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RESEARCH ARTICLE

Software Reliability Growth Model Selection by Using VIKOR Method Based on q-Rung Orthopair Fuzzy Entropy and Divergence Measures

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ABSTRACT Software reliability growth models (SRGMs) are widely used for predicting the reliability of software systems during testing and debugging. Selecting the most appropriate SRGM solutions is a multi-criteria decision-making (MCDM) problem that is very difficult when criteria are imprecise or ambiguous. The paper introduces a novel approach to SRGM selection which relies on q-rung orthopair fuzzy sets (q-ROFS) and the compromise ranking method of VIKOR. New q-rung orthopair fuzzy (q ROF) entropy and divergence measures are proposed for criteria weights assignment and to select superior SRGMs. The VIKOR method is then applied on the q-ROF decision matrix to identify the optimal compromise SRGM solution. This approach provides a systematic framework for handling subjective criteria and modeling uncertainty during SRGM selection. The proposed MCDM methodology is illustrated on the example of a case study involving four common SRGMs evaluated on the four different criteria. Results are demonstrated to be in line with the latest q-rung measures which provide more accurate results than the previous intuitionistic fuzzy methods. The q-ROF VIKOR approach provides the software teams with a more robust information base for the reliability growth decision-making process. At the end of this manuscript, we do the comparison of the proposed theory with certain prevailing concepts to reveal the dominance and supremacy of this work. Whereas yet there are some expected limitations of the proposed work for instance it can't be helpful in the generalized structures of q-ROFS.

INDEX TERMS Software reliability growth model, MADM, VIKOR technique, q-rung orthopair fuzzy entropy, divergence measures, fuzzy set, decision making.

I. INTRODUCTION

Delivering highly reliable software systems is a critical imperative for organizations across industries. Software failures can have severe consequences, ranging from frustrated users to catastrophic system outages impacting operations and revenues. As software grows more complex and is deployed in increasingly high-stakes domains like healthcare, transportation, and finance, rigorous reliability testing and modeling during development becomes paramount. Software reliability growth models (SRGMs) provide a powerful analytical lens for predicting and improving system reliability over the testing lifecycle. By mathematically characterizing the fault detection process, SRGMs enable forecasting of future failure rates and volumes. However, the large number of available SRGM options, each with their own assumptions and characteristics, poses a significant model selection challenge for practitioners.

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Selecting an improper SRGM for a specific project environment and reliability level may result in wrong prognosis, time-consuming testing, and, finally, the delivered software may not meet the reliability goals. For instance, an e-commerce firm that is implementing new website features will likely target the short-term SRGMs that aim at attaining reliability growth while an aerospace firm that is implementing flight control software will prefer the long-term reliability growth models. The best SRGM depends on factors such as the type of software, the methodology used in developing the software, the maturity level of the test practices, and the risk tolerance levels of the stakeholders. SRGM's mathematically describe the failure process in software systems and predict improvements in reliability over time. Common SRGMs include exponential and S-shaped models based on Non-Homogeneous Poisson Processes (NHPP). The NHPP distribution allows modeling software failure intensity as a function of factors like testing time and fault detection/removal rates. This research explores using the VIKOR multi-criteria decision methodology to optimize selection of an appropriate SRGM by evaluating data-model fitness, predictive power, and computation complexity across NHPP-based models like the Goel-Okumoto, Musa-Okumoto Logarithmic Poisson, and Yamada Delayed S-Shaped models. However, determining the most accurate SRGM for a particular system remains difficult given the number and variety of models available. Multi-criteria decision-making techniques provide a systematic methodology for SRGM selection based on multiple weighted criteria. The q-rung orthopair fuzzy entropy and divergence measures provide enhanced capability in criteria weighting and performance scoring for ambiguous/uncertain metrics. The proposed approach facilitates more informed, accurate SRGM selection for reliability growth assessment.

One of the earliest papers on the S-shaped SRGM was by Musa et al. [1] They proposed a three-parameter model that assumed that the detection of software fault and removal process could be modeled using an S-shaped curve. The three parameters of the model were the initial number of faults, the saturation level, and the inflection point. The model was shown to fit data from several large software projects reasonably well. Since then, many researchers have proposed variations of the S-shaped SRGM, including different functional forms and additional parameters. For example, based on the presumption that the pace of fault detection and removal is related to the number of faults still present, Goel and Okumoto [2] suggested a two-parameter model. Yamada et al. [3], [4] introduced the software reliability models reflected the test effort put out during phase called as exponential and Rayleigh models. These models even take into consideration the testing resources employed in the earlier established model. Two data sets were used to show that the new model, proposed by Pham and Zhang [5], Compared to the previous models, the non-homogeneous Poisson process-based model fits and makes predictions much better. A S-shaped curve model based on NHPP having increasing cumulative number of software failures was proposed by Zhang et al. [6]. Kim et al. [7] propose a software reliability model based on the assumption of dependent software failures. They derive this model considering the number of software failures and fault detection rate, assuming point symmetry. Jin and Jin [8] proposed a ground-breaking optimization technique known as Quantum Particle Swarm Optimization (QPSO) to alter reliability parameters. Gupta [9] developed the Weighted Distance-Based technique (WDBA) MCDM technique by modeling the as an MCDM problem. The SRGM selection procedure's solution is computed using a mathematical technique Garg [10], [11], [12]. Mahmudova [13] used the TOPSIS method concept to increase software efficiency.

q-Rung Orthopair Fuzzy Sets (q-ROFs) is a relatively recent development in fuzzy sets that has shown promise in modeling uncertain and imprecise data in SRGM's. q-ROFs combines the Orthopair fuzzy sets with q-Rung Fuzzy Set to represent both the degree of non-membership and membership of an element in a set. A method for identifying decision-making issues based on entropy weight and aggregation operators was described by Garg [10], [11], [12]. Entropy measures for IFSs were introduced in their separate and generalized form Garg [10], [11], [12]. Yager [14] presented the idea of the q-rung orthopair fuzzy set, a generalization of the Pythagorean and intuitionistic fuzzy sets. In Q-rung fuzzy sets, the membership function of an element in a set is represented by a sequence of real numbers that describe its degree of membership at different granularities or levels of abstraction. Orthopair fuzzy sets were introduced by Zhu et al. [15] as an extension of fuzzy sets that allows for the representation of uncertain and conflicting information. Using the tangent inverse function, Khan et al. [16] propose the knowledge measure for q-ROFS. This method quantifies the knowledge related to qROFS. The knowledge measure is defined by the membership and non-membership functions, as well as the hesitation margin, allowing it to consider both knowledge and fuzziness. To handle the decision data in circumstances where they might reflect a wider range of uncertain data because the aggregated of the qth powers of the degrees of both non-membership and membership is less than or equal to 1, Liu et al. [17] presented the q-ROF weighted geometric operator and the q-rung orthopair fuzzy weighted averaging operator. Complex q-ROF's (Cq-ROFS), a novel idea that is a useful tool for dealing with ambiguous and complex information, was introduced by Garg et al. [11].

Akram et al. [18] analyze aggregation operators (AOs) within the q-rung orthopair fuzzy environment using Einstein norm operations. Gou et al. [19] investigate Pythagorean fuzzy information, analyzing change values of Pythagorean fuzzy numbers (PFNs) as variables and dividing them into eight regions based on PFN operations. They also develop Pythagorean fuzzy functions and scrutinize their fundamental properties, including continuity, derivability, and differentiability. Pethaperumal et al. [20] investigate the algebraic properties of q-rung orthopair multi-fuzzy sets (MFSs) and examine their corresponding operational laws.

Gou et al. [21] introduce hesitant fuzzy linguistic entropy and cross-entropy measures and develop a model for determining criteria weights. This model accounts for both the individual and interactive effects of hesitant fuzzy linguistic elements (HFLEs) with respect to each criterion. Uluçay et al. [22] develop a novel multi-criteria decision-making (MCDM) method using the VIKOR approach based on Q-single-valued neutrosophic sets. Rani and Kumar [23] propose an innovative distance measure designed to quantify the dissimilarity between Q-rung orthopair fuzzy sets (Q-ROFSs). Akram et al. [24] introduce linguistic q-rung orthopair fuzzy graphs (Lq-ROFGs) and explore efficient methods for complex multi-attribute group decision-making (MAGDM) scenarios. This approach utilizes the distance measure to establish a similarity measure between Q-ROFSs. For instance, Park et al. [25] proposes a new method to calculate the divergence between intuitionistic fuzzy sets (IFSs) based on their three-dimensional representation, and explores the relationship between similarity and distance measures of IFSs. Qin et. al. [26] developed a novel multi-attribute decision-making (MADM) method for q-rung orthopair hesitant fuzzy information, utilizing the Choquet integral. Shahzadi et al. [27] propose a study to develop a decision-making strategy that addresses the significant tendencies of the conventional TOPSIS method within the framework of (p, q)-rung orthopair fuzzy sets. Akram et al. [28] works on the development and application of the measurement and ranking of the alternatives based on the compromise solution under the context of 2-tuple linguistic q-rung picture fuzzy sets. Mishra et al. [29] have proposed a four-stage hybrid DSS known as q-ROFR-SPC-RANCOM-MULTIMOORA. This system combines the q-rung orthopair fuzzy rough sets (q-ROFRS), the symmetry points of criterion (SPC), the ranking comparison (RANCOM), and the multiattribute multiobjective optimization based on ratio assessment (MULTIMOORA). Akram and Shumaiza [30] also improve the PROMETHEE method by applying q-rung orthopair fuzzy sets (q-ROFS) which is a generalization of Pythagorean fuzzy sets (PFS). This approach enhances the management of ambiguous and imprecise information, which in turn enhances the quantification of uncertainty in other ratings. Ariafar et al. [31] propose the Grey Hungarian Algorithm (GHA) to apply the grey preference degrees to the classic Hungarian algorithm for solving the Linear Assignment Models (LAM) in conditions of uncertainty. Pethaperumal et al. [20] propose two new distance metrics specifically for q-rung orthopair multi-fuzzy sets (q-ROMFSs) of order k, which improves the ability to differentiate between the sets. Gou et al. [21] introduce hesitant fuzzy linguistic entropy and cross-entropy measures, which create a model to calculate criteria weights. This model considers the direct impact of each hesitant fuzzy linguistic element (HFLE) and the combined impact of any two of them with regard to each criterion. Mahalakshmi et al. [32] put forward two new distance measures for q-rung orthopair multi-fuzzy sets (q-ROMFSs) of dimension k to improve the distinguishing capability of distance measures. Gou et al. [33] propose the probabilistic double hierarchy linguistic term sets (PDHLTS) which is more general. They also present enhanced operations for PDHLTSs and a distance measure for two PDHLTSs, as well as an adjustment method that makes the probability distributions of two PDHLTSs equal. This research aims to address the problem of SRGM selection by proposing a novel fuzzy multi-criteria decision-making (MCDM) approach that can effectively handle imprecise and subjective assessments of the models. The primary research questions are: 1) How can we systematically evaluate and rank SRGMs considering multiple criteria while accounting for the inherent uncertainty and vagueness in the decision-making process? 2) Can advanced fuzzy set theory and entropy measures provide more accurate and nuanced modeling of ambiguity compared to existing techniques?

The key research motivations are:

1. To develop a systematic SRGM selection framework to handle imprecise assessments
2. To apply advanced fuzzy set theory to model ambiguity and complexity
3. To introduce new q-rung orthopair fuzzy measures for criteria weights and evaluations
4. To implement a structured MCDM technique (VIKOR) for compromise ranking

The proposed fuzzy MCDM approach aims to provide more accurate and flexible SRGM selection compared to existing techniques by better capturing the subjective and uncertain nature of criteria evaluations. The novelty of this study lies in the integration of advanced q-ROF set theory with the VIKOR method, facilitating a robust and systematic decision-making process for SRGM selection while accounting for ambiguity and imprecision. The existing methods of selecting SRGM are still heavily dependent on subjective expert opinions, which often lack consistency and do not properly deal with the uncertainty involved in assessing trade-offs across multiple criteria. Unplanned selection process contributes to over or under estimation of the true reliability levels. The introduction of the proposed q-ROF VIKOR methodology with a structured, data-driven approach that explicitly captures subjective uncertainty for identifying the best SRGM is done. This enables the software teams to have better reliability decisions that are tailored to the product specifics and the organizational priorities. On the other hand, a company concentrating on short-term financial trading software stability would consider SRGMs against criteria like early prediction accuracy. A healthcare device manufacturer, on the other side, would put greater emphasis on long-term reliability projections because of risk aversion. The fuzzy entropy and divergence metrics can be used to model these possibly contradicting priorities in a refined way. Using the fuzzy set theory capability to treat ambiguity with the VIKOR compromise ranking algorithm, the method has a systematic framework for analyzing interdependencies of all the relevant criteria. The ability to do so allows for a more accurate assessment of the tradeoffs inherent in each project context and thus enables the practitioners to select the most appropriate SRGMs for their projects instead of relying on rudimentary techniques that cannot capture the full complexity of the decision process.

The main contributions of this study are:

1. Developing a systematic SRGM selection framework that can handle imprecise assessments and capture the complexity and ambiguity involved in the decision-making process.
2. Introducing novel q-rung orthopair fuzzy (q-ROF) entropy and divergence measures that can model uncertainty and subjectivity in a more nuanced way compared to previous intuitionistic or Pythagorean fuzzy methods.
3. Implementing the VIKOR compromise ranking technique, a structured MCDM method, to identify the best SRGM based on the fuzzy decision matrix and criteria weights derived from the proposed q-ROF measures.

With the ever-increasing societal reliance on software systems, rigorous reliability engineering has become a business-critical competency. This research offers software teams a novel and robust decision support tool for one of the most fundamental reliability tasks - optimal growth model selection tailored to their specific domain requirements and priorities.

The following categories apply to the article: The foundations of the VIKOR Methodology, q-ROF entropy and divergence measure are briefly reviewed in Section II. The suggested VIKOR methodology and divergence and entropy metrics of q-ROFs are discussed in Section III. The suggested measures have been validated using mathematical calculations. Section IV has completed the application of the research methodology and its implementation for the selection of SRGMs. The suggested methodology has been compared to the current methodology in Section V. Section VI concludes the article and provides direction for more analyses.

II. PRELIMINARIES

The definition, operational guidelines, comparison technique, divergence measure, and q-ROF's aggregation operator are explained in this section, which will be employ in the upcoming study, in order to make this work as self-contained as feasible.

Yager [14]presented a more general form, called q-ROFS, and proposes q-ROFS's operations based on the IFS and PFS.

Definition 1: Let X be a universe of discourse. A q-ROFS Ụ' over X is given by

$$Ụ' = \left\{ < \upsilon, \theta_{Ụ'}(\upsilon), \vartheta_{Ụ'}(\upsilon) > \middle| \upsilon \in X \right\}$$

where $\theta_{Ụ'} : X \rightarrow [0,1]$ signifies the degree of membership and $\vartheta_{Ụ'} : X \rightarrow [0,1]$ denotes the degree of non-membership of element $\upsilon \in X$ respectively with the condition that $0 \leq \theta^{q}_{Ụ'}(\upsilon) + \vartheta^{q}_{Ụ'}(\upsilon) \leq 1 \ (q \geq 1)$. The extent of indeterminacy is $\pi_{Ụ'}(\upsilon) = \left(1 - \left(\theta_{Ụ'}(\upsilon)\right)^{q} - \left(\vartheta_{Ụ'}(\upsilon)\right)^{q}\right)^{\frac{1}{q}}$. For expediency, Yager [14]referred $Ụ' = \left(\theta_{Ụ'}, \vartheta_{Ụ'}\right)$ as q-rung orthopair fuzzy set (q-ROFS). The main difference between PFNs, IFNs, and q-ROFNs is their corresponding constraint conditions. The set of all q-rung fuzzy set represented by $q - \mathrm{ROFS}(X)$

Definition 2: [34]If Ụ' and $\tilde{V} \in$ q-ROFS(X), then;

i. $Ụ'^{c} = \left\{ \upsilon, \theta_{Ụ'}(\upsilon), \vartheta_{Ụ'}(\upsilon) \middle| \upsilon \in X \right\}$;

ii. $Ụ' \subseteq \tilde{V}$ if and only if $\forall \upsilon \in X$, $\vartheta_{Ụ'}(\upsilon) \leq \theta_{Ụ'}(\upsilon)$ *and* $\upsilon_{Ụ'}(x) \geq \upsilon_{\tilde{V}}(x)$

iii. $Ụ' = \tilde{V}$ if and only if $\forall\ \upsilon \in X$, $\vartheta_{Ụ'}(\upsilon) = \theta_{Ụ'}(\upsilon)$ *and* $\upsilon_{Ụ'}(x) = \upsilon_{\tilde{V}}(x)$

iv. $Ụ' \cup \tilde{V} = \{\max(\vartheta_{Ụ'}(\upsilon), \vartheta_{\tilde{V}}(\upsilon)), \min(\theta_{Ụ'}(\upsilon), \theta_{\tilde{V}}(\upsilon)) | \upsilon \in X\}; \forall \upsilon \in X;$

v. $Ụ' \cap \tilde{V} = \{\min(\vartheta_{Ụ'}(\upsilon), \vartheta_{\tilde{V}}(\upsilon)), \max(\theta_{Ụ'}(\upsilon), \theta_{\tilde{V}}(\upsilon)) | \upsilon \in X\}; \forall \upsilon \in X;$

Definition 3: A map $\grave{Y} : q - \mathrm{ROFS}(X) \times q - ROFS(X) \rightarrow [0,1]$ is as identified divergence measure if the underneath axioms hold

(D1) $0 \leq \grave{Y}(Ụ', \tilde{V}) \leq 1$

(D2) $\grave{Y}(Ụ', \tilde{V}) = \grave{Y}(\tilde{V}, Ụ')$

(D3) $\grave{Y}(Ụ', \tilde{V}) = 0$ *if* $Ụ' = \tilde{V}$

(D4) *If* $Ụ' \subseteq \tilde{V} \subseteq \hat{Z}$, *and* $\grave{Y}\left(Ụ', \tilde{V}\right) \leq \grave{Y}\left(\tilde{V}, \hat{Z}\right)$ *and* $\grave{Y}(\tilde{V}, \hat{Z}) \leq \grave{Y}(Ụ', \hat{Z})$

To evaluate and rank alternatives based on several criteria, the multi-criteria decision-making (MCDM) technique VIKOR is utilized. It helps in selecting the best compromise solution when there are conflicting objectives. The following steps are needed to implement the VIKOR method:

- Identification of Decision Problem: The decision-making problem and the desired results should be stated in clear terms. Establish a list of alternatives for evaluation and decide on the relevant criteria for the problem.
- Criteria Definition: Determine the factors that are critical for assessing the options. These standards ought to be quantifiable, distinct, and exclusive. Give each criterion the proper weights to indicate their relative relevance.
- Normalize the criterion: Normalize the raw values to a common scale (typically 0 to 1). There are different normalization formulas that can be used.
- Decision Matrix Construction: Calculate the performance measure of each alternative under each criterion. Organize this into a matrix with alternatives as rows and criteria as columns.
- Determine the criteria's weights: Based on the relative importance of each criterion, determine its weights. Numerous techniques, such as the Analytic Hierarchy Process (AHP) and the Simple Additive Weighting (SAW) method, can be used to accomplish this.
- Determine the best and worst values. Identify the best (maximum) and worst (minimum) performance scores among the alternatives for each criterion.

- Determine the S- and R-values: Use formulas to calculate S (utility measure) and R (regret measure) for each alternative. These depend on the normalized scores, criteria weights, and best/worst values.
- Calculate the VIKOR index and order the options: Compute the VIKOR index (Q) for each alternative from S, R and v (weight for strategy of maximum group utility).
- Final Ranking: Using the rankings, choose the compromise that best solves the decision problem and achieves the intended goals

III. PROPOSED ENTROPY AND DIVERGENCE MEASURES FOR q-ROFS

In this section, a thorough explanation of the suggested measure has been presented, which is a crucial part of our study. Here, we want to clarify the significance and potential impact of our new method as we delve into its complexities in order to address the numerous difficulties at hand. Our suggested measure promises to alter current paradigms and open the door for more efficient multi-criteria decision-making processes, representing a step forward in the search for novel solutions.

A. PROPOSED ENTROPY AND DIVERGENCE MEASURES

Initially, we must consider the obvious proposition of entropy and divergence for the q-ROFS. Throughout this article, $q-ROFS(X)$ will represent collection of q-ROFSs.

Proposition 1: Let Ụ' and $\tilde{V}$ be two q-ROFS(X); Ụ' $= \left\{\mathscr{v},\theta_{Ụ'}(\mathscr{v}), \vartheta_{Ụ'}(\mathscr{v}) \middle| \mathscr{v}\in X\right\}$ and $\tilde{V}=\{\mathscr{v},\theta_{\tilde{V}}(\mathscr{v}),$ $\vartheta_{\tilde{V}}(\mathscr{v}) |\mathscr{v}\in X\}$ for $E: q-ROFS(X) \to [0,1]$, has to possess the following qualities in order to be a crisp function:

(E1) Confindness: $0\le\theta^{q}_{Ụ'}(\mathscr{v}), \vartheta^{q}_{Ụ'}(\mathscr{v}) \le 1$

(E2) Asymmetry: $E(Ụ'^{c}) = E(Ụ')$

(E3) Crispness: if $\theta^{q}_{Ụ'}(\mathscr{v}) = 0, \vartheta^{q}_{Ụ'}(\mathscr{v}) = 1$ or $\theta^{q}_{Ụ'}(\mathscr{v}) = 1, \vartheta^{q}_{Ụ'}(\mathscr{v}) = 0$ then *E(P) = 0*

(E4) Severable: if $\theta^{q}_{Ụ'}(\mathscr{v}) = \vartheta^{q}_{Ụ'}(\mathscr{v}) = 1$, than *E(P) = 1*

(E5) Disparity: $E(Ụ') \le E(\tilde{V})$, if $\theta^{q}_{Ụ'}(\mathscr{v}) \le \theta^{q}_{\tilde{V}}(\mathscr{v})$, $\vartheta^{q}_{Ụ'}(\mathscr{v}) \le \vartheta^{q}_{\tilde{V}}(\mathscr{v})$ or $\theta^{q}_{Ụ'}(\mathscr{v}) \ge \theta^{q}_{\tilde{V}}(\mathscr{v}), \vartheta^{q}_{Ụ'}(\mathscr{v}) \ge \vartheta^{q}_{\tilde{V}}(\mathscr{v})$ *for all* $\mathscr{v}\in X$

Proposition 2: Let assume a non-empty set X and Ụ', $\tilde{V}, \hat{Z} \in q-ROFS(X)$; a function $\mathbb{DIV}$: $q-ROFS \times q-ROFS(X) \to [0,1]$ satisfies is the divergence measure between Ụ' and $\tilde{V}$:

(D1) Confindness: $0\le Div(Ụ', \tilde{V})\le 1$

(D2) Severable: $Div\left(Ụ', \tilde{V}\right) = 0 \leftrightarrow Ụ' = \tilde{V}$

(D3) Slanting: $Div\left(Ụ', \tilde{V}\right) = Div\left(\tilde{V}, Ụ'\right)$

(D4) Disparity: If Ụ' $\subseteq \tilde{V} \subseteq \hat{Z}$, then $\grave{Y}\left(Ụ', \tilde{V}\right) \le \grave{Y}\left(\tilde{V}, \hat{Z}\right)$ and $\grave{Y}(\tilde{V}, \hat{Z}) \le \grave{Y}(Ụ', \hat{Z})$

The weight of the constituent's $\mathscr{v}_i \in X$ must be considered in a number of situations. For instance, while making decisions, the attributes typically have different significance and should be given different weights. As a result, we suggest the following entropy measurements:

$$E_{\tilde{V}ROF}(Ụ') = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \tan\frac{\left\{\left|\theta^{q}_{Ụ'}(\mathscr{v}_i) - \vartheta^{q}_{Ụ'}(\mathscr{v}_i)\right|\pi\right\}}{4}\right] \quad (1)$$

$$E_{W\tilde{V}ROF}(Ụ') = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \tan\frac{\{|\theta^{q}_{Ụ'}(\mathscr{v}_i) - \vartheta^{q}_{Ụ'}(\mathscr{v}_i)|\pi\}}{4}\right] \quad (2)$$

Accordingly, we suggest tangent divergence measures for q-ROFSs as:

$$D_{\tilde{V}ROF}\left(Ụ', \tilde{V}\right) = 1 - \frac{1}{n}\sum_{i=1}^{n} tan \left[\frac{\pi}{4} - \frac{\pi}{12}\left\{\begin{array}{l}\left|\theta^{q}_{Ụ'}(\mathscr{v}_i) - \theta^{q}_{\tilde{V}}(\mathscr{v}_i)\right| + \\ \left|\vartheta^{q}_{Ụ'}(\mathscr{v}_i) - \vartheta^{q}_{\tilde{V}}(\mathscr{v}_i)\right| + \\ \left|\eta^{q}_{Ụ'}(\mathscr{v}_i) - \eta^{q}_{\tilde{V}}(\mathscr{v}_i)\right|\end{array}\right\}\right] \quad (3)$$

$$D_{W\tilde{V}ROF}\left(Ụ', \tilde{V}\right) = 1 - \frac{1}{n}\sum_{i=1}^{n} \omega_i tan \left[\frac{\pi}{4} - \frac{\pi}{12}\left\{\begin{array}{l}\left|\theta^{q}_{Ụ'}(\mathscr{v}_i) - \theta^{q}_{\tilde{V}}(\mathscr{v}_i)\right| + \\ \left|\vartheta^{q}_{Ụ'}(\mathscr{v}_i) - \vartheta^{q}_{\tilde{V}}(\mathscr{v}_i)\right| + \\ \left|\eta^{q}_{Ụ'}(\mathscr{v}_i) - \eta^{q}_{\tilde{V}}(\mathscr{v}_i)\right|\end{array}\right\}\right] \quad (4)$$

where $\eta^{q}_{\mathfrak{B}_{Ụ'}}(\mathscr{v}_i) = \sqrt{1 - \theta^{q}_{Ụ'}(\mathscr{v}_i) - \vartheta^{q}_{Ụ'}(\mathscr{v}_i)}$ and $\eta^{q}_{\tilde{V}}(\mathscr{v}_i) = \sqrt{1 - \theta^{q}_{\tilde{V}}(\mathscr{v}_i) - \vartheta^{q}_{\tilde{V}}(\mathscr{v}_i)}$ and $\omega = (\omega_1, \omega_2, \omega_3 \ldots \omega_n)^T$ and $\omega_K \in [0,1]$, $k = 1, 2, \ldots, n$, $\sum_{k=1}^{n}, \omega_k = 1$ if $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots \frac{1}{n}\right)^T$ if we take $\omega_k = 1$ then $E_{\tilde{V}ROF}(Ụ') = E_{W\tilde{V}ROF}(Ụ')$ Similarly, the same can be rechecked that $D_{\tilde{V}ROF}\left(Ụ', \tilde{V}\right) = D_{W\tilde{V}ROF}\left(Ụ', \tilde{V}\right)$.

Theorem 1: Both entropy measure $E_{\tilde{V}ROF}(Ụ')$ and $E_{W\tilde{V}ROF}(Ụ')$ defined in equation (1) and (2) are the valid/reliable entropy measure of q-ROFSs :

It is of the utmost importance to demonstrate that the suggested entropy function satisfies all axioms of the entropy measures stated above, in order to establish its validity as an information measure. Proof is as follows:

(E1) Confindness: $0\le E_{\tilde{V}ROF}(Ụ'), E_{W\tilde{V}ROF}(Ụ') \le 1$

Proof: For $E_{\tilde{V}ROF}(Ụ')$: By the definition of q-ROFs, we have $0 \le \theta_{Ụ'}(\mathscr{v}) \le 1$ and $0 \le \vartheta_{Ụ'}(\mathscr{v}) \le 1$. This implies that $0 \le \theta^{q}_{Ụ'}(\mathscr{v}) \le 1$ and $0 \le \vartheta^{q}_{Ụ'}(\mathscr{v}) \le 1$. We have $0\le|\theta^{q}_{Ụ'}(\mathscr{v}) - \vartheta^{q}_{Ụ'}(\mathscr{v})|\le 1$

$$0\le\frac{\left\{\left|\theta^{q}_{Ụ'}(\mathscr{v}) - \vartheta^{q}_{Ụ'}(\mathscr{v})\right|\pi\right\}}{4} \le \frac{\pi}{4}$$

$$\Longrightarrow 0 \leq 1 - \tan\left[\frac{\left\{\left|\theta^q_{\text{Ụ}}(\upsilon) - \vartheta^q_{\text{Ụ}}(\upsilon)\right|\pi\right\}}{4}\right] \leq 1$$

$$\Longrightarrow 0 \leq \frac{1}{n}\sum_{i=1}^{n}\left[1 - \tan\frac{\left\{\left|\theta^q_{\text{Ụ}}(\upsilon) - \vartheta^q_{\text{Ụ}}(\upsilon)\right|\pi\right\}}{4}\right] \leq 1$$

$\Longrightarrow 0 \leq E_{\tilde{\text{V}}ROF}(\text{Ụ}) \leq 1$. So, we can prove similarly the measure of $E_{W\tilde{\text{V}}ROF}(\text{Ụ})$.

(E2) Asymmetry: $E(\text{Ụ}^C) = E(\text{Ụ})$

The proofs are simple and readily apparent.

(E3) Crispness: $E_{\tilde{\text{V}}ROF}(\text{Ụ}), E_{W\tilde{\text{V}}ROF}(\text{Ụ}) = 0$ if Ụ is the crisp set.

Proof: For $E_{\tilde{\text{V}}ROF}(\text{Ụ})$ if $\theta^q_{\text{Ụ}}(\upsilon) = 0$, $\vartheta^q_{\text{Ụ}}(\upsilon) = 1$ or $\theta^q_{\text{Ụ}}(\upsilon) = 1$, $\vartheta^q_{\text{Ụ}}(\upsilon) = 0$ then $|\theta^q_{\text{Ụ}}(\upsilon) - \vartheta^q_{\text{Ụ}}(\upsilon)| = 1$

$$\Longrightarrow \frac{1}{n}\sum_{i=1}^{n}\left[1 - \tan\frac{\left\{\left|\theta^q_{\text{Ụ}}(\upsilon) - \vartheta^q_{\text{Ụ}}(\upsilon)\right|\pi\right\}}{4}\right] = 0$$

$\Longrightarrow E_{\tilde{\text{V}}ROF}(\text{Ụ}) = 0$ So, we can prove similarly the measure of $E_{W\tilde{\text{V}}ROF}(\text{Ụ})$.

(E4) Severable: if $\theta^q_{\text{Ụ}}(\upsilon) = \vartheta^q_{\text{Ụ}}(\upsilon) = 1$, than E(A) = 1

Proof: For $E_{\tilde{\text{V}}ROF}(\text{Ụ})$: for all $\upsilon \in X$, if $\theta^q_{\text{Ụ}}(\upsilon) = \vartheta^q_{\text{Ụ}}(\upsilon)$ or $\theta^q_{\text{Ụ}}(\upsilon) = \vartheta^q_{\text{Ụ}}(\upsilon)$ then $\left|\theta^q_{\text{Ụ}}(\upsilon) = \vartheta^q_{\text{Ụ}}(\upsilon)\right| = 0$. Hence,

$$\tan\frac{\left\{\left|\theta^q_{\text{Ụ}}(\upsilon) = \vartheta^q_{\text{Ụ}}(\upsilon)\right|\pi\right\}}{4} = 0$$

$$\Longrightarrow \frac{1}{n}\sum_{i=1}^{n}\left[1 - \tan\frac{\left\{\left|\theta^q_{\text{Ụ}}(\upsilon) - \vartheta^q_{\text{Ụ}}(\upsilon)\right|\pi\right\}}{4}\right] = 1$$

Therefore, $E_{\tilde{\text{V}}ROF}(\text{Ụ}) = 1$. If $E_{\tilde{\text{V}}ROF}(\text{Ụ}) = 1$, this indicates

$$\left[1 - \tan\frac{\left\{\left|\theta^q_{\text{Ụ}}(\upsilon) - \vartheta^q_{\text{Ụ}}(\upsilon)\right|\pi\right\}}{4}\right] = 0$$

$$\Longrightarrow \left|\theta^q_{\text{Ụ}}(\upsilon) - \vartheta^q_{\text{Ụ}}(\upsilon)\right| = 0$$

Therefore, $\theta^q_{\text{Ụ}}(\upsilon) = \vartheta^q_{\text{Ụ}}(\upsilon)$, Hence $\theta_{\text{Ụ}}(\upsilon) = \vartheta_{\text{Ụ}}(\upsilon)$. So, we can prove similarly the measure of $E_{W\tilde{\text{V}}ROF}(\text{Ụ})$.

Theorem 2: Both divergence measure $D_{\tilde{\text{V}}ROF}\left(\text{Ụ}, \tilde{\text{V}}\right)$ and $D_{W\tilde{\text{V}}ROF}\left(\text{Ụ}, \tilde{\text{V}}\right)$ defined in equation (3) and (4) are the valid/reliable entropy measure of q-ROFSs:

Proof: These properties shown in Proposition 2 can be validated for the divergence measure $D_{\tilde{\text{V}}ROF}\left(\text{Ụ}, \tilde{\text{V}}\right)$ and $D_{W\tilde{\text{V}}ROF}\left(\text{Ụ}, \tilde{\text{V}}\right)$ in accordance with the contentions made above.

B. PROPOSED VIKOR METHODOLOGY

VIKOR is a MCDM technique used to solve the problem of optimizing a complex system with conflicting and non-measurable criteria and is most effective when the decision maker is not able to state his preferences at the initial stage of the design phase. VIKOR method is used when one has to select from a set of alternatives in the context of conflicting criteria, and it offers a maximum of 'group utility' for the 'majority' and a minimum of 'individual regret' for the 'opponent'. The method also presents the multi-criteria ranking index using the specific measure of 'distance' from the 'ideal' solution. The VIKOR method is initiated by constructing the decision matrix, in which the performance ratings of the alternatives are offered for each criterion. Next, the method determines the group utility and individual regret for each of the alternatives. These values are then used to rank the alternatives based on the VIKOR index which is the weighted sum of the group utility and the individual regret. The main strengths of the VIKOR method include the consideration of conflict between the criteria, efficiency of calculations, and determination of the compromise solution which is the closest to the ideal one. The method is most effective when the decision-makers have different preferences or priorities and it is required to identify a solution that would be equally undesirable for all of them. The VIKOR method has been used in different areas of application, including environmental management, energy, transportation, and project selection, among others.

The VIKOR method will help us in decision-making in respect of selection of appropriate SRGM's over the dataset used in this study. The suggested method is shown as a flowchart in Figure 1. The following steps will be used for ranking via VIKOR method:

Step 1: Decision matrix has been normalized, list the SRGM's (alternatives) evaluated during the study based on q-ROF's based on criteria for the selection.

Step 2: Entropy Calculation of each criteria using entropy equation (1).

Step 3: weights ω_i for each mentioned criterion for each C_i can be calculated using the following equation.

$$\omega_i = \frac{1 - E_{\tilde{\text{V}}ROF}(C_i)}{\sum_{i=1}^{n}(1 - E_{\tilde{\text{V}}ROF}(C_i))} \tag{5}$$

Step 4: The following equations can be used to calculate the virtual positive ideal solutions and the virtual negative ideal solutions for the attribute A_i:

$$\ddot{\Upsilon}_j^+ = max_i\left(\ddot{\Upsilon}_{ij}\right) \tag{6}$$

$$\ddot{\Upsilon}_j^- = min_i(\ddot{\Upsilon}_{ij}) \tag{7}$$

Step 5: Group utility computation of S_i and S_i' can be calculated using the following equation:

$$S_i = \sum_{j=1}^{n}\omega_j\frac{d(\ddot{\Upsilon}_j^+, \ddot{\Upsilon}_{ij})}{d(\ddot{\Upsilon}_j^+, \ddot{\Upsilon}_j^-)} \tag{8}$$

$$S_i' = \max\left\{\omega_j\frac{d\left(\ddot{\Upsilon}_j^+, \ddot{\Upsilon}_{ij}\right)}{d\left(\ddot{\Upsilon}_j^+, \ddot{\Upsilon}_j^-\right)}\right\} \tag{9}$$

TABLE 1. Description of SRGM'S for comparison.

Attribute	*Model Name*	*Model Type*	*Mean Value Function* $\boldsymbol{m(t)}$
$\boldsymbol{A_1}$	Goel and Okumoto Model [2]	*Concave*	$m(t) = a(1 - e^{-bt})$
$\boldsymbol{A_2}$	Yamada Rayleigh [3]	*S-Shape*	$m(t) = a\left(1 - e^{-r\alpha(1-e^{\frac{-bt^2}{2}})}\right)$
$\boldsymbol{A_3}$	Kim et. al. (DPF2) [7]	*S-Shape Dependent*	$m(t) = \dfrac{a}{1 + \frac{a}{h}\left(\frac{1+c}{c+e^{bt}}\right)^{a}}$
$\boldsymbol{A_4}$	ATF Model (Proposed)	*S-Shape Dependent*	$m(t) = \dfrac{a}{1-\beta}\left[1 - e^{-(1-\beta)(\frac{bt^3}{3}+bct)}\right]$

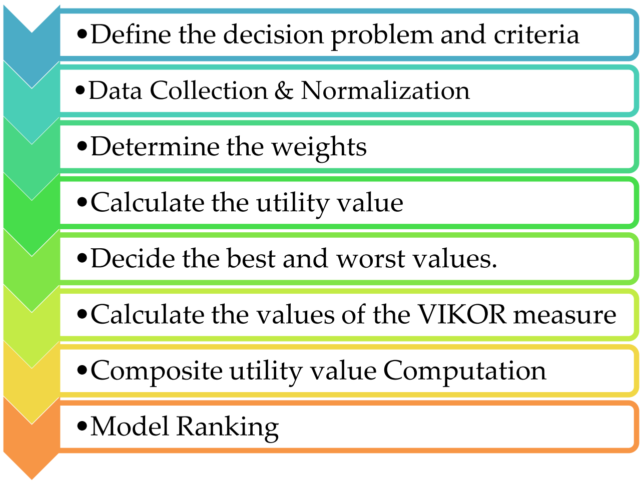

FIGURE 1. VIKOR approach flow chart.

Step 6: The results of $\tilde{V}_i$ will be computed by the following equation:

$$\tilde{V}_i = \frac{\alpha\,(S_i - min\,(S_i))}{(max\,(S_i) - min\,(S_i))} + \frac{(1-\ \alpha)\,(S'_i - min\,(S'_i))}{(maxS'_i - min\,(S'_i))} \tag{10}$$

Step 7: Rank the SRGMs based on their composite utility values, with the SRGM having the highest value ranked first.

In the equation (10), the symbol α is a balance parameter that can strike a balance between collective utility and personal regret. Three possibilities exist:

a. If $\alpha > 0.5$ symbolize that the minimal individual regret is greater than maximum group utility.
b. If $\alpha < 0.5$ symbolize that the maximum group utility is greater than the minimal individual regret.
c. If $\alpha = 0.5$ symbolize that the maximum group utility and the minimal individual regret have equal priority.

IV. SELECTION OF SRGM'S BY RANKING USING VIKOR APPROACH

During the testing and debugging process, SRGM's based on NHPP (Non-Homogeneous Poisson Process) are widely used to anticipate the reliability of software systems. It is

TABLE 2. Decision matrix of alternative & criteria.

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} (0.7,0.3) & (0.3,0.4) & (0.9,0.5) & (0.6,0.3) \\ (0.4,0.6) & (0.7,0.3) & (0.8,0.2) & (0.4,0.6) \\ (0.9,0.1) & (0.1,0.3) & (0.7,0.3) & (0.7,0.2) \\ (0.6,0.4) & (0.5,0.4) & (0.6,0.3) & (0.9,0.1) \end{bmatrix} \end{array}$$

TABLE 3. Entropy of each criteria.

Criteria	$\boldsymbol{E_{QROF}(C_i)}$
$\boldsymbol{C_1}$	0.2286
$\boldsymbol{C_2}$	0.0583
$\boldsymbol{C_3}$	0.2788
$\boldsymbol{C_4}$	0.2258

TABLE 4. Weighted values of each criteria.

Criteria	ω_i
$\boldsymbol{C_1}$	0.288774
$\boldsymbol{C_2}$	0.073684
$\boldsymbol{C_3}$	0.352216
$\boldsymbol{C_4}$	0.285326

assumed that software faults arise using the Poisson process that underlies this model, which has a time-dependent failure intensity function. This study focusses on evaluating, preference, and SRGM ratings in relation to the testing phase of SDLC. As a result, a thorough investigation into the operation and categorization of SRGMs was carried out, concentrating on distinct categories i.e. NHPP and failure-rate models. Delivering highly dependable software solutions is a critical skill for software engineers in today's cutthroat corporate environment. Software engineers frequently utilize SRGMs to estimate reliability. This study's primary objective is to frame the SRGMs selection problem as a difficult MCDM problem.

The Maximum Likelihood Estimation (MLE) approach is often used in SRGM's to estimate parameters. By maximizing the likelihood function, which measures the likelihood of getting the observed data given the model and its parameters, the MLE statistical technique estimates the model's parameters.

TABLE 5. Positive & negative ideal solutions.

	PIS & NIS	C_1	C_2	C_3	C_4
Decision-Makers	A^+	(0.9,0.5)	(0.8,0.6)	(0.9,0.3)	(0.9,0.4)
	A-	(0.3,0.3)	(0.4,0.6)	(0.1,0.1)	(0.5,0.6)

TABLE 6. Values of the group utility.

Symbol	Values	Symbol	Values
S_1	0.8911	S'_1	0.2999
S_2	0.9942	S'_2	0.3501
S_3	0.8423	S'_3	0.3028
S_4	0.8412	S'_4	0.3012

TABLE 7. q-computation against different attributes.

Q_i	Values
Q_1	0.1306
Q_2	0.8
Q_3	0.0257
Q_4	0.0105

TABLE 1 shows the list of four different types of SRGM's – these are Goel and Okumoto model (A_1), Yamada Rayleigh (A_2), Yamada exponential Model (A_3) and V-tub shaped fault detection rate model (A_4) with the aim of optimal selection researchers commonly employ four selection indices: Mean Square Error (C_1), $R^2(C_2)$, Predictive Ratio Risk (C_3) and Predictive Power (C_4). In this study, we have used the VIKOR's approach subsequent phases to access the most appropriate SRGM considering the above-mentioned criteria.

SRGM's can be ranked using the VIKOR approach, a multi-criteria decision-making (MCDM) technique, based on a variety of criteria or features. Criteria such as prediction accuracy, computational complexity, model simplicity, and robustness. The computation based on the VIKOR method following the procedural steps on the factors stated above, the best SRGM's may be determined:

Step 1: Normalized the decision matrix, Table 2 list the SRGM's (alternatives) evaluated during the study based on q-ROF's based on above mentioned criteria for the selection. This table presents the decision matrix, which lists the alternatives (different SRGM's) being evaluated against the selected criteria. The alternatives are represented by A1 to A4, while the criteria are C1 to C4. The entries in the matrix are in q-rung orthopair fuzzy number (q-ROFN) format, which captures both the membership and non-membership degrees to model uncertainty and ambiguity. Each cell contains a pair (x, y) representing the membership degree x and non-membership degree y for that particular alternative-criteria combination.

Step 2: The TABLE 3 calculates the entropy values $\boldsymbol{E_{QROF}(C_i)}$ for each criterion C_i using the proposed q-ROF entropy measure from Equation (2). Higher entropy indicates more ambiguity or uncertainty in that criterion's evaluations across the alternatives. Lower entropy suggests the criterion evaluations are crisper and more certain. These entropy values will be used in the next step to determine the relative importance weights of the criteria.

Step 3: Based on the entropy values computed in TABLE 3 this table derives the criteria weight values ω_i using equation (5). The weights represent the relative importance of each criterion in the overall SRGM selection decision. Criteria with lower entropy (more certain evaluations) get higher weights. These weights will be used in subsequent computations to determine the rankings of the SRGM alternatives.

Step 4: The TABLE 5 identifies the positive ideal solution (PIS) and negative ideal solution (NIS) for each criterion across the alternatives computed using Equation (6) and (7). The PIS represents the best/maximum criterion value across all alternatives, acting as an anchor for comparisons. The NIS represents the worst/minimum criterion value across all alternatives. These ideal solutions, derived from the decision matrix, enable calculation of divergences from each alternative to reference ideal scenarios.

Step 5: The TABLE 6 computes the group utility values S_i and individual regret values S'_i for each alternative A_i using the equation (8) and (9) respectively. S_i represents the overall utility or performance score of an alternative considering all criteria. S'_i captures the maximum regret or opportunity cost for that alternative across the worst-case criterion scenario.

These values will be used in the next step to derive the compromise rankings balancing collective and individual priorities.

Step 6: The TABLE 7 calculates the final VIKOR index value Q_i for each alternative A_i based on equation (10), using the group utility S_i and individual regret S'_i from TABLE 6. The index Q_i represents the compromise ranking score, balancing the group utility and individual regret through the weight parameter alpha. The alternative with the minimum Q_i value is considered the overall best compromise solution.

Step 7: The TABLE 8 presents the final ranking of the SRGM alternatives based on sorting the Q_i values computed in TABLE 7 in ascending order. It provides three ranking perspectives based on S_i (group utility), S'_i(maximum regret), and Q_i (compromise score). The top-ranked alternative based on Q_i represents the recommended optimal SRGM selection per the proposed methodology.

The VIKOR method enables the computation of group utility, allowing decision-makers to make informed choices considering multiple criteria or attributes. The ranking results have been listed in TABLE 8 against the four models. These four models have been compared against four different

TABLE 8. Ranking results.

Symbols	A_1	A_2	A_3	A_4	Ranking	Compromised Solution
S	0.8911	0.9942	0.8423	0.8412	$A_2 > A_1 > A_3 > A_4$	A_2
S'	0.2999	0.3501	0.3028	0.3012	$A_2 > A_3 > A_4 > A_1$	A_2
Q	0.1306	0.8	0.0257	0.0105	$A_2 > A_1 > A_3 > A_4$	A_2

TABLE 9. Comparison between proposed & existing approach.

Distance Measure	A_1	A_2	A_3	A_4
Park et al. **[25]**	Immeasurable	Immeasurable	Immeasurable	Immeasurable
Li et al. **[35]**	Immeasurable	Immeasurable	Immeasurable	Immeasurable
Arora & Naithani **[36]**	Immeasurable	Immeasurable	Immeasurable	Immeasurable
Proposed Measure	0.8911	0.9942	0.8423	0.8412

criteria. The VIKOR method offers a robust and systematic approach to decision-making by providing S-values that facilitate the ranking and selection of alternatives in multi-criteria scenarios. Decision-making can be enhanced through a considerate attitude towards different aspects, which are both positive and negative, enabling decision-makers to be guided by informed choices that suit conflicting criteria. Grasping how S-values are captured and their interpretation using the VIKOR method provides decision-makers with the tools to solve knotty decision-making challenges. These values enable the setup of a hierarchy that makes it possible to select the most suitable plans, and higher S-values encompass the best alternative. The VIKOR method is judicious as it considers not only benefits but also demerits of each option in its decision making while the trade-offs between them are also considered.

V. COMPARISON OF EXISTING METHODOLOGY WITH PROPOSED METHODOLOGY

The previous studies have contributed significantly to our understanding of various phenomena, but they often encounter limitations when it comes to measuring certain attributes. According to the previous studies Park et al [25]suggested the divergence measure using intuitionistic fuzzy sets. Li et al. [35]and the Arora & Naithani [36]uses the methodology of Pythagorean fuzzy sets for computation of the divergence measure. This suggests a consistent pattern across multiple studies where the techniques' differences are challenging to assess using the employed measures. The TABLE 9 below presents a comparison of divergence measures between different techniques based on multiple studies. The findings indicate that while some studies couldn't quantify the differences, whereas the proposed study suggested a novel measure that assigns numerical values to the divergence between these techniques.

While existing fuzzy MCDM methods offer some improvements over classical crisp techniques, the proposed methodology represents a cohesive advance in tackling higher-order subjective ambiguities inherent in the software reliability domain. The innovative integration of new q-ROFS measures with principled weight derivation and aggregation operators provides a flexible and rational decision framework for SRGM selection. The key differentiator is the methodology's ability to systematically capture subjective uncertainty beyond traditional fuzzy set representations. By leveraging the properties of q-ROF's, it can model more complex and nuanced degrees of membership, non-membership, and hesitation that often arise in software reliability evaluations. Furthermore, the proposed approach introduces a novel technique for determining criteria weights based on the ambiguity quantified by the q-ROFS entropy measure. This ensures that more certain criteria are prioritized appropriately during the decision-making process, enhancing the credibility of the final rankings.

By tailoring the widely-used VIKOR method to the SRGM selection problem and integrating it with the advanced q-ROF theory, this research delivers a cohesive and structured multi-criteria decision analysis solution specifically designed for the software reliability domain. The ability to balance group utility and individual regret considerations aligns well with the trade-offs and risk postures inherent in reliability engineering decisions. The comparative performance against other fuzzy MCDM techniques remains an area for further investigation, the proposed methodology's strong theoretical foundations and tailored design for the target problem domain position it as a promising solution for software teams grappling with subjective ambiguity in their reliability growth modeling efforts. The comparison between the proposed and the Existing approach are mentioned in TABLE 9. However, by the computation using the q-ROF Entropy and proposed Divergence Measure the four previously unmeasurable attributes that are valid and have a significant computation a better SRGM's selection have been suggested. Divergence measures proposed in this article were designed to satisfy certain properties and axioms relevant to software reliability modeling. Some existing measures may not fulfill the same foundations. Furthermore, the new measures enabled direct comparisons to previous Intuitionistic or Pythagorean fuzzy techniques applied in this domain existing q-ROF measures would not facilitate these insightful comparisons.

VI. CONCLUSION

This research introduced a fuzzy MCDM approach for selecting the most appropriate SRGM tailored to the specific priorities and ambiguity inherent in subjective evaluations. By integrating q-rung orthopair fuzzy set theory with the VIKOR compromise ranking technique, the proposed methodology provides a structured framework for SRGM selection that can effectively capture nuanced trade-offs across criteria. The new entropy and divergence measures of q-ROF are the theoretical contributions that help to expand the uncertainty modeling compared to the conventional fuzzy approaches. The entropy measure enables the differentiation of weights of criteria in terms of their fuzziness, and the divergence measure enables the comparison of SRGMs in the fuzzy criteria space. Applying the VIKOR method on this fuzzy decision information then gave the following compromise ranking which is the balance of both the collective and individual regrets. From a practical standpoint, the proposed methodology enables software organizations to make a rational decision on one of the most strategic reliability decisions, namely, the choice of an appropriate SRGM. Instead of relying on ad-hoc processes that can be inconsistent and biased, this approach helps practitioners with high confidence to find growth models that are appropriate for their specific product environments, development methodologies, and risk appetites. While the proposed techniques show strong potential, there are several avenues for future expansion and open questions warranting further investigation:

1. Validating the methodology on larger datasets covering more SRGM options and evaluation criteria would increase confidence in its universal applicability across software domains.
2. Exploring intuitive visualization techniques beyond the mathematical representations could enhance human comprehensibility when interpreting the fuzzy computations.
3. Integrating feedback loops and sensitivity analyses within the fuzzy VIKOR flow may provide insights on uncertainty propagation and assist in refining subjective evaluations.
4. Extending the fuzzy decision constructs to concepts like bipolar or spherical fuzzy sets could expand descriptive capability for modeling higher-order uncertainty.
5. Developing decision support tools that automate and streamline the fuzzy VIKOR calculations would facilitate easier industry adoption of the methodology.
6. Investigating how the q-rung measures influence computational complexity and algorithm scalability limitations when dealing with extremely large SRGM sets.
7. Examining real-world case studies comparing the fuzzy VIKOR SRGM recommendations against conventional techniques could validate its practical predictive accuracy.

Despite these outstanding questions, the research makes a valuable contribution towards addressing the ambiguity challenges in SRGM's - a crucial consideration as our societal reliance on robust software systems continues to escalate. The fuzzy MCDM techniques open new pathways for data-driven reliability decision-making capable of navigating subjective trade-offs. Potential applicability extends well beyond SRGM selection to other software domains grappling with multi-criteria prioritization under uncertainty. Overall, this paper delivers an innovative solution to a critical software engineering challenge leveraging advanced fuzzy sets and multi-criteria decision analysis. The fuzzy VIKOR approach represents a significant step forward in modeling ambiguity for improved SRGM selection. The expansion of this theory to include the concepts of bipolar complex fuzzy sets [37], [38], bipolar complex fuzzy soft sets [38], and bipolar complex spherical fuzzy sets [39] is one of our future goal.

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