

Probabilistic Estimation of Cadence and Walking Speed From Floor Vibrations

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This work was supported by the National Institute on Aging of the National Institute of Health under Award R01AG067395.

This work involved human subjects or animals in its research. Approval of all ethical and experimental procedures and protocols was granted by the University of South Carolina Institutional Review Board (IRB), Office of Research Compliance, under Application No. 00114842, April 11, 2021.

ABSTRACT Objective: This research aims to extract human gait parameters from floor vibrations. The proposed approach provides an innovative methodology on occupant activity, contributing to a broader understanding of how human movements interact within their built environment. Methods and Procedures: A multilevel probabilistic model was developed to estimate cadence and walking speed through the analysis of floor vibrations induced by walking. The model addresses challenges related to missing or incomplete information in the floor acceleration signals. Following the Bayesian Analysis Reporting Guidelines (BARG) for reproducibility, the model was evaluated through twenty-seven walking experiments, capturing floor vibration and data from Ambulatory Parkinson's Disease Monitoring (APDM) wearable sensors. The model was tested in a real-time implementation where ten individuals were recorded walking at their own selected pace. Results: Using a rigorous combined decision criteria of 95% high posterior density (HPD) and the Range of Practical Equivalence (ROPE) following BARG, the results demonstrate satisfactory alignment between estimations and target values for practical purposes. Notably, with over 90% of the 95% HPD falling within the region of practical equivalence, there is a solid basis for accepting the estimations as probabilistically aligned with the estimations using the APDM sensors and video recordings. Conclusion: This research validates the probabilistic multilevel model in estimating cadence and walking speed by analyzing floor vibrations, demonstrating its satisfactory comparability with established technologies such as APDM sensors and video recordings. The close alignment between the estimations and target values emphasizes the approach's efficacy. The proposed model effectively tackles prevalent challenges associated with missing or incomplete data in real-world scenarios, enhancing the accuracy of gait parameter estimations derived from floor vibrations. Clinical impact: Extracting gait parameters from floor vibrations could provide a non-intrusive and continuous means of monitoring an individual's gait, offering valuable insights into mobility and potential indicators of neurological conditions. The implications of this research extend to the development of advanced gait analysis tools, offering new perspectives on assessing and understanding walking patterns for improved diagnostics and personalized healthcare. Clinical and Translational Impact Statement: This manuscript introduces an innovative approach for unattended gait assessments with potentially significant implications for clinical decision-making. By utilizing floor vibrations to estimate cadence and walking speed, the technology can provide clinicians with valuable insights into their patients' mobility and functional abilities in real-life settings. The strategic installation of accelerometers beneath the flooring of homes or care facilities allows for uninterrupted daily activities during these assessments, reducing the reliance on specialized clinical environments. This technology enables continuous monitoring of gait patterns over time and has the potential for integration into healthcare platforms. Such integration can enhance remote monitoring, leading to timely interventions and personalized care plans, ultimately improving clinical outcomes. The probabilistic nature of our model enables uncertainty quantification in the estimated parameters, providing clinicians with a nuanced understanding of data reliability.

INDEX TERMS Walking-induced floor vibrations, gait parameters, ambulatory metrics, uncertainty quantification, probabilistic multilevel models, health monitoring, at-home assessments.

I. INTRODUCTION

WALKING patterns provide valuable insights into human health. Gait parameters, including walking speed and cadence, are robust predictors of survival, all-cause mortality, fall risk, physical activity, physical functional decline, and post-acute care setting [1], [2]. Furthermore, deviations from baseline gait patterns can indicate aging, walking capabilities, cognition, and other health-related markers [3].

Previous studies have correlated gait parameters with changes in health status. For example, a study involving 5,000 adults with a median age of 70.6 years showed that a cadence exceeding 100 steps/minute could be associated with a fifteen-year increase in survival, while a cadence below 100 steps/minute could indicate a ten-year increase [4]. Additionally, a cadence exceeding 100 steps/minute predicted a 21% reduction in all-cause mortality, with each additional ten steps further reducing mortality by 4%.

Walking speed, on the other hand, is a reliable clinical marker across various disease populations [3]. Measurements taken over a 4-meter distance reflect changes in health conditions beyond measurement errors, with variations of approximately 0.11 m/s for medium-speed walkers and 0.14 m/s for fast-speed walkers [5]. Research also shows that increases in walking speed are proportional to changes in cadence, step length, and metabolic intensity [6], [7].

Interpreting the variability of reported gait measurements and their interplay presents significant challenges in correlating them with changes in health status. This variability includes systematic errors, different technologies used for assessment [8], [9], random errors during assessment, biased estimations, and inherent variations in subjects' patterns [5]. Typically, gait parameters are measured by having individuals walk on the floor in a straight line or on a treadmill for a specific time [5]; which, as controlled measurements, may not accurately reflect natural gait variations [8]. The Hawthorne effect, the subjective influence of perceived observation, can introduce biases and increase measurement variability, affecting the identification of correlations between gait parameters and health changes.

Unlike controlled clinical settings, home environments reflect individuals' daily challenges, such as varying terrain and environmental conditions, which can impact gait patterns. Conducting assessments in familiar settings allows for a more comprehensive analysis of gait variability and its relation to health changes [10]. Performing assessments at home holds promise for reducing the burden on individuals and healthcare systems, enabling more frequent monitoring. This can be particularly beneficial for longitudinal studies or tracking disease progression, such as Parkinson's or Alzheimer's disease [11]. However, at-home assessments using the most popular available technologies face privacy concerns and challenges related to compliance with device usage, particularly in the case of wearable technologies and patients with cognitive disorders [12]. Forgetfulness or inconsistent use of

these devices can impact the ability to provide an accurate snapshot of individuals' gait changes.

The measurement of floor vibrations is one of the newest methodologies to identify individuals' walking patterns from the structure's response. Floor vibrations caused by the impact of footsteps during walking create deformations in the floor that sensors like accelerometers or geophones can detect [12]. Once step events are correctly identified from the measurements (e.g., acceleration signals), valuable information can be extracted from complete walking cycles. Various techniques, including the time of arrival methods (ToA) [13], force estimation methods [14], [15], signal-energy-based algorithms [16], and transfer learning [17], have been developed to address challenges associated with event extraction, such as wave dispersion or low signal-to-noise ratios (SNR), which refers to the signal of interest being buried by noise.

Some studies have explored gait parameter extraction using floor vibrations as a sort of stopwatch, measuring the time between the first and last events identified in a controlled setting where the walking distance is known [18]. Others have addressed gait balance symmetry using ground reaction forces with ToA methods employed for localization [19]. However, these methods rely on available energy dissipation throughout the system, which is sensitive to multipath effects. These effects, where signals reflect and arrive via multiple paths, can cause inaccuracies propagating through the estimations. Additionally, the energy dissipation throughout the system requires step events to be reachable by the receiver at all times. This can be problematic if the energy is too low, leading to the complete removal of the event from the signal during filtering [20]. Existing energy-based vibro-localization methods often do not consider the uncertainty of the localization success or failure by providing a measure of the reliability of the collected data, which is imperative in uncontrolled scenarios, especially if there is missing information. ToA methods also require sensor synchronization, which can be challenging in real-home settings where the area to cover is more significant than a hallway or if the furniture is completely rearranged within the home. Studies that address obstructions, such as [21], may heavily depend on sensor placement and the structural characteristics of the building, impacting the reliability of the localization results. These methods may also require sophisticated signal processing algorithms and computational resources, making real-time deployment challenging in resource-constrained environments.

Some of the most advanced techniques for step localization, accounting for localization uncertainty, are presented in [22], [23], and [24]. Although these techniques have yet to be tested in unattended scenarios, they offer a probabilistic approach to localization that can enhance gait extraction.

As previous research has shown, floor vibrations present a unique opportunity to advance at-home gait assessments compared to other technologies, owing to their non-intrusive

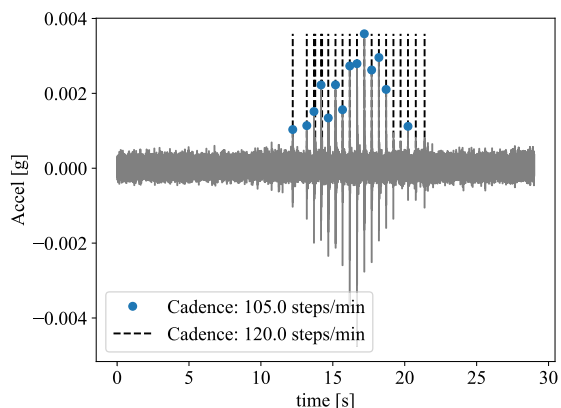


FIGURE 1. Differences in cadence estimation from acceleration data with missing step events.

and unobtrusive monitoring nature. However, implementation remains in its early stages and faces significant challenges, particularly in event identification within unattended environments. For instance, the acceleration signal presented in Figure 1 contains gait data, documenting a record of walking-induced vibrations. From the acceleration data collected, incomplete step events identification, represented by the blue markers, and correct step events identification, represented by the dashed line, provide two completely different estimations of cadence, namely 105 steps/min and 120 steps/min, with a difference of 13%. Such discrepancies can pose problems when attempting to correlate changes in health status with changes in cadence. Tracking all events from an acceleration record containing gait information is not always possible, especially if the person walks away from a sensor; thus requiring an estimation of cadence that can account for the missing information.

Similarly, methodologies aiming to remove noise contributions may inadvertently eliminate the event altogether, especially when the event’s frequency band is similar to that of the noise [20]. Considering the possibility of missing step events in the acceleration signal due to direct source collection or signal enhancement, this research aims to extract cadence and walking speed using a probabilistic approach.

The proposed probabilistic model offers an approach for at-home assessments, providing insights that can significantly impact clinical decision-making. Sensors can be strategically installed beneath the flooring of a home or care facility, allowing individuals to carry out their daily activities without interruption or conscious effort to participate in gait assessments. Integrating this technology into healthcare platforms empowers providers to monitor patients’ gait parameters remotely, significantly enhancing the timeliness of interventions and personalized care plans, ultimately leading to improved clinical outcomes. Furthermore, the probabilistic nature of the model allows for uncertainty quantification in the estimated parameters, providing clinicians with a nuanced understanding of data reliability and enabling more informed decisions and tailored interventions based on objective gait data obtained from at-home assessments.

TABLE 1. List of items for the Bayesian analysis reported guidelines (BARG) (adapted from [25], Table 1).

Location in this document	Reporting Items for the BARG
section II	Why Bayesian Analysis Goals of the analysis
	Model
section IV	Data variables
subsection IV-B	Likelihood function
subsection IV-A	Likelihood model comparison Prior distribution Formal specification of likelihood and prior Prior predictive check
	Details of the computation
subsection IV-C	Software MCMC chain convergence MCMC chain resolution
	Posterior Distribution
subsection IV-D	Posterior predictive check Summary of posterior PDFs
	Report decisions
subsection IV-D	ROPE
section V	Estimated values Decision threshold and model probabilities
	Sensitivity Analysis
subsection IV-E	Broad Priors Decisions
	Reproducibility
section VII	Software Script and data MCMC chains Auxiliary files

The probabilistic multilevel model herein accounts for nested sources of variability and limited information using Bayesian analysis. The model consists of two-level predictors: the time of steps events (cadence level) and the walking speed level predictor as a function of cadence. The first level can estimate cadence from acceleration signals by modeling possible missing steps as random variables and estimating their distributions according to the available information. The second level predictor uses available data from previous measurements on cadence and walking speed.

To ensure reproducibility of the results, we present the model using the Bayesian Analysis Reporting Guidelines (BARG) [25], which proposes reporting specific items from the Bayesian analysis to assure transparency of the analysis. The guidelines include model information, computations, posterior distributions, model comparison, and sensitivity analysis. Table 1 presents the list of items requested by the guidelines addressed throughout the manuscript. The manuscript is divided into four parts: (1) describes the multilevel model for cadence and walking speed following the BARG, (2) presents the experimental framework for collecting walking-induced floor acceleration data and Ambulatory Parkinson’s Disease Monitoring (APDM) sensors data to validate the model, (3) presents the implementation of the model with a group of subjects walking freely in an unattended setting, and (4) discusses the research results and corresponding decisions about the gait parameters estimations.

II. UNCERTAINTY QUANTIFICATION

Whether epistemic or aleatory, the assessment of uncertainty plays a critical role in informed decision-making [26]. The epistemic uncertainty is associated with variability that can be reduced once more information is collected. Aleatory uncertainty, however, represents an irreducible variability, even with access to unlimited data. For example, natural variations of walking speed and cadence observed in consecutive trials for the same individual exemplify aleatory uncertainty.

The Bayesian and the frequentist approaches to estimate uncertainty, including their differences, are well discussed in many papers [27], [28]. In the frequentist approach, the probability is defined as the frequencies of occurrences [28]; epistemic and aleatory uncertainty are present, and model parameters have a *true* value. In the Bayesian approach, however, probability represents a state of knowledge [29], and all uncertainty is considered epistemic. In the Bayesian approach, model parameters represent knowledge, probability is an expression of belief, and prior or expert knowledge is essential.

This study uses the Bayesian approach due to its suitability for handling small datasets and the variations in gait parameters derived from limited data, particularly given the challenges associated with extracting all step events from acceleration signals. Additionally, this approach offers flexibility when addressing missing information, which is crucial for accurate cadence estimation and subsequent speed estimation. For example, in the case that the number of missing steps from acceleration signals exceeds the number of accurately identified step events, higher uncertainty can be expected compared to cases where all step events are correctly extracted. A significant advantage of this approach is its ability to provide credible intervals for model parameters and predictive model quantities, which is crucial for representing the limited amount of data necessary for ambulatory metrics.

The goal of the proposed model is to find the distribution of cadence (Ω) and walking speed (\hat{V}) that best represents the time values of steps extracted from a floor acceleration signal. The purpose of the model is to use nested sources of information on two levels. The first level estimates cadence from observations, and the second uses cadence to predict walking speed. The second level is built based on pre-existing research linking both markers. Combined criteria using the high-posterior density (HPD) and the range of practical equivalence (ROPE) are used to test the target values. The criteria not only provide a credible interval of the cadence and walking speed estimations and their most likely value according to a predefined level of certainty, but they also test what percentage of the distribution is practically equivalent to the target values.

III. EXPERIMENTAL FRAMEWORK

Experiments were conducted in the structural laboratory at the University of South Carolina to validate and assess the results of the multilevel model using walking-induced floor vibrations. Twenty-seven repetitive trials were performed

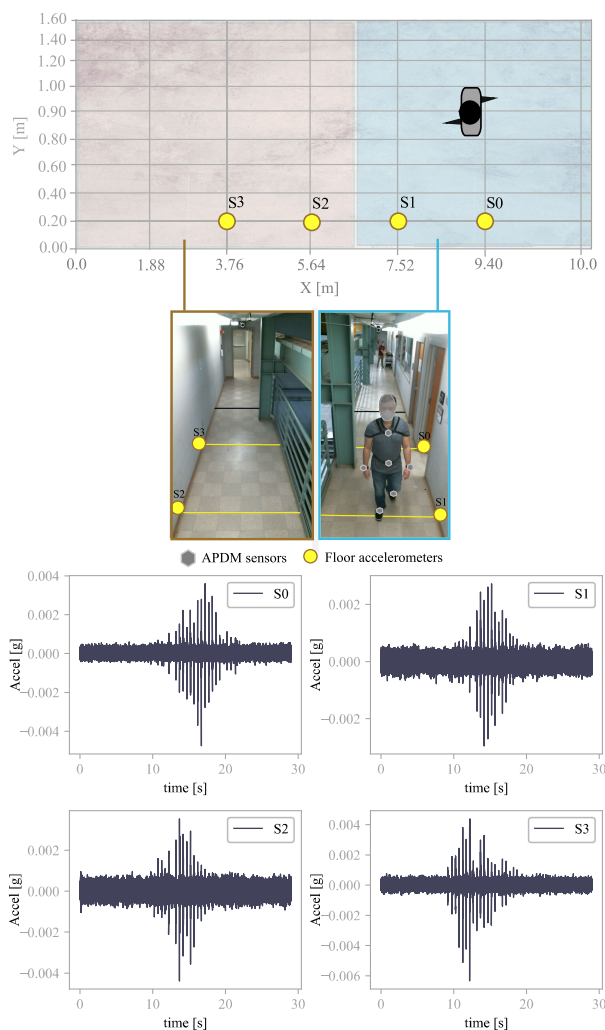


FIGURE 2. Walking test. Floor accelerations and APDM wearable sensors data collection. A trial using a metronome at 120 BPM.

by a 36-year-old participant wearing training shoes with a full rubber outsole. The participant walked on a 10-meter walkway with approximately 2 meters for acceleration and deceleration phase, respectively. Floor accelerations were recorded at a sampling frequency of 1706.67 Hz. The experimental setup, shown in Figure 2, featured four PCB 393B31 seismic accelerometers positioned next to a hallway separated 1.88 m from each other. These sensors have a sensitivity of $1.02 \text{ V}/(\text{m}/\text{s}^2)$ ($\pm 5\%$) and a frequency range of 0.1 – 200 Hz ($\pm 5\%$).

During the experiment, the individual wore six OPAL-APDM wireless body-mounted inertial sensors and followed a metronome at three different tempos: 90, 120, and 130 beats per minute (BPM). The participant stroked the ground with a left or right foot at each beat. Floor acceleration records and APDM measurements were collected during the tests. The APDM cadence and walking speed measurements are considered baselines (target values) for evaluating the estimations of the multilevel model. The OPAL-APDM sensors were selected for this research based on being widely used in

clinical trials for gait analysis. The collected data, including floor acceleration data from each sensor and the APDM sensor measurements, has been made available in the Open Science Framework (OSF) repository [30]. This research study was conducted with the approval and oversight of the Institutional Review Board (IRB) at the University of South Carolina. Informed consent was obtained to publish the participant's image (Figure 2) in an online open-access publication.

IV. MULTILEVEL PROBABILISTIC CADENCE-SPEED MODEL

Multilevel modeling allows the manipulation of complex patterns and nested sources of variability. Thus, when using hierarchy, it is necessary to consider the variability associated with each nesting level. In multilevel models, random variables incorporate the variation between different groups. By assuming that the random effect is generated under the same distribution, one can share information between the different levels, improving the precision of the prediction for groups with little data [31]. Consider, for example, the random variable cadence (Ω) for the first level model presented in Equation 1. Here, t_{s_i} represents the time of the i -th step from $i = \{1, 2, \dots, n\}$ of n steps, t_{s_0} the time of the first step, and $\Delta_t = 60/\Omega$ represents the time between consecutive steps as a function of cadence. Cadence (Ω) only depends on the time each foot touches the floor from consecutive steps. However, walking speed requires spatial information of the foot's location or, at minimum, the total distance walked. When collecting floor vibrations through multiple sensors in the time domain, information on the localization of the steps is not immediately available, which makes the estimation of walking speed from only time data impossible if sensor placement is not controlled. However, extensive literature shows that cadence, a walking pattern defined by the number of steps per minute, can be correlated with walking speed. Developing a multilevel model to transition from time (cadence) to space (walking speed) presents an opportunity to use all available data to make inferences for both markers, even with little or no information on the localization of the steps. By utilizing existing knowledge from literature, nested sources of information can be employed to estimate one marker based on the other. In this research, data from Shahar and Agmon [3] were used to inform the model to derive walking speed from cadence estimation using the second level predictor presented in Equation 2. The multilevel model, with both level predictors connected, is presented in Figure 3.

$$t_{s_i} = t_{s_0} + \Delta_t i \quad (1)$$

$$\hat{\Pi} = \alpha \Omega - \beta \quad (2)$$

When analyzing walking-induced vibration signals, the time of the step events can be determined using Equation 1. When the time differences between detected step events exceed the most frequently occurring value, there are likely missing steps. Consequently, additional random variables describing

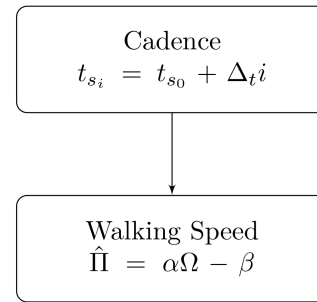


FIGURE 3. Multilevel model of the time of step events (cadence) with $\Delta_t = 60/\Omega$ and walking speed.

the possible missing step's time are included in the analysis. In the walking speed level, α and β are unknown random variables representing slope and intercept, respectively, of the proportional relationship between cadence and walking speed. The multilevel model's random variables are Ω , α , and β and different missing steps times t_j for each floor vibration signal. Here, α and β manage the uncertainty linked to the step length.

A. PRIORS & PRIOR PREDICTIVE CHECK

The selection of the prior distribution for cadence was based on the principle of maximum entropy [32], and the largest entropy was considered according to the data provided. The literature suggests a range of human cadence between 20 steps/minute and 140 steps/minute [3], [9]. Thus, a uniform distribution between these values, although a broad prior distribution, best represents current knowledge.

A prior predictive check of this model parameter is reported to assess that the chosen prior is consistent with current knowledge [33]. Figure 4 (a) shows the prior predictive check for cadence and its relationship to the predicted time interval between step events Δ_t , y-axis. It is noteworthy that the distribution's mass is tightly concentrated around possible values for the time between steps, supported by scientific reasoning. Extreme values of cadence (120-140 steps/minute) indicate that steps occur every 0.4-0.5 seconds. Values lower than this range would suggest running, rendering the distribution for cadence invalid in this analysis. Similarly, lower cadence values indicate that the person has completed at least one cycle, meaning the same foot has contacted the floor twice. The minimum time between steps would be three seconds, indicating the person is not moving.

The priors selected for the α and β parameters of the walking speed level predictor were two skew-normal distributions, defined as $\alpha \sim f(x | \mu = 0.009, \sigma = 0.005, \nu = 3)$ and $\beta \sim f(x | \mu = 0.2, \sigma = 0.01, \nu = 6)$. These informative priors were chosen based on an initial fitting of the data collected from the literature. The skew-normal distributions were selected to align with the positive nature of both parameters, with μ , σ , and ν representing the respective distributions' mean, standard deviation, and skewness.

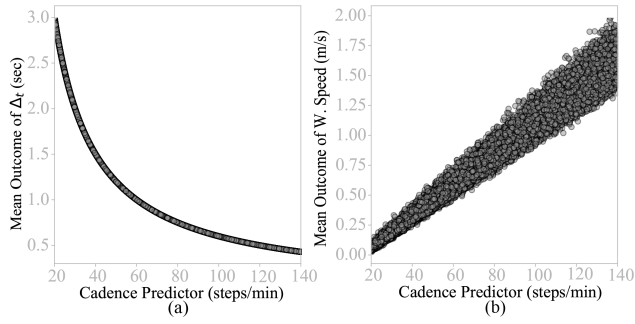


FIGURE 4. Prior predictive check for model parameters. a) Δ_t vs. Ω , b) $\hat{\Pi}(\alpha \ \& \ \beta)$ vs. Ω .

To understand the interplay of these prior distributions in the inference process, we assessed the effect of the multivariate distribution in the walking speed predictions [33]. The data generated by both distributions leads to the results presented in Figure 4.b, showing that walking speed values given cadence are physically plausible. The lowest value of cadence that represents zero movement leads to a walking speed marker of 0 m/s. Similarly, the extreme values of walking speed are associated with cadence for walking. Speed values over 2.0 m/s are associated with running, which is outside the scope of this study. Prior predictive checks on multilevel model parameters indicate that the prior predictive distributions have mass around extreme but plausible data, and there is no mass in implausible data.

B. LIKELIHOOD

The Bayesian formulation presented in Equation 3 estimates the probability of the multilevel model parameters θ given the data. The data for the first level defined as $\mathbf{D}_1 = \{t_1, t_2, \dots, t_n\}$ represent the time of the detected step events from acceleration signals. The data for the second level defined as $\mathbf{D}_2 = \{[\hat{\Omega}_1, \bar{\Pi}_1], [\hat{\Omega}_2, \bar{\Pi}_2], \dots, [\hat{\Omega}_n, \bar{\Pi}_n]\}$ represent the information collected from the literature on cadence ($\hat{\Omega}_n$) and associated walking speed values ($\bar{\Pi}_n$). The data consist of cadence and walking speed derived from an 8-10 meter walkway test with 60 adults, ranging in age from 18 to 80, recording at least fifteen cycles per participant [3].

$$\overbrace{P(\theta|\mathbf{D}M)}^{\text{Posterior}} = \overbrace{P(\theta|M)}^{\text{Prior}} \frac{\overbrace{P(\mathbf{D}|\theta M)}^{\text{Likelihood}}}{P(\mathbf{D}|M)} \quad (3)$$

The likelihood $P(\mathbf{D}|\theta, M)$ is a probability density function that describes the probability of the data given the model M and the parameters θ . Given the unknown nature of the likelihood function, an evaluation of multiple distributions using leave-one-out (LOO) cross-validation and Pareto Smoothed Importance Sampling (PSIS) was performed. This assessment involved using the same defined prior distributions and available data. Out-of-sample predictive accuracy measures how well a model predicts new, unseen data points, helping to determine which likelihood function results in models

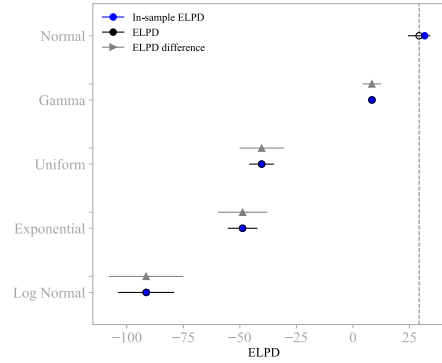


FIGURE 5. Expected log point-wise predictive density (ELPD) for different likelihood functions (In-sample ELPD: ELPD without penalization for the number of parameters. ELPD difference: standard error of the difference in ELPD between the model and the best model. A dashed line indicates the best function).

that make the most accurate predictions. This estimation was calculated for each likelihood function using posterior simulations. The likelihoods evaluated are presented in Figure 5 with their respective expected log point-wise predictive density (ELPD). The selected distributions are positively supported, aligning with the positive nature of the observations. From this comparison, the Gaussian likelihood presented in Equation 4 and Equation 5 could be considered the best assumption for the inference process. Thus, the likelihood for step times (cadence) was defined as:

$$P(\mathbf{D}_1|\theta M) = \prod_{i=1}^n \frac{1}{\sigma_t \sqrt{2\pi}} \times \exp \left[-\frac{1}{2} \frac{(t_i - t_{s_i}(\theta M))^2}{\sigma_t^2} \right] \quad (4)$$

And for walking speed as:

$$P(\mathbf{D}_2|\theta M) = \prod_{i=1}^n \frac{1}{\sigma_{ws} \sqrt{2\pi}} \times \exp \left[-\frac{1}{2} \frac{(\hat{\Pi}_i - \bar{\Pi}_i(\theta M))^2}{\sigma_{ws}^2} \right] \quad (5)$$

Parameters σ_t and σ_{ws} are also considered random variables defined by exponential prior distributions as $\sigma_t \sim f(x | \lambda = 1/\Delta_{t_d})$ and $\sigma_{ws} \sim f(x | \lambda = 10)$, respectively. Where Δ_{t_d} represents the lowest time interval between detected steps events from the acceleration signal.

C. MARKOV CHAIN MONTE CARLO (MCMC) SIMULATIONS

In Bayesian analysis, Markov Chain Monte Carlo (MCMC) simulations are indispensable for sampling from complex probability distributions, particularly when analytical solutions are impractical or when handling high-dimensional joint posteriors. These methods iteratively generate samples that systematically converge to the target distribution $P(\theta|\mathbf{D}, M, I)$.

To approximate the joint posterior distribution, we used 4,000 samples and ran five parallel chains to ensure convergence. Convergence analysis utilized the Potential Scale

Reduction Factor (PSRF) [34], a diagnostic tool that compares variances across multiple sequences. A PSRF value lower than 1.1 indicates that the within and between chain variances are sufficiently close, suggesting convergence to the same distribution. The MCMC analysis was implemented using Python 3.8 libraries [35], [36]. Convergence was achieved, with PSRF values below 1.1 for all parameters [30].

D. POSTERIOR PREDICTIVE CHECK

Two decision-making methods were used to evaluate the posterior distributions of cadence and walking speed. The first method employed was the 95% High Posterior Density (HPD) in conjunction with reference values. The HPD represents the range of most credible values, while the reference values are the cadence and walking speed measurements obtained from the APDM sensors. The second criterion was the region of practical equivalence (ROPE) [37], which defines a range of practical values considered satisfactory for practical purposes. Our analysis combines the HPD and ROPE to form the decision rule, following the principles outlined in [37]. For the decision rule, if more than 90% of the 95HPD falls inside the ROPE, we can accept the target values for practical purposes. If less than 90% of the 95HPD falls entirely outside ROPE, we can reject the target value of the marker. For any other case, we withhold the decision. The decision rule 95HPD + ROPE is more practical than using the reference values and the HPD, given that it considers the uncertainty of the estimation and evaluates how close or far from the practical values these estimations are.

The ROPE is the range of parameter values equivalent to the null value for practical purposes. Employing ROPE as a decision rule requires the most credible values of the marker to be sufficiently close to the null value to accept it or sufficiently far to reject it [37]. This study defined the ROPE based on the cadence and walking speed values obtained from the APDM sensors. Since the ultimate goal of the proposed models is to track changes in a cadence that correlates with changes in health, a variation of 10 steps/minute was used based on current literature [4]. Thus, the ROPEs for cadence in trials at 90, 120, and 130 BPM were defined as [80,100], [110-130], and [120-140] steps/minute, respectively. For walking speed, the literature correlates changes in health conditions with changes of 0.14 m/s [5]; thus, the ROPE for this marker in trials at 90, 120, and 130 BPM were selected as [0.69,1.0], [1.24,1.55], and [1.33,1.67] m/s, respectively.

The results for trials 4, 10, and 21 at 90, 120, and 130 BPM are presented in Figure 6-(a,b,c), respectively. The first plot illustrates the acceleration record, with vertical lines marking the identified steps and red lines indicating missing steps. The second plot displays the posterior distribution of cadence, with the red band representing the Region of Practical Equivalence (ROPE). The third plot presents the estimated walking speed, along with the ROPE. Table 2 and Table 3 summarize the results for the 27 trials and the decisions reached.

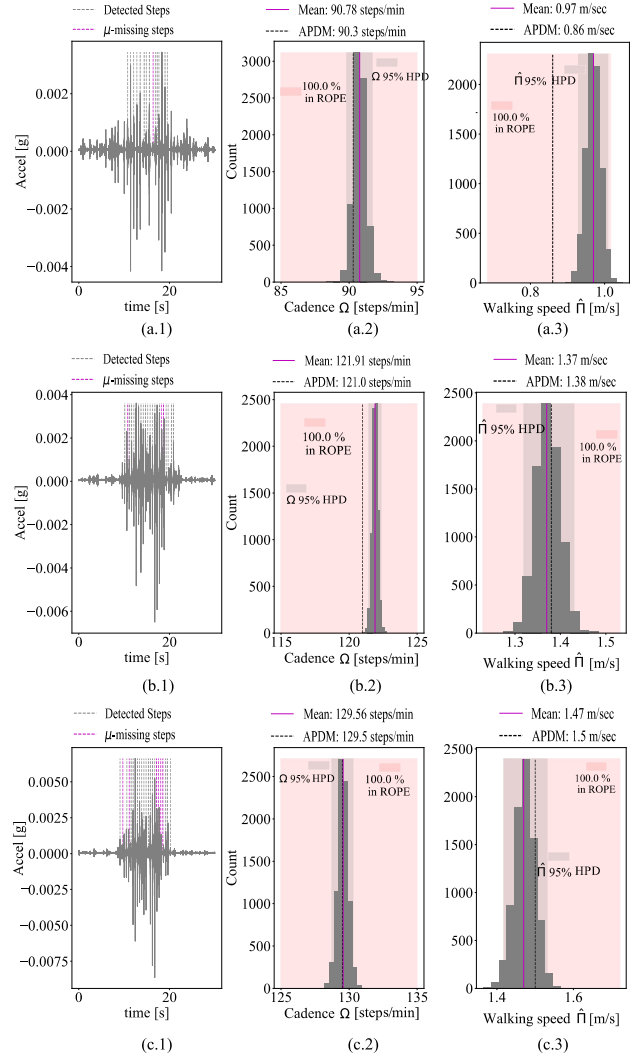


FIGURE 6. Posterior distributions for cadence and walking speed for trials 4,10, and 21, at 90 (a), 120 (b), and 130 (c) BPM. Distribution for walking speed and cadence, APDM value, 95% HPD, and ROPE.

The 95% HPD, the APDM reference value, and ROPE are highlighted on each graph. Similarly, Figure 7-(a,b,c) shows the posterior distribution of the random variables describing the time of the missing steps events from the acceleration signal with their 95% HPD and ROPE [30].

E. SENSITIVITY ANALYSIS

This analysis aims to evaluate the robustness of the model and the consistent decisions when using different prior distributions. Priors distribution were defined as, $\sigma_t \sim \text{Uniform}(x \mid a = 0.5\Delta_{t_d}; b = 1.5\Delta_{t_d})$, $\alpha \sim \text{Weibull}(x \mid \lambda = 3, K = 0.012)$; $\beta \sim \text{Weibull}(x \mid \lambda = 6, K = 0.23)$ and $\sigma_{WS} \sim \text{Weibull}(x \mid \lambda = 6, K = 2)$. Limits of these priors represent plausible times of step events and walking speed values. The results presented thus far will be addressed from this point forward as the results of the first analysis (A_1). The second set of priors will be addressed as the results of the second analysis (A_2).

TABLE 2. Characteristics of the cadence posterior distribution for trials at 90, 120, and 130 BPM, respectively.

Trial information		Reference value	Posterior distribution of cadence Ω			
Trial No	Metronome (BPM)	Cadence from APDM (steps/minute)	Mean (steps/min)	95 %HPD interval (steps/min)	% inside ROPE	Decision
1	90	90.1	91.6	[91.1-92.0]	100	Accept
2		90.2	91.3	[90.5-92.0]	100	Accept
3		90.2	91.8	[91.4-92.1]	100	Accept
4		90.3	90.8	[89.9-91.7]	100	Accept
5		89.5	91.05	[87.8-94.0]	100	Accept
6		90	90.9	[90.0-91.8]	100	Accept
7		90.4	89.1	[88.3-90.0]	100	Accept
8		90.7	90.9	[90.6-91.3]	100	Accept
9	120	120	121	[121-122]	100	Accept
10		121	122	[121-122]	100	Accept
11		120	121	[120-121]	100	Accept
12		121	122	[121-123]	100	Accept
13		120	121	[120-121]	100	Accept
14		121	122	[122-123]	100	Accept
15		120	121	[121-122]	100	Accept
16		119	120	[119-120]	100	Accept
17		120	121	[1201-122]	100	Accept
18	130	130	131	[130-131]	100	Accept
19		128	129	[128-130]	100	Accept
20		129	129	[128-129]	100	Accept
21		130	130	[129-130]	100	Accept
22		131	130	[130-131]	100	Accept
23		128	129	[129-130]	100	Accept
24		130	131	[131-132]	100	Accept
25		130	129	[129-130]	100	Accept
26		130	132	[131-132]	100	Accept
27		132	132	[131-132]	100	Accept

TABLE 3. Characteristics of walking speed posterior distribution for trials at 90, 120, and 130 BPM, respectively.

Trial information		Reference value	Posterior distribution of walking speed Π			
Trial No	Metronome (BPM)	Walking speed from APDM (m/sec)	Mean (m/sec)	95 %HPD interval (m/sec)	% inside ROPE	Decision
1	90	0.83	0.98	[0.94-1.02]	99.17	Accept
2		0.88	0.98	[0.94-1.02]	99.41	Accept
3		0.82	0.98	[0.94-1.02]	98.92	Accept
4		0.86	0.97	[0.93-1.01]	100	Accept
5		0.83	0.97	[0.91-1.02]	99.43	Accept
6		0.84	0.97	[0.93-1.01]	100	Accept
7		0.82	0.95	[0.91-0.99]	100	Accept
8		0.83	0.97	[0.93-1.01]	100	Accept
9	120	1.38	1.36	[1.31-1.41]	100	Accept
10		1.38	1.37	[1.32-1.43]	100	Accept
11		1.38	1.35	[1.30-1.40]	100	Accept
12		1.37	1.37	[1.32-1.42]	100	Accept
13		1.37	1.35	[1.31-1.41]	100	Accept
14		1.37	1.38	[1.33-1.43]	100	Accept
15		1.39	1.37	[1.32-1.42]	100	Accept
16		1.33	1.35	[1.30-1.40]	100	Accept
17		1.36	1.37	[1.31-1.42]	100	Accept
18	130	1.57	1.48	[1.43-1.54]	100	Accept
19		1.55	1.47	[1.41-1.52]	100	Accept
20		1.53	1.46	[1.41-1.51]	100	Accept
21		1.50	1.47	[1.42-1.53]	100	Accept
22		1.53	1.48	[1.43-1.54]	100	Accept
23		1.55	1.47	[1.42-1.52]	100	Accept
24		1.57	1.50	[1.44-1.55]	100	Accept
25		1.58	1.47	[1.41-1.52]	100	Accept
26		1.54	1.50	[1.44-1.55]	100	Accept
27		1.55	1.50	[1.44-1.55]	100	Accept

V. DISCUSSION

The analysis of twenty-seven trials at different tempos demonstrates satisfactory results for estimating cadence and walking speed using the proposed multilevel model.

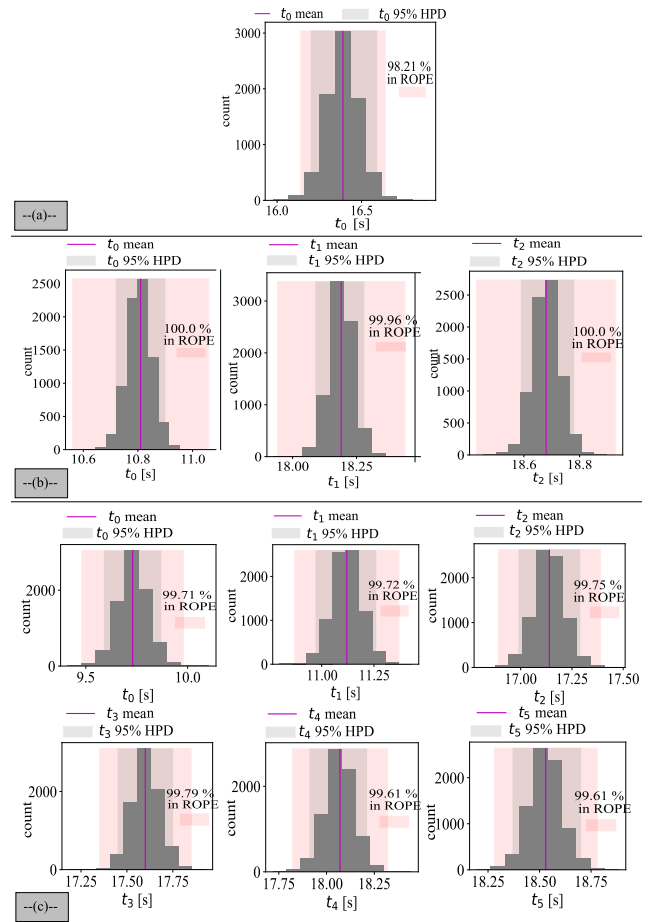


FIGURE 7. Missing steps times posterior distributions, with (a) missing steps from Figure 6 (a.1), (b) missing steps from Figure 6 (b.1), (c) missing steps from Figure 6 (c.1).

The baseline (target value) of the parameters, which is the ADPM sensor measurements for cadence and walking speed, falls inside the 95% HPD, and more than 90% of this range falls within the ROPE for all cases, further indicating the practicality and reliability of the estimations. The mean values of the posterior distribution for cadence and walking speed, presented in Table 2 and Table 3, provide the best estimates for these parameters and are practically equivalent to the target values.

The results of the distribution for cadence, which is directly estimated from the acceleration record, can be attributed to including missing step times as random variables in the analysis. Data imputation has provided more flexibility for the parameter values, reducing bias in cadence estimations. Figure 7 shows that 95HPD of these stochastic variables are within the ROPE in a percentage greater than 95% for 96% of the trials.

While the results for walking speed are satisfactory, there is more variation in the results associated with a cadence value of 90 steps per minute. This variation may be attributed to the insufficient information extracted from the literature for walking speeds between 0.85 and 1.0 m/s. Creating a

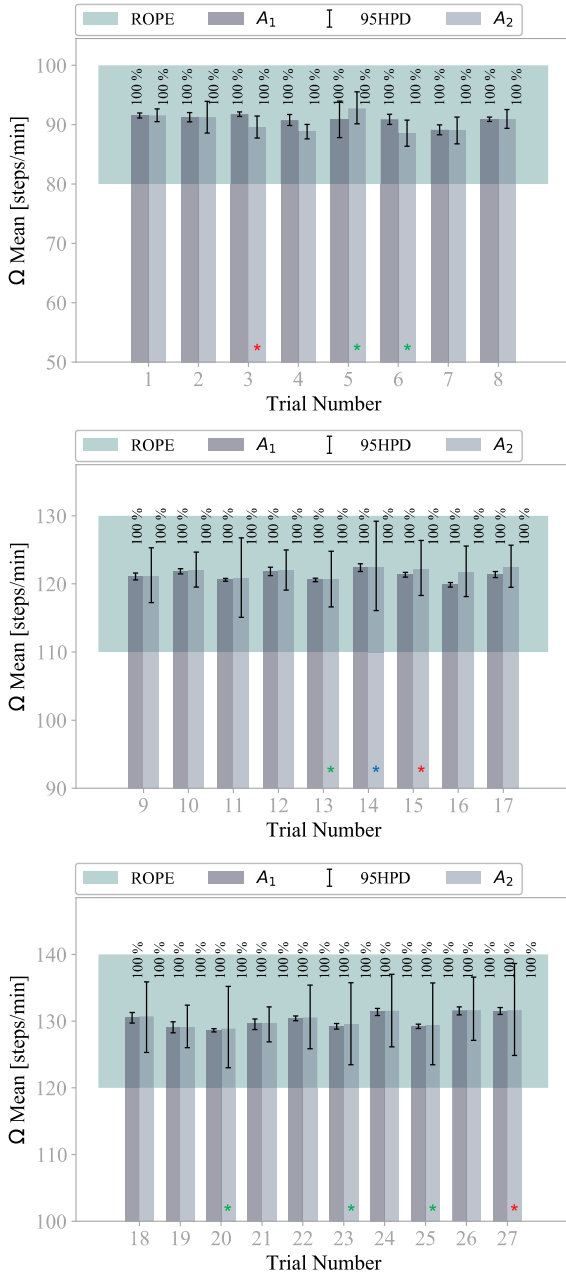


FIGURE 8. 95 % HPD and % inside ROPE for cadence distribution. (a), (b) and (c) represent trials analyzed for 90, 120, and 130 BPM—comparing analysis A_1 and A_2 with two sets of priors. Marker *, **, *: 5K, 7K, and 10K samples, respectively.

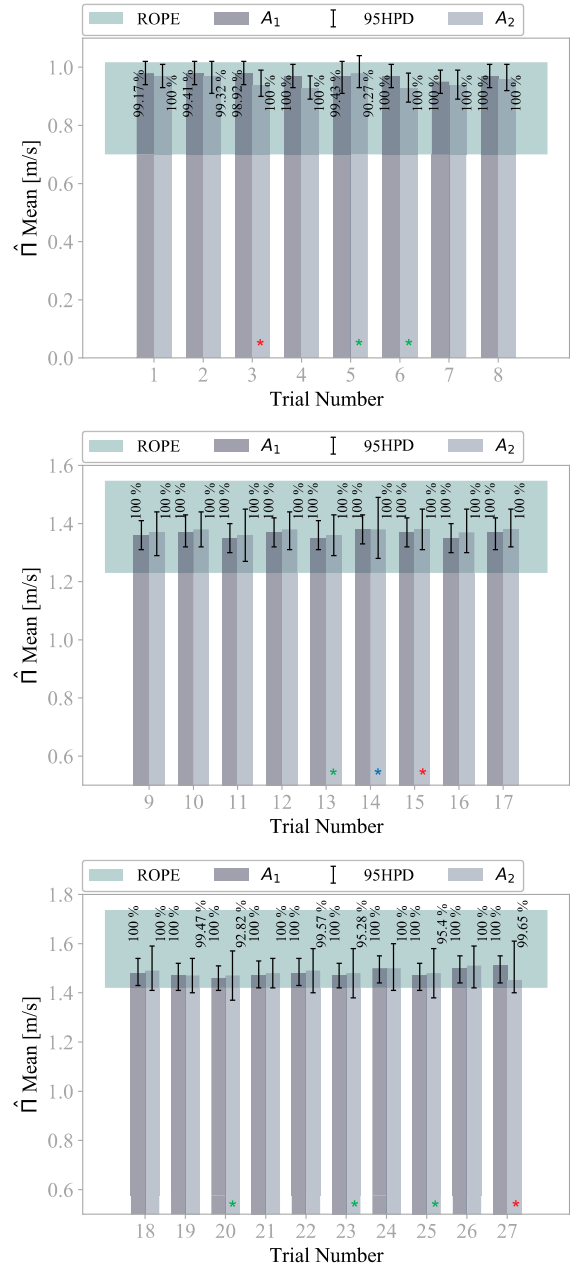


FIGURE 9. 95 % HPD and % inside ROPE for walking speed distribution. (a), (b) and (c) represent trials analyzed for 90, 120, and 130 BPM—comparing analysis A_1 and A_2 with two sets of priors. Marker *, **, *: 5K, 7K, and 10K samples, respectively.

unified model representing the variability among individual subjects or incorporating spatial dimensions when dealing with data from multiple individuals can further reduce uncertainty in the estimation. It's important to note that higher uncertainty in the estimation stems from incomplete data rather than modeling errors. For example, if spatial data, such as step localization, were available, it would contribute to a more accurate estimation of an individual's walking speed by directly accounting for step length, which is influenced by subject-specific characteristics such as age or height [38].

The sensitivity analysis, performed using a different set of priors, confirms that the decision about cadence remains consistent, with posterior distributions of the model parameters falling within the range of practical values over 90%. Figure 8 and Figure 9 present the results of twenty-seven trials for cadence and walking speed, respectively. While the decisions for cadence and walking speed remain unchanged, it should be noted that for trials marked with (*), the inference in A_2 was insufficient under the same conditions as A_1 . In these cases, convergence was achieved only after increas-

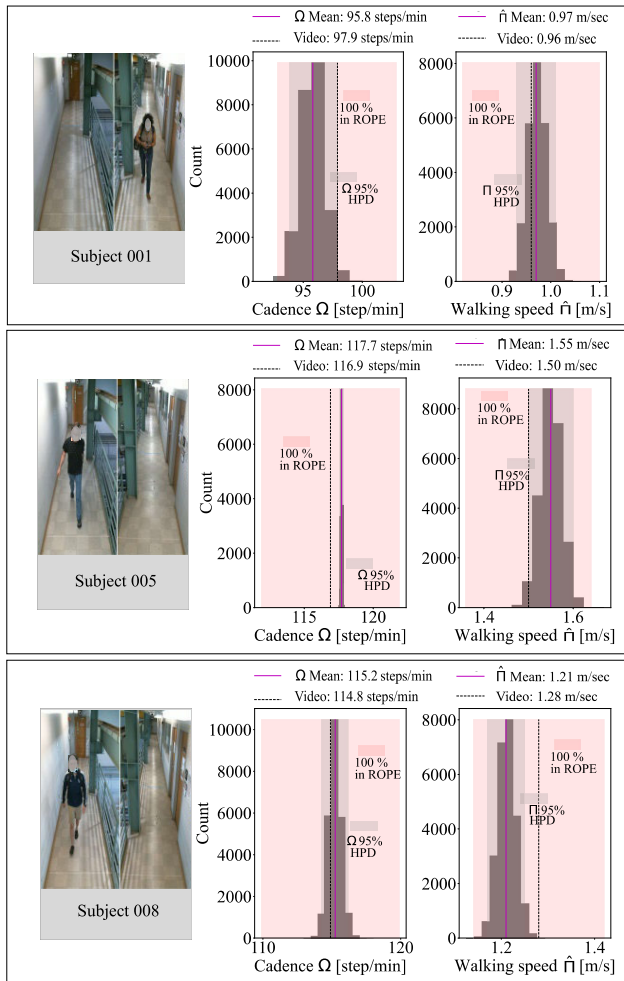


FIGURE 10. Individuals walking at self-selected speeds and posterior distributions of cadence and walking speed.

ing the number of draws to 5K, 7K, and 10K for specific trials. The complete results of the second analysis, including convergence diagnostics, marginals, and joint posterior distributions, can be found on the OSF platform [30].

VI. UNATTENDED IMPLEMENTATION

An unattended implementation was conducted to demonstrate the model’s efficacy in uncontrolled monitoring scenarios where velocity is not regulated. For this purpose, two Logitech C615 1080p cameras were strategically positioned on the ceiling to capture individuals walking, providing ground truth validation. A threshold-crossing acceleration trigger was implemented to synchronize data collection, simultaneously initiating acceleration recording and video capture. The cameras’ field of view covered the entire corridor, including two demarcated lines representing the start and end of a ten-meter walkway. Data collection involved gathering video recordings and acceleration data from ten individuals walking at various self-selected speeds. While subjects were informed about the system collecting video and floor vibrations by

TABLE 4. Characteristics of walking speed and cadence posterior distributions for all subjects.

Information	Reference value	Posterior distribution of Cadence Ω				
		Cadence from Video (steps/min)	Mean (steps/min)	95 %HPD interval (steps/min)	% inside ROPE	Decision
Subject ID						
001	97.9	95.8	[93.9-97.9]	100	Accept	
002	90.5	91.6	[91.2-91.9]	100	Accept	
003	114	111	[110-112]	100	Accept	
004	107	103	[102-103]	90.3	Accept	
005	117	118	[118-119]	100	Accept	
006	108	110	[110-111]	100	Accept	
007	107	105	[104-106]	100	Accept	
008	115	115	[114-116]	100	Accept	
009	103	104	[104-105]	100	Accept	
010	120	117	[116-118]	100	Accept	
Information	Reference value	Posterior distribution of walking speed \hat{v}				
		Walking speed from Video (m/sec)	Mean (m/sec)	95 %HPD interval (m/sec)	% inside ROPE	Decision
Subject ID						
001	0.96	0.97	[0.93-1.01]	100	Accept	
002	1.01	0.92	[0.89-0.95]	100	Accept	
003	1.19	1.16	[1.12-1.19]	100	Accept	
004	1.38	1.33	[1.28-1.37]	100	Accept	
005	1.50	1.55	[1.50-1.60]	100	Accept	
006	1.15	1.20	[1.12-1.19]	100	Accept	
007	1.19	1.09	[1.05-1.12]	100	Accept	
008	1.28	1.21	[1.17-1.25]	100	Accept	
009	1.44	1.35	[1.31-1.39]	100	Accept	
010	1.33	1.23	[1.20-1.27]	100	Accept	

IRB regulations, they were not instructed to walk at specific speeds or times. Recordings were made randomly, ensuring subjects remained unaware of being monitored in real-time. As a result, some subjects engaged in dual tasks, such as holding a backpack or looking at their cell phones while walking.

From the probabilistic model formulated, estimations for both walking speed and cadence were derived. These estimations were then compared with values extracted from the video recordings. Figure 10 illustrates the posterior distributions of cadence and walking speed for three subjects, demonstrating that the credible interval falls within the Region of Practical Equivalence (ROPE) in a percentage greater than 90%. This indicates the estimations are practically equivalent to those extracted from the video recordings. Posterior distributions for all subjects can be found in [30]. A summary of mean values, credible intervals, and the percentage of ROPE agreement is presented in Table 4 for both parameters and all subjects. Acceleration data and the probabilistic analysis for this implementation can be accessed in [30].

The uncertainty of the walking speed estimations is greater than the uncertainty associated with the cadence estimation in all cases. However, the criteria used to evaluate the uncertainty draw an acceptable decision for both distributions. Notably, the subjects identified in the data collection process have different heights, an additional parameter expected to affect walking speed estimations for similar cadence values. The step length of each individual is intrinsically incorporated

in the joint distribution of parameters α and β from the model formulation. Incorporating this additional layer of information in the multilevel model offers the opportunity to reduce uncertainty and estimate the step length for each individual.

VII. CONCLUSION

The approach presented in this manuscript expands our use of structural vibrations under human excitation to extract relevant insights about occupants. This manuscript introduces the first known probabilistic multilevel model that uses floor acceleration signals induced by walking to estimate gait parameters, including cadence and walking speed. By accounting for missing or incomplete information in the acceleration signal, the proposed model offers a more realistic estimation of these parameters, addressing common challenges such as missing steps and low signal-to-noise ratios in at-home assessments. Estimating cadence using signals with missing step events can result in inaccurate cadence and walking speed estimations, impeding their utility in correlating changes in health. The formulated model tackles this issue by providing a more realistic analysis of gait parameters while considering the available data and the limitations of the technology.

The results of the cadence and walking speed estimations demonstrate that in the twenty-seven cases evaluated using equivalence testing, the 95% HPD of cadence fell within more than 90% of the range of practical equivalence (ROPE). This indicates that the estimated cadence and walking speed values are practically equivalent to the baseline values measured with the APDM sensors. Similarly, including missing step events as stochastic variables in the analysis reduced bias in the cadence estimations, improving their practical relevance.

The combination of 95HPD and ROPE as decision criteria proved to be more stringent in accepting the target values. Although the ADPM measurements fell within the 95% HPD for all cases, the ROPE criteria evaluated the extent to which the distribution approached the null value, considering the uncertainty of the estimation. This approach enables a more informed decision regarding the parameters. The sensitivity analysis demonstrated that using different priors for all multilevel model parameters resulted in the same decision as the initial analysis. However, the complete analysis required an increase in the number of samples for specific trials.

While the posterior distribution for walking speeds associated with a cadence value of 90 steps per minute fell within the region of practical equivalence according to the ROPE criteria, it appeared to be less consistent with the variability of other cadence-walking speed combinations. The authors believe this outcome indicates incomplete data rather than problems with the model formulation. Expanding the model to incorporate spatial information will further reduce uncertainty.

To further validate the model, a real-time monitoring implementation was conducted. Video recordings and acceleration data were collected from ten individuals walking at a self-selected pace. All estimations derived from the

probabilistic model formulated here resulted in accepted decisions based on 95%HPD and ROPE criteria combined. These results suggest that the model can reliably estimate these markers in unattended acquisition and produce estimations practically equivalent to those obtained from APDM and video recordings.

The model presented herein demonstrates robust performance, contingent upon detecting a minimum of three-step events (even with background noise), collectively representing a complete walking cycle. However, supplementary signal enhancement techniques may be necessary to extract the gait signal in scenarios where all step events are entirely obscured by noise. These techniques could involve the application of filtering methodologies to distinguish the gait signal from the ambient noise. Following signal enhancement, the model is anticipated to function as intended.

DATA AVAILABILITY STATEMENT

The Bayesian analysis of the multilevel model was conducted utilizing the PyMC3 library in Python Language Reference, version 3.8 [35]. All datasets produced during the present study, along with Python scripts, MCMC chains, and additional supporting files, have been deposited in the Open Science Framework (OSF) repository [30].

ACKNOWLEDGMENT

The authors would like to thank Garrett Hainline for assisting in collecting the APDM data used in this work.

AUTHOR CONTRIBUTIONS STATEMENT

Yohana MejiaCruz: Performed conceptualization, methodology, formal analysis, research, data curation, writing an original draft, writing, review, editing, and data visualization. Juan M. Caicedo: Performed conceptualization, methodology, research, review and editing, supervision, project administration, and funding acquisition. Zhaoshuo Jiang: Completed review and editing, supervision, project administration, and funding acquisition. Jean M. Franco: Experimental setup and data collection.

DISCLAIMERS

The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institutes of Health.

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