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RESEARCH ARTICLE

Interval-Valued Spherical Fuzzy Extension of DEMATEL and Its Application in Early-Stage Investment

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ABSTRACT Decision-making trial and evaluation laboratory (DEMATEL) is a practical method used to analyze the interrelationships between criteria while evaluating their importance in multi-criteria decision-making (MCDM) problems. This paper proposes an extension to DEMATEL using a fuzzy set—specifically, an interval-valued spherical fuzzy set (IVSFS)—which is one of the latest fuzzy developments. In contrast to the fuzzy extensions of DEMATEL reported in the literature, this study proposes a new IVSFS version of the DEMATEL methodology that incorporates experts' weights. The proposed IVSFS–DEMATEL methodology uniquely manages the weights of experts using the form of IVSFSs, differing from existing DEMATEL methodologies. Furthermore, it aims to maintain the fuzziness of operations until the later stages, unlike existing methodologies that implement defuzzification operations at the early stages. A real case study of an early-stage investment process is conducted to justify the feasibility and applicability of the proposed methodology. Sensitivity and comparison analysis are performed to confirm the validity of the results.

INDEX TERMS DEMATEL, interval-valued spherical fuzzy set (IVSFS), multi-criteria decision-making, experts' weights, early-stage investment.

I. INTRODUCTION

The decision-making trial and evaluation laboratory (DEMATEL) method was developed at the Battelle Memorial Institute of Geneva by Gabus and Fontela [1] in 1972. It is a well-known multi-criteria decision-making (MCDM) method that has been widely used in various fields to solve decision-making problems [2], [3], [4], [5], [6], [7]. Unlike other MCDM methods, such as the analytic hierarchy process with the assumption that criteria are independent, which typically assumes the criteria are mutually independent, the DEMATEL method does not require this assumption but further helps the decision-makers in identifying the causal relationships among the criteria [3], [4], [5], [6]. The fundamental principle of DEMATEL is to analyze and visualize the interrelationship (i.e., influence series) among the criteria, then results in their classification into cause-and-effect groups while evaluating their importance provided [4], [8], [9], [10]. Moreover, the outputs include clarifications of interdependencies among the criteria and the construction of diagrams that reflect these connections. Similar to other classical MCDM methods, DEMATEL requires a group of experts to provide judgments against criteria using evaluation scales, usually in the form of pairwise judgments [4], [8]. Gabus and Fontela [1] developed a crisp five-level assessment scale, in which 0 represented no influence, 1 represented low influence, 2 represented medium influence, 3 represented high influence, and 4 represented very high influence. However, crisp measures cannot handle the intrinsic fuzziness, uncertainty, or hesitancy in human consciousness while experts express their judgments [9], [10], [11]. To address this problem, the DEMATEL method has been expanded to address uncertain and fuzzy environments using the fuzzy set theory [12] and extensions to solve real-world decision-making problems comprehensively and properly.

Fuzzy sets and their extensions with single- or interval-valued sets are commonly used to model uncertainty and hesitancy in the decision-making evaluation process. This was followed by the presentation of ordinary fuzzy sets (FSs) by Zadeh [12]. Turksen [13] proposed interval-valued fuzzy sets

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(IVFSs) by replacing real numbers with intervals in the FSs. In other words, IVFSs can combine the judgments of different experts by allowing them to assign intervals for parameters rather than single and constant values [14]. Therefore, this feature extends the representation power of the FSs while demonstrating and quantifying the experts' judgments [14]. Over the past decades, researchers have developed extensions of ordinary FSs using single-valued or interval-valued sets. A latest detailed review on several developed extensions of ordinary FSs is given by Gündoğdu and Kahraman [14]. Single-valued intuitionistic FSs (SVIFSs) [15], interval-valued intuitionistic FSs (IVIFSs) [16], single-valued neutrosophic sets (SVNSs) [17], interval-valued neutrosophic sets (IVNSs) [18], single-valued hesitant FSs (SVHFS) [19], interval-valued hesitant fuzzy set (IVHFSs) [20], single-valued Pythagorean FSs (SVPYFSs) [21], interval-valued Pythagorean FSs (IVPYFSs) [22], single-valued picture FSs (SVPFSs) [23], and interval-valued picture FSs (IVPFSs) [24] are developed and applied in different areas.

The most recent development of FSs is the single-valued spherical FSs (SVSFSs), offering an independent hesitation degree from the other factors [25]. In SVSFSs, experts can assign independent three-dimensional FSs to membership, nonmembership, and hesitancy degrees (the sum of these three values must be between 0 and 1) [26]. The main advantage of SFSs is its capability to collect the positive features of other fuzzy set extensions into a unique theory that accepted aspects of SVNSs and SVPFS by excluding the criticized aspect of the neutrosophic theory (i.e., a sum membership, nonmembership, and hesitancy degrees larger than 1) and the criticized aspect of the Pythagorean fuzzy theory (i.e., disregarding an independent hesitancy) [25]. Lately, interval-valued spherical fuzzy sets (IVSFSs) first proposed by Gündoğdu and Kahraman [14] extend this advantage by assigning three-dimensional intervals to independent membership, nonmembership, and hesitancy degrees. IVSFSs allow experts to independently assign their hesitancies to a decision environment with a larger domain of intervals, rather than exact numbers. Thus, the uncertainty and hesitation hidden in expert judgments can be quantified and processed more comprehensively. Therefore, IVSFS are superior to the other fuzzy extensions with a more flexible characterization to solve MCDM evaluation problems [14], [27].

In the DEMATEL literature, there are many applications of FSs and their single-valued and IVFS extensions. For instance, the DEMATEL method has been expanded with the following fuzzy sets: FSs [10], [28], [29], IVFSs [30], SVIFSs [3], [31], IVIFSs [4], [32], SVHFSs [33], IVHFSs [8], SVPFSs [34], SVPYFSs [35], [36], IVPYFSs [37], SVNSs [38], [39], IVNSs [39], and SVSFSs [7], [9]. A related discussion of the current literature on fuzzy extensions of DEMATEL is presented in Section II. However, three-dimensional intervals of independent membership, nonmembership, and hesitancy degrees are limited. In addition, most research focusing on contemporary extensions of DEMATEL performs early defuzzification operations to convert fuzzy numbers to crisp numbers and does not keep the entire process fuzzy, as desired. This motivated us to explore possible mergers of IVSFS and DEMATEL (IVSFS–DEMATEL).

In addition, the increasing difficulty of MCDM evaluation problems and the shortage of experts, who usually come from various fields with different knowledge, skills, and years of experience, have led to diverse judgments of experts [4]. Moreover, experts usually make judgments according to their experience and expertise [35]. Thus, there is a need to consider expert weights when solving MCDM evaluation problems. However, research on fuzzy set extensions in the DEMATEL literature has neglected to present expert weights using IVSFSs with a three-dimensional interval domain. Thus, the second motivation was to determine the experts' weights by considering their experience and using the IVSFS entropy method.

In light of the constraints outlined in prior academic investigations into fuzzy research, this research aims to enhance the efficacy of handling uncertainty and hesitancy judgments in MCDM evaluations by proposing an IVSFS–DEMATEL methodology that incorporates expert weights derived from a linguistic evaluation scale based on IVSFSs. The contributions of this study are delineated as follows:

- Proposal of IVSFSs within the DEMATEL framework to address uncertainty and hesitation judgments, facilitating decision experts in expressing their assessments comprehensively while evaluating potential influences among the criteria in MCDM problems and incorporating expert weights.
- Utilization of the IVSFS entropy method to quantify expert weights and then integration into the DEMATEL methodology.
- Implementation of the IVSFS–DEMATEL methodology, which aims to retain fuzzy operations as close to the end as feasible, in contrast to prevalent practices in existing literature, which often employ early defuzzification operations.
- Demonstration of the effectiveness of the proposed IVSFS–DEMATEL model through a real-world case study analyzing the criteria for early-stage investments.

The remainder of this paper is organized as follows. Section II presents an overview of related work. Section III summarizes the preliminaries of the DEMATEL method and the IVSFS theory. Section IV describes the proposed IVSFS–DEMATEL model. Section V applies the IVSFS–DEMATEL method to analyze the interrelationships among the criteria in the early-stage investment process and includes a sensitivity analysis and a comparison analysis. Finally, the conclusions are presented in the last section.

II. LITERATURE REVIEW ON FUZZY EXTENSIONS OF DEMATEL

DEMATEL and its fuzzy extensions are widely used methodologies for MCDM. The DEMATEL technique using FSs has been comprehensively reviewed in previous studies. Si et al. [40] conducted an excellent state-of-the-art study of different forms of DEMATEL by collecting studies covering 346 papers published between 2006 and 2016, such as crisp, fuzzy, and grey. Other reviews presented by Gül [9], Giri et al. [35], and Zhu et al. [7] support the finding of increasing interest in the developed DEMATEL integrated with various FS forms.

This study focuses on a state-of-the-art review of contemporary fuzzy set extensions of the DEMATEL method and its applications over the past three years. Table 1 presents a comprehensive literature survey of the DEMATEL method using FSs and their extensions. Table 1 summarizes the fuzzy set type considered by each study, whether the study included expert weights, and the application areas.

According to Table 1, the fuzzy set extensions of the DEMATEL method have been applied to many decision problems in the areas of business management, finance, engineering, technology, energy, and health. Another interesting finding from the literature listed in Table 1 is that interest in fuzzy extensions of DEMATEL has increased over the last couple of years. Moreover, it can be obtained that most of the studies have focused on single types of FSs (i.e., single-valued sets).

Focusing on the spherical fuzzy extension of DEMATEL, which is a relatively new type of fuzzy set (as depicted in Fig. 1), it can be observed that considering the weights of experts in group decision-making problems is still limited. Most studies that have applied DEMATEL along with SVSFSs to group decision-making problems have not considered the effect of different expert knowledge and expertise in the final decision. Considering this, Gül [9], Yuan et al. [41], Aytekin et al. [42], Kou et al. [43], and Yüksel et al. [44] applied SVSFS-DEMATEL methodologies to address several decision issues without paying attention to expert weights (see Table 1). Zhu et al. [7] employed the SVSFS-DEMATEL methodology by considering expert weights as crisp forms in engineering applications. Other studies by Le and Nhieu [45], Wu et al. [46], and Pandey et al. [47] analyzed different decision issues using SVSFS-DEMATEL methodologies. They calculated experts' weights using SVSFSs operators to represent their experience and degree of knowledge. Only one study, by Jiang et al. [48], used DEMATEL with IVSFSs, considering the known crisp weights of the experts. However, measuring experts' weights using the IVSFSs forms in DEMATEL studies is limited when the number of studies is compared with the other extensions (i.e., SVSFSs). Moreover, DEMATEL with IVSFS [48] performed defuzzification operations that result in fuzzy information loss during the decision process.

Compared to the other fuzzy set extensions in Table 1, 4SFSs can express evaluation information more flexibly to expand the expression of experts and measure their weights in DEMATEL studies. The main advantage of IVSFSs is that they allow experts to independently assign their hesitancies to decision environments with a larger domain of intervals rather than exact numbers [14], [49]. Thus, this feature extends the representation power of FSs while demonstrating and quantifying expert judgments. Moreover, IVSFSs are preferred when experts are unsure of the values of membership, nonmembership, and hesitancy degrees in their judgments. Thus, integrating IVSFSs with the DEMATEL methodology contributes positively to the effectiveness of the analysis.

Therefore, to fill the aforementioned gaps, this study proposes a new IVSFS version of the DEMATEL methodology. The proposed methodology handles the weights of experts based on the IVSFSs. Furthermore, defuzzification operations are applied close to the end to maintain the entire fuzzy process as much as possible. In the following sections, the theoretical background of the IVSFS–DEMATEL methodology is presented, and its applicability to a real case study on early-stage investment criteria is discussed.

III. PRELIMINARIES

A. TRADITIONAL DEMATEL

This section presents the procedural steps of classical DEMATEL. This can be summarized in five steps as follows [56], [57]:

Step 1: Generate an initial direct-relationship (IDR) matrix for n criteria within the system. Suppose a system consists of n criteria, and the degree of influence between criteria c_i and c_j can be expressed as $x_{ij} \in 0, 1, 2, 3; i, j = 1, 2, ..n$. The IDR matrix that describes the degree of direct influence between all pairs of the criteria in the system is defined as $X = \left[x_{ij}\right]_{n\times n}$, where $x_{ij} = 0$ for $i = j$. The degree of influence between two criteria is '$0 - 3$', where '0' means there is "no influence" between the two criteria, '1' means there is a "low influence" between the two criteria, '2' means there is a "medium influence" between the two criteria, and '3' means there is a "very high influence" between the two criteria.

Step 2: Normalize the IDR matrix. The normalized IDR matrix of X was determined using the following equation:

$$M = \left[m_{ij}\right]_{n\times n} = \frac{X}{\kappa}, \tag{1}$$

where $\kappa = max\left(\max_{1\leq i\leq n} \sum_j x_{ij}, \max_{1\leq j\leq n} \sum_i x_{ij}\right)$.

Step 3: Obtain the total relation matrix. The total relationship matrix T is obtained using the following equation:

$$T = \left[t_{ij}\right]_{n\times n} = M\left(I - M\right)^{-1}, \tag{2}$$

where I is an identity matrix.

Step 4: Calculate the prominence and relationship of each criterion. The sum of rows R and the sum of columns D of the total relation matrix T are calculated. According to $T = \left[t_{ij}\right]_{n\times n}$, r_i is the total influence of criterion c_i on the other criteria, $R = [r_i]_{n\times 1} = \sum_j t_{ij}; i, j = 1, 2, ..n$. Additionally, d_j is the total influence of criterion c_i on another criterion, $D = \left[d_j\right]_{1\times n} = \sum_i t_{ij}; i, j = 1, 2, ..n. r_i + d_i$ is defined as the prominence, showing the degree of importance

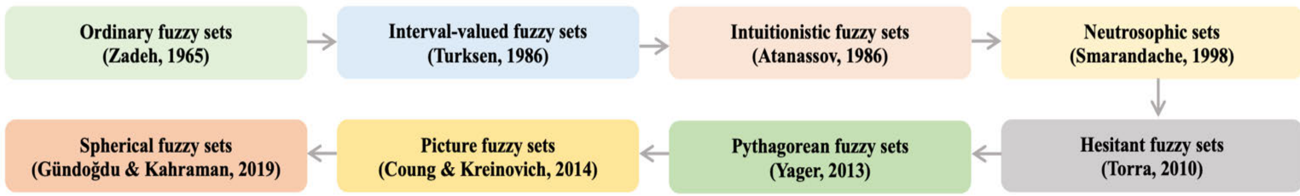

FIGURE 1. Fuzzy set (FS) extension flowchart (adopted from Gündoğdu and Kahraman [14]).

TABLE 1. Studies on contemporary fuzzy extensions of DEMATEL.

Author(s) and year	Type of FSs	Weights of experts	Application
Gül [9]	SVSFSs	-	Analyzing influences among the attributes in a building contractor selection problem.
Ocampo and Yamagishi [31]	SVIFSs	-	Modeling the lockdown relaxation protocols in response to the COVID-19 pandemic.
Guo *et al.* [33]	SVHFSs	SVHFS forms	Selecting sites of floating photovoltaic power generation projects.
Kilic *et al.* [50]	SVNSs		Assessing leanness to aid a company's lean transformation.
Lahane and Kant [51]	SVPYFSs	Crisp forms	Evaluating the circular supply chain implementation barriers.
Li *et al.* [30]	IVFSs	-	Identifying key factors influencing sustainable elements in healthcare waste management.
Yuan *et al.* [41]	SVSFSs	-	Defining strategies to increase green nuclear energy investments.
Aytekin *et al.* [42]	SVSFSs	-	Determining the importance level of criteria in the health information system selection.
Giri *et al.* [35]	SVPYFSs	-	Selecting suppliers in sustainable supply chain management.
Gül [34]	SVPFSs	-	Evaluating educational quality.
Karasan *et al.* [39]	SVNSs	-	Studying the relationships among the technical factors toward customer-oriented product design.
Kou *et al.* [43]	SVSFSs	-	Generating inventive maps of innovative carbon emission reduction strategies for investment projects.
Le and Nhieu [45]	SVSFSs	SVSFS forms	Assessing operational strategies for logistics.
Paul *et al.* [36]	SVPYFSs	SVPYFS forms	Evaluating the criteria weights for the selection of CO_2 storage sites in geological media.
Shafiee *et al.* [52]	SVPYFSs	-	Studying causality analysis of risks to perishable product supply chain networks during the COVID-19 outbreak.
Sundareswaran et al [38]	SVNSs	SVNS forms	Assessing the structural cracks in buildings.
Wu *et al.* [46]	SVSFSs	SVSFS forms	Choosing agritourism locations focusing on sustainability.
Yüksel and Dinçer [53]	IVPYFSs	-	Identifying the strategic priorities of nuclear energy investments.
Yüksel *et al.* [44]	SVSFSs	-	Evaluating the significant risks in geothermal energy investments.
Chen *et al.* [32]	IVFSs	IVFS forms	Assessing resumption risks amid COVID-19 prevention.
Mao *et al.* [37]	IVPYFSs	IVPYFS forms	Making investment decisions for offshore wind-PV-SPS power plants.
Pandey *et al.* [47]	SVSFSs	SVSFS forms	Evaluating and identifying issues in wearable apps.
Tsai *et al.* [54]	IVHFSs	-	Investigating the critical barriers to blockchain technology adoption in the Vietnamese agricultural supply chain.
Yu *et al.* [55]	IVPYFSs	-	Investigating the evolutionary process of the product service ecosystem.
Zhu *et al.* [7]	SVSFSs	Crisp forms	Identifying maintenance significant items for machine tools.
Jiang *et al.* [48]	IVSFSs	Crisp forms	Discovering the sustainable challenges of biomass energy.

that factor c_i plays in the system $r_i - d_i$ shows the net influence that factor c_i contributes to the system. If $r_i - d_i > 0$, the criterion c_i influences other criteria; thus, it will be a member of the "cause" group, and if $r_i - d_i < 0$, the factor c_i the criterion c_i influenced by others; thus, it will belong to the "effect" group.

Step 5: Building a network relation map (NRM). This visualization is based on the prominence $r_i + d_i$ and the relation $r_i - d_i$ values.

B. INTERVAL-VALUED SPHERICAL FUZZY SETS

The formal definitions of SVSFSs and IVSFSs are summarized in this subsection, followed by the definitions of basic operations and aggregation operators for interval-valued spherical fuzzy numbers (IVSFNs).

Definition 1: A SVSFS $\tilde{s}$ of the universe of discourse X is provided as [58]

$$\tilde{s} = \{< x, \mu_{\tilde{s}}(x), \nu_{\tilde{s}}(x), \pi_{\tilde{s}}(x) >, x \in X\}, \quad (3)$$

where $\mu_{\tilde{s}}(x) : X \longrightarrow [0, 1], \nu_{\tilde{s}}(x) : X \longrightarrow [0, 1]$ and $\pi_{\tilde{s}}(x) : X \longrightarrow [0, 1]$, which define the degrees of membership, nonmembership, and hesitancy of x to $\tilde{s}$, respectively, and $0 \leq \mu_{\tilde{s}}^2(x) + \nu_{\tilde{s}}^2(x) + \pi_{\tilde{s}}^2(x) \leq 1, \forall x \in X$. The refusal degree is calculated as

$$\zeta_{\tilde{s}}(x) = \sqrt{1 - \left(\mu_{\tilde{s}}^2(x) + \nu_{\tilde{s}}^2(x) + \pi_{\tilde{s}}^2(x)\right)}.$$

In the following, the definition of IVSFS is summarized, and some basic operations and aggregation operators for IVSFNs are defined.

Definition 2: An IVSFS $\tilde{s}$ of the universe of discourse X is defined as follows [49]:

$$\tilde{s} = \left\{ < x, \left[\mu_{\tilde{s}}^{L}(x), \mu_{\tilde{s}}^{U}(x)\right], \left[\nu_{\tilde{s}}^{L}(x), \nu_{\tilde{s}}^{U}(x)\right], \left[\pi_{\tilde{s}}^{L}(x), \pi_{\tilde{s}}^{U}(x)\right] >, \forall x \in X \right\}, \tag{4}$$

where $\left[\mu_{\tilde{s}}^{L}(x), \mu_{\tilde{s}}^{U}(x)\right] \subseteq [0,1]$, $\left[\nu_{\tilde{s}}^{L}(x), \nu_{\tilde{s}}^{U}(x)\right] \subseteq [0,1]$, $\left[\mu_{\tilde{s}}^{L}(x), \mu_{\tilde{s}}^{U}(x)\right] \subseteq [0,1]$, and $\mu_{\tilde{s}}^{U}(x)$, $\nu_{\tilde{s}}^{U}(x)$, $\pi_{\tilde{s}}^{U}(x)$ denote the upper membership, nonmembership, and hesitancy of x to $\tilde{s}$, respectively, and $0 \le \left(\mu_{\tilde{s}}^{U}(x)\right)^2 + \left(\nu_{\tilde{s}}^{U}(x)\right)^2 + \left(\pi_{\tilde{s}}^{U}(x)\right)^2 \le 1$. Additionally, $\mu_{\tilde{s}}^{L}(x)$, $\nu_{\tilde{s}}^{L}(x)$, and $\pi_{\tilde{s}}^{L}(x)$ denote the lower membership, nonmembership, and hesitancy of x to $\tilde{s}$, respectively. The refusal degree for the upper degrees of membership functions is defined as

$$\zeta_{\tilde{s}}^{U}(x) = \sqrt{1 - \left(\left(\mu_{\tilde{s}}^{U}(x)\right)^2 + \left(\nu_{\tilde{s}}^{U}(x)\right)^2 + \left(\pi_{\tilde{s}}^{U}(x)\right)^2\right)}.$$

The refusal degree for the lower degrees of membership functions is defined as

$$\zeta_{\tilde{s}}^{L}(x) = \sqrt{1 - \left(\left(\mu_{\tilde{s}}^{L}(x)\right)^2 + \left(\nu_{\tilde{s}}^{L}(x)\right)^2 + \left(\pi_{\tilde{s}}^{L}(x)\right)^2\right)}.$$

The pair $([\mu_{\tilde{s}}^{L}(x), \mu_{\tilde{s}}^{U}(x)], [\nu_{\tilde{s}}^{L}(x), \nu_{\tilde{s}}^{U}(x)], [\pi_{\tilde{s}}^{L}(x), \pi_{\tilde{s}}^{U}(x)])$ is an IVSFN. For convenience, an IVSFN is denoted by α. Its formal definition is provided by Jin et al. [49] as $\alpha = ([a,b], [c,d], [e,f])$, where $[a,b] \subseteq [0,1]$, $[c,d] \subseteq [0,1]$, $[d,f] \subseteq [0,1]$, and $0 \le (b)^2 + (d)^2 + (f)^2 \le 1$.

Definition 3: Let $\alpha = ([a,b], [c,d], [e,f])$, $\alpha_1 = ([a_1, b_1], [c_1, d_1], [e_1, f_1])$, and $\alpha_2 = ([a_2, b_2], [c_2, d_2], [e_2, f_2])$ be three IVSFNs, then basic operators [49]:

Union:

$$\alpha_1 \cup \alpha_2 = \left\{ \begin{array}{l} [max\{a_1, a_2\}, max\{b_1, b_2\}], [min\{c_1, c_2\}, min\{d_1, d_2\}], \\ [min\{e_1, e_2\}, min\{f_1, f_2\}] \end{array} \right\}. \tag{5}$$

Intersection:

$$\alpha_1 \cap \alpha_2 = \left\{ \begin{array}{l} [min\{a_1, a_2\}, min\{b_1, b_2\}], [max\{c_1, c_2\}, max\{d_1, d_2\}], \\ [min\{e_1, e_2\}, min\{f_1, f_2\}] \end{array} \right\}. \tag{6}$$

Addition:

$$\alpha_1 \oplus \alpha_2 = \left\{ \left[\left((a_1)^2 + (a_2)^2 - (a_1)^2 (a_2)^2\right)^{\frac{1}{2}}, \left((b_1)^2 + (b_2)^2 - (b_1)^2 (b_2)^2\right)^{\frac{1}{2}} \right], [c_1 c_2, d_1 d_2], \left[\left(\left(\left(1-(a_2)^2\right)(e_1)^2 + \left(1-(a_1)^2\right)(e_2)^2 - (e_1)^2 (e_2)^2\right)^{\frac{1}{2}}, \left(\left(1-(b_2)^2\right)(f_1)^2 + \left(1-(b_1)^2\right)(f_2)^2 - (f_1)^2 (f_2)^2\right)^{\frac{1}{2}}\right) \right] \right\}. \tag{7}$$

Multiplication:

$$\alpha_1 \otimes \alpha_2 = \left\{ [a_1 a_2, b_1 b_2], \left[\left((c_1)^2 + (c_2)^2 - (c_1)^2 (c_2)^2\right)^{\frac{1}{2}}, \left((d_1)^2 + (d_2)^2 - (d_1)^2 (d_2)^2\right)^{\frac{1}{2}} \right], \left[\left(\left(\left(1-(c_2)^2\right)(e_1)^2 + \left(1-(c_1)^2\right)(e_2)^2 - (e_1)^2 (e_2)^2\right)^{\frac{1}{2}}, \left(\left(1-(d_2)^2\right)(f_1)^2 + \left(1-(d_1)^2\right)(f_2)^2 - (f_1)^2 (f_2)^2\right)^{\frac{1}{2}}\right) \right] \right\}. \tag{8}$$

Multiplication by a scalar: $\lambda > 0$

$$\lambda.\alpha = \left\{ \left[\left(1 - \left(1-a^2\right)^{\lambda}\right)^{1/2}, \left(1 - \left(1-b^2\right)^{\lambda}\right)^{1/2} \right], \left[c^{\lambda}, d^{\lambda}\right], \left[\left(\left(1-a^2\right)^{\lambda} - \left(1-a^2-e^2\right)^{\lambda}\right)^{1/2}, \left(\left(1-b^2\right)^{\lambda} - \left(1-b^2-f^2\right)^{\lambda}\right)^{1/2} \right] \right\}. \tag{9}$$

λ th power of α $\mathcal{V}$ $\lambda \geq 0$

$$(\alpha)^{\lambda} = \left\{ \left[a^{\lambda}, b^{\lambda}\right], \left[\left(1-\left(1-c^2\right)^{\lambda}\right)^{1/2}, \left(1-\left(1-d^2\right)^{\lambda}\right)^{1/2} \right], \left[\left(\left(1-c^2\right)^{\lambda} - \left(1-c^2-e^2\right)^{\lambda}\right)^{1/2}, \left(\left(1-d^2\right)^{\lambda} - \left(1-d^2-f^2\right)^{\lambda}\right)^{1/2} \right] \right\}. \tag{10}$$

Definition 4: The score function of IVSFN $\alpha = ([a,b], [c,d], [e,f])$ is defined as [25]

$$SC(\alpha) = \frac{a^2 + b^2 - c^2 - d^2 - \left(\frac{e}{2}\right)^2 - \left(\frac{f}{2}\right)^2}{2}, \tag{11}$$

where $SC(\alpha) \in [-1, 1]$.

Definition 5: The interval-valued spherical weighted arithmetic mean (IVSWAM) method with respect to a weights vector is denoted as $w_j = (w_1, w_2, \ldots, w_n)$, where $w_i \in [0,1], i = 1, 2, \ldots n$ and $\sum_{j=1}^{n} w_j = 1$. $\alpha_j = \left(\left[a_j, b_j\right], \left[c_j, d_j\right], \left[e_j, f_j\right]\right)$ is a collection of IVSWAM and is presented as [25] and [49]:

$$IVSWAM_w = w_1 \cdot \alpha_1 \oplus w_2 \cdot \alpha_2 \oplus \ldots \oplus w_n \cdot \alpha_n = \left\{ \left[\left(1 - \prod_{j=1}^{n}\left(1-a_j^2\right)^{w_j}\right)^{\frac{1}{2}}, \left(1 - \prod_{j=1}^{n}\left(1-b_j^2\right)^{w_j}\right)^{\frac{1}{2}} \right], \right.$$

$$\left[\prod_{j=1}^{n}(c_j)^{w_j}, \prod_{j=1}^{n}(d_j)^{w_j}\right], \left[\left(\prod_{j=1}^{n}\left(1-a_j^2\right)^{w_j} - \prod_{j=1}^{n}\left(1-a_j^2-e_j^2\right)^{w_j}\right)^{1/2}, \left(\prod_{j=1}^{n}\left(1-b_j^2\right)^{w_j} - \prod_{j=1}^{n}\left(1-b_j^2-f_j^2\right)^{w_j}\right)^{1/2}\right]\Bigg\}. \tag{12}$$

IV. PROPOSED IVSFS–DEMATEL METHODOLOGY

In this section, an integrated model combining IVSFSs and DEMATEL is proposed to analyze the interrelationships among the criteria in MCDM evaluation problems. Methods that combine the DEMATEL method by considering the weights of decision experts in a spherical fuzzy environment were not found in the literature review. This study makes an important contribution in this direction.

Assume that $C = \{C_1, C_2, \ldots C_n\}$ is a finite set of the evaluation criteria, and $w = \{w_1, w_2, \ldots, w_n\}$ comprises the vector of weights for the investigated criteria, satisfying $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. If $E = \{E_1, E_2, \ldots E_k\}$ comprises the group of experts participating in the study, then experts' weights are given by the vector $\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_k\}$, satisfying $0 \leq \sigma_p \leq 1$ and $\sum_{p=1}^{k} \sigma_p = 1$. Notably, for the purposes of this research, the vector weights for both the criteria and experts are unknown. The framework of the proposed interval-valued spherical fuzzy extension of DEMATEL is illustrated in Fig. 2. This flowchart is unidirectional (downward), and each step is performed only once. Subsequently, the computational steps of the proposed model are introduced in detail.

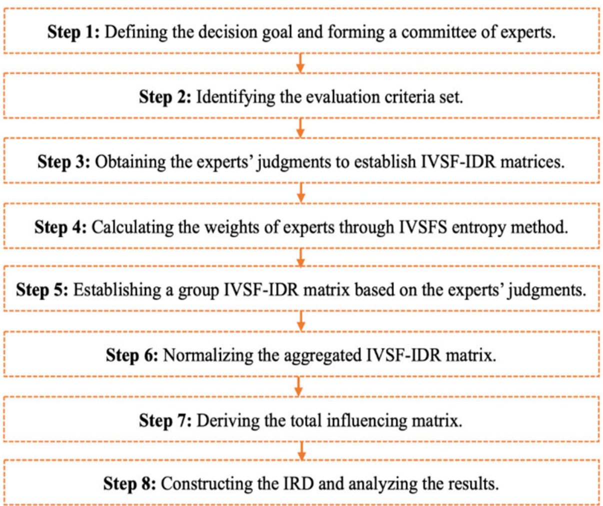

FIGURE 2. Proposed interval-valued spherical fuzzy extension of the DEMATEL model.

Step 1: Define the decision goal and form a committee of experts. The starting point defines a decision goal related to the problem under study. A group of experts $E = \{E_1, E_2, \ldots E_k\}$, whose opinions and judgments would serve to construct and analyze the problem, was established.

TABLE 2. IVSF linguistic terms of influence and their respective IVSFNs. Adopted from Mandal and Seikh [59].

Linguistic terms	IVSFNs
No influence (N)	([0.20, 0.30], [0.70, 0.80], [0.15, 0.25])
Low influence (L)	([0.30,0.40], [0.60,0.70], [0.25,0.35])
Medium influence (M)	([0.45,0.55], [0.40,0.50], [0.35,0.45])
High influence (H)	([0.60,0.70], [0.30,0.40], [0.25,0.35])
Very high influence (VH)	([0.70,0.80], [0.20,0.30], [0.15,0.25])

Step 2: Identify the evaluation criteria. Then, the evaluation attributes were clarified. They were considered when attempting to solve an MCGDM problem involving the building of a network that demonstrates the influences between them. The evaluation criteria $C = \{C_1, C_2, \ldots C_n\}$ were identified through literature review.

Step 3: Obtain expert judgments to establish IVSF initial direct-relation (IVSF–IDR) matrices. First, a group of $E = \{E_1, E_2, \ldots E_k\}$ experts was asked to evaluate the degree of influence of each criterion on the others. To allow them to indicate their evaluations, for each *pth* expert $(p = 1, 2, ..k)$, a pairwise matrix using experts' pairwise comparisons was formed based on an IVSF linguistic term scale of influence and its respective IVSFNs, as listed in Table 2. Then, an IVSF–IDR matrix was designed to represent the relationships between $C = \{C_1, C_2, \ldots C_n\}$ for *pth* expert $(p = 1, 2, ..k)$, which can be expressed as

$$H^{(p)} = \left[h_{ij}^{(p)}\right]_{n\times n} = \begin{bmatrix} 1 & h_{12}^{(p)} & \ldots & h_{1n}^{(p)} \\ h_{21}^{(p)} & h_{22}^{(p)} & \ldots & h_{2n}^{(p)} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1}^{(p)} & h_{n2}^{(p)} & \ldots & 1 \end{bmatrix}. \tag{13}$$

According to Eq. (13), $h_{ij}^{(p)} = ([a_{ij}^{(p)}, b_{ij}^{(p)}], [c_{ij}^{(p)}, d_{ij}^{(p)}], [e_{ij}^{(p)}, f_{ij}^{(p)}])$ is an IVSFN that refers to the judgment of the *pth* expert regarding how much C_i affects C_j in the selection process, where $i; j = 1, 2, \ldots n$.

Step 4: Calculate the weights of experts using the IVSFS entropy method. The corresponding expert weights should be calculated in advance because the DMs' importance differs.

To develop a measure of the importance of each expert in the group in terms of linguistic expressions, three main criteria were considered. (1) Experience refers to the number of years that each expert has been in the professional field since the date of their first job. (2) Knowledge refers to the

TABLE 3. **IVSF linguistic terms for the importance of experts and their respective IVSFNs. Adopted from Mandal and Seikh [59].**

Linguistic terms	IVSFNs
Moderately important (MI)	([0.45, 0.55], [0.25, 0.30], [0.20, 0.25])
Important (I)	([0.55,0.65], [0.20,0.25], [0.15,0.20])
Very important (VI)	([0.65,0.75], [0.15,0.20], [0.10,0.15])
Extremely important (EI)	([0.75,0.85], [0.10,0.15], [0.05,0.10])

educational level, and (3) responsibilities and position. Then, the importance of experts is described by the linguistic terms in Table 3.

The linguistic terms and their IVSFN are used to determine the weight of each expert, as shown in Table 3. The linguistic terms enable the importance of each expert to be estimated between "moderately important (MI)" and "extremely important (EI)."

Assume $\lambda^{(p)} = \left(\left[a^{(p)}, b^{(p)}\right], \left[c^{(p)}, d^{(p)}\right], \left[e^{(p)}, f^{(p)}\right]\right)$ is an IVSFN for rating the *pth* expert, and σ_p is the weight of *pth* expert.

The weight of *pth* expert σ_p is obtained by the IVSFS entropy operators in Eq. (14) [59].

$$\sigma_p = \frac{SC\left(\lambda^{(p)}\right)}{\sum_{p=1}^{k} SC\left(\lambda^{(p)}\right)} \tag{14}$$

where SC is the score function defined in Eq. (11). Then, experts' weights are given by the vector $\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_k\}$, satisfying $0 \leq \sigma_p \leq 1$ and $\sum_{p=1}^{k} \sigma_p = 1$.

Step 5: Establish a group IVSF–IDR matrix based on expert judgments. The IVSF–IDR matrices $H^{(p)}$ of multiple experts are aggregated into a group IVSF–IDR matrix H. Assume $H^{(p)}$ is an IVSF–IDR matrix for the *pth* expert, and the vector $\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_k\}$ is the weights for all experts, satisfying $0 \leq \sigma_p \leq 1$ and $\sum_{p=1}^{k} \sigma_p = 1$. To aggregate the different IVSF–IDR matrices $H^{(p)}$ into a group IVSF–IDR matrix H, IVSWAM is used as follows:

$$\begin{aligned} h_{ij} &= IVSWAM_{\sigma}(h_{ij}^{(p)}) = \sigma_1 \cdot h_{ij}^{(1)} \oplus \sigma_2 \cdot h_{ij}^{(2)} \oplus \ldots \oplus \sigma_k \cdot h_{ij}^{(k)} \\ &= \left\{ \left[\left(1 - \prod_{p=1}^{k} \left(1 - \left(a_{ij}^{(p)}\right)^2\right)^{\sigma_p}\right)^{\frac{1}{2}}, \left(1 - \prod_{p=1}^{k} \left(1 - \left(b_{ij}^{(p)}\right)^2\right)^{\sigma_p}\right)^{\frac{1}{2}} \right], \left[\prod_{p=1}^{k} \left(c_{ij}^{(p)}\right)^{\sigma_p}, \prod_{p=1}^{k} \left(d_{ij}^{(p)}\right)^{\sigma_p} \right], \right. \\ &\quad \left[\left(\prod_{p=1}^{k} \left(1 - \left(a_{ij}^{(p)}\right)^2\right)^{\sigma_p} - \prod_{p=1}^{k} \left(1 - \left(a_{ij}^{(p)}\right)^2 - \left(e_{ij}^{(p)}\right)^2\right)^{\sigma_p} \right)^{1/2}, \right. \\ &\quad \left. \left. \left(\prod_{p=1}^{k} \left(1 - \left(b_{ij}^{(p)}\right)^2\right)^{\sigma_p} - \prod_{p=1}^{k} \left(1 - \left(b_{ij}^{(p)}\right)^2 - \left(f_{ij}^{(p)}\right)^2\right)^{\sigma_p} \right)^{1/2} \right] \right\}. \end{aligned} \tag{15}$$

A group IVSF–IDR matrix can be defined as

$$H = \left[h_{ij}\right]_{n \times n} = \begin{bmatrix} 1 & h_{12} & \ldots & h_{1n} \\ h_{21} & h_{2n} & \cdots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n1} & h_{n2} & \ldots & 1 \end{bmatrix}. \tag{16}$$

According to Eq. (16), $h_{ij} = \left(\left[a_{ij}, b_{ij}\right], \left[c_{ij}, d_{ij}\right], \left[e_{ij}, f_{ij}\right]\right)$ is an IVSFN that refers to the group judgment of how much C_i affects C_j in the selection process, where $i; j = 1, 2, \ldots n$.

Step 6: Normalize the aggregated IVSF–IDR matrix H. Since there are three membership functions with six elements given in each comparison pair, the aggregated IVSF–IDR matrix H is divided into six submatrices: H^a, H^b, H^c, H^d, H^e, and H^f, which are composed of the lower and upper memberships, nonmemberships, and hesitancy of the aggregated IVSF–IDR matrix H, respectively, as follows [5], [7]:

$$H_a = \begin{bmatrix} 1 & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{2n} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \ldots & 1 \end{bmatrix},$$

$$H_b = \begin{bmatrix} 1 & b_{12} & \ldots & b_{1n} \\ b_{21} & b_{2n} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \ldots & 1 \end{bmatrix},$$

$$H_c = \begin{bmatrix} 1 & c_{12} & \ldots & c_{1n} \\ c_{21} & c_{2n} & \ldots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \ldots & 1 \end{bmatrix},$$

$$H_d = \begin{bmatrix} 1 & d_{12} & \ldots & d_{1n} \\ d_{21} & d_{2n} & \cdots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \ldots & 1 \end{bmatrix},$$

$$H_e = \begin{bmatrix} 1 & e_{12} & \ldots & e_{1n} \\ e_{21} & e_{2n} & \cdots & e_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ e_{n1} & e_{n2} & \ldots & 1 \end{bmatrix},$$

$$H_f = \begin{bmatrix} 1 & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{2n} & \cdots & f_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{n1} & f_{n2} & \ldots & 1 \end{bmatrix}. \quad (17)$$

Submatrices H_a, H_b, H_c, H_d, H_e, and H_f were normalized. The normalized IVSF–IDR submatrix $S_a = \left[(s_a)_{ij}\right]_{n\times n}$ was calculated as

$$S_a = \left[(s_a)_{ij}\right]_{n\times n} = \frac{H_a}{\kappa_a}, \quad (18)$$

where $\kappa_a = \max\left(\max_{1\le i\le n}\sum_j a_{ij}, \max_{1\le j\le n}\sum_i a_{ij}\right)$. Similarly, the normalized IVSF–IDR submatrices S_b, S_c, S_d, S_e, and S_f can be obtained using Eqs. (17) and (18).

Step 7: Derive the total influencing matrix T. First, the total influencing submatrix $T_a = \left[(t_a)_{ij}\right]_{n\times n}$ is obtained based on the normalized IVSF–IDR S_a. Considering all the direct and indirect influences, the total influencing submatrix T_a is computed as follows:

$$T_a = S_a\left(I - S_a\right)^{-1}, \quad (19)$$

where I is the $(n \times n)$ identity matrix. Similarly, the total influencing submatrices T_b, T_c, T_d, T_e, and T_f can be obtained using the submatrices S_b, S_c, S_d, S_e, and S_f, respectively.

The total influencing matrix T can be obtained by aggregating T_a, T_b, T_c, T_d, T_e, and T_f as $T = \left[t_{ij}\right]_{n\times n}$, where $t_{ij} = \left(\left[(t_a)_{ij}, (t_b)_{ij}\right], \left[(t_c)_{ij}, (t_d)_{ij}\right], \left[(t_e)_{ij}, (t_f)_{ij}\right]\right)$ is the IVSFN.

Then, the sum of rows R and columns C is computed from the total influencing matrix T through the operation rules in Eq. (7), as described in Section III-B.

$$R = (r_i)_{n\times 1} = \left(\sum_{j=1}^{n} t_{ij}\right)_{n\times 1} \quad (20)$$

$$C = (c_j)_{n\times 1} = \left(\sum_{i=1}^{n} t_{ij}\right)_{1\times n} \quad (21)$$

Through the operation rules in Eq. (7), we obtain

$$\begin{aligned}&\left(\left[(t_a)_{11}, (t_b)_{11}\right], \left[(t_c)_{11}, (t_d)_{11}\right], \left[(t_e)_{11}, (t_f)_{11}\right]\right)\\ &\quad\oplus \left(\left[(t_a)_{12}, (t_b)_{12}\right], \left[(t_c)_{12}, (t_d)_{12}\right], \left[(t_e)_{12}, (t_f)_{12}\right]\right)\\ &= \Bigg\{\Bigg[\left(((t_a)_{11})^2 + ((t_a)_{12})^2 - ((t_a)_{11})^2((t_a)_{12})^2\right)^{1/2},\\ &\quad \left(((t_b)_{11})^2 + ((t_b)_{12})^2 - ((t_b)_{11})^2((t_b)_{12})^2\right)^{1/2}\Bigg],\\ &\quad \left[(t_c)_{11}(t_c)_{12}, (t_d)_{11}(t_d)_{12}\right], \Bigg[\Bigg(\left(\left(1 - ((t_a)_{12})^2\right)((t_e)_{11})^2\right.\\ &\quad \left.+\left(1-((t_a)_{11})^2\right)((t_e)_{12})^2 - ((t_e)_{11})^2((t_e)_{12})^2\right)^{1/2},\\ &\quad \left(\left(1-((t_b)_{12})^2\right)((t_f)_{11})^2\right.\\ &\quad \left.+\left(1-((t_b)_{11})^2\right)((t_f)_{12})^2 - ((t_f)_{11})^2((t_f)_{12})^2\right)^{1/2}\Bigg)\Bigg]\Bigg\}.\end{aligned}$$

Based on the calculation results of the rows and columns, IVSFN should be defuzzified by performing the score function given in Eq. (11) for each row and column sum and adding value '1' [48] as $SC(R) = \left(r_i^{SC} + 1\right)_{n\times 1}$ and $SC(C) = \left(c_j^{SC} + 1\right)_{n\times 1}$, where $SC(R)$ and $SC(C)$ are the scores for R and C, respectively.

Step 8: Construct the influential relation diagram (IRD) and analyze the results. The horizontal axis, $(SC(R) + SC(C))$, named "prominence," indicates the degree of importance of each criterion, while the vertical axis, $(SC(R) - SC(C))$, named "relation," shows the contribution of criterion i to the system. To visualize the relationships among the criteria, the IRD was plotted with $(SC(R) + SC(C))$ as the horizontal axis and $(SC(R) - SC(C))$ as the vertical axis, as shown in Fig. 3. The cutoff point of the horizontal axis is determined as $\alpha_x = \frac{\max(SC(R)+SC(C))+\min(SC(R)+SC(C))}{2}$, and the vertical cutoff point is calculated as $\alpha_y = \frac{\max(SC(R)-SC(C))+\min(SC(R)-SC(C))}{2}$ [60].

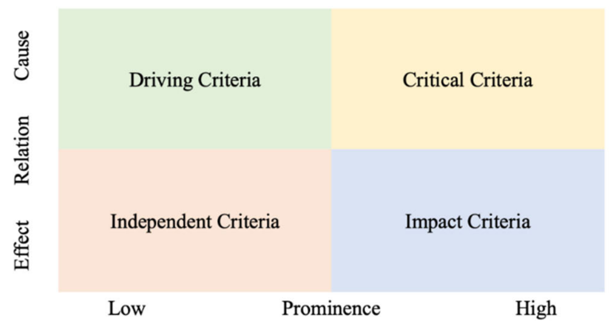

FIGURE 3. An influential relation diagram (IRD) [60].

In Fig. 3, there are four categories representing the critical, driving, independent, and impact criteria. The critical criteria include a significant relationship and high influence, the driving criteria include a significant relationship with low influence, the independent criteria entail a minor relationship and low influence, and the impact criteria entail a minor relationship but strong influence. In addition, if the value of $(SC(R) - SC(C))$ is positive, the criterion is grouped into the cause group. On the other hand, if the value of $(SC(R) - SC(C))$ is negative, the criterion is grouped into the effect group.

V. APPLICATION OF A REAL CASE: EARLY-STAGE INVESTMENT PROCESS

A real case study was conducted and analyzed to demonstrate the applicability and feasibility of the proposed IVSFS–DEMATEL methodology. In this section, an IVSFS–DEMATEL evaluation of the early-stage investment process is considered a case study. First, brief information on the decision problem of the early-stage investment process is provided, followed by the definitions of the investment criteria. The problem was solved using the proposed IVSFS–DEMATEL methodology. Subsequently, a sensitivity analysis and a comparison analysis were performed.

A. BACKGROUND

It is well documented that start-up businesses are an important component of economic growth [61]. They play key roles not only in creating jobs, income, and wealth but also in innovation and industrial renewal [62], [63]. However, start-up businesses face challenges in raising funds because they lack access to capital markets, expertise, and industry [64]. Early-stage investors fill the funding gap by providing capital to these start-up businesses to boost their growth and global impact. Early-stage investors use investment criteria to select promising start-up companies [65]. Multiple criteria are measured by a group of experts (i.e., early-stage investor experts) to make an informative decision in the evaluation process [66]. Thus, the early-stage investment process can be addressed as MCDM [67], [68].

Crisp and fuzzy MCDM models have been proposed to analyze early-stage investment processes. Afful-Dadzie et al. [68] applied the fuzzy preference ranking organization method for enrichment evaluation (Fuzzy PROMETHEE) to evaluate how early-stage investors choose which start-ups to fund. Afful-Dadzie and Afful-Dadzie [66] propose the SVIFS extension of the technique for order preference by similarity to the ideal solution (TOPSIS) method to evaluate the selection of start-up businesses. Dhochak and Sharma [69] identify the critical criteria influencing early-stage investors' investment decisions by applying an analytical hierarchical process (AHP) approach. Tian et al. [67] proposed the SVIFS' extinction of the TOPSIS method to select promising enterprises for early-stage investors. Nevertheless, previous models may not be appropriate for the early-stage investment process when uncertainty and hesitancy are hidden in expert judgments. Furthermore, early-stage investment process literature neglects handling experts' weights using three-dimensional interval measurements (e.g., IVSF linguistic terms), which are more appropriate than exact measurements. Therefore, this study presents a newly developed IVSFS–DEMATEL methodology that considers expert weights to analyze early-stage investment processes. The proposed methodology is used to determine the relative importance of early-stage investment criteria while modeling the interdependent relationships among them and incorporating expert weights. In the application part of this study, early-stage investment criteria were used based on a literature review and are briefly summarized below.

- **Start-ups' management team characteristics (C_1)** (Petty and Gruber [70]; Warnick et al. [65]; Block et al. [71]; Gompers et al. [64]; Dhochak and Sharma [69]; Kim and Lee [72]): This criterion is given by indicators, such as education, experience, expertise, and personality of the team.
- **Product or service characteristics (C_2)**(Petty and Gruber [70]; Block et al. [73]; Gompers et al. [64]; Dhochak and Sharma [69]; Kim and Lee [72]): This is defined as the innovativeness of a start-up's product or service and its competitive advantage.
- **Financial characteristics (C_3)** (Petty and Gruber [70]; Hsu et al. [74]; Block et al. [73]; Dhochak and Sharma [69]; Kim and Lee [72]): This criterion refers to the start-up's potential financial status, such as the expected rate of return and revenue growth.
- **Market characteristics (C_4)** (Petty and Gruber [70]; Block et al. [71]; Dhochak and Sharma [69]; Kim and Lee [72]): This criterion seeks to determine the market size, market growth, and market acceptance of a product or service.
- **Social features (C_5)** (Block et al. [71]; Dunbar [75]): This criterion describes the possibility of a product or a service providing social benefits (e.g., employment creation, promoting diversity, equity, and well-being in a start-up's team).
- **Environmental features (C_6)** (Matos [76]; Kim and Lee [72]): This criterion defines the possibility of a product or service contributing to environmental issues (e.g., climate change, pollution, and natural environmental crises).

B. PROPOSED METHODOLOGY IMPLEMENTATION

In this subsection, the proposed IVSFS–DEMATEL methodology is applied to real data to establish a model for analyzing the early-stage investment process. In this study, all calculations, analyses, and visual representations were performed using MATLAB software. The data that support the findings of this study are available from the corresponding author upon reasonable request.

Step 1: After setting the main goal, a group of early-stage investor experts was invited to participate. The group was composed of five experts from five investment firms in the United Kingdom, $E = \{E_1, E_2, \ldots E_5\}$. Details about the experts' educational background, years of experience, and position in firms are shown in Table 4.

TABLE 4. Background and expertise for the early-stage investor experts' group.

Expert	Position	Experience	Educational level	Education area(s)
E_1	Associate	3–5 years	Bachelor's degree	Economic, Law, Social
E_2	Partner	More than 10 years	PhD degree	Economic, Humanities
E_3	Partner	6–9 years	Master's degree	Economic
E_4	Analyst	6–9 years	Bachelor's degree	Humanities
E_5	Analyst	3–5 years	Bachelor's degree	Economic

Step 2: In the current study, six criteria are defined in Section V-B, namely entrepreneur and management team

characteristics (C_1), product or service characteristics (C_2), financial characteristics (C_3), market characteristics (C_4), social features (C_5), and environmental features (C_6), and are used.

Step 3: The experts' judgments were collected through an online questionnaire designed and administered using the Qualtrics XM software package. The purpose of this questionnaire was to allow experts to measure the degree of influence of each early-stage investment criterion on the others. The questionnaire was structured into three parts. First, it outlined the study's objectives and explained the early-stage investment criteria extensively. Second, it gathered demographic data from the respondents' experts. Lastly, it contained questions about the early-stage investment criteria within the IVSFS–DEMATEL framework using the IVSF linguistic terms.

The IVSF linguistic term scale, presented in Table 2, was used to measure the degree of influence. Subsequently, five IVSF–IDR matrices were established using the same number of experts. The data under analysis comprised $5 \times 6 \times 6 = 180$ elements, each of which corresponded to a linguistic term, as listed in Table 5. Using Table 2, the linguistic judgments in the IVSF–IDR matrices were converted into IVSFNs.

Step 4: The IVSF linguistic term scale presented in Table 3 was used to assign importance to the experts. The weight of each expert, $\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_5\}$, in terms of the linguistic terms and IVSFNs were presented in Table 6. The IVSFS entropy method, given by Eq. (14), was applied to calculate the weights.

Step 5: A group IVSF–IDR matrix was established by utilizing the IVSWAM method with respect to the expert weights, as shown in Eq. (12). The IVSF–IDR matrices are listed in Table 7.

Step 6: After obtaining the group IVSF–IDR matrix, the normalized IVSF–IDR submatrices S_a, S_b, S_c, S_d, S_e, and S_f were obtained using Eqs. (17) and (18) and are listed in Table 8.

Step 7: The total influencing submatrices T_a, T_b, T_c, T_d, T_e, and T_f were derived using Eq. (19), and the total influencing matrix T was structured. Then, the sums of the rows and columns of the total influencing matrix T were calculated using Eqs. (17) and (18) and the operation rules in Eq. (7). The sums of rows (R) and columns (C) are listed in Table 9. The IVSFNs are defuzzified by performing the score function given in Eq. (11) for each row and column sum as $SC(R)$ and $SC(C)$.

Step 8: To construct the IRD, the values of $SC(R) + SC(C)$ and $SC(R) - SC(C)$ were computed, as listed in Table 10. The IRD is depicted in Fig. 4, where the values of the horizontal and vertical cutoff points are obtained as 2.061 and 0.034, respectively.

Table 10 provides several results pertaining to the early-stage investment criteria. First, the start-up management team characteristics (C_1) criterion is the most important criterion because it has the highest prominence value and is the most influential criterion because it has the highest value of relation. This means that it is the most important investment criterion for early-stage investors when selecting promising start-up businesses. This suggests that investors

TABLE 5. IVSF–IDR matrices.

Expert	Criterion	C_1	C_2	C_3	C_4	C_5	C_6
E_1	C_1	N	H	VH	H	L	M
	C_2	M	N	L	L	M	H
	C_3	M	H	N	H	VH	VH
	C_4	H	H	H	N	M	H
	C_5	H	M	H	H	N	H
	C_6	H	L	H	L	H	N
		C_1	C_2	C_3	C_4	C_5	C_6
E_2	C_1	N	H	VH	M	VH	H
	C_2	H	N	M	H	H	H
	C_3	H	H	N	VH	M	M
	C_4	H	H	VH	N	M	M
	C_5	M	H	H	H	N	H
	C_6	H	M	H	M	H	N
		C_1	C_2	C_3	C_4	C_5	C_6
E_3	C_1	N	L	L	H	M	N
	C_2	L	N	H	H	L	L
	C_3	H	L	N	H	L	L
	C_4	M	H	M	N	M	M
	C_5	H	VH	VH	VH	N	M
	C_6	H	H	H	L	L	N
		C_1	C_2	C_3	C_4	C_5	C_6
E_4	C_1	N	H	VH	L	L	M
	C_2	VH	N	H	H	M	L
	C_3	M	M	N	H	M	M
	C_4	H	H	M	N	M	M
	C_5	H	H	M	H	N	H
	C_6	M	H	M	H	L	N
		C_1	C_2	C_3	C_4	C_5	C_6
E_5	C_1	N	L	M	M	H	M
	C_2	H	N	H	M	M	L
	C_3	L	H	N	H	M	M
	C_4	M	VH	L	N	H	M
	C_5	M	VH	L	H	N	H
	C_6	M	H	L	M	M	N

TABLE 6. Importance of the experts in terms of the linguistics terms and related weights.

Expert	Linguistic Terms	IVSFNs	Weight (σ)
E_1	I	([0.55,0.65], [0.20,0.25], [0.15,0.20])	0.18
E_2	EI	([0.65,0.75], [0.15,0.20], [0.10,0.15])	0.25
E_3	VI	([0.75,0.85], [0.10,0.15], [0.05,0.10])	0.37
E_4	MI	([0.45, 0.55], [0.25, 0.30], [0.20, 0.25])	0.10
E_5	MI	([0.45, 0.55], [0.25, 0.30], [0.20, 0.25])	0.10

TABLE 7. Group IVSF–IDR matrix.

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	([0.20,0.30], [0.70, 0.80], [0.15, 0.25])	([0.50,0.60], [0.42,0.53], [0.26,0.36])	([0.59,0.69], [0.33,0.44], [0.21,0.31])	([0.54,0.64], [0.36,0.46], [0.29,0.39])	([0.53,0.63], [0.37,0.48], [0.27,0.37])	([0.44,0.54], [0.46,0.57], [0.28,0.38])
C_2	([0.51,0.61], [0.39,0.50], [0.26,0.36])	([0.20,0.30], [0.70, 0.80], [0.15, 0.25])	([0.53,0.63], [0.37,0.47], [0.28,0.38])	([0.56,0.66], [0.35,0.46], [0.27,0.37])	([0.46,0.56], [0.44,0.54], [0.3,0.4])	([0.47,0.57], [0.45,0.56], [0.26,0.36])
C_3	([0.54,0.64], [0.35,0.45], [0.28,0.38])	([0.51,0.61], [0.40,0.51], [0.27,0.37])	([0.20,0.30], [0.70, 0.80], [0.15, 0.25])	([0.63,0.73], [0.28,0.38], [0.23,0.33])	([0.48,0.59], [0.42,0.52], [0.29,0.39])	([0.48,0.59], [0.42,0.52], [0.29,0.39])
C_4	([0.54,0.64], [0.35,0.45], [0.30,0.40])	([0.62,0.72], [0.29,0.39], [0.25,0.34])	([0.56,0.66], [0.34,0.44], [0.28,0.37])	([0.20,0.30], [0.70, 0.80], [0.15, 0.25])	([0.47,0.57], [0.39,0.49], [0.34,0.44])	([0.49,0.59], [0.38,0.49], [0.34,0.44])
C_5	([0.56,0.66], [0.34,0.44], [0.29,0.39])	([0.64,0.74], [0.27,0.37], [0.23,0.32])	([0.62,0.72], [0.29,0.39], [0.23,0.33])	([0.65,0.75], [0.26,0.36], [0.22,0.32])	([0.20,0.30], [0.70, 0.80], [0.15, 0.25])	([0.56,0.66], [0.34,0.44], [0.29,0.39])
C_6	([0.58,0.68], [0.32,0.42], [0.28,0.37])	([0.53,0.63], [0.37,0.47], [0.28,0.38])	([0.57,0.67], [0.30,0.40], [0.27,0.37])	([0.41,0.51], [0.49,0.59], [0.30,0.40])	([0.48,0.58], [0.43,0.54], [0.27,0.37])	([0.20,0.30], [0.70, 0.80], [0.15, 0.25])

TABLE 8. Normalized IVSF–IDR submatrices.

S_a	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.06	0.15	0.18	0.17	0.16	0.14
C_2	0.16	0.06	0.17	0.17	0.14	0.15
C_3	0.17	0.16	0.06	0.2	0.15	0.15
C_4	0.17	0.19	0.17	0.06	0.15	0.15
C_5	0.17	0.20	0.19	0.2	0.06	0.17
C_6	0.18	0.17	0.18	0.13	0.15	0.06

S_b	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.08	0.16	0.18	0.17	0.17	0.14
C_2	0.16	0.08	0.17	0.17	0.15	0.15
C_3	0.17	0.16	0.08	0.19	0.15	0.15
C_4	0.17	0.19	0.17	0.08	0.15	0.15
C_5	0.17	0.19	0.19	0.2	0.08	0.17
C_6	0.18	0.17	0.18	0.13	0.15	0.08

S_c	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.26	0.15	0.12	0.13	0.13	0.17
C_2	0.14	0.26	0.13	0.13	0.16	0.16
C_3	0.13	0.15	0.26	0.1	0.15	0.15
C_4	0.13	0.11	0.12	0.26	0.14	0.14
C_5	0.12	0.1	0.1	0.09	0.26	0.12
C_6	0.12	0.13	0.12	0.18	0.16	0.26

S_d	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.24	0.16	0.13	0.14	0.14	0.17
C_2	0.15	0.24	0.14	0.14	0.16	0.16
C_3	0.13	0.15	0.24	0.11	0.15	0.15
C_4	0.13	0.12	0.13	0.24	0.15	0.14
C_5	0.13	0.11	0.12	0.11	0.24	0.13
C_6	0.13	0.14	0.13	0.18	0.16	0.24

S_e	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.09	0.15	0.13	0.18	0.16	0.17
C_2	0.16	0.09	0.17	0.16	0.18	0.15
C_3	0.17	0.16	0.09	0.14	0.17	0.17
C_4	0.18	0.15	0.17	0.09	0.21	0.2
C_5	0.17	0.14	0.14	0.13	0.09	0.18
C_6	0.17	0.17	0.16	0.18	0.16	0.09

S_f	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0.11	0.16	0.14	0.17	0.16	0.17
C_2	0.16	0.11	0.17	0.16	0.18	0.16
C_3	0.17	0.16	0.11	0.15	0.17	0.17
C_4	0.18	0.15	0.17	0.11	0.2	0.19
C_5	0.17	0.14	0.14	0.14	0.11	0.17
C_6	0.17	0.17	0.16	0.18	0.17	0.11

TABLE 9. Sums of the rows and columns of the total influencing matrix T and its score values.

	R	C	$SC(R)$	$SC(C)$
C_1	$([0.14,0.11], [1.33 \times 10^{-11}, 4.87 \times 10^{-12}], [1.10 \times 10^{-3}, 6.32 \times 10^{-4}])$	$([0.19,0.16], [3.43 \times 10^{-11}, 1.57 \times 10^{-11}], [9.45 \times 10^{-4}, 4.30 \times 10^{-4}])$	1.09636	1.01578
C_2	$([0.11,0.09], [4.30 \times 10^{-11}, 1.56 \times 10^{-11}], [1.81 \times 10^{-4}, 1.04 \times 10^{-4}])$	$([0.07,0.06], [5.55 \times 10^{-11}, 1.71 \times 10^{-11}], [8.30 \times 10^{-4}, 3.92 \times 10^{-4}])$	1.02812	1.01005
C_3	$([0.12,0.10], [3.43 \times 10^{-11}, 1.47 \times 10^{-11}], [1.17 \times 10^{-3}, 5.40 \times 10^{-4})$	$([0.07,0.06], [1.74 \times 10^{-10}, 5.11 \times 10^{-11}], [4.36 \times 10^{-4}, 2.12 \times 10^{-4}])$	1.00415	1.01258
C_4	$([0.12,0.10], [9.14 \times 10^{-11}, 2.77 \times 10^{-11}], [2.74 \times 10^{-3}, 1.35 \times 10^{-4}])$	$([0.05,0.04], [2.19 \times 10^{-10}, 8.03 \times 10^{-11}], [8.18 \times 10^{-4}, 4.41 \times 10^{-4}])$	1.00237	1.01323
C_5	$([0.15,0.13], [3.86 \times 10^{-11} 1.25 \times 10^{-11}], [2.18 \times 10^{-3}, 1.14 \times 10^{-4}])$	$([0.12,0.10], [7.16 \times 10^{-10}, 2.89 \times 10^{-10}], [1.08 \times 10^{-3}, 6.70 \times 10^{-4}])$	1.01174	1.01998
C_6	$([0.11,0.10], [1.54 \times 10^{-9}, 5.60 \times 10^{-11}], [3.14E \times 10^{-3}, 1.75 \times 10^{-4}])$	$([0.08,0.06], [2.50 \times 10^{-12}, 6.77 \times 10^{-13}], [3.18 \times 10^{-3}, 1.76 \times 10^{-3}])$	1.00481	1.01111

perceive the characteristics of the management team in start-up businesses as pivotal elements in their investment deliberations. This observation aligns with earlier scholarly investigations emphasizing the pivotal role of management team criteria in selection determinations [64], [73], [78].

Product or service characteristics (C_2) are the second most important criteria in the early stages of the investment process. This finding is consistent with that of Zacharakis and Meyer [77], who indicated that product characteristics were critical to investment decisions. Thus, start-up

TABLE 10. Prominence and relation values.

	Prominence $SC(R)+SC(C)$	Rank	Relation $SC(R)-SC(C)$	Influence group	Rank
C_1	2.11215	1	0.08058	Cause	1
C_2	2.03817	2	0.01807	Cause	2
C_3	2.01673	4	−0.00843	Effect	5
C_4	2.01561	6	−0.01086	Effect	6
C_5	2.03172	3	−0.00824	Effect	4
C_6	2.01601	5	−0.00622	Effect	3

TABLE 11. Scenarios for sensitivity analysis with changing expert weights.

Scenarios	E_1	E_2	E_3	E_4	E_5
Normal	0.18	0.25	0.37	0.10	0.10
Scenario 1	0.20	0.20	0.20	0.20	0.20
Scenario 2	0.50	0.125	0.125	0.125	0.125
Scenario 3	0.125	0.50	0.125	0.125	0.125
Scenario 4	0.125	0.125	0.50	0.125	0.125
Scenario 5	0.125	0.125	0.125	0.50	0.125
Scenario 6	0.125	0.125	0.125	0.125	0.50

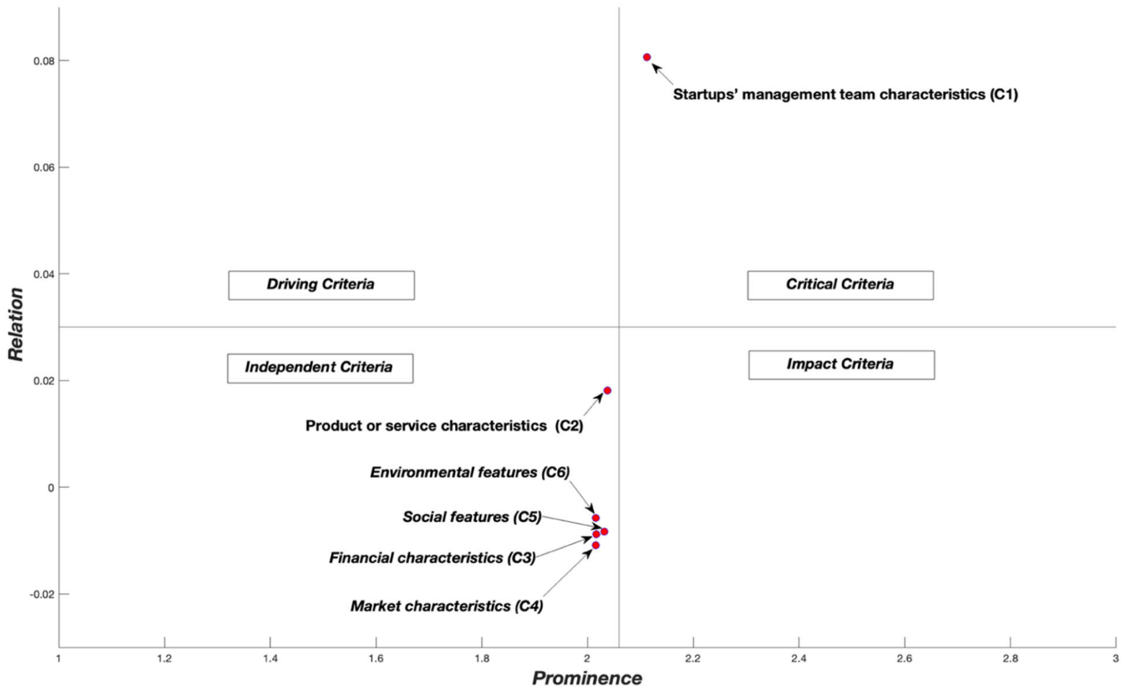

FIGURE 4. An IRD for early-stage investment criteria.

management team characteristics and the product or service characteristics should be given more attention by young and high-potential entrepreneurs in their start-up businesses. Second, the market characteristics criterion (C_4) has the lowest relation value; therefore, it is obviously influenced and impacted by other criteria. Moreover, the criterion market characteristics (C_4) is the least important criterion with the lowest value of prominence. These findings suggest that investors do not place significant emphasis on factors such as product/service size, growth potential, or market acceptance when making investment decisions. These results echo those of previous studies, which have concluded that market characteristics are only of minor importance in investment decisions [78], [79].

Based on the degree of importance (see the values of prominence $SC(R) + SC(C)$ in Table 10), the rank order

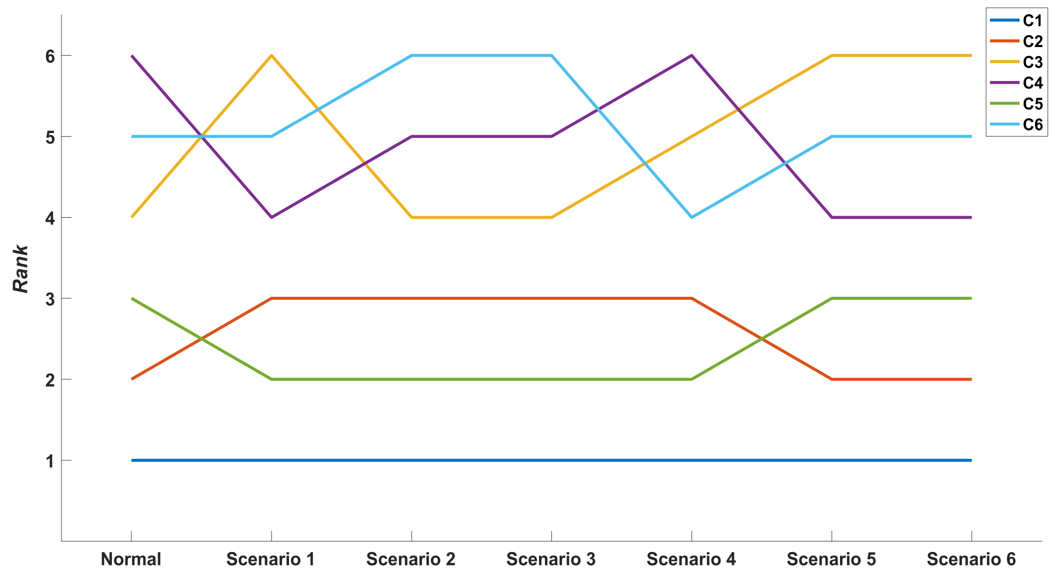

FIGURE 5. Results of sensitivity analysis with changing expert weights.

TABLE 12. Comparative analysis.

	Prominence rank			Influence group		
	IF-DEMATEL [80]	IVFS-DEMATEL [32, 81]	The proposed methodology	IF-DEMATEL [80]	IVFS-DEMATEL [32, 81]	The proposed methodology
C_1	2	2	1	Cause	Cause	Cause
C_2	1	1	2	Cause	Cause	Cause
C_3	4	3	4	Effect	Effect	Effect
C_4	6	6	6	Effect	Effect	Effect
C_5	3	4	3	Effect	Effect	Effect
C_6	5	5	5	Effect	Effect	Effect

of the criteria is as follows: $C_1 > C_2 > C_5 > C_3 > C_6 > C_4$. The cause-and-effect groups of the criteria were identified, as shown in Table 10. The results in Table 10 show two criteria for the cause group and four criteria for the effect group. The cause group criteria were sequenced as start-up management team characteristics (C_1), and the product or service characteristics (C_2) and the effect group criteria were sequenced as the environmental features (C_6), the social features (C_5), financial characteristics (C_3), and the market characteristics criterion (C_4). In essence, investors consider start-up businesses that incorporate environmental, social, financial, and market characteristics criteria, recognizing their potential emergence at later stages, as they may be influenced by both the criteria of the management team and the criteria associated with product characteristics. Consequently, nascent and high-potential entrepreneurs in the start-up domain should increase focus toward the attributes of their management team as well as the distinctive features of their product or service offerings.

The IRD shown in Fig. 4 was developed based on the data in Table 10. In Fig. 5, the criteria are clustered into four categories: critical, driving, impact, and independent. The critical criteria category contains one criterion (start-up management team characteristics (C_1)), which is the most influential and critical criterion, as it lies in the cause group and is a high-influence criterion. The independent criteria category comprises five criteria: product or service characteristics (C_2), financial characteristics (C_3), market characteristics (C_4), social features (C_5), and environmental features (C_6). This indicates that these criteria were not affected by other challenges.

C. SENSITIVITY ANALYSIS

Different weights for experts can be used to ensure the robustness of the proposed IVSFS–DEMATEL methodology. Therefore, a sensitivity analysis was conducted by evaluating early-stage investment criteria with different expert weights. The six scenarios represent different weight assignments for the experts. In the first scenario of the sensitivity analysis, equal weights were assumed to be assigned to the experts. For the remaining five scenarios, the highest weight was successively assigned to each expert. The results from those six scenarios were compared with the results from a based scenario "normal," which are the results obtained in Section V-B. Thus, the "normal" scenario considered the experts' weight calculated from step 3 in Section V-B. Table 11 lists all scenarios used in the sensitivity analysis.

Fig. 5 presents the results of the sensitivity analysis considering the changes in the experts' weights. The results indicate that start-up management team characteristics (C_1) are the most important criteria in all scenarios. Notably, Fig. 5 shows that the product or service characteristics (C_2) and social feature (C_5) criteria were ranked second or third in all scenarios. However, the rankings of the remaining criteria based on their importance weights changed across the six scenarios. Thus, the overall sensitivity results show that the proposed model is stable to some extent, but the lowest importance criteria are sensitive to the weights of the experts. Therefore, the extension of the IVSFS–DEMATEL by considering expert weights in an interval-valued spherical fuzzy environment is reasonable for MCDM evaluation problems.

D. COMPARISON ANALYSIS

To further demonstrate the feasibility and validity of the proposed methodology, a comparison with the existing approaches was carried out under a fuzzy environment. Thus, the comparative analysis was conducted using common methods called IF-DEMATEL [80] and IVFS-DEMATEL [32], [81]. The results corresponding to the comparison analysis are represented in Table 12. With respect to this analysis, it is obvious that the result of the existing approaches is consistent with the compromise solutions obtained from the proposed methodology. With respect to this analysis, it is obvious that the results of the existing approaches are consistent with the compromise solutions obtained from the proposed methodology.

VI. CONCLUSION AND FUTURE WORK

DEMATEL is a commonly used method for analyzing the structure of MCDM problems that involve interrelationships among the criteria while evaluating their importance. A literature review revealed several modifications to DEMATEL with different fuzzy set extensions. Although all these modifications of DEMATEL offer enhanced modeling of the uncertainty compared to the classical DEMATEL, none handles the uncertainty and hesitance hidden in expert judgments to be quantified and processed more comprehensively. Additionally, none of them addressed or measured the unknown weights of experts using a three-dimensional interval domain. Furthermore, they lost fuzzy information during the decision process because they performed defuzzification operations during the early stages. This study addresses these issues by proposing a new IVSFS version of the DEMATEL methodology by considering the experts' weights, offering an improved representation of the uncertainty.

The contributions and advantages of the study can be summarized as follows: I) This is the first time that IVSFSs and DEMATEL have been combined to deal with the uncertainty and hesitancy hidden in experts' judgments by considering the experts' weights. II) Compared with the DEMATEL method in the literature, the IVSFS entropy method was utilized to measure expert weights in the form of the IVSFS and incorporated with the DEMATEL method. III) In this study, the proposed methodology aims to keep the fuzzy operations as close to the end as possible to preserve the entire fuzzy process. IV) A real case study is conducted for the first time in an early-stage investment process to demonstrate the effectiveness and applicability of the proposed approach, followed by an applied sensitivity analysis and comparison analysis to show the validity of the obtained results from a real-world case study.

However, the proposed methodology has some limitations that must be addressed. The degree of influence of the criteria was evaluated subjectively, with uncertainties. In future research, using probabilistic methods (e.g., The Fermatean probabilistic hesitant fuzzy set (FPHFS) [82]) would be helpful for obtaining the influence degrees objectively, combining objective and subjective influence degrees and expanding the IVSFS–DEMATEL methodology. In addition, the proposed methodology considers only subjective weights for experts. Thus, objective weights for experts could be calculated via the knowledge-based fuzzy entropy measures [83] in future research work. Moreover, combining objective and subjective weights for experts will be considered in future investigations by utilizing a combination weighting of game theory (CWGT) [84].

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