraneous details to allow the solution to be replicated by humans, machines, and computers. The computational thinking process can be divided into four components or stages, namely 1) decomposition, 2) pattern recognition, 3) abstraction, and 4) algorithmic thinking. Decomposition is meant to divide the problem into smaller and easily understandable segments. In pattern recognition connections are found between different segments of the problem. They help to identify similar trends among the decomposed segments. Abstracting involves extracting the key information from each broken-down problem, helping to generalize what needs to be done precisely to solve the entire problem. This phase of computational thinking assists students in recognizing how these crucial details can be applied to resolve other aspects of the same problem. The ultimate element or zenith of computational thinking is algorithmic thinking. This involves establishing a systematic solution to the problem, ensuring it can be reproduced for a consistent and dependable result. In the context of an engineer's contemporary understanding of computational thinking this solution comprises a sequence of steps that can be executed either by computers or humans partially or fully [4], [5], [6].

By applying the four components of computational thinking (CT), a student can readily attain the six hierarchical stages of cognitive learning outlined in Bloom's Taxonomy. This approach also meets the criteria for STEM-based education. STEM education, BT, and CT intersect to promote a holistic approach to transforming engineering education, emphasizing critical thinking, problem-solving, and creativity in the discipline [7], [8], [9], [10].

Researchers in [11] conducted a systematic review of CT in math education, examining studies from 2006 to 2021 found in the Web of Science database. They selected 24 articles for detailed analysis based on education levels, contexts, programming tools, and learning outcomes. Their findings highlighted that geometrized programming and student-centered approaches enhance learning in both CT and math, emphasizing the interactive process of reasoning mathematically and computationally.

In [12], researchers highlighted the simultaneous development of mathematical and CT-related concepts and practices across four math domains. They identified two key interactions: using mathematical knowledge to create CT artifacts and generating new mathematical knowledge through CT practice. The study offered three new insights: (1) Mathematical problem solutions should not be immediately obvious to enhance learning; (2) Dynamic representations and immediate visual feedback from programming tools aid student learning; (3) Customization options in both problems and tools enhance educational outcomes.

In [13], computational thinking (CT) is viewed as a boundary object that bridges mathematics and computer science in a school problem-solving context. The authors investigated middle school students' engagement in mathematical problem-solving within the block-based programming environment, Scratch, treating CT as an embedded boundary object. By analyzing the boundary-crossing features of CT in students' Scratch artifacts related to symmetry and arithmetic sequences, the study uncovered new avenues for exploring CT as a boundary object in integrated STEM pedagogy.

Recent research has increasingly focused on integrating computational thinking (CT) into mathematics education. Despite many studies, there remains a lack of clear explanations on how CT supports mathematics learning. Addressing this research gap, the primary focus of this research is to explore the integration of Bayesian theory within the framework of computational thinking (CT) to enhance understanding and application in probabilistic scenarios. The motivation for this research stems from the need to bridge the gap between abstract statistical concepts and practical computational skills, thereby fostering a more intuitive and robust approach to data analysis and decision-making processes. The main contributions of this work are aimed at providing a comprehensive understanding of how Bayesian theory can be revisited and applied through the lens of computational thinking. The research is motivated by the following key objectives:

- **Revisiting Bayes' Theorem and Its Probabilities:**
  The research begins with a detailed examination of Bayes' theorem, emphasizing its application as a CT problem. This revisitation is crucial for demystifying the theorem and making it accessible to learners and practitioners who may find traditional statistical approaches challenging.
- **Integration of CT Components:**
  The study investigates how the four core components of computational thinking—decomposition, pattern recognition, abstraction, and algorithmic thinking—can be utilized to learn and comprehend probabilistic scenarios. By decomposing complex problems into manageable parts, recognizing patterns in data, abstracting key concepts, and developing algorithmic solutions, the research aims to create a structured approach to Bayesian analysis.
- **Bayesian Computational Thinking:**
  The research explores various probabilistic situations where Bayesian computational thinking can be effectively applied. It delineates the steps necessary to achieve a comprehensive understanding and application of Bayesian methods, providing a clear pathway for learners to follow.
- **Bayesian CT Learning Model:**
  A significant contribution of this work is the development of Bayes' CT learning model, which is designed to enhance both descriptive and inferential statistical learning. This model serves as a practical guide for implementing Bayesian methods within a CT framework, thereby making statistical learning more interactive and engaging.
- **Correlation with Bloom's Cognitive Learning Paradigm:**
  The research examines the dynamics of applying Bayesian CT in relation to Bloom's cognitive learning paradigm. By aligning Bayesian CT with the different cognitive levels of Bloom's taxonomy, the study aims to enhance the educational impact of Bayesian learning, ensuring that it caters to various stages of cognitive development.
- **Alignment with STEM Learning Paradigm:**
  Lastly, the research aligns the Bayesian CT learning model with the STEM (Science, Technology, Engineering, and Mathematics) learning paradigm. This alignment underscores the interdisciplinary nature of the research, highlighting its relevance and applicability across different STEM fields. By integrating Bayesian methods into STEM education, the research seeks to foster a more holistic and practical approach to learning and problem-solving.

This research is driven by the need to make Bayesian theory more accessible and applicable through computational thinking. By revisiting fundamental probabilistic concepts, integrating core CT components, and aligning the learning model with established educational paradigms, the study aims to enhance both the teaching and application of Bayesian methods. This work not only contributes to the academic understanding of Bayesian computational thinking but also provides practical tools and models for educators and practitioners in various fields.

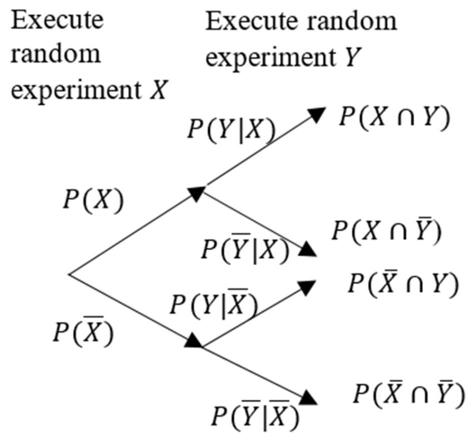

FIGURE 1. Investigating Y after X.

II. COMPUTATIONAL THINKING LEARNING MODEL DEVELOPMENT OF BAYES' THEOREM

A. DECOMPOSITION

To remember and understand (1), instead of memorizing it, a simple decomposition of the problem can be made by considering two distinct scenarios.

Cogitate two progressive and statistically dependent random experiments "X" and "Y". Random experiment "X" can result in either outcome "X" or compliment "$\overline{X}$". Likewise random experiment "Y" can yield "Y" or compliment "$\overline{Y}$". The probabilities of all potential events can be illustrated through a tree diagram in Fig. 1. Likewise, initial execution is made for random experiment "Y" followed by the execution of "X". Fig.2 presents the methodology of X investigation after Y.

Using basic set theory and rules for the probability of the intersection of statistically dependent events, Fig. 1 and Fig. 2 can be related, and the following cases can be obtained.

Case 1:

$$Branch\ 1(Fig.\ 1.) = Branch\ 1(Fig.\ 2.)$$

$$P(X \cap Y) = P(Y \cap X)$$

$$P(X)\,P(Y \mid X) = P(Y)\,P(X|Y)$$

$$P(Y \mid X) = \frac{P(Y)\,P(X|Y)}{P(X)} \tag{2}$$

$$\text{Here } P(X) = P(Y)\,P(X \mid Y) + P\left(\overline{Y}\right)P\left(X \mid \overline{Y}\right) \tag{3}$$

Similarly,

$$P(X \mid Y) = \frac{P(X)\,P(Y \mid X)}{P(Y)} \tag{4}$$

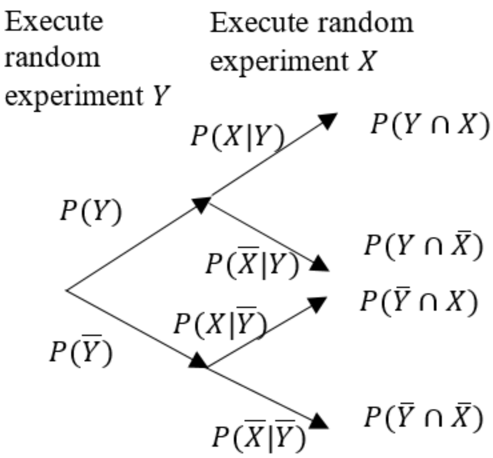

FIGURE 2. Investigating *X* after *Y*.

$$\text{Here } P(Y) = P(X)\, P(Y \mid X) + P\left(\overline{X}\right) P\left(Y \mid \overline{X}\right) \tag{5}$$

Case 2:

$$\textit{Branch } 2\,(\textit{Fig. } 1.) = \textit{Branch } 3(\textit{Fig. } 2.)$$

$$P\left(X \cap \overline{Y}\right) = P(\overline{Y} \cap X)$$

$$P(X)\, P\left(\overline{Y} \mid X\right) = P\left(\overline{Y}\right) P(X|\overline{Y})$$

$$P\left(\overline{Y} \mid X\right) = \frac{P\left(\overline{Y}\right) P(X|\overline{Y})}{P(X)} \tag{6}$$

$$\text{Here } P(X) = P(Y)\, P(X \mid Y) + P\left(\overline{Y}\right) P\left(X \mid \overline{Y}\right) \tag{7}$$

Similarly,

$$P\left(X \mid \overline{Y}\right) = \frac{P(X)\, P\left(\overline{Y} \mid X\right)}{P(\overline{Y})} \tag{8}$$

$$\text{Here } P\left(\overline{Y}\right) = P(X)\, P\left(\overline{Y} \mid X\right) + P\left(\overline{X}\right) P\left(\overline{Y} \mid \overline{X}\right) \tag{9}$$

Case 3:

$$\textit{Branch } 3\,(\textit{Fig. } 1.) = \textit{Branch } 2(\textit{Fig. } 2.)$$

$$P\left(\overline{X} \cap Y\right) = P(Y \cap \overline{X})$$

$$P\left(\overline{X}\right) P\left(Y \mid \overline{X}\right) = P(Y)\, P(\overline{X}|Y)$$

$$P\left(Y \mid \overline{X}\right) = \frac{P(Y)\, P(\overline{X}|Y)}{P\left(\overline{X}\right)} \tag{10}$$

$$\text{Here } P\left(\overline{X}\right) = P(Y)\, P\left(\overline{X} \mid Y\right) + P\left(\overline{Y}\right) P\left(\overline{X} \mid \overline{Y}\right) \tag{11}$$

Similarly,

$$P\left(\overline{X} \mid Y\right) = \frac{P\left(\overline{X}\right) P\left(Y \mid \overline{X}\right)}{P(Y)} \tag{12}$$

$$\text{Here } P(Y) = P(X)\, P(Y \mid X) + P\left(\overline{X}\right) P\left(Y \mid \overline{X}\right) \tag{13}$$

Case 4:

$$\textit{Branch } 4\,(\textit{Fig. } 1.) = \textit{Branch } 4(\textit{Fig. } 2.)$$

$$P\left(\overline{X} \cap \overline{Y}\right) = P(\overline{Y} \cap \overline{X})$$

$$P\left(\overline{X}\right) P\left(\overline{Y} \mid \overline{X}\right) = P\left(\overline{Y}\right) P(\overline{X}|\overline{Y})$$

$$P\left(\overline{Y} \mid \overline{X}\right) = \frac{P\left(\overline{Y}\right) P(\overline{X}|\overline{Y})}{P\left(\overline{X}\right)} \tag{14}$$

$$\text{Here } P\left(\overline{X}\right) = P(Y)\, P\left(\overline{X} \mid Y\right) + P\left(\overline{Y}\right) P\left(\overline{X} \mid \overline{Y}\right) \tag{15}$$

Similarly,

$$P\left(\overline{X} \mid \overline{Y}\right) = \frac{P\left(\overline{X}\right) P\left(\overline{Y} \mid \overline{X}\right)}{P\left(\overline{Y}\right)} \tag{16}$$

$$\text{Here } P\left(\overline{Y}\right) = P(X)\, P\left(\overline{Y} \mid X\right) + P\left(\overline{X}\right) P\left(\overline{Y} \mid \overline{X}\right) \tag{17}$$

Observing the aforementioned instances, it is noteworthy that (2), (4), (6), (8), (10), (12), (14) and (16) represent different expressions derived from the Bayes' theorem. The analysis above is focused on a straightforward scenario with only two outcomes in the execution of each random experiment. However, this can be expanded similarly to accommodate a higher level of complexity with more than two mutually exclusive outcomes.

B. PATTERN RECOGNITION AND PROBLEM ABSTRACTION

In all the Bayes theorem instances obtained from (2) to (16), many recurring patterns can be identified showing a common abstraction possible to various Bayes' probability problems. These common patterns are acknowledged as follows.

1) Bayes' probabilities are consistently identified within categorical variables encompassing at least two categories.
2) Within any Bayes' problem, the unknown probability invariably takes the form of a conditional probability. Moreover, the question or problem in hand will consistently furnish a counterpart in the reverse form of the unknown conditional probability.
3) The problem can always be decomposed into two probability trees. One tree will contain all the probabilities derived directly from the provided information, acting as an input or data tree. This tree will also contain the prior event. The unknown probability, as specified in the question, becomes a branch in this second tree and represents a conditional probability in it. This tree can be termed an unknown tree or an output tree. Representing and abstracting the problems through probability trees simplifies the numerical problem-solving process. After making these probability trees, it is now only required to identify the two intersections between known and unknown trees which are equal to each other. Subsequently, by applying algebraic manipulations, the probability of the unknown branch can be determined as outlined in (2) to (16).
4) Since the whole purpose of Bayes' theorem is to find how the prior probability of an event is updated in terms of posterior probability after seeing the data. Hence the final probability would always be determined such that it will change the prior in any direction. Any alteration in the prior results in the corresponding change in the posterior. This aspect/ pattern is crucial to consider when addressing Bayes' theorem problems. Prior is equivalent to the idea of perspective. In this context, different priors

or perspectives can also be tested to evaluate their impact on the posterior.

C. ALGORITHMIC THINKING

To streamline the process of solving Bayes' theorem probabilities, a methodical step-by-step approach reflecting algorithmic thinking can be devised.

The following steps are employed when the question provides probabilities instead of explicit datasets, which would otherwise provide counts for different categories of categorical variables.

i. Start with the given provided conditional probability. This conditional probability elucidates the initial random experiment and would help develop the known or input probability tree. For example, $P(X|Y)$ is interpreted first random experiment and has mutually exclusive events as Y or $\bar{Y}$ and these are then followed by the random experiment having mutually exclusive outcomes as X or $\bar{X}$.
ii. Now create a probability tree diagram based on the given conditional probability, such as $P(X \mid Y)$. All the other probability values mentioned in the question will be incorporated into this probability tree.
iii. In a similar way create an output probability tree. The output tree is the opposite of the probability tree made in (ii). I.e., the subsequent random experiment in the input probability tree would be the initial random experiment in the output tree, and the initial random experiment of the input tree would be the subsequent random experiment of the output tree.
iv. Identify the branch in the output probability tree that contains the unknown or required probability.
v. Now, establish the relationship between the intersecting branches of the trees depicted in (ii) and (iii) that encompass our unidentified conditional event. Accordingly, the required Bayes' probability can now be found using procedures outlined in (2) to (16).

In the case of explicit datasets containing counts of the categories, we make use of cross-tabulation tables and clustered bar charts for solving Bayes' theorem probabilities. This solution provides all the probabilities to complete the branches of unknown and unknown trees, hence solving Bayes' theorem probabilities.

D. WORKING EXAMPLE

Reference: Sedlmeir, [14].

"The probability that a woman who undergoes a mammography will have breast cancer is 1%. If a woman undergoing a mammography has breast cancer, the probability that she will test positive is 80%. If a woman undergoing mammography does not have cancer, the probability that she will test positive is 10%. What is the probability that a woman who has undergone a mammography actually has breast cancer if she tests positive?"

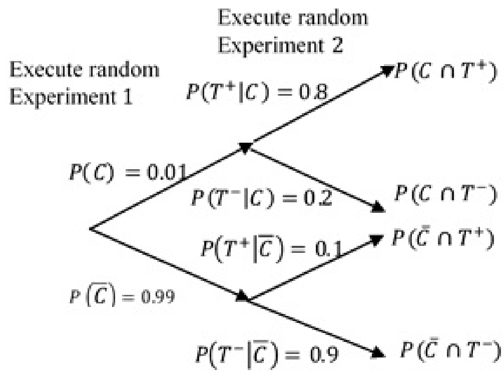

FIGURE 3. Probability tree for the given information.

Solution Steps:

Consider the information provided in the above scenario as follows:

C = Cancer is present
$\overline{C}$ = Cancer is not present
T^+ = Cancer is tested positive
T^- = Cancer is tested is negative

i. Following the given problem, we need to determine $P(C|T^+)$, which is the inverse of the provided information, $P(T^+| C)$. Thus, the task at hand involves solving this problem using Bayes' theorem.
ii. The given data shows that random experiments having mutually exclusive outcomes as C or $\overline{C}$ occurred earlier which is then subsequently followed by the random experiment then the event of T^+ or T^-. This leads us to the following probability tree diagram in Fig. 3. This is the known tree.
iii. The tree diagram, including the entailed conditional probability probabilities, is as in Fig. 4.
iv. Unknown probability is $P(C|T^+)$, whihx is in the second branch of Fig. 6 after the event T^+.
v. Relating two trees as stated in Section II,

$$P\left(C\cap T^{+}\right)=P\left(T^{+}\cap C\right)$$
$$P\left(C\right)P\left(T^{+}\mid C\right)=P(T^{+})P(C|T^{+})$$
$$P(C|T^{+})=\frac{P\left(C\right)P\left(T^{+}\mid C\right)}{P\left(T^{+}\right)} \tag{18}$$

Here,

$$P\left(T^{+}\right)=P\left(C\right)P\left(T^{+}\mid C\right)+P\left(\overline{C}\right)P\left(T^{+}\mid\overline{C}\right) \tag{19}$$

Utilizing Fig. 3. in (18) and (19),

$$P\left(C\mid T^{+}\right)=0.0747 \tag{20}$$

As can be seen from (20), there has been an increase in $P(C)$, i.e., chances of occurrence of cancer from 1% (given) to 7.47% when the test result is positive.

It is evident from Bayes'(1) and the discourse spanning from (2) to (20) that the numerical value of the posterior probability is closely linked to the prior. The notion of prior aligns seamlessly with the concept of perspective. Specifically in (2) and in all the subsequent instances from Bayes'(2)

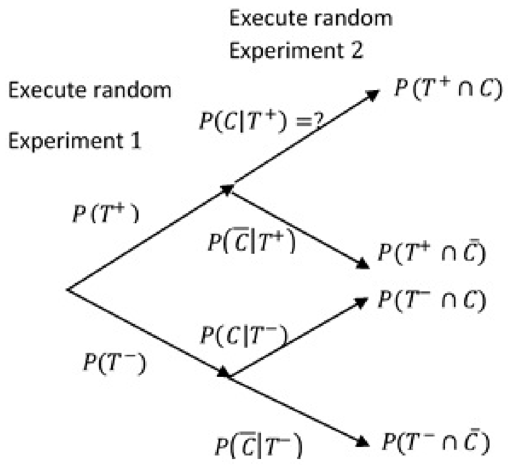

FIGURE 4. The unknown tree for given problem.

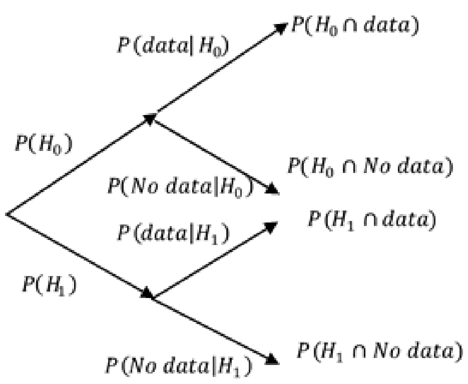

FIGURE 5. The known tree for the given problem.

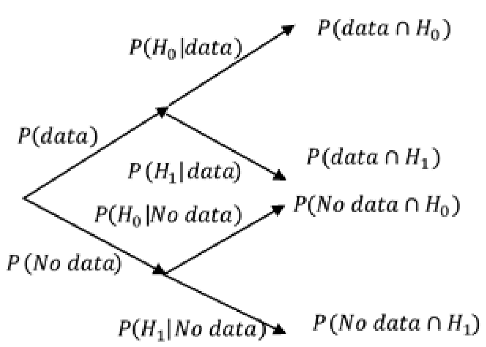

FIGURE 6. The unknown tree for the given problem.

to (20), the posterior is directly proportional and consequently equivalent to the product of the prior and an updating factor, as articulated in (21).

$$posterior = prior \times updating\ factor \tag{21}$$

From (21), the updating factor can be written as (22)

$$updating\ factor = \frac{likelihood}{evidence} \tag{22}$$

Hence (21) becomes (23) as in,

$$posterior = prior \times \frac{likelihood}{evidence} \tag{23}$$

It is evident from (23) that within the entire evidence curve, an increase in the area under the curve of likelihood corresponds to a higher updating factor for altering the prior. To grasp this concept, let us revisit the illustrated example. In it the determined posterior probability was as follows.

$$P\left(C \mid T^{+}\right) = \frac{P\left(C\right) P\left(T^{+} \mid C\right)}{P\left(T^{+}\right)}$$

Rewriting it to separate the prior and updating factor as in (23), gives (24),

$$P\left(C \mid T^{+}\right) = P\left(C\right) \times \frac{P\left(T^{+} \mid C\right)}{P\left(T^{+}\right)} \tag{24}$$

Substituting $P\left(T^{+}\right)$ from (19) gives (25) as in,

$$P\left(C \mid T^{+}\right) = P\left(C\right) \times \frac{P\left(T^{+} \mid C\right)}{P\left(C\right) P\left(T^{+} \mid C\right) + P\left(\overline{C}\right) P\left(T^{+} \mid \overline{C}\right)} \tag{25}$$

Concentrating on the fraction indicating the updating factor in (25), it is apparent that the event of T^{+} can occur either after C or $\overline{C}$ events. Its intersection is present in both of them. Furthermore, evidence supporting the outcome C will also increase the product of $P\left(C\right)$ and $P\left(T^{+} \mid C\right)$. Hence, the fraction indicating the updating factor will increase. On the contrary, evidence supporting the outcome $\overline{C}$ will increase the product of $P\left(\overline{C}\right)$ and $P\left(T^{+} \mid \overline{C}\right)$. This will decrease the value of the updating factor. Therefore, the effect of increasing and decreasing the updating factor alters the prior respectively. This change in prior in the presence of evidence is depicted in the form of posterior probability.

III. POSTERIOR AS A DISTRIBUTION

It is to be noted that the known or input tree in Fig. 1. has a parameter in the first execution and then the data in the second execution. On the contrary, in Fig. 2. i.e., the unknown or output tree has data in the first execution and parameters in the second execution. The parameter is a quantity from the population that we are always interested in finding from the given data that we collect. Since it is unknown, hence it is estimated statistically and this estimation process is called statistical inference. It is interesting to note that although Bayes (1) to (17) can be mathematically manipulated in any way to find any conditional probability, the most obvious usage is to find a conditional probability that lies on one of the branches of an unknown tree. Explicitly using Bayes' theorem in this way reflects the idea of posterior probability as an estimated value of the parameter. The notion of Bayesian thinking is, that we cannot estimate its value from just nothing. There should be a perspective or prior available to have an estimation about the parameter. This estimated parameter value, in actuality, is called the posterior. In this regard, the known tree provides the prior information. It also provides us with evidence of a total marginal effect. Therefore, the known tree can also be called the prior tree. It is understandable that an unknown parameter cannot fully depend on prior information. However, there will be a degree of dependency reflecting its uncertainty. Hence a prior must be defined by a probability distribution.

The data fitting within the population defined by the parameter has a certain likelihood. Likelihood is a conditional probability of finding data or test statistics given the parameter in the prior tree. It tells how well the data or test statistics fit in the population defined by the parameter and is obtained from the likelihood function of the parameter. However, it's not a probability distribution function or PDF because its total area under the curve does not equal one. Multiplying the prior with the likelihood transforms the latter into a PDF. It peaks on the maximum likelihood value, which defines the median of the likelihood function. Using this median as a central value an interval can be defined on the PDF to contain the required population parameter. This interval is called the *credible interval or CI.*

The more informative the prior, the more posterior distribution for parameter estimation will follow the prior. The more uninformative the prior, the more posterior distribution will follow the likelihood of the data. An informative prior means we have a good idea of the parameter of interest. Hence posterior depends on the prior makes in a logical way.

In Bayesian statistics, there isn't typically a focus on finding a single true value for a parameter. Instead, Bayesian analysis provides a framework for quantifying uncertainty about parameters by representing them as probability distributions. Based on observed data through Bayes ' theorem, these distributions incorporate prior beliefs and update them. The result is a posterior distribution representing the updated uncertainty about the parameter after considering the data. This approach acknowledges and quantifies uncertainty rather than aiming for a single true value. [15].

In frequentist statistics, parameters are fixed quantities, whereas in Bayesian statistics, the true value of a parameter can be thought of as being a random variable to which a probability distribution is assigned. Hence, giving the final answer as a point value is not coherent with the idea of Bayesian statistics. According to Bayesian statistics, an interval showing some percentage of occurrence of the posterior outcome w.r.t to prior follows the Bayesian thinking, and only then can predictions and decision-making be made. In this context of statistical inference, a parameter has a certain degree of association between both the testing population and the alternative population. Updating factor containing the likelihood information in (22), besides updating the prior also defines the fitness of data in the population of interest w.r.t to the other (alternative) population.

IV. POSTERIOR ESTIMATION COMPUTATION MODEL

The ideas discussed in the previous section will now be simulated to understand how change in priors can affect the posterior and, more importantly, how Bayesian inferencing works to accomplish this task. The working example discussed previously in (II) can be taken again for this purpose. Our objective is to estimate a range of values of $P(C)$. This is the unknown parameter of the population. To estimate its values, we need to have observable data. Afterward, an appropriate statistical model can be used to relate the observable data to the unknown population parameter. The statistical model must be probabilistic. In this example, we used diagnostic tests to check the change in the probability of the event of detecting cancer, i.e., equation sits complement $\bar{C}$. Therefore, diagnostic tests are observable data. It can be either obtaining positive tests, i.e. $T^+ \left| C, T^+ \right| \bar{C}$ or negative tests, i.e. $T^-|C, T^-|\bar{C}$. Out of this, for illustration purposes, we consider only one observable data, i.e. $T^+|C$.

We want to see how the number of obtaining $T^+|C$ change the $P(C)$ in the population. A mathematical model that can be used to relate the unknown parameter $P(C)$ w.r.t no. of $T^+|C$ can be a Binomial distribution model from a Bernoulli process. The assumptions satisfying the Bernoulli process for this scenario are 1) Event of obtaining C is independent 2) Either we can have C or $\bar{C}$ as success or failure and 3) $P(C)$ and $P(\bar{C})$ (either of which can be success or failure) have a constant probability. Since we are interested in no. of successes of $(T^+|C)$ out of a given no. trials from an experiment that follows a Bernoulli process, hence Binomial distribution model can be used to relate the observed data $(T^+|C)$ with the unknown parameter value (C). The objective is to estimate a range of values of $P(C)$ using the observed data follows a model comparison process using Bayesian inferencing. The BCLM we made for Bay's theorem in the previous section (II) for performing estimations as a statistical inference problem can be obtained as follows.

A. PROBLEM DECOMPOSITION

1. We start with the initial hypotheses, the null hypothesis (H_0) and alternate hypothesis (H_1)
2. Using Bayes' theorem (1 to 17), We can find $P(H_0|data)$ and $P(H_1|data)$. This would mean making known and unknown probability trees as done previously in Fig. 5. and Fig. 6.
3. The Bayes' theorem (1) can be generalized for any hypothesis using appropriate intersections from Fig.5. and Fig. 6. as in (26)

$$P(H|data) = P(H) \times \frac{P(data|H)}{P(data)} \tag{26}$$

Here,
$P(H|data) = posterior$ → degree of belief in H after observing the data.
$P(H) = prior$ → degree of belief in H before observing the data.
$P(data|H) = likelihood$ → the degree to which observed data is likely under H.
$P(data) = prior\ predictive$ → weighted average of probabilities of observing data under all models being considered.

4. We start by writing Bayes' theorem for two models, H_0 and H_1 as in (27) and (28)

$$P(H_0|data) = P(H_0) \times \frac{P(data|H_0)}{P(data)} \quad (27)$$

$$P(H_1|data) = P(H_1) \times \frac{P(data|H_1)}{P(data)} \quad (28)$$

5. In order to compare the model in (27) and (28), posterior odd is formulated as in (29) and (30).

$$\frac{P(H_0|data)}{P(H_1|data)} = \frac{P(H_0)}{P(H_1)} \times \frac{P(data|H_0)}{P(data|H_1)} \quad (29)$$

In (29),
$\frac{P(data|H_0)}{P(data|H_1)}$ → is called updating factor or Bayes Factor (BF_{01})
$\frac{P(H_0)}{P(H_1)}$ → is called prior odds

Hence (29) can also be written as in (30)

$$\frac{P(H_0|data)}{P(H_1|data)} = \frac{P(H_0)}{P(H_1)} \times BF_{01} \quad (30)$$

i.e.

$$Posterior\ odds_{01} = prior\ odds_{01} \times Bayes\ Factor_{01}$$

Posterior odds defined in (29) are for H_0 compared to H_1, i.e. how likely H_0 is true compared to H_1 with the given data or test statistics. To formulate posterior odds H_1 compared to H_0, i.e. how likely H_1 is true compared to H_0 with the given data or test statistics.

$$\frac{P(H_1|data)}{P(H_0|data)} = \frac{P(H_1)}{P(H_0)} \times \frac{P(data|H_1)}{P(data|H_0)} \quad (31)$$

Or,

$$\frac{P(H_1|data)}{P(H_0|data)} = \frac{P(H_1)}{P(H_0)} \times BF_{10} \quad (32)$$

i.e.

$$Posterior\ odds_{10} = prior\ odds_{10} \times Bayes\ Factor_{10}$$

Equations (30) and (32) are strictly comparisons between two models defined by null hypothesis and alternate hypothesis. These modeling give two conceptual definitions for the Bayes factor: 1) the factor by which the observed data is more likely under one model/ hypothesis compared to the other, and 2) the factor by which the prior odds between models are updated after observing data. Odds are ratios of probabilities. The prior odds can be conceptualized such that e.g. for odds to 1 : 1 → $\frac{P(H_0)}{P(H_1)} = 1$. This means both models are equally likely. Similarly, if the odds are 3 : 1 → $\frac{P(H_0)}{P(H_1)} = 3$. This means H_0 is 3 times more likely than H_1. These are a priori knowledge. Required posterior probability can now be easily determined as the subsequent CT steps explain.

B. PATTERN RECOGNITION AND ABSTRACTION

After obtaining posterior odds, posterior probability can be abstracted and estimated using the pattern obtained from model comparison, (30) and (32). A simple derivation can be illustrated from (33) to (37) as,

Using axioms of probability theory.

$$P(H_0/data) + P(H_1/data) = 1 \quad (33)$$

Therefore,

$$P(H_1/data) = 1 - P(H_0/data) \quad (34)$$

This gives,

$$Posterior\ odds_{01} = \frac{P(H_0/data)}{1 - P(H_0/data)} \quad (35)$$

A little algebraic manipulation gives (36)

$$P(H_0/data) = \frac{Posterior\ odds_{01}}{1 + Posterior\ odds_{01}} \quad (36)$$

Likewise,

$$P(H_1/data) = \frac{Posterior\ odds_{10}}{1 + Posterior\ odds_{10}} \quad (37)$$

In general,

$$Posterior\ probability = \frac{Posterior\ odds}{1 + Posterior\ odds} \quad (38)$$

C. ALGORITHMIC THINKING

Abstraction of the problem suggests that the Bayes Factor or BF is the key idea in Bayesian inferencing. Using this, both posterior odds and posterior probability can be determined easily. Bayes Factor can be found for both BF_{01} and BF_{10} case. Algorithmic intuition can be developed using the following steps.

1. Decide upon prior odds or prior. General practice is to start with a uniform prior, i.e., 1:1. Hence, both models are equally probable.
2. For a particular Bayes Factor (i.e. either for BF_{01} and BF_{10}), start by plotting prior and posterior on the same plot.
3. Find the value on the vertical axis for $P(H)$, which is $P(H_0)$ on both the prior and posterior curves for BF_{01} and $P(H_1)$ on both the prior and posterior curves for BF_{10}. Bayes Factor can be found by finding the magnitude of change from before posterior values of $P(H)$. The values of prior and posterior can be read directly from their corresponding curves in this plot.
4. For example, for BF_{01} case, depending on whether prior or posterior has a higher value for the testing value of $P(H_0)$, will suggest if given data has increased or decreased our belief in the testing model/ hypothesis ($P(H_0)$ in this case).
5. After finding the Bayes Factor, apply (36) or (37) to find posterior probability and its distribution in a certain credible interval (CI).

D. WORKING EXAMPLE

"The probability that a woman who undergoes a mammography will have breast cancer is 1%. If a woman undergoing a mammography has breast cancer, the probability that she will test positive is 80%. If a woman undergoing mammography does not have cancer, the probability that she will test positive is 10%. What is the probability that a woman who has undergone a mammography actually has breast cancer if she tests positive?"

Solution Steps:

Consider the information provided in the above scenario as follows:

C = Cancer is present
$\overline{C}$ = Cancer is not present
T^+ = Cancer is tested positive
T^- = Cancer is tested is negative

It is required to find the probability of detecting cancer provided memmpgraphy is obtained positive, i.e. $P(C|T^+)$.

Earlier, we solved this problem using Bayes'theorem. This time, we will apply Bayesian inferencing methods. $P(C)$ is the parameter we want to obtain using the data which is having a postitive mammography test, i.e. $P(C|T^+)$ is required. Previously there was just $P(T^+|C)$ available. However, we now want to solve this as an inferential problem by estimating the unknown parameter $P(C)$ using prior information about it as provided in the question and mammography test results as data. For this, we need a sample of data containing a no. of $(T^+|C)$ in the total sample size. Hence from the given data, we can then infer $P(C)$ which is the same as finding $P(C/T^+)$. The ultimate goal is to find how data or evidence is going to change the chance of occurance of the event. As discussed in developing the BCLM, we proceed using the following steps.

1. We start by conducting a model comparison using (29), i.e. finding $\frac{P(H_0/data)}{P(H_1/data)} \rightarrow Posterior\ odds$

We need two ingredients to complete the recipe of finding posterior odds: 1) prior odds and 2) Bayes Factor. For prior odds, we make the following hypotheses;

$$P(H_0) = P(C) = 0.01$$
$$P(H_1) = P(\bar{C}) \neq 0.01$$

These hypotheses are developed using the information provided in the given working problem. We aim to compare H_0 against H_1, hence we will use $\frac{P(H_0)}{P(H_1)}$ as our prior odds and BF_{01} as Bayes Factor. To perform this we computer simulations on JASP®.

2. In order to apply Bayes inferencing to find the uncertainty in $P(C)$ using the positive test results, we need to have a probability distribution to model this uncertainty. As explained earlier, getting a cancer patient in the trial can be taken as Bernoulli trials, hence our unknown population parameter $P(C)$ can be taken as the following Binomial PDF. We make a few hypothetical assumptions to simulate this idea and also perform the algorithmic thinking specified in (B and C). These are given as case 1, case 2, and case 3. Alterations

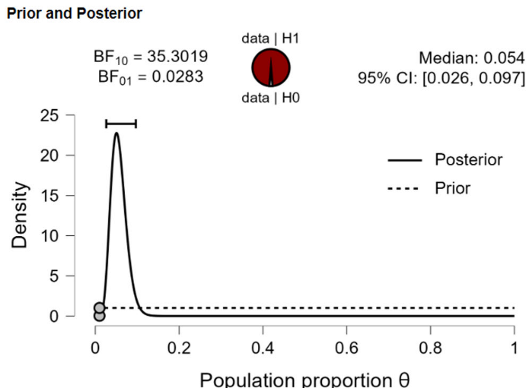

FIGURE 7. Bayesian binomial test with uniform prior.

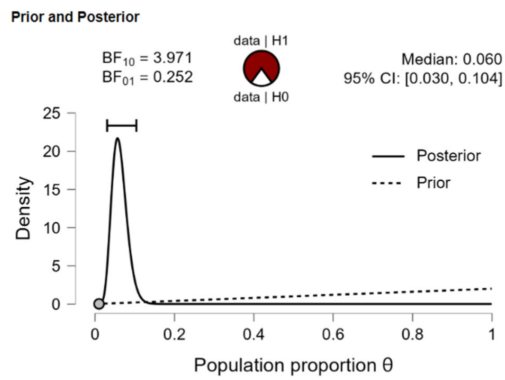

FIGURE 8. Bayesian binomial test with non-uniform prior (2:1).

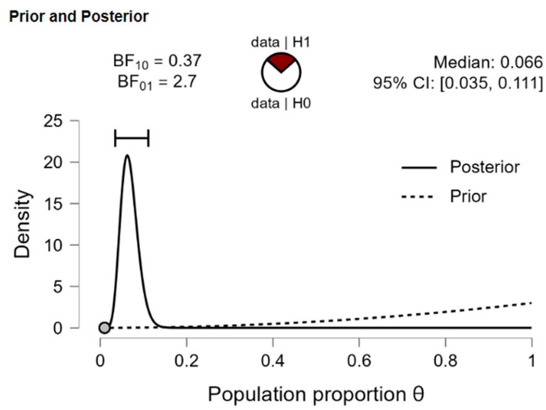

FIGURE 9. Bayesian binomial test with non-uniform prior (3:1).

in the cases are made by trying three different priors. In all cases, simulations were made considering there were in total 158 subjects and 8 were successes, i.e. having $(T^+|C)$.

In all the simulated three cases, BF_{01} is changed by changing prior. This shows prior effects significantly on posterior probability. Progression from cases 1, 2 to 3, shows how BF_{01} changes our belief in H_0 from low to high value. Alongside Fig. 7, Fig. 8 and Fig. 9 also indicated the estimated posterior probability values at 95% CI. Posterior distribution follows the likelihood shape of Binomial distribution for this particular example, with a peak at $P(H_0) = P(C) = .01 or 1\%$.

***Case* 1:*Uniform Prior* (1:1)**
***Case* 2:*Non Uniform Prior* 2:1**
***Case* 3:*Non Uniform Prior* (3:1)**

V. DISCUSSION

The study presented in this paper investigates the integration of Bayes' theorem within the framework of computational thinking (CT) to enhance probabilistic reasoning. This approach aims to bridge the gap between abstract statistical concepts and practical computational skills, fostering a more intuitive and robust method for data analysis and decision-making processes. The research highlights several key contributions. First, it revisits Bayes' theorem, emphasizing its application as a computational thinking problem, which helps demystify the theorem and make it more accessible to learners. Second, it explores how the four components of CT—decomposition, pattern recognition, abstraction, and algorithmic thinking—can be utilized to understand and solve probabilistic scenarios. By decomposing complex problems into manageable parts, recognizing patterns, abstracting key concepts, and developing algorithmic solutions, the study provides a structured approach to Bayesian analysis.

Furthermore, the research develops a Bayesian CT learning model that enhances both descriptive and inferential statistical learning. This model serves as a practical guide for implementing Bayesian methods within a CT framework, making statistical learning more interactive and engaging. The study also examines the alignment of Bayesian CT with Bloom's cognitive learning paradigm and the STEM learning paradigm, ensuring that the educational impact of Bayesian learning is maximized across different cognitive levels and STEM fields.

STEM education incorporates computational thinking as a fundamental skill, especially in technology and engineering fields [16], [17]. The BCLM model proposed in this work encourages students to approach problems systematically and analyze the data using algorithmic working steps, which are essential skills in STEM disciplines. STEM paradigm also aligns with the cognitive levels of BT. Researchers believe that the top four levels of cognitive BT (applying, analyzing, evaluating, and creating) are best reflected in STEM-based lessons and classes. The proposed BCLM allows students to apply Bayes' formula (1) to any relevant scenario if it can be mapped and fit with the requirements and conditions for BCLM. This provides students with a drill to analyze, apply, and evaluate which problem is actually a Bayesian question and which is not. These working methodologies alleviate students beyond remembering and understanding to higher degrees of the cognitive learning ladder. The significant focus of STEM is also critical thinking and being able to evaluate multiple solutions to a problem and finally find the best fit for the problem. Hence, STEM and cognitive levels of BT are well groomed in the proposed BCLM. Hence, by using BCLM, instructors can enterprise STEM-based lessons, CEPs, and PBL activities that incorporate computational thinking stratagems aligned and progressing with Bloom's Taxonomy for cognitive learning.

In essence, this research offers a comprehensive understanding of how Bayesian theory can be revisited and applied through computational thinking, providing practical tools and models for educators and practitioners in various fields. This work not only contributes to the academic understanding of Bayesian computational thinking but also promotes a more holistic and practical approach to learning and problem-solving in probabilistic contexts.

Algorithmic thinking outlined in Section II summarizing the proposed BCLM as descriptive statistics, is presented in the pseudocode as follows:

```
BEGIN

// Start with the given conditional probability
INPUT conditionalProbability

// Interpret the initial random experiment
initialExperiment < − INTERPRET conditionalProbability

// Create input probability tree based on the given conditional probability
inputProbabilityTree <
−CREATE_PROBABILITY_TREE(conditionalProbability)

// Incorporate all given probability values into the input probability tree
INCORPORATE_VALUES(inputProbabilityTree, givenProbabilities)

// Create output probability tree (reverse of input tree)
outputProbabilityTree <
− CREATE_OUTPUT_PROBABILITY_TREE(
inputProbabilityTree)

// Identify the branch in the output probability tree that contains the unknown or required probability
unknownProbabilityBranch <-
IDENTIFY_BRANCH(outputProbabilityTree, unknownProbability)

// Establish the relationship between intersecting branches of the input and output trees
relationship <- ESTABLISH_RELATIONSHIP
(inputProbabilityTree, outputProbabilityTree, unknownProbabilityBranch)

// Calculate the required Bayes' probability using established relationships
requiredBayesProbability <-
CALCULATE_BAYES_PROBABILITY(relationship, procedures)
// Output the required Bayes' probability
OUTPUT requiredBayesProbability

END
```

Algorithmic thinking outlined in Section IV summarizing the proposed BCLM as inferential statistics, is presented in the pseudocode as follows:

```
BEGIN

// Step 1: Decide upon prior odds or prior
priorOdds <- 1:1 // Uniform prior
```

```
// Step 2: Plot prior and posterior for Bayes Factor (BF_01 and BF_10)
PLOT priorCurve, posteriorCurve

// Step 3: Find the value on the vertical axis for P(H) for both prior and posterior curves
priorP_H0 <- FIND_VALUE(priorCurve, P(H_0))
posteriorP_H0 <- FIND_VALUE(posteriorCurve, P(H_0))
priorP_H1 <- FIND_VALUE(priorCurve, P(H_1))
posteriorP_H1 <- FIND_VALUE(posteriorCurve, P(H_1))

// Step 4: Calculate Bayes Factor by finding the magnitude of change from prior to posterior values
IF PLOT_TYPE == "BF_01" THEN
   bayesFactor < −
CALCULATE_MAGNITUDE_CHANGE(priorP_H0, posteriorP_H0)
ELSE
   bayesFactor < −
CALCULATE_MAGNITUDE_CHANGE(priorP_H1, posteriorP_H1)
END IF

// Step 5: Analyze whether belief in the hypothesis has increased or decreased
IF priorP_H0 < posteriorP_H0 THEN
   beliefChange <- "increased"
ELSE
   beliefChange <- "decreased"
END IF

// Step 6: Apply formulas (36) or (37) to find posterior probability and its distribution
posteriorProbability <- APPLY_FORMULA(bayesFactor, formula_36_or_37)
credibleInterval < −
FIND_CREDIBLE_INTERVAL(posteriorProbability)

// Output the results
OUTPUT bayesFactor, posteriorProbability, credibleInterval, beliefChange

END
```

VI. CONCLUSION

In this work, we proposed a Bayesian Computational Learning Model as an auxiliary tool to learn better and understand Bayesian statistics. We have produced a STEM-based learning paradigm in BCLM, simultaneously satisfying the needs of BT for cognitive learning inclusive of remembering, understanding, applying, analyzing, evaluating, and creating. The STEM and BT accomplishment is made possible using computational thinking (CT). STEM education, Bloom's Taxonomy, and computational thinking are interconnected through their shared focus on fostering critical thinking, problem-solving skills, and analytical abilities. Computational thinking aligns with various levels of BT. For instance, remembering and understanding involve recognizing patterns and understanding algorithmic concepts, while applying and analyzing in BCLM require students to use computational thinking to solve the problem at hand and evaluate solutions. Furthermore, creating involves designing innovative solutions in any domain of study using BCLM.

We anticipate that the proposed BCLM for learning Bayesian statistics will have a significant impact on Engineering Education in the paradigm of data sciences and analysis. However, at present there are limitations leading to future directions of the research.

The Bayesian Computational Learning Model (BCLM) may be challenging for students with limited prior knowledge of Bayesian statistics or computational thinking. The complexity of integrating multiple disciplines (STEM, Bloom's Taxonomy, and computational thinking) might overwhelm beginners. Effective implementation of BCLM in diverse educational settings requires significant resources, including trained educators, appropriate technological infrastructure, and curriculum adjustments. Schools with limited resources may find it difficult to adopt and sustain this model. Assessing students' performance in such an integrated learning model can be complex. Traditional assessment methods might not fully capture the development of skills in computational thinking and the application of Bayesian statistics. The model's effectiveness may vary across different fields of study and educational levels. While it shows promise in engineering and data sciences, its adaptability and impact in other domains need further investigation. The process of mastering both Bayesian statistics and computational thinking within this integrated framework can be time-consuming, potentially leading to slower progress in the initial stages of learning.

Future research should focus on developing comprehensive curricula that gradually introduce Bayesian statistics and computational thinking concepts, tailored to different educational levels and fields of study. Establishing professional development programs to train educators in implementing BCLM effectively will be crucial. These programs should cover both the theoretical aspects and practical applications of the model. Focus must be made to investigate scalable strategies for implementing BCLM in diverse educational settings, particularly in resource-constrained environments. This could involve developing low-cost technological solutions and creating adaptable teaching materials. Concentration is required to develop innovative assessment tools that can accurately measure the development of students' skills in computational thinking and Bayesian statistics within the BCLM framework. These tools should be able to capture both the cognitive and practical aspects of learning. Research should be initiated to conduct longitudinal studies to evaluate the long-term impact of BCLM on students' learning outcomes and career trajectories. This will provide valuable insights into the model's effectiveness and areas for improvement. It is entailed to explore the application of BCLM in other interdisciplinary fields beyond engineering and data sciences. Understanding how this model can be adapted and applied in various contexts will enhance its utility and impact.

Leverage can be initiated in advancements in educational technology, such as artificial intelligence and virtual reality, to create immersive learning environments that support the BCLM framework. These technologies can provide personalized learning experiences and enhance students' engagement.

By addressing these limitations and pursuing these future directions, the potential of the Bayesian Computational Learning Model to transform education, particularly in the context of data sciences and engineering, can be fully realized.

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