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Low-complexity signal detection for massive MIMO systems via trace iterative method

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Abstract: Linear minimum mean square error (MMSE) detection has been shown to achieve near-optimal performance for massive multiple-input multiple-output (MIMO) systems but inevitably involves complicated matrix inversion, which entails high complexity. To avoid the exact matrix inversion, a considerable number of implicit and explicit approximate matrix inversion based detection methods is proposed. By combining the advantages of both the explicit and the implicit matrix inversion, this paper introduces a new low-complexity signal detection algorithm. Firstly, the relationship between implicit and explicit techniques is analyzed. Then, an enhanced Newton iteration method is introduced to realize an approximate MMSE detection for massive MIMO uplink systems. The proposed improved Newton iteration significantly reduces the complexity of conventional Newton iteration. However, its complexity is still high for higher iterations. Thus, it is applied only for first two iterations. For subsequent iterations, we propose a novel trace iterative method (TIM) based low-complexity algorithm, which has significantly lower complexity than higher Newton iterations. Convergence guarantees of the proposed detector are also provided. Numerical simulations verify that the proposed detector exhibits significant performance enhancement over recently reported iterative detectors and achieves close-to-MMSE performance while retaining the low-complexity advantage for systems with hundreds of antennas.

Keywords: signal detection, low-complexity, linear minimum mean square error (MMSE), massive multiple-input multiple-output (MIMO), trace iterative method (TIM).

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1. Introduction

Massive multiple-input multiple-output (MIMO) technology serves as a cornerstone for modern wireless communication systems, enabling the maximization of throughput, coverage, and spectral efficiency within the available radio spectrum [1−3]. Unfortunately, the benefits are achieved at the expense of increased computational complexity due to a multitude of antennas at the base station (BS) and user terminals. In particular, the complexity of signal detection is a critical challenge for realizing a practical massive MIMO receiver. The multiple bit streams transmitted by users experience multipath propagation, where undesired copies of the signal arriving from various directions and with different delays interfere with the direct signal at the BS [4,5]. This interference corrupts the received data. Consequently, a MIMO receiver often employs a data detector to separate these streams and mitigate the effects of interference The optimal non-linear maximum likelihood detection suffers from high computational cost which scales exponentially with the number of users and order of modulation, which hinders its implementation in practice for systems with a large number of antennas. Linear minimum mean square error (MMSE) achieves good performance for the systems with massive antenna arrays [6] at low-complexity. However, it involves matrix inversion operation, whose computations increase cubically with the number of users. This motivates the design of detection algorithms capable of obtaining a good balance between performance and complexity in massive MIMO systems.

Iterative detection algorithms have recently attracted considerable attention for massive MIMO systems. Iterative detectors entail a significantly lower computational complexity than the exact matrix inversion method while delivering almost the same performance. One of the earli-

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est detectors that approximate the inverse of a matrix by converting it to a series of matrix-vector multiplications is Neumann series approximation (NSA) [7]. The NSA only achieves a marginal reduction in complexity. Thus, to obtain a reasonable balance of complexity and performance, several methods such as Newton iteration [8,9], Richardson method [10,11], Jacobi [12], recursive conjugate-gradient (RCG) [13] iterative sequential detection [14], Gauss-Seidel [15,16], and successive over relaxation (SOR) [17] have been introduced. The existing detection techniques need more number of iterations to attain near-MMSE performance that unfortunately increases complexity, or computations are difficult to parallelize when estimating each symbol from users due to high correlations. Thus, it is still a challenging open issue to design low-complexity detectors with good detection performance.

To mitigate the problem of high detection complexity at the BS in massive MIMO, this paper presents a low-complexity linear detector. We exploit the combination of explicit (requires to compute inverse of a matrix) and implicit (directly computes the solution vector) methods to enable high performance detection for systems with hundreds of antennas. The relationship between these schemes is analyzed first and then based on that analysis we develop an improved Newton iteration detector. Since complexity of the Newton iteration is high for higher-order iterations, it is only utilized for first two iterations. For subsequent iterations, we propose a detection algorithm designed based on the trace iterative method (TIM). It has significantly lower computational complexity than higher Newton iterations and reduces an order of magnitude overall complexity of linear MMSE virtually at no loss in detection performance. The proposed algorithm is mathematically demonstrated to be convergent and its computational complexity is analyzed in detail. The ability of the proposed detector to efficiently solve the problem of matrix inverse iteratively for attaining the desired detection results is demonstrated in the numerical results. Analysis illustrate that the proposed algorithm achieves near-MMSE error rate performance with substantially low-complexity.

The remainder of the paper is structured as follows: Section 2 provides a review of the uplink massive MIMO system and the classical linear MMSE exact matrix inversion detection approach. The proposed algorithm is presented and analyzed in Section 3. The numerical results are provided in Section 4. Finally, this paper is concluded in Section 5.

2. System model

Consider a $B\times U$ massive (or large) MIMO system in which U user terminals simultaneously transmit data symbols to a BS with $B\gg U$. Let $\boldsymbol{s}$ represents the transmitted symbol vector with each symbol being drawn independently from a given constellation set such as quadrature amplitude modulation (QAM). Then, the received symbol vector $\boldsymbol{y}$ at the BS [18] can be expressed as

$$\boldsymbol{y}=\boldsymbol{H}\boldsymbol{s}+\boldsymbol{n} \tag{1}$$

where entries of the additive noise vector $\boldsymbol{n}$ are drawn from the complex-valued circularly-symmetric Gaussian distribution and $\boldsymbol{H}$ is the channel matrix. The each entry $h_{i,j}$ in $\boldsymbol{H}$ is independent and identically distributed with variance σ^2 and mean zero.

The objective of the MIMO detector is to determine the optimal symbol vector $\boldsymbol{s}$ from the received data $\boldsymbol{y}$. Linear MMSE is one of the most attractive detectors that achieves near-optimal performance. The estimated symbol vector $\hat{\boldsymbol{s}}$ using MMSE [19] is given by

$$\hat{\boldsymbol{s}}=\left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}+\sigma^2\boldsymbol{I}_U\right)^{-1}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{y}=\boldsymbol{W}^{-1}\hat{\boldsymbol{y}} \tag{2}$$

where $\boldsymbol{W}=\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}+\sigma^2\boldsymbol{I}_U$ and $\hat{\boldsymbol{y}}=\boldsymbol{H}^{\mathrm{H}}\boldsymbol{y}$ represent MMSE filtering matrix and the matched-filter output, respectively. The major challenge for linear MMSE is to compute the high-dimensional matrix inversion $\boldsymbol{W}^{-1}$.

3. Proposed detector for massive MIMO systems

First, this section discusses the relationship between the approximate matrix inversion methods and the iterative methods. Next, based on that discussion, we improve the Newton iteration, which approximate the matrix inversion with low-complexity, and then exploit it to detect the signal for first two iterations. We then propose a novel iterative method based on the TIM to efficiently solve the detection problem for subsequent iterations. Finally, convergence proof and complexity analysis of the proposed algorithm are presented.

3.1 Relationship between explicit and implicit methods

Various explicit methods such as Newton iteration that needs to compute the matrix inversion, and iterative methods that directly computes the estimates of the signals emitted by different users without the need for computing the matrix inversion such as Richardson iteration have been recently reported for massive MIMO systems. To compute the linear MMSE estimation with (2) is equivalent to finding the solution $\hat{\boldsymbol{s}}$ to

$$\boldsymbol{W}\boldsymbol{s} = \hat{\boldsymbol{y}} \tag{3}$$

where $U \times U$ MMSE filtering matrix $\boldsymbol{W}$ is symmetric positive definite for massive MIMO systems. Direct computation of (3) involves matrix inversion which is very costly in case of large multi antenna systems. Therefore, we exploit combination of the explicit method and the implicit method to solve (3). For any splitting $\boldsymbol{W} = \boldsymbol{M} - \boldsymbol{N}$, where $\boldsymbol{M}$ is the iteration matrix, the iteration scheme to solve the linear system of (3) is given by

$$\boldsymbol{M}\boldsymbol{s}^{(i+1)} = \boldsymbol{N}\boldsymbol{s}^{(i)}\hat{\boldsymbol{y}} \tag{4}$$

where $i = 1, 2, \cdots, N_{\text{iter}}$. This iterative routine converges to the unique solution of a system in (3) for appropriately chosen initial vector if and only if $\rho(\boldsymbol{M}^{-1}\boldsymbol{N}) < 1$ [20], where ρ denotes the spectral radius of matrix $\boldsymbol{W}$. Equivalently, the iteration in (4) can be represented as

$$\boldsymbol{s}^{(i+1)} = \boldsymbol{B}\boldsymbol{s}^{(i)} + \boldsymbol{k} \tag{5}$$

where $\boldsymbol{s}^{(0)}$ is initial solution and iteration matrix $\boldsymbol{B}$ and vector $\boldsymbol{k}$ are given by

$$\begin{cases} \boldsymbol{B} = \boldsymbol{M}^{-1}\boldsymbol{N} = \boldsymbol{I} - \boldsymbol{M}^{-1}\boldsymbol{W} \\ \boldsymbol{k} = \boldsymbol{M}^{-1}\hat{\boldsymbol{y}} \end{cases}. \tag{6}$$

If the initial matrix $\boldsymbol{s}^{(0)}$ is $\boldsymbol{s}^{(0)} = \boldsymbol{M}^{-1}\hat{\boldsymbol{y}}$, then the estimated solution after ith iteration in the iteration process is equal to the ith order ($i+1$ terms) expansion in NSA [9], which is a well-known approximate matrix inversion method. Moreover, the result of $2^i - 1$ iterations in the implicit scheme is equivalent to i iterations in Newton iteration.

Different combinations of $\boldsymbol{M}$ and $\boldsymbol{N}$ in (4) will lead to different methods. For example, $\boldsymbol{M}=\frac{1}{w}\boldsymbol{I}$ and $\boldsymbol{N}=\frac{1}{w}\boldsymbol{I} - \boldsymbol{W}$ gives Richardson method (where w is relaxation parameter), and $\boldsymbol{M} = \boldsymbol{D}^{-1}$ and $\boldsymbol{N} = \boldsymbol{D}^{-1} - \boldsymbol{W}$ leads to Jacobi iteration (where $\boldsymbol{D}$ denotes the diagonal entries of $\boldsymbol{W}$). The Richardson iteration method's optimal relaxation parameter [21] can be expressed as

$$w = \frac{2}{\lambda_{\min} + \lambda_{\max}} \tag{7}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum eigenvalue of matrix $\boldsymbol{W}$, respectively. In general, Richardson iteration converges faster as compared to Jacobi iteration [22].

3.2 Improved newton iteration

In [8], Newton iteration method has been applied to find the approximation of the matrix inverse involved in MMSE detection. If $\boldsymbol{M}_0^{-1}$ is the original estimation of the matrix $\boldsymbol{M}^{-1}$, then the ith estimate of Newton iteration [6] is given as

$$\boldsymbol{M}_i^{-1} = \boldsymbol{M}_{i-1}^{-1}\left(2\boldsymbol{I} - \boldsymbol{W}\boldsymbol{M}_{i-1}^{-1}\right) \tag{8}$$

which converges if

$$\left\|\boldsymbol{I} - \boldsymbol{W}\boldsymbol{M}_0^{-1}\right\| < 1. \tag{9}$$

It is pointed out in [8] that, compared to the NSA, the Newton iteration converges faster. In this subsection, we improve the existing Newton iteration and then apply it to approximate the matrix inversion.

Since MMSE filtering matrix $\boldsymbol{W}$ is a diagonally dominant for massive MIMO systems, the matrix $\boldsymbol{M}$ can be set as $\boldsymbol{D}$. As discussed above there is a positive correlation between the iterative methods and the approximate matrix inversion method. Moreover, it has also been mentioned that the Jacobi iteration with $\boldsymbol{M} = \boldsymbol{D}$ converges slowly than Richardson iteration method with $\boldsymbol{M} = \frac{\lambda_{\min}+\lambda_{\max}}{2}\boldsymbol{I}$. Correspondingly, the convergence of Newton iteration with $\boldsymbol{M} = \frac{\lambda_{\min}+\lambda_{\max}}{2}\boldsymbol{I}$ should be faster as compared with $\boldsymbol{M} = \boldsymbol{D}$.

As initial estimation determines the number of iterations needed for the iteration process to converge, the initial matrix inversion solution $\boldsymbol{M}_0^{-1}$ should be chosen properly. Therefore, we improve the Newton iteration by replacing the existing initialization $\boldsymbol{M}_0^{-1} = \boldsymbol{D}^{-1}$ with novel initialization matrix $\boldsymbol{M} = \frac{\lambda_{\min}+\lambda_{\max}}{2}\boldsymbol{I}$. Then, $\boldsymbol{s}^{(0)}$ of the proposed algorithm can be formulated as

$$\boldsymbol{s}^{(0)} = \boldsymbol{M}_0^{-1}\hat{\boldsymbol{y}}. \tag{10}$$

Next we compute approximate matrix inversions $\boldsymbol{M}_1^{-1}$ and $\boldsymbol{M}_2^{-1}$ first, and then we obtain the estimated signals $\boldsymbol{s}^{(1)}$ and $\boldsymbol{s}^{(2)}$. The matrix $\boldsymbol{M}_1^{-1}$ of the designed detector utilizing (8) is given by

$$\boldsymbol{M}_1^{-1} = \boldsymbol{M}_0^{-1}\left(2\boldsymbol{I} - \boldsymbol{W}\boldsymbol{M}_0^{-1}\right) = \boldsymbol{M}_0^{-1} + \left(-\boldsymbol{M}_0^{-1}\boldsymbol{N}\right)\boldsymbol{M}_0^{-1} \tag{11}$$

where $\boldsymbol{N} = \boldsymbol{M}_0 - \boldsymbol{W}$. Then, $\boldsymbol{M}_0^{-1}\boldsymbol{N}$ is given by

$$\boldsymbol{M}_0^{-1}\boldsymbol{N} = \boldsymbol{I} - \boldsymbol{M}_0^{-1}\boldsymbol{W}. \tag{12}$$

Similarly, the second iteration $\boldsymbol{M}_2^{-1}$ using (8) can be computed as

$$\boldsymbol{M}_2^{-1} = \boldsymbol{M}_1^{-1}\left(2\boldsymbol{I} - \boldsymbol{W}\boldsymbol{M}_1^{-1}\right). \tag{13}$$

Since $\boldsymbol{M}_1^{-1}$ has already been computed in (11), by simply substituting the value of $\boldsymbol{M}_1^{-1}$ in (13), we get

$$\boldsymbol{M}_2^{-1} = \left(\boldsymbol{M}_0^{-1} - \boldsymbol{M}_0^{-1}\boldsymbol{N}\boldsymbol{M}_0^{-1}\right)\left(2\boldsymbol{I} - \boldsymbol{W}\left(\boldsymbol{M}_0^{-1} - \boldsymbol{M}_0^{-1}\boldsymbol{N}\boldsymbol{M}_0^{-1}\right)\right). \tag{14}$$

By putting $\boldsymbol{W} = \boldsymbol{M} - \boldsymbol{N}$ and after some calculation, we obtain $\boldsymbol{M}_2^{-1}$ [9] as

$$\boldsymbol{M}_2^{-1} = \boldsymbol{M}_1^{-1} + \left(\boldsymbol{M}_0^{-1}\boldsymbol{N}\right)^2\boldsymbol{M}_1^{-1}. \tag{15}$$

Consequently, the detected signals $\boldsymbol{s}^{(1)}$ and $\boldsymbol{s}^{(2)}$ can be obtained as

$$s^{(1)} = M_1^{-1}\hat{y} = s^{(0)} + (I - M_0^{-1}W)s^{(0)}. \quad (16)$$

By putting the MMSE filtering matrix W and $M_0^{-1} = \frac{2}{\lambda_{\min}+\lambda_{\max}}$ into (16), we get the estimation of $s^{(1)}$ as follows:

$$s^{(1)} = s^{(0)} + \left(I - \frac{2}{\lambda_{\min}+\lambda_{\max}}W\right)s^{(0)}. \quad (17)$$

Similarly, $s^{(2)}$ can be obtained as

$$s^{(2)} = M_2^{-1}\hat{y} = s^{(1)} + (I - M_0^{-1}W)^2 s^{(1)}. \quad (18)$$

By substituting the values of W and M_0^{-1} into (18), and after some calculations, we get

$$s^{(2)} = s^{(1)}\left[\left(\frac{2}{\lambda_{\min}+\lambda_{\max}}W\right)^2 - 2\left(\frac{2}{\lambda_{\min}+\lambda_{\max}}W\right) + I\right]s^{(1)}. \quad (19)$$

Note that the proposed $s^{(1)}$ and $s^{(2)}$ depends on $\lambda_{\min}$ and $\lambda_{\max}$. Determining eigenvalues in practice proves challenging. Nonetheless, given that the elements of H are independent and identically distributed complex Gaussian random variables, $H^{\mathrm{H}}H$ is a complex central Wishart matrix. Thus, as B increases, the smallest and the largest eigenvalue of W converges to a deterministic value [6] as follows:

$$\begin{cases} \lambda_{\min} = B\left(1 - \sqrt{\frac{B}{U}}\right)^2 \\ \lambda_{\max} = B\left(1 + \sqrt{\frac{B}{U}}\right)^2 \end{cases}. \quad (20)$$

From (20), it can be noted that the $\lambda_{\min}$ and $\lambda_{\max}$ now can be computed approximately using system parameters. Since the higher Newton iterations are computationally expensive, we propose a novel detection algorithm for subsequent (i.e., $i \geqslant 3$) iterations that has low-complexity.

3.3 TIM detector

According to statistical matrix theory, as the size of random channel matrix H grows, the distribution of singular values will become independent of the statistical distribution of its elements and will only depend on the ratio U/B. This phenomenon is called channel hardening. The characteristics of a small-scale fading can be canceled by exploiting this phenomenon and it becomes more dominant if the BS-to-user antenna ratio gets larger. Intuitively, as the dimensions of massive MIMO system gets larger, the diagonal entries of $H^{\mathrm{H}}H$ will become larger compared to off-diagonal entries, i.e., $H^{\mathrm{H}}H$ becomes Hermitian positive definite. Hence, in massive MIMO systems, iterative techniques can attain near-optimal detection performance within a few numbers of iterations [23]. Inspired by this, we can exploit TIM iteration for low-complexity signal detection in the proposed algorithm for $i \geqslant 3$. Unlike MMSE that directly computes W^{-1}, TIM employs an iterative procedure to achieve the estimates of the transmitted symbols.

The generalized stationary iteration method based on (4) can be written as

$$Mx^{(i)} = -\left(Ax^{(i-1)} + b\right). \quad (21)$$

If the nonsingular matrix $M = -\frac{\mathrm{Tr}(A)}{\beta}I$, we have the TIM iteration defined [24] as follows:

$$\frac{\mathrm{Tr}(A)}{\beta}x^{(i)} = \left(b - Ax^{(i-1)}\right) \quad (22)$$

where $\beta > 0$ is relaxation parameter. Since H is asymptotically orthogonal in large MIMO systems, we can take the benefit of iterative structure of TIM and exploit it for estimating the transmitted symbols without computing the matrix inversion as

$$s^{(i)} = \left(I - \frac{\beta}{\mathrm{Tr}(W)}W\right)s^{(i-1)} + \frac{\beta}{\mathrm{Tr}(W)}\hat{y}. \quad (23)$$

The optimum value of relaxation parameter β will be defined in Section 4. Equation (21) iteratively refines the initial solution and this procedure is carried out until a specific iteration stopping criterion is met. The most appropriate stopping criterion is the maximum iteration numbers. We achieve the approximated MMSE solution without computing the matrix inverse after the convergence of the iteration. In the proposed algorithm, we employ TIM for $i \geqslant 3$.

This detection algorithm attains significantly faster convergence to the final solution compared to conventional iterative methods. This enhancement has been validated through numerical simulations detailed in Section 4. Summarizes the proposed detector as shown in Algorithm 1.

Algorithm 1 Proposed algorithm

Input $H, y, N_{\text{iter}}, \beta, \sigma^2$
Output Detected signal $\hat{s}$
1 Preprocessing and initialization
2 $\lambda_{\min} = B\left(1 - \sqrt{\frac{B}{U}}\right)^2$; $\lambda_{\max} = B\left(1 + \sqrt{\frac{B}{U}}\right)^2$
3 $w = \frac{2}{\lambda_{\min}+\lambda_{\max}}$
4 $M = wI = \frac{\lambda_{\min}+\lambda_{\max}}{2}I$
5 $W = H^{\mathrm{H}}H + \sigma^2 I_U$

$\hat{\boldsymbol{y}} = \boldsymbol{H}^{\mathrm{H}}\boldsymbol{y}$

$\boldsymbol{s}^{(0)} = \boldsymbol{M}_0^{-1}\hat{\boldsymbol{y}}$

$\boldsymbol{s}^{(1)} = \boldsymbol{s}^{(0)} + \left(\boldsymbol{I} - \frac{2}{\lambda_{\min}+\lambda_{\max}}\boldsymbol{W}\right)\boldsymbol{s}^{(0)}$

$\boldsymbol{s}^{(2)} = \boldsymbol{s}^{(1)}\left[\left(\frac{2}{\lambda_{\min}+\lambda_{\max}}\boldsymbol{W}\right)^2 - 2\left(\frac{2}{\lambda_{\min}+\lambda_{\max}}\boldsymbol{W}\right) + \boldsymbol{I}\right]\boldsymbol{s}^{(1)}$

Iteration

For $i = 3, 4, \cdots, N_{\text{iter}}$

$\boldsymbol{s}^{(i)} = \left(\boldsymbol{I} - \frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}\boldsymbol{W}\right)\boldsymbol{s}^{(i-1)} + \frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}\hat{\boldsymbol{y}}$

end

3.4 Convergence

The convergence guarantees for the proposed detection algorithm has been provided in this subsection. Since TIM is applied for higher iterations, convergence of the proposed detector completely depends on it. Thus, only convergence of the TIM iteration is discussed below.

Theorem 1 The proposed TIM-based massive MIMO data detector converges for $0 < \beta < \left(\frac{2}{\lambda_{\max}}\right)\mathrm{Tr}(\boldsymbol{W})$ with any initial solution if all eigenvalues λ_k are real-valued and satisfy

$$0 < \lambda_{\min} \leqslant \lambda_k \leqslant \lambda_{\max},\ \ k = 1, 2, \cdots, n. \tag{24}$$

Proof Ensuring the convergence of the iterative scheme requires that the spectral radius be less than unity. Let $\lim_{i\to\infty} \boldsymbol{s}^{(i)} = \hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{s}} = \boldsymbol{R}\hat{\boldsymbol{s}} + \boldsymbol{d}$ for any initial solution $\boldsymbol{s}^{(0)}$, where $\boldsymbol{d} = \frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}\hat{\boldsymbol{y}}$ and $\boldsymbol{R} = \boldsymbol{I} - \frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}\boldsymbol{W}$ is iteration matrix of the TIM iteration whose eigenvalues are $\mu_k = 1 - \frac{\beta\lambda_k}{\mathrm{Tr}(\boldsymbol{W})}$, and satisfy $1 - \frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}\lambda_{\max} \leqslant \mu_k \leqslant 1 - \frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}\lambda_{\min}$. The matrix $\boldsymbol{W}$ is positive definite and symmetric since entries of the channel matrix $\boldsymbol{H}$ are identically distributed complex Gaussian random variables. Therefore, $\frac{\lambda_k}{\lambda_{\max}} < 1$ and we know that

$$\sum_{k=1}^{n} \lambda_k = \mathrm{Tr}(\boldsymbol{W}). \tag{25}$$

Moreover,

$$\begin{cases} 1 - \frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}\lambda_{\min} < 1 \\ 1 - \frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}\lambda_{\max} > -1 \end{cases}, \tag{26}$$

then $|\mu_k < 1|$ for all k and the method is convergent. Consequently, for all $\beta > 0$ the first condition is satisfied since $\lambda_{\min} > 0$, and the second condition is true if

$$\beta < \left(\frac{2}{\lambda_{\max}}\right)\mathrm{Tr}(\boldsymbol{W}).$$ □

3.5 Complexity

We first discuss the approximate computations in terms of the number of complex-valued multiplications involved in various steps of the proposed algorithm and then compare it with recently developed methods. Since all detectors require to compute $\boldsymbol{W}$ and $\hat{\boldsymbol{y}}$, we focus on the complexity of iterative cycles. It is easy to see that $\boldsymbol{s}^{(0)} = \boldsymbol{M}_0^{-1}\hat{\boldsymbol{y}}$ of the proposed algorithm requires $U+7$ multiplications. For the first iteration of the proposed Newton method, we first compute $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{s}^{(0)}$ and $\sigma^2\boldsymbol{s}^{(0)}$, and then we calculate $\boldsymbol{M}_0^{-1}(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{s}^{(0)} - \sigma^2\boldsymbol{s}^{(0)})$, where $(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}\boldsymbol{s}^{(0)} - \sigma^2\boldsymbol{s}^{(0)})$ is a vector. Note that the proposed initialization matrix $\boldsymbol{M}_0^{-1}$ is already obtained for $\boldsymbol{s}^{(0)}$. Thus, $2U^2 + 2U$ total multiplications are needed to obtain $\boldsymbol{s}^{(1)}$. We follow the similar procedure for $\boldsymbol{s}^{(2)}$ to obtain approximate computational complexity. It can be observed from (19) that, it requires $6U^2 + 7U$ multiplications to compute second iteration $\boldsymbol{s}^{(2)}$ of the improved Newton iteration.

Next we analyze the computational complexity of the designed TIM detector. Iteration routine of the TIM in (23) can also be expressed as follows:

$$\boldsymbol{s}^{(i)} = \boldsymbol{s}^{(i-1)} + \frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}\left(\hat{\boldsymbol{y}} - \boldsymbol{W}\boldsymbol{s}^{(i-1)}\right). \tag{27}$$

Based on (27), the computation of $\boldsymbol{s}^{(i)}$ involves the multiplication of $U \times U$ matrix $\boldsymbol{W}$ and a vector $\boldsymbol{s}^{(i-1)}$ of size $U \times 1$, multiplication of $\frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}$ with a vector $(\hat{\boldsymbol{y}} - \boldsymbol{W}\boldsymbol{s}^{(i-1)})$ and one division to obtain $\frac{\beta}{\mathrm{Tr}(\boldsymbol{W})}$. Hence, (25) requires $U^2 + U + 1$ multiplications to compute $\boldsymbol{s}^{(i)}$. The computational complexity of the proposed algorithm is summarized in Table 1. The complexity comparison of various methods is provided in Section 4.

Table 1 Number of multiplications for different steps of the proposed algorithm

Step	Complexity
Initialization matrix	$U+7$
Improved Newton ($i = 2$)	$2U^2+2U$
Improved Newton ($i = 3$)	$6U^2+7U$
TIM iteration	U^2+U+1

4. Numerical results

In Section 4, numerical results of the proposed detector are presented and compared with existing iterative methods such as Chebyshev iteration [12], non-stationary

Richardson (NSR) method [10], Newton iteration (NI) [8] and recursive-conjugate-gradient (RCG) [13]. Linear MMSE with exact matrix inversion is also compared. Numerical results are provided for different massive MIMO antenna configurations and the transmitted symbols of each user are randomly selected from the same given constellation, for example, 16-QAM or 64-QAM. Channel matrices are generated using flat rayleigh fading channel model for all simulations. We also compare the complexity of the developed algorithm with recently developed iterative algorithms.

We first present the computational complexity comparison of the designed detection algorithm with other recently introduced iterative detectors. We also present the complexity of the linear MMSE detection with exact matrix inversion serving as the benchmark. Fig. 1 demonstrates the number of complex-valued multiplications against number of users. The following observations can be found.

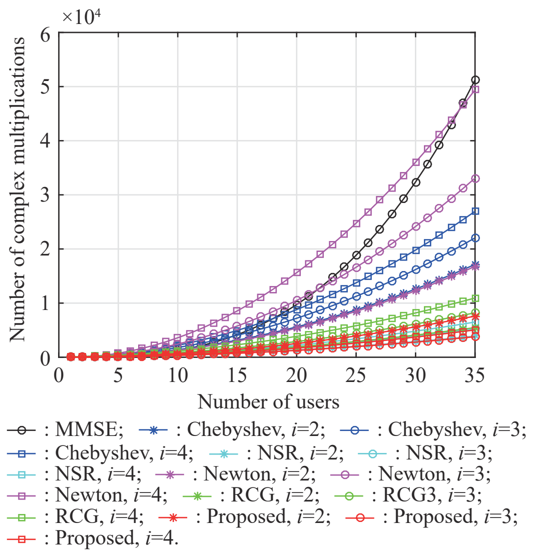

Fig. 1 Computational complexity comparison for various detection methods

(i) It can be seen that, compared to all methods, the proposed algorithm has lowest computational complexity for $i = 3$ and $i = 4$.

(ii) For $i = 2$, the proposed technique shows higher complexity than the proposed TIM detector. This complexity further increase if we apply proposed improved Newton method for higher iterations. Thus, we introduced the novel method for higher iterations, which significantly reduce the multiplications as can be seen from Fig. 1.

(iii) It can be observed that the proposed improved Newton iteration greatly reduces the complexity of the conventional Newton detector for $i = 2$. The proposed new initialization matrix is the main reason of this complexity reduction.

(iv) Fig. 2 demonstrates that the RCG exhibits lower complexity than conventional Newton and Chebyshev methods. In addition, the complexity of the conventional Newton iteration is very high (even higher than linear MMSE exact matrix inverse scheme for $i = 4$) followed by Chebyshev detector. The results are presented for various signal to noise ratio (SNR) values with $i = 4$.

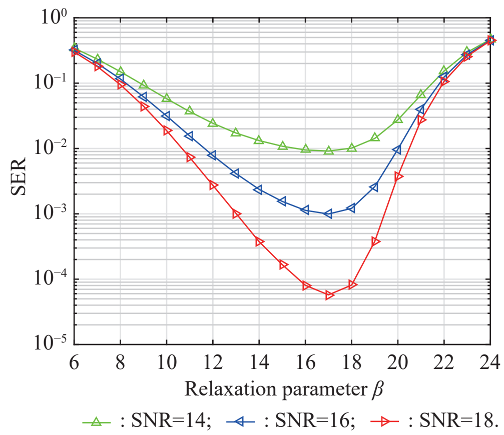

Fig. 2 SER performance of the TIM detector against β for $B \times U = 128 \times 16$ massive MIMO system with 64-QAM modulation technique.

In summary, Fig. 1 shows that the designed detector has lower complexity than all aforementioned iterative detectors. In addition, it significantly reduces the computations of linear MMSE at virtually same error rate performance which has been verified in next subsections.

We now study symbol error ratio (SER) versus different values of relaxation parameter to find optimal β in Fig. 2. For $B \times U = 128 \times 16$ antenna configuration with 64-QAM modulation technique, Fig. 2 illustrates that the SER performance improves at first and reaches the minimum when $\beta \approx 17$ and then starts to degrade as β further increases. We can conclude that the proposed detector obtains the best performance with $\beta = 17$. Thus, optimal relaxation parameter $\beta = 17$ is chosen for the proposed TIM to achieve best results with less number of iterations.

In Fig. 3, the performance of the proposed algorithm with linear MMSE detection is compared. The $B \times U = 128 \times 32$ massive MIMO system and 16-QAM modulation is considered for this simulation. It can be seen from figure that the performance of the proposed detector improves with the increasing number of iterations. It achieves almost similar performance to that of the linear MMSE with exact matrix inversion for $i = 6$. The performance gap between the proposed method and linear MMSE is just 0.12 dB at 10^{-3} for $i = 6$. Note that the complexity of the proposed detector is still very low for $i = 6$ as compared to MMSE exact inversion method.

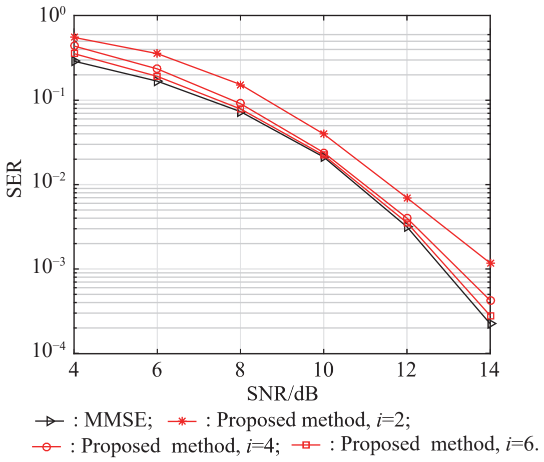

Fig. 3 Comparison of the proposed algorithm with linear MMSE

Next, SER of the proposed algorithm is compared with other recently developed iterative detectors. Fig. 4 shows the SER performance against SNR for Chebyshev iteration, NSR method, conventional Newton iteration, RCG and the proposed method for $B \times U = 128 \times 16$ large MIMO system. Fig. 4 shows that the Chebyshev iteration exhibits slow convergence for $i = 2$ and $i = 3$, but it shows better performance for $i = 4$. We see that the conventional Newton detector and RCG have almost similar performance for $i = 3$ and $i = 4$. Further we note that the proposed method outperforms than aforementioned iterative methods, and only four iterations are sufficient for it to realize the performance of the linear MMSE.

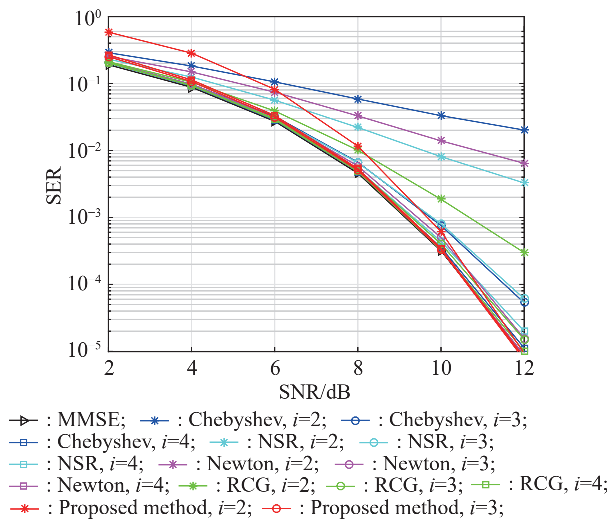

Fig. 4 SER of various massive MIMO detectors for a system with $B \times U$= 128×16 antennas

In Fig. 5, we reduce the BS-to-user antenna ratio by increasing the number of antennas. In this case, the $B \times U = 144 \times 24$ antenna system (BS-to-user antenna ratio = 6) is considered to compare the SER performance against SNR. It can be noted that the proposed algorithm outperforms Chebyshev and conventional Newton detector by a significant margin. These methods exhibit insufficient convergence with noticeable performance loss in this scenario due to BS-to-user antenna ratio is relatively larger than that of the Fig. 4. The RCG shows performance improvement over Chebyshev and Newton based detectors. However, the proposed detector exhibits superior performance than other iterative approaches. It can be further noted that only the proposed algorithm can approach the MMSE performance with four iterations whereas other iterative methods require a higher number of iterations to achieve the same performance.

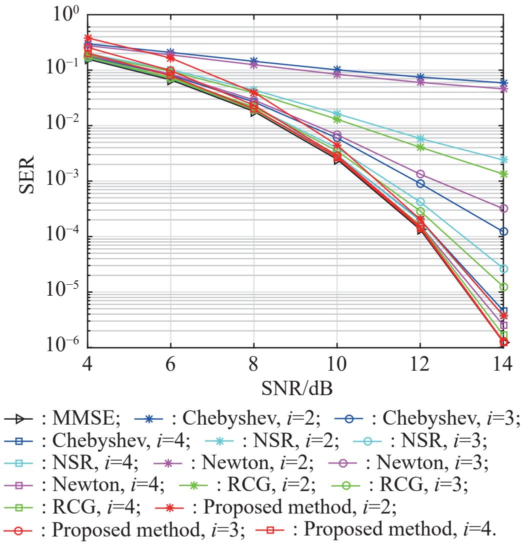

Fig. 5 Comparison of the performance of the proposed algorithm with recently reported detection algorithms for $B \times U$= 144×24 massive MIMO system

Fig. 6 illustrates the simulation results depicting the SER performance as a function of the number of iterations. Specifically, the proposed technique is contrasted with other state-of-the-art methods in a scenario featuring 128 antennas at the BS and 32 users utilizing 16-QAM modulation, with the SNR set at 12 dB. It can be observed from Fig. 6 that the NSR based detector shows the slow convergence compared to other detectors. The main reason is its sensitively to relaxation parameter. It can further be seen that the performance of RCG is better at the start up to three iterations compared to Chebyshev, NSR and Newton based detectors. However, for $i > 3$, RCG and Newton method have almost similar performance. Moreover, the proposed algorithm demonstrates the quickest convergence among all mentioned iterative methods, achieving satisfactory results within just a few iterations. This observation suggests that the proposed detector outperforms all compared iterative detectors in terms of both convergence speed and error performance. In summary, we can say that the proposed algorithm can realize linear MMSE detection within only a few numbers of iterations under different antenna scenarios. Additionally, owing to the high convergence rate, it can also maintain detection performance as users increase.

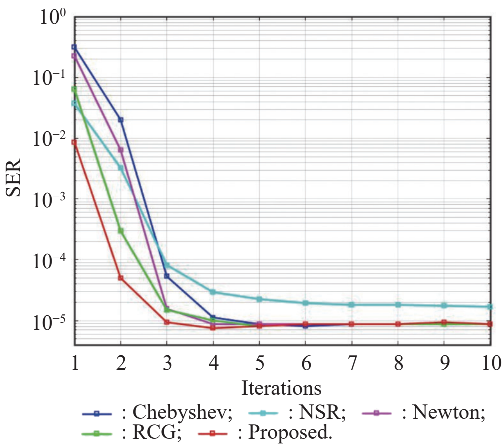

Fig. 6 SER Performance against number of iterations of various detection algorithms for $B\times U=128\times 32$ massive MIMO system at 12 dB SNR employing 16-QAM modulation

5. Conclusions

We have proposed a novel data detection algorithm tailored for massive MIMO systems, which is based on the combination of explicit and implicit methods. We have first analyzed the relationship between explicit and implicit methods and then based on that analysis an improved Newton detector is developed. It has been shown that the improved Newton detector greatly reduces the complexity of the conventional Newton detector and simultaneously provides improved error rate performance. Since the complexity of Newton iteration is high for higher iterations, we have proposed TIM for subsequent iterations, which is a novel low-complexity signal detection algorithm designed specifically for massive MIMO systems. Analysis illustrates that the proposed TIM is convergent for any initialization when relaxation parameter is properly chosen. It has been shown through numerical simulations that the combined explicit and implicit methods based detection algorithm achieves much better performance than conventional iterative methods and approaches the performance of linear MMSE detection with significantly reduced complexity. Thus, the proposed algorithm is an attractive solution for massive MIMO receiver implementation.

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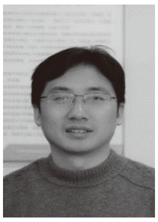

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