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RESEARCH ARTICLE

Dynamic Analysis of an Economic and Financial Supply Chain System Using the Supervised Neural Networks

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ABSTRACT This study presents the dynamic analysis for the fractional order economic and financial supply chain dynamical system using the supervised neural network performances aided with scale conjugate gradient. The investigations based on the fractional derivatives have been implemented to achieve the realistic and accurate performances of the fractional order economic and financial supply chain dynamical system. The mathematical form of the fractional order economic and financial supply chain dynamical system is categorized into three dynamics: rate of interest, investment cost, and price index. Three different variations based on the fractional order form of the economic and financial supply chain dynamical system have been numerically presented using the supervised neural network performances based on the scale conjugate gradient scheme. The selection of the data for solving the economic and financial supply chain dynamical system is taken as 80% for training, and 12% for testing, and 8% for endorsement. The accuracy of the proposed stochastic scheme is presented using the obtained and referenced Adam results. Rationality, capability and perfection are performed through the supervised neural network with scale conjugate gradient scheme performances-based together with the performances of correlation/regression, mean square error, state transition measures, and error histograms. Finally, a comparison of numerical results is examined and observed that the range of absolute error is between 10^{-05} to 10^{-07} which indicates that the proposed stochastic computing model can effectively analyze the economic and financial supply chain dynamical system.

INDEX TERMS Financial supply chain, fractional order, scaled conjugate gradient, neural networks, numerical performances.

I. INTRODUCTION

Supply chain finance describes a set of technology-based solutions that aim to reduce financing costs and improve business efficiency for buyers and sellers linked in a sales transaction [1]. As global supply chains become more complex, Financial Supply Chain Management (FSCM) is increasingly essential for businesses [2]. One of the critical problems that businesses frequently face is financial constraints [1], [3]. Supply chain finance is gaining more and more attention from academia and industry owing to its success in reducing financing costs and increasing financing efficiency and effectiveness, particularly for small and medium-sized businesses [4], [5]. In supply chain financing, suppliers with limited budgets and low credit can request loans from financial institutions in collaboration with the trustworthy central company of the supply chain, loan eligibility is primarily based on transaction records, including orders, invoicing, and account receivability [6]. The most important stage in supply chain finance services is the instant

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sharing of massive amounts of transaction-related data, such as data related to orders, data related to warehouses, and logistic data, which allows the authenticity verification of transaction papers [6]. However, while many data technologies have been utilized to encourage the rapid sharing of data related to consumer transactions with improved access, transparency, and automation of supply chains, the manipulation of transaction papers remains an essential issue to be addressed [7]. The ongoing application of information technology in supply chain finance and economics has resulted in an increasing number of electronic transaction documents. As a result, economic and financial organizations remain at risk of various disruptions in economic and financial supply chains due to the falsification of transaction documents [7]. Moreover, substantial expenses are incurred to confirm the integrity of such documents and the associated logistical data [8].

However, the uncertainty in supply chain systems creates challenging tasks for control and management systems [9], [10], [11]. Thus, in supply chain finance systems, control and management system play a significant role in industry survival. Recently, several published studies related to this topic have attracted a great deal of attention [12], [13], [14], [15]. Owing to the day-to-day complexity of financial systems, the identification and forecasting of nonlinear systems play a significant role in their control [12], [16], [17], [18]. Additional dynamical behavior investigations of economic and financial supply chain real-world systems will explore more efficient and reliable outcomes that can enhance management systems' decision-making [19]. However, for better and more accurate decisions in operations and management, researchers should understand the dynamic changes in supply chain systems and management processes. Therefore, many researchers have proposed various techniques for the synchronization and regulation of these financial systems by utilizing and taking advantage of various theories [13], [15], [20], [21], [22]. Notably, game theory is an important theory that shows a higher level of reference performance for solving models such as contract design, inventory, and ordering [22]. Currently, several theories related to the game theory system are designed based on the first kind of differential equation model, which is linked to that decision by predicting the outcomes [13], [15], [20], [21], [22]. Each enterprise's demand requires minor enterprises. Due to several unpredictable components, the supply chain system has a significantly longer forecasting time, low precision, and poor dependability. This will cause the supply chain to be out of alignment, which will, in turn, generate misleading information and result in step-by-step amplification, creating a bullwhip effect [23]. The well-known models based on the supply chain are always considered to be random complexes, and most of the time supply chain financial systems have limited observations and some of the available data are noisy [24], [25]. Thus, to predict state changes and future behavior based on different supplies of supply chain networks, this study investigates the fractional order economic and financial supply chain model using a supervised neural network with a scale conjugate gradient scheme (SNN-SCGS).

Therefore, the main contributions or research questions of this study are summarized as follows.

I. The dynamic analysis of the fractional order economic and financial supply chain dynamical system is numerically investigated to predict state changes and future behaviour based on the different rates of interest, $v(\tau)$, investment cost, $w(\tau)$ index price, a indicates saving amount, b the cost per investment, and c the demand for goods.

II. These investigations present the numerical solutions for the fractional order economic and financial supply chain dynamical system using a supervised neural network performances based on the scale conjugate gradient (SNN-SCGS).

III. The solution of this model has never been proposed by using the designed stochastic structure, which is a research gap in this study that provided a chance to the authors to present the results through this designed stochastic SNN-SCGS structure to increase the financing efficiency and effectiveness in the overall network.

IV. The validity and correctness of the proposed stochastic structure has been observed by various statistical measures.

The remaining paper is organized as follows. Section II provides some related literature and background on the fractional order economic and financial supply chain dynamical system. Section III. mathematical development of a fractional order financial supply chain model with novel features. Section IV presents the designed fractional order economic and financial supply chain model procedure. Section V provides the results of the fractional order economic and financial supply chain model and Section VI presents the conclusions and possible avenues for future research.

II. RELATED LITERATURE WORKS

In recent years, many researchers have significantly focused on the mathematical development of financial supply models. These financial supply chain models are highly complex and contain various components [23]. To deal with the complexity of these financial supply chain models, many researchers have developed various nonlinear differential models consisting of various components to understand the dynamic behaviors of financial supply chain systems and explore their production mechanisms [23]. Some of them are summarized as, in the study of Zhang et al. [26] explored the stability of a financial supply chain model, Serletic [27] investigated chaos in the economic system, and Lin et al. [28] conducted a detailed analysis of chaotic behavior in a financial complex model. Furthermore, Ma and Chen [29] studied the chaotic phenomenon and bifurcation of a finance model. Nowadays, many scholars have found that fractional calculus, which is a generalization of ordinary differentiation and integration, has potential applications in various fields such as economics, physics, heat transfer, and chemical engineering [30],

[31], [32], [33]. Many researchers argue that it is more reasonable to describe natural phenomena using fractional-order differential equations rather than integer-order differential equations, as fractional-order differential equations can better describe memory characteristics and historical dependence. Considering that financial coefficients possess very long memory, and the variation of financial coefficients is closely connected with the previous and current times, it is important to establish fractional-order financial systems. In recent years, there have been numerous articles investigating fractional-order financial systems with some uncertainty parameters [30], [34], [35], [36], [37]. Recently many researchers also explored various stochastic solvers for these existing models. Such as modelling the dynamics of supply chains [38], [39] dynamic study of the supply chain financial system using fractional delay [23] the dynamic non-linear models of the coronavirus [40], biological non-linear leptospirosis system [41] models for singular functional systems [42], the models for HIV infections [43], three-echelon supply chain network with emulation of a fractional-order-chaotic system [44], the nonlinear three-species stochastic food chain system model [45], and genetic and optimization algorithm for the three-species food chain non-linear model with neural networks [46]. The main motivation for the implementation of different types of fractional derivatives was to obtain more realistic and accurate results. The minute particulars in fractional order networks using the superfast/super low transition are studied with a comprehensive description based on the system dynamics with the use of fractional calculus, which is considered difficult to examine based on integer order. Additionally, using the theory of fractional order calculus, a system dynamics index is produced. In the present situation, fractional order derivatives exhibit significantly superior outcomes than the integer order derivatives. To verify the performance effectiveness of the model evaluation process using real-world applications, fractional order derivatives were utilized [47], [48]. Furthermore, fractional order derivatives have been widely researched to find solutions and address various real-life applications through control networks, engineering management, and economic and financial, physical, and mathematical systems. The substantial operators have been frequently employed to implement fractional calculus over the past 30 years in various studies such as [49], [50], [51], [52], and [53]. Each of these operators is valuable and significant in their own way. However, the most popular definition of the Caputo derivative can be used to solve both homogeneous and non-homogeneous initial conditions. Compared to other definitions, the Caputo derivatives are simpler to implement. For the applications of fractional order derivatives, researchers are interested in developing a fractional order economic and financial supply chain model and providing numerical performance based on supervised neural network performances, aided by the scale conjugate gradient.

III. MATHEMATICAL DEVELOPMENT OF A FRACTIONAL ORDER FINANCIAL SUPPLY CHAIN MODEL WITH NOVEL FEATURES

In this section, the mathematical development of the financial supply model is provided. The financial supply chain model is a highly complex nonlinear system consisting of various components [20]. The complexity of the model does not lie in its various factors, but rather in its characteristics and interactions among factors [20]. When the system is disturbed by the outside world, the instability of economic and financial markets can lead to chaos and unpredictable economic behavior. To understand the dynamic behavior of financial supply chain systems and explore their production mechanisms, several researchers have designed and developed various financial models, including integer order, fractional order, and fractional order with time delay [38], [39], [40], [41]. However, recently, Ma and Li designed and developed a financial supply chain model based on these three variables [23]. A well-established financial supply chain structure consisting of the building blocks of manufacturing, currency, securities, and labor was employed. Thus, the mathematical form of the economic and financial supply chain model based on the three variables is presented as [20], [42], and [43]:

$$\begin{cases} \dfrac{du(\tau)}{d\tau} = w(\tau) + v(\tau)u(\tau) - au(\tau), u_0 = i_1, \\ \dfrac{dv(\tau)}{d\tau} = 1 - bv(\tau) - [u(\tau)]^2, v_0 = i_2, \\ \dfrac{dw(\tau)}{d\tau} = -u(\tau) - cw(\tau), w_0 = i_3, \end{cases} \tag{1}$$

where $u(\tau)$ represents the rate of interest, $v(\tau)$ represents investment cost, $w\tau$ show the index price, these represents the state of the system at time τ, a indicates saving amount, b denotes the cost per investment, and c shows the demand for goods in the system. The mathematical form of the fractional order financial supply chain model is represented as:

$$\begin{cases} \dfrac{d^\alpha u(\tau)}{d\tau^\alpha} = w(\tau) + v(\tau)u(\tau) - au(\tau), u_0 = i_1, \\ \dfrac{d^\alpha v(\tau)}{d\tau^\alpha} = 1 - bv(\tau) - [u(\tau)]^2, v_0 = i_2, \\ \dfrac{d^\alpha w(\tau)}{d\tau^\alpha} = -u(\tau) - cw(\tau), w_0 = i_3, \end{cases} \tag{2}$$

where α is the fractional form of the Caputo derivative to present the fractional order financial supply chain model represented by Equation (2). The values of α are taken from 0 and 1 of these fractional order derivatives. To present and analyze the behavior of the fractional order economic and financial supply chain model. The derivative of the fractional order economic and financial supply chain model in Equation (2) are incorporated to examine the super's low evolution and superfast transients minute particulars, which is difficult to evaluate with the integer order as presented in model system (1). The fractional calculus has been used in numerous submissions throughout the last few years, such

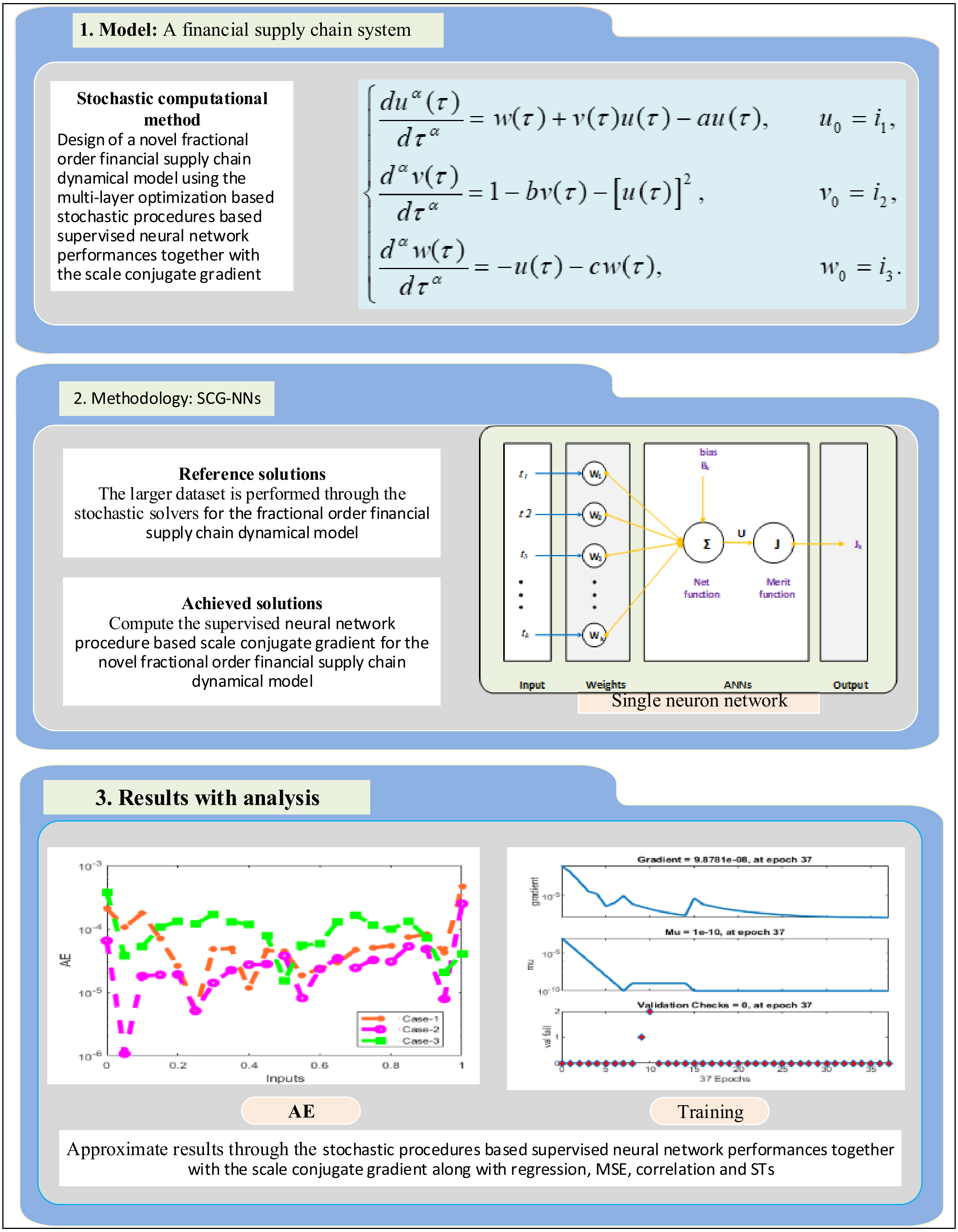


FIGURE 1. **The overall workflow of the fractional order economic and financial supply chain model.**

as fractional-order chaotic financial system [44] anomalous heat transfer [45], dynamical analysis of fractional-order finance system [46], fractional order financial system with time delay [47], a variety of Belousov-Zhabotinskii reaction

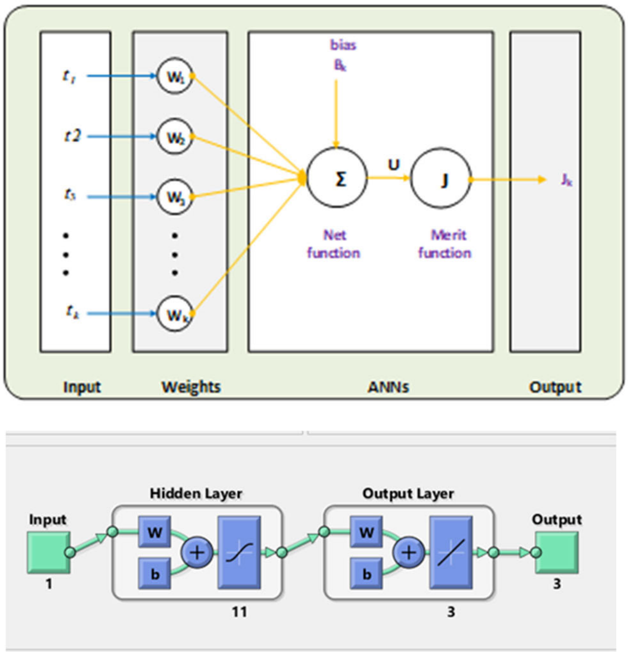


FIGURE 2. Single neuron structure and input/output/hidden layers structure for the fractional order economic and financial supply network model.

systems are examined for some spatiotemporal patterns [48], Chaos, Hopf bifurcation and control of a fractional-order delay financial system [49] and model for biologically based population growth utilizing the carrying capacity [50].

The SNN-SCGS model was developed to solve the fractional order economic and financial supply chain model and has the novel features outlined below:

I. The development of a fractional order economic and financial supply chain model is presented to evaluate and analyze the system's accuracy and realistic performance.

II. Dynamical analysis, which has never been implemented in the literature, is employed to investigate the fractional order economic and financial supply chain models with SNN-SCGS.

III. Mathematical simulations of the economic and financial supply chain model applying the fractional order derivatives between the [0, 1] interval is carried out using stochastic computing with SNN-SCGS.

IV. The accuracy of the stochastic computing with SNN-SCGS is demonstrated by the comparison analysis of the obtained and reference solutions.

V. The results of the absolute error (AE) in accurate measures are demonstrated, along with the accuracy and competence of the dynamic analysis of SNN-SCGS for solving the fractional order economic and financial supply chain mathematical model.

VI. The error histograms (EHs), state transitions (STs), correlation, regression and mean squared error (MSE) values designate the consistency of the SNN-SCGS for the fractional order economic and financial supply chain mathematical model.

TABLE 1. Performances of the SNN-SCGS for the fractional order economic and financial supply chain model.

Case	MSE			Epoch	Performance	Gradient	Mu	Time
	Testing	Training	Validation					
1	1.98×10^{-08}	3.82×10^{-09}	1.39×10^{-09}	37	3.83×10^{-09}	9.88×10^{-08}	1×10^{-10}	03
2	4.48×10^{-10}	3.524×10^{-10}	2.74×10^{-09}	20	3.52×10^{-10}	7.51×10^{-08}	1×10^{-11}	03
3	8.68×10^{-09}	4.35×10^{-09}	6.94×10^{-11}	08	4.14×10^{-09}	3.59×10^{-08}	1×10^{-10}	01

IV. DESIGNED FRACTIONAL ORDER ECONOMIC AND FINANCIAL SUPPLY CHAIN MODEL PROCEDURE

This section presents and outlines the framework for computing the stochastic analysis of SNN-SCGS for the fractional order economic and financial supply chain mathematical model, as defined in the system of Equations (1). The overall evaluation procedure of the mathematical fractional order economic and financial supply chain model with SNN-SCGS computing scheme is presented through a workflow diagram in Figure 1, based on the three levels such as level first represents the mathematical modelling of the economic and financial supply chain system, level second represents the development of the methodology, and the final level represents the result outcome analyses. Two measures performances of the design model are provided below.

(i) Based on the SNN-SCGS significant procedures are provided.

(ii) The operational procedures using the SNN-SCGS for the economic and financial supply chain model are provided.

The Adam technique was used to offer substantial generalization procedures, and default parameter settings have been used for the numerical procedures to produce the model dataset. Splitting the data into three sets to solve the fractional order economic and financial supply chain dynamical system. 80% of the data is for training, 12% for testing, and 8% for endorsement. Eleven numbers of hidden neurons have been selected to solve the fractional order economic and financial supply chain dynamical system. The complexity, premature convergence, overfitting, and underfitting scenarios were all performed with the best cooperation using artificial intelligence-based supervised learning SNN-SCGS. Additionally, these network parameters were selected and fixed after extensive simulation analysis, experiences, understandings, care, and even minor changes to these settings can cause the networks to perform inadequately. The single neuron model-based generic perception is used and shown in Figure 2, to express the stochastic analysis of SNN-SCGS in second phase. Figure 2 depicts the general perception based single neuron model and it is used to express the stochastic analysis of SNN-SCGS in second phase. To solve the economic and financial supply chain mathematical model, the subfigure 2(a) shows a single-layered neural network

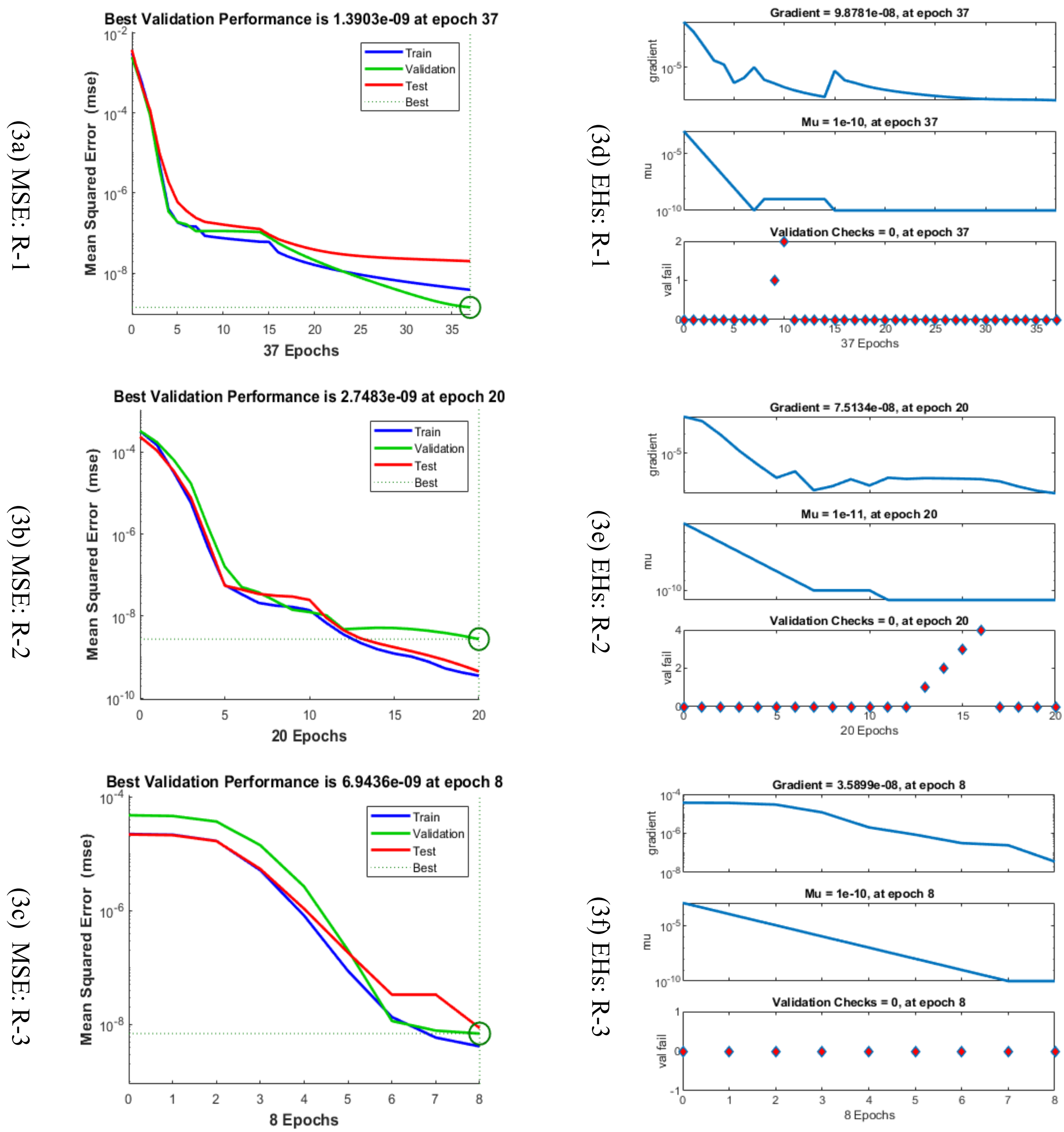


FIGURE 3. MSE and ST results for the fractional order economic and financial supply chain model.

design having a specific layer construction, one input layer vector, the hidden layer contains fifteen neurons, and the outer layer is the consequence of three. Choosing the relevant hidden neuron sections using the ''Matlab'' program (nftool command), statistical testing, learning strategies, static verifications, and stochastic based SNN-SCGS were implemented.

V. RESULTS OF THE FRACTIONAL ORDER ECONOMIC AND FINANCIAL SUPPLY CHAIN MODEL

This section presents three different variations of the fractional order based on SNN-SCGS for solving the economic and financial supply chain model, which are mathematically classified as follows:

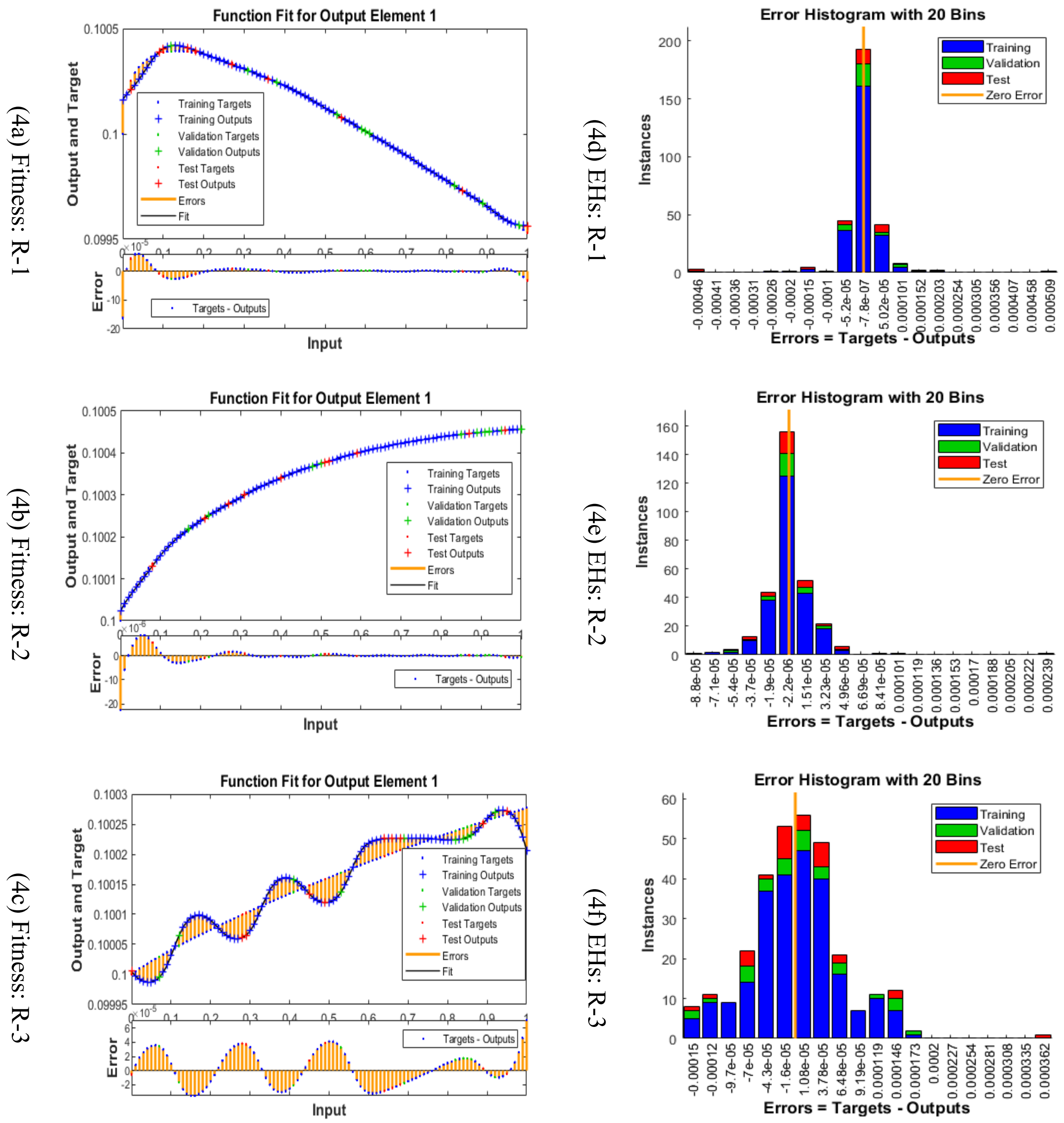


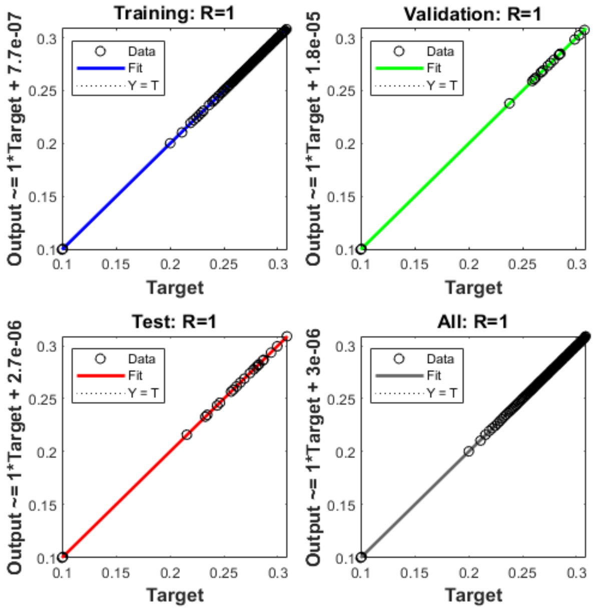
FIGURE 4. Fitness measures and EHs results of the fractional order economic and financial supply chain model.

Case 1: Consider the values $\alpha = 0.5, a = 3$, $\mathrm{b} = 0.1$, $\mathrm{c} = 1$, $i_1 = 0.1$, $i_2 = 0.2$ and $i_3 = 0.3$ in Equation (1), given as:

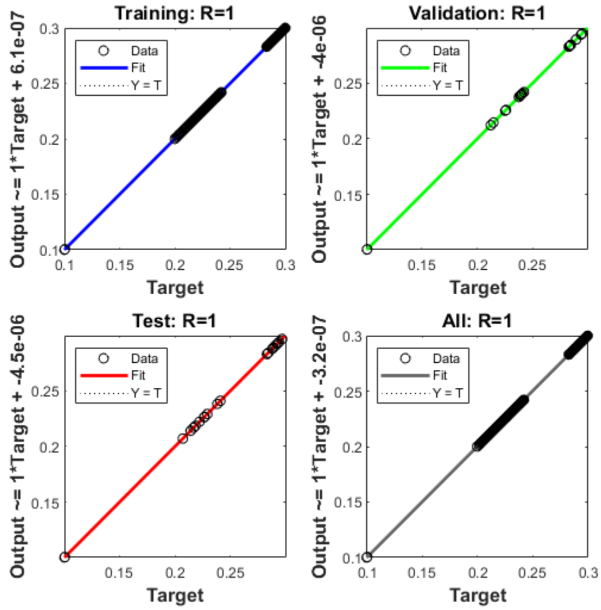
$$\begin{cases} \dfrac{d^{0.5}u(\tau)}{d\tau^{0.5}} = w(\tau) + v(\tau)\,u(\tau) - 3u(\tau)\,, u_0 = 0.1, \\ \dfrac{d^{0.5}v(\tau)}{d\tau^{0.5}} = 1 - 0.1v(\tau) - [u(\tau)]^2\,, v_0 = 0.2, \\ \dfrac{d^{0.5}w(\tau)}{d\tau^{0.5}} = -u(\tau) - 1w(\tau)\,, w_0 = 0.3, \end{cases} \tag{3}$$

Case 2: Consider the values $\alpha = 0.7, a = 3$, $\mathrm{b} = 0.1$, $\mathrm{c} = 1$, $i_1 = 0.1$, $i_2 = 0.2$ and $i_3 = 0.3$ in Equation (1), given as:

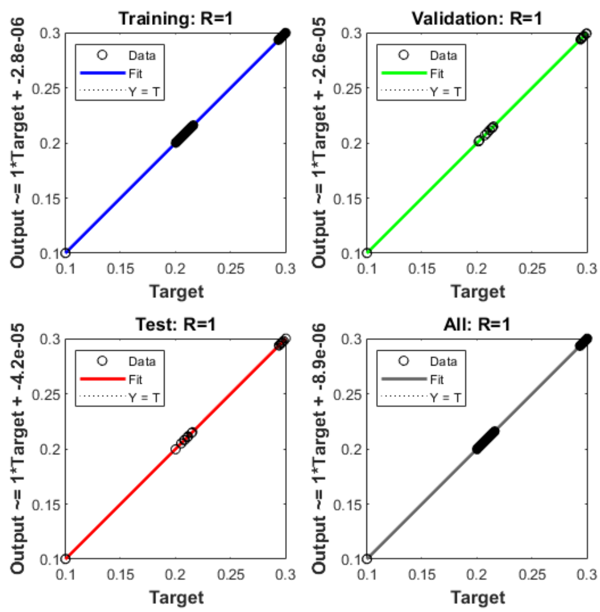
$$\begin{cases} \dfrac{d^{0.7}u(\tau)}{d\tau^{0.7}} = w(\tau) + v(\tau)\,u(\tau) - 3u(\tau)\,, u_0 = 0.1, \\ \dfrac{d^{0.7}v(\tau)}{d\tau^{0.7}} = 1 - 0.1v(\tau) - [u(\tau)]^2\,, v_0 = 0.2, \\ \dfrac{d^{0.7}w(\tau)}{d\tau^{0.7}} = -u(\tau) - 1w(\tau)\,, w_0 = 0.3, \end{cases} \tag{4}$$



(i) Regression: R-1



(ii) Regression: R-2



(iii) Regression: R-3

FIGURE 5. **Regression performance results of the fractional order economic and financial supply chain model.**

Case 3: Consider the values $\alpha = 0.9$, $a = 3$, $\mathrm{b} = 0.1$, $\mathrm{c} = 1$, $i_1 = 0.1$, $i_2 = 0.2$ and $i_3 = 0.3$ in Equation (1), given as:

$$\begin{cases} \dfrac{d^{0.9}u(\tau)}{d\tau^{0.9}} = w(\tau) + v(\tau)u(\tau) - 3u(\tau), u_0 = 0.1, \\ \dfrac{d^{0.9}v(\tau)}{d\tau^{0.9}} = 1 - 0.1v(\tau) - [u(\tau)]^2, v_0 = 0.2, \\ \dfrac{d^{0.9}w(\tau)}{d\tau^{0.9}} = -u(\tau) - 1w(\tau), w_0 = 0.3, \end{cases} \tag{5}$$

Figure 3 represent the SNN-SCGS stochastic scheme for the supply chain economic and financial model. The STs performances and the optimal values of the supply chain economic and financial model are provided in Figure 3. The results based on STs and MSE in terms of substantiation, best curve and training are presented in Figure 3, depending on the SCGGNNs stochastic procedure for the fractional supply chain economic and financial model. The best calculated results of the model based on the economic and financial supply chain have been derived at epochs 37, 20 and 08

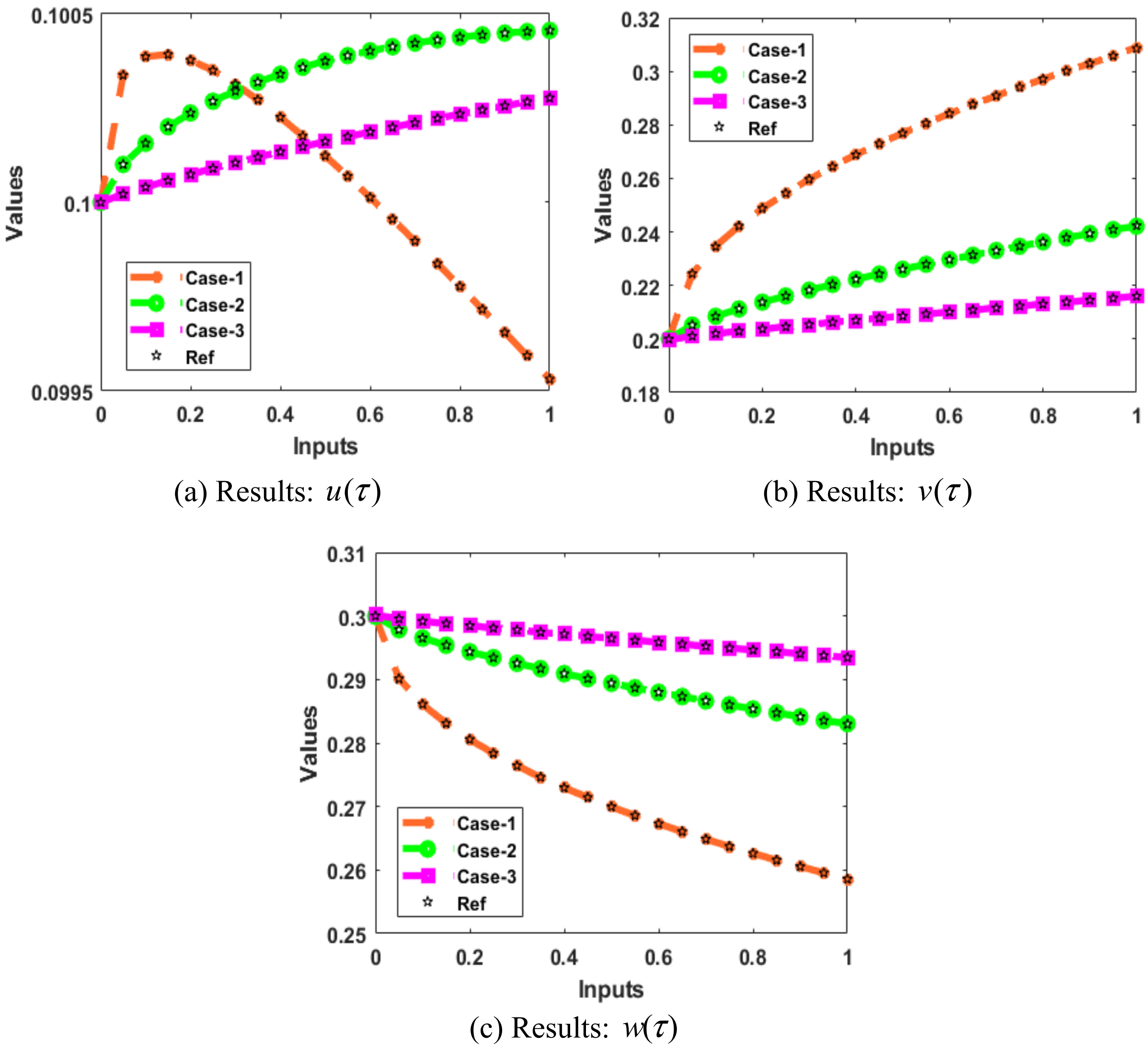
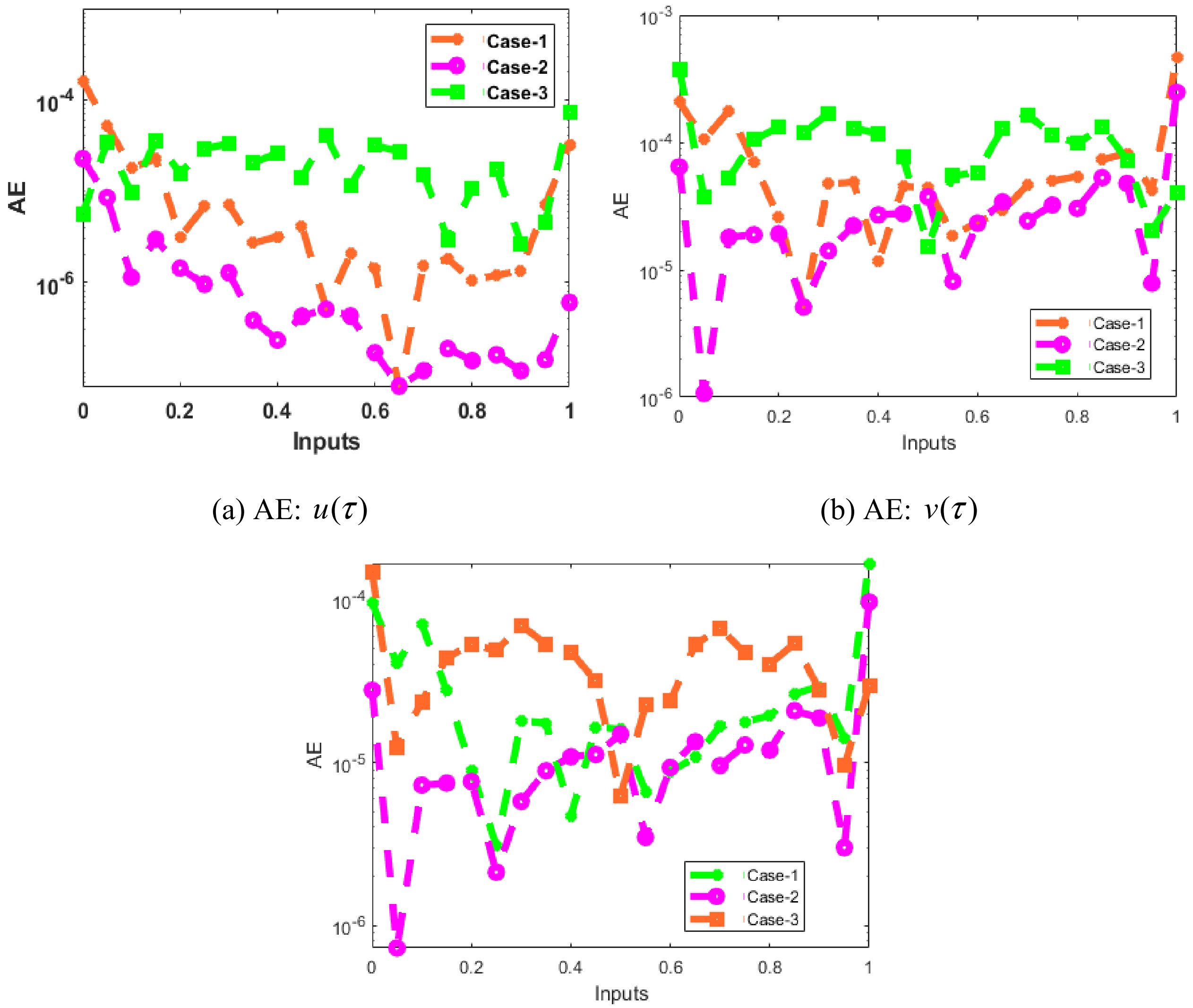


FIGURE 6. Matching of the results for the fractional order economic and financial supply chain system.

performed as 3.83×10^{-09}, 3.52×10^{-10} and 4.14×10^{-09} respectively. The second part of the Figure 3 indicates the gradient measures using the SNN-SCGS stochastic solver for the nonlinear fractional dynamical system. The gradient measures were calculated as 9.88×10^{-08}, 7.51×10^{-08}, and 3.59×10^{-08}. These illustrations depict the precision and convergence of the proposed SNN-SCGS in economic and financial supply chain systems. The assessment of the results using the testing/training/validation outputs, errors, and the values of the fitness curves are provided in the first part of Figure 4. The errors and fitness curves are crucial for assessing the model's predictive accuracy and its ability to capture the underlying economic and financial patterns of the supply chain. Lower error values and well-fitted curves indicate a successful modeling approach, which is supported by the optimal values and gradient measures discussed earlier. Overall, Figures 3 and 4 collectively demonstrate the precision, convergence, and robustness of the SNN-SCGS stochastic scheme in modeling the economic and financial system of the supply chain. The detailed analysis of epochs, gradient measures, and various output assessments emphasizes the model's effectiveness and reliability in handling complex economic and financial dynamics within supply chains.

The EHs derived are shown in the second part of Figure 4 using the training, authentication, zero, and test error measures to solve the economic and financial dynamic model. The performance-based EHs are reported as -7.6×10^{-07}, -2.2×10^{-06}, and -1.6×10^{-05} for solving the economic and financial dynamical supply chain model using the proposed SNN-SCGS solver. Figure 5 shows the correlation operator measures through training/validation/testing for the mathematical model based on the economic and financial supply chain model. Using the fractional order mathematical economic and financial supply chain model, the correlation



(a) AE: $u(\tau)$ (b) AE: $v(\tau)$

(c) AE: $w(\tau)$

FIGURE 7. **AE results of fractional order economic and financial supply chain model.**

is 1. These illustrations signify the exactness of the SNN-SCGS stochastic solver for the mathematical economic and financial supply chain systems. The convergence based on MSE represents the complexity performance, training, backpropagation, authentication checks, generations, and testing provided in Table 1 using the economic and financial supply chain dynamical model.

The SNN-SCGS stochastic solver has been shown to be robust and effective in modeling economic and financial supply chains. It can achieve high precision with low error magnitudes, perfect correlation, and efficient convergence, making it suitable for complex supply chain management scenarios.

Detailed results and metrics presented in Figures 3-5 and Table 1 demonstrate the solver's ability to effectively handle nonlinear fractional dynamical systems. This discussion emphasizes the solver's strength in providing accurate, reliable, and efficient solutions for economic and financial supply chain models, making it a valuable tool for researchers and practitioners in the field. The comparative representations of the results, along with the values of the AEs, are shown in Figs. 6–7 for the fractional order economic and financial supply chain model. The accuracy of the SNN-SCGS stochastic solver using outcome matching is depicted in Figure 6 for each category of the economic and financial supply chain fractional model. Furthermore, AE performance for each dynamic of the fractional order economic and financial supply chain model based on the SNN-SCGS stochastic approach are shown in Fig. 7. The AEs for the first dynamic of the model are 10^{-05} to 10^{-07}, 10^{-06} to 10^{-07}, and 10^{-04} to 10^{-05} for variations 1 to 3, respectively, of the fractional order economic and financial supply chain model. The values for the second dynamic of the system are 10^{-04} to 10^{-05}, 10^{-04} to 10^{-06}, and 10^{-03} to 10^{-05} for cases 1 to 3, respectively, of the economic and financial supply chain fractional order model. The third system dynamics of are$10^{-05}$to 10^{-06}, 10^{-05} to 10^{-07}, and 10^{-04} to 10^{-05} for cases 1 to 3, respectively of the fractional order economic and financial supply chain model. These AE depictions represent the exactness of stochastic SNN-SCGS for fractional order economic and financial supply chain mathematical model. The detailed analysis of outcome matching, and AEs provided in Figures 6 and 7, along with previous performance metrics, showcases the SNN-SCGS stochastic solver as a powerful tool for modeling and managing the fractional order economic and financial supply chain. Its high accuracy, robust performance, and validated stochastic approach make it a valuable outcome for researchers and practitioners in the field.

VI. CONCLUSION

In these numerical investigations, a reliable computing framework based on supervised neural network performances in conjunction with the scale conjugate gradient is presented for solving a novel fractional order economic and financial supply chain systems. The fractional order economic and financial supply chain dynamical model is divided into the rate of interest, investment cost, and price index. Some concluding remarks of this study are proposed as follows:

•Fractional order investigations have been conducted to obtain more realistic solutions as compared to integer order for the economic and financial supply chain system.

•A supervised neural network in conjunction with the scale conjugate gradient has been presented successfully for the novel fractional order economic and financial supply chain systems.

•An Adam numerical method has been used to get the dataset of the fractional order economic and financial supply chain system.

•The dataset has been divided into 80% for training, 12% for testing, and 8% for endorsement to reduce the mean square error.

•The neurons are selected as eleven in the hidden layers for solving the fractional order economic and financial supply chain differential models. Greater neurons present over-lapping, while smaller neurons premature convergence. Hence eleven neurons are used to provide the better performance.

•The correctness is perceived through the overlapping of the results along with reducible performances of the absolute error.

•The regression is performed as 1 for each case of the model, which depicts the accuracy of the solver.

While the paper proposes advanced methods to determine nonlinear chaotic characteristics, the validation of these methods remains an area for further research. The accuracy and reliability of the combined testing methods need to be continuously improved and verified with different datasets and market conditions. In the future, the proposed stochastic procedure-based supervised neural network performance can be implemented to solve the non-linear and fractional order dynamical economic and financial supply chain systems with real-time data and some models related to impact of sales [63].

DATA AVAILABILITY STATEMENT

The authors confirm that data supporting the findings of this study are available within the article.

CONFLICT OF INTEREST

Author Shahid Ahmad Bhat declares that he has no conflict of interest. Author Tariq Aljuneidi declares that he has no conflict of interest. Author Zhaojun Li declares that he has no conflict of interest.

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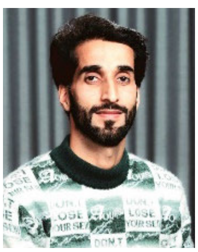

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