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RESEARCH ARTICLE

A Distributed-Decentralized Tri-Layer Game-Based Transactive Energy Framework Including Adaptive Proximal ADMM and Scale-Independent DRO

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ABSTRACT Transactive energy (TE) as a market-based mechanism provides a practical framework to fully manage and control local energy networks with a high penetration of distributed energy resources (DERs). However, the existing designed TE frameworks can rarely encourage prosumers to participate fairly owing to neglecting competition and cooperation. This paper proposes a tri-layer hybrid game-based TE framework, wherein the cooperation of prosumers to trade energy in a peer-to-peer (P2P) fashion is considered in the first layer using the Nash Bargaining Game (NBG) Theory. The competition among prosumers to trade with the most affordable aggregator is modeled at the second layer using the evolutionary game (EG). The third layer also models the competition among aggregators and the competition between aggregators and the cooperation of prosumers by developing a non-cooperative game. Besides, a scale-independent distributionally robust optimization (DRO) is developed based on the Wasserstein ambiguity set to allow prosumers to manage their uncertainty using all potential historical data while ensuring tractability. Finally, a new adaptive proximal alternative direction method of multipliers (ADMM) is introduced to develop a distributed-decentralized decision-making scheme for satisfying network constraints and energy trading in a P2P manner to accelerate the solution procedure and preserve privacy. The tests and implementations demonstrate that the proposed tri-layer TE framework lowered the overall costs for prosumers by 11 % and 2.85 % compared to the total costs in non-cooperative TE and cooperative TE, respectively.

INDEX TERMS Adaptive proximal ADMM, cooperative game, evolutionary game (EG), Nash bargaining game (NBG), non-cooperative game, scale-independent DRO, transactive energy framework, tri-layer game.

NOMENCLATURE

MAIN ABBREVIATIONS *ES*/*DG*/*D* Energy storage/ Distributed generator/ Demand.

SETS AND INDICES

- *h*/*k*/*n* Index of iterations/Index of aggregators/ Index of nodes.
- $i, j/t/v$ Index of prosumers/Index of time/Index of historical data.
- E^{Net} Set of the feasibility area of network constraints.
- Set of mapping i^{th} prosumer to n^{th} node.

 $a_{t,k}^B$, b_t^B Cost and risk coefficients regarding the purchased power from prosumers by *k th* aggregator at time *t*.

FUNCTIONS

- C_i^G $\int_{i,t}^{G}$ Generation cost of DG of i^{th} prosumer at time *t*.
- Penalty cost of *i th* prosumer at time *t*.

 $C^P_{i,t}$
 $C^T_{i,t,k}$ Total cost of i^{th} prosumer at time t due to trading with *k th* aggregator.

 \overline{C}_t^T *^t* Average total cost at time *t*.

- $\check{C}^T_{i,t,k}$ The value of total cost of *i th* prosumer at time *t* due to trading with k^{th} aggregator.
- $\widetilde{C}_i^{(\star)}$ The value of total cost of *i th* prosumer without P2P trading.
- $\hat{C}_i^{(\star)}$ *i* The value of total cost of *i th* prosumer with P2P trading.
- $P_{t,k}^B$ Total purchased power of *k th* aggregator at time *t*.
- $\frac{P^S_{t,k}}{\bar{P}^B_{t,k}}$ Total sold power of *k th* aggregator at time *t*.
- Maximum capacity of the purchased power of *k th* aggregator at time *t*.
- $\bar{P}_{t k}^{S}$ *f*_{*t*},*k* Maximum capacity of the sold power of k^{th} aggregator at time *t*.
- $U_{t,k}^B$ Total risk cost of *k th* aggregator at time *t* regarding the purchased power.
- $U_{t,k}^S$ Total risk cost of *k th* aggregator at time *t* regarding the sold power.

VARIABLE

- $E_{i,t}$ Energy of energy storage of i^{th} prosumer at time *t*.
- $P_{i,t}^{ch}/P_{i,t}^{dch}$ Charging and discharging power of energy storage of *i th* prosumer at time *t*.

 $P_{i,t}^E/P_{i,t}^D$. $\big/P^G_{i,t}$ Output power of energy storage/ consumption/DG of *i* prosumer at time *t*.

P Net n,*t* /*Q Net n*,*t* Injected/absorbed active and reactive power to/from *n th* node at time *t*.

$$
P_{ij,t}^{P2P}
$$
 P2P transaction power from *i*th prosumer to
*j*th prosumer at time *t*.

- $P_{i,t}^T/Q_{i,t}^T$ Injected/absorbed power by inverter of *i th* prosumer at time *t*.
- $\overline{\mathcal{Q}}_i^T$ *i*,*t* Inverter maximum capacity of reactive power of *i th* prosumer at time *t*.
- $\overline{r}_{i,t}/\overline{r}_{i,t}$ Upward and downward reserve capacities of energy storage of *i th* prosumer at time *t*.
- $\Delta p_{i,t}^{ES} / \Delta p_{i,t}^D$. $\Delta p_{i,t}^G$ Power adjustment of energy
	- storage/consumption/DG of *i th* prosumer at time *t*.
	- Contribution of energy storage/ consumption/DG of *i th* prosumer at time *t* to compensate for the uncertainty of *i th* prosumer.
- $\omega_{i,t}$ Random variable to model the uncertainty of *i th* prosumer at time *t*.

 Φ_{ii} P2P trading cost.

I. INTRODUCTION

 $\frac{\tau_{i,t}^{ES}/\tau_{i,t}^{D}}{\tau_{i,t}^{G}}$

The rapid expansion of distributed energy resources (DERs) within local networks marks a transformative shift towards prosumption, reshaping the power systems' landscape [\[1\].](#page-12-0) This presents an exciting business opportunity to aggregate the capacity of local networks, allowing them to participate in the distribution or transmission-level markets actively [\[2\].](#page-12-1) However, establishing a sustainable business model for local networks is not without significant challenges, as it requires effectively balancing the interests of both aggregators and prosumers while ensuring profitability. With this respect, transactive energy (TE) offers a market-based methodology that respects prosumers' preferences and aligns with aggregators' financial targets [\[3\]. Co](#page-12-2)nsequently, the design of a TE framework to fairly harness the prosumption capacity of local networks has become a focal point of attention and research.

In this context, a particular group of studies has focused on devising an auction-based TE market, where prosumers and aggregators submit their bids to fulfill their respective prosumption requirements. For instance, [4] [and](#page-12-3) [5] [pre](#page-12-4)sents a double auction-based algorithm for peer-to-peer (P2P) trading, incorporating aggregators' roles. Additionally, [\[6\]](#page-13-0) proposes a pricing algorithm aimed at maximizing aggregators' profits and minimizing prosumers' billing costs. Two-stage and bi-level TE markets are also suggested in [\[7\]](#page-13-1) and [\[8\], re](#page-13-2)spectively. To enhance market stability, [\[9\]](#page-13-3) takes into account aggregators' performance in upperlevel markets, and [\[10\]](#page-13-4) considers prosumers' discomfort cost. However, the primary objective of these markets is often centered around minimizing total costs or maximizing overall profits, which may not sufficiently provide a fair TE framework for local networks due to the disregard of individual preferences, thereby potentially discouraging prosumer participation.

To address this challenge, another set of studies has introduced a non-cooperative TE framework, treating the conflict of interest between aggregators and prosumers as a competitive scenario. Among non-cooperative game approaches, Stackelberg games (SG) have been widely employed. In the SG setup, an entity acts as the leader (e.g., the aggregator), setting prices first, while others act as followers (e.g., prosumers), adjusting their prosumption accordingly. For instance, [\[11\]](#page-13-5) and [\[12\]](#page-13-6) presents a bi-level model where aggregators maximize profits at the upper level, and prosumers minimize costs at the lower level. Additionally, [\[13\]](#page-13-7) and [\[14\]](#page-13-8) develop an SG for voltage support by prosumers, and [\[15\]](#page-13-9) formulates an SG-based TE market to improve network imbalance. To consider multiple aggregators and their competition, a multi-leader multi-follower SG is suggested in [\[16\]](#page-13-10) and [\[17\].](#page-13-11) To preserve prosumers' privacy in the non-cooperative TE market, a distributed decision-making scheme is suggested in [\[18\].](#page-13-12) Additionally, [\[19\]](#page-13-13) proposes a non-cooperative game for the participation of heat ventilation and air conditioning (HVAC) capacity in the TE market. Further, [\[20\]](#page-13-14) introduces a Bertrand game to account for competition among prosumers, and [\[21\]](#page-13-15) devises a Cournot Nash game to offer a bidding strategy for prosumers in the TE market.

On the other hand, some studies have demonstrated the potential efficiency of prosumers in increasing their profits through cooperation among themselves [\[22\],](#page-13-16) [\[23\],](#page-13-17) [\[24\],](#page-13-18) [\[25\],](#page-13-19) [\[26\],](#page-13-20) [\[27\],](#page-13-21) [\[28\],](#page-13-22) [\[29\],](#page-13-23) [\[30\]. C](#page-13-24)onsequently, the coalitional game, as a cooperative game approach, has been employed in [\[22\]](#page-13-16) and [\[23\]](#page-13-17) to encourage prosumers towards P2P trading, albeit at the expense of neglecting network constraints. However, the coalition game TE framework often fails to divide profits fairly as it utilizes the Sharply method for profit propagation [\[24\]. T](#page-13-18)o overcome this challenge, a TE market based on the Nash Bargaining Game (NBG) is developed in [\[25\],](#page-13-19) wherein profits are distributed based on each prosumer's contribution to cost reduction. Nevertheless, this NBG-based model comes with a considerable computational burden. In response, $\lfloor 26 \rfloor$ and $\lfloor 27 \rfloor$ presents an exact decomposition method specifically designed to alleviate the computational complexity associated with the NBG-based TE market. Furthermore, [\[28\]](#page-13-22) and [\[29\]](#page-13-23) investigations emphasize the significance of prioritizing NBG-based P2P trading for operators, not only due to the financial advantages it offers but also to avoid transferring the complexity and uncertainty of local networks to higher levels. To foster viable cooperation among prosumers, [\[30\]](#page-13-24) addresses a bi-level model, encompassing network constraints at the upper level and NBG at the lower level. However, a notable limitation of these studies lies in their oversight of the competition dynamics between prosumers and aggregators within the suggested TE market.

From the reviewed literature, The following gaps are identified to the best of the authors' knowledge: 1) Finding a comprehensive TE framework that can effectively optimize individual preferences while considering both existing cooperations and competitions in local networks is challenging. Non-cooperative TE frameworks often overlook the bargaining potential of prosumers, leading to higher costs and lower profits for them [\[27\]. O](#page-13-21)n the other hand, cooperative TE frameworks that neglect competition between prosumers and aggregators may produce optimistic solutions that are difficult to sustain in practice [\[31\]. M](#page-13-25)oreover, aggregators utilizing available cooperative TE frameworks can wield market power, as the clearing prices of prosumer cooperation become influenced by the aggregators' bids [\[32\]. T](#page-13-26)o address these limitations, only a limited number of studies have presented hybrid forms of cooperative and non-cooperative TE markets. For instance, [\[31\]](#page-13-25) and [\[32\]](#page-13-26) introduce a bi-level TE market where the upper level incorporates a multileader multi-follower Stackelberg game, accounting for competition between aggregators and prosumers, while the lower level considers prosumer bargaining power. However, this approach neglects competition among prosumers, and achieving the Nash point using this model remains controversial due to its mixed-integer linear programming (MILP) nature. 2) Uncertainties in prosumptions are often overlooked in many studies, which results in avoiding the TE framework to reach the Nash point. Although stochastic programming (SP) is utilized in [1] [to](#page-12-0) address uncertainty, it requires access to exact probability density functions (PDFs). On the other hand, the suggested robust optimization (RO) in [\[33\]](#page-13-27) tends to be excessively conservative, resulting in significant cost increases and questionable achievement of the Nash point. Consequently, motivating prosumers to participate in such designed TE frameworks becomes difficult. To overcome these issues, a data-driven distributionally robust optimization (DRO) approach is addressed in [\[34\]](#page-13-28) and [\[35\], a](#page-13-29)lthough its performance can vary significantly depending on the scale of the historical data. Large datasets become computationally intractable, while small-scale data compromise accuracy. Thus, achieving the Nash point in this

context remains uncertain, further complicating prosumer engagement in the designed TE framework. 3) Preserving the privacy of prosumers presents another challenge in the TE framework. Although the alternating direction method of multipliers (ADMM) is used in [\[36\]](#page-13-30) for preserving prosumer privacy, its convergence speed and scalability raise concerns, particularly when dealing with a considerable number of prosumers [\[37\].](#page-13-31) Additionally, initializing the penalty factor in the regular ADMM is highly challenging since it can significantly affect convergence speed and accuracy [\[10\].](#page-13-4)

To remedy the aforementioned shortcomings, This paper proposes the following contributions:

• **Proposing a hybrid tri-layer TE framework.** Proposing the tri-layer TE framework, including all cooperations and competitions, contributes to fairly encouraging prosumers to participate. In the first layer, prosumers' cooperation and bargaining power are considered using NBG. The second layer introduces competition among prosumers as they buy and sell their prosumptions to aggregators at the most affordable prices, modeled through an evolutionary game (EG). In the third layer, the modeling encompasses the competition between aggregators and the cooperation among prosumers, alongside considering the competition among aggregators. This layer is represented as a developed noncooperative game, taking into account the performance of aggregators in the upper-level market. The hybrid tri-layer TE framework has been addressed in this study for the first time.

• **Developing a scale-independent DRO based on Wasserstein ambiguity set for uncertainty management of this problem.** Having a scale-independent DRO is crucial because it removes the dependence of the proposed TE framework on the number of samples. However, the DRO method presented in [9] [an](#page-13-3)d [\[38\]](#page-13-32) shows a high sensitivity to changes in the number of samples.

• **Developing a new adaptive proximal ADMM.** The proposed model demonstrates lower sensitivity to initialization and significantly faster convergence compared to other available algorithms. Furthermore, a distributeddecentralized algorithm is developed based on the proposed adaptive proximal ADMM. This algorithm efficiently oversees network security constraints by the operator in a distributed fashion, while also facilitating decentralized P2P trading among prosumers in a decentralized manner.

II. ILLUSTRATION EXAMPLE OF THE PROPOSED HYBRID TRI-LAYER GAME

To illustrate the tri-layer game, Fig. [1](#page-3-0) presents a scenario with three prosumers and two aggregators. Let's consider a specific time instance: Prosumer 1 generates 150 kW, while Prosumers 2 and 3 consume 100 kW and 50 kW, respectively. In this situation, a dynamic emerges with prosumers possessing bargaining power, as Prosumer 1 can sell its power at a premium (e.g., 9 \$/kWh) compared to the

aggregators' buying price (i.e., 8 \$/kWh). Simultaneously, the consumers have the opportunity to purchase power at a more economical rate than the aggregator selling prices, a phenomenon well-documented in various papers [\[27\]. N](#page-13-21)ow, let's consider another scenario: Prosumer 1 generates 100 kW, while Prosumers 2 and 3 consume 250 kW and 300 kW. This introduces an additional layer to the dynamics. Beyond the bargaining power among prosumers, there arises competition between Prosumers 2 and 3. Both prosumers aim to trade with Aggregator 1, which proves more lucrative considering its capacity compared to Aggregator 2. This competition among prosumers adds complexity to the interplay. Moreover, there is another layer of competition in play. Aggregators vie for superiority based on their expected profit and the overall performance of both other aggregators and prosumers in the system. Thus, the tri-layer game unfolds as follows: the cooperation among prosumers forms the first layer, the competition among prosumers constitutes the second layer, and the competition among aggregators shapes the third layer. To develop the hybrid tri-layer game, the model of prosumer is presented in the next section. Meanwhile, the formulation of the tri-layer game is presented in Section [IV.](#page-5-0)

FIGURE 1. Illustrative Example of tri-layer game.

III. PROSUMER PROBLEM FORMULATION

Prosumers can be equipped with distributed generators (DGs), energy storages (ESs), and demands. All of these resources can have uncertainties; hence, Prosumers need to take their uncertainties by allocating sufficient flexibilities similar to [\[34\]. T](#page-13-28)o simplify the understanding, the formulations are classified into separate parts below:

A. DG CONSTRAINTS

The constraints of DGs can be considered for *i th* prosumer at *t th* time as follows:

$$
P_{i,t}^G - \underline{r}_{i,t}^G \ge \underline{P}_i^G, \quad \forall i, t
$$
\n(1a)

$$
P_{i,t}^G + \overline{r}_{i,t}^G \le \overline{P}_i^G, \quad \forall i, t
$$
 (1b)

$$
\overline{r}_{i,t}^G, \underline{r}_{i,t}^G \ge 0, \quad \forall i, t \tag{1c}
$$

$$
C_{i,t}^G = \alpha_i^G \times (P_{i,t}^G + \Delta p_{i,t}^G)^2
$$

+ $\beta_i^G \times (P_{i,t}^G + \Delta p_{i,t}^G) + \gamma_i^G$, $\forall i, t$ (1d)

 $(1a)$ - $(1c)$ model the minimum and maximum boundaries for DGs with respect to adjustments (i.e., $\overline{r}_{i,t}^G$ and $\underline{r}_{i,t}^G$) [\[34\]. T](#page-13-28)he cost of DGs is also modeled in [\(1d\)](#page-4-2) as a quadratic function considering adjustments [\[1\].](#page-12-0)

B. ES CONSTRAINTS

The constraints of ESs are presented in $(2a)-(2i)$ $(2a)-(2i)$. As ESs can work either in charging mode or in discharging mode, the convex model of ES is utilized here [\[34\], r](#page-13-28)ather than the MILP one.

$$
P_{i,t}^{ES} = P_{i,t}^{dch} - P_{i,t}^{ch}, \quad \forall i, t
$$
\n
$$
(2a)
$$

$$
P_{i,t}^{ch} + \underline{r}_{i,t}^{ES} \le \overline{P}_i^{ch}, \quad \forall i, t
$$
 (2b)

$$
\underline{r}_{i,t}^{ES} \le \overline{P}_i^{ch} + P_{i,t}^{dch}, \quad \forall i, t
$$
\n(2c)

$$
P_{i,t}^{dch} + \overline{r}_{i,t}^{ES} \le \overline{P}_i^{dch}, \quad \forall i, t
$$
 (2d)

$$
\overline{r}_{i,t}^{ES} \le \overline{P}_i^{dch} + P_{i,t}^{ch}, \quad \forall i, t
$$
\n(2e)

$$
E_{i,t} = E_{i,t-1} + (\eta_i^{ch} \times P_{i,t}^{ch} - P_{i,t}^{dch}/\eta^{dch}), \quad \forall i, t \qquad (2f)
$$

$$
\underline{E}_{i,t} \le E_{i,t} \le \overline{E}_{i,t}, \quad \forall i, t \tag{2g}
$$

$$
P_{i,t}^{ch}, P_{i,t}^{dch}, \overline{r}_{i,t}^{ES}, \underline{r}_{i,t}^{ES} \ge 0, \quad \forall i, t
$$
 (2h)

$$
C_{i,t}^{ES} = \delta_i^{ES} \times (P_{i,t}^{ES} + \Delta p_{i,t}^{ES})^2, \quad \forall i, t
$$
 (2i)

where $P_{i,t}^{ES}$ is the output of ESs that is equal to the subtraction of discharging power (i.e., $P_{i,t}^{dch}$) and charging power (i.e., $P_{i,t}^{ch}$ in [\(2a\)](#page-4-3) [\[1\]. In](#page-12-0) [\(2b\)](#page-4-5)[-\(2e\),](#page-4-6) the boundaries of $P_{i,t}^{ch}/P_{i,t}^{ch}$ are presented with respect to adjustment (i.e., $\overline{r}_{i,t}^{ES}$ and $\underline{r}_{i,t}^{ES}$) to compensate for uncertainties [\[34\]. T](#page-13-28)he energy capacity of ESs (i.e., $E_{i,t}$) and the boundaries are contemplated in $(2f)-(2g)$ $(2f)-(2g)$. The degradation cost of ESs is presented in $(2i)$ to avoid over-utilization [\[1\].](#page-12-0)

C. DEMAND CONSTRAINTS

The constraints of demand and its flexibilities are presented as follows [\[34\]:](#page-13-28)

$$
P_{i,t}^D - \overline{r}_{i,t}^D \ge \underline{P}_i^D, \quad \forall i, t
$$
 (3a)

$$
P_{i,t}^D + \underline{r}_{i,t}^D \le \overline{P}_i^D, \quad \forall i, t
$$
 (3b)

$$
\sum_{t} P_{i,t}^{D} = E_i^{D}, \quad \forall i
$$
 (3c)

$$
\overline{r}_{i,t}^D, \underline{r}_{i,t}^D \ge 0, \quad \forall i, t
$$
\n(3d)\n
$$
S^D = P \quad (2P \quad \rightarrow P \quad \hat{r}^D)^2 \quad \forall i, t
$$

$$
C_{i,t}^{D} = \sigma_{i,t}^{D} \times (P_{i,t}^{D} - \Delta p_{i,t}^{D} - \hat{P}_{i,t}^{D})^{2}, \quad \forall i \qquad (3e)
$$

The boundaries of demands are considered in $(3a)-(3b)$ $(3a)-(3b)$ with respect to demand adjustments (i.e., $\overline{r}_{i,t}^D$ and $\underline{r}_{i,t}^D$). [\(3c\)](#page-4-11) indicates that the required energy of prosumers should be satisfied. [\(3e\)](#page-4-12) also models the discomfort cost for prosumers, which is presented in [\[28\].](#page-13-22)

D. AFFINE ADJUSTMENT ALLOCATION CONSTRAINTS

To accurately specify adjustments, the rescheduling powers of DGs $(\Delta p_{i,t}^G)$, ESs $(\Delta p_{i,t}^{DG})$, and demands $(\Delta p_{i,t}^D)$ should compensate for uncertainties based on affine formulation [\[34\]. U](#page-13-28)ncertainties, in this paper, are defined as errors in the predicted output of renewables (i.e., $P_{i,t}^{RE}$ + $\omega_{i,t}$). $\omega_{i,t}$ is a random variable representing uncertainties. The rescheduling powers can be determined based on [\(4a\)](#page-4-13)[-\(4c\),](#page-4-14) wherein $\tau_{i,t}^G$, $\tau_{i,t}^{ES}$, and $\tau_{i,t}^D$ are participation factors [\[34\].](#page-13-28)

$$
\Delta p_{i,t}^G = \tau_{i,t}^G \times \omega_{i,t}, \quad \forall i, t \tag{4a}
$$

$$
\Delta p_{i,t}^{ES} = \tau_{i,t}^{ES} \times \omega_{i,t}, \quad \forall i, t
$$
 (4b)

$$
\Delta p_{i,t}^D = \tau_{i,t}^D \times \omega_{i,t}, \quad \forall i, t \tag{4c}
$$

$$
- \underline{r}_{i,t}^G \le \Delta p_{i,t}^G \le \overline{r}_{i,t}^G, \quad \forall i, t
$$
\n(4d)

$$
- \underline{r}_{i,t}^{ES} \le \Delta p_{i,t}^{ES} \le \overline{r}_{i,t}^{ES}, \quad \forall i, t
$$
\n
$$
- \overline{r}_{i,t}^{D} < \Delta p_{i,t}^{D} < r_{i,t}^{D} \quad \forall i, t
$$
\n
$$
(46)
$$

$$
-\overline{r}_{i,t}^D \le \Delta p_{i,t}^D \le r_{i,t}^D, \quad \forall i, t
$$
\n(4f)
\n
$$
\tau_{i,t}^G + \tau_{i,t}^{ES} + \tau_{i,t}^D = 1, \quad \forall i, t
$$
\n(4g)

$$
0 \le \tau_{i,t}^{G}, \tau_{i,t}^{ES}, \tau_{i,t}^{D} \le 1, \quad \forall i, t
$$
 (4h)

[\(4d\)-](#page-4-15)[\(4f\)](#page-4-16) declare that the rescheduling powers should be supported by allocated adjustments. $(4g)$ also states that the summation of participation factors must be equal to one [\[34\].](#page-13-28)

$$
C_{i,t}^{P} = \left[\vartheta_{i,t}^{cr} \times \left([\tau_{i,t}^{G} \times \omega_{i,t} - \overline{r}_{i,t}^{G}]^{+} + [\tau_{i,t}^{ES} \times \omega_{i,t} - \overline{r}_{i,t}^{ES}]^{+} + [\tau_{i,t}^{D} \times \omega_{i,t} - \underline{r}_{i,t}^{D}]^{+} \right) + \vartheta_{i,t}^{sh} \times \left([-\tau_{i,t}^{G} \times \omega_{i,t} - \underline{r}_{i,t}^{G}]^{+} + [-\tau_{i,t}^{ES} \times \omega_{i,t} - \underline{r}_{i,t}^{ES}]^{+} + [-\tau_{i,t}^{D} \times \omega_{i,t} - \overline{r}_{i,t}^{D}]^{+} \right) \right]
$$
\n(4i)

To facilitate modeling uncertainty, $(4a)$ - $(4c)$ can be replaced in $(4d)$ - $(4f)$, and then they can be transferred as a penalty cost presented in [\(4i\).](#page-4-18) This penalty cost expresses that the curtailment cost, in which $\vartheta_{i,t}^{cr}$ is the curtailment coefficient, should be considered when more rescheduling capacity is considered than the adjustment capacity (i.e., the predicted error is more than the real error). Reversely, load shedding cost, in which $\vartheta_{i,t}^{sh}$ is the load shedding coefficient, should be applied to $(4i)$ when the allocated rescheduling capacity is less than the adjustment capacity similar to [\[34\]](#page-13-28) (i.e., the predicted error is less than the real error).

E. INTERNAL SETTINGS

The internal settings of prosumers in the connection point to the network are presented here.

$$
P_{i,t}^T = P_{i,t}^{RE} + P_{i,t}^G + P_{i,t}^{ES} - P_{i,t}^D, \quad \forall i, t \tag{5a}
$$

$$
-\overline{Q}_{i,t}^T \le Q_{i,t}^T \le \overline{Q}_{i,t}^T, \quad \forall i, t
$$
\n
$$
-\overline{T}^2 \qquad \qquad T^2 \qquad T^2 \qquad (5b)
$$

$$
\overline{Q}_{i,t}^{T^2} \le S_{i,t}^{T^2} - P_{i,t}^{T^2}, \ \forall i, t
$$
 (5c)

$$
\overline{Q}_{i,t}^T \le P_{i,t}^T \times \tan(\varphi_i), \ \forall i, t \tag{5d}
$$

$$
\overline{Q}_{i,t}^T \ge 0, \quad \forall i, t \tag{5e}
$$

[\(5a\)](#page-5-1) declares that the injected/absorbed active power to/from the network is the summation of all resources, wherein $P_{i,t}^{RES}$ is the predicted output of renewables. In this paper, it is assumed that prosumers are connected to the network through an inverter. The inverter can provide reactive power based on its capacity. $(5a)$ - $(5e)$ model this capacity $[28]$.

F. COSTS MODELING

Buying and selling costs and the total cost are presented here.

$$
C_{i,t,k}^{B/S} = \left[[P_{i,t}^D - P_{i,t}^G - P_{i,t}^{RE} - P_{i,t}^{ES} + \sum_j P_{ij,t}^{P2P}]^+ \times \lambda_{t,k}^S - [P_{i,t}^G + P_{i,t}^{RE} + P_{i,t}^{ES} - P_{i,t}^D - \sum_j P_{ij,t}^{P2P}]^+ \times \lambda_{t,k}^B \right] (6a)
$$

 $C^{B/S}_{i\, t\, k}$ $i_{i,t,k}^{B/3}$ is the buying and selling costs including two terms in $(6a)$. The first term states that prosumers should buy their demanded power based on selling prices of aggregators when $P_{i,t}^D - P_{i,t}^G - P_{i,t}^{RE} - P_{i,t}^{ES} + \sum_j P_{ij,t}^{P2P}$ is positive. The positive value means that prosumers need the power for which they need to purchase. The second term declares that in the case that this value is negative (i.e., prosumers have extra power), they should sell their extra power based on the buying prices of aggregators. It is worth mentioning that $[\dagger]^{+}$ means $max{0, \dagger}$, and a convex model for it is presented in [\[39\].](#page-13-33)

$$
C_{i,t,k}^T = C_{i,t}^G + C_{i,t}^{ES} + C_i^D + C_{i,t,k}^{B/S} + \sup_{\mathbb{P} \in \hat{\mathbb{P}}_{\mathbb{V}}} \mathbb{E}_{\mathbb{P}}[C_{i,t}^P(r, \omega)]
$$
\n(6b)

The total cost of prosumers includes DG generation costs, ES degradation costs, discomfort costs, buying/selling costs, and the expected penalty cost on the worst distribution in the Wasserstein ambiguity set.

IV. THE PROPOSED HYBRID TRI-LAYER TE FRAMEWORK

The proposed TE framework consists of three layers, namely, NBG for modeling cooperation among prosumers, evolutionary game (EG) for modeling competition among prosumers, and non-cooperative game (NCG) for modeling competition among aggregators and between aggregators and the created cooperation.

The NBG is utilized in the first layer to encourage prosumers to P2P trading. To decrease the computational burden of NBG, it can be exactly decomposed into two problems: power scheduling problem and payment problem [\[28\]. T](#page-13-22)he power scheduling problem is formulated in [\(7\)](#page-5-4) to minimize costs subject to the balance of P2P trading and coupling injected/absorbed powers on the prosumer and network sides [\[28\].](#page-13-22) $P_{n,t}^{Net}$ and $Q_{n,t}^{Net}$ are active and reactive power on the network side and E^{Net} is the conic DistFlow set, presented in [\[40\]. T](#page-13-34)he optimization in [\(7\)](#page-5-4) is over Θ = ${P, Q, Q, \overline{r}, \underline{r}, \tau, \omega}.$

Min
$$
C_i^{(\star)} = \sum_{t,k} C_{i,t,k}^T
$$

\ns.t. (1) – (3), (4*d*) – (4*i*), (5), (6)
\n $P_{ij,t}^{P2P} + P_{ji,t}^{P2P} = 0 : (\mu_{ij,t}^{P2P}), \forall i, j, t$
\n $P_{i,t}^T = \sum_{n \in \Omega} P_{n,t}^{Net} : (\mu_{i,t}^P), \forall i, t$
\n $Q_{i,t}^T = \sum_{n \in \Omega} Q_{n,t}^{Net} : (\mu_{i,t}^Q), \forall i, t$
\n $P_{n,t}^{Net}, Q_{n,t}^{Net} \in \Xi^{Net}$ (7)

The payment problem is also can be formulated based on [\(8\),](#page-5-7) $\widetilde{C}_i^{(*)}|_{P^{P2P}=0}$ is the value of the objective function [\(7\)](#page-5-4) without P2P trading. $\hat{C}_i^{(\star)}$ $i^{(*)}$ is also the value of the objective function of [\(7\)](#page-5-4) with P2P trading. Φ_{ij} means the P2P trading fee between prosumers *i* and *j*. The constraint of this problem is the balance of P2P trading fees among prosumers.

$$
\begin{aligned}\n\text{Max}_{\Phi} \quad & \prod_{i} (\widetilde{C}_{i}^{(\star)}|_{P^{p_2 p} = 0} - \widehat{C}_{i}^{(\star)} - \sum_{j} \Phi_{ij}) \\
\text{s.t. } \Phi_{ij} + \Phi_{ji} = 0\n\end{aligned} \tag{8}
$$

B. SECOND LAYER: EVOLUTIONARY GAME (EG)

The EG is utilized here to model the competition among prosumers to buy/sell their energy from/to the most affordable aggregator. EG is utilized since prosumers behave similarly to each other in their buying/selling to minimize their costs. Hence, their behavior is like the behavior of a population. The EG is developed to model the behavior of a population with various strategies [\[41\]. T](#page-13-35)he basics of EG is presented in [\[42\].](#page-13-36) In EG, three factors are critical: the probability of trading with k^{th} aggregator (i.e., $z_{t,k}^{h}$), the average cost (i.e., $\overline{C_{t}^{T}}$), and the replicator. $\sum_{k} z_{t,k}^{h} = 1$ is a crucial condition in EG. In EG, The first step is to solve the problem [\(7\)](#page-5-4) and then calculate the selling power (i.e., $P_{t,k}^S$) and the buying power (i.e., $P_{t,k}^B$) of each aggregator based on $(9a)-(9b)$ $(9a)-(9b)$, wherein $\lbrack \star \rbrack$ means the determined optimal value of variables by solving [\(7\).](#page-5-4)

$$
P_{t,k}^{S} = z_{t,k}^{h} \times \sum_{i} [\check{P}_{i,t}^{D} - \check{P}_{i,t}^{G} - \check{P}_{i,t}^{RE} - \check{P}_{i,t}^{ES} + \sum_{j} \check{P}_{ij,t}^{P2P}]^{+}
$$
\n(9a)

$$
P_{t,k}^{B} = z_{t,k}^{h} \times \sum_{i} [\check{P}_{i,t}^{G} + \check{P}_{i,t}^{RE} + \check{P}_{i,t}^{ES} - \check{P}_{i,t}^{D} - \sum_{j} \check{P}_{ij,t}^{P2P}]^{+}
$$
\n(9b)

Meanwhile, the capacity factors should be calculated as $\iota_{t,k}^S =$ $\bar{P}_{t,k}^S/P_{t,k}^S$ and $\iota_{t,k}^B = \bar{P}_{t,k}^B/P_{t,k}^B$ in which $\bar{P}_{t,k}^S$ and $\bar{P}_{t,k}^B$ are the buying and selling capacity of aggregator *k*. In the case that these factors are greater or equal to one for *k th* aggregator, it can satisfy buying and selling requests. Otherwise, The trading capacity of prosumers with this aggregator should be fixed on its capacity multiplied by its capacity factor, followed by recalculating the total cost by prosumers. After calculating the total cost by prosumers, the average cost can be calculated using [\(10\).](#page-6-1)

$$
\bar{C}_t^T = \sum_k z_{t,k}^h \times \sum_i \check{C}_{i,t,k}^T
$$
 (10)

The replicator of EG by which the dynamic of EG can be managed is modeled in (11) . This replicator has a stable dynamic, resulting in convergence to the Evolutionary Nash Equilibrium (ENE) for which proof is presented in Appendix [A.](#page-0-0) It is worth noting that *h* is the iteration index.

$$
\frac{\partial z_{t,k}^h}{\partial h} = z_{t,k}^h \times (\sum_i \check{C}_{i,t,k}^T - \bar{C}_t^T)
$$
(11)

[\(11\)](#page-6-2) can be simplified to [\(12\),](#page-6-3) wherein ℓ_t is the modification factor. The iterative process is stopped when the condition in [\(13\)](#page-6-4) is satisfied.

$$
z_{t,k}^{h+1} = z_{t,k}^h + \ell_t \times z_{t,k}^h \times (\sum_i \check{C}_{i,t,k}^T - \bar{C}_t^T) \qquad (12)
$$

$$
|\sum_{i} \check{C}_{i,t,k}^T - \bar{C}_t^T| < \varepsilon \tag{13}
$$

The proposed explanations are summarized in Algorithm [1.](#page-6-5)

C. THIRD LAYER: NON-COOPERATIVE GAME (NCG)

The utility function of aggregators can be modeled as $(14a)-(14b)$ $(14a)-(14b)$, which includes two items. The first term is the cost and revenue of buying and selling power to prosumers $(i.e., -\lambda_{t,k}^B \times \bar{P}_{t,k}^B \text{ and } \lambda_{t,k}^S \times \bar{P}_{t,k}^S$). The second term is the risk of selling the purchased power and buying the sold power in the upper-level market. This risk is modeled as a quadratic cost function meaning that the risk of selling $\bar{P}_{t,k}^B$ /buying $\bar{P}_{t,k}^S$ to/from the wholesale market to cover prosumers using historical data. $a_{t,k}^B$ and $b_{t,k}^B$ are risk coefficients of selling $\bar{P}_{t,k}^B$ to the wholesale market, and $a_{t,k}^S$ and $b_{t,k}^S$ are risk coefficients of buying $\bar{P}_{t,k}^S$ from the market. The objective of aggregators is to minimize costs and maximize revenue.

$$
U_{t,k}^{B} = -\lambda_{t,k}^{B} \times \bar{P}_{t,k}^{B} - a_{t,k}^{B} \times (\bar{P}_{t,k}^{B})^{2} + b_{t,k}^{B} \times \bar{P}_{t,k}^{B} \quad (14a)
$$

$$
U_{t,k}^{S} = \lambda_{t,k}^{S} \times \bar{P}_{t,k}^{S} - a_{t,k}^{S} \times (\bar{P}_{t,k}^{S})^{2} - b_{t,k}^{S} \times \bar{P}_{t,k}^{S} \quad (14b)
$$

Based on the objective function of aggregators, the update method of prices and capacities can be extracted using [\(15a\)-](#page-6-8)[\(15d\).](#page-6-9)

$$
(\lambda_{t,k}^B)^{new} - (\lambda_{t,k}^B)^{old} = \ell_k^B \times (\iota_{t,k}^B - 1) \tag{15a}
$$

Algorithm 1 Pseudocode for the Proposed Evolutionary Game

1: $k = 1$ and $h = 1$ 2: **while** $k < K$ **do** 3: **while** [\(13\)](#page-6-4) is not satisfied **do** 4: Solve Problem $(7) \rightarrow$ $(7) \rightarrow$ (**Algorithm 3**) 5: Calculate $P_{t,k}^S$ and $P_{t,k}^B$ based on [\(9a\)-](#page-5-8)[\(9b\)](#page-6-0) 6: Calculate $t_{t,k}^{S} = \bar{P}_{t,k}^{S} / P_{t,k}^{S}$ and $t_{t,k}^{B} = \bar{P}_{t,k}^{B} / P_{t,k}^{B}$ 7: **if** $\iota_{t,k}^S \ge 1$ **then** 8: Keep $C_{i,t,k}^T$ 9: **else** 10: Multiply $\iota_{t,k}^S$ in $P_{t,k}^S$ and recalculate $C_{i,t,k}^T$ 11: **end if** 12: **if** $\iota_{t,k}^B \ge 1$ **then** 13: **Keep** $C_{i,t,k}^T$ 14: **else** 15: Multiply $\iota_{t,k}^B$ in $P_{t,k}^B$ and recalculate $C_{i,t,k}^T$ 16: **end if** 17: Calculate $\sum \check{C}_{i,t,k}^T$ and \bar{C}_t^T based on [\(10\)](#page-6-1) *i* 18: Update $z_{t,k}^h$ based on [\(12\)](#page-6-3) 19: $k \leftarrow k + 1 \text{ and } h \leftarrow h + 1$ 20: **end while** 21: **end while**

$$
(\lambda_{t,k}^S)^{new} - (\lambda_{t,k}^S)^{old} = \ell_k^S \times (\iota_{t,k}^S - 1)
$$
\n
$$
-(\lambda_{t,k}^B)^{new} + \kappa_k^B
$$
\n(15b)

$$
\bar{P}_{t,k}^{B} = \frac{-(\lambda_{t,k}^{B})^{new} + b_{t,k}^{B}}{2 \times a_{t,k}^{B}}
$$
 (15c)

$$
\bar{P}_{t,k}^{S} = \frac{-(\lambda_{t,k}^{S})^{new} + b_{t,k}^{S}}{2 \times a_{t,k}^{S}}
$$
 (15d)

In [\(15a\)-](#page-6-8)[\(15b\),](#page-6-10) new prices are determined based on the request of prosumers, which is a function of their cooperation. In [\(15c\)](#page-6-11)[-\(15d\),](#page-6-9) new capacities are calculated based on prices and risk factors. The stopping criteria for this iterative process is the equality of the capacities to one, which means that the ordered capacity from the market and the traded capacity with prosumers are equal for each aggregator.

$$
|t_{t,k}^B - 1| < \varepsilon, \quad |t_{t,k}^S - 1| < \varepsilon \tag{16}
$$

The designed NCG has a non-empty core and it converges to the Nash equilibrium for which proof is presented in Appendix [B.](#page-0-0) The algorithm of the proposed NCG is presented in Algorithm [2.](#page-7-1)

V. SCALE-INDEPENDENT DRO REFORMULATION BASED ON WASSERSTEIN AMBIGUITY SET

Wasserstein ambiguity set can be defined as [\(17\)](#page-6-12) to find the infimum distance between historical samples $(\hat{\omega}_v)$ on an empirical distribution (\mathbb{P}_{V}) and the random samples (ω) on a worst-case distribution (\mathbb{P}). \mathbb{P}_{V} also satisfies $\sum_{v} \hat{\omega}_{v} \times \Upsilon_{v}$, wherein ϒ*^v* is Diac function.

$$
W(\mathbb{P}, \hat{\mathbb{P}}_{\mathbb{V}}) = \inf_{\psi} \left\{ \int_{\psi} d(\omega, \hat{\omega}_{\mathbb{V}}) \Psi(d\omega, d\hat{\omega}_{\mathbb{V}}) \right\}
$$
(17)

- 1: Publish initial $\lambda_{t,k}^B$, $\lambda_{t,k}^S$, $\bar{P}_{t,k}^B$, $\bar{P}_{t,k}^S$ by Aggregators
- 2: **while** [\(16\)](#page-6-13) is not satisfied **do**
- 3: Run **Algorithm [1](#page-6-5)**
-
- 4: Extract $\iota_{t,k}^{S}$ and $\iota_{t,k}^{B}$
5: Update $\lambda_{t,k}^{B}$ and $\lambda_{t,k}^{S}$ based on [\(15a\)](#page-6-8) and [\(15b\)](#page-6-10)
- 6: Update $\overline{P}_{t,k}^B$ and $\overline{P}_{t,k}^S$ based on [\(15c\)](#page-6-11) and [\(15d\)](#page-6-9)
- 7: Publish new $\lambda_{t,k}^B$, $\lambda_{t,k}^S$, $\bar{P}_{t,k}^B$, $\bar{P}_{t,k}^S$ by Aggregators
- 8: **end while**
- 9: Solve problem (8) by the operator

The ambiguity set can be defined in [\(18\)](#page-7-2) based on the radius of the Wasserstein ball (ϵ) .

$$
\varsigma := \{ \mathbb{P} \in \mathcal{R} | W(\mathbb{P}, \hat{\mathbb{P}}_{\mathbb{V}}) \le \epsilon \}
$$
 (18)

 ϵ can be determined based on [\(19\)](#page-7-3) if there is a set on ω as $||\omega||_{\infty} \leq \pi$. $\hat{\pi}$ is also the average of historical data. χ is known as the confidence factor [\[43\]. T](#page-13-37)o determine *D*, a useful discussion can be found in [\[43\].](#page-13-37)

$$
\epsilon = \min\{\pi - \hat{\pi}, \pi + \hat{\pi}, D \times \sqrt{\frac{2}{V} \ln \frac{1}{1 - \chi}}\}
$$
(19)

Finally, DRO reformulation using dual theory can be developed in [\(20\)](#page-7-4) [\[43\].](#page-13-37)

$$
\sup_{\mathbb{P}\in\hat{\mathbb{P}}_{\mathbb{V}}} \mathbb{E}_{\mathbb{P}}[C_{i,t}^{P}(r,\omega)] = \inf_{\kappa \ge 0} {\{\kappa \times \epsilon \}}
$$

+
$$
\frac{1}{V} \sum_{\nu=1}^{V} \sup_{\omega} (C_{i,t}^{P}(r,\omega) - \kappa \times ||\omega - \hat{\omega}_{\nu}||) {\} } \tag{20}
$$

The reformulation can be simplified by defining an auxiliary variable (ϱ_v) in [\(21\).](#page-7-5)

$$
\inf_{\kappa \ge 0} \kappa \times \epsilon + \frac{1}{V} \sum_{\nu=1}^{V} \varrho_{\nu}
$$
\n
$$
\text{s.t. } \sup_{\omega} (C_{i,t}^{P}(r, \omega) - \kappa \times ||\omega - \hat{\omega}_{\nu}||) \le \varrho_{\nu} \qquad (21)
$$

[\(21\)](#page-7-5) is further simplified in [\[34\]](#page-13-28) considering this point that the extreme points of [\(21\)](#page-7-5) are boundaries of ω and $\hat{\omega}_v$. Hence, if we assume $\omega \leq \omega \leq \overline{\omega}$, [\(21\)](#page-7-5) can be simplified as follows:

$$
\inf_{\kappa \ge 0} \quad \kappa \times \epsilon + \frac{1}{V} \sum_{\nu=1}^{V} \varrho_{\nu}
$$
\n
$$
\text{s.t. } (C_{i,t}^{P}(r, \overline{\omega}) - \kappa \times ||\overline{\omega} - \hat{\omega}_{\nu}||) \le \varrho_{\nu}; \ \forall \nu
$$
\n
$$
(C_{i,t}^{P}(r, \underline{\omega}) - \kappa \times ||\underline{\omega} - \hat{\omega}_{\nu}||) \le \varrho_{\nu}; \ \forall \nu
$$
\n
$$
C_{i,t}^{P}(r, \hat{\omega}_{\nu}) \le \varrho_{\nu}; \ \forall \nu
$$
\n(22)

It is observable that (22) is intractable when a significant amount of historical data is available. However, $C_{i,t}^P(r, \omega)$ is a piece-wise function (*q* is the index of pieces) for which the following formulation can be proposed:

$$
\sum_{i,t} C_{i,t}^P(r,\omega) = \sum_{i,t} \sum_q (y_q(\vartheta_{i,t}, \tau_{i,t}) \times \omega + o_q(r_{i,t})) \quad (23)
$$

Algorithm 3 Pseudocode for Adaptive Proximal ADMM

- 1: Keep decisions at the previous time as initialization
- 2: **while** $\Gamma_{pri} > \varepsilon$ and $\Gamma_{dul} > \varepsilon$ **do**
- 3: Update *X* using [\(26\)](#page-7-7) by prosumers
- 4: Update ρ regarding P2P trading using [\(28\)](#page-8-0) by prosumers
- 5: Update μ regarding P2P trading using [\(29\)](#page-8-1) by prosumers
- 6: Update *Y* using [\(27\)](#page-7-8) by the operator
- 7: Update ρ regarding network constraints using [\(28\)](#page-8-0) by the operator
- 8: Update μ regarding network constraints using [\(29\)](#page-8-1) by the operator
- 9: **end while**

Based on (23) , (22) can be exactly reformulated to (24) for which proof is presented in Appendix [C.](#page-0-0)

$$
\inf_{\kappa,\zeta,\xi\geq 0} {\{\kappa \times \epsilon + \xi \times \overline{\omega} - \zeta \times \underline{\omega} \atop + \frac{1}{V} \sum_{\nu=1}^{V} (\sum_{i,t} C_{i,t}^P(r, \hat{\omega}_{\nu})) + (\xi - \zeta) \times \hat{\omega}_{\nu} \}}
$$
\ns.t. $||\xi - \zeta - y_q||_* \leq \kappa$; $\forall q$ (24)

[\(24\)](#page-7-10) is a scale-independent DRO since the index of samples is transferred from constraints to the objective.

VI. DISTRIBUTED-DECENTRALIZED ALGORITHM BASED ON ADAPTIVE PROXIMAL ADMM

In this paper, the operator oversees network security constraints in a distributed fashion and prosumers can handle their P2P trading using a decentralized manner. To keep the generality, the compact format is utilized here. Problem [\(7\)](#page-5-4) is written in a compact format in (25) , wherein Λ is the feasibility set of [\(7\).](#page-5-4)

$$
\lim_{\Theta \in \Lambda} C^*(X)
$$

s.t. $X = Y$: (μ) (25)

Using consensus ADMM the above problem can be reformulated to the following problems in which proximal terms $(\frac{1}{2\rho^{r}}||X - \check{X}^{r}||_2^2$ and $\frac{1}{2\rho^{r}}||Y - \check{Y}^{r}||_2^2$ are considered to accelerate the convergence $[44]$. To update X, which is a set of variables under prosumers' control, the following problem would be solved:

Min
$$
C^*(X) + \mu^r \times X + \frac{\rho^r}{2} ||X - \check{Y}^r||_2^2 + \frac{1}{2\rho^r} ||X - \check{X}^r||_2^2
$$
 (26)

To update Y, which is a set of variables under the operator's control, the following problem should be solved in which E^{Net} is the set of network constraints:

$$
\underset{\Theta \in \Xi^{\text{Net}}} {\text{Min}} \ - \mu^r \times Y + \frac{\rho^r}{2} ||\check{X}^{r+1} - Y||_2^2 + \frac{1}{2\rho^r} ||Y - \check{Y}^r||_2^2 \tag{27}
$$

FIGURE 2. Average convergence results of the tri-layer TE framework for (a) probabilities ($z^h_{t,k}$), (b) prices ($\lambda^B_{t,k},\lambda^S_{t,k}$), (c) capacities ($\bar{P}^B_{t,k},\bar{P}^S_{t,k}$), (d) average cost $(\bar{C}_{t}^{T}),$ (e) selling ratio $(\iota_{t,k}^{S}),$ and (f) buying ratio $(\iota_{t,k}^{B}).$

To decrease the sensitivity of initialization and improve the convergence speed, the penalty factor (ρ) has a dynamic based on dual $(\Gamma_{dul} = |\mu^{r+1} - \mu^r|)$ and primal $(\Gamma_{pri} =$ $|\check{X}^{r+1}-\check{X}^{r}|$) residuals. This dynamic is presented below [\[10\]:](#page-13-4)

$$
\rho^{r+1} = \begin{cases}\n\rho^r \times e & \Gamma_{pri} \times l \ge \Gamma_{dul} \\
\rho^r / e & \Gamma_{pri}/l < \Gamma_{dul} \\
\rho^r & \text{otherwise}\n\end{cases} \tag{28}
$$

Finally, the control signals can be updated as follows:

$$
\mu^{r+1} = \mu^r + \rho^{r+1} \times (\check{X}^{r+1} - \check{Y}^{r+1})
$$
 (29)

The stopping criteria for the proposed adaptive proximal ADMM is reaching small values for dual and optimal residuals. Algorithm [3](#page-7-0) illustrates the iterative process and the extensive format of these formulations is presented in [\[45\].](#page-13-39)

TABLE 1. Input data for aggregators [\[27\].](#page-13-21)

VII. PERFORMANCE EVALUATION AND DISCUSSIONS

In this section, a modified IEEE 15-bus system is utilized to confirm the performance of the proposed TE framework and the developed DRO for which data is elaborated in [\[45\]. T](#page-13-39)his approach also considers two aggregators, with corresponding data presented in Table [1.](#page-8-2) Also, a modified IEEE 123-bus system is utilized to confirm the scalability of the proposed framework for which data can be found in [\[45\].](#page-13-39)

TABLE 2. Comparison between the extracted results of the proposed tri-layer game-based TE and the literature.

	Case I $[9]$, $[10]$	Case II [11], [12]	Case III $\left[31\right]$	Case IV $\left[28\right]$	$Pros.*$
P ₂ P Buying Price (S/kWh)				10.126	11.358
P ₂ P Selling Price (S/kWh)				11.545	13.108
Buying Price (S/kWh)	10	10.347	11.054	10	11.232
Selling Price (S/kWh)	12	13.821	13.564	12	13.407
Total Cost $(x10^3)$	81.105	82.409	79.362	76.352	74.234

*: Proposed

A. CONVERGENCE OF THE PROPOSED TE FRAMEWORK

The convergence procedure of the proposed tri-layer game is demonstrated in Fig. [2.](#page-8-3) The convergence procedure includes the convergences of probabilities (Fig. $2(a)$), prices $(Fig.2(b))$, capacities $(Fig. 2(c))$, the average cost $(Fig. 2(d))$, and capacity to the traded prosumption ratios (Fig. [2 \(e\)](#page-8-3) and Fig. [2 \(f\)\)](#page-8-3). It was assumed that the probability of trading with two aggregators is equal to 0.5 in the first iteration. However, the probability of trading with the first aggregator increased after convergence since trading with this aggregator is more affordable for prosumers. Moreover, it was observed that the buying and selling prices of aggregators converged to the same value, representing the competition between aggregators and prosumers. In fact, it was observed that aggregators adjusted their prices and the ordered capacities from the upstream market to maximize their profit. The proof of this statement is observable in the convergence

of the ordered capacities to the traded capacities ratios, which illustrates that aggregators try to create a balance between the ordered capacities and the traded capacities. This framework, hence, is significantly beneficial for aggregators, while they should offer a constant price based on the proposed framework in [\[28\]. T](#page-13-22)he calculations demonstrated that the profits of aggregators in the proposed model are equal to 14,501.48 \$ 12,191.41 \$, while it is equal to 10,304.53 \$ and 9,836.24 \$ using [\[28\]](#page-13-22) for the first and second aggregators, respectively.

FIGURE 3. Total cost and P2P clearing price of prosumers with respect to the price change of aggregators.

B. COMPARISON OF THE PROPOSED TE FRAMEWORK

In this subsection, the proposed tri-layer hybrid game is compared to the four existing TE frameworks, namely, optimizing the total preferences [\[9\],](#page-13-3) [\[10\]](#page-13-4) (*Case I*), noncooperative Stackalberg game [\[11\],](#page-13-5) [\[12\]](#page-13-6) (*Case II*), the suggested hybrid bi-level [\[31\]](#page-13-25) (*Case III*), and the suggested cooperative game [\[28\]](#page-13-22) (*Case IV*). Three factors are compared here: the average P2P clearing price, the average offered prices of aggregators, and the total cost of prosumers. It was observed that the total cost of prosumers using the suggested method in *Case II* is the highest. The reason is ignoring the bargaining power of prosumers and increasing the prices of aggregators considering the high demand for trading with them. Also, the price in *Case I* is not affordable for prosumers since the bargaining power is ignored, while the prices of aggregators are assumed to be constant. It was observed that the total cost decreased in *Case III* due to considering the impact of bargaining power on the aggregators' prices, while the P2P trading is neglected. Using the cooperative game in *Case IV*, the total cost is reduced significantly due to considering P2P trading and providing a cooperative scheme to determine its price by which P2P clearing prices are more attractive for prosumers than the prices of

TABLE 3. A comparison between the proposed scale-independent DRO and the exiting methods.

*: Proposed

aggregators, resulting in a substantial cost reduction. This even improved in the proposed TE framework by considering both cooperative P2P trading and Non-cooperative price adjustment of aggregators. The summary of these results is presented in Table [2.](#page-8-4) The problem with the suggested cooperative TE in [\[28\]](#page-13-22) is not limited to the higher total cost. It, indeed, can contribute to the power market for aggregators. Fig. [3](#page-9-0) illustrates this statement, where changing the prices of aggregators significantly impacts the P2P clearing price and the total cost of prosumers. Nevertheless, aggregators cannot create market power in the proposed TE since they have to adjust their prices to maximize their profit.

C. COMPARISON OF THE DEVELOPED SCALE-INDEPENDENT DRO

In this subsection, the performance of the proposed scale-independent DRO is evaluated. To do so, the proposed scale-independent DRO is compared to the suggested stochastic programming (SP) in $[1]$, the suggested robust optimization (RO) in $[33]$, and the suggested DRO in $[34]$. Sample extraction in this subsection is explained in [\[45\]](#page-13-39) for these methods based on Weibull distribution. According to the achieved results, the calculated expected cost in SP is less than its value in other methods. However, the calculated cost based on SP is hazardous to be utilized by prosumers. To illustrate this, the average and expected penalty costs using SP are equal to 12.403 $\frac{6}{5} \times 10^3$ and 36.931 $\frac{6}{5} \times 10^3$. This significant difference between these values declares that prosumers are exposed to high penalty costs if they cannot estimate an exact PDF from their uncertainties. On the other hand, RO provides a robust solution with the highest cost. This high cost is expected since the worst-case scenario is considered in RO. Also, the solution procedure of RO is time-independent, as the boundary of uncertainties is critical in RO, not the number of samples. However, this time is substantial (36.12 *sec*) compared to the running time of other algorithms due to the iterative solution. In contrast to RO and SP, DRO provides a point between them considering the nature of samples and confidence factor [\[34\]. H](#page-13-28)ence, the calculated total cost by DRO is higher than SP and less than RO. More specifically, the calculated total cost using DRO is 8.9 % more than the calculated total cost by SP, and it is 14.4% less than the calculated total cost by RO.

FIGURE 4. The total cost with respect to increasing samples.

FIGURE 5. A runtime comparison between available DRO and the proposed scale-independent DRO.

Nonetheless, the performance of DRO significantly depends on the number of samples. It was observed that increasing the number of samples helps the Westrerien ambiguity set to improve the accuracy of estimation of the worstcase PDF. For instance, increasing samples from 500 to 5,000 (ten times) decreases the total cost by 4.34%. However, the suggested DRO in [\[34\]](#page-13-28) is not capable of considering a large number of samples. More specifically, there is no output of the suggested DRO in $[34]$ when the number of samples is higher than 5,000 due to the memory error. Nevertheless, the proposed DRO does not have this flaw due to the independence of the number of samples. The simulations illustrated that improving the number of samples from 500 to 100,000 decreases the total cost by 7.05%. Please note that there is a saturation in considering a high number of samples. In other words, increasing the number of samples cannot tangibly improve the accuracy after a specific number. For example, the difference between the total costs of 100,000 and 200,000 consideration is only 0.11%. Hence, the performance of increasing samples should be tasted by prosumers which is possible by the proposed scaleindependent DRO. The results and comparisons are presented in Table [3](#page-9-1) and the saturation is demonstrated in Fig. [4.](#page-10-0) Moreover, the runtimes of the proposed scale-independent DRO and the suggested DRO in [\[34\]](#page-13-28) are depicted in Fig. [5.](#page-10-1) Accordingly, it is observable that increasing the number of samples does not have a significant impact on the proposed scale-independent DRO since the dimensions of samples are transferred to the objective. On the other hand, the runtime of the suggested DRO in [\[34\]](#page-13-28) highly changes when the number of samples increases.

D. ANALYSIS OF THE PROPOSED ADAPTIVE PROXIMAL ADMM

FIGURE 6. (a) The operator's control signals using different algorithms for prosumer 112 [\[45\], \(](#page-13-39)b) The P2P trading control signals using different algorithms for transaction between prosumers 112 and 114 [\[45\].](#page-13-39)

In this subsection, IEEE 123-bus is utilized to test the proposed adaptive proximal ADMM. To evaluate the performance of the proposed adaptive ADMM, it is compared to regular ADMM [\[1\], fa](#page-12-0)st ADMM (F-ADMM) [\[38\], a](#page-13-32)nd adaptive ADMM (A-ADMM) [\[10\]. O](#page-13-4)ne of the defects of the regular ADMM is its slow convergence rate in large systems as is visible in Table [4.](#page-11-0) However, this challenge is appropriately tackled by F-ADMM. Nonetheless, there is still a serious problem regarding both regular ADMM and F-ADMM which is the sensitivity of their convergence and accuracy to the value of the penalty factor as is observable in Table [4.](#page-11-0) Hence, it is challenging for the operator to optimize the value of the penalty factor in practice. Therefore, A-ADMM is proposed to tackle the sensitivity challenge, while the convergence issue remains a challenge. On the other hand, the proposed adaptive proximal ADMM can improve both sensitivity and convergence rate. It is faster than F-ADMM since there are two-step updates of control signals in F-ADMM to accelerate the algorithm, which contributes to a higher computational burden in large systems. However, the proposed adaptive proximal ADMM only has one step update and its proximal term accelerates the algorithm without posing a computational burden. Besides, its accuracy is similar to A-ADMM since it is equipped with the adaptive penalty factor to remove the sensitivity on initialization and improve convergence. An example of the operator's control

signals for satisfying system constraints and P2P trading control signals for P2P transactions is presented in Fig. [6.](#page-10-2)

TABLE 4. A comparison among the proposed adaptive proximal ADMM and the existing algorithms.

Algms.	ρ	Runtime		Total Cost	Accuracy
		(sec)	$($ Iter# $)$	$(\$ \times 10^3)$	
ADMM	0.01	38.43	302	108.346	96%
	0.02	37.84	297	107.354	97%
	0.05	35.26	282	110.605	94%
F-ADMM	0.01	18.16	112	109.624	95%
	0.02	17.52	108	108.320	96%
	0.05	1734	106	109.427	95%
A-ADMM	0.01	19.12	194	106.253	98%
	0.02	19.14	190	105.195	99%
	0.05	19.44	186	106.458	98%
Proposed	0.01	15.21	89	106.354	98%
	0.02	15.16	84	105.416	99%
	0.05	15.13	82	107.475	97%

VIII. CONCLUSION

This paper proposed a tri-layer hybrid game-based transactive energy (TE) framework, including a distributed-decentralized optimization for satisfying network constraints and energy trading in a peer-to-peer (P2P) fashion along with a scale-independent distributional robust optimization (DRO) for uncertainty management. It was evaluated that the proposed tri-layer hybrid game-based TE framework is more effective than other existing frameworks since the cooperation among prosumers for P2P energy trading is considered using Nash Bargaining Game (*first layer*), the competition among prosumers to trade energy with the most affordable aggregator is considered using Evolutionary Game (*second layer*), and the competitions among aggregators and between aggregators and prosumers are contemplated using a non-cooperative game (*third layer*). This demonstrated that utilizing the proposed hybrid tri-layer game-based TE not only can improve the profit of aggregators and reduce the costs of prosumers but also can avoid the market power of aggregators for controlling P2P transactions in local markets. Besides, an adaptive proximal alternative direction method of multipliers (ADMM) was proposed, which has a higher convergence rate and accuracy compared to available algorithms. A scale-independent DRO is also developed for this problem to remove the limitations of utilizing all potential data. The outcomes of this research are highlighted below:

- 1) The proposed tri-layer TE framework reduced the total costs of prosumers by 11% and 2.85% compared to the total cost in non-cooperative TE and cooperative TE, respectively.
- 2) The proposed adaptive proximal ADMM is 52% faster than the regular ADMM, while its accuracy is higher and its sensitivity to the penalty factor is lower.
- 3) Considering a huge number of historical data helps to decrease the total cost by increasing the accuracy of estimating worst-case distribution, which is possible by the developed scale-independent DRO.

APPENDIX A

CONVERGENCE PROOF OF EVOLUTIONARY GAME

Consider an error function as $e_{t,k}^h = z_{t,k}^* - z_{t,k}^h$, wherein $z_{t,k}^*$ is the final value of probability. Accordingly, the Lyapunov function can be defined as $\Psi(t) = (e_{t,k}^h)^2/2$ [\[46\]. A](#page-13-40)ccording to the Lyapunov function, the EG problem converges to EGE if the Lyapunov function satisfies three conditions [\[47\]: 1](#page-13-41)) $\Psi(t)$ is positive, 2) $\dot{\Psi}(t)$ is negative, and 3) $\Psi(t) \rightarrow \infty$ if $e_{t,k}^{h} \rightarrow \infty$. Based on the Lyapunov function, the first and third conditions can be simply satisfied. Hence, we need to prove the third condition as follows:

$$
\begin{split} \dot{\Psi}(t) &= \frac{\partial (e_{t,k}^h)^2/2}{\partial t} = e_{t,k}^h \times \frac{\partial e_{t,k}^h}{\partial t} = -e_{t,k}^h \times \frac{\partial z_{t,k}^h}{\partial t} \\ &= -z_{t,k}^h \times (z_{t,k}^* - z_{t,k}^h)(\sum_i \check{C}_{i,t,k}^T - \bar{C}_i^T) \\ &= -z_{t,k}^h \times (z_{t,k}^* - z_{t,k}^h)(\sum_i \check{C}_{i,t,k}^T - \sum_k z_{t,k}^h \times \sum_i \check{C}_{i,t,k}^T) \end{split} \tag{30}
$$

For affordable aggregators, we always have $z_{t,k}^* \ge z_{t,k}^h$. Also, the second term of the above equation for affordable aggregators is always positive. Hence, $\dot{\Psi}(t)$ is negative. Hence, the replicator has convergence for the affordable aggregators. Thus, the replicator has also convergence for non-affordable aggregators since their convergence completely depends on the convergence of other aggregators. As a result, the proposed replicator converges to the EGE.

APPENDIX B PROOF OF NONEMPTY CORE OF NON-COOPERATIVE GAME

A non-cooperative game has a nonempty core and Nash equilibrium if it has the following conditions [\[48\]: 1](#page-14-0)) The player set is finite. 2) The strategy sets are bounded and convex. 3) The profit functions/ cost functions are quasiconcave.

Here, it is clear that the first two conditions are valid in this problem since the feasibility area is convex and the number of aggregators is pre-defined for the algorithm. Thus, we only need to prove the third condition. For simplicity, $U_{t,k}^S$ is chosen to prove that it is quasi-concave throughout the feasible strategies. It is worth noting that the same proof can be inspired for $U_{t,k}^B$. When $\iota_{t,k}^S$ is less or equal to 1 (i.e., all available capacity has been sold and demand is greater than the available capacity), $U_{t,k}^S$ can be modified as follows:

$$
U_{t,k}^{S} = \frac{(\lambda_{t,k}^{S} - b_{t,k}^{S})^{2}}{4a_{t,k}^{S}}
$$
(31)

 $\frac{dU_{t,k}^{S}}{d\lambda_{t,k}^{S}}$ then, can be calculated as follows:

$$
\frac{dU_{t,k}^S}{d\lambda_{t,k}^S} = \frac{\lambda_{t,k}^S - b_{t,k}^S}{2a_{t,k}^S}
$$
(32)

Since the $U_{t,k}^S$ at least should be zero, the following statement is true: $\lambda_{t,k}^S \geq b_{t,k}^S$. Therefore, $\frac{dU_{t,k}^S}{d\lambda_{t,k}^S}$ is positive in this case.

On the other hand, if $\iota_{t,k}^{S}$ is greater or equal to 1 (i.e., capacity is greater than the demand), $\frac{dU_{t,k}^{S}}{d\lambda_{t,k}^{S}}$ can be calculated as follows after some simplifications with $D_{t,k}^S$ demand level for *k th* aggregator:

$$
\frac{dU_{t,k}^{S}}{d\lambda_{t,k}^{S}} = \lambda_{t,k}^{S} \frac{dD_{t,k}^{S}}{d\lambda_{t,k}^{S}} - b_{t,k}^{S} + D_{t,k}^{S} - \bar{P}_{t,k}^{S}
$$
(33)

From [\(6a\),](#page-5-3) it is clear that the value of $D_{t,k}^S$ is a non-increasing *t*,*k* with respect to $\lambda_{t,k}^S$. This means $\frac{dD_{t,k}^S}{d\lambda_{t,k}^S} < 0$. Also, we know that $D_{t,k}^S \le \bar{P}_{t,k}^S$. Therefore, we can say $\frac{dU_{t,k}^S}{d\lambda_{t,k}^S} < 0$. As a result, $U_{t,k}^S$ is quasi-concave and the same proof can be proposed for $U_{t,k}^B$. Thus, the proposed non-cooperative game has a nonempty core and Nash Equilibrium. ■

APPENDIX C PROOF OF SCALE-INDEPENDENT DRO REFORMULATION

In the first step, the proof of reformulation is proposed. $(6b)$ can be reformulated as [\[49\],](#page-14-1)

$$
\inf_{\kappa \ge 0} \ \kappa \times \epsilon + \frac{1}{V} \sum_{\nu=1}^{V} \sup_{\omega} \{ C_{i,t}^{P}(r,\omega) - \kappa \times ||\omega - \hat{\omega}_{\nu}|| \} \tag{34}
$$

The above reformulation can be expressed as follows:

$$
\inf_{\kappa \ge 0} \quad \kappa \times \epsilon + \frac{1}{V} \sum_{\nu=1}^{V} \varrho_{\nu}
$$
\n
$$
\text{s.t. } \sup_{\omega} \{ C_{i,t}^{P}(r, \omega) - \kappa \times ||\omega - \hat{\omega}_{\nu}|| \} \le \varrho_{\nu} \qquad (35)
$$

The above reformulation also can be expressed as follows [\[49\]:](#page-14-1)

$$
\inf_{\kappa \ge 0} \kappa \times \epsilon + \frac{1}{V} \sum_{\nu=1}^{V} \varrho_{\nu}
$$
\n
$$
\text{s.t. } \sup_{\omega} \{ C_{i,t}^{P}(r, \omega)_{q} - \max_{\|z_{\nu,q}\| \le \kappa} z_{\nu,q} \times (\omega - \hat{\omega}_{\nu}) \} \le \varrho_{\nu} \quad (36)
$$

where $C_{i,t}^P(r, \omega)_q$ can be defined as $\sum_{i,t}$ $C^P_{i,t}(r, \omega)$ = \sum *i*,*t* \sum $\sum_{q} (y_q(\vartheta_{i,t}, \tau_{i,t}) \times \omega + o_q(r_{i,t}))$. The upper bound reformulation for the above reformulation can be defined as follows [\[50\]:](#page-14-2)

$$
\inf_{\kappa \ge 0} \kappa \times \epsilon + \frac{1}{V} \sum_{\nu=1}^{V} \varrho_{\nu}
$$
\n
$$
\text{s.t. } \sup_{\omega} \{ C_{i,t}^{P}(r, \omega)_{q} - \max_{||z_{q}|| \le \kappa} z_{q} \times (\omega - \hat{\omega}_{\nu}) \} \le \varrho_{\nu} \qquad (37)
$$

By applying conjugation of $C_{i,t}^P(r, \omega)$, the above upper level can be re-expressed as follows [\[49\]:](#page-14-1)

$$
\inf_{\kappa \ge 0} \kappa \times \epsilon + \frac{1}{V} \sum_{\nu=1}^{V} \varrho_{\nu}
$$

s.t.
$$
[-C_{i,t}^P(r,\omega)_q]^*(z_q - w_q) + \Theta(w_q) - z_q \times \hat{\omega}_v \le \varrho_v
$$
\n(38)

Considering the above reformulation, we can say that,

$$
[-C_{i,t}^{P}(r,\omega)_{q}]^{*}(z_{q}-w_{q})
$$

=
$$
\begin{cases} 0 & z_{q}-w_{q}=-y_{q} \\ \infty & otherwise \end{cases}
$$
 (39)

$$
\Theta(w_q) = \begin{cases} \sup_{\omega} w_q \times \omega \\ \text{s.t. } \underline{\omega} \le \omega \le \overline{\omega} \end{cases}
$$
 (40)

After writing the dual form of (40) , which is the definition of the support function $[49]$, it can be presented as follows:

$$
\Theta(w_q) = \begin{cases} \sup_{\xi,\zeta} \xi \times \overline{\omega} - \zeta \times \underline{\omega} \\ \text{s.t. } \xi - \zeta = w_q \end{cases}
$$
(41)

After replacing (41) and (39) in (38) , we can achieve:

$$
\inf_{\kappa,\zeta,\xi\geq 0} \{ \kappa \times \epsilon + \xi \times \overline{\omega} - \zeta \times \underline{\omega} \n+ \frac{1}{V} \sum_{\nu=1}^{V} \left(\sum_{i,t} C_{i,t}^{P}(r, \hat{\omega}_{\nu}) + (\xi - \zeta) \times \hat{\omega}_{\nu} \right) \ns.t. \quad ||\xi - \zeta - y_{q}||_{*} \leq \kappa; \quad \forall q
$$
\n(42)

Please note that $||.||_*$ is dual norm. Now, we need to prove that the solution of the upper band reformulation is equal to the solution of (37) to illustrate the exactness. Considering this point that the maximum value of ϵ is less or equal to $\{\pi - \hat{\pi}, \pi + \hat{\pi}\}\$ based on the defined ambiguity set in [\(19\)](#page-7-3) in the manuscript, there is a proof in [\[51\]](#page-14-3) to demonstrate the result of [\(42\)](#page-12-10) for $z_{v,q}$ is $-y_q$, which is independent to *v*. The solution of [\(37\)](#page-12-9) for $z_{v,q}$ is $-y_q$ [\[51\]. H](#page-14-3)ence, the solution of upper-bound reformulation is exact.

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