Received 27 May 2024, accepted 31 May 2024, date of publication 4 June 2024, date of current version 12 June 2024.
Digital Object Identifier 10.1109/ACCESS.2024.3409651

RESEARCH ARTICLE

A Heuristic Solution Algorithm for a Comprehensive Optimal Phasor Measurement Unit Placement Considering Zero-Injection Buses and Practical Constraints in Power System State Observability Problem

SAYED HAMID HOSSEINI DOLATABADI[1], TANVEER HOSSAIN BHUIYAN[1], AND MOHAMMAD ESMAIL HAMEDANI GOLSHAN[2], (Senior Member, IEEE)
[1]Department of Mechanical Engineering, The University of Texas at San Antonio, San Antonio, TX 78249, USA
[2]Department of Electrical and Computers Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran
Corresponding author: Tanveer Hossain Bhuiyan (tanveer.bhuiyan@utsa.edu)

ABSTRACT Accurate power system observability relies on the optimal phasor measurement unit (PMU) placement to minimize costs while ensuring complete state observability. However, modeling the effect of zero-injection (ZI) buses remains a key challenge. Existing approaches use simplifying assumptions about ZI buses that compromise measurement redundancy and connectivity. We propose a novel heuristic methodology to address these shortcomings through comprehensive ZI bus modeling. This research provides a computationally-efficient heuristic optimization strategy for power systems with complex ZI bus topologies. The approach systematically analyzes ZI bus connectivity scenarios to develop enhanced observability evaluation. The proposed comprehensive observability evaluation approach is also incorporated to consider practical limitations including single outage contingencies of current measurement channels of PMUs, transmission lines, single PMUs, limited budget, lack of PMU positioning site, and incomplete observability with partial observability and depth of one unobservability for identifying optimal PMU placements that balance cost and observability. Case studies on IEEE 14, 30, 57, and New-England 39 bus test systems demonstrate the efficiency of the proposed approach in finding high-quality solutions for normal operating conditions and the capability of our proposed heuristic solution algorithm to consider different practical situations in power network observability analysis. Compared to the existing studies, our proposed solution algorithm achieves up to 21% and 29% fewer PMUs for obtaining complete state observability under normal operating conditions and for single outage contingency of PMUs, respectively.

INDEX TERMS Optimal PMU placement, state observability, zero injection buses, incomplete observability, N-1 contingency.

NOMENCLATURE

SETS

Sets	Description
$\mathfrak{B}$	Set of all network buses, indexed by b.
$\mathscr{Z}$	Set of all ZI buses, indexed by z.
$\mathscr{Q}$	Set of NZI buses which are connected to at least one ZI bus, indexed by q.

Sets	Description
$\mathscr{N}$	Set of NZI buses which are not connected to any ZI bus, indexed by n.
$\mathscr{Q}_z$	Set of buses connected to ZI bus $z \in \mathbb{Z}$ including z, indexed by q.

The associate editor coordinating the review of this manuscript and approving it for publication was Jahangir Hossain.

$\mathscr{X}_r$ Set of all ZI buses connected to each bus $r \in \mathscr{X} \cup \mathscr{Q}$.

S_0 Set of buses without communication infrastructure, indexed by s.

PARAMETERS

Parameters	Description
C	$N \times N$ bus-bus connectivity matrix.
N	Number of all network buses.
Z	Number of ZI buses.
W_b	Number of buses connected to bus $b \in \mathfrak{B}$.
N_B^{PMU}	Maximum number of available PMUs due to the limited budget.

VARIABLES

Variables	Description
x_b	Binary decision variable for PMU placement on bus $b \in \mathfrak{B}$ ($1 = install$).
f_b	observability status of bus $b \in \mathfrak{B}$.
g_q^z	observability status of bus $q \in \mathscr{Q}_z$ according to applying KCL on bus $z \in \mathscr{X}$.
u_r	observability status of bus $r \in \mathscr{X} \cup \mathscr{Q}$ according to applying KCL on all its adjacent ZI buses.
h_i	observability status of bus $i \in \mathscr{N} \cup \mathscr{Q}$ according to depth of one unobservability.

I. INTRODUCTION

A. MOTIVATION

The ability of PMUs to calculate and obtain voltage and current synchrophasors has made it possible to continuously monitor the complete state and operation of the power system [1].

The possibility of completely solving the state estimation (SE) problem to determine all the voltage and current phasors of the power system according to the provided measurements from different measurement devices, such as PMUs dispersed across the network is called power system state observability. Therefore, if a set of PMUs installed on different buses of the network can provide sufficient voltage and current phasor information to the state estimator which can solve the SE problem and find all the state variables of the power network, the related power system is completely observable [2].

While the installation of PMUs on each of the network buses provides comprehensive monitoring capabilities, the high cost of PMUs necessitates the problem of finding the minimum and optimal placement of PMUs (OPP), balancing economic considerations with the need for complete power system observability. The OPP problem is an NP-Complete combinatorial optimization problem in a very large search space. In a large search space (i.e., especially for large-scale power networks), an optimization algorithm is necessary for efficiently solving the problem [3], [4].

State observability, which is the main constraint of the OPP problem, can be evaluated using topological state observability analysis proposed in 1980 for the first time. It is based on the network graph concept [2] and is the most common method of evaluating state observability in the OPP problem. In 1993, the OPP problem of attaining complete power system observability [3] was studied for the first time, and from then, numerous studies concentrated on the concept of the OPP problem for power system observability which is one of the main aspects of power system operation studies.

The OPP problem has been solved using various exact and heuristic approaches. The existence of ZI buses in the power network makes it possible to apply KCL to them. Considering ZI buses in the OPP problem is one of the most challenging issues. Numerous studies modeled the effect of ZI buses in the OPP problem and presented mixed integer linear optimization problems with some simplifying assumptions, such as neglecting measurement redundancy and studying the combination of connected ZI buses. Therefore, while exact approaches aim to reach the global optimal solution, they often rely on simplified assumptions for modeling the effect of ZI buses. As a result, the final PMU placement obtained by solving these models may not necessarily guarantee complete state observability. On the other hand, heuristic approaches can facilitate more accurate modeling of ZI buses without such simplifying assumptions. Moreover, heuristic approaches can solve the problem computationally much faster than the exact optimization models. Therefore, developing an effective heuristic methodology for the OPP problem is a suitable solution for considering a comprehensive and realistic modeling of ZI buses. As a result, the motivation of this study is to present an in-depth study on different possible observability scenarios due to the presence of ZI buses to comprehensively model their effect in the OPP problem, and provide a practical solution approach that balances optimality with accurate system representation to ensure complete state observability.

B. LITERATURE REVIEW

There are numerous studies on the OPP problem that can be divided into four general categories (1) analyzing fundamental state observability [5] and considering practical constraints such as N-1 contingencies, investment limitation, incomplete observability, and measurement redundancy [6], [7]; (2) improving heuristic and meta-heuristic algorithms for solving the OPP problem, such as genetic algorithm (GA), particle swarm optimization (PSO), and tabu search (TS) [8], [9] and providing a linear mathematical formulation for the OPP problem [4]; (3) extending the OPP problem to consider different applications of PMUs such as fault location [10], reliability [11], [12], and small signal stability [5]; and (4) providing more accurate modeling of ZI buses in the OPP problem, which is the scope of this study.

Some studies provided a mixed-integer linear mathematical formulation for the OPP problem. Theodorakatos et al. [13] proposed the use of the Branch-and-Bound algorithm for solving the OPP problem formulated as a mixed-integer linear programming model. Their approach aims to minimize the number of PMUs required to achieve complete observability of the power grid while considering various practical constraints. The authors demonstrated the effectiveness of their method in finding optimal solutions for different test cases and compared its performance with other solution techniques. In [14], a novel approach to solve the OPP problem is proposed using a binary polynomial optimization problem. Maximizing the measurement redundancy is achieved in [13] and [14] to improve the reliability of the measurement system. However, they did not consider the effect of ZI buses on power system observability analysis [15].

In [16], a PMU placement algorithm is presented that takes into account the network sparsity and the limited communication bandwidth. In [17], an advanced measurement placement method for power system observability using semidefinite programming is proposed that aims to find the optimal placement of measurement devices, including PMUs and conventional measurements, to ensure complete observability of the system considering various practical constraints, such as the number of available measurement channels and the cost of installing measurement devices. In [18], ZI buses are modeled to provide new current measurement information to the system. In an almost similar way, [19] assigned a pseudo measurement for each of the ZI buses in the network and proposed a three-stage PMU placement heuristic approach.

The initial introduction of ZI buses into the linear mathematical representation of the OPP problem began with [4] and [11] based on this statement that the observability status of each ZI bus and all its adjacent buses can be evaluated together by applying KCL to ZI buses. The authors also considered the concept of incomplete observability, the effect of existing conventional measurements, and single or multi-PMU loss in the observability analysis. A similar method for considering ZI buses is used in [20] based on the concept of multi-stage PMU placement. By knowing the final optimal solution, this study provides the best sub-optimal PMU placement for a multi-stage investment in the phasor measurement system.

In [21], by using a similar rule for ZI buses, different contingencies including a single outage of transmission lines and PMUs and limitations on communication channels of PMUs are also considered in the problem. The concept of line-wise observability which is related to different considerations in WAMS like restoration and dynamic line monitoring is proposed in [22] in which a similar method is used to consider ZI buses and assigns current measurements to the adjacent lines of PMUs. In [23], based on the same method for ZI buses, the connectivity matrix which is a bus-bus incidence matrix is modified and single-line outage contingencies is considered in the problem. The equivalent linear formulation for exhaustive search is proposed in [24] which implicitly followed the basic rule for ZI buses and considered different N-1 contingencies in the problem. By using the same method for OPP linearization, the problem is studied in a real case of the Qatar grid in [25] in which the presence of injection measurements is also added to the problem. A multi-stage PMU placement considering the probability of observation of each bus is proposed in [12] by using the basic method for considering ZI buses. The concept of depth of ZI observation is proposed in [26] assuming that the reliability of ZI observation becomes weaker in modern complex power networks with new topologies and then, the authors considered different N-1 contingencies in the problem and modeled the effect of ZI buses similar to [11]. A similar formulation for considering ZI buses is used in [27] in which the effect of different contingencies and communication channel limitation is also studied. In [28], existing conventional measurements are also studied in the OPP problem with a completely similar formulation to the previous studies. A new approach called the sine cosine algorithm is proposed in [29] for the OPP problem considering the effect of ZI buses in the observability analysis of the power system similar to the previous studies. Using the same method for considering ZI buses in the problem, and the new concept of the resiliency of bus connection, a stochastic approach is used in [30] to consider N-1 contingencies.

Due to deficiencies in considering the effect of ZI buses on the OPP problem, a few works tried to study ZI buses more carefully. In [31], instead of using a basic formulation for considering ZI buses, new variables are defined to maximize the measurement redundancy. A special case of unobservability by considering two-connected ZI buses is studied in [32] based on an iterative approach that starts from the placement of PMUs on buses with high priority and modifying the solution to find the optimal placement. In [8], a set of unobservable connected ZI buses is studied based on a proposed new rule for observability analysis considering ZI buses. However, this is a special case of connected ZI buses in which the effect of measurement redundancy is neglected. A similar approach for considering connected unobservable ZI buses is used in [33] along with considering N-1 contingencies in the problem formulation. Rashidi et al. [34] also improved the mathematical formulation of considering ZI buses by proposing some new rules for the problem. However, some of the rules proposed in this paper cannot be generally true. For example, rule #6 in [34], cannot be applicable in all cases and may not lead to an optimal solution with complete observability, especially for systems with numerous connected ZI buses. Based on previous studies and modifications, other researchers extended the OPP problem to consider some other aspects of the problem, such as measurement channel limitation and cyber-attacks. With almost the same formulation as [33], the limitation on current measurement channels of PMUs is studied in [35]. In [36], a multi-objective OPP by adding maximum observability and considering single PMU loss to the problem is studied. However, the proposed method in [36] for considering the effect of ZI buses is similar to [31]. The formulation of [35] and [37] is used in [38] to consider the coordination of PMUs with other communication sensors in a power network with radial lines. In [39], based on the proposed formulation in [8], a probabilistic OPP under various contingencies like cyber-attacks is studied which has similar shortcomings in terms of modeling ZI buses.

In summary, although existing studies on OPP problem have proposed some models for considering the effect of ZI buses, three important aspects of modeling ZI buses in the OPP problem still need to be improved. **First**, since some of the network buses may be observed by two or more PMUs, measurement redundancy should be considered accurately in modeling ZI buses. **Second**, since series-connected multiple ZI buses (SCMZIBs) can increase the number of observable buses, accurately modeling the effect of SCMZIBs is crucial for solving the OPP problem. **Third**, since special cases of non-zero injection (NZI) buses between some ZI buses can result in observing new buses of the network, this aspect needs to be incorporated in solving the OPP problem.

C. CONTRIBUTION

This paper focuses on modeling the effect of ZI buses in power system observability analysis to fill the gaps in the existing literature on the OPP problem. We propose a novel methodology for accurately modeling the effect of ZI buses by relaxing the limiting assumptions of the previous approaches including (1) measurement redundancy, (2) SCMZIBs, and (3) special cases of NZI buses between ZI buses. Therefore, this paper advances the current literature by overcoming the shortcomings of the previous works in modeling ZI buses and providing the following contributions.

1) We propose a more comprehensive OPP problem formulation that considers the impact of ZI buses.
2) The proposed OPP problem formulation is solved by a proposed heuristic solution algorithm enabling to consecutively evaluate the observability of ZI buses.
3) The proposed OPP formulation is extended to consider some important practical limitations in the OPP problem including limited investment budget, depth of one unobservability, limited communication infrastructure in some of the network buses, single transmission line, single loss of current measurement channels of PMUs, and single PMU outage contingencies.
4) The numerical results of applying our proposed solution algorithm on IEEE standard test networks provide insights for the system planner on the effect of inaccurate modeling of ZI buses and the effectiveness of comprehensively modeling ZI buses in reducing the number of required PMUs, optimal solutions of the OPP problem considering single outage contingencies and trade-offs between investment budget and observability, and the effect of lack of communication infrastructure on the ability of PMUs to observe the power system.

By addressing the challenges in comprehensively modeling ZI buses, our research significantly enhances the understanding and practical implementation of OPP for power system observability. The remainder of this paper is organized as follows: in section II, a general description of the OPP problem, the existing approach with regard to the basic formulation of considering ZI buses in the power system, and the shortcomings in modeling the effect of ZI buses are provided. Section III, presents the proposed heuristic solution algorithm to consider the effect of ZI buses in the OPP formulation. Section IV, studies important practical limitations to be considered in the OPP problem. In section V, the proposed methodology is applied to different standard test networks and the results are compared with the existing approaches. Section VI, concludes the paper.

II. PROBLEM DESCRIPTION

The OPP problem basically looks for finding the minimum number and optimal placement of PMUs to attain complete power system observability. Different buses of the network can be mainly categorized as ZI and NZI buses. NZI buses are the buses with unknown generation/consumption which have to be determined after running SE. ZI buses are buses with no generation and/or consumption.

There are three important shortcomings in modeling ZI buses in the OPP problem considered in the previous references including (1) measurement redundancy, (2) SCMZIBs, and (3) existing NZI buses between ZI buses.

To demonstrate the shortcomings of the common modeling of ZI buses in the OPP problem, the existing general rules for the OPP problem considering ZI buses are reviewed in the following subsections.

A. BASIC MODEL OF THE OPP PROBLEM

Fig. 1 shows a PMU installed on bus j that can measure the phasor voltage of V_j and the current phasors of I_{jk}, I_{ij}, and I_{Lj}.

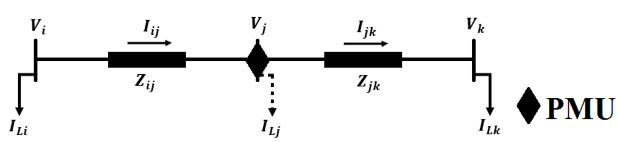


FIGURE 1. A portion of a network with a PMU installed on bus j.

By applying KVL and Ohm law and knowing line impedances, the voltage phasors of V_i and V_k can be calculated according to Eqs. (1).

$$V_i = V_j + Z_{ij}I_{ij} \quad (1a)$$
$$V_k = V_j - Z_{jk}I_{jk} \quad (1b)$$

As a result, the state of buses i, j, and k are observable by this PMU. Based on this result, one of the basic principles of state observability is that each PMU installed on bus i has the capability of observing the state of bus i and all its connected buses.

Since bus j is a ZI bus in Fig. 1, in addition to Eqs. (1), Eq. (2) is obtained by applying KCL.

$$I_{ij} - I_{jk} = 0 \tag{2}$$

In this set of three equations, there are five unknown variables. If two of these five variables have been determined by the installed PMUs across the network, a set of three equations and three unknowns is derived, and therefore, the other three unknowns can be calculated by solving this set of equations. As a result, if two voltage phasors have been observed, the other voltage phasor becomes observable by applying KCL. Based on this explanation and generalizing it to similar cases, another principle of state observability is that for a group of buses consisting of a ZI bus and its adjacent buses, if only one of them is not observed by the installed PMUs, that bus will be observed by applying KCL on the ZI bus. The above analysis and explanations are based on topological state observability. For a power system that works normally and close to the nominal ratings, topological observability can guarantee the accuracy of state estimation and numerical observability. Although some cases may exist that topological observability cannot guarantee numerical observability [40], [41], these cases are rare [42], [43].

Based on the above basic principles of state observability, the general formulation for the OPP problem [4], [11], [20] can be written as model (3).

$$min \quad \sum_{b \in \mathfrak{B}} x_b \tag{3a}$$

$$s.t. \quad F = C \times X \tag{3b}$$

$$f_n \geq 1, \ \forall n \in \mathscr{N} \tag{3c}$$

$$\sum_{q \in \mathscr{Q}_z} f_q \geq W_z, \ \forall z \in \mathscr{Z} \tag{3d}$$

In Eq. (3a), x_b is a binary decision variable to decide whether a PMU is installed on bus b or not. If a PMU is installed on bus b, x_b equals one, otherwise it equals zero. Moreover, in Eq. (3b), C is an $N \times N$ connectivity matrix. Each element of C is defined as Eq. (4).

$$C_{a,b} = \begin{cases} 1 & a = b \, or \, bus \, b \, is \, connected \, to \, bus \, a \\ 0 & bus \, b \, is \, not \, connected \, to \, bus \, a \end{cases} \tag{4}$$

X is an $N \times 1$ vector of PMU installation decision variables (i.e., x_b), and F is the observability status vector. Each element of F, f_b, represents the observability status of bus b according to installed PMUs on the power network considering the basic principles of state observability. Each PMU installed on any of the network buses can observe that bus and all its adjacent buses. Therefore, for bus b, if a PMU is installed on bus b, or any of its adjacent buses, f_b will be equal to or greater than 1 representing the observability of that bus. Moreover, for a ZI bus and its adjacent buses, if only one of them is unobservable, that bus is actually observable by applying KCL. Therefore, all the network buses can be divided into three categories including (1) ZI buses ($\mathscr{Z}$), NZI buses which are not connected to any of the ZI buses ($\mathscr{N}$), and (3) NZI buses which are connected to at least one ZI bus ($\mathscr{Q}$). For each of the buses belonging to $\mathscr{N}$, Eq. (3c) can be written as these buses should be observable directly by installed PMUs across the network. For buses belonging to $\mathscr{Z}$ and $\mathscr{Q}$, Eq. (3d) is written to model the principle of state observability for group of buses connected to ZI buses. In Eq. (3d), $\mathscr{Q}_z$ is the set of network buses connected to a ZI bus ($z \in \mathscr{Z}$) including the ZI bus (i.e., z), and W_z is the number of elements of $\mathscr{Q}_z$ minus 1 (i.e., the number of buses connected to bus z). For example, for bus j in Fig. 1 which is a ZI bus, $\mathscr{Q}_z = \{i, j, k\}$ and $W_z = 2$. Therefore, Eq. (3d) is represented as Eq. (5) for this part of the network.

$$f_i + f_j + f_k \geqslant 2 \tag{5}$$

which means that in the group of bus j and all of its connected buses, at least two of these three buses must be observable.

B. DEFECTS OF BASIC MODEL OF THE OPP PROBLEM

The basic model of the OPP problem explained in section II-A has some important defects and therefore, the existing approach to the OPP problem cannot lead to minimum number of PMUs while ensuring complete state observability. These defects are explained in the following subsections.

1) MEASUREMENT REDUNDANCY

The basic formulation used in the existing approaches for the OPP problem is based on the assumption that F is a binary vector that demonstrates whether a bus is observed directly by a PMU or not. However, according to Eq. (3b), f_b is the multiplication of b^{th} row of C (connectivity matrix) and vector X (PMU placement decisions). Since each of the rows of C may have multiple elements with the value of one, and PMUs are installed on more than one of the network buses, f_b can have non-negative integer values. It means that f_b can take any integer value even more than one.

Having a value more than one for f_b represents the measurement redundancy for bus b which means bus b is observed by more than one PMU across the network due to its connectivity in the topology of the network. However, this fact may result in inaccurate satisfaction of Eq. (3d) which is written based on the observability principle for groups of buses connected to ZI buses. Consider Fig. 1 without the PMU installed on bus j. If bus i has measurement redundancy such that $f_i = 2$, and buses j and k are not observed directly by any of the PMUs, meaning $f_j, f_k = 0$, Eq. (5) is still satisfied while two of the buses in this group are unobservable and therefore, buses j and k cannot be observable by applying KCL. As it is seen, Eq. (3b) can be satisfied whereas the entire buses are not observable. Therefore, it cannot guarantee the complete state observability of the network.

2) SERIES-CONNECTED MULTIPLE ZI BUSES

Sometimes there are situations in a power network when multiple ZI buses are sequentially connected. In these situations,

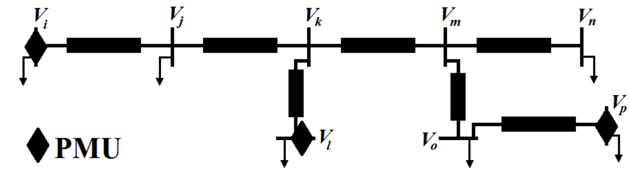


FIGURE 2. A portion of a network with SCMZIBs.

KCL can be consecutively applied to series-connected ZI buses to observe new buses of the network.

Consider Fig. 2 as a portion of a network with SCMZIBs without measurement redundancy. According to the installed PMUs and Eq. (3b), $f_i, f_j, f_l, f_k, f_p, f_o = 1$ and $f_m, f_n = 0$. Since buses k and m are ZI buses, Eq. (3d) can be written for them as Eqs. (6) and (7), respectively,

$$f_j + f_k + f_l + f_m \geq 3 \tag{6}$$

$$f_k + f_m + f_o + f_n \geq 3 \tag{7}$$

Since $f_j, f_l, f_k = 1$, Eq. (6) is satisfied which represents the observability of the remaining unobservable bus m. Moreover, since $f_k, f_o = 1$ and $f_m, f_n = 0$, Eq. (7) is not satisfied. However, based on the observability principle for the groups of buses connected to ZI buses, since bus m is observable due to applying this principle to bus k, for the group of bus m and its adjacent buses, the only unobservable remaining bus n becomes observable. Therefore, although Eq. (7) is not satisfied, all of the buses in Fig. 2 are observable. As a result, Eq. (3b) in the basic formulation can be unsatisfied whereas the entire buses are observable. Therefore, using the basic model for the OPP problem may not be able to lead to the minimum number of PMUs for complete state observability under these conditions.

3) NZI BUSES BETWEEN ZI BUSES

In some cases when an NZI bus is between some ZI buses, there can be an opportunity to observe more buses of the network by applying KCL on ZI buses.

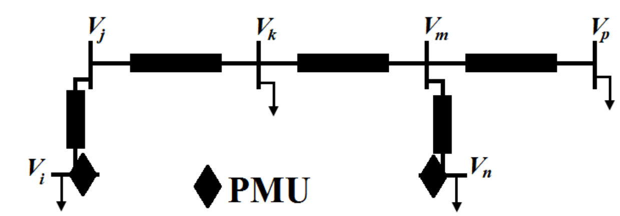


FIGURE 3. A portion of a network with an NZI between two ZI buses.

Consider Fig. 3 as a portion of a network with an NZI bus between two ZI buses. Based on the PMUs installed on buses i and n, and Eq. (3b), $f_i, f_j, f_n, f_m = 1$ and $f_k, f_p = 0$. Since buses j and m are ZI buses, Eq. (3d) can be written for them as Eqs. (8) and (9), respectively,

$$f_i + f_j + f_k \geq 2 \tag{8}$$

$$f_k + f_m + f_n + f_p \geq 3 \tag{9}$$

Since $f_i, f_j = 1$, Eq. (8) is satisfied and therefore, bus k becomes observable. Moreover, since $f_n, f_m = 1$ and $f_k, f_p = 0$, Eq. (9) is not satisfied. However, based on the observability principle for bus m, the only remaining unobservable bus p becomes observable in the group of buses connected to m. It is seen that in this case, Eq. (9) is not satisfied although the entire buses in Fig. 3 are observable according to the installed PMUs. Therefore, in addition to the situations of SCMZIBs, there are other situations in the power network that Eq. (3d) cannot comprehensively model the complete state observability by the minimum number of PMUs.

III. PROPOSED METHODOLOGY FOR OPP CONSIDERING THE ACCURATE MODELING OF ZI BUSES

A. PROPOSED FORMULATION

As explained in section II-B, Eq. (3d) cannot accurately model the effect of ZI buses and therefore, the existing approach to the OPP problem presented in model (3), may lead to incomplete state observability or non-minimum required PMUs for complete state observability. Overcoming the defects of the basic model in ZI buses modeling is crucial for achieving comprehensive observability evaluation and reducing the number of required PMUs. Therefore, a new strategy to overcome these issues based on a consecutive heuristic approach to solve the OPP problem is proposed in this section.

To overcome the shortcomings discussed in section II-B1 as measurement redundancy in modeling OPP problem with ZI buses, it is necessary to limit the upper-bound of F vector by one to avoid inaccurate satisfaction of Eq. (3d). Therefore, Eq. (3b) is modified as Eq. (10) to overcome this shortcoming.

$$f_b = min\left\{\sum_{i=1}^{N} C_{b,i}x_i,\ 1\right\}, \quad \forall b \in \mathfrak{B} \tag{10}$$

where $C_{b,i}$ is the element in the b^{th} row and i^{th} column of matrix C. This modification limits the observability status variable, f_b, to be at most equal to one. Therefore, it can prevent the occurrence of situations similar to the one explained in section II-B1.

Moreover, as explained in sections II-B2 and II-B3, there are situations that although Eq. (3d) is not satisfied, the entire buses belong to the group of buses connected to a ZI bus become observable. To overcome these challenging situations, the observability status of different buses across the network can be consecutively evaluated. For example, if in the situation explained in section II-B2 we update the value of f_m after satisfying Eq. (6), Eq. (7) can be also satisfied and therefore, the entire buses of Fig. 2 become observable.

To update the F vector consecutively, it is necessary to modify Eq. (3d) such that it calculates the observability status of all the buses connected to each of the ZI buses across the network. This modification is applied as Eqs. (11) and (12).

$$g_q^z = \begin{cases} 0 & \sum_{q\in\mathscr{Q}_z} f_q - W_z + 1 \leq 0 \\ 1 & \sum_{q\in\mathscr{Q}_z} f_q - W_z + 1 \geq 1 \end{cases}, \forall z \in \mathscr{Z}, \forall q \in \mathscr{Q}_z \quad (11)$$

$$u_r = min\left\{\sum_{z\in\mathscr{Z}_r} g_r^z + f_r, 1\right\}, \forall r \in \mathscr{Z} \cup \mathscr{Q} \quad (12)$$

where $\mathscr{Z}_r$ is the set of all ZI buses that are connected to bus r. Some of the network buses can be connected to more than one ZI bus and therefore, it is possible that the observability principle for groups of buses connected to ZI buses can be applied multiple times for them. Therefore, Eq. (11) evaluates the observability status of all buses connected to each of the ZI buses. If two or more of the buses in the group of buses connected to the ZI bus z ($\mathscr{Q}_z$) are not directly observed by the installed PMUs across the network, the term $\sum_{q\in\mathscr{Q}_z} f_q - W_z + 1$ is equal or less than zero and therefore, none of the buses in this group can become observable by applying KCL on the ZI bus z. Otherwise, the term $\sum_{q\in\mathscr{Q}_z} f_q - W_z + 1$ is greater than one, and therefore, the entire buses of $\mathscr{Q}_z$ are observable. The network buses belong to the set $\mathscr{N}$ which are NZI buses and not connected to any of the ZI buses and must become observable directly by the installed PMUs across the network. However, the other buses of the network can become observable directly by PMUs and/or by applying KCL on ZI buses. Therefore, Eq. (12) evaluates the observability of all the buses belonging to the sets $\mathscr{Z}$ and $\mathscr{Q}$. Each of these buses can be observable according to applying KCL on their adjacent ZI buses ($\sum_{z\in\mathscr{Z}_r} g_r^z$), and/or directly by the installed PMUs (f_r). As a result, f_n for $n \in \mathscr{N}$ and u_r for $r \in \mathscr{Z} \cup \mathscr{Q}$ are binary variables evaluating the observability status of different buses across the network.

B. PROPOSED HEURISTIC SOLUTION ALGORITHM

According to the situations explained in sections II-B2 and II-B3, after applying the above-mentioned modifications, it is necessary to update the observability status of the network buses and consecutively apply KCL and evaluate whether new buses can become observable according to Eqs. (11) and (12) or not. Therefore, we introduce a new heuristic algorithm for solving the OPP problem that is based on a consecutive observability evaluation to accurately model the effect of ZI buses and to find the optimal PMU placement to achieve complete power system observability. The OPP problem that we define in this section is as follows.

$$min \quad \sum_{b\in\mathfrak{B}} x_b \quad (13a)$$

$$s.t. \quad Evaluate\ the\ observability\ of\ all\ buses \quad (13b)$$

$$f_b \geq 1, \forall b \in \mathfrak{B} \quad (13c)$$

In the model (13), we use the proposed concept explained in this section to evaluate the observability status of the network buses. This proposed concept can consider the effect of measurement redundancy and evaluates the observability status of each of the network buses individually. The terms f_b are binary variables that represent the observability status of different buses. However, as we discussed in section II-B, it is possible to observe more buses by consecutively applying KCL on different ZI buses across the network. Therefore, Eqs. (11) and (12) are only enough for the first time of evaluating the observability status of buses connected to ZI buses. However, it is possible to observe more buses of the network after consecutively applying KCL. Therefore, to achieve a comprehensive evaluation of observability status, it is needed to run a consecutive observability evaluation until there would be no other observable bus. To achieve this goal, for each of the PMU placement strategies, the observability status of all the network buses are consecutively evaluated to make sure that for each of the PMU placement strategies, maximum observable buses are determined. The proposed consecutive approach for a given PMU placement strategy to evaluate the observability status of all the network buses (constraint (13b)) is presented as a flowchart shown in Fig. 4

In the proposed framework, for a given set of PMUs, we first evaluate the observability status of all the network buses based on Eq. (10). In this step, we can determine the observability status of all network buses according to the installed PMUs. Then, we split the vector F into three vectors $F_{\mathscr{N}}$, $F_{\mathscr{Z}}$, and $F_{\mathscr{Q}}$. The vector $F_{\mathscr{N}}$ is associated with all of the buses $n \in \mathscr{N}$ which should be observed directly by the installed PMUs. Therefore, vector $F_{\mathscr{N}}$ remains unchanged even after applying KCL on ZI buses. For the other buses of the network (i.e., $\forall r \in \mathscr{Z} \cup \mathscr{Q}$) which are ZI buses or buses connected to ZI buses, applying KCL can result in the observability of new buses of the network according to Eqs. (11) and (12). As a result, in the next step, we evaluate the observability status of ZI buses and all the buses connected to ZI buses based on the Eqs. (11) and (12). Therefore, we obtain the binary observability status vector, u_r, for all of the buses $r \in \mathscr{Z} \cup \mathscr{Q}$. This is the first stage of the proposed consecutive heuristic approach. At this stage, a check is performed to compare the modified observability status u_r with the original observability status f_r before applying KCL on ZI buses. If u_r is found to be identical to f_r for $\forall r \in \mathscr{Z} \cup \mathscr{Q}$, it can be inferred that the applied observability evaluation did not result in more observable buses. Therefore, we again combine the splitted vectors to obtain the maximum observable buses according to the given PMU placement. Otherwise, it indicates the presence of new observable buses after applying the proposed observability evaluation of ZI buses. As explained in detail in sections II-B2 and II-B3, it is possible to observe more buses of the network by applying KCL on ZI buses consecutively. Therefore, we replace f_r with u_r to re-evaluate the observability status of all the buses belonging to $\mathscr{Z} \cup \mathscr{Q}$. This helps the framework to apply KCL consecutively on ZI buses. After re-evaluating the observability status of all ZI buses and buses connected to

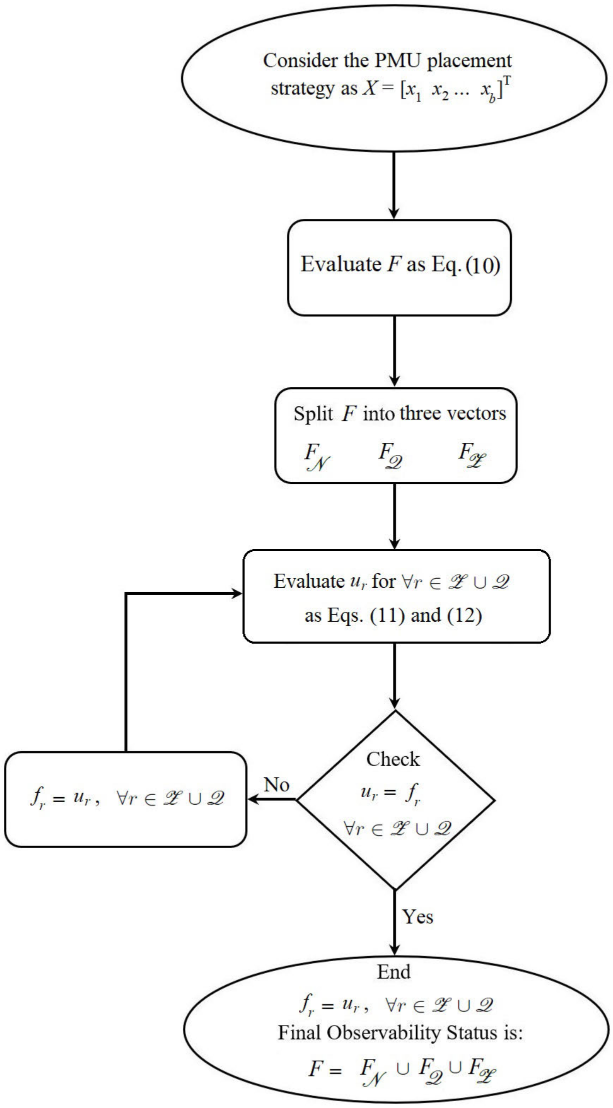


FIGURE 4. Flowchart of the proposed consecutive approach.

ZI buses by applying Eqs. (11) and (12), the new resulting binary variables u_r is compared to the f_r for $\forall r \in \mathscr{Z} \cup \mathscr{Q}$ to check the existence of other observable buses across the network. The consecutive approach continues until there is no additional observable buses, which indicates the final solution is reached.

Then, we use the proposed consecutive approach shown in Fig. 4 to evaluate the observability status of all network buses in the model (13) to solve the OPP problem. However, the presence of any loop in a mathematical formulation prevents it from being considered as a MILP formulation and being solved by the state-of-the-art optimization solvers, such as CPLEX. Therefore, we use GA to solve the model (13). It is worth mentioning that any other heuristic/meta-heuristic algorithms including but not limited to PSO, SA, and TS can be also applicable to solve the model (13).

IV. MODELING PRACTICAL LIMITATIONS

In practice, several practical constraints and unforeseen events can impact power systems. These encompass, but are not restricted to, factors such as (1) constrained investment budgets, (2) insufficient communication infrastructure or the absence of optimal PMU placement sites, (3) individual failures in the current measurement channels of PMUs, (4) contingencies arising from branch outages, and (5) PMU losses. Ensuring the reliability of a measurement system requires a comprehensive inclusion of these significant practical limitations and contingencies within the OPP problem. Our proposed formulation and solution algorithm has the capability of considering all the practical limitations of the OPP problem. Therefore, within this section, we present adapted OPP problems, grounded in the proposed model (13), designed to address crucial practical considerations in WAMS.

A. LIMITED INVESTMENT

In this section, we consider the practical constraint of the limited budget that affects the power system observability. Due to the limited investment, it may be impossible to have the entire power system observable, and therefore, the power network can have partial observability. Two approaches for partial observability can be chosen [4]. In the first approach, it is needed to have the maximum observability (i.e. maximum number of observable buses) with respect to the limited investment cost (i.e., limited number of PMUs). In this approach, the purpose of the OPP problem is to achieve the maximum number of observable buses across the network with the given number of PMUs. To incorporate this approach in the OPP problem, model (13) can be modified as model (14).

$$max \quad \sum_{b \in \mathfrak{B}} J_b \tag{14a}$$

$$s.t. \quad \sum_{b \in \mathfrak{B}} x_b = N_B^{PMU} \tag{14b}$$

$$\textit{Evaluate the observability of all buses} \tag{14c}$$

$$f_b = J_b, \ \forall b \in \mathfrak{B} \tag{14d}$$

In model (14), the objective function is to maximize the number of observable buses. As f_b is a binary variable, it can take values of zero or one, representing the unobservability and observability status of bus b, respectively. Therefore, based on the limited budget in constraint (14b), the model (14) finds the maximum number of observable buses in the power system with the optimal PMU placement. By changing N_B^{PMU} in Eq. (15), we can find the optimal solutions of the OPP problem for different levels of investment.

$$1 \leq N_B^{PMU} \leq \hat{N}_B^{PMU} \tag{15}$$

The value of $\hat{N}_B^{PMU}$ (the upper bound of N_B^{PMU}) is the minimum number of PMUs required for complete observability which is determined after solving model (13).

In the second approach, the concept of depth of unobservability is incorporated [44]. In the depth of one unobservability, the state of a bus can be estimated from the two adjacent observable buses. In this concept, the load of some of the NZI buses can be estimated based on the historical data and the state of their adjacent buses. Therefore, as the amount of load can be predicted, these buses can be considered as ZI buses and KCL can be applicable for them. Therefore, if after running the observability evaluation according to the proposed heuristic approach (shown in Fig. 4), there are still some NZI buses that their adjacent buses are observable, they can be considered as buses with depth of one unobservability. For the NZI buses that can become observable by the depth of one unobservability, all of their adjacent buses should be observable. Therefore, they can be considered as special ZI buses that all their adjacent buses should be observable. To incorporate this approach in the OPP problem, model (13) can be modified as model (16).

$$min \quad \sum\nolimits_{b\in\mathfrak{B}} x_b \tag{16a}$$

$$s.t. \; \textit{Evaluate the observability of all buses} \tag{16b}$$

$$h_i = \begin{cases} 0 & \sum\nolimits_{j\in\mathscr{Q}_i} f_j - W_i + 1 \leq 0 \\ 1 & \sum\nolimits_{j\in\mathscr{Q}_i} f_q - W_i + 1 \geq 1 \end{cases}, \quad \forall i \in \mathscr{N} \cup \mathscr{Q} \tag{16c}$$

$$u_i = max\,\{f_i, h_i\}, \; \forall i \in \mathscr{N} \cup \mathscr{Q} \tag{16d}$$

$$f_z \geq 1, \; \forall z \in \mathscr{Z} \tag{16e}$$

$$u_i \geq 1, \; \forall i \in \mathscr{N} \cup \mathscr{Q} \tag{16f}$$

In model (16), the objective is to minimize the required number of PMUs. However, as we incorporate the concept of depth of one unobservability, the investment (i.e., the number of required PMUs) can be less than the solution of model (13). Constraint (16b), evaluates the observability status of all the network buses according to the consecutive heuristic approach shown in Fig. 4. Then, constraint (16c) evaluates the situation of depth of one unobservability for all the NZI buses across the network. If all the adjacent buses to bus $i \in \mathscr{N} \cup \mathscr{Q}$ are observable, $\sum_{j\in\mathscr{Q}_i} f_j - W_i + 1$ is greater than one and therefore, the state of bus i can be estimated from its adjacent observable buses. As a result, constraint (16d) determines that bus i is either observable by the installed PMUs or with the depth of one unobservability. Finally, constraints (16e) and (16f) ensure that all the network buses are observable.

B. LIMITED COMMUNICATION INFRASTRUCTURE

In this section, we consider the practical constraint of the limited communication infrastructure which results in the lack of PMU positioning on some of the network buses. Considering this limitation will result in limiting the problem from placing PMU on specific buses of the network. Therefore, the proposed model (17) for this problem is as follows.

$$min \quad \sum\nolimits_{b\in\mathfrak{B}} x_b \tag{17a}$$

$$s.t. \; x_s = 0, \; \forall s \in S_0 \tag{17b}$$

$$\textit{Evaluate the observability of all buses} \tag{17c}$$

$$f_b \geq 1, \; \forall b \in \mathfrak{B} \tag{17d}$$

In model (17), the objective function is to minimize the required number of PMUs for complete state observability accounting for the limitation that it is impossible to place PMUs on some of the network buses according to constraint (17b). In constraint (17b), S_0 is the set of network buses without communication infrastructure. Therefore, this decision-making problem cannot place a PMU on these buses. Then, constraints (17c) and (17d) evaluates the observability of all the buses according to the proposed heuristic approach shown in Fig. 4.

C. BRANCH OUTAGE CONTINGENCIES

In this section, we consider one of the most important and common practical contingencies of the power system which is transmission line outage contingency. To have a more reliable power system, it is needed to have power system observability under at least single outage contingencies of the transmission lines. Transmission line outage will result in changing the topology of the power system that may make some of the network buses unobservable. Therefore, in order to make sure that the power system is still observable under any possible single outage of the transmission lines, different topologies of the power system after the outage of each of the transmission lines should be studied. Therefore, the proposed model for this problem is presented as follows.

$$min \quad \sum\nolimits_{b\in\mathfrak{B}} x_b \tag{18a}$$

$$s.t. \; \textit{remove each of the branches iteratively} \tag{18b}$$

$$\textit{Evaluate the observability of all buses for each of the contingencies} \tag{18c}$$

$$f_b \geq 1, \; \forall b \in \mathfrak{B}, \; \forall \textit{contingency} \tag{18d}$$

In model (18), the objective is to minimize the required number of PMUs for complete state observability during the normal operation and single outage of transmission lines. Therefore, constraints (18b) to (18d) evaluates the observability of all the network buses using the heuristic approach shown in Fig. 4 considering the single outage of each of the transmission lines.

D. PMU LOSS CONTINGENCIES

In this section, we consider another practical contingency of the system which is the loss of PMUs due to cyber-attacks or loss of communication infrastructure. In this problem, the measurement system should be robust to at least single outages of the PMUs. In other words, by considering the outage of any of the PMUs, the entire power system should be still observable. To consider this practical constraint, for a given set of PMUs, the outage scenarios of each of the PMUs should be modeled and the power system observability should be satisfied. Therefore, the proposed model for this problem is presented as follows.

$$min \quad \sum\nolimits_{b\in\mathfrak{B}} x_b \tag{19a}$$
$$s.t. \quad remove\ each\ of\ the\ PMUs\ iteratively \tag{19b}$$
$$Evaluate\ the\ observability\ of\ all\ buses$$
$$for\ each\ of\ the\ contingencies \tag{19c}$$
$$f_b \geq 1,\ \forall b \in \mathfrak{B},\ \forall\, contingency \tag{19d}$$

Model (19) similar to model (18) evaluates the observability of all the network buses with iteratively removing each of the PMUs for the given set of PMUs with the objective of minimizing the required number of PMUs.

E. LOSS OF CURRENT MEASUREMENT CHANNELS

In this section, another practical contingency of the system considered is the single loss of a current measurement channel of one of the PMUs. This contingency will limit the capability of phasor measurement systems by disabling one of the current measurement channels of a PMU installed across the network. This limitation has less effect compared to the single PMU outage. Therefore, the proposed model for this problem is presented as follows.

$$min \quad \sum\nolimits_{b\in\mathfrak{B}} x_b \tag{20a}$$
$$s.t. \quad remove\ each\ of\ the\ current\ measurement$$
$$channels\ of\ each\ of\ the\ PMUs\ iteratively \tag{20b}$$
$$Evaluate\ the\ observability\ of\ all\ buses$$
$$for\ each\ of\ the\ contingencies \tag{20c}$$
$$f_b \geq 1,\ \forall b \in \mathfrak{B},\ \forall\, contingency \tag{20d}$$

In model (20), similar to the models in sections IV-C and IV-D, the observability status of all the network buses are evaluated considering each of the contingencies of removing one of the current measurement channels of PMUs.

V. CASE STUDY

To evaluate the effectiveness of our proposed heuristic solution algorithm, we conducted numerical experiments and provided insights using the IEEE 14, 30, 39, and 57 bus standard test networks. Moreover, to prove the efficacy of our proposed approach, we also included a large-scale network, the Polish 2383-bus network, in our simulation results. Additionally, we compared the results of the proposed heuristic approach with the previous studies to show the superior performance of our proposed approach in terms of reducing the minimum required number of PMUs for complete power system observability.

A. STANDARD TEST NETWORKS

In this section, we introduce the standard test networks that we used in this paper to evaluate the effectiveness of the proposed methods.

1) IEEE 14-Bus: This is an IEEE standard test network with 14 buses, 15 lines, and one ZI bus.
2) IEEE 30-Bus: This is an IEEE standard test network with 30 buses, 34 lines, and six ZI buses.
3) New-England 39-Bus: This a standard test network widely studied in the literature with 39 buses, 34 lines, and 12 ZI buses.
4) IEEE 57-Bus: This is an IEEE standard test network with 57 buses, 62 lines, and 17 ZI buses.
5) Polish 2383-Bus: This is a large-scale test network with 2383 buses, 2896 lines, and 552 ZI buses.

A summary of the key information for these test networks is shown in Table 1.

TABLE 1. Key information of the test networks.

Test Network	Base Data	
	Number of network buses (N)	Number of ZI buses (Z)
IEEE 14-Bus	14	1
IEEE 30-Bus	30	6
New-England 39-Bus	39	12
IEEE 57-Bus	57	17
Polish 2383-Bus	2383	552

B. NUMERICAL RESULTS

To assess the performance of our proposed model, we implemented the basic formulation of the existing model (3) using GAMS language ver. 25.0.2 and CPLEX solver [45]. Moreover, our proposed heuristic approach was executed using MATLAB with GA with the population of 1000, mutation rate of 0.1, and a uniform crossover function. The computational times for solving the proposed OPP problems using our proposed heuristic approach are from 3.6 seconds for IEEE 14-Bus in normal operating condition to 478.3 seconds for IEEE 57-Bus under single-line outage contingency for complete observability. Moreover, the computation time for the Polish 2383-bus test system in normal operating condition is 637.6 seconds. We implemented the OPP problems on a personal computer running Windows 10 with Intel Core i9 @2.5 gigahertz with 32 gigabytes of installed RAM. To measure the effectiveness of our proposed model, we compared the results of our proposed heuristic approach with the basic MILP formulation of the OPP problem (i.e., model (3)) to the four standard test networks described in section V-A. The results are described in Table 2.

TABLE 2. Minimum number of PMUs obtained by the existing model and our proposed heuristic approach.

Test Networks	Proposed heuristic approach (13)		Model (3)	
	Minimum required PMUs	Number of redundant observable buses	Minimum required PMUs	Number of redundant observable buses
IEEE 14-Bus	3	2	3	2
IEEE 30-Bus	7	5	7	5
New-England 39-Bus	8	6	9	8
IEEE 57-Bus	11	9	14	13
Polish 2383-Bus	549	362	672	438

TABLE 3. Minimum number of PMUs obtained by our proposed heuristic approach considering each of the shortcomings of ZI buses separately.

Test Networks	Minimum required number of PMUs for complete observability considering			
	Measurement redundancy	NZI buses between ZI buses	SCMZIBs	Complete model
New-England 39-Bus	10	9	8	8
IEEE 57-Bus	15	13	12	11

Table 2 clearly shows the effectiveness of our proposed heuristic approach to reduce the number of required PMUs for complete power system observability.

The IEEE 14-Bus network has only one ZI bus and therefore, model (3) can converge to the optimal solution. Although IEEE 30-Bus network has six ZI buses, the maximum number of connected ZI buses in this power network equals two. Therefore, model (3) could track the observability status of all the network buses and our proposed method could not achieve lower number of required PMUs. However, for the New-England 39-Bus network, our proposed heuristic approach obtains a better solution than model (3). The existence of practical situations of ZI buses such as SCMZIBs in this power network made it difficult for model (3) to track the observability status of all the network buses. The final solution of the proposed heuristic approach for IEEE 57-bus power network resulted in almost 21% fewer number of PMUs than model (3), which is about 21% lower cost. Moreover, our proposed heuristic approach resulted in 18.3% fewer PMUs compared to model (3) for the Polish 2383-bus network. This less reduction in the number of PMUs compared to the IEEE 57-bus network is mainly because of the percentage of ZI buses in the power system. For the IEEE 57-bus network, almost 30% of the network buses are ZI buses compared to the Polish 2383-bus network in which 23% of the network buses are ZI buses.

It is worth mentioning that although we changed the observability status variable from integer to binary to avoid the inaccurate effect of measurement redundancy on the observability analysis considering ZI buses, there still can be some of the network buses with measurement redundancy. In other words, the state of some of the network buses can be observable with more than one PMU across the network. Therefore, we also derived the number of buses that can be observed with more than one PMU or through more than one observability path which is shown in Table 2. It clearly shows that our proposed heuristic approach wich resulted in fewer number of PMUs has also less number of redundant observable buses compared to model (3).

Moreover, Table 3 presents the minimum required number of PMUs obtained by our proposed heuristic solution algorithm considering each of the explained shortcomings of modeling ZI buses in the basic formulation separately. As explained in section II-B1, not considering measurement redundancy of some of the network buses may result in inaccurate satisfaction of state observability and therefore, the obtained PMU placement cannot guarantee the complete state observability of the power system. As seen from Table 3, considering the effect of measurement redundancy resulted in a higher number of PMUs for complete state observability which reflects that the optimal solution of model (3) led to inaccurate solution of the OPP problem. However, considering the other two shortcomings including NZI buses between ZI buses and SCMZIBs resulted in lower number of PMUs. Moreover, Table 3 effectively shows that the inclusion of SCMZIBs has the highest effect on reducing the number of PMUs compared to other situations. Specifically, for IEEE 57-Bus network with the maximum number of SCMZIBs being 11 (which means that in this network there is a situation that 11 ZI buses are connected together in series), 20% reduction in required number of PMUs is obtained. Furthermore, considering all the shortcomings explained in section II-B1 in our proposed heuristic method (13), the required number of PMUs can be reduced while ensuring complete state observability of the power networks.

Although a larger number of network buses cannot definitely result in a larger number of PMUs for power network observability, the effectiveness of our proposed heuristic approach for power networks with larger number of buses and ZI buses can be clearly seen in Table 2. Therefore,

TABLE 4. Comparing the solutions of single outage contingencies obtained by our proposed heuristic algorithm and basic formulation.

Test Network	Minimum required number of PMUs for complete state observability considering single outage contingency					
	Transmission line		PMU		Current measurement channel	
	Proposed model (18)	Basic formulation	Proposed model (19)	Basic formulation	Proposed model (20)	Basic formulation
IEEE 14-Bus	7	7	7	7	7	7
IEEE 30-Bus	11	12	14	16	11	13
New-England 39-Bus	12	14	17	21	14	17
IEEE 57-Bus	19	24	22	31	19	25

TABLE 5. Selected buses without communication infrastructure.

Test Network	Buses without communication infrastructure
IEEE 14-Bus	4-10-13
IEEE 30-Bus	3-5-7-12-19-23
New-England 39-Bus	2-8-11-17-23-26-29-39
IEEE 57-Bus	1-3-9-13-21-25-36-39-51-53-57

TABLE 6. Minimum number of PMUs considering lack of communication infrastructure in some of the network buses.

Test Networks	Number of buses without communication infrastructure	Minimum required number of PMUs	
		Complete state observability	Depth of one unobservability
IEEE 14-Bus	3	3	3
IEEE 30-Bus	6	8	5
New-England 39-Bus	8	11	10
IEEE 57-Bus	11	13	8

comprehensively modeling the effect of ZI buses has a significant impact on reducing the cost of PMU installment in the power network.

C. RESULTS OF PRACTICAL LIMITATIONS

In this section, we apply the proposed models (14), (16), (17), (18), (19), and (20) on the four test networks. The optimal solutions of considering single outage contingencies of PMUs, transmission lines, and current measurement channels of PMUs considering different levels of investment budget are presented in Tables 9 to 20, respectively in appendix.

Tables 9 to 20 effectively show how increased level of investment budget can increase the observability of power networks considering the single outage contingencies. Given the optimal solutions by changing the investment budget, the system planner can observe the optimal trade-off between the investment and observability. Moreover, Table 4 presents the minimum required number of PMUs for complete state observability considering each of the single outage contingencies obtained by using our proposed heuristic solution algorithm and the basic formulation that has shortcomings in terms of accurately modeling the effect of ZI buses. Our proposed heuristic algorithm results in up to 29% reduction in the required number of PMUs for IEEE 57-Bus network compared to the basic formulation for outage contingency of single PMUs. Furthermore, as seen in Table 4, single outage contingency of PMUs has more limitations on the OPP problem and therefore, considering this contingency to improve the observability of power system requires more number of PMUs.

We also applied the practical constraint of the lack of communication infrastructure in some of the network buses proposed in model (17) for all of the standard test networks described in section V-A. To illustrate the lack of PMU positioning sites, the buses without the possibility of placing PMUs on them are selected as in Table 5. The selection of these buses is designed in a way that a subset of buses associated with the optimal solution for complete state observability in normal operating conditions is included in these sets. Therefore, considering the inability of the problem

TABLE 7. Selected buses without communication infrastructure.

Test Network	Buses without communication infrastructure
IEEE 14-Bus	2-7-8-9
IEEE 30-Bus	1-4-10-24-29-30
New-England 39-Bus	3-6-14-20-25-28-34-38
IEEE 57-Bus	2-5-10-11-12-14-15-23-31-34-42-44-54-56

TABLE 8. Minimum required number of PMUs considering lack of communication infrastructure in more network buses.

Test Networks	Number of buses without communication infrastructure	Complete state observability		Depth of one unobservability	
		Minimum required PMUs	Number of unobservable buses	Minimum required PMUs	Number of unobservable buses
IEEE 14-Bus	7	5	2	3	1
IEEE 30-Bus	12	8	1	7	-
New-England 39-Bus	16	10	4	9	4
IEEE 57-Bus	25	14	2	9	-

to install PMUs on those buses, its capability to achieve the best solution is evaluated by taking into account the buses without communication infrastructure.

Since the lack of PMU positioning on some of the network buses may result in higher number of PMUs for complete state observability, we also applied the concept of depth of one unobservability proposed in model (16). The results of applying the proposed models for the situations described in Table 5 are presented in Table 6.

It is seen that having limited communication infrastructure results in an increased required number of PMUs for complete state observability and incomplete observability with depth of one unobservability and therefore, increased investment in placing PMUs. For example, New-England 39-Bus test network requires eight PMUs for complete state observability in the normal operating condition. However, having eight buses without communication infrastructure resulted in increasing the minimum required number of PMUs to 11 and 10 for complete state observability and for depth of one unobservability, respectively. It is also seen that considering the depth of one unobservability resulted in less number of required PMUs compared to the complete state observability situation. It is also worth mentioning that although IEEE 57-Bus requires higher PMUs for complete state observability than New-England 39-Bus, the number of PMUs required for depth of one unobservability for IEEE 57-Bus is less than that of New-England 39-Bus. Therefore, for IEEE 57-Bus, the concept of depth of one unobservability facilitates estimating the state of many of the network buses with a good approximation from the adjacent observable buses.

Another situation to be considered is that with an increase in the number of buses without communication capabilities, conditions may arise such that even if PMUs are deployed in all possible buses, complete state observability may not be achievable even considering the depth of one unobservability. In such cases, the problem-solving process should be directed towards finding solutions that minimize the number of unobservable buses. To achieve this objective, it is necessary to consider both partial observability with maximum number of observable buses and depth of one unobservability. Therefore, in this case, we increase the number of buses without communication infrastructure and consider the combination of models (17), (16), and (14). The new sets of buses without communication infrastructure for the test networks are shown in Table 7. The selection of these buses is also based on considering more number of buses from the optimal solution for complete state observability in normal operating conditions.

The results of applying the proposed models with the heuristic approach for observability evaluation are demonstrated in Table 8.

The results in Table 8 indicate that increasing the constraints on communication capabilities leads to an increase in the required number of PMUs in certain scenarios. Additionally, it is observed that considering depth of one unobservability, some network buses remain unobservable. These buses are those that do not satisfy any of the observability depth situations. The only buses that do not comply with the situation even with an increase in observability depth are the terminal buses. For example, bus 34 in the New-England 39-Bus network is a terminal bus, and since the installation of a PMU in buses 20 and 34 is not possible due to the lack of communication capabilities, this bus remains unobservable even with considering depth of one unobservability. Furthermore, the inability to observe some buses results in a lower required number of PMUs to observe the rest of the network buses. Another important observation from Table 8 is the significance of analyzing incomplete observability and understanding the concept of observability depth. Although in the analysis of complete observability for IEEE 57-Bus network, the possibility of observing two buses from the network buses does not exist, the concept of depth of one unobservability allows obtaining an arrangement of PMUs in which the phasors of unobservable buses are estimated from the state of adjacent buses.

VI. CONCLUSION

In this paper, we proposed a novel heuristic solution algorithm for the OPP problem by comprehensively modeling ZI buses. We presented the shortcomings of the existing OPP models in considering ZI buses including (1) measurement redundancy, (2) SCMZIBs, and (3) special cases of NZI buses between ZI buses. The core novelty of this paper lies in consecutively analyzing ZI bus topologies to develop enhanced observability evaluation analysis and proposing a new heuristic approach for the OPP problem.

TABLE 9. Optimal solutions for IEEE 14-Bus power network for single transmission line outage contingencies and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
3	5	2-6-9
4	8	2-9-11-13
5	12	1-4-9-11-13
6	13	2-4-6-9-11-13
7	14	2-4-5-9-11-12-13

TABLE 10. Optimal solutions for IEEE 30-Bus power network for single transmission line outage contingencies and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
7	18	1-7-10-12-19-24-27
8	23	1-5-8-10-12-19-24-29
9	26	1-6-7-10-12-15-19-24-29
10	29	1-5-6-10-12-15-16-19-24-30
11	30	1-3-5-6-10-12-15-17-19-24-29

TABLE 11. Optimal solutions for New-England 39-Bus power network for single transmission line outage contingencies and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
8	22	3-8-10-16-20-23-25-29
9	31	6-13-16-18-20-23-25-29-39
10	34	6-13-16-18-20-21-23-25-29-39
11	37	2-6-8-13-16-18-20-21-23-25-29
12	39	3-6-8-13-16-20-21-23-25-26-29-39

TABLE 12. Optimal solutions for IEEE 57-Bus power network for single transmission line outage contingencies and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
11	31	1-4-9-20-25-29-32-38-51-54-56
12	34	1-5-9-18-25-27-29-32-38-51-53-56
13	42	1-6-9-18-23-25-29-32-38-47-51-54-56
14	46	1-6-9-14-18-20-25-27-29-32-38-51-54-56
15	50	1-6-12-18-20-25-27-29-32-38-41-47-51-54-56
16	52	1-6-12-14-18-20-25-27-29-31-36-38-41-51-54-56
17	54	1-6-12-19-20-25-27-29-30-32-38-41-47-51-53-54-56
18	55	1-6-9-12-14-18-20-25-27-29-30-32-38-41-50-53-54-56
19	57	1-3-6-9-12-14-18-20-27-29-30-32-38-41-50-51-53-54-56

TABLE 13. Optimal solutions for IEEE 14-Bus power network for single PMU outage contingencies and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
3	2	2-6-9
4	7	4-5-6-9
5	11	2-4-6-9-13
6	13	2-4-6-9-11-13
7	14	2-4-5-6-9-10-13

TABLE 14. Optimal solutions for IEEE 30-Bus power network for single PMU outage contingencies and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
7	8	1-7-10-12-19-24-27
8	10	1-2-10-12-13-18-24-27
9	17	1-2-6-10-12-13-15-19-27
10	23	2-3-6-10-12-13-15-19-24-30
11	26	1-7-10-12-13-15-17-19-24-27-29
12	28	1-2-5-10-12-13-15-17-19-24-27-30
13	29	1-2-5-10-12-13-15-17-18-19-24-27-29
14	30	1-2-4-7-10-12-13-15-17-18-20-24-27-30

TABLE 15. Optimal solutions for New-England 39-Bus power network for single PMU outage contingencies and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
8	4	8-13-16-18-20-23-25-29
9	13	3-8-12-13-16-20-23-25-29
10	19	6-8-13-16-18-20-23-25-29-30
11	23	2-4-8-11-13-16-20-23-25-26-29
12	29	6-8-12-16-18-20-23-25-26-29-30-32
13	33	2-6-8-10-13-16-18-20-23-25-26-29-35
14	34	2-3-6-8-10-13-16-17-20-23-25-29-34-35
15	37	3-6-8-10-12-16-20-21-23-25-26-29-30-34-36
16	38	2-6-8-12-13-16-18-20-23-25-26-29-34-35-36-38
17	39	2-6-8-12-16-18-20-21-23-25-26-29-32-34-36-37-38

TABLE 16. Optimal solutions for IEEE 57-Bus power network for single PMU outage contingencies and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
11	10	1-4-9-20-25-28-32-38-51-53-56
12	19	1-5-9-19-25-29-32-38-47-51-54-56
13	25	1-6-9-18-20-25-29-32-38-46-51-53-56
14	33	1-6-9-18-20-25-27-29-32-38-46-51-54-56
15	40	1-6-9-18-20-25-27-29-32-35-38-47-51-54-56
16	41	1-6-9-12-18-22-25-27-29-31-32-38-46-50-54-56
17	48	1-6-9-18-20-25-27-29-31-32-37-38-41-46-51-53-56
18	51	1-6-9-12-18-20-25-27-29-32-35-38-42-47-51-53-54-56
19	52	1-4-6-12-14-20-25-27-29-31-32-38-39-41-49-51-53-54-56
20	54	1-3-4-6-12-14-20-25-27-29-31-32-38-39-41-49-51-53-54-56
21	55	1-3-6-9-12-18-20-25-27-29-30-32-33-35-38-41-47-51-53-54-56
22	57	1-3-6-9-12-18-20-25-27-29-30-32-33-35-38-41-47-50-51-53-54-56

We applied the proposed heuristic solution algorithm to four standard test networks: IEEE 14-bus, IEEE 30-bus, IEEE 57-bus, and the New-England 39-Bus power networks. The numerical results demonstrate that our proposed heuristic approach outperforms the existing approaches in terms of reducing the required number of PMUs for complete state observability. Specifically, our proposed heuristic approach reduces the number of PMUs for IEEE 57-bus power network by 21% compared to the existing studies, which is a significant achievement due to the high cost of PMUs.

TABLE 17. Optimal solutions for IEEE 14-Bus power network for single outage contingencies of current measurement channels and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
3	5	2-6-9
4	8	2-9-11-13
5	12	2-4-9-11-13
6	13	1-4-9-11-12-13
7	14	2-4-5-6-9-11-13

TABLE 18. Optimal solutions for IEEE 30-Bus power network for single outage contingencies of current measurement channels and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
7	21	1-7-10-12-18-24-27
8	23	2-3-5-10-12-18-24-27
9	27	2-3-5-10-12-15-19-24-27
10	29	2-3-5-10-12-15-16-19-24-27
11	30	2-3-7-10-12-13-15-16-19-23-27

TABLE 19. Optimal solutions for New-England 39-Bus power network for single outage contingencies of current measurement channels and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
8	18	3-8-13-16-20-23-25-29
9	25	6-8-13-16-18-20-23-25-29
10	32	6-10-16-18-20-21-23-25-29-39
11	36	3-7-10-16-20-22-23-26-29-37-39
12	37	3-7-10-16-20-22-23-26-29-37-38-39
13	38	3-7-10-16-20-22-23-26-29-34-37-38-39
14	39	3-7-10-16-20-22-23-26-29-34-36-37-38-39

TABLE 20. Optimal solutions for IEEE 57-Bus power network for single outage contingencies of current measurement channels and different investment budgets.

Number of PMUs	Maximum number of observable buses	Placement strategy
11	31	1-4-9-20-25-28-32-38-51-53-56
12	40	1-4-9-14-20-25-29-32-38-51-54-56
13	43	1-5-9-14-18-23-25-29-32-38-51-54-56
14	47	1-5-9-14-19-23-25-29-32-38-41-51-54-56
15	49	1-4-9-12-14-20-25-27-29-32-38-41-51-54-56
16	51	1-4-9-12-19-23-25-29-32-38-41-46-51-53-54-56
17	54	1-6-9-12-14-19-23-27-29-32-38-42-51-53-54-56
18	56	1-2-6-12-19-23-27-29-30-32-38-41-47-50-51-53-55-56
19	57	1-2-6-12-19-23-27-29-30-32-33-38-41-47-50-51-53-55-56

This reduction in the number of PMUs with comprehensively modeling the effect of ZI buses is a testament to the superior performance of our proposed method.

Moreover, we studied important practical aspects of the OPP problem including (1) limited investment budget; (2) lack of PMU positioning sites; (3) incomplete observability; and (4) single outage contingencies of PMUs, transmission lines, and current measurement channels. We proposed new optimization models for the OPP problem considering each of the practical limitations using our proposed heuristic solution algorithm. Our proposed heuristic solution algorithm reduces the number of required PMUs for complete state observability under single outage contingencies up to 29% compared to the existing studies. Furthermore, studying the optimal solutions considering single outage contingencies by changing the investment budget provides insights into the trade-offs between investment and observability, and determining the most constrained practical limitation of the OPP problem which is the single outage of PMUs.

In conclusion, this research has demonstrated the superior performance of our proposed observability analysis approach and OPP problem in addressing the effect of ZI buses to reduce the minimum required number of PMUs for complete state observability. This study can be further extended to maximize the redundancy of the measurements as illustrated in [13] and [14] while accurately modeling the effect of ZI buses as an indication of the reliability of the phasor measurement system.

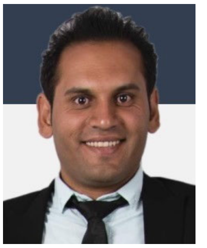
APPENDIX
OPTIMAL SOLUTIONS OF SINGLE OUTAGE CONTINGENCIES

To avoid making the manuscript very long, this Appendix presents the optimal solutions for single outage contingencies considering different levels of investment budget for the four test networks (i.e., IEEE 14-Bus, IEEE 30-Bus, New-England 39-Bus, and IEEE 57-Bus) in Tables 9 to 20.

REFERENCES

[1] A. G. Phadke and J. S. Thorp, *Synchronized Phasor Measurements and Their Applications*, vol. 1. New York, NY, USA: Springer, 2008.
[2] G. Krumpholz, K. Clements, and P. Davis, "Power system observability: A practical algorithm using network topology," *IEEE Trans. Power App. Syst.*, vol. PAS-99, no. 4, pp. 1534–1542, Jul. 1980.
[3] T. L. Baldwin, L. Mili, M. B. Boisen, and R. Adapa, "Power system observability with minimal phasor measurement placement," *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 707–715, May 1993.
[4] B. Gou, "Generalized integer linear programming formulation for optimal PMU placement," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1099–1104, Aug. 2008.
[5] M. K. Arpanahi, H. H. Alhelou, and P. Siano, "A novel multiobjective OPP for power system small signal stability assessment considering WAMS uncertainties," *IEEE Trans. Ind. Informat.*, vol. 16, no. 5, pp. 3039–3050, May 2020.
[6] R. F. Nuqui and A. G. Phadke, "Phasor measurement unit placement techniques for complete and incomplete observability," *IEEE Trans. Power Del.*, vol. 20, no. 4, pp. 2381–2388, Oct. 2005.
[7] M. A. R. S. Cruz, H. R. O. Rocha, M. H. M. Paiva, J. A. L. Silva, E. Camby, and M. E. V. Segatto, "PMU placement with multi-objective optimization considering resilient communication infrastructure," *Int. J. Electr. Power Energy Syst.*, vol. 141, Oct. 2022, Art. no. 108167.
[8] X.-C. Guo, C.-S. Liao, and C.-C. Chu, "Enhanced optimal PMU placements with limited observability propagations," *IEEE Access*, vol. 8, pp. 22515–22524, 2020.
[9] M. E. Hamedani Golshan, S. H. Hosseini Dolatabadi, and S. M. Tabatabaei, "Determining minimum number and optimal placement of PMUs for fault observability in one-terminal algorithms," *IET Gener., Transmiss. Distrib.*, vol. 12, no. 21, pp. 5789–5797, Nov. 2018.
[10] S. H. Hosseini Dolatabadi and M. E. Hamedani Golshan, "Fault location observability rules for impedance-based fault location algorithms," *Electric Power Syst. Res.*, vol. 224, Nov. 2023, Art. no. 109771.
[11] N. H. Abbasy and H. M. Ismail, "A unified approach for the optimal PMU location for power system state estimation," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 806–813, May 2009.
[12] F. Aminifar, M. Fotuhi-Firuzabad, M. Shahidehpour, and A. Khodaei, "Probabilistic multistage PMU placement in electric power systems," *IEEE Trans. Power Del.*, vol. 26, no. 2, pp. 841–849, Apr. 2011.
[13] N. P. Theodorakatos, R. Babu, and A. P. Moschoudis, "The branch-and-bound algorithm in optimizing mathematical programming models to achieve power grid observability," *Axioms*, vol. 12, no. 11, p. 1040, Nov. 2023.
[14] N. P. Theodorakatos, A. P. Moschoudis, and R. Babu, "Calculating global minimum points to binary polynomial optimization problem: Optimizing the optimal PMU localization problem as a case-study," *J. Phys., Conf. Ser.*, vol. 2701, no. 1, Feb. 2024, Art. no. 012001.
[15] N. U. Nimbalkar and Dr. P. M. Joshi, "Optimal PMU placement using ILP and ACO: A comparative study," in *Proc. Global Conf. for Advancement Technol. (GCAT)*, Oct. 2019, pp. 1–4.
[16] M. Korkali and A. Abur, "Impact of network sparsity on strategic placement of phasor measurement units with fixed channel capacity," in *Proc. IEEE Int. Symp. Circuits Syst.*, May 2010, pp. 3445–3448.
[17] N. M. Manousakis and G. N. Korres, "An advanced measurement placement method for power system observability using semidefinite programming," *IEEE Syst. J.*, vol. 12, no. 3, pp. 2601–2609, Sep. 2018.
[18] R. Andreoni, D. Macii, M. Brunelli, and D. Petri, "Tri-objective optimal PMU placement including accurate state estimation: The case of distribution systems," *IEEE Access*, vol. 9, pp. 62102–62117, 2021.
[19] B. K. Saha Roy, A. K. Sinha, and A. K. Pradhan, "An optimal PMU placement technique for power system observability," *Int. J. Electr. Power Energy Syst.*, vol. 42, no. 1, pp. 71–77, Nov. 2012.
[20] D. Dua, S. Dambhare, R. K. Gajbhiye, and S. A. Soman, "Optimal multistage scheduling of PMU placement: An ILP approach," *IEEE Trans. Power Del.*, vol. 23, no. 4, pp. 1812–1820, Oct. 2008.
[21] F. Aminifar, A. Khodaei, M. Fotuhi-Firuzabad, and M. Shahidehpour, "Contingency-constrained PMU placement in power networks," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 516–523, Feb. 2010.
[22] M. Khodadadi Arpanahi, R. Torkzadeh, A. Safavizadeh, A. Ashrafzadeh, and F. Eghtedarnia, "A novel comprehensive optimal PMU placement considering practical issues in design and implementation of a wide-area measurement system," *Electric Power Syst. Res.*, vol. 214, Jan. 2023, Art. no. 108940.
[23] K. S. K. Reddy, D. A. K. Rao, A. Kumarraja, and B. R. K. Varma, "Implementation of integer linear programming and exhaustive search algorithms for optimal PMU placement under various conditions," in *Proc. IEEE Power, Commun. Inf. Technol. Conf. (PCITC)*, Oct. 2015, pp. 850–855.
[24] S. Azizi, A. S. Dobakhshari, S. A. N. Sarmadi, and A. M. Ranjbar, "Optimal PMU placement by an equivalent linear formulation for exhaustive search," *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 174–182, Mar. 2012.
[25] M. Mahgoub, A. Gastli, and L. Ben-Brahim, "Optimal PMU placement for large practical systems Qatar grid as an example," in *Proc. 9th IEEE-GCC Conf. Exhib. (GCCCE)*, May 2017, pp. 1–9.
[26] C. Lu, Z. Wang, M. Ma, R. Shen, and Y. Yu, "An optimal PMU placement with reliable zero injection observation," *IEEE Access*, vol. 6, pp. 54417–54426, 2018.
[27] S. Azizi, G. B. Gharehpetian, and A. S. Dobakhshari, "Optimal integration of phasor measurement units in power systems considering conventional measurements," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 1113–1121, Jun. 2013.
[28] D. R. Shrivastava, S. A. Siddiqui, and K. Verma, "Optimal PMU placement for complete power system observability under (P–1) contingency," in *Proc. IEEE Int. WIE Conf. Electr. Comput. Eng. (WIECON-ECE)*, Dec. 2017, pp. 129–132.
[29] M. Laouamer, A. Kouzou, R. D. Mohammedi, and A. Tlemçani, "Optimal PMU placement in power grid using sine cosine algorithm," in *Proc. Int. Conf. Appl. Smart Syst. (ICASS)*, Nov. 2018, pp. 1–5.
[30] M. Mandich, T. Xia, and K. Sun, "Optimal PMU placement using stochastic methods," in *Proc. IEEE Power Energy Soc. Gen. Meeting (PESGM)*, Aug. 2019, pp. 1–5.
[31] M. Esmaili, K. Gharani, and H. A. Shayanfar, "Redundant observability PMU placement in the presence of flow measurements considering contingencies," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3765–3773, Nov. 2013.
[32] P. K. Ghosh, S. Chatterjee, and B. K. Saha Roy, "Optimal PMU placement solution: Graph theory and MCDM-based approach," *IET Gener., Transmiss. Distrib.*, vol. 11, no. 13, pp. 3371–3380, Sep. 2017.
[33] E. Abiri, F. Rashidi, and T. Niknam, "An optimal PMU placement method for power system observability under various contingencies," *Int. Trans. Electr. Energy Syst.*, vol. 25, no. 4, pp. 589–606, Apr. 2015.

[34] F. Rashidi, E. Abiri, T. Niknam, and M. R. Salehi, "Optimal placement of PMUs with limited number of channels for complete topological observability of power systems under various contingencies," *Int. J. Electr. Power Energy Syst.*, vol. 67, pp. 125–137, May 2015.
[35] E. Abiri, F. Rashidi, T. Niknam, and M. R. Salehi, "Optimal PMU placement method for complete topological observability of power system under various contingencies," *Int. J. Electr. Power Energy Syst.*, vol. 61, pp. 585–593, Oct. 2014.
[36] S. P. Singh and S. P. Singh, "A novel multi-objective PMU placement method for power system state estimation," in *Proc. Int. Electr. Eng. Congr. (iEECON)*, Mar. 2018, pp. 1–4.
[37] A. Enshaee, R. A. Hooshmand, and F. H. Fesharaki, "A new method for optimal placement of phasor measurement units to maintain full network observability under various contingencies," *Electric Power Syst. Res.*, vol. 89, pp. 1–10, Aug. 2012.
[38] A. Bashian, M. Assili, and A. Anvari-Moghaddam, "A security-based observability method for optimal PMU-sensor placement in WAMS," *Int. J. Electr. Power Energy Syst.*, vol. 121, Oct. 2020, Art. no. 106157.
[39] X.-C. Guo, C.-S. Liao, and C.-C. Chu, "Probabilistic optimal PMU placements under limited observability propagations," *IEEE Syst. J.*, vol. 16, no. 1, pp. 767–776, Mar. 2022.
[40] G. N. Korres, N. M. Manousakis, T. C. Xygkis, and J. Löfberg, "Optimal phasor measurement unit placement for numerical observability in the presence of conventional measurements using semi-definite programming," *IET Gener., Transmiss. Distribution*, vol. 9, no. 15, pp. 2427–2436, Nov. 2015.
[41] N. M. Manousakis and G. N. Korres, "Optimal allocation of phasor measurement units considering various contingencies and measurement redundancy," *IEEE Trans. Instrum. Meas.*, vol. 69, no. 6, pp. 3403–3411, Jun. 2020.
[42] A. Monticelli, "Electric power system state estimation," *Proc. IEEE*, vol. 88, no. 2, pp. 262–282, Feb. 2000.
[43] A. J. Monticelli, A. V. Garcia, and I. W. Slutsker, "Handling discardable measurements in power system state estimation," *IEEE Trans. Power Syst.*, vol. 7, no. 3, pp. 1333–1340, Oct. 1992.
[44] N. P. Theodorakatos, R. Babu, and A. P. Moschoudis, "An incomplete observability-constrained PMU allocation problem by using mathematical and evolutionary algorithms," *J. Phys., Conf. Ser.*, vol. 2701, no. 1, Feb. 2024, Art. no. 012013.
[45] IBMI Complex, "V12. 1: User's manual for CPLEX," *Int. Bus. Mach. Corp.*, vol. 46, no. 53, p. 157, 2009.



SAYED HAMID HOSSEINI DOLATABADI was born in Isfahan, Iran, in 1987. He received the B.Sc. and M.Sc. degrees in electrical engineering from Isfahan University of Technology, in 2009 and 2015, respectively. He is currently pursuing the Ph.D. degree with the Department of Mechanical Engineering, The University of Texas at San Antonio. His research interests include power system operation and planning, operations research and optimization, local energy markets, machine learning, and renewable and flexible resources.

TANVEER HOSSAIN BHUIYAN received the Ph.D. degree in industrial and systems engineering from the University of Tennessee, Knoxville. He is currently an Assistant Professor with the Department of Mechanical Engineering, The University of Texas at San Antonio. His expertise is in developing data-driven optimization and big-data analytics methods for solving complex decision-making problems in the areas of power systems, transportation and supply chain, and security of cyber-physical systems.

MOHAMMAD ESMAIL HAMEDANI GOLSHAN (Senior Member, IEEE) was born in Isfahan, Iran, in 1964. He received the B.Sc. degree in electrical engineering from Isfahan University of Technology, in 1987, the M.Sc. degree in electrical engineering from Sharif University of Technology, in 1992, and the Ph.D. degree in electrical engineering from Isfahan University of Technology, in 1998. He is currently a Professor with the Department of Electrical and Computer Engineering, Isfahan University of Technology. His major research interests include power system analysis, power system dynamics, power system protection, power quality, smart grid, dispersed generation, and flexible ac transmission systems.