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RESEARCH ARTICLE

PNSND: A Novel Solution for Dynamic Nonlinear Equations and Its Application to Robotic Arm

SONGJIE HUANG, PEIXUAN ZHANG, KEER WU, AND XIUCHUN XIAO
School of Electronic and Information Engineering, Guangdong Ocean University, Zhanjiang 524088, China

Corresponding author: Xiuchun Xiao (xiaoxc@gdou.edu.cn)

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ABSTRACT Dynamic nonlinear equations (DNEs) are essential for modeling complex systems in various fields due to their ability to capture real-world phenomena. However, the solution of DNEs presents significant challenges, especially in industrial settings where periodic noise often compromises solution fidelity. To tackle this challenge, we propose a novel approach called Periodic Noise Suppression Neural Dynamic (PNSND), which leverages the gradient descent approach and incorporates velocity compensation to overcome the limitations of the traditional Gradient Neural Dynamic (GND) model. Additionally, the PNSND model aims to suppress periodic noise by addressing its harmonic properties according to the method of Fourier decomposition of harmonics. In the paper, we explore the performance of convergence and robustness of the PNSND model. Moreover, we demonstrate the effectiveness of the PNSND model in addressing dynamic nonlinear problems under periodic noise interference through its application to robotic arm, highlighting its practical significance in industrial contexts.

INDEX TERMS Dynamic nonlinear equations (DNEs), periodic noise suppression, neural dynamic, gradient descent method, robotic arm.

I. INTRODUCTION

Compared to simple linear frameworks [1], nonlinear systems provide a more precise understanding of complex problems [2]. Therefore, the comprehension and utilization of nonlinear systems are paramount for addressing varied challenges within contemporary scientific and industrial domains [3]. Nonlinear systems have found extensive application across various domains, including optimization, control theory, signal processing, robotics, and more [4], [5], [6], [7]. In the domain of nonlinear systems, academic and industrial research converge on the resolution of dynamic nonlinear equations (DNEs) [8], [9], [10], [11].

Iterative methods, particularly the Newtonian iteration method, have historically been the primary approach for tackling DNEs [12], [13], [14]. In order to improve the Newton method, many scholars have conducted further research on Newton-type iterations.

Advanced iterative methods are described in [15], combining Newton's approach and its derivatives, achieving fifth and eighth-order convergence for tackling DNEs, its efficiency is supported by computational results. Madhu et al. present three novel iterative approaches of orders four, five, and six for tackling DNEs. These methods demonstrate enhanced convergence and decreased computational complexity, and their effectiveness is validated through numerical experiments and applications to Chandrasekhar's equation and the 1-D Bratu problem [16]. In [17], an anti-noise Newton-Raphson-based method for addressing dynamic Lyapunov equations is introduced. This algorithm showcases enhanced convergence accuracy and noise robustness compared to traditional methods, as validated through numerical simulations and its application to robotic motion tracking. Meanwhile, Wang et al. introduce a control-theoretic Newtonian iterative

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TABLE 1. Comparative analysis of various models for the DNEs problem.

	Derivative information involved	Dsigned for dynamic problem	Anti-noise	Suppress periodic noise
traditional GND model (4)	No	No	No	No
ZND model (6)	Yes	Yes	No	No
IEZNN model (7)	Yes	Yes	Yes	No
CRNN model (8)	Yes	Yes	Yes	Yes
accelerate GND model (15)	Yes	Yes	No	No
improved GND model (16)	Yes	Yes	No	No
The proposed PNSND model (21)	Yes	Yes	Yes	Yes

approach for addressing linear equations in the presence of noise [18]. This method demonstrates improved convergence and accuracy compared to traditional approaches, albeit with a longer convergence duration under noisy conditions.

Neural dynamic models are currently undergoing rapid development, with a variety of innovative structures being introduced to tackle dynamic nonlinear problems [19], [20]. These models showcase superior computational capacity compared to traditional Newtonian or Newton-like iterations and find extensive utilization in addressing DNEs within scientific and engineering fields [21], [22], [23]. Scholars have proposed the Gradient Neural Dynamic (GND) model, leveraging the gradient descent optimization technique. The traditional GND model has proven particularly effective in managing large-scale, static computational challenges [24]. However, its design primarily caters to dynamic problems and lacks incorporation of derivative information, resulting in significant lag errors when applied to dynamic scenarios [25]. In response, subsequent scholars have proposed the Zeroing Neural Dynamic (ZND) model as a viable alternative for dynamic problem-solving [26]. The ZND model has demonstrated notable success in improving convergence performance and solution accuracy [27], [28], [29].

In industrial environments, noise frequently undermines the precision of solutions [30]. Addressing the challenge of real-time execution of DNEs amid noise has been a focal point of research recently. However, previous research on neural dynamic models has primarily focused on the accuracy and speed of addressing DNEs [31], frequently neglecting the robustness of these models [9]. To address the detrimental impact of noise, the integration-enhanced zeroing neural network (IEZNN) was introduced in [32]. Engineered to execute dynamic matrix inversion amidst diverse noise sources, the IEZNN model effectively mitigates their effects, showcasing superior real-time computational performance and noise resistance. However, it's essential to note that common periodic noises encountered in practical engineering, such as square and triangular waves, often have high frequencies. These high-frequency emissions can induce electromagnetic interference, potentially disrupting the functionality of electronic components and corrupting computational accuracy. Consequently, the aforementioned model may encounter challenges in achieving precise solutions to DNEs amidst periodic noise. Therefore, devising strategies to mitigate the deleterious effects of periodic noise on system performance is imperative. Introducing a circadian rhythms neural network (CRNN) addresses challenges associated with periodic noise, effectively addressing robotic arm problems caused by such disturbances [33]. Differing from other neural dynamic models, CRNN introduces a compensation term in the error term opposite to the periodic noise period, offsetting its effect and accelerating the approach of the error towards nil. However, while these approaches contribute to alleviating the impact of periodic noise, they have not thoroughly analyzed the inherent properties of periodic noise in their suppression strategies. Consequently, efficiency significantly degrades when confronted with high-frequency periodic noise. In summary, addressing periodic noise challenges with advanced anti-noise neural dynamic models primarily focuses on residual aspects rather than the periodic noise itself. Accurately obtaining and analyzing the properties of periodic noise remains a challenge in dealing with such disturbances.

In the paper, the PNSND model is proposed to enhance the traditional GND model for accurately addressing the DNEs problem under periodic noise interference. Firstly, leveraging the gradient design method [21], which drives the DNEs problem residuals towards zero along the negative gradient direction as rapidly as possible, we propose a GND model for addressing DNEs. Secondly, in the absence of noise, we introduce an improved GND model that effectively eliminates lagging errors by incorporating derivative information from related equations [34]. Consequently, the accelerated GND model presented in this paper can also handle dynamic problems, demonstrating better performance compared to the ZND model. Furthermore, by leveraging Fourier expansion to represent periodic noise, we design harmonic compensation terms by combining the most significant frequency characteristics of periodic noise and error function information [32], [33]. These tailored harmonic terms counteract the interference caused by periodic noise. Based on the improved GND model and noise compensation terms, we develop the PNSND model, which addresses periodic noise interference from a harmonic perspective, thereby improving the model's accuracy in addressing DNEs under periodic noise interference. Specifically, Table (1) illustrates a comparative analysis of the characteristics of several excellent models and the PNSND model.

The primary works presented in the paper are delineated below:

- Proposing a neural dynamic model (PNSND) to address DNEs problems under periodic noise interference. Constructed from the perspective of noise itself, the PNSND model demonstrates superior performance compared to existing anti-noise models.

- Through mathematical analysis and proofs, the validity and robustness of the PNSND model in addressing DNEs problems are strictly analyzed and proven. Theoretical analysis confirms the model's excellent ability to handle the impact of periodic noise.
- Demonstrating the feasibility of the PNSND model in practical applications through numerical simulations and successful application to a robot arm. This verification confirms the model's effectiveness in handling periodic noise interference in real-world scenarios.

The following outlines the composition of the paper. Section II summarizes the formulation of DNEs problems, introduces previous advanced anti-noise models, and applies to the DNEs problem. Section III introduces the PNSND model which can efficiently address the DNEs problem under periodic noise. The performance of the PNSND model under periodic noise interference is proved by rigorous theoretical analysis in Section IV. In Section V, simulation experiments of various models are conducted, illustrating the superior capabilities of the PNSND model via a comparative analysis. Additionally, the effectiveness of the PNSND model is validated through its application to a robotic arm. Eventually, the conclusion of the paper is provided in Section VI.

II. RELATED WORK

A. PROBLEM FORMULATION

With a guarantee of generality, the DNEs can be expressed as [35]:

$$\Theta(y(t), t) = 0, t \in [t_s, t_e], \tag{1}$$

in which the variable t denotes time, $\Theta(\cdot) \in R^m$ is a nonlinear mapping and $y(t) \in R^n$ signifies the sought solution to equations (1), with $n = m$. This indicates that equation (1) constitutes a well-posed problem, guaranteeing a unique solution for each unknown function without ambiguity in its determination. The variables t_s and t_e respectively represent the start time and the end time. Clarity is enhanced by stating that the objective of addressing equations (1) is to ensure the value of $y(t)$ approaches the expected theoretical limit $y^*(t)$ at any time $t \in [t_s, t_e] \subseteq [0, +\infty)$.

B. TRADITONAL GND MODEL

In this subsection, a traditional GND model for addressing DNEs (1) is derived in detail. Firstly, to derive the GND model for addressing DNEs (1), the scalar function $\Xi(t)$ is chosen as the performance index of the GND model [24]. Below is the expression of the energy function $\Xi(t)$ based on the norm.

$$\Xi(t) = \frac{||\Theta(y(t), t)||_2^2}{2}, \tag{2}$$

where $||\,\Theta(y(t), t)\,||_2$ denotes the 2-norm of $\Theta(y(t), t)$. The GND model is derived by minimizing $\Xi(t)$ along its negative gradient direction $-\partial\Xi(t)/\partial y(t)$:

$$\dot{y}(t) = \frac{dy(t)}{dt} = -\gamma(\frac{\partial\Xi(t)}{\partial y(t)}), \tag{3}$$

where $\gamma > 0$ is utilized to control the rate of convergence. Traditional GND model can be expanded by (3):

$$\dot{y}(t) = -\gamma(\frac{\partial\Theta(y(t), t)}{\partial\, y(t)})^T\, \Theta(y(t), t) \;= -\gamma J^T\Theta(y(t), t), \tag{4}$$

where J^T represents the transpose of the matrix J, given that the Jacobian matrix $J = \frac{\partial\Theta(y(t),t)}{\partial y(t)} \in R^{m\times n}$ has full row rank.

C. ZND MODEL

The ZND model for dealing with DNEs (1) problem is presented in advance [26]:

$$\dot{e}(t) = -\gamma e(t). \tag{5}$$

The function used to quantify error is represented by $e(t) = \Theta(y(t), t)$ and monitor the addressing process of DNEs (1). The ZND model can be expanded:

$$J\dot{y}(t) = -\gamma\Theta(y(t), t) - \dot{\Theta}_t(y(t), t). \tag{6}$$

D. ANTI-NOISE MODEL

To assess the capability of the PNSND model (21) to suppress periodic noise, two anti-noise models, the IEZNN model and the CRNN model, are given to address DNEs(1) under periodic noise [32], [33]. The IEZNN model can be expressed as follows in addressing DNEs (1):

$$\begin{aligned} J\dot{y}(t) = &-\gamma\Theta(y(t), t) - \dot{\Theta}_t(y(t), t) \\ &- \alpha\int_0^t \Theta(y(g), g)dg + N(t), \end{aligned} \tag{7}$$

and the CRNN model is given as:

$$J\dot{y}(t) = -\gamma\Theta(y(t), t) - \dot{\Theta}_t(y(t), t) - x(t) + N(t), \tag{8}$$

where $x(t)$ adheres to the following principles:

$$x(t) = \begin{cases} x(t-T) + \alpha\Theta(y(t), t), \text{if } t \geq T \\ 0, \text{if } 0 < t < T, \end{cases} \tag{9}$$

where $T(= 2\pi f)$ represents the period of the periodic noise.

III. METHODOLOGY

A. ACCELERATED GND MODEL

To leverage the efficiency of the ZND model in addressing dynamic problems, we incorporate derivative information into the solution process of the GND model. The velocity compensation term $s(t)$ is introduced to address the dynamic aspects of the problem. This hybrid approach combines the strengths of both the GND and ZND models, resulting in a more efficient solution process for dynamic challenges. By considering the derivative information through the velocity compensation term $s(t)$, the accelerated GND model

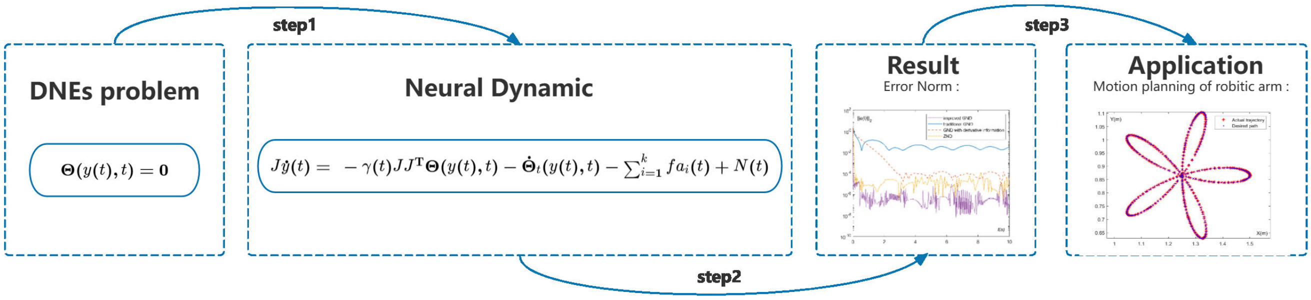

FIGURE 1. The construction process of PNSND model.

becomes more capable of effectively addressing DNEs (1) problems:

$$\dot{y}(t) = -\gamma J^{\mathrm{T}}\Theta(y(t), t) + s(t). \tag{10}$$

Achieving the ideal solution to the problem entails reaching a state of zero error, denoted as $e(t) = 0$, which also means that $\Theta(y(t), t) = 0$. Equation (10) reaches equilibrium and the following states can be obtained:

$$\dot{y}(t) = s(t). \tag{11}$$

Equation (10) is deformed by multiplying both sides by a Jacobian matrix J:

$$J\dot{y}(t) = -\gamma JJ^{\mathrm{T}}\Theta(y(t), t) + Js(t). \tag{12}$$

Subsequently, when the error function reaches its minimum value and stabilizes, the time derivative of the error $\dot{e}(t)$ is given as:

$$\dot{e}(t) = \dot{\Theta}_t(y(t), t) + J\dot{y}(t) = 0, \tag{13}$$

the following states is obtained:

$$\dot{\Theta}_t(y(t), t) = -J\dot{y}(t). \tag{14}$$

Substituting (11) and (14) into (12) generates the following accelerated GND model that can handle dynamic problems as well as the ZND model (6) and outperforms the ZND model (6):

$$J\dot{y}(t) = -\gamma JJ^{\mathrm{T}}\Theta(y(t), t) - \dot{\Theta}_t(y(t), t). \tag{15}$$

B. IMPROVED GND MODEL

In [36], the neural dynamic model can benefit from a proper selection of γ, which will increase the convergence rate. However, we discovered a neural dynamic with a parameter-changing function can be faster than one utilizing a fixed parameter in terms of convergence rate through many experiments. Building on this research, we propose a novel parameter-changing function that significantly outperforms other functions in terms of convergence capability, and we implement it in model (15) within the paper. As done above, replacing fixed parameters in model (15) with a parameter-changing function $\gamma(t) \geq 0$ yields the improved GND model with a better convergence rate. To analyze the performance of the improved GND model for addressing DNEs (1), we have the improved GND model:

$$J\dot{y}(t) = -\gamma(t)JJ^{\mathrm{T}}\Theta(y(t), t) - \dot{\Theta}_t(y(t), t), \tag{16}$$

where $\gamma(t) = p \arctan t + p^3, p \in N^+$.

C. PNSND MODEL

Our objective was to design a periodic noise suppression neural dynamic model, with a particular focus on the performance of the model under periodic noise. The construction process of PNSND model is shown in Figure 1. In mathematics, a periodic function is accurately represented through the Fourier series, which expresses it as an infinite sum of sine and cosine components, each with distinct amplitude and frequency. This representation is given by the following series:

$$N(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty} \left[a_i \cos(iw_0 t) + b_i \sin(iw_0 t)\right], \tag{17}$$

where $N(t)$ represents periodic noise, $\frac{a_0}{2}$ represents the direct component, a_i and b_i are the Fourier coefficients determined by properties of the function, i is an integer, w_0 is the angular frequency ($2\pi f$, where f is the frequency of periodic function). Significantly, for the PNSND model, the key to suppressing periodic noise lies in the suppression of the superposition of its harmonic components. The periodic function can be effectively reconstructed through the combination of a finite number of these terms, with increased accuracy achieved by using additional terms when necessary. However, it is not essential to deliberately introduce an excessive number of harmonic terms in pursuit of extreme precision, as the desired effect can be achieved within a limited set. Figure 2 demonstrates the use of a sawtooth wave, a form of periodic noise, as an example of harmonic term expansion. In practice, it is unnecessary to add an excessive number of compensatory terms, which would only serve to increase the computational load unnecessarily. When conditions permit, that is, when periodic noise is a singular function, that is, the direct component and the cosine component are zero, the harmonic term of periodic noise can be completely represented by the superposition of finite sine

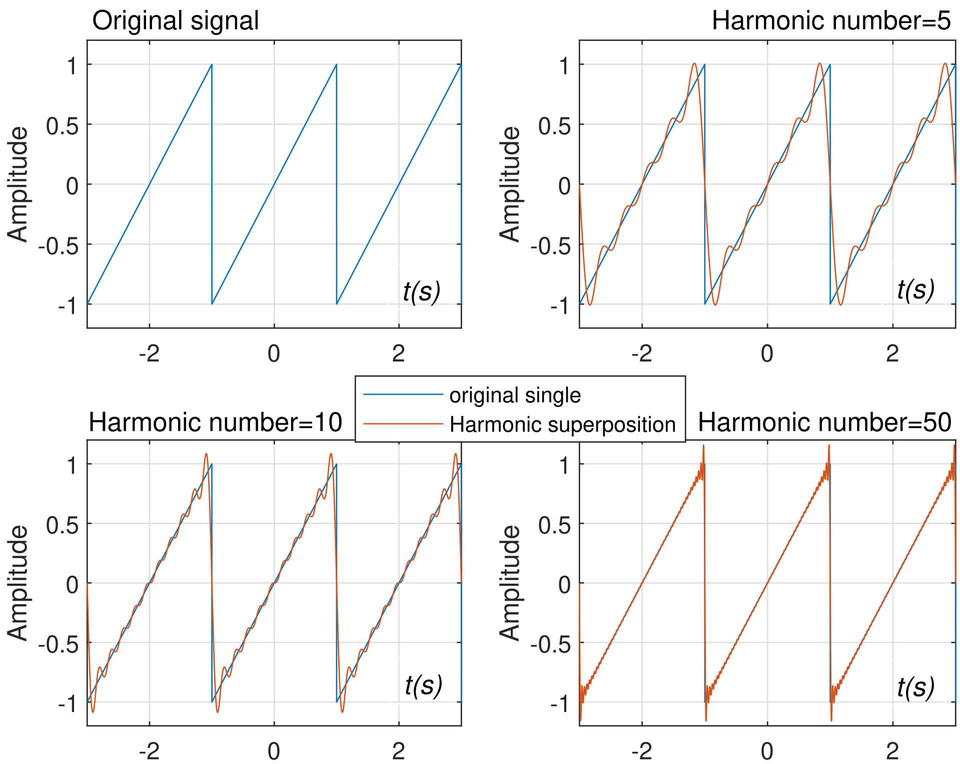

FIGURE 2. Taking sawtooth wave noise as an example, the Fourier of periodic noise is expanded into a finite number of harmonic terms, and then these harmonic terms are superimposed to restore the original signal.

functions as shown below:

$$N(t) = \sum_{i=1}^{k} b_i \sin(2i\pi f t + \phi_i),\ k \in [1, \infty], \tag{18}$$

where k is the number of harmonic terms, ϕ_i is unknown phase. Taking the derivative of the noise twice can be obtained:

$$\ddot{N}(t) = -\sum_{i=1}^{k} 4i^2\pi^2 f^2 b_i \sin(2i\pi f t + \phi_i),\ k \in [1, \infty]. \tag{19}$$

It follows that $\ddot{N}(t)$ is proportional to $N(t)$, which is a characteristic of periodic noise. We proceed by simulating the behavior of periodic noise, harnessing its distinctive properties in conjunction with the error function to construct harmonic terms. The following is the specific information of the harmonic terms:

$$\begin{cases} \dot{a}_i(t) = q_i(t) + 4\pi^2 f_i^2 \Theta(y(t), t), \\ \dot{q}_i(t) = -4\pi^2 f_i^2 a_i(t), \\ f_i = f \cdot i,\ i \in [1, k], \end{cases} \tag{20}$$

where variables $a_i(t)$ and $q_i(t)$ are crafted to leverage the unique property that the second derivative of periodic noise correlates with the noise itself, effectively intertwining the error function with periodic noise data. This design serves to integrate the dynamics of both the error and the noise, enhancing the responsiveness of model to periodic disturbances. According to the above method, the appropriate harmonic terms can be designed by analyzing the characteristics of the periodic noise, and combining the information of the error function. These harmonic compensation terms are specifically crafted to counteract the effects of the periodic noise, effectively reducing its impact on the solution process. The PNSND model is constructed by combining the improved GND model (16) and the harmonic compensation term to effectively suppress the periodic noise, which is constructed as follows:

$$\begin{cases} J\dot{y}(t) = -\gamma(t) J J^{\mathrm{T}} \Theta(y(t), t) - \dot{\Theta}_t(y(t), t) \\ \qquad - \sum_{i=1}^{k} f a_i(t) + N(t), \\ \dot{a}_i(t) = q_i(t) + 4\pi^2 f_i^2 \Theta(y(t), t), \\ \dot{q}_i(t) = -4\pi^2 f_i^2 a_i(t), \\ f_i = f \cdot i,\ i \in [1, k], \end{cases} \tag{21}$$

where $\sum_{i=1}^{k} f a_i(t)$ is the harmonic compensation term for periodic noise $N(t)$. By incorporating the designed harmonic term into the model, the periodic noise can be suppressed, leading to improved performance and accuracy in addressing the problem.

IV. THEORETICAL ANALYSIS

In this section, the efficacy of the PNSND model (21) for addressing the DNEs (1) with periodic noise is proved through the following theorems. First, we analyze the convergence of the PNSND model (21) without noise in Theorem 1. In addition, we further analyze the robustness of the PNSND model (21) under periodic noise in Theorem 2.

A. CONVERGENCE OF PNSND

Theorem 1, along with its detailed proof, is provided to comprehensively analyze the convergence performance of the PNSND model (21).

Theorem 1: The solution of DNEs derived from the PNSND model (21) converges exponentially to the theoretical solution (1) in the absence of noise.

Proof: First, when the PNSND model (21) is in the case of no noise, that is, $N(t) = 0$ and $f = 0$, then the harmonic compensation term is naturally zero. The expression of PNSND model (21) in the absence of noise is identical to the one presented for the improved GND model (16), with additional transformations, we derive the subsequent equation:

$$J\dot{y}(t) + \dot{\Theta}_t(y(t), t) = -\gamma(t)JJ^{\mathrm{T}}\Theta(y(t), t). \tag{22}$$

Next, combining (13) and (22) leads to:

$$\dot{e}(t) = J\dot{y}(t) + \dot{\Theta}_t(y(t), t) = -\gamma(t)JJ^{\mathrm{T}}e_i(t). \tag{23}$$

Elementwisely, the i th subsystem is obtained and $\lambda_i > 0$ represents the i th eigenvalue of the matrix JJ^{T}:

$$\dot{e}_i(t) + \lambda_i\gamma(t)e_i(t) = 0, \tag{24}$$

where $i \in [1, n]$. The proof of convergence for this system can be converted to solving linear homogeneous differential equations of first order and its general solution is as follows:

$$\begin{aligned} e_i(t) &= C_0 e^{-\lambda_i \int \gamma(t)dt} \\ &= C_0 e^{-\lambda_i(p^3 t + t\arctan(t) - \frac{ln(t^2+1)}{2} + C_1)}, \end{aligned} \tag{25}$$

where C_0 and C_1 are all constants, let $\kappa(t) = p^3t + t\arctan(t) - \frac{ln(t^2+1)}{2} + C_1$, and $\dot{\kappa}(t)$ can be represented:

$$\dot{\kappa}(t) = p^3 + \arctan(t) > 0. \tag{26}$$

Then $\kappa(t)$ is an increasing function, when $t \to \infty$, $\kappa(t) \to \infty$, and we can get:

$$\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} C_0 e^{-\lambda_i\kappa(t)} = 0. \tag{27}$$

The following inference can be made:

$$\lim_{t\to\infty} ||e(t)||_2 = 0. \tag{28}$$

Therefore, it can be summarized and generalized that the residual error of the PNSND model (21) in addressing DNEs (1) exponentially converges to zero in the absence of noise.

B. PNSND IN THE PRESENCE OF PERIODIC NOISE

In this section, Theorem 2 and the corresponding proof are provided to investigate the performance of the PNSND model (21) under the interference of periodic noise.

Theorem 2: The solution of DNEs derived from the PNSND model (21) converges exponentially to the theoretical solution (1) under the interference of periodic noise $N(t) = \sum_{i=1}^{k} b_i \sin(2i\pi ft + \phi_i)$, where amplitude b_i and phase ϕ_i remain unspecified.

Proof: Based on the properties of the introduced periodic noise $N(t) = \sum_{i=1}^{k} b_i \sin(2i\pi ft + \phi_i)$, adding the following periodic noise information:

$$\begin{cases} \dot{N}(t) = \sum_{i=1}^{k} 2i\pi f b_i \sin(2i\pi ft + \phi_i) \\ \qquad = 2\pi f_i N(t) = n(t), \\ \dot{n}(t) = \sum_{i=1}^{k} 4i^2\pi^2 f^2 b_i \sin(2i\pi ft + \phi_i) \\ \qquad = -4\pi^2 f_i^2 N(t). \end{cases} \tag{29}$$

Multiple noise compensation terms are represented by $\Delta(t)$, that is, $\Delta(t) = \sum_{i=1}^{k} fa_i(t)$. The PNSND model (21) can be represented as follows:

$$\begin{cases} J\dot{y}(t) = -\gamma(t)JJ^{\mathrm{T}}\Theta(y(t), t) - \dot{\Theta}_t(y(t), t) \\ \qquad\quad -\Delta(t) + N(t), \\ \dot{a}(t) = q(t) + 4\pi^2 f_i^2 \Theta(y(t), t), \\ \dot{q}(t) = -4\pi^2 f_i^2 a(t), \\ \dot{N}(t) = n(t), \\ \dot{n}(t) = -4\pi^2 f_i^2 N(t). \end{cases} \tag{30}$$

By defining

$$\begin{cases} b(t) = \Delta(t) - N(t), \\ c(t) = q(t) - n(t), \end{cases} \tag{31}$$

a more concise and simple expression can be obtained:

$$\begin{cases} J\dot{y}(t) = -\gamma(t)JJ^{\mathrm{T}}\Theta(y(t), t) - \dot{\Theta}_t(y(t), t) - b(t), \\ \dot{b}(t) = c(t) + 4\pi^2 f^2 \Theta(y(t), t), \\ \dot{c}(t) = -4\pi^2 f_i^2 b(t). \end{cases} \tag{32}$$

Combining the definition of $\dot{e}(t)$, the above matrix representation can be converted into the subsequent n-dimensional subsystem:

$$\begin{cases} \dot{e}_i(t) = -\lambda_i\gamma(t)e_i(t) - b_i(t), \\ \dot{b}_i(t) = c_i(t) + 4\pi^2 f^2 e_i(t), \\ \dot{c}_i(t) = -4\pi^2 f_i^2 b_i(t), \end{cases} \tag{33}$$

where $i \in [1, n]$ and $\lambda_i > 0$ denotes the i th eigenvalue of the matrix JJ^T. Barbalat's lemma can be used to analyze the aforementioned system because it can be thought of as a non-autonomous dynamic system. The immediate corollary in [37]: the convergence of states in the system can be examined by employing a Lyapunov-like lemma, which is commonly utilized to assess the stability of dynamic systems. For this purpose, the Lyapunov function can be defined as follow:

$$V_2(t) = \frac{e_i^2(t)}{2} + \frac{b_i^2(t)}{8\pi^2 f_i^2} + \frac{c_i^2(t)}{32\pi^4 f_i^4}. \tag{34}$$

We get $\dot{V}_2(t)$ with (33):

$$\begin{aligned} \dot{V}_2(t) &= \dot{e}_i(t)e_i(t) + \frac{\dot{b}_i(t)b_i(t)}{4\pi^2 f_i^2} + \frac{\dot{c}_i(t)c_i(t)}{16\pi^4 f_i^4} \\ &= -\lambda_i\gamma(t)e_i^2(t) \le 0. \end{aligned} \tag{35}$$

It can be obtained that $V_2(t) \le V_2(0)$, and $e_i(t)$, $b_i(t)$, and $c_i(t)$ are bounded. Due to the nonautonomous nature of the dynamic system (33), the direct application of the invariant set theorem for analysis is not possible. To meet the requirements of Barbalat's lemma, it is necessary to establish the uniform continuity of $\dot{V}_2(t)$ and provide proof for it. Then we take the derivative of $\dot{V}_2(t)$ and get $\ddot{V}_2(t)$

$$\ddot{V}_2(t) = -\lambda_i \left\{ e_i(t) \left[\dot{\gamma}(t) - \lambda_i \gamma^2(t) \right] - \gamma(t) b_i(t) \right\}, \quad (36)$$

both $\gamma(t)$ and $\dot{\gamma}(t) = \frac{p}{1+t^2}$ are abounded. Since it has been demonstrated that $e_i(t)$ and $b_i(t)$ are bounded, it follows that $\ddot{V}_2(t)$ is also bounded. Consequently, the uniform continuity of $\dot{V}_2(t)$ can be inferred. Last, it can be summarized from the Barbalats lemma:

$$\lim_{t\to\infty} e_i(t) = 0. \quad (37)$$

So it can be deduced:

$$\lim_{t\to\infty} ||e(t)||_2 = 0. \quad (38)$$

Therefore, the residual error of the PNSND model (7) in addressing (1) globally converges to zero with the interference of periodic noise.

V. SIMULATION EXPERIMENT

A. NUMERICAL SIMULATION

In the previous sections, theoretical analysis was conducted to prove that the PNSND model (7) exhibits both convergence and robustness. This section presented numerical simulations to compare the above models' performance in addressing the DNEs (1) problem under different conditions. For convenience, a simple system of nonlinear equations was directly chosen for this task. The following are the equations and their corresponding theoretical solutions:

$$\Theta(y(t), t) = \begin{bmatrix} \ln(y_1(t)) - \frac{1}{t+1} \\ y_1(t)y_2(t) - e^{\frac{1}{t+1}} \sin(t) \\ y_1^2(t) - \sin(t)y_2(t) + y_3(t) - 2 \\ y_1^2(t) - y_2^2(t) + y_3(t) + y_4(t) - t \end{bmatrix}, \quad (39)$$

$$y^*(t) = \begin{bmatrix} e^{\frac{1}{t+1}} \\ \sin(t) \\ 2 - e^{\frac{2}{t+1}} + \sin^2(t) \\ t-2 \end{bmatrix}. \quad (40)$$

The corresponding computer simulation of DNEs (1) is carried out considering the following three cases, that is, no noise interference, square wave noise interference, and triangular wave noise interference. The model parameters $\gamma = 100, p = 100, \alpha = 100, k = 6$, and the simulation duration is 10 s. For controlling the convergence speed, we have chosen to standardize the coefficient across all models with γ. Specifically, for the PNSND model, which typically employs a variable parameter function to control convergence, we have unified its parameter p with γ to ensure a single variable is used for convergence control. Additionally, for the anti-noise component of models, where α is a coefficient related to the noise reduction capability, we have set α to a uniform value for two anti-noise models in order to facilitate the comparison of the anti-noise performance. For the number of harmonic terms k, we have determined that a finite number of terms, as illustrated in Figure 2, can achieve the desired effect, thus k takes a value that is small and still achieves the desired performance.

1) IN THE ABSENCE OF NOISE INTERFERENCE

Simulation results of the traditional GND model (4), the ZND model (6), the accelerated GND model (15) and the improved GND model (16) in the absence of noise interference are shown in Figure 3. The four models are applied to address dynamic nonlinear equations under the same conditions for comparison purposes. As depicted in Figure 3 (a), the traditional GND model (4) generates large lagging errors, thus the solution is not well. The accelerated GND model (15) almost lost its ability to converge within the initial three seconds due to an exceedingly slow rate of convergence. The ZND model is specially used to address this kind of dynamic problem, so the solution effect is bound to be better than the GND model. Whereas the improved GND model (16) has better convergence accuracy and faster convergence speed compared with the above models, even surpasses that of the ZND model (6) as depicted in Figure 3. The proposed improved GND model (16) features a velocity compensation term $s(t)$ which is $\dot{\Theta}(y(t), t)$ from a control perspective, and it can control $\Theta(y(t), t) = 0$ well. In short, every solution of the improved GND model (16) converges to the theoretical solution and achieves good results, as demonstrated in Figure 3 (b).

2) UNDER THE INTERFERENCE OF PERIODIC NOISE

The section presented numerical simulations to evaluate and contrast the performance of all the models in the paper under periodic noise. First, the simulations are carried out to assess the sensitivity of CRNN model (7) and PNSND model (21) to high-frequency noise. Figure 4 depicts that the residual resulting from the CRNN model (7) undergoes notable harmonic oscillatory behavior. The oscillation becomes more pronounced as the frequency increases, with the order of oscillation reaching up to the order of 10^0. However, the PNSND model (21) ensures that the residual converges to the order of 10^{-4} regardless of the noise frequency. Subsequent simulations are conducted to compare the resistance of PNSND model (21), IEZNN model (7), and CRNN model (8) to various types of periodic noise. In this comparative simulation study, square wave and triangular wave noises, which are prevalent in industrial applications, are selected to validate the robustness of the aforementioned models. The waveforms of these two distinct periodic noises are depicted in Figure 5.

- Under the interference of square wave noise (in Figure 5 (a)): Simulation outcomes for the trio of anti-noise models are shown in Figure 6. The residual error of the IEZNN model (7) exhibits a periodic characteristic, as observed in the simulation results. Because the construstion of the model only considered the change of $e(t)$ and did not involve any information about noise. Conversely, the CRNN model (8) incorporates the

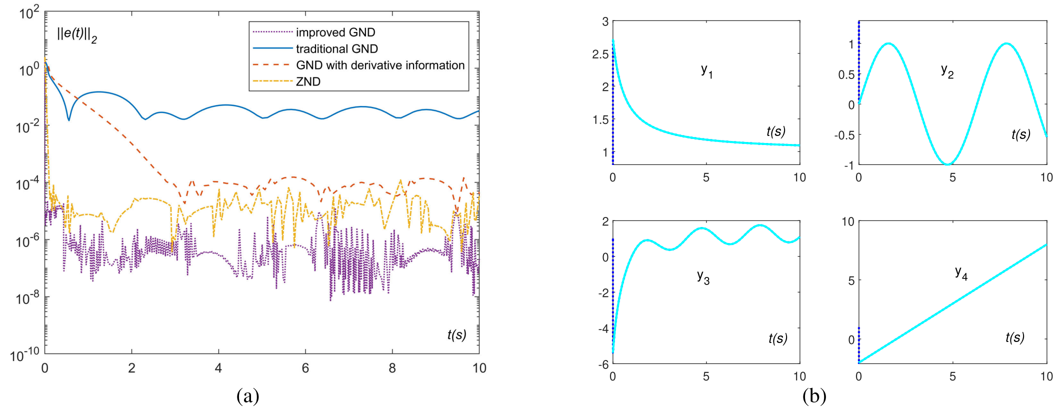

FIGURE 3. (a) The residual error of four models for addressing DNEs (1) in the absence of noise. (b) The theoretical solution (cyan-solid line) and computed solution (blue-dotted line) of the improved GND model (16).

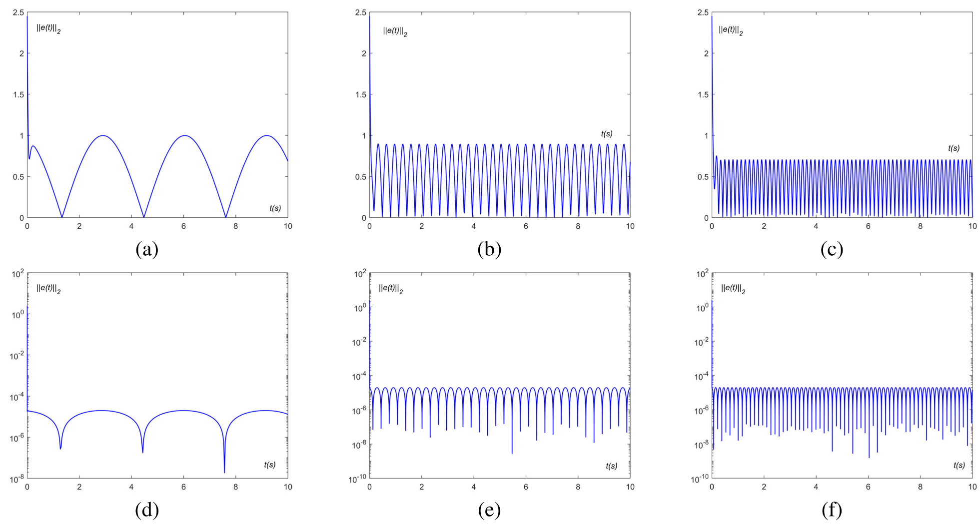

FIGURE 4. The residual error of PNSND and CRNN for addressing DNEs (1) under the interference of the sine function with an amplitude of 10 and different frequency(f_1). (a) CRNN (8) with $f_1 = 1/2\pi$. (b) CRNN (8) with $f_1 = 5/\pi$. (c) CRNN (8) with $f_1 = 10/\pi$. (d) PNSND (21) with $f_1 = 1/2\pi$. (e) PNSND (21) with $f_1 = 5/\pi$. (f) PNSND (21) with $f_1 = 10/\pi$.

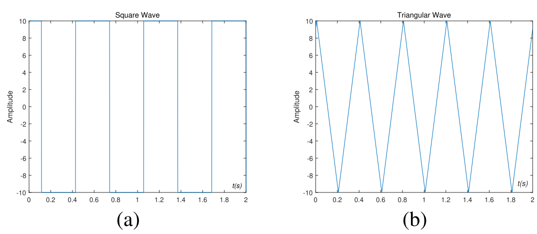

FIGURE 5. Two distinct types of periodic disturbances. (a) Square wave noise. (b) Triangular wave noise.

periodic noise information, thereby the convergence effect of residual error is better than the IEZNN model (7). However, in comparison to the aforementioned models, the PNSND model (21) obtains the most important frequency characteristics of periodic noise and combines it with the information of error function to suppress the noise from the harmonic perspective of the periodic noise, which makes the residual convergence faster and more accurate.

- Under the interference of triangular wave noise (in Figure 5 (b)): It is theoretically established that the PNSND model (7) is capable of efficiently suppressing any form of periodic noise. Under the interference of triangular wave noise, the PNSND model (21) proposed in this paper still maintains excellent robustness compared with the previous two models in Figure 7 (a). In this situation, the PNSND model (7) exhibits the most superior convergence characteristics and achieves the order of 10^{-4}, as can be observed from Figure 7 (b), it is obvious that the PNSND model (21) has a good addressing ability to address DNEs (1) problem.

In general, when comparing the PNSND model (21) with the IEZNN model (7) and the CRNN model (8), it can be concluded that the PNSND model (21) is the optimal

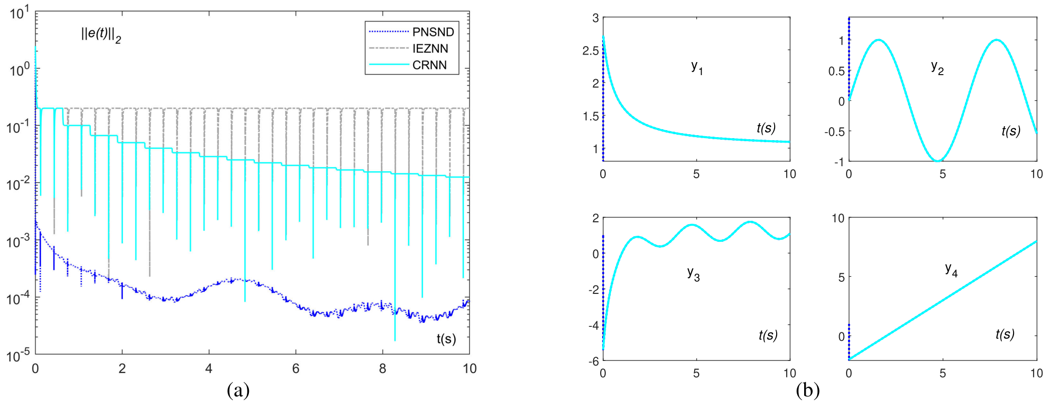

FIGURE 6. **The residual error of three models for addressing DNEs (1) under the interference of square wave noise. (b) The theoretical solution (cyan-solid line) and computed solution (blue-dotted line) of the PNSND model (21) under the interference of square wave noise.**

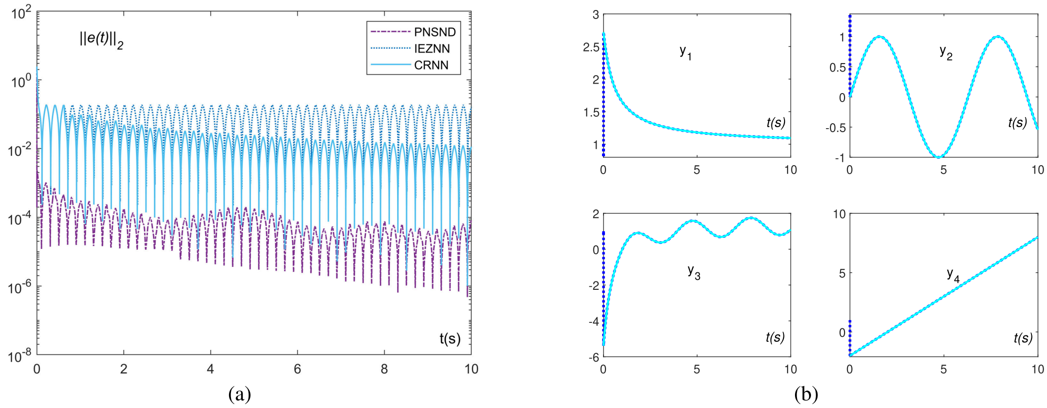

FIGURE 7. **The residual error of three models for addressing DNEs (1) under the interference of triangular wave noise. (b) The theoretical solution (cyan-solid line) and computed solution (blue-dotted line) of the PNSND model (21) under the interference of triangular wave noise.**

approach for addressing DNEs problems under the interference of periodic noise. The PNSND model (21) exhibits the highest accuracy and the fastest convergence rate among the aforementioned models. Since the PNSND model (21) combines the information of the error function and frequency characteristics of periodic noise, the noise interference is suppressed from the harmonic angle of periodic noise. Therefore, in terms of efficiency and effectiveness, the PNSND model (21) outperforms the IEZNN model (7) and the CRNN model (8) in addressing DNEs problems under the interference of periodic noise.

B. APPLICATION OF ROBOTIC ARM

In the preceding sections, the PNSND model (21) was employed to address the DNEs issue (1). In this section, we further validate the efficacy of the PNSND model (21) by applying it to a robotic arm for real-world applications. A robotic arm can be characterized by the linkage between the joint angles and the location of its end effector, which is described through forward kinematics [38], [39], [40]. The location of the end effector is derived from the joint angles, and this linkage is encapsulated in the following formula:

$$\epsilon(\theta(t)) = l(t) \in R^m. \tag{41}$$

$\theta(t) \in R^n$ denotes the joint angle of the robotic arm, $l(t)$ denotes the location of the end effector with $n = m$, and $\epsilon(\cdot)$ represents a nonlinear forward kinematics mapping function.

In this simulation, the two-link planar robotic arm is used to draw a five-leaf grass pattern under the interference of square wave noise (in Figure 5 (a)). The robotic arm has two connecting rods, each measuring 1 meter in length, and the initial joint state is $[0, \pi/3]^T$ rad. The parameters of the PNSND model (21) are identical to those utilized in the preceding simulations. The simulation results of the PNSND model (21) applied to the motion trajectory of the robotic arm are illustrated in figure 8. Figure 8 (a) and (c) reveal that the actual trajectory produced by the PNSND model (21) is highly close to the desired path and the given task is completed well under the interference of periodic noise. The maximum error of the corresponding location of the end effector is also well controlled within 10^{-3} m. Furthermore, it is observed that the joint angles ultimately revert to their

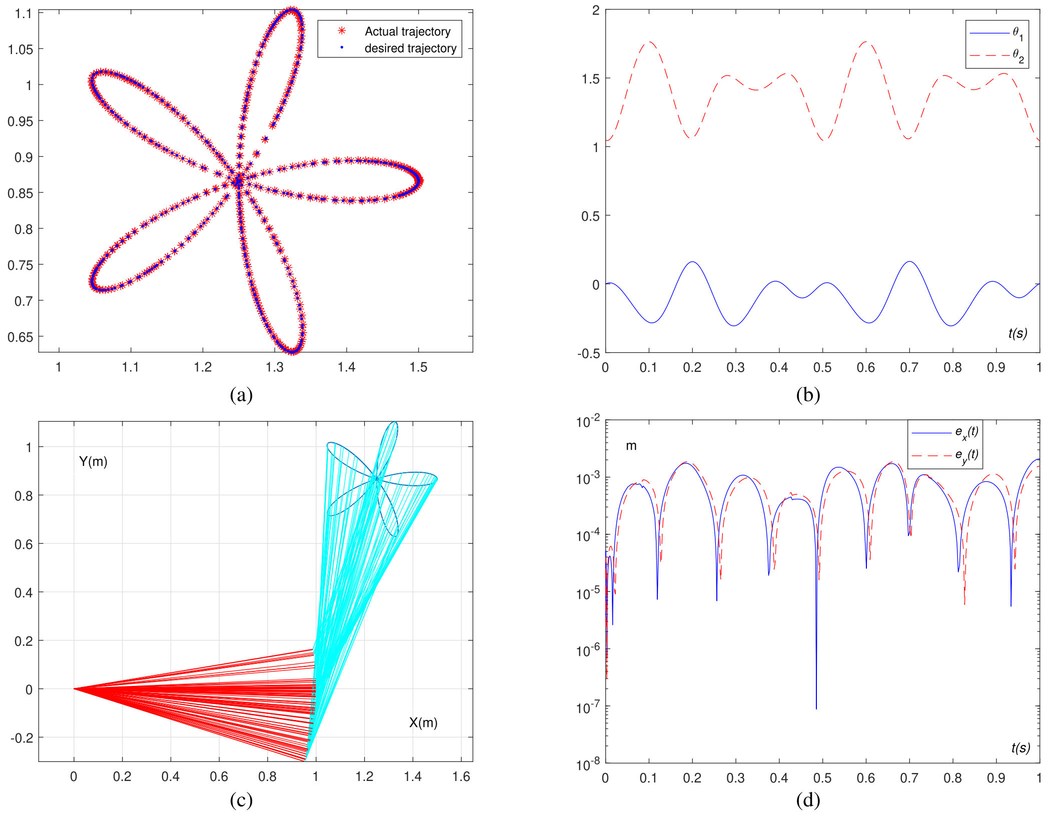

FIGURE 8. Application of the PNSND model (13) to kinematic control of a two-link planar robotic arm: (a) Desired path and actual trajectory. (b) Joint-angle profiles $\theta(t)$. (c) Motion trajectories. (d) Corresponding location error of end-effector $e(t)$.

original positions. This indicates that both the resolution of kinematic redundancy and the occurrence of joint drift have been altered due to the influence of periodic noise. Through this simulation experiment, it can be concluded that the PNSND model (21) can address dynamic nonlinear problems effectively under the interference of periodic noise. The practical use of the PNSND model (21) in the industrial setting further confirms its efficiency and advantageous performance.

VI. CONCLUSION AND FUTURE

The paper proposes the PNSND model, designed to accurately address DNEs in the presence of periodic noise interference. To refine solution precision, the traditional GND model, previously unable to address dynamic issues, is extended by incorporating a velocity compensation mechanism. This modification enables the GND model to effectively handle dynamic problems. Additionally, we propose a novel parameter-varying function coefficient to enhance the model's convergence capacity and speed. By leveraging Fourier expansion of periodic noise, harmonic compensation is employed to nullify the effects of periodic noise interference. The proposed PNSND model demonstrates superior performance in eliminating periodic noise interference compared to existing noise suppression neural dynamic models, leveraging the frequency characteristics of the periodic noise and the variation of the residual error. Comparative simulations confirm the PNSND model's enhanced capability in suppressing periodic noise during DNEs handling, outperforming other neural dynamic models. Furthermore, robotic arm experiments validate the effectiveness of the PNSND model. Theoretically, it expands the scope of neural dynamics by introducing a novel approach to addressing DNEs under periodic noise, providing a robust framework for future studies in the field. Practically, the PNSND model serves as a robust tool for engineers and practitioners, significantly enhancing the stability and precision of dynamic systems amidst noise. It optimizes control systems and offers a crucial solution for precise dynamic system modeling and control, boosting the efficiency and effectiveness of engineering systems.

In the forthcoming research, we intend to delve deeper into the potential of PNSND model, particularly in addressing the balance between the number of harmonic compensation terms and computational capacity. As computational power continues to grow, we can enhance the performance of model to suppress periodic noise by increasing the number of compensation terms. However, it is crucial to identify the optimal trade-off between computational complexity and solution accuracy. This may involve algorithmic optimizations to ensure that the model remains efficient even with an increased number of compensation terms. Given the specialized design of our model to counteract periodic noise, the current limitation is that it can only be adapted to scenes with mainly periodic noise interference. This targeted approach, while effective for periodic interference, may not extend as effectively to environments rife with non-periodic noise. We aim to extend the applicability of the model to not only periodic noise but also non-periodic disturbances with periodic characteristics.

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SONGJIE HUANG received the B.E. degree from the College of Electronics and Information Engineering, Guangdong Ocean University, Zhanjiang, China, in 2019, where he is currently pursuing the M.E. degree in electronic information with the College of Electronics and Information Engineering. His current research interests include neural dynamic, robotics, and optimization.

PEIXUAN ZHANG is currently pursuing the degree with the Communication Engineering Program, School of Electronics and Information Engineering, Guangdong Ocean University, Zhanjiang, China. His current research interests include neural networks and optimization algorithms.

KEER WU received the master's degree from Shantou University, in 2023. She is currently employed as a full-time Teacher with the Department of Electronic and Information Engineering, Guangdong Ocean University. Her research interests include image processing and computer vision.

XIUCHUN XIAO received the Ph.D. degree in communication and information system from Sun Yat-sen University, Guangzhou, China, in 2013. He is currently a Full Professor with the School of Electronics and Information Engineering, Guangdong Ocean University, Zhanjiang, China. His current research interests include artificial neural networks, image processing, and computer vision.