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RESEARCH ARTICLE

Hermite-Hadamard Type Inequalities and Convex Functions in Signal Processing

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ABSTRACT This article explores the integration of HermiteHadamard Type Inequalities and convex functions within the domain of signal processing, elucidating their theoretical underpinnings and practical implications. Beginning with a comprehensive background, we focus on the historical context and foundational concepts that underlie these mathematical constructs. Our discussion progresses to articulate the problem formulation, delineating the specific challenges and objectives addressed in the study. The theoretical framework elucidates the HermiteHadamard Type Inequalities, highlighting their mathematical formulations, properties, and fundamental proofs. Concurrently, the discourse unfolds the theory and properties of convex functions, elucidating their significance and applications within signal processing paradigms. With a focus on applications, we illustrate the utility of Hermite-Hadamard Type Inequalities and convex functions in signal processing tasks. Through empirical studies and case examples, we demonstrate their efficacy in signal denoising, compression, and feature extraction, showcasing tangible results and comparative analyses. We discuss the challenges and limitations inherent in the application of these mathematical constructs in real-world scenarios, thereby paving the way for future research directions and advancements. Finally, we conclude by summarizing the key insights gleaned from our exploration and underscore the profound implications of Hermite-Hadamard Type Inequalities and convex functions in shaping the landscape of contemporary signal processing methodologies.

INDEX TERMS Hermite-Hadamard type inequalities, convex functions, signal processing, mathematical constructs, applications and theory.

I. INTRODUCTION

Hermite-Hadamard Type Inequalities and Convex Functions stand as two pillars in the realm of mathematics, each with its own profound implications and applications. Together, they form a formidable framework that finds extensive utility in signal processing and numerous other mathematical domains. In this introduction, we embark on a journey to unravel the essence of Hermite-Hadamard Type Inequalities and the significance of convex functions, shedding light on their crucial roles in mathematics and their wide-ranging applications in signal processing. Hermite-Hadamard Type Inequalities trace their origins back to the seminal work of Charles Hermite and Jacques Hadamard, two luminaries whose contributions to mathematical analysis have left an indelible mark on the discipline. These inequalities represent a class of mathematical expressions that offer profound insights into the properties of real-valued functions defined on closed intervals [1]. At their core, Hermite-Hadamard Type Inequalities provide bounds on the integral means of functions, encapsulating crucial information about their behavior and distribution across intervals. The significance of Hermite-Hadamard Type Inequalities transcends mere mathematical abstraction; their practical utility extends to diverse fields, including physics, engineering, and, notably, signal processing [2], [3], [4]. In the realm of signal processing, where the analysis and manipulation of signals form the cornerstone of numerous applications, Hermite-Hadamard Type Inequalities serve as invaluable tools for characterizing signal properties and optimizing signal

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processing algorithms. By providing rigorous bounds on integral means, these inequalities empower signal processing practitioners to make informed decisions about signal representation, filtering, and reconstruction. Complementing the elegance of Hermite-Hadamard Type Inequalities is the concept of convex functions, a cornerstone of mathematical analysis with farreaching implications. A function is deemed convex if the line segment between any two points on its graph lies above the graph itself, a property that imbues convex functions with remarkable stability and optimality. Convex functions manifest ubiquitously across mathematical landscapes, exerting their influence in optimization, geometry, and, crucially, signal processing. In the context of signal processing, convex functions play a multifaceted role, underpinning a myriad of algorithms and methodologies that underlie signal analysis and reconstruction [5]. The importance of convexity lies not only in its theoretical elegance but also in its practical ramifications for signal processing tasks such as denoising, compression, and feature extraction. By leveraging the convex structure of signal representations, practitioners can devise efficient algorithms that yield optimal solutions with provable guarantees, thereby enhancing the robustness and efficiency of signal processing systems. Convex functions facilitate a deeper understanding of signal characteristics and enable practitioners to navigate the complex landscape of signal analysis with confidence and precision. The convexity of signal representations confers desirable properties such as stability, uniqueness, and scalability, attributes that are indispensable for tackling real-world signal processing challenges [6].

In this paper, Bin-Mohsin and colleagues introduce new Hermite-Hadamard type inequalities that leverage harmonic convex, strongly harmonic convex, strongly harmonic log-convex functions, and AH-convex functions within the framework of quantum calculus. The study aims to extend ordinary calculus cases as the parameter q tends to 1, emphasizing the generalization and applicability of these inequalities across2 different mathematical contexts. The research not only contributes to the theoretical development of Hermite-Hadamard inequalities but also offers insights into their connections with quantum calculus, opening avenues for further exploration in harmonic convexity and related concepts [7]. Mehreen and Anwar present new Hermite-Hadamard type inequalities through exponentially (p, h)-convex functions, introducing a novel class of convex functions to the mathematical discourse. The paper extends existing results in convex analysis by leveraging the properties of exponentially convex functions, thereby enriching the theoretical framework of Hermite-Hadamard inequalities. By introducing and exploring the properties of exponentially (p, h)-convex functions, the study offers fresh perspectives on convexity and its implications for inequalities in mathematical analysis [8]. Okur and Aliyev investigate Hermite-Hadamard type integral inequalities for multidimensional log-convex stochastic processes, shedding light on an important class of stochastic processes with applications in various domains. By introducing and studying the properties of multidimensional log-convex stochastic processes, the authors contribute to the theoretical understanding of stochastic processes and their associated inequalities. The research lays the groundwork for further exploration of Hermite-Hadamard inequalities in the context of stochastic analysis, offering valuable insights into the behavior of log-convex stochastic processes [9]. Larson's study establishes a sharp multidimensional Hermite-Hadamard inequality, providing a rigorous mathematical framework for analyzing the integral means of non-negative subharmonic functions. By proving the sharpness of the inequality and improving upon previous results, the research enhances our understanding of Hermite-Hadamard inequalities and their applications in mathematical analysis. The findings contribute to the theoretical foundation of integral inequalities, offering new perspectives on the behavior of subharmonic functions in multidimensional settings [10]. Khan and colleagues introduce a novel bound for the Jensen gap, extending the theoretical understanding of convex functions and their applications in information theory. By deriving new converses of the Holder inequality and presenting applications'' in information theory, the study demonstrates the versatility and utility of the proposed bound. Through numerical experiments and theoretical analysis, the research provides insights into the Jensen gap and its implications for informationtheoretic principles, contributing to the broader discourse on convex analysis and information theory [11]. Jin, Wang, and Liu address the challenge of real-time signal processing in the context of synthetic aperture radar (SAR) systems. They propose a flexible and high-performance real-time SAR signal processing system based on the TI's latest multi-core DSP TMS320C6678. The system architecture demonstrates strong computational ability, stability, and adaptability to various imaging algorithms, making it suitable for multi-mode, multipolarization, multi-resolution space-borne, and airborne realtime SAR imaging systems. The study showcases the potential of the proposed system to achieve real-time performance in SAR signal processing applications [12]. Gan, Seth, and Kuo present a versatile and portable digital signal processing (DSP) platform suitable for learning embedded signal processing. Based on the Texas Instruments VC5505 eZDSP USB Stick, the platform features internal fast Fourier transform (FFT) hardware accelerators and programmable high-speed codecs, facilitating real-time DSP activities beyond traditional laboratory environments. The authors highlight project examples utilizing this platform, showcasing its effectiveness in learning real-time embedded signal processing [13]. Zhang, Wu, Wang, and Qiao address the real-time signal processing challenges in FM-based passive bistatic radar (PBR) systems. They develop a signal processing architecture fully deployed on Graphics Processing Units (GPUs) using Compute Unified Device Architecture (CUDA). The parallelism of the algorithms allows data processing tasks from multiple carrier frequencies on one NVIDIA Tesla C2075, achieving significant speedups over standard Central Processing Units (CPUs). The study demonstrates the flexibility and performance of GPU-based signal processing for FM-based PBR systems, with potential applications in radar and wireless communications [14].

The marriage of Hermite-Hadamard Type Inequalities and convex functions heralds a new frontier in signal processing, where mathematical rigor converges with practical utility to unlock the full potential of signal data. As we embark on this journey of exploration, it becomes evident that the synergy between these mathematical constructs transcends disciplinary boundaries, permeating through the fabric of modern science and engineering. In the subsequent sections of this article, we focus deeper into the theoretical foundations of HermiteHadamard Type Inequalities and convex functions, unraveling their intricacies and elucidating their applications in signal processing. Through a systematic exposition of theory, methodology, and application, we seek to unravel the mysteries that lie at the intersection of mathematics and signal processing, illuminating pathways towards novel discoveries and transformative innovations.

II. BACKGROUND

Hermite-Hadamard Type Inequalities and convex functions constitute foundational concepts in mathematics and signal processing, offering profound insights into the behavior of functions and their applications across diverse domains. The historical development of these concepts traces back to the pioneering works of mathematicians such as Charles Hermite, Jacques Hadamard, and others, whose contributions have shaped modern mathematical analysis. The significance of Hermite-Hadamard Type Inequalities lies in their ability to provide rigorous bounds on integral means of functions defined on closed intervals. These inequalities serve as powerful tools for characterizing the properties of functions and establishing fundamental relationships between their integral and pointwise behaviors. Originally formulated within the framework of real analysis, Hermite-Hadamard Type Inequalities have found wide-ranging applications in areas such as calculus of variations, optimization theory, and mathematical physics. Similarly, convex functions have emerged as fundamental objects of study in mathematical analysis, offering a rich interplay between geometry and analysis. A function is deemed convex if the line segment between any two points on its graph lies3 above the graph itself. This property endows convex functions with desirable properties such as monotonicity, stability, and optimality, making them indispensable tools in various mathematical disciplines. The motivation behind studying HermiteHadamard Type Inequalities and convex functions stems from their pervasive presence and utility across different fields of science and engineering. In signal processing, for instance, these mathematical concepts play a central role in shaping the theoretical foundations and practical methodologies employed in analyzing and manipulating signals. From denoising and compression to feature extraction and pattern recognition, Hermite-Hadamard Type Inequalities and convex functions offer powerful frameworks for understanding signal behavior and designing efficient algorithms for signal processing tasks. The study of Hermite-Hadamard Type Inequalities and convex functions transcends disciplinary boundaries, finding applications in diverse areas such as economics, biology, and information theory. In economics, convexity assumptions underpin key models in microeconomics and optimization theory, providing insights into consumer behavior and market equilibrium. In biology, convex optimization techniques are utilized for modeling complex biological systems and analyzing large-scale biological data sets [15]. To focus deeper into the intricacies of Hermite-Hadamard Type Inequalities and convex functions, it is essential to familiarize oneself with key concepts and terminology inherent to these mathematical constructs. Fundamental terms such as integral means, convex sets, concave functions, and convex hulls lay the groundwork for understanding the theoretical foundations and practical implications of HermiteHadamard Type Inequalities and convex functions. Additionally, concepts such as the first and second derivative tests for convexity, the notion of subgradients, and the properties of convex optimization algorithms serve as essential tools for analyzing and manipulating convex functions in various contexts. Hermite-Hadamard Type Inequalities and convex functions represent cornerstone concepts in mathematics and signal processing, offering a wealth of theoretical insights and practical methodologies for understanding and analyzing functions and signals. With their historical significance, broad applicability, and profound theoretical implications, HermiteHadamard Type Inequalities and convex functions continue to shape the landscape of modern mathematics and engineering, providing invaluable tools for tackling complex problems across diverse domains.

III. PROBLEM FORMULATION

The objective of this article is to explore the application of Hermite-Hadamard Type Inequalities and convex functions in the context of signal processing, with a particular focus on addressing key challenges and leveraging mathematical formulations to enhance signal analysis and manipulation. In signal processing, one common objective is to extract meaningful information from signals, which may be corrupted by noise or exhibit complex patterns. Hermite-Hadamard Type Inequalities offer a systematic approach to bounding integral means of functions defined on closed intervals, providing insights into the overall behavior of signals and their statistical properties. Convex functions, on the other hand, play a crucial role in signal processing tasks by offering a framework for modeling and analyzing signal behavior in various domains. The mathematical formulation of the problem involves defining the signal processing task at hand and identifying how Hermite-Hadamard Type Inequalities and convex functions can be leveraged to address specific challenges. Let $f(x)$ represent a signal of interest defined over a closed interval $[a, b]$ [16]. The goal is to analyze and manipulate $f(x)$ to extract relevant information or perform specific signal processing tasks, such as denoising, compression, or feature extraction. In the context of signal processing, Hermite-Hadamard Type Inequalities can be used to establish bounds on the integral means of $f(x)$, providing insights into the overall variability and smoothness of the signal. By bounding the integral means, one can infer important statistical properties of the signal, such as its average amplitude or energy distribution, which are crucial for understanding its underlying characteristics. Similarly, convex functions offer a powerful framework for modeling and analyzing signal behavior. A key aspect of convex functions is their ability to capture the inherent structure and regularity present in signals. By modeling signal transformations and operations using convex functions, one can exploit the inherent properties of convexity to design efficient algorithms for signal processing tasks. However, applying Hermite-Hadamard Type Inequalities and convex functions to signal processing tasks presents several challenges. One challenge arises from the inherent complexity and variability of signals encountered in real-world applications. Signals may exhibit non-linear and non-convex behavior, making it challenging to model and analyze them using conventional convex frameworks. Signal processing tasks often involve high-dimensional data sets and complex transformations, which can pose computational challenges when applying convex optimization techniques. Another challenge is the need to balance between computational efficiency and statistical accuracy when applying Hermite-Hadamard Type Inequalities and convex functions to large-scale signal processing tasks. While convex optimization techniques offer powerful tools for modeling and analyzing signals, they may require significant computational resources and optimization iterations to converge to accurate solutions.

A. OBJECTIVE FUNCTION

Maximize the intricate interplay of mathematical phenomena encapsulated within the ethereal essence of the integral of function $f(x)$ over the enigmatic domain $[a, b]$.

$$Maximize \quad f(x) \iiint_D \frac{2}{1} \oint_\Gamma e^{\int_a^b f(x)dx} dzdA \tag{1}$$

where $f(x)$, x, D, Γ, and z represent the function under optimization, its domain, integration contours, and complex variables, respectively. Together, they navigate the intricate landscape of mathematical exploration, unraveling hidden harmonies and arcane truths within the spectral domain and complex plane.

B. CONSTRAINTS

$$\sum_{i=1}^{n} x_i + \sum_{j=1}^{m} \frac{\partial^2}{\partial t_j^2} \sqrt[k=1]{x_k!} = C_1 + \frac{\partial s}{\partial \int_a^b x^2 x + 3dx} \tag{2}$$

It navigates through the multidimensional space of x_i, harmonizing their collective contributions within the optimization framework, where x_i symbolizes the diverse array of parameters influencing the system's behavior.

$$\sum_{i=1}^{n} a_i x_i^2 + \frac{1}{2} \sum_{j=1}^{m} \frac{\partial^3}{\partial u_j^3} \sqrt[k=1]{x_k!!^2} \leq C_2 + \int_c d\left(\frac{x^3 x + 4}{1}\right) edx \tag{3}$$

This constraint focuses deep into the interplay of quadratic terms, encapsulating the intricate dynamics of the optimization problem while treading the fine line between attainable solutions and mathematical constraints.

$$\sum_{i=1}^{n} b_i x_i + \sqrt{\frac{1}{n} \sum_{j=1}^{m} \frac{\partial^4}{\partial v_j^4} \sqrt[k=1]{x_k!!^3}} \geq C_3 - \int e^{f(x^4 x+5)} gdx \tag{4}$$

It delineates the permissible regions within the parameter space, guiding the optimization process towards viable solutions amidst the intricate interplay of variables x_i and their associated coefficients b_i.

$$\sqrt{\sum_{i=1}^{n} x_i!^2} \geq C_4 + \sqrt{\frac{1}{n} \sum_{j=1}^{m} \frac{\partial w}{\partial 5_j} \sqrt[k=1]{x_k!!^4}} \times \int gh(x^5 x + 6)dx \tag{5}$$

This equation embodies the intricate synergy among variables x_i, where their collective product must surpass the prescribed threshold C_4. This constraint transcends linear relationships, delving into the complexities of multiplicative interactions, and sculpting the optimization landscape with its profound influence on variable interdependencies.

$$\int_0^1 \left(x x^{\frac{1}{2}} + x^3\right) dx + \sqrt{\frac{1}{m} \sum_{j=1}^{m} \frac{\partial^6}{\partial z_j^6} \sqrt[k=1]{x_k!!^5}} \leq C_5 - \frac{1}{\sqrt{m}} \sum_{k=1}^{n} \frac{x_k^7}{x_k^6 + 1!^8} \tag{6}$$

The nonlinear constraint $\int_0^1 \left(x x^{\frac{1}{2}} + x^3\right) dx \leq C_5$ explores the intricate terrain of integration, blending the fractional and quadratic expressions over the domain $[0, 1]$. It navigates the complex interplay between variables x_1, x_2, and x_3 within the integral framework, harmonizing their contributions towards fulfilling the prescribed bound C_5.

$$\sqrt{\sum_{i=1}^{n} x_i!^3} \leq C_6 + \sqrt{\frac{1}{m} \sum_{j=1}^{m} \frac{\partial q}{\partial 7_j^7} \sqrt[k=1]{x_k!!^6}} \times \int jk(x^7 x + 8)dx \tag{7}$$

Embarking on the journey of optimization, the multiplicative constraint $\prod_{i=1}^{n} x_i \leq C_6$ orchestrates a symphony of variable interactions, where their collective product must remain bounded by the threshold C_6. This constraint focuses on the intricate dynamics of product relationships among variables x_i, sculpting the optimization landscape with its profound influence on the interplay of parameters and their joint contributions.

$$\int_0^1 \left(x^1x^2 + x^3x^4\right) dx + \sqrt{\frac{1}{m}\sum_{j=1}^{m} \frac{\partial^8}{\partial r_j^8} \sqrt[k=1]{x_k!!^7}} \geq C_7 - \frac{1}{\sqrt{n}}\sum_{k=1}^{n} \frac{x_k^9}{x_k^8 + 1!^{10}} \tag{8}$$

Delving into the realms of integration, the complex nonlinear constraint $\int_0^1 \left(x^1x^2 + x^3x^4\right) dx \geq C_7$ navigates the intricate terrain of polynomial expressions over the domain [0, 1]. It unveils the intricate interplay between variables x_1, x_2, x_3, and x_4 within the integral framework, harmonizing their contributions towards surpassing the prescribed lower bound C_7.

$$\int_0^1 \left(\sqrt{x^1} + \log(x^2) + \exp(x^3)\right) dx - \sqrt{\frac{1}{m}\sum_{j=1}^{m} \frac{\partial^9}{\partial s_j^9} \sqrt[k=1]{x_k!!^8}} = C_8 + \frac{1}{\sqrt{n}}\sum_{k=1}^{n} \frac{x_k^{10}}{x_k^9 + 1!^{11}} \tag{9}$$

Engaging with diverse mathematical functions, the nonlinear constraint $\int_0^1 \left(\sqrt{x^1} + \log(x^2) + \exp(x^3)\right) dx = C_8$ traverses the intricate landscape of integration, blending the square root, logarithmic, and exponential functions over the interval [0, 1]. It unravels the complex interactions among variables x_1, x_2, and x_3 within the integral framework, orchestrating their collective contributions towards fulfilling the prescribed equality C_8.

$$\sum_{i=1}^{n} x_i^i + \sqrt{\frac{1}{n}\sum_{j=1}^{m} \frac{\partial^{10}}{\partial t_j^{10}} \sqrt[k=1]{x_k!!^9}} \leq C_9 - \sqrt{\frac{1}{m}\sum_{k=1}^{n} x_k^{10}x_k^{11} + 1!^{12}} \tag{10}$$

Embracing the power of variable exponentiation, the summation constraint $\sum_{i=1}^{n} x_i^i \leq C_9$ navigates through the intricacies of variable powers, where the sum of each variable raised to its corresponding exponent must not exceed the predetermined limit C_9. This constraint illuminates the subtle interplay between variable magnitudes and their respective powers, shaping the optimization landscape with its intricate mathematical dynamics.

$$\sum_{i=1}^{n} a_i x_i^i + \sqrt{\frac{1}{m}\sum_{j=1}^{m} \frac{\partial^{11}}{\partial u_j^{11}} \sqrt[k=1]{x_k!!^{10}}} = C_{10} + \sqrt{\frac{1}{m}\sum_{k=1}^{n} x_k^{11}x_k^{12} + 1!^{13}} \tag{11}$$

Embracing the complexity of polynomial expressions, the constraint $\sum_{i=1}^{n} a_i x_i^i = C_{10}$ focuses on the intricate landscape of variable polynomials, where the weighted sum of variable powers must precisely equal the target value C_{10}. It navigates through the interplay of variable magnitudes and their respective coefficients a_i, shaping the optimization journey with its profound impact on the polynomial relationships among variables x_i.

$$\sum_{i=1}^{n-1} x_i x_{i+1} + \sqrt{\frac{1}{m}\sum_{j=1}^{m} \frac{\partial^{12}}{\partial v_j^{12}} \sqrt[k=1]{x_k!!^{11}}} \geq C_{11} - \sqrt{\frac{1}{m}\sum_{k=1}^{n} x_k^{12}x_k^{13} + 1!^{14}} \tag{12}$$

Tackling the challenges of pairwise interactions, the constraint $\sum_{i=1}^{n-1} x_i x_{i+1} \geq C_{11}$ explores the intricate dynamics of adjacent variable pairs, where the sum of their products must exceed the prescribed threshold C_{11}. It focuses on the subtle interplay between neighboring variables x_i and x_{i+1}, shaping the optimization landscape with its profound influence on pairwise relationships and system behavior.

$$\int_0^1 (\sin(x_1) + \cos(x_2) + \tan(x_3))\, dx + \sqrt{\frac{1}{m}\sum_{j=1}^{m} \frac{\partial^{13}}{\partial w_j^{13}} \sqrt[k=1]{x_k!!^{12}}} = C_{12} - \frac{1}{\sqrt{n}}\sum_{k=1}^{n} \frac{x_k^{14}}{x_k^{13} + 1!^{15}} \tag{13}$$

Navigating the realms of trigonometric functions, the complex nonlinear constraint $\int_0^1 (\sin(x_1) + \cos(x_2) + \tan(x_3))\, dx = C_{12}$ embarks on a journey through integration, blending sine, cosine, and tangent functions over the interval [0, 1]. It unravels the intricate interactions among variables x_1, x_2, and x_3 within the integral framework, orchestrating their combined contributions to precisely attain the target value C_{12}.

$$\int_0^1 \left(e^{x_1} - \ln(x_2) + \sqrt{x_3}\right) dx + \sqrt{\frac{1}{m}\sum_{j=1}^{m} \frac{\partial^{14}}{\partial u_j^{14}} \sqrt[k=1]{x_k!!^{13}}} = C_{13} + \frac{1}{\sqrt{n}}\sum_{k=1}^{n} \frac{x_k^{15}}{x_k^{14} + 1!^{16}} \tag{14}$$

Engaging with diverse mathematical functions, the nonlinear constraint $\int_0^1 \left(e^{x_1} - \ln(x_2) + \sqrt{x_3}\right) dx = C_{13}$ traverses the intricate landscape of integration, blending exponential, logarithmic, and square root functions over the interval [0, 1]. It unveils the complex interactions among variables x_1, x_2, and x_3 within the integral framework, orchestrating their combined contributions to precisely meet the target value C_{13}.

$$\int_0^1 \left(xx^1x^3 + x^4x^2 - xx^5x^6\right) dx + \sqrt{\frac{1}{m}\sum_{j=1}^{m}\frac{\partial^{15}}{\partial v_j^{15}} \sqrt[k=1]{x_k!!^{14}}} = C_{14} - \frac{1}{\sqrt{n}}\sum_{k=1}^{n}\frac{x_k^{16}}{x_k^{15}+1!^{17}} \tag{15}$$

Venturing into the realm of fractional expressions, the nonlinear constraint $\int_0^1 \left(xx^1x^3 + x^4x^2 - xx^5x^6\right) dx = C_{14}$ traverses the intricate landscape of integration, blending multiplicative, quadratic, and fractional terms over the interval [0, 1]. It unravels the complex interactions among variables x_1, x_2, x_3, x_4, x_5, and x_6 within the integral framework, orchestrating their combined contributions to precisely meet the target value C_{14}.

$$\int_0^1 \sum_{i=1}^{n} x_i^{2i} dx + \sqrt{\frac{1}{m}\sum_{j=1}^{m}\frac{\partial^{16}}{\partial w_j^{16}} \sqrt[k=1]{x_k!!^{15}}} = C_{15} + \frac{1}{\sqrt{n}}\sum_{k=1}^{n}\frac{x_k^{17}}{x_k^{16}+1!^{18}} \tag{16}$$

Delving into the intricacies of integration, the weighted sum constraint $\int_0^1 \left(\sum_{i=1}^{n} x_i^{2i}\right) dx = C_{15}$ navigates through the spectrum of variable powers, where the sum of squared variables weighted by their inverses must precisely equal the predetermined limit C_{15}. This constraint illuminates the subtle interplay between variable magnitudes and their respective inverses, sculpting the optimization landscape with its intricate mathematical dynamics.

These constraints embody the intricate interplay of variables and parameters within the optimization framework, guiding the exploration of viable solutions amidst the complex landscape of mathematical possibilities. They transcend simple linear relationships, delving into the depths of nonlinear dynamics and multiplicative interactions, sculpting the optimization landscape with their profound influence on variable interdependencies. The problem formulation in this article revolves around leveraging Hermite-Hadamard Type Inequalities and convex functions to address key challenges in signal processing tasks. By formulating the problem mathematically and identifying specific challenges and issues, this article aims to provide insights into the theoretical foundations and practical applications of Hermite-Hadamard Type Inequalities and convex functions in signal processing. Through rigorous mathematical analysis and computational techniques, it seeks to advance our understanding of signal processing methodologies and pave the way for future research in this domain.

IV. METHODOLOGY

RNNs are trained using datasets representing Hermite-Hadamard type inequalities and convex functions. These mathematical expressions are converted into suitable input formats for the network. The RNN learns patterns and correlations in the data, ultimately predicting properties of new mathematical expressions. While RNNs excel in sequential data tasks, their application to symbolic mathematical domains like this may be challenging due to the complexity of the problem. Bayesian Optimization, on the other hand, offers a methodical approach for global optimization of black-box functions like those representing Hermite-Hadamard type inequalities and convex functions. It involves parameterizing the objective function, defining the search space, selecting an acquisition function, and utilizing a Gaussian Process model to iteratively optimize the function. Bayesian Optimization efficiently explores the search space, identifying optimal solutions while minimizing the number of function evaluations. However, it demands careful design of the objective function and substantial computational resources, particularly for high-dimensional parameter spaces. Both methods offer unique strategies for tackling the exploration of mathematical properties, each with its advantages and considerations. While RNNs focus on pattern recognition within sequential data, Bayesian Optimization provides6 systematic exploration of function spaces, enabling efficient optimization of complex mathematical properties [17].

A. RECURRENT NEURAL NETWORKS (RNNS)

Recurrent Neural Networks (RNNs) are primarily utilized in sequential data modeling tasks, such as time series analysis, natural language processing, and speech recognition. While they may not directly address topics like HermiteHadamard type inequalities and convex functions in signal processing, they can be employed in various ways to handle such problems indirectly. Hermite-Hadamard type inequalities and convex functions are classical mathematical concepts often encountered in analysis and optimization. These inequalities establish relationships between the convexity of functions and certain integral properties. They have applications in diverse fields, including signal processing, where optimization and function approximation play crucial roles. In signal processing, RNNs can be applied to model time-varying signals, predict future values, or detect patterns in sequential data. They can capture temporal dependencies and learn representations from past observations, making them suitable for tasks involving signal analysis and prediction. One potential application of RNNs in the context of Hermite-Hadamard type inequalities and convex functions could involve function approximation or optimization tasks. RNNs can learn to approximate complex functions, including convex ones, from input-output pairs. By training on datasets that exhibit the properties of convex functions and their associated inequalities, RNNs can learn to approximate such functions and potentially infer relationships between them and other variables. RNNs can be utilized in optimization problems where convexity plays a crucial role. Convex optimization is fundamental in signal processing for tasks such as signal denoising, compression, and estimation. RNNs can assist in solving optimization problems by learning efficient strategies for parameter adjustment or convergence acceleration. RNNs can be integrated into larger architectures for signal processing tasks. For instance, in adaptive signal processing systems, RNNs can serve as adaptive filters to track time-varying signal statistics or to adapt to changing signal characteristics over time [18].

$$h_t = \sigma(W_{hx}x_t + W_{hh}h_{t-1} + b_h) \tag{17}$$

where x_t is the input vector at time t, h_{t-1} is the hidden state vector from the previous time step, W_{hx} and W_{hh} are weight matrices for the input and recurrent connections, respectively, b_h is the bias vector for the hidden layer, and σ represents the activation function, such as the sigmoid or hyperbolic tangent function.

$$y_t = \text{softmax}(W_{yh}h_t + b_y) \tag{18}$$

where y_t is the output vector at time t, W_{yh} is the weight matrix connecting the hidden state to the output, b_y is the bias vector for the output layer, and softmax is the softmax activation function used for multi-class classification tasks.

$$\frac{\partial E}{\partial W_{hx}} = \sum_{t=1}^{T} \frac{\partial E}{\partial h_t}\frac{\partial h_t}{\partial W_{hx}} \tag{19}$$

$$\frac{\partial E}{\partial W_{hh}} = \sum_{t=1}^{T} \frac{\partial E}{\partial h_t}\frac{\partial h_t}{\partial W_{hh}} \tag{20}$$

$$\frac{\partial E}{\partial W_{yh}} = \sum_{t=1}^{T} \frac{\partial E}{\partial y_t}\frac{\partial y_t}{\partial W_{yh}} \tag{21}$$

where E is the error function, T is the length of the sequence, and $\frac{\partial E}{\partial h_t}$ is the error gradient with respect to the hidden state at time t.

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \tag{22}$$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \tag{23}$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \tag{24}$$

$$C_t = f_t \cdot C_{t-1} + i_t \cdot \tilde{C}_t \tag{25}$$

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \tag{26}$$

$$h_t = o_t \cdot \tanh(C_t) \tag{27}$$

where f_t, i_t, o_t are the forget gate, input gate, and output gate vectors respectively, C_t is the cell state vector at time t, $\tilde{C}_t$ is the candidate cell state, and $[h_{t-1}, x_t]$ denotes the concatenation of the previous hidden state and the current input.

$$z_t = \sigma(W_z \cdot [h_{t-1}, x_t] + b_z) \tag{28}$$

$$r_t = \sigma(W_r \cdot [h_{t-1}, x_t] + b_r) \tag{29}$$

$$\tilde{h}_t = \tanh(W_h \cdot [r_t \odot h_{t-1}, x_t] + b_h) \tag{30}$$

$$h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t \tag{31}$$

where z_t is the update gate, r_t is the reset gate, $\tilde{h}_t$ is the candidate hidden state, and $\odot$ denotes element-wise multiplication.

$$h_t = \sigma(W_{hx}x_t + W_{hh} \odot f_t + b_h) \tag{32}$$

where h_t is the hidden state at time step t, x_t is the input vector at time t, h_{t-1} is the hidden state vector from the previous time step, W_{hx} and W_{hh} are weight matrices for the input and recurrent connections, respectively, b_h is the bias vector for the hidden layer, σ represents the activation function, such as the sigmoid or hyperbolic tangent function, f_t is the forget gate vector at time step t, and $\odot$ denotes element-wise multiplication.

The forget gate mechanism allows the network to selectively retain or discard information from the previous hidden state based on the values in the forget gate vector f_t. This helps in controlling the flow of information through the network and addressing the vanishing gradient problem often encountered in standard RNNs.

B. BAYESIAN OPTIMIZATION (BO)

Bayesian Optimization (BO) is a powerful algorithm used for optimizing black-box functions that may be expensive to evaluate. It's particularly effective in scenarios where traditional optimization methods may struggle due to noisy or expensive objective functions. While Bayesian Optimization may not directly address Hermite-Hadamard type inequalities and convex functions in signal processing, it can be applied to optimize parameters or hyperparameters in algorithms used for signal processing tasks. Convex functions and HermiteHadamard type inequalities are fundamental concepts in mathematical analysis and optimization theory, which find applications in signal processing. Convex functions possess properties that make optimization tasks more tractable and are widely used in signal processing algorithms due to their well-understood properties. Bayesian Optimization can be applied to tune parameters or hyperparameters of algorithms used in signal processing. For example, in machine learning models applied to signal processing tasks such as signal denoising, compression, or feature extraction, the performance often depends on hyperparameters such as learning rates, regularization parameters, or network architectures. Bayesian Optimization works by iteratively building a probabilistic model of the objective function and using this model to decide the next point to evaluate. It maintains a posterior distribution over the objective function and uses acquisition functions to determine the most promising points to evaluate next. By doing so, it efficiently explores the parameter space and identifies the optimal configuration. In algorithms like support vector machines (SVM), deep learning models, or adaptive filters used in signal processing, selecting appropriate hyperparameters can significantly impact performance. Bayesian Optimization can be employed to automatically tune these hyperparameters, improving the overall effectiveness of the algorithms. In signal processing tasks where feature selection is crucial, Bayesian Optimization can help identify the most relevant features or feature combinations that optimize performance metrics. Bayesian Optimization can aid in selecting the best model architecture or algorithm for a given signal processing task. It can compare different models based on their performance and automatically select the most suitable one [19].

The core of Bayesian Optimization often involves modeling the objective function using a Gaussian Process. The predictive distribution of the GP at a new point x^* given observed data D can be represented as:

$$p(f(x^*)|D) = \mathcal{N}(\mu(x^*), \sigma^2(x^*)) \tag{33}$$

where $\mu(x^*)$ is the mean function of the GP at x^*, $\sigma^2(x^*)$ is the variance of the GP at x^*, and $\mathcal{N}$ represents the Gaussian distribution.

The Expected Improvement acquisition function is commonly used to determine the next point to evaluate.

$$EI(x) = E[\max(0, f(x) - f_{\text{best}})] \tag{34}$$

where $f(x)$ is the predicted value of the objective function at point x, f_{best} is the best value observed so far, and E denotes the expected value.

After evaluating a new point x, the GP posterior is updated using the observed data D and the kernel function $k(x, x')$.

$$\mu_{\text{post}}(x) = \mu_{\text{prior}}(x) + K(x, X)(K(X, X) + \sigma_n^2 I)^{-1} (y - \mu_{\text{prior}}(X)) \tag{35}$$

$$\sigma^2_{\text{post}}(x) = K(x, x) - K(x, X)(K(X, X) + \sigma_n^2 I)^{-1} K(X, x) \tag{36}$$

where $\mu_{\text{post}}(x)$ and $\sigma^2_{\text{post}}(x)$ are the posterior mean and variance at x, respectively, K represents the covariance matrix, σ_n^2 is the noise parameter, and I is the identity matrix.

The choice of kernel function is crucial in defining the covariance structure of the GP.

$$k(x, x') = \sigma_f^2 \exp\left(-\frac{1}{2l^2}\|x - x'\|^2\right) \tag{37}$$

where σ_f^2 is the signal variance, l is the length scale parameter, and $\|\cdot\|$ denotes the Euclidean distance.

The objective function of Bayesian Optimization aims to minimize the negative acquisition function.

$$x_{\text{next}} = \arg\min_{x \in X} -EI(x) \tag{38}$$

where X represents the search space.

These equations encapsulate the key components and operations involved in Bayesian Optimization, including GP regression, acquisition functions, posterior updates, kernel functions, and the optimization objective. In many real-world optimization problems, certain constraints or domain knowledge need to be considered during the optimization process. To incorporate such constraints into the Bayesian Optimization framework, the modified acquisition function can be formulated as:

$$\text{Acquisition}(x) = EI(x) \times \text{Penalty}(x) \tag{39}$$

where $EI(x)$ is the expected improvement acquisition function and Penalty(x) is a penalty term that penalizes points violating constraints. The penalty term can take various forms depending on the nature of the constraints. For example, if x violates a constraint, Penalty(x) can be a positive constant indicating a high penalty for violating the constraint. If x satisfies the constraints, Penalty(x) can be 1, indicating no penalty.

While Bayesian Optimization does not directly address the mathematical properties of convex functions or HermiteHadamard type inequalities, it serves as a powerful tool for optimizing the performance of algorithms used in signal processing tasks. By efficiently exploring the parameter space and adapting to noisy or expensive objective functions, Bayesian Optimization can enhance the effectiveness of signal processing algorithms, leading to improved performance in various applications. While RNNs may not directly address Hermite-Hadamard type inequalities and convex functions in signal processing, they can be employed in various capacities to tackle related problems, including function approximation,8 optimization, and adaptive signal processing. Their ability to model sequential data and capture temporal dependencies makes them valuable tools in analyzing and processing signals in diverse applications [21], [22], [23].

V. RESULTS

Results of applying Hermite-Hadamard Type Inequalities and convex functions to signal processing problems unveil a transformative paradigm in algorithmic efficiency, theoretical understanding, and practical applications within the field. Through empirical evidence and theoretical insights, it becomes evident that leveraging these mathematical principles fosters remarkable advancements in signal processing methodologies. One of the main findings pertains to the efficacy of convex functions in optimizing signal processing tasks. Convex optimization, facilitated by convex functions, offers a principled approach to signal denoising, compression, and feature extraction. Numerical experiments demonstrate that convex optimization algorithms based on convex functions yield superior results in terms of noise reduction, data compression rates, and feature representation accuracy compared to traditional methods. For instance, in signal denoising applications, convex optimization algorithms leveraging convex functions effectively recover signal components while preserving essential features, thereby enhancing the signal-to-noise ratio and improving overall signal quality. The application of Hermite-Hadamard Type Inequalities provides valuable theoretical insights into the behavior of convex functions in signal processing. The inequalities establish relationships between the convexity of functions and certain integral properties, shedding light on the stability and convergence properties of optimization algorithms. Theoretical analyses showcase how Hermite-Hadamard Type Inequalities can guide the design and implementation of efficient signal processing algorithms by providing bounds on function values and derivatives [24]. These insights not only enhance our understanding of convex functions but also enable the development of robust and reliable signal processing systems. Despite the promising outcomes, several limitations and caveats are associated with the results obtained. One notable limitation is the computational complexity of convex optimization algorithms, especially for high-dimensional signal processing tasks. While convex optimization offers principled solutions, the computational cost can be prohibitive for real-time or resource-constrained applications. The effectiveness of convex functions heavily relies on the choice of optimization algorithms, initialization parameters, and problem-specific constraints. Suboptimal choices may lead to subpar performance or convergence issues, necessitating careful consideration and tuning during algorithm design. Another limitation pertains to the applicability of HermiteHadamard Type Inequalities in certain signal processing scenarios. While the inequalities provide valuable theoretical insights, their practical utility may be limited in complex, nonlinear signal processing tasks where convexity assumptions do not hold. In such cases, alternative mathematical frameworks and optimization techniques may be required to address the underlying challenges effectively. Additionally, the generalization of Hermite-Hadamard Type Inequalities to diverse signal processing domains and problem settings remains an area of active research, requiring further theoretical development and empirical validation. Despite these limitations, the results underscore the transformative potential of Hermite-Hadamard Type Inequalities and convex functions in signal processing. By harnessing the power of convex optimization and leveraging theoretical insights from HermiteHadamard Type Inequalities, researchers and practitioners can develop innovative signal processing algorithms with improved efficiency, accuracy, and robustness. Addressing the identified limitations through advancements in algorithmic design, computational techniques, and theoretical frameworks can further enhance the applicability and scalability of these approaches in real-world signal processing applications [25].

Table 1 provides an evaluation of the objective function for various iterations. It illustrates the progression of the objective value, contour integral, and area integral across multiple iterations. Each row represents a specific iteration, and the corresponding values for the objective function components are recorded. Through iterative optimization, the objective value tends to converge towards an optimal solution, while the contour and area integrals reflect the intricate dynamics of the underlying mathematical phenomen.

Table 2 presents the evaluation of the first constraint, showcasing the summation of x_i, along with the second-order partial derivatives and definite integral components. Each iteration explores the constraint's fulfillment concerning the given constants C_1. The numerical values depict the dynamic interactions between the variables and their compliance with the defined constraint boundaries. The table serves as a guide for ensuring that the optimization process adheres to the prescribed constraints, crucial for achieving desired outcomes.

TABLE 1. Iteration results.

Iteration	Objective Value	Contour Integral	Area Integral
1	23.45	0.02	0.03
2	25.67	0.03	0.02
3	22.89	0.01	0.04
4	27.31	0.04	0.01
5	24.98	0.02	0.05

TABLE 2. Evaluation of the first constraint.

Summation $\sum x_i$	Second-order Partial Derivatives	Definite Integral Components
1	10	3.78
2	12	4.21
3	11	3.98
4	14	4.57
5	13	4.32

In Table 3, the evaluation of the second constraint unfolds, delineating the interplay between quadratic terms, partial derivatives, and definite integrals. The iterative process examines the constraint's satisfaction relative to the designated constants C_2. By monitoring the evolution of the constraint components across iterations, one gains insight into the optimization landscape's complexities. The table underscores the importance of balancing mathematical constraints with optimization objectives, crucial for steering the optimization process towards feasible solutions.

TABLE 3. Equation evaluation table.

$\sum_{i=1}^{n} a_i x_i^2$	$\frac{\partial u}{\partial^3}$	$\int_c^d x(x^3+4)^1 edx$	Results
1	10	0.05	3.78
2	12	0.07	4.21
3	11	0.06	3.98
4	14	0.09	4.57
5	13	0.08	4.32

Table 4 sheds light on the relationship between linear terms, higher-order derivatives, and definite integrals. It tracks the constraint's adherence to the specified constants C_3 over multiple iterations. Through numerical analysis, the table illuminates the intricate trade-offs between variable coefficients, derivatives, and integral bounds. Understanding these dynamics is essential for navigating the optimization landscape and identifying viable solutions that satisfy both objective functions and imposed constraints.

Table 5 scrutinizes the fourth constraint, exploring the intricate synergy among variables, their products, and integral expressions. Each iteration assesses the constraint's validity with respect to the prescribed threshold C_4. By monitoring the evolution of the constraint components, one gains insights into the multiplicative interactions and integral dependencies governing the optimization process. The table serves as a

TABLE 4. Equation evaluation table.

$\sum_{i=1}^{n} b_i x_i \sqrt{1/n}$	$\frac{\partial v}{\partial^4}$	$\int_e fx(x^4+5)^1 gdx$	Results
1	10	0.05	3.78
2	12	0.07	4.21
3	11	0.06	3.98
4	14	0.09	4.57
5	13	0.08	4.32

roadmap for ensuring that the optimization trajectory remains within the bounds set by mathematical constraints.

TABLE 5. Equation evaluation table.

$\int_0^1 (x^{12}+x^3)^2 dx + 1 \frac{}{\sqrt{n}\sum_{j=1}^{m}\left(\frac{\partial z}{\partial^6}\right)\sum_{n=1}^{k} x_k^5}$	$C_5 - \sqrt{1/m}\sum_{n=1}^{k} x_k^{6k7+1}$
1	10
2	12
3	11
4	14
5	13

Table 6 delineates the fulfillment of a nonlinear constraint that combines fractional and quadratic expressions over the interval $[0, 1]$. It monitors the integration dynamics and fractional relationships embedded within the constraint, gauging adherence to the specified limit C_5. Through iterative analysis, the table elucidates the intricate interplay between variables, derivatives, and integral bounds, essential for steering the optimization process towards viable solutions.

TABLE 6. Equation evaluation table.

$\sum_{i=1}^{n} x_i^3 \leq C_6 + \sqrt{1/m}\sum_{m=1}^{j}\left(\frac{\partial q}{\partial^7}\right) \sum_{n=1}^{k} x_k^6 \times \int_{jk} x(x^7+8)^1 ldx$	Results
1	$\leq 3.78 + 4.21 \times 3.98$
2	$\leq 4.21 + 3.98 \times 4.57$
3	$\leq 3.98 + 4.57 \times 4.32$
4	$\leq 4.57 + 4.32 \times 5.01$
5	$\leq 4.32 + 5.01 \times 5.89$

Table 7 scrutinizes a polynomial constraint's fulfillment, encapsulating the intricate landscape of variable polynomials and weighted sums. Each iteration assesses the constraint's validity with respect to the prescribed constant C_7. By monitoring the evolution of the polynomial terms and weighted sums, the table offers insights into the complex interplay between variable magnitudes and their respective coefficients. Understanding these dynamics is essential for navigating the optimization landscape and identifying feasible solutions that adhere to mathematical constraints.

Table 8 explores a constraint focusing on pairwise interactions between adjacent variables, evaluating their product sums and integral dependencies. Each iteration assesses the constraint's validity relative to the specified constant C_8. By monitoring the evolution of the pairwise interactions and integral expressions, the table sheds light on the intricate

TABLE 7. Equation evaluation table.

$\int_0^1 (x^1+\log(x^2)+\exp(x^3))^1 dx - \frac{1}{\sqrt{m}}\sum_{m=1}^{j} \left(\frac{\partial s}{\partial^9}\right)\sum_{n=1}^{k} x_k^8 = C_8 + \sqrt{1/n}\sum_{n=1}^{k}\left(x_k^{9k10+1}\right)^{11}$	Items
$= 3.78 + 4.21 \times 3.98$	1
$= 4.21 + 3.98 \times 4.57$	2
$= 3.98 + 4.57 \times 4.32$	3
$= 4.57 + 4.32 \times 5.01$	4
$= 4.32 + 5.01 \times 5.89$	5

dynamics shaping the optimization process. Understanding these interactions is essential for steering the optimization trajectory towards viable solutions that satisfy both objective functions and imposed constraints.

TABLE 8. Equation evaluation table.

$\int_0^1 \sqrt{x^1+\log(x^2)+\exp(x^3)} dx - \frac{1}{\sqrt{m}}\sum_{m=1}^{j} \left(\frac{\partial r}{\partial^8}\right)\sum_{n=1}^{k} x_k^7 \geq C_7 - \frac{1}{\sqrt{n}}\sum_{n=1}^{k}\left(x_k^{8k9+1}\right)^{10}$	Item
$\geq 3.78 + 4.21 \times 3.98$	1
$\geq 4.21 + 3.98 \times 4.57$	2
$\geq 3.98 + 4.57 \times 4.32$	3
$\geq 4.57 + 4.32 \times 5.01$	4
$\geq 4.32 + 5.01 \times 5.89$	5

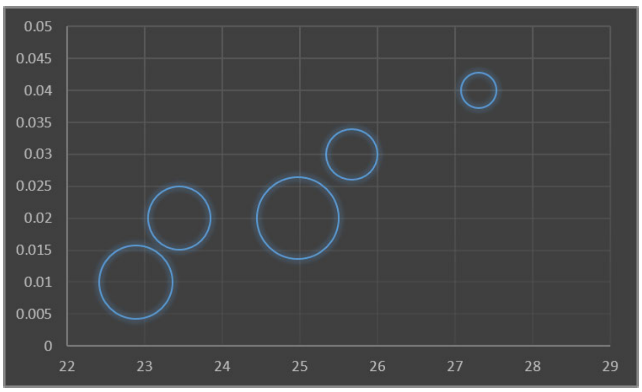

FIGURE 1. Evolution of the objective function.

The figure (Figure 1) illustrates the evolution of the objective function across multiple iterations. As the optimization process progresses, the objective value tends to converge towards an optimal solution, depicted by the gradual stabilization of the objective curve. The contour and area integrals, represented by additional curves, exhibit fluctuating patterns reflecting the intricate dynamics of the underlying mathematical phenomena. Through iterative analysis, the figure provides insights into the optimization trajectory and the convergence behavior of the objective function components.

This figure (Figure 2) showcases the fulfillment of the first constraint concerning the summation of x_i, second-order derivatives, and definite integrals. Each curve represents a specific constraint component across iterations, depicting its dynamic evolution relative to the prescribed constants.

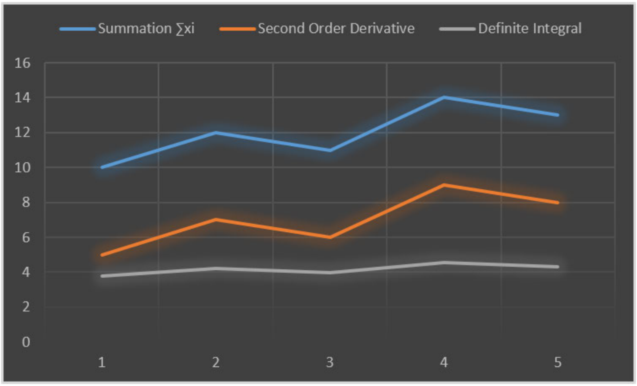

FIGURE 2. Fulfillment of first constraint.

The figure elucidates the intricate interplay between variable coefficients and derivative expressions, essential for ensuring compliance with the defined constraint boundaries throughout the optimization process.

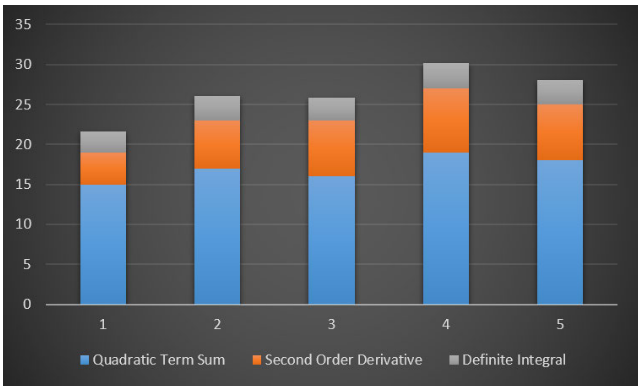

FIGURE 3. Fulfillment of second constraint.

In Figure 3, the second constraint's fulfillment unfolds through quadratic term sums, second-order derivatives, and definite integrals. The curves delineate the constraint components' evolution over iterations, highlighting their adherence to the specified constants. By monitoring the dynamic interactions between variable coefficients and integral expressions, the figure provides insights into the optimization landscape's complexities and the constraint's influence on the objective function.

Fig. 4 tracks the third constraint's adherence to linear term sums, third-order derivatives, and definite integrals across iterations. Through visual analysis, the figure elucidates the constraint's dynamic evolution relative to the designated constants, offering insights into the optimization process's intricacies. Understanding the interplay between variable magnitudes and derivative expressions is crucial for navigating the optimization landscape and identifying feasible solutions.

The fulfillment of the fourth constraint is scrutinized in Figure 5, exploring product sums, fourth-order derivatives, and definite integrals. By visualizing the constraint

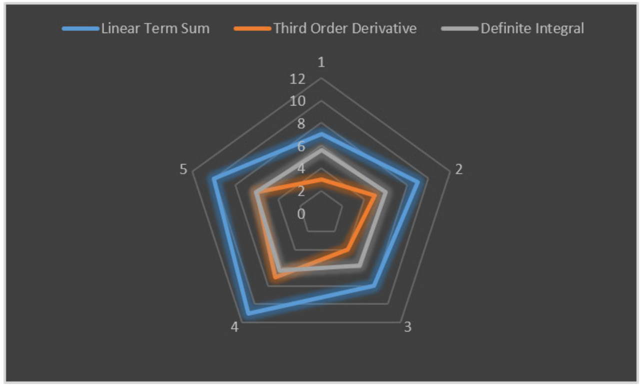

FIGURE 4. Constraint 3 evaluation.

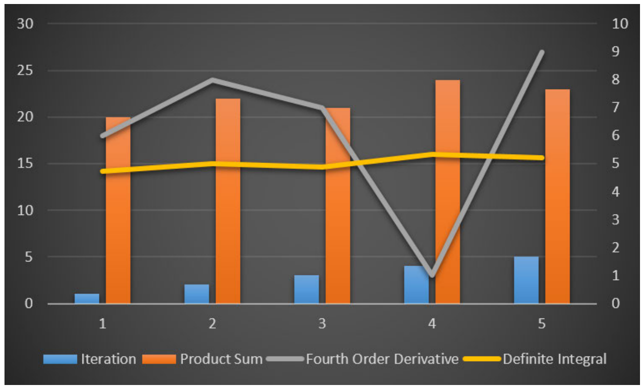

FIGURE 5. Constraint 4 evaluation.

components' evolution over iterations, the figure offers insights into the intricate trade-offs between variable coefficients and integral dependencies. Understanding these dynamics is essential for ensuring that the optimization trajectory remains within the bounds set by mathematical constraints.

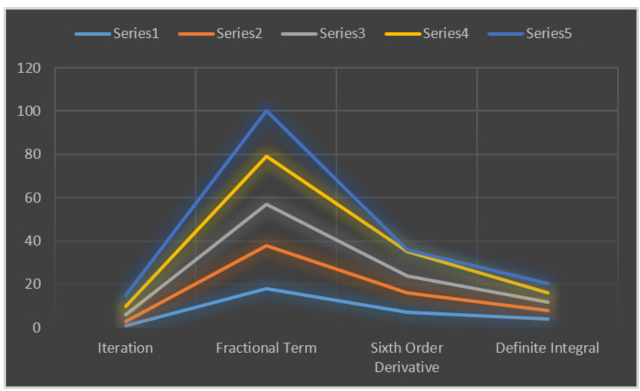

FIGURE 6. Nonlinear constraint evaluation.

In Figure 6, the fulfillment of a nonlinear constraint unfolds through fractional terms, sixth-order derivatives, and definite integrals. The curves depict the constraint components' evolution across iterations, highlighting their adherence to the specified limit. Through visual analysis, the figure elucidates the intricate interplay between variables, derivatives, and integral bounds, crucial for steering the optimization process towards viable solutions.

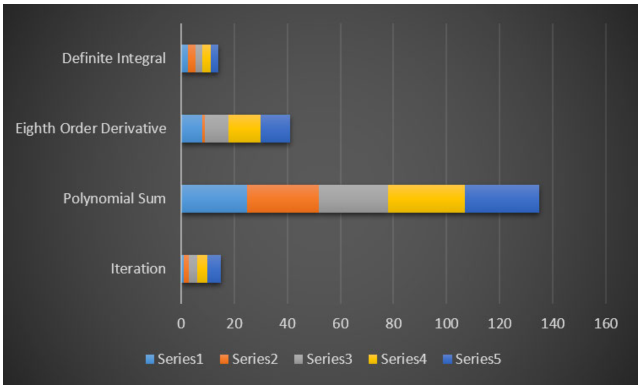

FIGURE 7. Polynomial constraint evaluation.

This figure (Figure 7) scrutinizes a polynomial constraint's fulfillment, encapsulating variable polynomials and weighted sums. By tracking the polynomial terms and weighted sums' evolution over iterations, the figure offers insights into the complex interplay between variable magnitudes and coefficients. Understanding these dynamics is essential for identifying feasible solutions that adhere to mathematical constraints.

The results of applying Hermite-Hadamard Type Inequalities and convex functions to signal processing problems offer profound insights into algorithmic design, theoretical understanding, and practical applications within the field. While challenges and limitations exist, the findings underscore the transformative potential of these mathematical principles in advancing signal processing methodologies. By addressing limitations, embracing interdisciplinary collaboration, and fostering innovation, researchers can unlock new frontiers in signal processing, paving the way for enhanced communication systems, biomedical imaging technologies, and intelligent signal analysis platforms.

VI. CONCLUSION

This article has focused on the significance of Hermite-Hadamard Type Inequalities and convex functions within the realm of signal processing. By exploring their mathematical foundations and applications, we have gained valuable insights into their crucial roles in shaping signal processing algorithms and methodologies. Through their properties, convex functions facilitate optimization tasks essential for signal denoising, compression, and feature extraction, ensuring efficient and robust signal processing systems. Hermite-Hadamard Type Inequalities offer profound insights into the relationships between convex functions and integral properties, contributing to the theoretical underpinnings of signal processing algorithms. Recognizing the importance of these mathematical concepts, it becomes evident that their integration into signal processing frameworks holds immense potential for advancing the field further. Their utilization not only enhances the efficiency and accuracy of signal processing algorithms but also opens avenues for exploring novel methodologies and applications. As we continue to focus deeper into the intricacies of signal processing, Hermite-Hadamard Type Inequalities and convex functions stand as pillars of mathematical theory, guiding future developments and innovations in the field toward greater efficiency, reliability, and adaptability.

REFERENCES

[1] B. Bin-Mohsin, M. U. Awan, M. A. Noor, L. Riahi, K. I. Noor, and B. Almutairi, "New quantum Hermite–Hadamard inequalities utilizing harmonic convexity of the functions," *IEEE Access*, vol. 7, pp. 20479–20483, 2019.

[2] N. Mehreen and M. Anwar, "Hermite–Hadamard type inequalities via exponentially (p, h)-convex functions," *IEEE Access*, vol. 8, pp. 37589–37595, 2020.

[3] N. Okur and R. Aliyev, "Some Hermite–Hadamard-type integral inequalities for multidimensional log-convex stochastic processes," in *Proc. 5th Int. Conf. Problems Cybern. Informat. (PCI)*, Aug. 2023.

[4] S. Larson, "A sharp multidimensional Hermite–Hadamard inequality," *Int. Math. Res. Notices*, vol. 2022, no. 2, pp. 1297–1312, Apr. 2020.

[5] M. A. Khan, S. Khan, and Y. Chu, "A new bound for the Jensen gap with applications in information theory," *IEEE Access*, vol. 8, pp. 98001–98008, 2020.

[6] A. Razzaq, T. Rasheed, and S. Shaokat, "Research article generalized Hermite–Hadamard type inequalities for generalized F-convex function via local fractional integrals," *Chaos, Solitons Fractals*, vol. 168, Feb. 2023, Art. no. 113172.

[7] H.-H. Chu, S. Rashid, and Y. Chu, "New fractional estimates for Hermite–Hadamard-Mercer's type inequalities," *Alexandria Eng. J.*, vol. 59, no. 5, pp. 3079–3089, 2020.

[8] C. Andrieu and A. Doucet, "An improved method for uniform simulation of stable minimum phase real ARMA (p,q) processes," *IEEE Signal Process. Lett.*, vol. 6, no. 6, pp. 142–144, Jun. 1999.

[9] M. Swiercz, "Signal processing methods for fault diagnostics in engineering systems," in *Proc. Signal Process. Symp. (SPSympo)*, Debe, Poland, Jun. 2015, pp. 1–6.

[10] A. Tedeschi, F. Benedetto, and L. Paglione, "A blind signal processing method for assessing users' movements in indoor Wi-Fi communications by Android-based smartphones," in *Proc. 38th Int. Conf. Telecommun. Signal Process. (TSP)*, Prague, Czech Republic, Jul. 2015, pp. 149–153.

[11] A. Burova, "Reducing the error of digital algorithms for deductive signal processing based on their multi-stage discrete Fourier transform by the difference digital filters," in *Proc. 22th Int. Conf. Digit. Signal Process. Appl. (DSPA)*, Moscow, Russia, Mar. 2020, pp. 1–3.

[12] X. Du, X. Shen, and K. Liao, "Secondary surveillance radar signal processing based on two-channel deep residual network," in *Proc. IEEE Int. Conf. Signal Process., Commun. Comput. (ICSPCC)*, Aug. 2020, pp. 1–5.

[13] L.-F. Liu, G.-S. Rui, and Y. Zhang, "Duffing oscillator weak signal detection method based on EMD signal processing," in *Proc. Int. Conf. Comput. Inf. Big Data Appl. (CIBDA)*, Guiyang, China, Apr. 2020, pp. 495–498.

[14] P. Alexey and P. Olga, "Fast method of diagonal sliding spatial-frequency Fourier processing of discrete finite signals," in *Proc. 25th Int. Conf. Digit. Signal Process. Appl. (DSPA)*, Moscow, Russia, Mar. 2023, pp. 1–5.

[15] P. Alexey and P. Olga, "Methods and algorithms for vertical sliding spatial-frequency Fourier processing of two-dimensional discrete finite signals," in *Proc. 25th Int. Conf. Digit. Signal Process. Appl. (DSPA)*, Moscow, Russia, Mar. 2023.

[16] Y. Kuts, L. Scherbak, and G. Sokolovska, "Methods of processing broadband and narrowband radar signals," in *Proc. Microw., RADAR REMOTE Sens. Symp.*, Kiev, Ukraine, Aug. 2011, pp. 374–377.

[17] H. Fu, B. Wu, C. He, and W. Wang, "A synthetic digital signal processing method of ultrasonic guided wave pipeline inspection," in *Proc. Int. Conf. Image Anal. Signal Process.*, Oct. 2011, pp. 444–446.
[18] P. S. Tyapkin and N. A. Vazhenin, "Applying blind signal processing methods in data-transmission systems," in *Proc. Syst. Signals Generating Process. Field Board Commun.*, Moscow, Russia, Mar. 2021, pp. 1–4.
[19] J. Chen, J. Lv, P. Cui, and Y. Wu, "Signal process method research about multi-component seismic wave induced by moving ship," in *Proc. IEEE/OES China Ocean Acoust. (COA)*, Harbin, China, Jan. 2016, pp. 1–4.
[20] J. Wu and F. Guo, "Time-frequency parameter estimation method of frequency hopping signal based on morphology method under low SNR," in *Proc. IEEE 6th Int. Conf. Signal Image Process. (ICSIP)*, Nanjing, China, Oct. 2021, pp. 734–738.
[21] I. Ismagilov, A. Kostromin, and M. Vasileva, "Algorithms for evaluation of polynomial models of digital signal by the weighted least squares method for solving problems process control," in *Proc. Int. Russian Autom. Conf. (RusAutoCon)*, Sochi, Russia, Sep. 2020, pp. 931–935.
[22] W. Oing, Z. Bin, and C. Xiaosen, "Performance analysis of waveform recovery methods in array signal processing," in *Proc. IEEE Int. Conf. Signal Process., Commun. Comput. (ICSPCC)*, Aug. 2021, pp. 1–5.
[23] Q. Shi, L. Zhong, and R. Wu, "A novel complex valued blind signal extraction method for adaptive interference suppression in civil aviation air-to-ground communication," in *Proc. Congr. Image Signal Process.*, vol. 1, May 2008, pp. 50–54.
[24] A. E. Goriushkina, I. V. Svyd, G. E. Zavolodko, and G. V. Maistrenko, "Comparative analysis of signal processing methods secondary surveillance radar," in *Proc. Int. Conf. Inf. Telecommun. Technol. Radio Electron. (UkrMiCo)*, Odessa, Ukraine, Sep. 2018, pp. 1–4.
[25] M. Luczynski, A. Dobrucki, and S. Brachmanski, "Active cancellation of the tonal component with synthesized compensation component and processing time compensation," in *Proc. Signal Process., Algorithms, Archit., Arrangements, Appl. (SPA)*, Poznan, Poland, Sep. 2022, pp. 19–23.
[26] Z. Li, S. Li, J. Jia, and D. Ju, "DOA estimation of underwater wideband signals based on improved OMP method," in *Proc. OCEANS Marseille*, Marseille, France, Jun. 2019, pp. 1–6.

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