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RESEARCH ARTICLE

A Deadlock Prevention Strategy for Petri Nets Through Tuning Time Constraints

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ABSTRACT Deadlocks are of paramount importance in resource allocation systems, which are usually treated from the perspective of discrete event systems. This paper develops a deadlock prevention strategy for a system modeled with Petri nets, by endowing appropriate time constraints with certain transitions to schedule the firing priority of enabled transitions in a Petri net such that transition sequences leading to deadlocks are prohibited, i.e., expanding an untimed Petri net model into a time Petri net to prevent deadlocks. To increase the system permissiveness of a time Petri net with time constraints endowed, a control place is designed, which does not expand the reachable space of the original Petri net. The predominant role of the control place is to convert continuously enabled transitions at certain markings into newly enabled transitions, which can prolong the firing time of transitions. Furthermore, we propose a method that merely enumerates deadlock prevention condition inequalities to derive a series of time constraints by probing the connection between deadlock prevention conditions and transitions. The developed method only needs to designate time constraints for partial transitions. Examples are provided to demonstrate the effectiveness of the presented methodology.

INDEX TERMS Time Petri net, deadlock prevention, time constraint, discrete event system.

I. INTRODUCTION

The presence of resource sharing and competition within the various resource allocation systems such as flexible manufacturing systems may result in the occurrence of deadlocks [1]. Graph theory [2], [3], automata [4], [5] and Petri nets [6], [7], [8], [9] are three essential mathematical vehicles for dealing with typical deadlock issues. In comparison with graph theory and automata, Petri net models [10] are natural and compact in explicitly representing the behavioral characteristics of a system such as synchronization, asynchronization, and concurrency. As shown, Petri nets are widely used in supervisory control of discrete event systems, especially in the realm of deadlock analysis and control of resource allocation systems [11], [12], [13], [14].

There are three primary deadlock prevention approaches in the framework of Petri nets, i.e., reachability graph analysis, structural analysis, and their combination. The computational complexity, the structural complexity, and the permissive behavior of a controlled system design are the three primary criteria for assessing the merits of a control strategy [15]. The reachability graph analysis [16] conventionally focuses on constructing a reachability graph of a Petri net, providing a comprehensive guide for system analysis and control such that, under certain conditions, a maximally permissive supervisor represented by Petri nets can be derived [17].

The study in [18] defines marking/transition separation instances in a reachability graph and presents a deadlock

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prevention policy based on the theory of regions [19], where, for a marking/transition separation instance, a control place, computed by solving a family of linear inequalities, is used to prevent from enabled certain transitions whose firing may reach dead or bad markings at dangerous markings.

The structural analysis techniques usually focus on particular Petri net structural objects, such as siphons [20], [21], [22] and place invariants [23], [24], by which a deadlock control policy is derived. A seminal work reported in [25] explores the straightforward cause of deadlocks in a class of flexible manufacturing systems whose Petri net model has an explicit place partition such that deadlocks and the presence of unmarked strict minimal siphons are closely tied. With a known result in net theory that a Petri net is deadlock-free if no strict minimal siphons are emptied, it is technically convenient to derive a generalized mutual exclusion constraint (expressed by a linear inequality concerning markings) on the non-emptying of siphons, serving as a condition that ensures the number of tokens in a siphon to be one or more [26]. Since the controlled siphons may be restrained to retain more than one token, supervisors obtained by using siphon control for deadlock prevention are in general not maximally permissive.

As the scale of a Petri net increases, the reachability space, the number of siphons, and marking/transition separation instances are subject to exponential growth [26]. The work in [27] proposes notions of elementary and dependent siphons by exploring the algebraic relationships between strict minimal siphons. An algorithm to compute elementary siphons for a class of Petri nets is reported in [28]. Then, a control place is derived for elementary siphons such that dependent siphons can be implicitly controlled under particular conditions that are usually satisfied automatically in real-world systems thanks to an initial system configuration.

The work in [29] presents a deadlock prevention strategy consisting of two procedures. A siphons control method is first used for an original net to develop controllers that make it optimally invariant controlled. Then the theory of regions is utilized for deriving a group of supervisors such that the net is deadlock-free. The method in [29] reduces the computational cost and the size of supervisors compared with the approach using the theory of regions alone.

A supervisor for a Petri net model is usually represented by place controllers, called monitors. There are also transition controllers [30], [31], [32] called recovery transitions. The research in [31] proposes a resource flow graph for representing the competition among different processes for shared resources. A group of recovery transitions that can fire at dead markings is derived based on loop graphs and partial dead markings. The approach developed in [31] circumvents the issue of state explosion caused by the computation of reachability graphs.

To simplify the structural complexity of supervisors, deadlock prevention based on time constraints is of particular interest, i.e, expanding an untimed Petri net into a time Petri net to effectively prevent deadlocks [33], [34]. The study in [33] endows time intervals with untimed Petri nets in the form of parameters and proposes a symbolic reachability graph. Then, a group of time constraints such that the resulting Petri net is deadlock-free can be derived by solving a set of inequalities containing the deadlock prevention conditions of dangerous markings as well as the permissiveness conditions of legal markings [35]. However, this approach significantly impacts the permissive behavior of a plant and in extreme cases may lead to infeasibility of specific work processes.

Compared with place and transition controllers [20], [30], the deadlock prevention based on time constraints not only allows a controlled system to have a simple structure but also provides an accurate analysis of the real-time performance of concurrent systems [36], [37]. The main contributions of this article are summarized as follows.

- This paper explores some cases that deadlock prevention conditions and permissiveness conditions are mutually exclusive [33], [35]. To address partial mutual exclusion, a control place that does not influence the reachable space of an original Petri net is developed before endowing time constraints with transitions, which can enhance the behavioral permissiveness of a deadlock-free TPN extension of the Petri net.
- This paper explores the relationship between deadlock prevention conditions and transitions. Compared with the research in [33], this paper provides a method for solving merely the set of inequalities with deadlock prevention conditions, rather than solving a combination of deadlock prevention and permissiveness conditions. Furthermore, the proposed method requires assigning time constraints only to a subset of transitions, which not only simplifies the process of obtaining time constraints but also provides the maximum range of time constraints that makes a Petri net deadlock-free.

The rest of this article is organized as follows. Section II reviews the basic definitions of Petri nets and time Petri nets. Section III formalizes the problem statement. Section IV proposes an algorithm to design a control place that prevents partial permissiveness conditions and deadlock prevention conditions from being mutually exclusive. Then we present a procedure that only calculates the deadlock prevention condition inequalities to obtain a set of time constraints and formulate a novel deadlock prevention policy by endowing appropriate time constraints with transitions. Section V provides examples to illustrate the deadlock prevention policy. Finally, Section VI concludes this article.

II. PRELIMINARIES

A. PETRI NET

A generalized Petri net [38] is a quadruple $N = (P, T, Pre, Post)$, where P and T are finite sets of places and transitions, respectively, with $P \cap T = \emptyset$ and $P \cup T \neq \emptyset$. $Pre : P \times T \rightarrow \mathbb{N}^{|P| \times |T|}$ is the pre-incidence function that specifies arcs from places to transitions, where $\mathbb{N}$ is the set of non-negative integers, and $|P|$ and $|T|$ are the numbers of places and transitions, respectively. If there exits an arc with weight w from p to t, then $Pre(p, t) = w$; otherwise $Pre(p, t) = 0$. $Post : T \times P \to \mathbb{N}^{|P|\times|T|}$ is the post-incidence function that specifies arcs from transitions to places. If there exists an arc with weight w from t to p, then $Post(t, p) = w$; otherwise $Post(t, p) = 0$.

A marking of net N is a mapping $M : P \to \mathbb{N}^{|P|}$ and $M(p)$ presents the number of tokens in place $p \in P$ at marking M. A place $p \in P$ is marked at a marking M if there exists at least one token in p, i.e., $M(p) > 0$. We denote the initial marking of N by M_0. A two-tuple (N, M_0) is called a net system. A transition $t \in T$ is said to be enabled at a marking M, in symbols $M[t\rangle$, if for all $p \in P$, $M(p) \geq Pre(p, t)$ holds. We denote the set of transitions enabled at a marking M by $En(M)$, i.e., $En(M) = \{t \in T \mid M \geq Pre(\cdot, t)\}$. A transition t enabled at M is able to fire, yielding a new marking M', i.e., $M' = M + Post(t, \cdot) - Pre(\cdot, t)$, where $Pre(\cdot, t)$ (resp. $Post(t, \cdot)$) is the column vector associated with transition t in Pre (resp. $Post$). We use $M[t\rangle M'$ to denote that M' is reached from M by firing t.

Given a transition sequence $\sigma = t_1t_2 \ldots t_n \in T^*$, σ is enabled at a marking M if there exist markings $M_1, M_2, \ldots, M_n$ such that $M[t_1\rangle M_1[t_2\rangle M_2 \ldots M_{n-1}[t_n\rangle M_n$ holds. The length of σ is denoted by $|\sigma|$. We denote the number of t in σ by $y_\sigma(t)$, i.e., $y_\sigma(t) = n$ if t appears n times in σ.

Given a net system (N, M_0), the set of all reachable markings of N from M_0 is defined as $R(N, M_0) = \{M \in \mathbb{N}^{|P|} \mid (\exists\sigma \in T^*)\ M_0[\sigma\rangle M\}$. A Petri net system (N, M_0) is bounded if for all $M \in R(N, M_0)$ and for all $p \in P$, there exists $K \in \mathbb{N}$ such that $M(p) \leq K$ holds. A Petri net is said to be ordinary if any arc is with weight 1.

Given two markings $M, M' \in R(N, M_0)$ and a transition $t_f \in T$ with $M'[t_f\rangle M$, a transition $t \in En(M)$ is said to be newly enabled if $t = t_f$ or $M' - Pre(\cdot, t_f) < Pre(\cdot, t)$; otherwise, t is said to be continuously enabled. The sets of all newly and continuously enabled transitions at M reached from M' by firing t_f are denoted by $Nw(M', t_f) = \{t \in En(M) \mid (M'[t_f\rangle M) \wedge (t = t_f \vee M' - Pre(\cdot, t_f) < Pre(\cdot, t)\}$ and $Cw(M', t_f) = \{t \in En(M) \mid M'[t_f\rangle M \wedge t \notin Nw(M', t_f)\}$, respectively.

A net system (N, M_0) contains a deadlock if there exists a marking $M \in R(N, M_0)$, at which no transition is enabled, i.e., $En(M) = \emptyset$. Such markings are called dead markings whose set is defined as $\mathcal{M}_f = \{M \in R(N, M_0) \mid En(M) = \emptyset\}$. A net system (N, M_0) is deadlock-free if $\mathcal{M}_f = \emptyset$.

Given a net system (N, M_0), a marking $M \in R(N, M_0)$ is said to be *bad* if $En(M) \neq \emptyset$ and there does not exist $\sigma \in T^*$ such that $M[\sigma\rangle M_0$ holds. We denote by $\mathcal{M}_b = \{M \in R(N, M_0) \mid En(M) \neq \emptyset \wedge (\nexists\sigma \in T^*)\ M[\sigma\rangle M_0 \wedge (\exists\sigma \in T^*, \exists M' \in \mathcal{M}_f)\ M[\sigma\rangle M'\}$ the set of all bad markings of (N, M_0). A marking $M \in R(N, M_0)$ is said to be *dangerous* if (1) there exists $\sigma \in T^*$ such that $M[\sigma\rangle M_0$ holds; (2) there exist $t \in T$ and $M' \in \mathcal{M}_f \cup \mathcal{M}_b$ such that $M[\sigma'\rangle M'$ holds. Formally, the set of dangerous markings of (N, M_0) is $\mathcal{M}_d = \{M \in R(N, M_0) \mid (\exists\sigma \in T^*)\ M[\sigma\rangle M_0 \wedge (\exists t \in T, \exists M' \in \mathcal{M}_f \cup \mathcal{M}_b)\ M[\sigma'\rangle M'\}$.

Given a net system (N, M_0), a marking $M \in R(N, M_0)$ is said to be *sound* if it is none of a dangerous, bad, and dead marking. The set of sound markings is denoted by $\mathcal{M}_g = \{M \in R(N, M_0) \mid M \notin \mathcal{M}_f \cup \mathcal{M}_b \cup \mathcal{M}_d\}$, i.e., $\mathcal{M}_g = R(N, M_0) \setminus (\mathcal{M}_f \cup \mathcal{M}_b \cup \mathcal{M}_d)$. A marking $M \in R(N, M_0)$ is said to be *legal* if it is either sound or dangerous. The set of legal markings is then $\mathcal{M}_l = \mathcal{M}_g \cup \mathcal{M}_d$.

B. TIME PETRI NET

A time Petri net (TPN) [39] is a pair $N_t = (N, Is)$, where $N = (P, T, Pre, Post)$ is a generalized Petri net and $Is : T \to \mathbb{R}^+ \times (\mathbb{R}^+ \cup \{\infty\})$ is a static firing interval function that associates an interval with a transition in T, where $\mathbb{R}^+$ is the set of non-negative real numbers. In the context, we denote by (N_t, M_0) a TPN system. An untimed Petri net is equivalent to a TPN in which the static firing time interval of any transition is $[0, \infty]$ [33].

Given a transition $t_i \in T$, write $Is(t_i) = [l_i, u_i]$, where $0 \leq l_i < \infty$ and $l_i \leq u_i \leq \infty$. The lower bound l_i is called the earliest firing time of t_i, while the upper bound u_i is called the latest firing time of t_i. A transition $t_i \in T$ is able to fire if it remains enabled for a time interval contained in $[l_i, u_i]$.

A state class [40] of a TPN system (N_t, M_0) is denoted by $S_k = (M_k, \Phi_k)$, where $M_k \in R(N, M_0)$ is a reachable marking and $\Phi_k = \{l_i^k \leq \phi_i \leq u_i^k\}$ is a set of inequalities that relates to the firing time domain of an enabled transition at M_k, with i being the index of transitions enabled at M_k. An inequality $l_i^k \leq \phi_i \leq u_i^k$ in Φ_k implies that a transition t_i can fire at S_k after l_i^k time units are elapsed and before u_i^k time units are elapsed. The initial state class is denoted by $S_0 = (M_0, \Phi_0)$, where M_0 is the initial marking and $\Phi_0 = \{Is(t) | t \in En(M_0)\}$ is a set of inequalities that represent the static firing intervals of transitions enabled at M_0.

III. PROBLEM STATEMENT

A. PARAMETRIC TPN

Given an untimed net system (N, M_0) that is liable to deadlocks, our control goal is to expand it into a deadlock-free TPN by endowing time intervals with particular transitions to manage the firing of transitions. To formalize the problem statement, an untimed Petri net is first transformed into a parametric TPN, where the static firing time intervals of transitions in a TPN are in the forms of parameters $[a_i, b_i]$. The study in [33] introduces a new structure called a symbolic reachability graph (SRG) whose nodes are called extended markings denoted by a couple $\alpha = (M, \Delta)$, where M is a marking and $\Delta : En(M) \to \Gamma$ with Γ being the set of linear combinations with integer coefficients.

Given a net system (N, M_0), the initial extended marking of its parametric TPN is denoted by $\alpha_0 = (M_0, \Delta_0)$, where M_0 is the initial marking and Δ_0 satisfies the fact that for all transitions $t, t' \in En(M_0)$, $Is(t) = [a, b]$, $Is(t') = [a', b']$ and $\Delta_0(t, t') = b - a'$. Starting from $\alpha_0 = (M_0, \Delta_0)$, the

Algorithm 1 Calculation of a Symbolic Reachability Graph

Input: A net system (N, M_0).
Output: A symbolic reachability graph $\mathcal{S}r(N, \alpha_0)$.

Initialize the root node $\alpha_0 = (M_0, \Delta_0)$ and tag α_0 with "new";
while there exists a node tagged with "new" **do**
  Select a node $\alpha_c = (M_c, \Delta_c)$ tagged with "new";
  for all $t_f \in En(M_c)$ **do**
    $M_n = M_c - Pre(\cdot, t_f) + Post(t_f, \cdot)$;
    if $|En(M_n)| \leq 1$ **then**
      Δ_n is not defined;
    end if
    if $|En(M_n)| \geq 2$ **then**
      for all $t_p, t_q \in En(M_n)$ **do**
        if $t_p \in Nw(M_c, t_f)$ and $t_q \notin Nw(M_c, t_f)$ **then**
          $\Delta_n(t_p, t_q) = b_p + \Delta_c(t_f, t_q)$;
        else if $t_p \notin Nw(M_c, t_f)$ and $t_q \in Nw(M_c, t_f)$ **then**
          $\Delta_n(t_p, t_q) = \Delta_c(t_p, t_f) - a_q$;
        else if $t_p, t_q \in Nw(M_c, t_f)$ **then**
          $\Delta_n(t_p, t_q) = b_p - a_q$;
        else
          $\Delta_n(t_p, t_q) = \Delta_c(t_p, t_q)$;
        end if
      end for
    end if
    $\alpha_n = (M_n, \Delta_n)$;
    if node α_n does not exist in the graph **then**
      add a new node $\alpha_n = (M_n, \Delta_n)$;
      add an edge from α_c to α_n labeled t_f;
      tag α_n to "new";
    end if
  end for
  tag α_c to "old";
end while
Remove all tags.

successor extended markings can be calculated by using the following rules.

Let $\alpha = (M, \Delta)$ be an extended marking. If there exists $t_f \in En(M)$ that is firable from α, then its successor extended marking, represented by $Su(\alpha, t_f)$, is $\alpha' = (M', \Delta')$, satisfying the following three conditions:

- $M' = M - Pre(\cdot, t_f) + Post(t_f, \cdot)$
- $\forall t, t' \in En(M'), Is(t) = [a, b], Is(t') = [a', b'], \Delta'(t, t')$

$$= \begin{cases} b - a' & \text{if } t, t' \in Nw(M, t_f) \\ b + \Delta(t_f, t') & \text{if } t \in Nw(M, t_f) \wedge t' \notin Nw(M, t_f) \\ \Delta(t, t_f) - a' & \text{if } t \notin Nw(M, t_f) \wedge t' \in Nw(M, t_f) \\ \Delta(t, t') & \text{otherwise} \end{cases}$$

- $\Delta'$is not defined if $|En(M')| \leq 1$

Note that $\Delta'(t, t')$ is an upper bound of the difference between the firing instants of enabled transitions t and t' at M'. That is to say, $\Delta'(t, t')$ is equal to the latest firing time of t minus the earliest firing time of t'. Thus, $\Delta'(t, t') < 0$ is a sufficient condition ensuring that t' cannot fire before t from α. On the contrary, $\Delta'(t, t') \geq 0$ is a necessary condition that ensures t' can fire from α [33]. These two inequalities are hereinafter applied to the deadlock prevention conditions and permissiveness conditions, respectively.

Given a net system (N, M_0), the set of all reachable extended markings of its parametric TPN from α_0 is defined as $\mathcal{S}r(N, \alpha_0) = \{\alpha = (M, \Delta) \,|\, (\exists \sigma \in T^*) \alpha = Su(\alpha_0, \sigma)\}$. The symbolic reachability graph of a parametric TPN can be generated by Algorithm 1 [35].

Example 1: Consider the Petri net (N_1, M_0) depicted in Fig. 1. The initial extended marking of its parametric TPN is $\alpha_0 = (M_0, \Delta_0)$, where $M_0 = [3\,0\,0\,3\,0\,0\,1\,1]^T$, $En(M_0) = \{t_1, t_4\}$, $\Delta_0(t_1, t_4) = b_1 - a_4$, and $\Delta_0(t_4, t_1) = b_4 - a_1$. If t_1 is firable from α_0, the successor extended marking of α_0 is $\alpha_1 = (M_1, \Delta_1)$, where $M_1 = [2\,1\,0\,3\,0\,0\,0\,1]^T$ and $En(M_1) = \{t_2, t_4\}$. The transitions $t_2 \in Nw(M_0, t_1)$ and $t_4 \in Cw(M_0, t_1)$ are enabled at M_1 reached from M_0 by firing t_1 due to $t_2 \notin En(M_0)$ and $M_0 - Pre(\cdot, t_1) \geq Pre(\cdot, t_4)$. Thus, it holds $\Delta_1(t_2, t_4) = b_1 + b_2 - a_4$ and $\Delta_1(t_4, t_2) = b_4 - a_1 - a_2$. If transition t_4 is firable from α_0, the successor extended marking $Su(\alpha_0, t_4)$ is $\alpha_2 = (M_2, \Delta_2)$, where $M_2 = [3\,0\,0\,2\,1\,0\,1\,0]^T$ and $En(M_2) = \{t_1, t_5\}$. The transitions $t_5 \in Nw(M_0, t_4)$ and $t_1 \in Cw(M_0, t_4)$ are enabled at M_2 reached from M_0 by firing t_4 thanks to $t_5 \notin En(M_0)$ and $M_0 - Pre(\cdot, t_4) \geq Pre(\cdot, t_1)$. Thus, $\Delta_2(t_1, t_5) = b_1 - a_4 - a_5$ and $\Delta_2(t_5, t_1) = b_4 + b_5 - a_1$. By the above calculations, the symbolic reachability graph SRG$_1$ of the parametric TPN (N_1, M_0) is constructed due to Algorithm 1, where the SRG$_1$ and its node information are demonstrated in Fig. 2 and Table 1. □

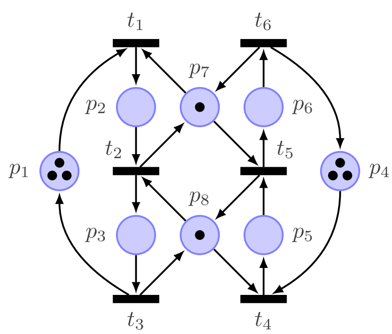

FIGURE 1. A petri net N_1 with $M_0 = [3\,0\,0\,3\,0\,0\,1\,1]^T$.

Since the proposed deadlock prevention strategy is based on reachable space analysis in this paper, the considered net system (N, M_0) is assumed to satisfy the following assumptions.

(A1) The net system (N, M_0) is bounded.

(A2) For all transitions $t \in T$ in (N, M_0), t is controllable.

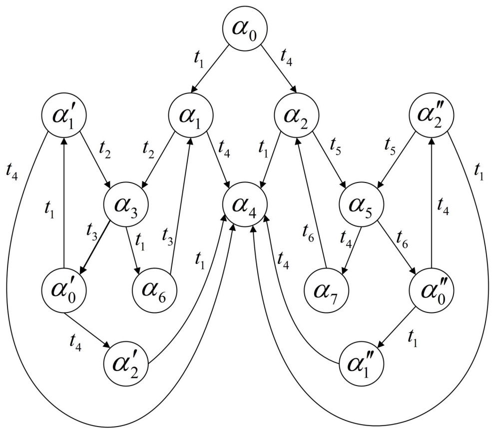

FIGURE 2. The symbolic reachability graph SRG_1 of N_1.

TABLE 1. Node information of SRG_1.

α	M	$\Delta(t, t')$
α_0	$M_0 = [3\,0\,0\,3\,0\,0\,1\,1]^T$	$\Delta_0(t_1, t_4) = b_1 - a_4$ $\Delta_0(t_4, t_1) = b_4 - a_1$
α_1	$M_1 = [2\,1\,0\,3\,0\,0\,0\,1]^T$	$\Delta_1(t_2, t_4) = b_1 + b_2 - a_4$ $\Delta_1(t_4, t_2) = b_4 - a_1 - a_2$
α_2	$M_2 = [3\,0\,0\,2\,1\,0\,1\,0]^T$	$\Delta_2(t_1, t_5) = b_1 - a_4 - a_5$ $\Delta_2(t_5, t_1) = b_4 + b_5 - a_1$
α_3	$M_3 = [2\,0\,1\,3\,0\,0\,1\,0]^T$	$\Delta_3(t_1, t_3) = b_1 - a_3$ $\Delta_3(t_3, t_1) = b_3 - a_1$
α_4	$M_4 = [2\,1\,0\,2\,1\,0\,0\,0]^T$	
α_5	$M_5 = [3\,0\,0\,2\,0\,1\,0\,1]^T$	$\Delta_5(t_4, t_6) = b_4 - a_6$ $\Delta_5(t_6, t_4) = b_6 - a_4$
α_6	$M_6 = [1\,1\,1\,3\,0\,0\,0\,0]^T$	
α_7	$M_7 = [3\,0\,0\,1\,1\,1\,0\,0]^T$	
α_0'	$M_0 = [3\,0\,0\,3\,0\,0\,1\,1]^T$	$\Delta_0'(t_1, t_4) = b_1 - a_3 - a_4$ $\Delta_0'(t_4, t_1) = b_4 + b_3 - a_1$
α_1'	$M_1 = [2\,1\,0\,3\,0\,0\,0\,1]^T$	$\Delta_1'(t_2, t_4) = b_1 + b_2 - a_3 - a_4$ $\Delta_1'(t_4, t_2) = b_4 + b_3 - a_1 - a_2$
α_2'	$M_2 = [3\,0\,0\,2\,1\,0\,1\,0]^T$	$\Delta_2'(t_1, t_5) = b_1 - a_3 - a_4 - a_5$ $\Delta_2'(t_5, t_1) = b_3 + b_4 + b_5 - a_1$
α_0''	$M_0 = [3\,0\,0\,3\,0\,0\,1\,1]^T$	$\Delta_0''(t_1, t_4) = b_1 + b_6 - a_4$ $\Delta_0''(t_4, t_1) = b_4 - a_6 - a_1$
α_1''	$M_1 = [2\,1\,0\,3\,0\,0\,0\,1]^T$	$\Delta_1''(t_2, t_4) = b_1 + b_2 + b_6 - a_4$ $\Delta_1''(t_4, t_2) = b_4 - a_1 - a_2 - a_6$
α_2''	$M_2 = [3\,0\,0\,2\,1\,0\,1\,0]^T$	$\Delta_2''(t_1, t_5) = b_1 + b_6 - a_4 - a_5$ $\Delta_2''(t_5, t_1) = b_4 + b_5 - a_1 - a_6$

Assumption (A1) ensures that the set of reachable markings $R(N, M_0)$ of the net system (N, M_0) is finite. Assumption (A2) guarantees that for any transition, we can endow it with a time constraint or modify its firing interval.

B. DEADLOCK PREVENTION AND PERMISSIVENESS CONDITIONS

Given an untimed net system (N, M_0), the SRG of its parametric TPN is useful to calculate the deadlock prevention conditions for dangerous markings and permissiveness conditions for legal markings. Given an extended marking $\alpha = (M, \Delta) \in \mathcal{S}r(N, \alpha_0)$, we denote by $\mathcal{O}(M)$ the set of enabled transitions whose firing can reach a bad or dead marking, i.e., $\mathcal{O}(M) = \{t \in En(M) \mid (\exists M' \in \mathcal{M}_b \cup \mathcal{M}_f)\, M[t\rangle M'\}$.

Given an extended marking $\alpha = (M, \Delta) \in \mathcal{S}r(N, \alpha_0)$, we denote by $\mathcal{K}(M)$ the set of enabled transitions whose firing can reach a legal marking, i.e., $\mathcal{K}(M) = \{t \in En(M) \mid (\exists M' \in \mathcal{M}_l)\, M[t\rangle M'\}$. For all markings $M \in \mathcal{M}_d$ ($\mathcal{M}_b$; $\mathcal{M}_g$), we have $En(M) = \mathcal{O}(M) \cup \mathcal{K}(M)$ and $\mathcal{O}(M) \cap \mathcal{K}(M) = \emptyset$ ($\mathcal{K}(M) = \emptyset$ and $\mathcal{O}(M) = En(M)$; $\mathcal{O}(M) = \emptyset$ and $\mathcal{K}(M) = En(M)$).

Suppose that the marking M of an extended marking $\alpha = (M, \Delta)$ is dangerous. The deadlock prevention conditions for M require that for all transitions $t_d \in \mathcal{O}(M)$ and $t_g \in \mathcal{K}(M)$, t_d should not fire in advance of t_g [33], which is represented as the following inequality:

$$\bigwedge_{t_d \in \mathcal{O}(M)} \bigvee_{t_g \in \mathcal{K}(M)} \Delta\left(t_g, t_d\right) < 0$$

Given a net system (N, M_0), its deadlock prevention conditions are the conjunction of the deadlock prevention conditions of all dangerous markings. This can be represented as follows:

$$\bigvee_{M_i \in \mathcal{M}_d} \bigwedge_{t_d \in \mathcal{O}(M_i)} \bigvee_{t_g \in \mathcal{K}(M_i)} \Delta_i\left(t_g, t_d\right) < 0$$

Given an extended marking $\alpha = (M, \Delta) \in \mathcal{S}r(N, \alpha_0)$, if M is a dangerous marking, then the permissiveness conditions for M imply that for all transitions $t_d \in \mathcal{O}(M)$ and $t_g \in \mathcal{K}(M)$, t_g must fire in advance of t_d. If M is a sound marking, then the permissiveness conditions for M imply that for all transitions $t \in En(M)$, t can fire from α [33]. They can be represented as follows:

$$\begin{cases} \bigwedge_{t_i, t_j \in En(M)} \Delta\left(t_i, t_j\right) \geq 0 & \text{if } M \in \mathcal{M}_g \\ \bigwedge_{t_d \in \mathcal{O}(M)\, t_g \in \mathcal{K}(M)} \Delta\left(t_d, t_g\right) \geq 0 & \text{if } M \in \mathcal{M}_d \end{cases}$$

Example 2: Consider the net system (N_1, M_0) portrayed in Fig. 1 and the SRG of its parametric TPN is shown in Fig. 2. Since the marking M_1 of the extended marking $\alpha_1 = (M_1, \Delta_1)$ is dangerous, $\mathcal{O}(M) = \{t_4\}$ and $\mathcal{K}(M) = \{t_2\}$. The deadlock prevention condition for M_1 implies that t_4 cannot fire from α_1 in advance of t_2, which can be formally represented as $\Delta_1(t_2, t_4) = b_1 + b_2 - a_4 < 0$. The permissiveness condition implies that t_2 can fire from α_1, which can be expressed as $\Delta_1(t_4, t_2) = b_4 - a_1 - a_2 \geq 0$. □

Compared with the traditional reachability graph, the SRG is designed to represent the state space of parameterized TPNs. The SRG can more conveniently show the upper bound of the firing date between individual enabled transitions under each state, which is beneficial for scheduling the priority of the firing of transitions.

IV. DEADLOCK PREVENTION STRATEGY

A. CONTROL PLACE

The deadlock prevention conditions and permissiveness conditions derived by an SRG may be mutually exclusive, which can extenuate the behavior of a system. In this section, we analyze several scenarios that may trigger mutual exclusion and investigate potential solutions to ameliorate their effects. Particularly, we introduce a control place called p_c to an original Petri net before endowing time constraints. The control place is utilized to modulate the firing time domains of certain transitions such that the firing priority of transitions can be realigned.

Definition 1: Let $\sigma = t_1 t_2 \ldots t_n \in T^*$ be a transition sequence and $M, M_1, M_2, \ldots, M_n \in R(N, M_0)$ be reachable markings in (N, M_0), satisfying $M[t_1\rangle M_1 \ldots M_{n-1}[t_n\rangle M_n$. A transition $t \in T$ is said to be *strongly continuously enabled* at M_n that reached from M_1 by firing σ, denoted as $t \in SC(M_1, \sigma)$, if the following two conditions hold:

(1) $t \in Cw(M, t_1) \cap Cw(M_1, t_2) \cap \ldots \cap Cw(M_{n-1}, t_n)$;

(2) there exist $t_f \in T$ and $M' \in R(N, M_0)$ with $M'[t_f\rangle M$ such that $t \in Nw(M', t_f)$ holds. ♢

Definition 2: Let $\sigma = t_1 t_2 \ldots t_{n-1} \in T^*$ be a transition sequence in (N, M_0) and $M_1, M_2, \ldots, M_n \in R(N, M_0)$ be reachable markings, satisfying $M_1[t_1\rangle M_2 \ldots M_{n-1}[t_{n-1}\rangle M_n$. Transition sequence σ is said to be a *newly enabled sequence* if the following two conditions hold: (1) there exist a marking $M \in R(N, M_0)$ and a transition $t_f \in T$ with $M[t_f\rangle M_1$ such that $t_1 \in Nw(M, t_f)$; (2) for all $t_i \in \sigma$, $i = \{2, 3, \ldots, n-1\}$, $t_i \in Nw(M_{i-1}, t_{i-1})$ holds. ♢

Example 3: Consider again the net system (N_1, M_0) portrayed in Fig. 1. Note that $\sigma_1 = t_6 t_1$ is an enabled sequence that can fire from M_5, i.e., $M_5[t_6\rangle M_0[t_1\rangle M_1$. Since $t_4 \in Nw(M_2, t_5) \cap Cw(M_5, t_6) \cap Cw(M_0, t_1)$, we have $t_4 \in SC(M_5, \sigma_1)$. Also, there exists an enabled sequence $\sigma_2 = t_2 t_3 t_4$ that can fire from M_1, i.e., $M_1[t_2\rangle M_3[t_3\rangle M_0[t_4\rangle M_2$. We say that σ_2 is newly enabled because of $t_2 \in Nw(M_0, t_1)$, $t_3 \in Nw(M_1, t_2)$, and $t_4 \in Nw(M_3, t_3)$. □

Proposition 1: Let (N, M_0) be a net system. Given extended markings $\alpha_d = (M_d, \Delta_d)$, $\alpha_g = (M_g, \Delta_g) \in \mathcal{S}r(N, \alpha_0)$ with $M_d \in \mathcal{M}_d$ and $M_g \in \mathcal{M}_g$, markings M, $M' \in R(N, M_0)$, and transitions t_{f1}, $t_{f2} \in T$ such that $M[t_{f1}\rangle M_d$ and $M'[t_{f2}\rangle M_g$, if there exist transitions t, t' such that $t, t' \in Nw(M, t_{f1}) \cap Nw(M', t_{f2}) \cap \mathcal{K}(M_g)$, $t \in \mathcal{K}(M_d)$, and $t' \in \mathcal{O}(M_d)$, then the permissiveness conditions for M_g are mutually exclusive with the deadlock prevention conditions for M_d.

Proof: Since t and t' are both newly enabled at α_d and α_g, the firing time domains of t and t' are equal to their static firing intervals. One of the deadlock prevention conditions for M_d is $\Delta_d(t, t') = b - a' < 0$. One of the permissiveness conditions for M_g is $\Delta_g(t, t') = b - a' \geq 0$. The two inequalities are mutually exclusive, which completes the proof. ■

Definition 3: Given two transition sequences $\sigma, \sigma' \in T^*$, σ is said to be a *subsequence* of σ' if $|\sigma| < |\sigma'|$ and for all transitions $t \in \sigma$, $t \in \sigma'$ holds. ♢

Example 4: Consider again the net system (N_1, M_0) portrayed in Fig. 1. Given two transition sequences $\sigma_1 = t_1 t_2 t_3 t_1$ and $\sigma_2 = t_2 t_3$, σ_2 is a subsequence of σ_1 since $|\sigma_2| < |\sigma_1$ and for all $t \in \sigma_2$, $t \in \sigma_1$. □

Proposition 2: Let (N, M_0) be a net system. Given extended markings $\alpha_d = (M_d, \Delta_d)$, $\alpha_g = (M_g, \Delta_g) \in \mathcal{S}r(N, \alpha_0)$ with $M_d \in \mathcal{M}_d$ and $M_g \in \mathcal{M}_g$, markings M, $M' \in R(N, M_0)$, and sequences σ_d, $\sigma_g \in T^*$ such that $M[\sigma_d\rangle M_d$ and $M'[\sigma_g\rangle M_g$, there exist $t \in \mathcal{K}(M_d) \cap \mathcal{K}(M_g)$ and $t' \in \mathcal{O}(M_d) \cap \mathcal{K}(M_g)$ such that the permissiveness conditions for M_g are mutually exclusive with the deadlock prevention conditions for M_d if (1) σ_d and σ_g are *newly enabled sequences*; (2) $t' \in SC(M, \sigma_d) \cap SC(M', \sigma_g)$; (3) σ_g is a *subsequence* of σ_d.

Proof: For the sake of convenience, we assume that $\sigma_g = t_j \ldots t_q$, $\sigma_d = t_i t_j t_q \ldots t_n$ and t is newly enabled at α_d and α_g. By conditions (1) and (2), t' remains continuously enabled during the firing of σ_g that is newly enabled. One of the permissiveness conditions for M_g is

$$\begin{aligned}\Delta_g\left(t, t'\right) &= b + \Delta_q(t_q, t)\\ &= b + b_q + \cdots + \Delta_j(t_j, t')\\ &= b + b_q + \cdots + b_j - a'\\ &= \tau_g - a' \geq 0 \end{aligned} \tag{1}$$

where $\tau_g = b + b_q + \cdots + b_j$.

One of the deadlock prevention conditions for M_d is

$$\begin{aligned}\Delta_d\left(t, t'\right) &= b + \Delta_n(t_n, t)\\ &= b + b_n + \cdots + \Delta_q(t_q, t')\\ &= b + b_n + \cdots + b_q + \Delta_j(t_j, t')\\ &= b + b_n + \cdots + b_q + b_j + b_i - a'\\ &= \tau_d - a' < 0 \end{aligned} \tag{2}$$

where $\tau_d = b + b_n + \cdots + b_q + b_j + b_i$.

By $|\sigma_g| < |\sigma_d|$, we have $\tau_g < \tau_d$. Apparently, inequalities (1) and (2) are mutually exclusive. ■

Proposition 3: Let (N, M_0) be a net system. Given extended markings $\alpha_d = (M_d, \Delta_d)$, $\alpha_g = (M_g, \Delta_g) \in \mathcal{S}r(N, \alpha_0)$ with $M_d \in \mathcal{M}_d$ and $M_g \in \mathcal{M}_g$, markings $M, M' \in R(N, M_0)$, and sequences σ_d, $\sigma_g \in T^*$ such that $M[\sigma_d\rangle M_d$ and $M'[\sigma_g\rangle M_g$, there exist $t \in \mathcal{K}(M_d)$, $t' \in \mathcal{O}(M_d) \cap \mathcal{K}(M_g)$ and $t'' \in K(M_g)$ such that the permissiveness conditions for M_g are mutually exclusive with the deadlock prevention conditions for M_d if (1) σ_d and σ_g are newly enabled; (2) $t' \in SC(M, \sigma_d) \cap SC(M', \sigma_g)$; (3) σ_g is a subsequence of σ_d and $t'' \in \sigma_d$.

Proof: To facilitate the proof, we assume that t is newly enabled at α_d, t'' is newly enabled at α_g, $\sigma_g = t_j \ldots t_q$, and $\sigma_d = t_i t_j t'', \ldots, t_n$. By the proof of Proposition 2, we can comfortably derive deadlock prevention conditions for M_d

and permissiveness conditions for M_g as follows:

$$\Delta_d(t,t') = b + b_n + \cdots + b'' + b_j + b_i - a' = b'' + \tau_d - a' < 0 \quad (3)$$

$$\Delta_g(t'',t') = b'' + b_q + \cdots + b_j - a' = b'' + \tau_g - a' \geq 0 \quad (4)$$

where $\tau_d = b + b_n + \cdots + b_j + b_i$ and $\tau_g = b_q + \cdots + b_j$.

By $|\sigma_g| < |\sigma_d|$, we have $\tau_g < \tau_d$. It is obvious that inequalities (3) and (4) are mutually exclusive. ■

Example 5: Consider two extended markings $\alpha_0' = (M_0, \Delta_0')$ and $\alpha_1' = (M_1, \Delta_1')$ in Fig. 2, where $M_0 \in \mathcal{M}_d$ and $M_1 \in \mathcal{M}_d$. There exist two sequences $\sigma_1 = t_3$ and $\sigma_2 = t_3t_1$ satisfying conditions (1)–(3) of Proposition 3 and thus the permissiveness conditions for M_0 and deadlock prevention conditions for M_1 are mutually exclusive. □

In the sense of deadlock control, the fact that the permissiveness condition is mutually exclusive with the deadlock prevention condition implies that we may sacrifice the partial legal behavior of a system for deadlock-freedom. Propositions 1, 2, and 3 enumerate three situations in which there exist deadlock prevention conditions and permissiveness conditions that are mutually exclusive. There may also exist other cases of mutual exclusion and we mainly address the case of Proposition 3 in this article. That is to say, the supervisor developed in this paper may be not optimal.

Theorem 1: Let (N, M_0) be a net system. Given extended markings $\alpha_d = (M_d, \Delta_d)$, $\alpha_g = (M_g, \Delta_g) \in \mathcal{S}r(N, \alpha_0)$ with $M_d \in \mathcal{M}_d$ and $M_g \in \mathcal{M}_g$, if there exist transitions $t \in \mathcal{O}(M_d) \cap \mathcal{K}(M_g)$, $t' \in \mathcal{K}(M_d)$ and $t'' \in \mathcal{K}(M_g)$ such that $\Delta_d(t', t) \geq \Delta_g(t'', t)$, then the permissiveness condition for M_g and the deadlock prevention condition for M_d are mutually exclusive.

Proof: It is obvious that $\Delta_d(t', t) = \tau_d - a$ and $\Delta_g(t'', t) = \tau_g - a$, where $\tau_d, \tau_g \in \mathbb{N}$. We have $\tau_d < a$ since $\Delta_d(t', t)$ is a deadlock prevention condition for M_d. Analogously, we have $\tau_g \geq a$, which also implies $\tau_d < \tau_g$. However, $\tau_d \geq \tau_g$ comes to be true by $\Delta_d(t', t) \geq \Delta_g(t'', t)$. Then, we conclude that $\tau_d < \tau_g$ and $\tau_d \geq \tau_g$ are mutually exclusive, which alludes to the fact that the permissiveness condition for M_g and deadlock prevention condition for M_d are mutually exclusive. ■

Theorem 1 states the fundamental reason that deadlock prevention conditions and permissiveness conditions are mutually exclusive. Let (N, M_0) be a net system. Given extended markings $\alpha = (M, \Delta)$, $\alpha' = (M', \Delta') \in \mathcal{S}r(N, \alpha_0)$ and a sequence $\sigma \in T^*$ with $\alpha' = Su(\alpha_0, \sigma)$, if there exists $t \in SC(M, \sigma) \cap \mathcal{O}(M')$, then the firing time domain of t at α' would progressively decrease until it converges to the time origin. To reduce the upper bound of the difference between the firing instants of other enabled transitions and t at α', t should be transformed to be newly enabled since the earliest firing time of a newly enabled transition is the largest.

Example 6: Consider again the net system (N_1, M_0) portrayed in Fig. 1. The symbolic reachability graph SRG_1 of (N_1, M_0) is depicted in Fig. 2 and the corresponding node information is illustrated in Table 1. Note that M_4 is a dead marking and one of the deadlock prevention conditions for M_4 is $\Delta_1(t_2, t_4) = b_1 + b_2 - a_4 < 0$, i.e., $b_1 + b_2 < a_4$. One of the permissiveness conditions for M_4 is $\Delta_0(t_1, t_4) = b_1 - a_4 \geq 0$, i.e., $b_1 \geq a_4$, which is necessary for a system execution. Since $\Delta_1(t_2, t_4) > \Delta_0(t_1, t_4)$, the two conditions are mutually exclusive by Theorem 1, implying that, to eliminate the mutual exclusion, we have to compromise some legal markings to ensure that the system is deadlock-free. □

Proposition 4: Let (N, M_0) be a net system. Given a transition $t_f \in T$ and markings $M, M' \in R(N, M_0)$ that satisfy $M'[t_f\rangle M$, if there exists a transition $t_d \in Cw(M', t_f)$, there exists a control place such that t_d at M is newly enabled.

Proof: Due to the definition of continuously enabled transitions, we have $M' - Pre(\cdot, t_f) \geq Pre(\cdot, t_d)$. To facilitate t_d being converted into a newly enabled transition at M, it is imperative to break the above relation. To this end, we introduce into (N, M_0) a control place p_c with arcs from t_f, t_d to p_c and from p_c to t_f, t_d with unity weight values. The resulting net system is denoted by (N_c, M_{0c}), where $M_{0c}(p_c) = 1$. Then the following equations are naturally witnessed:

$$Pre_{N_c}(\cdot, t_f) = \begin{bmatrix} Pre(\cdot, t_f) \\ 1 \end{bmatrix}, Pre_{N_c}(\cdot, t_d) = \begin{bmatrix} Pre(\cdot, t_d) \\ 1 \end{bmatrix}.$$

By the definition of newly enabled transitions, it comes

$$M'_{N_c} - Pre_{N_c}(\cdot, t_f) = \begin{bmatrix} M' \\ 1 \end{bmatrix} - \begin{bmatrix} Pre(\cdot, t_f) \\ 1 \end{bmatrix} = \begin{bmatrix} M' - Pre(\cdot, t_f) \\ 0 \end{bmatrix} < \begin{bmatrix} Pre(\cdot, t_d) \\ 1 \end{bmatrix} = Pre_{N_c}(\cdot, t_d).$$

Thus, t_d is turned to be newly enabled. ■

To this end, a place controller p_c connected to some transitions in a self-loop is introduced to an original Petri net before endowing time constraints based on Proposition 4. The introduction of p_c does not change (neither expand nor narrow down) the reachable space of the original Petri net because of its attachment in a self-loop mode. The predominant role of p_c is to modify the characteristics of certain transitions, i.e., to change the continuously enabled transitions to newly enabled transitions under dangerous markings, such that $M_0(p_c) = 1$ is sufficient. The control place added will reduce the computational overheads of the SRG. At the same time, the control place also solves some of the mutual exclusion cases. Therefore, the added control place can improve the permissiveness of the system.

Example 7: Consider again the net system (N_1, M_0) portrayed in Fig. 1. Given a control place p_c with $pre(p_c, t_1) = post(t_1, p_c) = 1$, $pre(p_c, t_4) = post(t_4, p_c) = 1$, and $M_0(p_c) = 1$, transition t_4 can be transformed to be newly enabled at M_2 that is reached from M_0 by firing t_1 due to Proposition 4. □

Algorithm 2 Calculation of the Arcs Connected to the Control Place p_c

Input: Net system (N, M_0), the sets of dead markings $\mathcal{M}_f$, dangerous markings $\mathcal{M}_d$, bad markings $\mathcal{M}_b$ and sound markings $\mathcal{M}_g$.
Output: A control place p_c.
1: $Pre(p_c, \cdot) := \mathbf{0}^{1\times n}$, $Post(\cdot, p_c) := \mathbf{0}^{1\times n}$.
2: **for all** $t \in En(M_0)$ **do**
3: let $Pre(p_c, t) := 1$.
4: let $Post(t, p_c) := 1$.
5: **end for**
6: **for all** $M \in \mathcal{M}_g \cup \mathcal{M}_d$ **do**
7: **for all** $t \in En(M)$ **do**
8: **if** $M[t\rangle M_0$ **then**
9: let $Pre(p_c, t) := 1$.
10: let $Post(t, p_c) := 1$.
11: **end if**
12: **end for**
13: **end for**
14: **for all** $M \in \mathcal{M}_d$ **do**
15: **for all** $t_i \in \mathcal{O}(M)$ **do**
16: **if** $\{t_i$ is continuously enabled at $M\}$ and {there exists a marking $M' \in \mathcal{M}_g$ such that $t_i \in En(M')$ and $||En(M')|| > 1\}$ **then**
17: let $Pre(p_c, t_i) := 1$.
18: let $Post(t_i, p_c) := 1$.
19: **for all** $M'' \in \mathcal{M}_g \cup \mathcal{M}_d$ **do**
20: **for all** $t_j \in En(M'')$ **do**
21: **if** $M''[t\rangle M$ **then**
22: let $Pre(p_c, t_j) := 1$.
23: let $Post(t_j, p_c) := 1$.
24: **end if**
25: **end for**
26: **end for**
27: **end if**
28: **end for**
29: **end for**
30: Output $Pre(p_c, \cdot)$ and $Post(\cdot, p_c)$.

Theorem 2: Given extended markings $\alpha_1 = (M_1, \Delta_1)$, $\alpha'_1 = (M_1, \Delta'_1)$, $\alpha_2 = (M_2, \Delta_2)$, $\alpha_3 = (M_3, \Delta_3) \in \mathcal{S}r(N, \alpha_0)$ and sequences σ, $\sigma' \in T^*$, with $\alpha_1 = Su(\alpha_2, \sigma)$ and $\alpha'_1 = Su(\alpha_3, \sigma')$, there exist t, $t' \in En(M_1)$ such that $\Delta_1(t, t') > \Delta'_1(t, t')$ and $\Delta_1(t', t) > \Delta'_1(t', t)$ if (1) σ is a subsequence of σ'; (2) $t \in Sc(M_2, \sigma)$ and $t \in Sc(M_3, \sigma')$.

Proof: We assume $\sigma = t_i \ldots t_j t_k$ and $\sigma' = t_p \ldots t_q t_m$. By condition (2), we have $\Delta_1(t, t') = b - \tau_1 - a'$ and $\Delta'_1(t, t') = b - \tau'_1 - a'$, where $\tau_1 = a_k + a_j + \cdots + a_i$ and $\tau'_1 = a_m + a_q + \cdots + a_p$. Since σ is a subsequence of σ', we have $\tau_1 < \tau'_1$ and $\Delta_1(t, t') > \Delta'_1(t, t')$. Analogously, $\Delta_1(t', t) > \Delta'_1(t', t)$ is obtained, which completes the proof. ■

With the above propositions, Algorithm 2 shows how to calculate a control place p_c that can enhance permissiveness

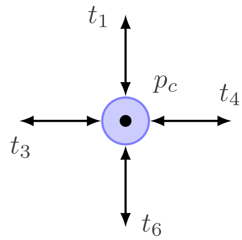

FIGURE 3. The control place p_c and its corresponding arcs of N_1.

after introducing time constraints. The principal body of Algorithm 1 is composed of two parts. In the first part, i.e., Steps 2 to 13, we keep enabled transitions at the initial marking as newly enabled transitions at all times, which can decrease the number of extended markings of the SRG of a parametric TPN. During the evolution of a TPN, it is difficult to backtrack to the initial state, as it may significantly increase the number of state classes and we have to reset the system through a reboot operation.

In the second part, i.e., Steps 14 to 29, if a transition t that should be forbidden at a dangerous marking but is enabled at other sound markings, then t should be treated as a newly transition enabled, which can prevent partial deadlock prevention conditions that are mutually exclusive with permissiveness conditions. In the worst case, for all transitions $t \in T$, we have $Pre(p_c, t) = Post(t, p_c) = 1$, which implies that each enabled transition at a marking is newly enabled. To illustrate the propositions introduced and Algorithm 2, a simple example is presented as follows.

Example 8: Consider again the net system (N_1, M_0) portrayed in Fig. 1. A control place p_c depicted in Fig. 3 is generated by using Algorithm 2. A new net system (N_{1c}, M_{0c}) can be constructed by introducing p_c. The symbolic reachability graph SRG_2 of (N_{1c}, M_{0c}) can be constructed by using Algorithm 1. The SRG_2 and corresponding node information are demonstrated in Fig. 4 and Table 2. Notice that the mutually exclusive case described in Proposition 3 is eliminated from Table 2. To guarantee that (N_{1c}, M_{0c}) is deadlock-free, the following constraints are obtained:

$$\begin{cases} \Delta_1(t_2, t_4) = b_2 - a_4 < 0 \\ \Delta_2(t_5, t_1) = b_5 - a_1 < 0 \\ \Delta_0(t_1, t_4) = b_1 - a_4 \geq 0 \\ \Delta_0(t_4, t_1) = b_4 - a_1 \geq 0 \\ \Delta_1(t_4, t_2) = b_4 - a_2 \geq 0 \\ \Delta_2(t_1, t_5) = b_1 - a_5 \geq 0 \\ \Delta_3(t_1, t_3) = b_1 - a_3 \geq 0 \\ \Delta_3(t_3, t_1) = b_3 - a_1 \geq 0 \\ \Delta_5(t_4, t_6) = b_4 - a_6 \geq 0 \\ \Delta_5(t_6, t_4) = b_6 - a_4 \geq 0 \end{cases}$$

Then a set of time constraints ensuring that (N_{1c}, M_{0c}) is deadlock-free is determined as follows: $Is(t_1) = Is(t_4) = [4, 5]$, $Is(t_2) = Is(t_5) = [0, 1]$, $Is(t_3) = Is(t_6) = [0, 4]$. □

B. DEADLOCK PREVENTION POLICY

Permissiveness conditions are necessary to guarantee that the legal behavior of a Petri net can be fulfilled. Deadlock

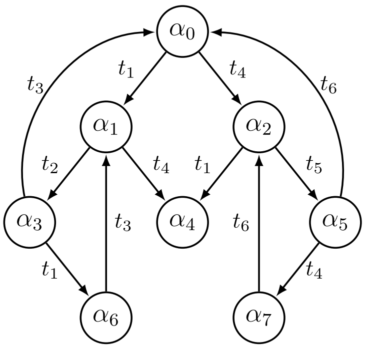

FIGURE 4. The symbolic reachability graph SRG_2 of (N_{1c}, M_{0c}).

TABLE 2. Node information of SRG_2.

α	M	$\Delta(t,t')$
α_0	$M_0 = [3\,0\,0\,3\,0\,0\,1\,1\,1]^T$	$\Delta_0(t_1,t_4) = b_1 - a_4$ $\Delta_0(t_4,t_1) = b_4 - a_1$
α_1	$M_1 = [2\,1\,0\,3\,0\,0\,0\,1\,1]^T$	$\Delta_1(t_2,t_4) = b_2 - a_4$ $\Delta_1(t_4,t_2) = b_4 - a_2$
α_2	$M_2 = [3\,0\,0\,2\,1\,0\,1\,0\,1]^T$	$\Delta_2(t_1,t_5) = b_1 - a_5$ $\Delta_2(t_5,t_1) = b_5 - a_1$
α_3	$M_3 = [2\,0\,1\,3\,0\,0\,1\,0\,1]^T$	$\Delta_3(t_1,t_3) = b_1 - a_3$ $\Delta_3(t_3,t_1) = b_3 - a_1$
α_4	$M_4 = [2\,1\,0\,2\,1\,0\,0\,0\,1]^T$	
α_5	$M_5 = [3\,0\,0\,2\,0\,1\,0\,1\,1]^T$	$\Delta_5(t_4,t_6) = b_4 - a_6$ $\Delta_5(t_6,t_4) = b_6 - a_4$
α_6	$M_6 = [1\,1\,1\,3\,0\,0\,0\,0\,1]^T$	
α_7	$M_7 = [3\,0\,0\,1\,1\,1\,0\,0\,1]^T$	

prevention conditions are sufficient to guarantee that a Petri net is deadlock-free [33]. In general, a set of time constraints is acquired by solving the deadlock prevention conditions and permissiveness conditions [35]. In this section, we show an alternative approach that merely enumerates the deadlock prevention condition inequalities.

From the SRG of a Petri net system (N, M_0), an intuitively derived deadlock prevention condition for a marking $M \in \mathcal{M}_d$ is $\Delta = \sum_{i=p}^{m} b_i - \sum_{j=q}^{n} a_j < 0$. For any transition $t \in T$ with the parametric static firing time interval $[a, b]$, if neither a nor b appears in Δ, then we say that t is not associated with the deadlock prevention conditions for M, which also implies that t does not interfere with the firing time of enabled transitions at M. In the following, Definitions 5 and 6 are proposed to formally expose the relationship between transitions and deadlock prevention conditions.

Theorem 3: Let (N, M_0) be a net system. Given a marking $M \in \mathcal{M}_d$, a sequence $\sigma = t_i \ldots t_j t_k$ is said to be associated with the firing time domains of the enabled transitions at M if there exist a marking $M' \in R(N, M_0)$ and a transition $t \in En(M)$ such that $M'[\sigma\rangle M$ and $t \in SC(M', \sigma)$.

Proof: Given markings $M_i, \ldots, M_j \in R(N, M_0)$, it holds $M'[t_i\rangle M_i \ldots M_j[t_k\rangle M$ by $M'[\sigma\rangle M$. Thanks to $t \in SC(M', \sigma)$, it comes to us $t \in Cw(M_j, t_k) \cap \cdots \cap Cw(M', t_i)$.

If $t \in \mathcal{O}(M)$, then there exists a transition $t' \in \mathcal{K}(M)$ such that one of the deadlock prevention conditions for M can be represented as follows:

$$\begin{aligned}\Delta\left(t', t\right) &= b' + \Delta_k(t_k, t) \quad t \in Cw(M_j, t_k)\\ &= b' + b_k + b_j + \cdots + \Delta_i(t_i, t) \quad t \in Cw(M', t_i)\\ &= b' + b_k + b_j + \cdots + b_i - a\end{aligned}$$

Since for all transitions $t_q \in \sigma$, their static firing upper bound are all in $\Delta(t', t)$, we say that σ is associated with the firing time domains of the enabled transitions at M. If $t \in \mathcal{K}(M)$, it is manifest that for all transitions $t_q \in \sigma$, their static firing lower bounds are all in $\Delta(t, t'')$ where $t'' \in \mathcal{O}(M)$. ■

Given a net system (N, M_0), due to Theorem 3, we denote by $\Sigma_d(M) = \{\sigma \in T^* | (\exists t \in En(M), \exists M' \in R(N, M_0))\ t \in SC(M', \sigma) \wedge M'[\sigma\rangle M\}$ the set of all sequences associated with the firing time domains of enabled transitions at marking $M \in \mathcal{M}_d$.

Definition 4: Given a net system (N, M_0), a transition $t \in T$ is said to be associated with deadlock prevention conditions of (N, M_0) if one of the following conditions holds: (1) there exists a marking $M \in \mathcal{M}_d$ such that $t \in \mathcal{K}(M)$; (2) there exists a sequence $\sigma \in \Sigma_d(M)$ such that $t \in \sigma$. ♢

Example 9: Consider the Petri net (N_2, M_0) portrayed in Fig. 5, where $M_0 = [2\,0\,0\,0\,0\,0\,2\,0\,0\,0\,1\,1\,1\,1\,1\,1\,1]^T$. Due to space limitation, we provide only a partial view of $R(N_2, M_0)$, as shown in Fig. 6. The marking M_{49} is a dangerous marking and $\sigma = t_6 t_0 t_1$ is a sequence that is associated with the firing time domains of t_3 at M_{49} since $M_{39}[\sigma\rangle M_{49}$ and $t_3 \in SC(M', \sigma)$. The transition t_1 is associated with deadlock prevention conditions of (N_2, M_0) since $\sigma \in \Sigma_d(M)$ and $t_1 \in \sigma$. Transitions t_0 and t_2 are also associated with deadlock prevention conditions of (N_2, M_0). □

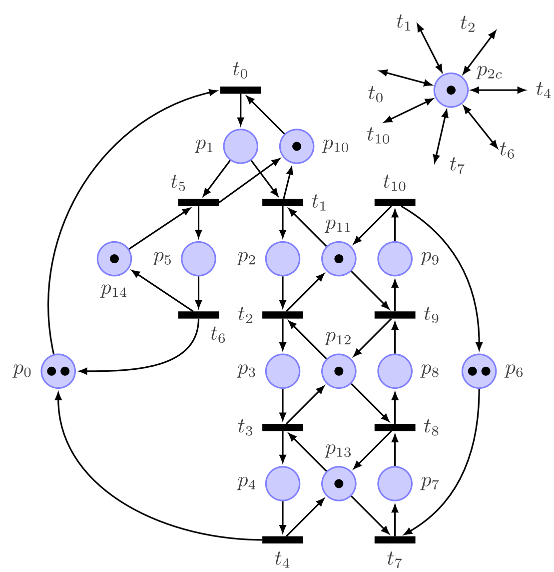

FIGURE 5. A petri net N_2 with $M_0 = [2\,0\,0\,0\,0\,0\,2\,0\,0\,0\,1\,1\,1\,1\,1\,1\,1]^T$.

Definition 5: Let (N, M_0) be a net system. Given a sequence $\sigma \in \Sigma_d(M)$, where $M \in \mathcal{M}_d$, the static firing lower

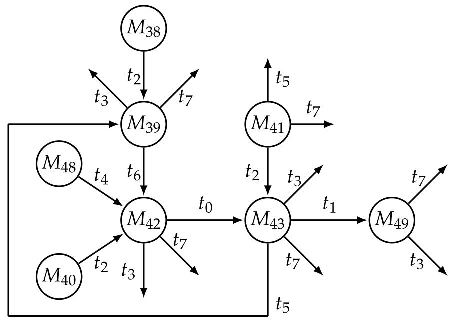

FIGURE 6. A partial view of $R(N_2, M_0)$.

bound of a transition $t \in \sigma$ is said to be associated with the firing time domains of enabled transitions at M if there exist a transition $t' \in \mathcal{K}(M)$ and a marking $M' \in R(N, M_0)$ such that $M'[\sigma\rangle M$ and $t' \in SC(M', \sigma)$ hold. ◊

Definition 6: Let (N, M_0) be a net system. Given a sequence $\sigma \in \Sigma_d(M)$, where $M \in \mathcal{M}_d$, the static firing upper bound of a transition $t \in \sigma$ is said to be associated with the firing time domains of enabled transitions at M if there exist a transition $t' \in \mathcal{O}(M)$ and a marking $M' \in R(N, M_0)$ such that $M'[\sigma\rangle M$ and $t' \in SC(M', \sigma)$ hold. ◊

Given a net system (N, M_0), the set of all transitions associated with deadlock prevention conditions is denoted as $T_d = \{t \in T | (\exists \sigma \in \Sigma_d(M)) t \in \sigma \vee (\exists M \in \mathcal{M}_d) t \in En(M)\}$ by Definition 4. Based on Definition 5, we denote by $\Sigma_a(M) = \{\sigma \in \Sigma_d(M) | (\exists M' \in R(N, M_0), \exists t \in \mathcal{K}(M))\ M'[\sigma\rangle M \wedge t \in SC(M', \sigma)\}$ the set of all sequences whose firing lower bounds are related to the firing time domains of enabled transitions at $M \in \mathcal{M}_d$. Based on Definition 6, we denote by $\Sigma_b(M) = \{\sigma \in \Sigma_d(M) | (\exists M' \in R(N, M_0), \exists t \in \mathcal{O}(M))\ M'[\sigma\rangle M \wedge t \in SC(M', \sigma)\}$ the set of all sequences whose firing upper bounds are associated with the firing time domains of enabled transitions at $M \in \mathcal{M}_d$.

Let T_a be the set of transitions whose firing lower bounds are related to the deadlock prevention conditions, i.e., $T_a = \{ t \in T | (\exists M \in \mathcal{M}_d)\ t \in \mathcal{O}(M) \vee (\exists \sigma \in \Sigma_a(M))\ t \in \sigma\}$. Let T_b be the set of transitions whose firing upper bounds are related to the deadlock prevention conditions, i.e., $T_b = \{t \in T | (\exists M \in \mathcal{M}_d)\ t \in \mathcal{K}(M) \vee (\exists \sigma \in \Sigma_b(M))\ t \in \sigma\}$.

As to be seen, Corollary 2 illustrates that it is not necessary to endow time constraints with all transitions, while only those related to the deadlock prevention conditions need to be associated with time constraints. Moreover, Corollary 2 provides a method to solve the set of deadlock prevention inequalities. The set of solutions obtained by Corollary 2 is the maximum range of feasible solutions that ensure a system is deadlock-free. In seeking to demonstrate Corollary 2 with more facility, it is necessary to review Corollary 1 originally from [33].

Corollary 1: [33] Let $N_t = (N, Is)$ and $N_t' = (N, Is')$ be two TPNs with the initial marking M_0, where $N = (P, T, Pre, Post)$ is a generalized Petri net. If for all transitions $t \in T$, $Is(t) \subseteq Is'(t)$, then $R(N_t, M_0) \subseteq R(N_t', M_0)$.

Corollary 2: Given a net system (N, M_0), a set of time constraints that are obtained by solving the deadlock prevention conditions and permissiveness conditions inequalities is denoted as Is. A deadlock-free TPN extension $N_t = (N, Is)$ can be generated by assigning time constraints Is to N. A set of time constraints acquired by solving only the deadlock prevention conditions inequalities is denoted as Is'. A deadlock-free TPN extension of N endowed Is' is denoted as $N_t' = (N, Is')$. Let (N_t, M_0) and (N_t', M_0) be two TPN systems. Then, $R(N_t, M_0) = R(N_t', M_0)$ if (1) for all $t \notin T_d$, $Is'(t) = [0, \infty]$; (2) for all $t \in T_d \setminus T_a$, $Is'(t) = [0, \tau_b]$; (3) for all $t \in T_d \setminus T_b$, $Is'(t) = [\tau_a, \infty]$, where τ_a and τ_b are derived by solving the deadlock prevention condition inequalities.

Proof: Permissiveness conditions are necessary to ensure the legal behavior of a Petri net system. Considering only deadlock prevention conditions may lead to some legal markings to be forbidden, we have $R(N_t, M_0) \supseteq R(N_t', M_0)$.

For all transitions $t \in T_d$, $Is(t) = Is'(t)$. The conditions (1)–(3) in Corollary 2 imply that N_t' has a maximum range of firing time domains to ensure that $t \notin T_d$ can fire, i.e., $Is(t) \subseteq Is'(t)$. In this case, we have $R(N_t, M_0) \subseteq R(N_t', M_0)$ by Corollary 1, which completes the proof. ■

Example 10: Consider the net system (N_1, M_0) depicted in Fig. 1. The deadlock prevention conditions of N_1 are as follows:

$$\begin{cases} \Delta_1(t_2, t_4) = b_2 - a_4 < 0 \\ \Delta_2(t_5, t_1) = b_5 - a_1 < 0 \end{cases}$$

We have $T_d = \{t_1, t_2, t_4, t_5\}$, $T_a = \{t_1, t_4\}$ and $T_b = \{t_2, t_4\}$. Based on Corollary 2, we can acquire another set of time constraints Is_1' that ensure (N_1, M_0) is deadlock-free: $t_1[2, \infty]$, $t_2[0, 1]$, $t_4[2, \infty]$, $t_5[0, 1]$. Compared with Is_1 in Example 8, we can identify for all transitions $t \in T$, $Is_1 \subseteq Is_1'$. □

With the above analysis, a deadlock prevention algorithm through tuning time constraints is proposed, as displayed in Algorithm 3. Next, let us see how Algorithm 3 expands an untimed Petri net to a TPN to achieve a deadlock-free net system. In Step 1, a control place p_c with the initial marking of one is initialized. Step 2 applies Algorithm 1 to obtain some arcs connected to p_c. Steps 3–7, if there do not exist arcs connected to p_c, implying that there would be no mutually exclusive case as in Proposition 3. Then, we calculate the SRG of the original Petri net. The supervisor obtained is a set of time constraints.

If there exist arcs connected to p_c, then we append p_c to the original Petri net at Step 9. Similarly, we calculate the SRG of the controlled net and obtain a set of time constraints by steps 10–13. The supervisor obtained is a combination of a control place and a group of time constraints.

Theorem 4: Net system (N_c, M_{0c}) due to Algorithm 3 is deadlock-free.

Proof: Based on Proposition 4, the introduction of a control place p_c generated by Algorithm 2 adjoined to an original net system (N, M_0) does not change the reachable space of (N, M_0). Deadlock prevention conditions

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Algorithm 3 Calculation of a Supervisor Ensuring That a Net System Is Deadlock-Free

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Input: A Petri net system (N, M_0) with deadlocks.
Output: A supervisor ensuring that (N_c, M_{0c}) is deadlock free.

Initialize a control place p_c with $M_{0c}(p_c) = 1$.
Calculate arcs connected to the control place p_c by applying Algorithm 2, i.e, $Pre(p_c, \cdot)$ and $Post(\cdot, p_c)$.
if $Pre(p_c, \cdot) = Post(\cdot, p_c) = \mathbf{0}^{1\times|T|}$ **then**
  Calculate the SRG of (N, M_0) by Algorithm 1;
  Enumerate the set of inequalities for the deadlock prevention conditions from the SRG;
  Apply Corollary 2 to solve the inequalities;
  Output a set of time constraints Is.
else
  Add p_c to (N, M_0) and acquire a controlled net called (N_c, M_{0c});
  Calculate the SRG of (N_c, M_{0c}) by applying Algorithm 1;
  Enumerate the set of inequalities for the deadlock prevention conditions from SRG;
  Apply Corollary 2 to solve the inequalities;
  Output a control place p_c and a set of time constraints Is.
end if

---

are sufficient to ensure that a net system is deadlock-free. We can obtain all deadlock prevention conditions from the SRG and a set of time constraints Is by Algorithm 3. Based on Corollary 2, a deadlock-free net system (N_c, M_{0c}) is constructed by introducing p_c and Is into (N, M_0). ■

V. EXPERIMENTAL EXAMPLES

In this section, two examples are considered to illustrate the effectiveness of the proposed deadlock prevention strategy.

A. EXAMPLE 1

Let us consider a Petri net model N_3 with $M_0 = [15\ 0\ 0\ 1]^T$ as shown in Fig. 7. There are 508 reachable markings, including one dead marking.

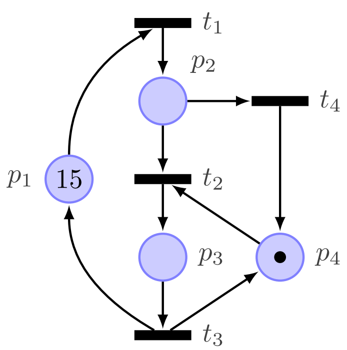

FIGURE 7. A petri net N_3 with $M_0 = [15\ 0\ 0\ 1]^T$.

First, by Algorithm 2, there is no control place being generated since there does not exist the mutually exclusive case in N_3 as claimed in Proposition 3. Next, Algorithm 1 is applied to construct a symbolic reachability graph of (N_3, M_0) and the deadlock prevention condition can be captured as follows:

$$\Delta_d(t_2, t_4) = b_2 - a_4 < 0$$

Then we have $T_d = \{t_2, t_4\}$, $T_a = \{t_2\}$, and $T_b = \{t_4\}$. An optimal solution can be obtained by Corollary 2: $Is(t_2) = [0, 1]$, $Is(t_4) = [2, 3]$, $Is(t_i) = [0, \infty]$, where $i \in \{1, 3\}$. A supervisor consisting of two time constraints, which makes N_3 deadlock-free and retains 507 reachable markings, can be obtained, i.e., the maximally permissive behavior of its original net model.

B. EXAMPLE 2

The layout of a flexible manufacturing cell is portrayed in Fig. 8 and its Petri net model N_4 with $M_0 = [3\,0\,0\,0\,0\,3\,0\,0\,0\,3\,0\,0\,1\,3\,1\,1]^T$ is shown in Fig. 9. The cell is equipped with two machines: M1 and M2. M1 has a capacity of three parts, but M2 can hold only one part at a time. Additionally, the cell is equipped with two robots R1 and R2, each capable of holding one part. Parts are introduced into the cell via three loading buffers I1–I3 and are removed from the cell through three unloading buffers O1–O3. Then the robots are responsible for managing the movement of parts. Specifically, R1 is responsible for transferring parts from M1 to O3 and I1 to M1. R2 is responsible for transferring parts between the two machines. The Petri net (N_4, M_0) has 391 reachable markings, 327 legal markings, 58 bad markings, and six dead markings.

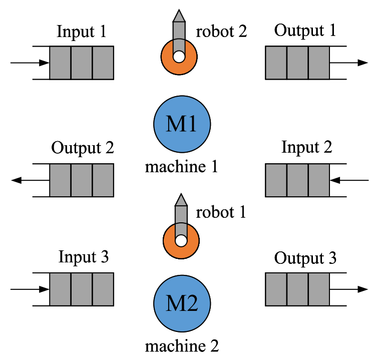

FIGURE 8. The layout of a flexible manufacturing cell.

By applying Algorithm 3, we can derive a control place p_c depicted in Fig. 10. A group of time constraints that make the system deadlock-free is that $Is(t_1) = [4, 5]$, $Is(t_2) = [2, 3]$, $Is(t_3) = [0, 4]$, $Is(t_6) = [3, 7]$, $Is(t_7) = [2, 5]$, $Is(t_8) = [0, 1]$, $Is(t_{10}) = [4, 5]$ and $Is(t_{11}) = [2, 3]$. Finally, we obtain a controlled net N_{4t} by introducing time constraints and the control place p_c. Verified by tool TINA [41], the controlled net N_{4t} has 258 reachable markings.

Consider the structure of the Petri net in Fig. 9. Table 3 compares the deadlock prevention methods in [33] with ours under different initial markings. We compare the method in [33] with ours in three dimensions: number of reachable states, number of controllers, and permissiveness ratio. We refer to the added control places as monitors and the time

TABLE 3. Comparison and analysis of performance based on different initial markings in Fig. 9.

initial markings	policy in [33]			ours			leagl/reachable markings of the original Petri net
	permissive state	controller count	permissiveness ratio	permissive state	controller count	permissiveness ratio	
$M_0 = [3\,0\,0\,0\,0\,3\,0\,0\,0\,3\,0\,0\,1\,3\,1\,1]^T$	73	12	22%	258	9	79%	327/391
$M_0 = [4\,0\,0\,0\,0\,4\,0\,0\,0\,4\,0\,0\,1\,4\,1\,1]^T$	144	12	23%	446	9	70%	636/747
$M_0 = [5\,0\,0\,0\,0\,5\,0\,0\,0\,5\,0\,0\,1\,5\,1\,1]^T$	197	12	18%	721	9	67%	1071/1247
$M_0 = [6\,0\,0\,0\,0\,6\,0\,0\,0\,6\,0\,0\,1\,6\,1\,1]^T$	277	12	17%	1191	9	72%	1653/1915
$M_0 = [7\,0\,0\,0\,0\,7\,0\,0\,0\,7\,0\,0\,1\,7\,1\,1]^T$	369	12	15%	1698	9	70%	2403/2775
$M_0 = [8\,0\,0\,0\,0\,8\,0\,0\,0\,8\,0\,0\,1\,8\,1\,1]^T$	473	12	14%	2303	9	69%	3342/3851
$M_0 = [9\,0\,0\,0\,0\,9\,0\,0\,0\,9\,0\,0\,1\,9\,1\,1]^T$	629	12	14%	3078	9	69%	4491/5167
$M_0 = [12\,0\,0\,0\,0\,12\,0\,0\,0\,12\,0\,0\,1\,12\,1\,1]^T$	1009	12	11%	6243	9	66%	9408/10795
$M_0 = [15\,0\,0\,0\,0\,15\,0\,0\,0\,15\,0\,0\,1\,15\,1\,1]^T$	1537	12	9%	11010	9	65%	16971/19447
$M_0 = [40\,0\,0\,0\,0\,40\,0\,0\,0\,40\,0\,0\,1\,40\,1\,1]^T$	19137	12	7%	156435	9	60%	258846/295947
$M_0 = [50\,0\,0\,0\,0\,50\,0\,0\,0\,50\,0\,0\,1\,50\,1\,1]^T$	35677	12	7%	294055	9	60%	491571/561947

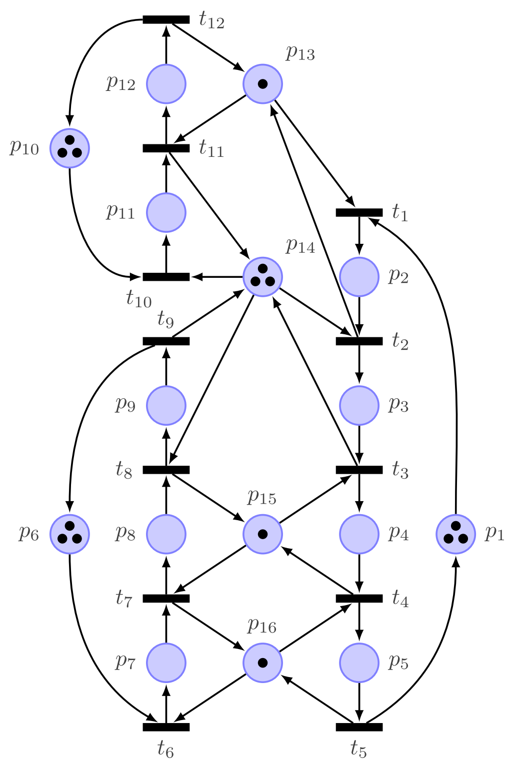

FIGURE 9. A petri net N_4 with $M_0 = [3\,0\,0\,0\,0\,3\,0\,0\,0\,3\,0\,0\,1\,3\,1\,1]^T$.

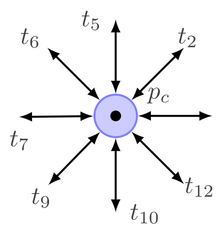

FIGURE 10. The control place p_c and its corresponding arcs of N_4.

constraints as timers. Moreover, the monitors and timers are collected as controllers. The ratio of reachable markings of the resulting Petri net system to the legal markings of the original Petri net is called the permissiveness ratio. The study in [33] develops a timer for each transition. By analyzing the data in Table 3, the proposed approach requires fewer controllers, while more permissive states are retained in the controlled system.

An optimal supervisor implies a permissiveness ratio of 100%. The data in Table 3 demonstrates that the supervisor developed in this paper is not optimal, since there are still some mutual exclusion cases that cannot be solved by adding the control place p_c. For example, there exist two markings M_{413} and M_{414} under the initial marking $M_0 = [4\,0\,0\,0\,0\,4\,0\,0\,0\,4\,0\,0\,1\,4\,1\,1]^T$, where $M_{413} = [1\,0\,1\,1\,1\,4\,0\,0\,0\,3\,1\,0\,1\,2\,0\,0]^T$ is a sound marking and $M_{414} = [1\,0\,3\,0\,0\,2\,1\,1\,0\,3\,1\,0\,1\,0\,0\,0]^T$ is a dangerous marking. Transitions t_1 and t_{11} are newly enabled transitions at M_{413} and M_{414}. Note that $b_{11} - a_1 < 0$ is a deadlock prevention condition and $b_{11} - a_1 \geq 0$ is a permissiveness condition since $t_1 \in \mathcal{O}(M_{414}) \cap \mathcal{K}(M_{413})$. These two conditions are mutually exclusive and cannot be resolved by the control place p_c, which means that we must prohibit the firing of t_1 at M_{413} even if the firing of t_1 can reach a legal marking. As a result, some successor markings that are reached by firing transition sequence $t_1\sigma$ at M_{413} would also be forbidden. Therefore, a permissiveness ratio of 100% cannot be achieved with this method.

VI. CONCLUSION

This paper proposes a novel deadlock prevention strategy through tuning time constraints, i.e., expanding an untimed Petri net into a TPN to ensure that the resulting Petri net is deadlock-free. First, we introduce a control place p_c

that prevents partial deadlock conditions and permissiveness conditions from being mutually exclusive, which can impair the legal behavior of a system with the addition of time constraints. Then we demonstrate that it is feasible to obtain a set of time constraints that make the Petri net deadlock-free by computing only the deadlock prevention conditions. The supervisor designed in this paper is composed of a control place p_c and a set of time constraints, which has a simpler structure than traditional deadlock prevention supervisors. However, it can only solve the partial mutual exclusion of deadlock prevention conditions and permissiveness conditions. In future research, we will focus on enforcing time constraints to Petri nets with uncontrollable transitions to ensure that a system is deadlock-free. Based on the proposed method, other methods, such as the theory of regions [16], [18], are explored to obtain a group of maximally permissive time constraints. The paper proposes a method to analyze the firing time of enabled transitions affecting dangerous markings in a sequence form, which may be useful for DES property analysis and state estimation. Future work will also consider the opacity enforcement problem in DES with partial observation [42], [43] under transition failures [44], [45].

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