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RESEARCH ARTICLE

# Estimation of Unmodeled Dynamics: Nonlinear MPC and Adaptive Control Law With Momentum Observer Dynamic

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**ABSTRACT** This article proposes an enhancement to estimate unmodeled dynamics within the simplified dynamic model of a quadcopter by integrating three key methodologies: Nonlinear Model Predictive Control (NMPC), a Momentum Observer Dynamics (MOD), and an adaptive control law. Termed as Adaptive NMPC with MOD, this integrated approach leverages NMPC, implemented using the CasADi framework, for real-time decision-making, while the momentum observer facilitates system state estimation and uncertainty mitigation. Simultaneously, the adaptive control law adjusts parameters to estimate errors in unmodeled dynamics. Through digital twin and Model in Loop (MiL) simulations, the effectiveness of this framework is demonstrated. Specifically, the study focuses on the simplified quadcopter model, acknowledging often overlooked inherent dynamics resulting from the simplification by not considering the nonlinearities induced by the drone's attitude angles. Addressing these unmodeled dynamics is critical, and the Adaptive NMPC with MOD method emerges as a robust solution, showcasing its potential across various scenarios.

**INDEX TERMS** NMPC, adaptive control, disturbance estimation, UAV dynamics, momentum observer, CasADi.

## I. INTRODUCTION

Several recent studies have demonstrated interest in developing advanced control algorithms for various applications using unmanned aerial vehicles (UAV) due to their extensive range of applications in different areas of compliance, such as transportation and logistics, civil, maintenance, security, and even military applications [1], [2]. One of the primary uses of UAVs is in executing specific tasks such as following a desired trajectory, position, and path tracking [3]. Thus, it becomes necessary to program UAVs to follow a predetermined path or to reach a specific location. This capability is crucial in many applications, such as surveying large areas of land or delivering goods to a particular destination that may be far away from the control center or pilot's location [4]. Another significant area of interest is visual servo control [5]. It involves using visual feedback information to control the motion of the UAV. This technology proves useful when the drone needs to interact with its environment, such as in search and rescue operations and when capturing aerial photos or videos [6]. It is also useful to detect points of interest like gates to pass through, where tools using computer vision and artificial intelligence have provided significant development [7]. Obstacle avoidance control using artificial intelligence is also a critical area of focus [8]. It uses AI algorithms to enable the UAV to detect and avoid obstacles.

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This technique is particularly important in ensuring the safety and reliability of UAV operations, especially in complex and unpredictable environments [9]. Finally, formation control of multiple UAVs is another exciting area of research [10]. This area involves coordinating the movement of a group of UAVs to work together to achieve a common goal. Therefore, it has potential applications in several areas, such as swarm robotics, where a group of robots work together to perform tasks that would be difficult or impossible for a single robot to accomplish [11]. On the other hand, one of the main challenges that come when working with UAVs in real environments is the presence of external disturbances. Most controller design approaches do not consider uncertainties in mathematical models [12] in either partially structured or unstructured spaces. These uncertainties can generate external disturbances such as noise, communication delays, wear and tear, flight system failures, and variation of the payload mass [13], [14]. These factors can significantly affect the performance and reliability of the UAV. Furthermore, the internal dynamics of the robot change at every instant of time due to the desired task. This means that the control system may no longer be effective, and the dynamic parameters must be updated despite the uncertainties to which the system may be subjected [15]. One solution is the use of adaptive control. It is a method used in automatic control systems that allows the controller to adapt to unknown or changing conditions [16]. This makes it an ideal option for UAVs, as it enables the control system to adjust to changes in the machine's internal dynamics during the execution of desired tasks [17].

### A. RELATED WORK

The development of UAV has had significant advancements with the integration of Model Predictive Control (MPC) to improve tracking performance in UAVs [18], in particular its variants Nonlinear Model Predictive Control (NMPC), and Adaptive Nonlinear Predictive Control, enhancing trajectory tracking, energy optimization, and safety, while focusing on a Moment Observer-Based approach to further improve the results of the models proposed.

NMPC, as a variant of MPC, is mainly used due to its accuracy and computational efficiency in trajectory tracking, offering substantial improvements over baseline feedback controllers [19]. Its application extends to sophisticated tasks such as obstacle avoidance [20], showcasing MPC's capability in allowing navigation through dynamic environments. The technology also proves adaptable across various UAV types, from fixed-wing to quadrotors, addressing challenges from precision landing [21] to agile flight [22].

Regarding safety and reliability in UAV deployment, NMPC provides practical strategies for collision avoidance [23], [24] and dynamic obstacle negotiation [25], [26]. It is important to also remark how NMPC allows stable and accurate control during visual-based tasks [27]. The methodology facilitates coordinated maneuvers, which is evident in studies on multi-UAV encirclement [28] and formation control [29] emphasizing its utility in complex operational scenarios, where fault-tolerant control provides a robust solution for handling faults and disturbances [30].

NMPCs also provide a solution to the energy efficiency concern, with reduced consumption in quad-rotor systems [31] and highlighting the importance of real-time implementation efficiency [32]. Computational burden may also decrease thanks to effective control performance through simulations and experiments with NMPC [18]. Additionally, system identification and collision avoidance integrated within NMPC frameworks [33] allow safety during flight without losing performance. Lastly, MPC-based navigation [34] and explicit MPC approaches [35] further validate the model's adaptability and effectiveness in real-world applications such as constrained environments, underscoring its potential to enhance UAV operational capabilities across various conditions and tasks.

Other control strategies derived from Model Predictive Control, such as adaptive and robust control frameworks and nonlinear and fuzzy logic approaches, are summarized, aiming at improving trajectory tracking, attitude stabilization, obstacle avoidance, and payload transportation in unmanned aerial vehicles (UAVs) and multi-rotor systems. Path tracking and several Adaptive Model Predictive Control (ANMPC) techniques applied in quadrotors have resulted in high precision in several tasks, like carrying unknown payloads [36], or following desired trajectories and stabilizing attitudes under dynamic conditions [37], enhancing the path tracking capabilities of autonomous systems [38]. These methodologies are improved by incorporating advanced algorithms like the Laguerre-based Adaptive MPC, or robust and nonlinear MPC strategies, which offer significant improvements in control accuracy and robustness against disturbances and uncertainties [39], [40] and also in cases that the model requires to adapt it to varying linear parameters.

Moreover, obstacle avoidance and mission planning for aerial platforms have been addressed through adaptive model predictive control strategies and differential evolution-based distributed MPC, facilitating safer and more efficient aerial operations, particularly in complex environments like autonomous ship landing [10], [15]. Integrating active disturbance rejection and backstepping-based adaptive control further emphasizes the focus on ensuring performance and enhancing stability against external perturbations [41]. This approach extends to autonomous load transporting systems, where adaptive control and model reference adaptive control techniques are pivotal in coordinating flight formations and managing load transportation with high stability and accuracy [42]. Another examination of nonlinear and adaptive intelligent control techniques and their application in various control systems, include quadrupedal robots and unmanned rotorcrafts, showcasing their versatility and effectiveness across different platforms [43], [44], [45]. It is worth noting that all presented works underscore the importance of performance, precision, and payload optimization in aerial systems through the development of robust adaptive control strategies and nonlinear model predictive path tracking approaches. These innovations not only enhance the control and maneuverability of UAVs but also ensure their adaptability to diverse and challenging operational scenarios [46], [47]. The next paragraph will focus on the approach of Observer-Based models as an additional tool to further improve the control system of UAVs using combined control methods.

Observer-based models choose one or more variables to analyze in order to control the effects of uncertainties or disturbances as an "observer" like outside the model. As this work focuses on a Momentum-Observer model, this subsection will showcase works that have researched this area in UAVs and other kinds of robots. Regarding Momentum-Observed Based (MOB) models, [48] introduces a MOB algorithm using LSTM for collision detection, learning model uncertainties without a precise dynamics model. This approach, validated on a real robot, improves traditional MOB methods by better handling model errors and friction effects. Another work proposes a Nonlinear Extended State Momentum Observer (NESMO) for sensor-less collision detection in robots under model uncertainties [49]. This paper presents NESMO for sensor less collision detection, addressing sensitivity and noise immunity challenges in current methods. Utilizing a fractional power function and time-varying damping ratio, NESMO improves monitoring bandwidth and noise immunity, with a novel TVT to distinguish collision signals from disturbances, proven effective on a 6-DOF robot. Finally, [50] reveals an SMO model for collaborative robots, leveraging sliding mode control for high-accuracy, minimal-delay collision detection without joint acceleration measurement, enhancing safety and reliability in human-robot interactions.

### B. MAIN CONTRIBUTIONS

The paper presents the following contributions in the field. Primarily, it introduces a cutting-edge approach by integrating adaptive laws with a MOD, thus significantly enhancing the controller's adaptability and precision in response to dynamic system changes. This fusion not only amplifies the robustness of the control strategy but also creates opportunities for real-time adjustments in the face of uncertainties or varying operating conditions. Additionally, the paper delves into the challenging task of estimating perturbation dynamics in the simplified UAV dynamic model, offering a comprehensive comparative analysis of various identification methods. By doing so, it provides valuable insights into effective techniques for addressing unknown or uncertain system models, thereby advancing the state-of-the-art in system identification and control methodologies.

### C. OUTLINE

The article is structured into chapters to present its content systematically. It begins with an introduction emphasizing the importance of accurately modeling dynamics in UAV frameworks. Chapters 2 and 3 explore the background of the topic, focusing on the kinematic and dynamic model of quadcopter. Chapters 4 through 7 present the formulation of Nonlinear Model Predictive Control, Adaptive MPC Controller, MPC with Dynamic Moment Observer, and Adaptive MPC with Moment Observer. Chapter 8, the results section, provides a detailed analysis of the case study, including a comparison of the experimental framework and the control laws utilized. Finally, Chapter 9 presents the conclusions drawn from the study

## II. KINEMATIC MODEL

Figure 1 shows the quadcopter platform, where the world-fixed inertial frame is represented by $\langle \mathbb{I} \rangle$ with the following unit vectors $\{\mathbf{I}_x, \mathbf{I}_y, \mathbf{I}_z\}$ and the body-fixed frame attached to quadcopter movements is defined by $\langle \mathbb{B} \rangle$ with the unit vectors $\{\mathbf{B}_x, \mathbf{B}_y, \mathbf{B}_z\}$, where the center of mass (CoM) is aligned.

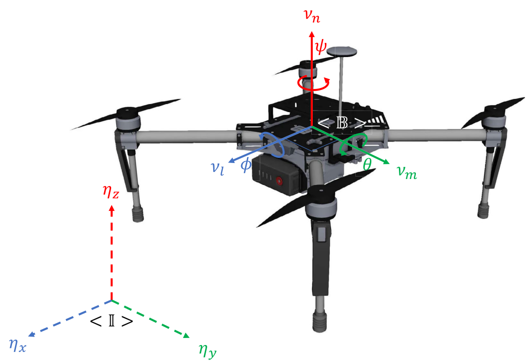


**FIGURE 1.** UAV reference frame DJI Matrice 100.

This work considers that it is enabled to rotate only in yaw ($\psi$) defined in the vertical axis $\mathbf{B}_z$ and does not consider the rotational movements of Pich and roll because the angles on the horizontal axis $\mathbf{B}_x$ and $\mathbf{B}_y$ are relatively small in flights that are not agile. In addition, the multirotor's low-level controller ensures stable hovering flight. The position and orientation of quadcopter is define by:

$$\begin{cases} \eta_x = \eta_{x_0} + a\cos(\psi) - b\sin(\psi) \\ \eta_y = \eta_{y_0} + a\sin(\psi) + b\cos(\psi) \\ \eta_z = \eta_{z_0} + c \\ \eta_\psi = \psi, \end{cases} \tag{1}$$

and defined vectorially by $\boldsymbol{\eta} = [\eta_x\ \eta_y\ \eta_z\ \eta_\psi]^\mathsf{T} \in \mathbb{R}^4$ respect to the frame $\langle \mathbb{I} \rangle$, where the values $(\eta_{x_0}, \eta_{y_0}, \eta_{z_0})$ are the locations of the center of mass and $(a, b, c)$ values define the displacement of the point of interest measured from the CoM. The evolution of the point of interest over time represents the instantaneous kinematics of the quadrotor, expressed in matrix form is:

$$\begin{bmatrix}\dot{\eta}_x\\ \dot{\eta}_y\\ \dot{\eta}_z\\ \dot{\eta}_\psi\end{bmatrix} = \begin{bmatrix}\cos(\psi) & -\sin(\psi) & 0 & -\rho_1\\ \sin(\psi) & \cos(\psi) & 0 & \rho_2\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\nu_l\\ \nu_m\\ \nu_n\\ \nu_\omega\end{bmatrix} \tag{2}$$

where $\boldsymbol{\nu} = \begin{bmatrix}\nu_l & \nu_m & \nu_n & \nu_\omega\end{bmatrix}^\intercal \in \mathbb{R}^4$ define the linear $(\nu_l, \nu_m, \nu_n)$ and angular $(\nu_\omega)$ velocities in $\langle\mathbb{B}\rangle$ frame; the expressions $\rho_1 = a\sin(\psi) + b\cos(\psi)$ and $\rho_2 = a\cos(\psi) - b\sin(\psi)$ represent the additional behavior considering the displacement of the point of interest. Equation (2) is expressed in the compact form as:

$$\dot{\boldsymbol{\eta}}(t) = \mathbf{J}(\psi(t))\boldsymbol{\nu}(t), \tag{3}$$

where $\mathbf{J}(\psi(t)) \in \mathbb{R}^{4\times4}$ is the Jacobian matrix which allows the linear mapping between the control maneuverability velocities $\boldsymbol{\nu}$ to the evolution of the point of interest $\dot{\boldsymbol{\eta}}$.

## III. DYNAMIC MODEL

Most unmanned aerial vehicles (UAVs) used in development are equipped with low-level PID controllers within their motors. These controllers adjust the voltages required for attitude and altitude control based on reference speeds [51]. This setup effectively leverages the advantages offered by software development kits (SDKs) provided by commercial drone manufacturers for developers and researcher.

The dynamic model (4) presented in [33] expresses a simplified dynamic model $\dot{\boldsymbol{\nu}}(t) = \mathbf{f}(\boldsymbol{\xi}, \boldsymbol{\nu}(t), \boldsymbol{\mu}(t))$, considering linear and rotational velocities as the input signals of the system.

$$\begin{bmatrix}\mu_l\\ \mu_m\\ \mu_n\\ \mu_\omega\end{bmatrix} = \begin{bmatrix}\xi_1 & 0 & 0 & \xi_2\\ 0 & \xi_3 & 0 & \xi_4\\ 0 & 0 & \xi_5 & 0\\ b\xi_6 & a\xi_7 & 0 & \xi_8(a^2+b^2)+\xi_9\end{bmatrix}\begin{bmatrix}\dot{\nu}_l\\ \dot{\nu}_m\\ \dot{\nu}_n\\ \dot{\nu}_\omega\end{bmatrix} + \begin{bmatrix}\xi_{10} & \omega\xi_{11} & 0 & a\omega\xi_{12}\\ \omega\xi_{13} & \xi_{14} & 0 & b\omega\xi_{15}\\ 0 & 0 & \xi_{16} & 0\\ a\omega\xi_{17} & b\omega\xi_{18} & 0 & \xi_{19}\end{bmatrix}\begin{bmatrix}\nu_l\\ \nu_m\\ \nu_n\\ \nu_\omega\end{bmatrix} \tag{4}$$

To simplify the notation, the dynamic model of the UAV can be written in its compact form as:

$$\boldsymbol{\mu}_n(t) = \mathbf{M}_n(\boldsymbol{\xi}, a, b)\dot{\boldsymbol{\nu}}(t) + \mathbf{C}_n(\boldsymbol{\xi}, \boldsymbol{\nu}, a, b)\boldsymbol{\nu}(t), \tag{5}$$

where $\boldsymbol{M}_n(\boldsymbol{\xi}, a, b) \in \mathbb{R}^{4\times4}$ is a positive definite matrix, which define the mass and inertia matrix of the quadcopter; $\mathbf{C}_n(\boldsymbol{\xi}, \boldsymbol{\nu}, a, b) \in \mathbb{R}^{4\times4}$ is the Coriolis and Centripetal matrix; $\boldsymbol{\mu} = \begin{bmatrix}\mu_l & \mu_m & \mu_n & \mu_\omega\end{bmatrix}^\intercal \in \mathbb{R}^4$ are the reference maneuverability velocities; and $\dot{\boldsymbol{\nu}} = \begin{bmatrix}\dot{\nu}_l & \dot{\nu}_m & \dot{\nu}_n & \dot{\nu}_\omega\end{bmatrix}^\intercal \in \mathbb{R}^4$ are the accelerations generated in the system. The dynamic model presented in (5) can be represented as a regressor matrix of the system $\boldsymbol{Y}_n$ and the vector of dynamic parameters $\boldsymbol{\xi}$,

$$\mathbf{M}_n(\boldsymbol{\xi}, a, b)\dot{\boldsymbol{\nu}}(t) + \mathbf{C}_n(\boldsymbol{\xi}, \boldsymbol{\nu}, a, b)\boldsymbol{\nu}(t) = \boldsymbol{Y}_n(\dot{\boldsymbol{\nu}}, \boldsymbol{\nu}, a, b)\boldsymbol{\xi}, \tag{6}$$

where, the vector $\boldsymbol{\xi} = \begin{bmatrix}\xi_1 & \xi_2 & .. & \xi_p\end{bmatrix}^\intercal \in \mathbb{R}^p$, with $p = 19$, is the vector of unknown dynamic parameters represented in Table 1, encompassing the internal dynamics of the quadrotor. These parameters constitute a combination of values associated with the physical, mechanical, electrical, and aerodynamic phenomena influencing the robotic system.

**TABLE 1.** Valores de $\xi$.

| | | | |
|---|---|---|---|
| $\xi_1 = 0.6756$ | $\xi_2 = 1.0000$ | $\xi_3 = 0.6344$ | $\xi_4 = 1.0000$ |
| $\xi_5 = 0.4080$ | $\xi_6 = 1.0000$ | $\xi_7 = 1.0000$ | $\xi_8 = 1.0000$ |
| $\xi_9 = 0.2953$ | $\xi_{10} = 0.5941$ | $\xi_{11} = -0.8109$ | $\xi_{12} = 1.0000$ |
| $\xi_{13} = 0.3984$ | $\xi_{14} = 0.7040$ | $\xi_{15} = 1.0000$ | $\xi_{16} = 0.9365$ |
| $\xi_{17} = 1.0000$ | $\xi_{18} = 1.0000$ | $\xi_{19} = 0.9752$ | |

However, (6) does not consider the unmodeled dynamics inherent in the simplified model and the incidence of external perturbations that may change the internal configuration of the UAV, so that an uncertainty velocities term $\mathbf{S}\boldsymbol{\tau}_u$ is introduced:

$$\boldsymbol{\mu}_T(t) = \mathbf{M}_n(\boldsymbol{\xi}, a, b)\dot{\boldsymbol{\nu}}(t) + \mathbf{C}_n(\boldsymbol{\nu}, \boldsymbol{\xi}, a, b)\boldsymbol{\nu}(t) + \mathbf{S}\boldsymbol{\tau}_u, \tag{7}$$

where $\boldsymbol{\mu}_T = \boldsymbol{\mu}_n + \boldsymbol{\mu}_{ext}$ represents the total velocities applied on the platform and it is the sum of the nominal active velocities $\boldsymbol{\mu}_n$, and the external perturbation velocities $\boldsymbol{\mu}_{ext} \in \mathbb{R}^4$.

## IV. MPC CONTROLLER

This section describes the formulation of the optimal control problem to plan the trajectory tracking over a finite prediction horizon $l \in [t, t+N]$, as shown in Figure 2. The prediction of the generalized nonlinear kinematic-dynamical system is defined as:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{\mu}(t),$$
$$A = \begin{bmatrix}0_{4\times4} & \mathbf{J}(\psi)\\ 0_{4\times4} & -\mathbf{M}_n^{-1}\mathbf{C}_n\end{bmatrix},\ B = \begin{bmatrix}0_{4\times4}\\ \mathbf{M}_n^{-1}\end{bmatrix}, \tag{8}$$

where $\boldsymbol{x}(t) = \begin{bmatrix}\boldsymbol{\eta}^\intercal & \boldsymbol{\nu}^\intercal\end{bmatrix}^\intercal \in \mathbb{X}$ and $\boldsymbol{\mu}(t) \in \mathbb{U}$ are the state and input to the system; and $\dot{\boldsymbol{x}}(t) = \begin{bmatrix}\dot{\boldsymbol{\eta}}^\intercal & \dot{\boldsymbol{\nu}}^\intercal\end{bmatrix}^\intercal$. An intermediate cost function $\boldsymbol{\ell}_t$ is defined as:

$$\boldsymbol{\ell}_t(\tilde{\boldsymbol{\eta}}, \boldsymbol{\mu}) = \frac{1}{2}(\tilde{\boldsymbol{\eta}}^\intercal(t)\mathbf{Q}\tilde{\boldsymbol{\eta}}(t) + \boldsymbol{\mu}^\intercal(t)\mathbf{R}\boldsymbol{\mu}(t)). \tag{9}$$

At the last instant of time the final prediction cost function $\boldsymbol{\ell}_f$ is defined as:

$$\boldsymbol{\ell}_f(\tilde{\boldsymbol{\eta}}) = \frac{1}{2}\tilde{\boldsymbol{\eta}}^\intercal(N)\mathbf{Q}\tilde{\boldsymbol{\eta}}(N). \tag{10}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are positive definite design gain matrices for the error and input system, respectively. The NMPC is defined as the solution to the optimal control

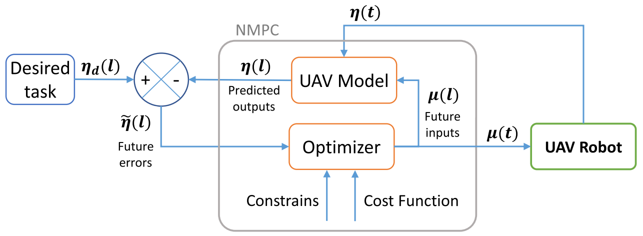


**FIGURE 2. Nominal NMPC general scheme.**

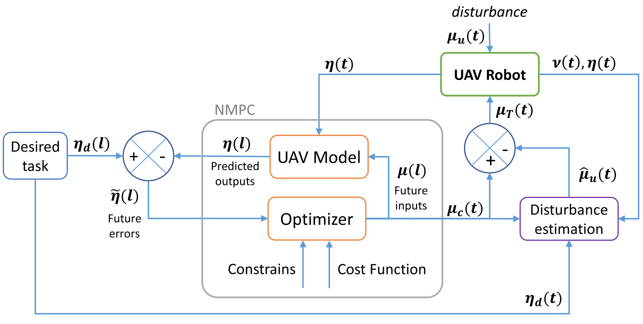


**FIGURE 3. Adaptive NMPC general scheme.**

problem (OCP):

$$\min_{\tilde{\eta}(.),\mu(.)} \ \ell_f(\tilde{\eta}(N)) + \int_t^N \ell_t(\tilde{\eta}(t), \mu(t))dt \tag{11a}$$

$$\text{subject to:} \quad \dot{x} = \mathbf{f}(x(t), \mu(t)) \tag{11b}$$

$$x(0) = x_0 \tag{11c}$$

$$\nu \in [\nu_{min}, \nu_{max}] \tag{11d}$$

$$\mu(t) \in \mathbb{U} \quad \forall t \in [0, N-1] \tag{11e}$$

$$x(t) \in \mathbb{X} \quad \forall t \in [0, N] \tag{11f}$$

where the NOCP (11a) is solved considering the initial conditions (11c) and translated into a nonlinear programming formulation (NLP) using the direct multiple shooting method [52]. (11b) define the system dynamics considered as a constraint. Equations (11d) to (11e) and (11f) define the input and state constraints, respectively.

## V. ADAPTIVE NMPC CONTROLLER

Figure 3 shows the general scheme for solving the trajectory tracking problem as the direct sum of the optimal control actions generated by the NMPC subject to the constraints and the estimation of the unmodeled dynamics that have an adaptive compensation character against the perturbations to which the quadrotor may be subjected.

Due to the changes in the internal dynamics and the effect of the uncertainties that affect the quadrotor dynamic model, velocity errors $\tilde{\nu} = \mu_c - \nu$ are generated, which are the difference between the velocities of the optimal controller and the actual velocities of the robot. The dynamic model (7) can be rewritten in the regressor matrix form:

$$\mu_T(t) = Y_n(\dot{\nu}, \nu)\xi_n + Y_u(\dot{\nu}, \nu)\xi_u. \tag{12}$$

From here onwards, the subscript '$n$' refers to the nominal dynamic model obtained by offline identification. The subscript '$u$' refers to the dynamics not modeled in the simplified dynamic model which includes the unknown external disturbances. Considering the error between the approximate uncertainty model and the real uncertainty, it is obtained:

$$Y_u(\dot{\nu}, \nu)\hat{\xi}_u \approx Y_u(\dot{\nu}, \nu)\xi_u + Y_u(\dot{\nu}, \nu)\tilde{\xi}_u, \tag{13}$$

where, $\hat{\xi}$ and $\xi$ are the estimated and real dynamics parameters, respectively, whereas $\tilde{\xi} = \hat{\xi} - \xi$ is the vector of parameter error. In order to estimate the uncertainties in the dynamic parameters and the non-modeled dynamics, the NMPC output is assumed to produce an optimal input $\mu_c(t) \approx Y_n(\dot{\nu}, \nu)\xi_n$, and the following Adaptive Non Linear MPC is proposed:

$$\mu_T(t) = \mu_c(t) + \mathbf{Y}_u(\dot{\nu}_r, \nu_r)\hat{\xi}_u \tag{14}$$

$$= \mu_c(t) + \hat{\mu}_u(t). \tag{15}$$

The adaptable component $\mathbf{Y}_u(\dot{\nu}_r, \nu_r)\hat{\xi}_u =: \hat{\mu}_u(t)$ serves two purposes by providing an estimation of the disturbance and generating the necessary adjustments in the system's dynamics. Also, the following parameter-updating law is proposed:

$$\mathbf{Y}_u(\dot{\nu}_r, \nu_r)\hat{\xi}_u = \hat{\mathbf{M}}_u(\xi)\dot{\nu}_r + \hat{\mathbf{C}}_u(\xi)\nu_r + \mathbf{K}_r\sigma, \tag{16}$$

$$\dot{\hat{\xi}}_u = \mathbf{\Gamma}^{-1}\mathbf{Y}_u^{\mathsf{T}}\sigma. \tag{17}$$

where, the undesirable steady-state position errors are constrained to lie on a sliding surface $\sigma = \tilde{\nu} + \mathbf{\Lambda}\tilde{\eta}$, where $\mathbf{\Lambda}$ is a constant diagonal gain matrix of velocity errors. Here, $\nu_r = \sigma + \nu$ represents the modified reference velocity.

To demonstrate the tracking error global convergence, the following Lyapunov candidate function is considered,

$$\mathbf{V} = \frac{1}{2}\sigma^{\mathsf{T}}\mathbf{M}_u\sigma + \frac{1}{2}\tilde{\xi}^{\mathsf{T}}\mathbf{\Gamma}\tilde{\xi}, \tag{18}$$

where $\mathbf{\Gamma} \in \mathbb{R}^{p\times p}$ is a positive definite diagonal matrix. From (18), the time derivative of the Lyapunov candidate along the system's trajectories is,

$$\dot{\mathbf{V}} = -\sigma^{\mathsf{T}}[\mu_u - \mathbf{M}_u\dot{\nu}_r - \mathbf{C}_u\nu_r] + \tilde{\xi}^{\mathsf{T}}\mathbf{\Gamma}\dot{\tilde{\xi}}. \tag{19}$$

From equations (16), (17), and (19), and considering that $\dot{\tilde{\xi}} = \dot{\hat{\xi}}$ due to the parameters are scalar values, then,

$$\dot{\mathbf{V}} = -\sigma^{\mathsf{T}}[\tilde{\mathbf{M}}_u\dot{\nu}_r + \tilde{\mathbf{C}}_u\nu_r + \mathbf{K}_r\sigma] + \tilde{\xi}^{\mathsf{T}}\mathbf{\Gamma}\dot{\hat{\xi}} \tag{20}$$

$$= -\sigma^{\mathsf{T}}\mathbf{K}_r\sigma + \tilde{\xi}^{\mathsf{T}}[\mathbf{\Gamma}\dot{\hat{\xi}} - \mathbf{Y}_u^{\mathsf{T}}\sigma] \tag{21}$$

$$= -\sigma^{\mathsf{T}}\mathbf{K}_r\sigma \leq 0. \tag{22}$$

The expression (22). indicates that the output errors approach the sliding surface over time.

$$\sigma = \tilde{\nu} + \mathbf{\Lambda}\tilde{\eta} = 0. \tag{23}$$

Furthermore, the expression (23) suggests that the sliding surface $\sigma$ approaches zero, indicating the system's convergence towards an equilibrium state. However, to affirm that the system attains a stable state where both the steady-state velocity error $\tilde{\boldsymbol{\nu}}$ and the steady-state position error $\tilde{\boldsymbol{\eta}}$ converge to zero, it is necessary to demonstrate the vanishing of the steady-state velocity error as well. This can be accomplished by substituting the desired velocity $\boldsymbol{\nu}_d$ with a virtual 'reference velocity' $\boldsymbol{\nu}_r$ [53]:

$$\boldsymbol{\nu}_r = \boldsymbol{\nu}_d + \boldsymbol{\Lambda}\tilde{\boldsymbol{\eta}}, \tag{24}$$

it is defined that,

$$\boldsymbol{\sigma} = \tilde{\boldsymbol{\nu}}_r = \boldsymbol{\nu}_r - \boldsymbol{\nu}$$

The control law and adaptation law become,

$$\hat{\boldsymbol{\mu}}_u(t) = \hat{\mathbf{M}}_u(\boldsymbol{\xi})\dot{\boldsymbol{\nu}}_d + \hat{\mathbf{C}}_u(\boldsymbol{\xi})\boldsymbol{\nu}_d + \mathbf{K}_r\tilde{\boldsymbol{\nu}}, \tag{25}$$

with,

$$\dot{\hat{\boldsymbol{\xi}}}_u = \boldsymbol{\Gamma}^{-1}\mathbf{Y}_u^{\mathsf{T}}\tilde{\boldsymbol{\nu}}. \tag{26}$$

Once more, global convergence of the tracking can be demonstrated by employing the Lyapunov function:

$$\mathbf{V} = \frac{1}{2}\tilde{\boldsymbol{\nu}}^{\mathsf{T}}\mathbf{M}_u\tilde{\boldsymbol{\nu}} + \frac{1}{2}\tilde{\boldsymbol{\xi}}^{\mathsf{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{\xi}}, \tag{27}$$

Similarly, in the previous demonstration, instead of (18), the result obtained is,

$$\dot{\mathbf{V}} = -\tilde{\boldsymbol{\nu}}^{\mathsf{T}}\mathbf{K}_r\tilde{\boldsymbol{\nu}} \leq 0 \tag{28}$$

The expression (28) suggests that the steady-state velocity error of the aircraft is zero. Consequently, this implies that $\tilde{\boldsymbol{\eta}} \rightarrow 0$ as $t \rightarrow \infty$. The adaptive controller designed for the perturbations defined by equations (16) and (17) achieves global asymptotic stability and ensures zero steady-state error for UAV positions.

## VI. MPC WITH MOMENTUM OBSERVER DYNAMICS

The generalized momentum of the platform $\mathbf{p} \in \mathbb{R}^4$ is:

$$\mathbf{p} = \mathbf{M}_n\boldsymbol{\nu}. \tag{29}$$

From model in (5) the time temporaly evolution of $\mathbf{p}$ is written as

$$\dot{\mathbf{p}} = \boldsymbol{\mu}_\tau + \dot{\mathbf{M}}_n\boldsymbol{\nu} - \mathbf{C}_n\boldsymbol{\nu} \tag{30}$$
$$= \boldsymbol{\mu}_\tau + \mathbf{C}_n^{\mathsf{T}}\boldsymbol{\nu}, \tag{31}$$

where the property $\dot{\mathbf{M}}_n = \mathbf{C}_n + \mathbf{C}_n^{\mathsf{T}}$ is used. As presented in [54], from (30), the dynamics of the momentum observer is deduced

$$\dot{\hat{\mathbf{p}}} = \boldsymbol{\mu} + \hat{\boldsymbol{\mu}}_{ext} + \mathbf{C}_n^{\mathsf{T}}\boldsymbol{\nu} \tag{32}$$
$$\dot{\hat{\boldsymbol{\mu}}}_{ext} = \mathbf{K}_o(\dot{\mathbf{p}} - \dot{\hat{\mathbf{p}}}), \tag{33}$$

where $\hat{\boldsymbol{\mu}}_{ext}$ is the external perturbation velocity estimation, and $\mathbf{K}_o$ is the positive diagonal gain matrix of the observer. The resulting output from the observer is the estimation of external disturbance, which is obtained by integrating (33) as follows,

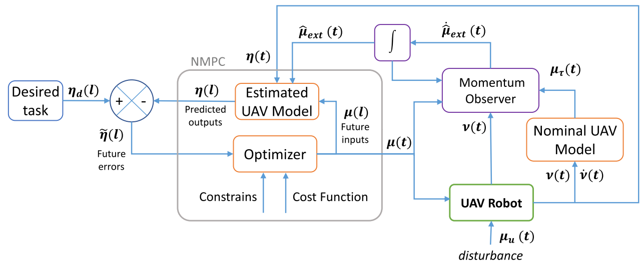


**FIGURE 4.** MOD-NMPC general scheme.

$$\hat{\boldsymbol{\mu}}_{ext} = \mathbf{K}_o\left(\mathbf{p}(t) - \int_{t_0}^{t}(\boldsymbol{\mu}_n + \hat{\boldsymbol{\mu}}_{ext} + \mathbf{C}_n^{\mathsf{T}}\boldsymbol{\nu})dt - \mathbf{p}(0)\right). \tag{34}$$

The monitored signal $\hat{\boldsymbol{\mu}}_{ext}$, is also referred to as the residual vector. Ideally, this vector, resembling an abstract sensor, allows the momentum observer to function like a virtual sensor, detecting external velocities across the entirety of the UAV's structure.

The dynamic relationship between the external velocities $\boldsymbol{\mu}_{ext}$ and $\hat{\boldsymbol{\mu}}_{ext}$ is expressed as:

$$\dot{\hat{\boldsymbol{\mu}}}_{ext} = \mathbf{K}_o(\boldsymbol{\mu}_{ext} - \hat{\boldsymbol{\mu}}_{ext}) \tag{35}$$

This equation represents a first-order, low-pass, stable, linear, and decoupled estimation of the external forces, with the property $\hat{\boldsymbol{\mu}}_{ext} \approx \boldsymbol{\mu}_{ext}$ as $\mathbf{K}_o$ approaches infinity.

Figure 4 shows the general scheme for solving the trajectory tracking problem as the direct sum of the optimal control actions generated by the NMPC subject to the constraints and the estimation of the unmodeled dynamics that have an adaptive compensation character against the perturbations to which the quadrotor may be subjected.

The combination of Momentum Observer Dynamics and NMPC, called MOD-NMPC, is defined as:

$$\min_{\tilde{\boldsymbol{\eta}}(.),\boldsymbol{\mu}(.)} \int_t^N \boldsymbol{\ell}_t(\tilde{\boldsymbol{\eta}}(t), \boldsymbol{\mu}(t))dt + \boldsymbol{\ell}_f(\tilde{\boldsymbol{\eta}}(N)) \tag{36a}$$
$$\text{subject to:} \quad \dot{\boldsymbol{x}} = \mathbf{f}(\boldsymbol{x}(t), \boldsymbol{\mu}_n(t), \hat{\boldsymbol{\mu}}_{ext}(t)) \tag{36b}$$
$$\boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{36c}$$
$$\boldsymbol{\nu} \in [\boldsymbol{\nu}_{min}, \boldsymbol{\nu}_{max}] \tag{36d}$$
$$\boldsymbol{\mu}(t) \in \mathbb{U} \quad \forall t \in [0, N-1] \tag{36e}$$
$$\boldsymbol{x}(t) \in \mathbb{X} \quad \forall t \in [0, N] \tag{36f}$$

In which it is considered that the estimated disturbance $\hat{\boldsymbol{\mu}}_{ext}$ at time $t$ remains the same throughout the horizon $[t, t+N]$.

## VII. ADAPTIVE NMPC WITH MOD SCHEME

We suppose that the Momentum Observer Dynamic is designed such that the difference $\|\boldsymbol{\mu}_{ext} - \hat{\boldsymbol{\mu}}_{ext}\|$ is minimized; however, there exists a residual error $\tilde{\boldsymbol{\mu}}_{ext}$ such that:

$$\hat{\boldsymbol{\mu}}_{ext} \approx \boldsymbol{\mu}_{ext} + \tilde{\boldsymbol{\mu}}_{ext} \tag{37}$$

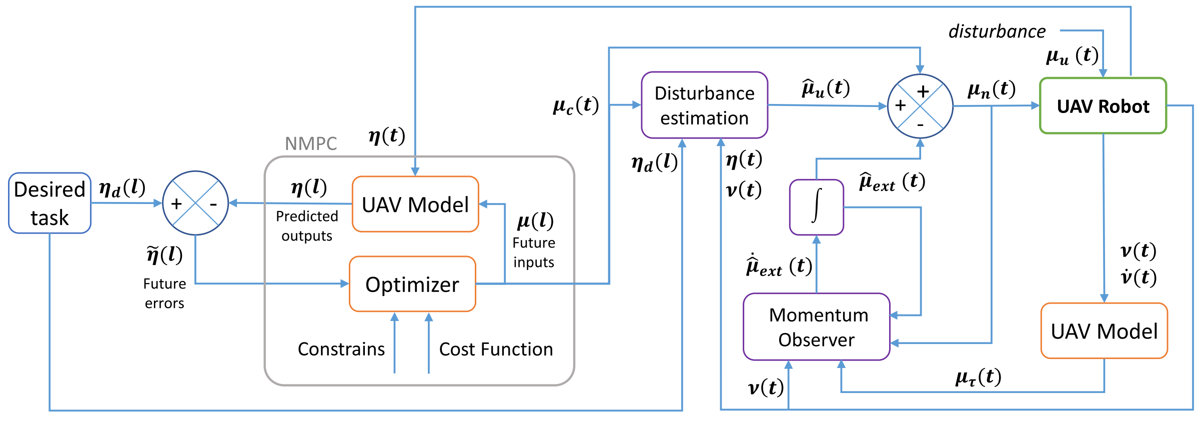


**FIGURE 5. Adaptive NMPC with MOD general scheme.**

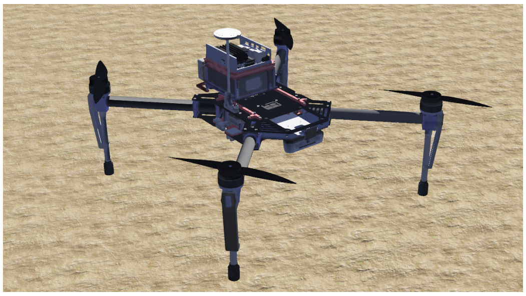

**FIGURE 6. Digital twin of DJI matrice 100.**

Figure 5 shows the general scheme that this work proposes as combination of adaptive control with NMPC and Momentum Observer Dynamics to generate a control action that minimizes, adapts, and estimates the external disturbances acting on the aircraft body, using control law 38.

$$\boldsymbol{\mu}_n(t) \approx \boldsymbol{\mu}_c(t) + \hat{\boldsymbol{\mu}}_u(t) - \hat{\boldsymbol{\mu}}_{ext}(t) \tag{38}$$

$$\approx \boldsymbol{\mu}_c(t) + \mathbf{Y}_u(\dot{\boldsymbol{\nu}}_r, \boldsymbol{\nu}_r)\hat{\tilde{\boldsymbol{\xi}}}_u - \hat{\boldsymbol{\mu}}_{ext} \tag{39}$$

If the error remains minimal, the observer term $\hat{\boldsymbol{\mu}}_{ext}$ approaches the actual uncertainty $\boldsymbol{Y}_u(\dot{\boldsymbol{\nu}}, \boldsymbol{\nu})\boldsymbol{\xi}_u$, then $\hat{\boldsymbol{\mu}}_u(t) \approx \tilde{\boldsymbol{\mu}}_{ext}$. Without consistently stimulating references, this suggests that $\hat{\boldsymbol{\mu}}_u(t) \not\approx \hat{\boldsymbol{\mu}}_{ext}(t)$. Nevertheless, it does indicate that the NMPC dynamics roughly approach the genuine dynamics.

## VIII. RESULT

The proposed controllers in this article are evaluated and validated through simulation experiments applied to the digital twin of the quadcopter DJI Matrice 100 quadrotor, utilizing a dynamic model identified within a real experimentation framework [19]. This analysis compares the proposals in a MiL framework [55] using the robotics simulation software Webots [56], as shown in Figure 6.

The experiment involves testing four baseline scenarios by applying perturbations:

1) Nominal NMPC
2) Adaptive NMPC
3) MOD-NMPC
4) Adaptive NMPC with MOD

Achieving ideal results is possible by using an NMPC problem that is fully aware of the disturbances applied to the model. The main goal of this work is to precisely estimate unmodeled dynamics and leverage them to enhance the robustness of the NMPC. All methods employ the same intermediate and final cost function, fine-tuned in terms of gain matrices to attain optimal performance in the nominal case.

In order to evaluate the runtime performance of the advanced control algorithm, the experiments are performed on a single ground station PC (Intel i7-8850H, 2.6 GHz, hexa-core 64-bit), with the loop control running at 30 Hz, this means a time horizon of 1s whit $N = 30$. The OCP solver is based in IPOP Multiple Shooting algorithm provided by CasADi [57]. The communication between the node controller and the UAV node is facilitated through ROS.

### 1) EXPERIMENTAL SETUP

For the implementation of the simulated experiments, the following constraints, initial conditions, and gain matrices are established for the proposed controller. Considering that the maximum velocity of the desired trajectory is reached when the derivative of the function is at its peak, it is inferred that $\boldsymbol{\mu}_{max} = \begin{bmatrix}4 & 4 & 4 & 0.5\end{bmatrix}^\intercal [\frac{m}{s}]$; $\boldsymbol{\mu}_{min} = -\boldsymbol{\mu}_{max}$; $\boldsymbol{\eta}_0 = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}^\intercal [m]$; Moreover, $\boldsymbol{\nu}_0 = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}^\intercal [\frac{m}{s}]$; weight matrix for cost function $\mathbf{Q} = 1.5I_{4\times4}$ and $\mathbf{R} = diag[1\ 1\ 1\ 1]$; weight matrix for adaptive law $\boldsymbol{\Omega} = 2I_{4\times4}$, $\boldsymbol{\Lambda} = 0.2I_{4\times4}$ and $\boldsymbol{\Gamma} = diag[1\ 1\ 1\ 1]$.

When a perturbation appears, the robot's acceleration is described by the following differential equation,

$$\dot{\boldsymbol{\nu}} = \mathbf{M}_n(\boldsymbol{\xi}, a, b)^{-1}[-\mathbf{C}_n(\boldsymbol{\xi}, \boldsymbol{\nu}, a, b)\boldsymbol{\nu} + \mathbf{S}\boldsymbol{\tau}_u + \boldsymbol{\mu}] \tag{40}$$

**TABLE 2.** Desired trajectory for experiments.

| Parameters | Values |
|---|---|
| $\eta_{x_d}$ | $4\sin(0.12t)+3$ |
| $\eta_{y_d}$ | $4\sin(0.24t)$ |
| $\eta_{z_d}$ | $2\sin(0.24t)+6$ |
| $\eta_{\psi_d}$ | $\operatorname{atan2}(\dot{y}_d, \dot{x}_d)$ |

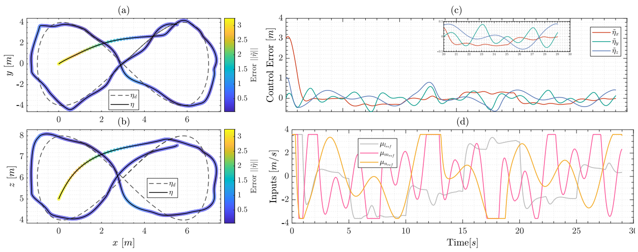


**FIGURE 7.** Nominal NMPC experiment results.

The perturbation, denoted by $\mathbf{S}\boldsymbol{\tau}_u = \begin{bmatrix}\tau_{u_x} & \tau_{u_y} & \tau_{u_z} & \tau_{u_\psi}\end{bmatrix}^\mathsf{T} \in \mathbb{R}^4$, is described by the function as follows:

$$
\tau_{u_x}(t) = \begin{cases} 0 & t < 5 \lor t > 25 \\ 2 & 5 \le t < 10 \\ 0.5 & 10 \le t < 15 \\ -1.8 & 15 \le t < 20 \\ 1.5 & 20 \le t < 25 \end{cases}
$$

$$
\begin{aligned}
\tau_{u_y}(t) &= 2\sin(0.1\,t) + 1.5\cos(0.06t) \\
\tau_{u_z}(t) &= 2\sin(0.05t) + 2\cos(-0.025t) \\
\tau_{u_\psi}(t) &= 1.5\mathrm{sign}(0.2\sin(0.04t) + 3\cos(-0.04\,t))
\end{aligned} \tag{41}
$$

The main objective of the controller proposed in this work is to track the reference trajectory defined in Table 2 over the frame $\langle \mathbb{I} \rangle$ during the incidence of disturbances not considered in the dynamic model.

### A. NOMINAL NMPC EXPERIMENT

The evaluation is conducted with consideration to perturbation functions acting as external signals modifying the system dynamics. The results depicted in Figures 7.a and 7.b illustrate that the NMPC with nominal model alone is not robust enough against perturbations. In Figures 7.c and 7.d, the error and control actions are observed respectively. This highlights the necessity of integrating adaptive control mechanisms, such as MOD, to enhance the system's error in real-world scenarios.

Figure 8 shows that the duration to solve the optimal control problem for the NMPC consistently falls below 5.2 ms for a 1-second horizon. It is recognized that computational performance is contingent upon the hardware employed, suggesting scalability with a more robust ground station PC.

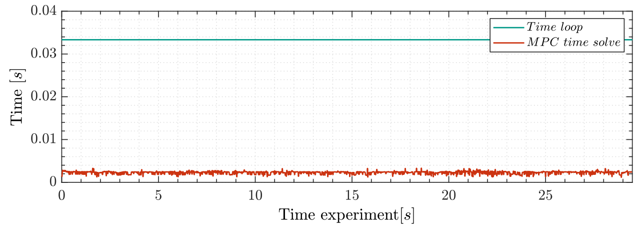


**FIGURE 8.** MPC solver execution time.

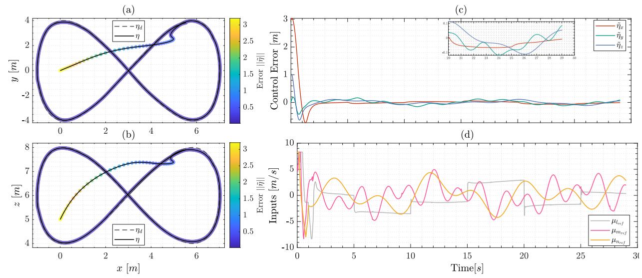


**FIGURE 9.** Adaptive NMPC experiment results.

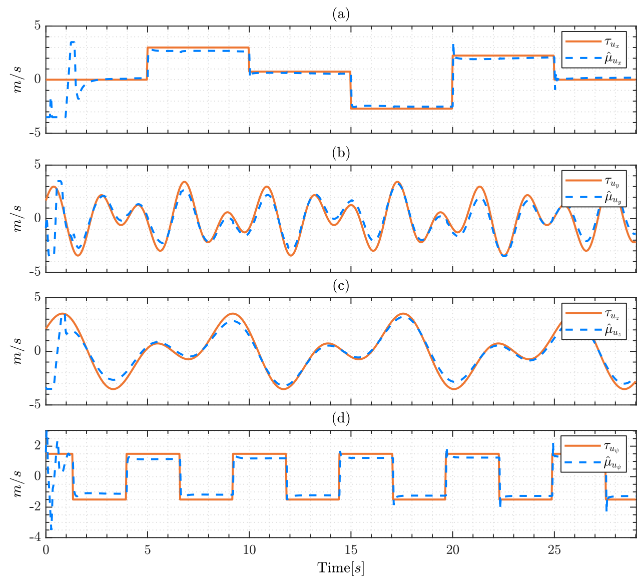


**FIGURE 10.** Adaptive law disturbance estimation results.

### B. ADAPTIVE NMPC EXPERIMENT

Figure 9.a and 9.b depict the obtained behavior closely aligned with the desired trajectory. This is because the errors observed in Figure 9.c converge to zero, and the control actions in Figure 9.d are bounded.

In Figure 10, a comparison is presented between the components of disturbances generated by (41) and those estimated by the adaptation law with NMPC. The illustration vividly demonstrates the algorithm's exceptional capability to approximate the disturbances introduced to the aircraft. This observation underscores the effectiveness of the adaptive con-

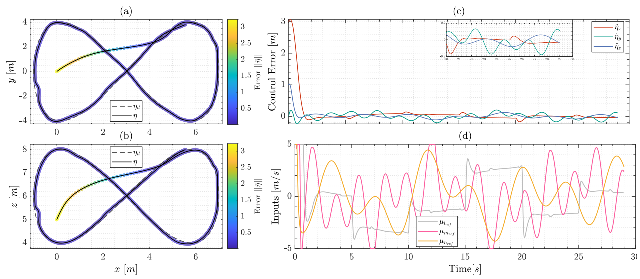


**FIGURE 11.** MOD-NMPC experiment results.

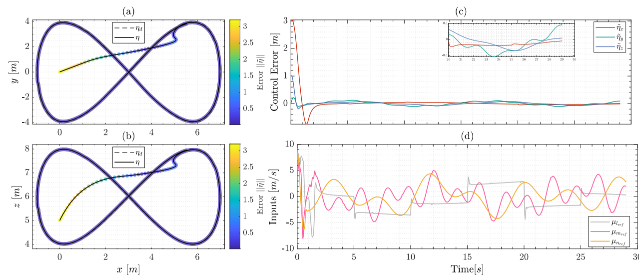


**FIGURE 13.** Adaptive NMPC with MOD experiment results.

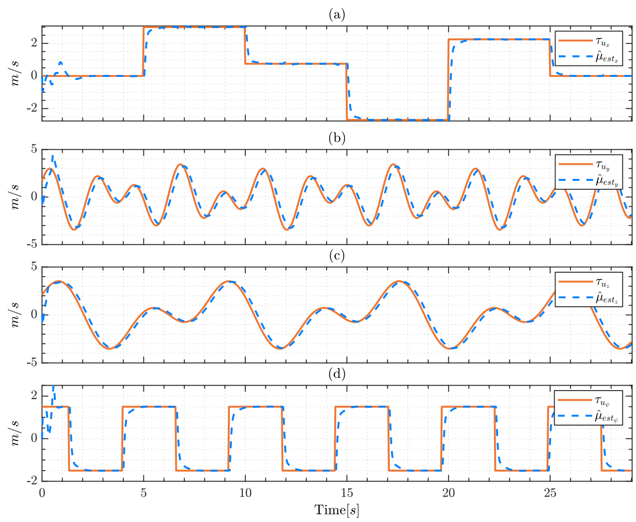


**FIGURE 12.** MOD disturbance estimation results.

trol mechanism in mitigating external disturbances, thereby enhancing the system's overall robustness and performance.

### C. MOD-NMPC EXPERIMENT

Figure 11.a and Figure 11.b illustrate the aircraft's behavior resulting from the implementation of MOD-NMPC as outlined in (36). The outcomes demonstrate that the proposed controller effectively mitigates perturbations affecting the system, even when they are highly agile. This is evident in how the errors remain bounded, as depicted in Figure 11.c, along with the control actions shown in Figure 11.d. Consequently, the quadrotor maintains its desired trajectory even in the presence of unknown disturbances.

Figure 12 depicts the comparison between estimated and real disturbances. This comparison offers a general understanding of the disturbances that can affect the system while also being estimable. It confirms that the moment observer effectively estimates the velocities impacting the system and altering the internal configuration of the quadrotor, thanks to the filtering characteristics ensuring a smooth approximation to the estimated values.

### D. ADAPTIVE NMPC WITH MOD EXPERIMENTS

In Figure 13.a and 13.b, an improved trajectory tracking is observed in the presence of disturbances. Figure 13.c demonstrates that the control error $\tilde{\boldsymbol{\eta}} = \begin{bmatrix}\tilde{\eta}_x \; \tilde{\eta}_y \; \tilde{\eta}_z \; \tilde{\eta}_\psi\end{bmatrix}^T \in \mathbb{R}^4$ converges to values close to zero in the presence of external disturbances applied in the simulation. Specifically, the control errors are bounded, achieving a final characteristic error $|\tilde{\boldsymbol{\eta}}| < 0.1$, and are different from zero, i.e., $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta}_d - \boldsymbol{\eta} \neq 0$. Finally, the control actions are depicted in Figure 13.d.

Figure 14 illustrates the estimation of perturbations affecting the internal configuration of the quadrotor during the real experimental test, showcasing the evident presence of non-modeled dynamics. While the MOD action estimates the external velocities perturbing the system, the adaptation law estimates the estimation error, ensuring its convergence to zero, i.e., $\hat{\boldsymbol{\mu}}_u(t) \approx \tilde{\boldsymbol{\mu}}_{ext}$.

The estimation of non-model dynamics, expressed as external disturbances, offers insight into phenomena not accounted for during the development of the mathematical model of the robotic system. The results confirm the robustness of the proposed controller, which is a combination of NMPC, the adaptation-estimation law, and the momentum observer dynamic.

### E. DISCUSSIONS

This work combines a dynamic momentum observer and an adaptive estimation law with a model-based predictive controller subject to constraints to estimate unmodeled disturbances in order to solve the trajectory tracking task. The results of the simulated tests show that the momentum observer estimation law and the adaptive control law allow estimating the component of the unmodeled dynamics that serves both to estimate the disturbance and to generate the necessary adaptation in the system dynamics to dissipate control errors. Furthermore, the NMPC is structured as a nonlinear programming problem, employing the multiple shooting method. Its cost function encompasses both the kinematic and dynamic models of the quadrotor.

The tests utilize the MiL framework and external disturbances were introduced to simulate disruptive behavior for the system. This allowed demonstrating the robustness of the proposed controller by combining the optimal input of the predictive controller and the estimation of the disturbance unknown to the system, ensuring that the steady-state error converges to values close to zero despite the incidence of external disturbances that modify the system's dynamics. The

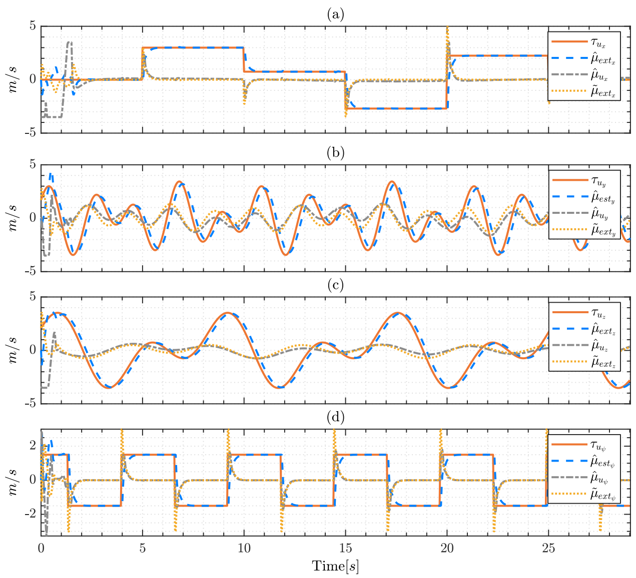


**FIGURE 14.** Results of adaptive law and MOD disturbance estimation.

use of the CasADi framework ensures that the time required to find the optimal solution remains below the sampling time, addressing a significant challenge in using predictive control schemes in critical flight system conditions.

## IX. CONCLUSION

This work develops and validates a robust controller, which combines NMPC incorporating a cost function that considers both the kinematic and dynamic models of the quadrotor to produce an optimal input for the nominal model, coupled with the momentum observer acting as a virtual sensor, along with an adaptive law for estimating non-modeled dynamics that affect and modify the system's dynamics, aiming to mitigate control errors caused by external disturbances. The proposed controller is evaluated through simulated experiments using the CasADi nonlinear programming framework, known for its high computational efficiency. In the simulated tests, fictitious disturbances are generated, and the adaptation law accurately estimates values close to the real ones, thus validating the functionality of the proposed controller.

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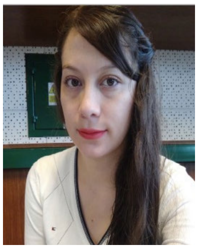

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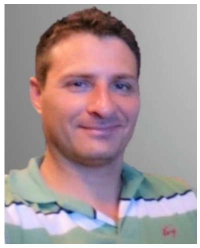

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