Spatial Constellation Design for MIMO Visible Light Communication Based on the Optimal Geometric Shaping

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Abstract—**As a wireless communication which combines illumination and communication, visible light communication (VLC) has attracted great attention. In indoor VLC systems, multiple illumination devices are commonly employed, forming a natural multi-input multi-output (MIMO) communication system. In this paper, a spatial constellation design based on the optimal geometric shaping is proposed for the MIMO-VLC system with the peak-power and total average-power constraints. Firstly, we derive the optimal geometric shaping region for the MIMO-VLC system under the aforementioned power constraints. Subsequently, we intersect the scaled integer lattice with the optimal geometric shaping region and combine it with minimum-energy mapping to obtain a spatial constellation. Simulation results for indoor MIMO-VLC systems verify the superiority of our approach compared to conventional methods.**

Index Terms—**Visible light communication (VLC), multi-input multi-output (MIMO), constellation design, constellation shaping.**

I. Introduction

VISIBLE light communication (VLC), as an emerging wireless communication technology that integrates illumination and data transmission, has witnessed rapid advancements in recent years [1], [2], [3]. VLC leverages the widespread deployment of light emitting diodes (LEDs) as transmitters and photodiodes (PDs) as receivers. Given the exponential growth and extensive utilization of LEDs, VLC is considered a promising solution for indoor wireless communication scenarios in the future [4], [5], [6]. The VLC system typically employs intensity-modulation direct-detection (IM/DD) for its simplicity. The transmitted signal is modulated onto the optical intensity emitted by LEDs and subsequently recovered through direct detection of the incoming light at PDs.

Additionally, in practical indoor environments, multiple LEDs are simultaneously employed to enhance illumination capabilities, thereby establishing the groundwork for potential integration of VLC with multi-input multi-output (MIMO) techniques. By leveraging MIMO techniques, VLC can be effectively utilized to achieve higher data rates and improved energy efficiency. Therefore, it is necessary to conduct research on MIMO-VLC to enhance the reliability and effectiveness of indoor VLC systems.

The MIMO-VLC model and the MIMO in radio frequency (RF) wireless communications channel model differ in two aspects: 1) The signal of the VLC system must be nonnegative, resulting in a disparity in signal space between the VLC system and the RF communication system. 2) The power of the signal in VLC is proportional to the intensity instead of its square in RF domain. Due to the two differnces, the singular-value decomposition (SVD) scheme used to transform MIMO-RF channels to parallel channels is not directly applicable in MIMO-VLC [7].

In the context of traditional MIMO techniques, repetition coding (RC) involves simultaneous transmission of identical signals from all LEDs, resulting in improved error performance in highly correlated channels [8]. In contrast, spatial multiplexing (SMP) is designed for independent signal transmission from each LED and exhibits superior error performance in low correlated channels [9]. Both schemes design transmitted signals irrespective of channel state information (CSI), and cannot adapt to the channel.

A novel constellation design is proposed in [10] for the MIMO-VLC system, which aims to optimize symbol collaboration in order to minimize average optical power while maintaining a fixed minimum Euclidean distance (MED). A new superposed odd-order 32 quadrature amplitude modulation (QAM) constellation design is introduced, where two LEDs transmit a 4QAM and geometricly shaped square 8QAM respectively [11]. Additionally, this design is extended to support higher-order QAM constellations of the form 2^n [12]. These schemes exhibit excellent performance and primarily focus on 2×2 MIMO-VLC systems. Furthermore, the channel-adaptive space-collaborative constellation (CASCC), as mentioned in [13], exhibits superior performance and broader applicability compared to conventional MIMO-VLC schemes, with its constraint condition being the average electronic power. Typically, in practical indoor VLC systems, the optical power transmitted from multiple LEDs installed at different positions need to be constrained considering factors such as LED nonlinearity, limited dynamic range, and user requirements. In this paper, motivated by the multidimensional constellation design

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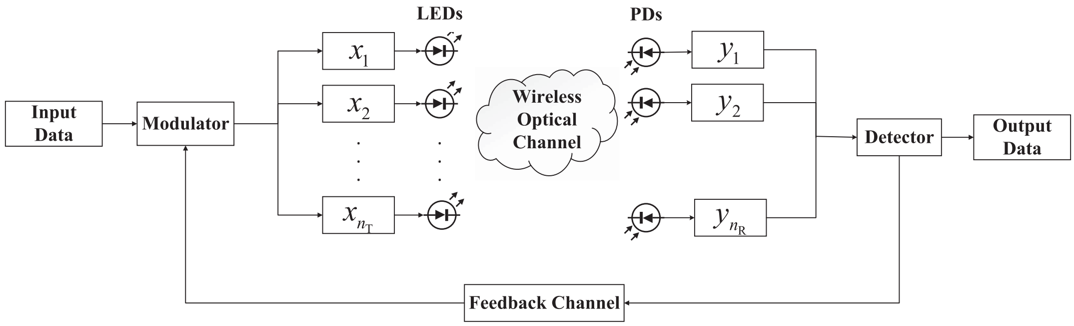


Fig. 1. System model of the MIMO-VLC system.

for single-input single-output (SISO) VLC system based on the optimal geometric shaping in [14] and the minimum-energy mapping (MEM) presented in [15], we proposed a spatial constellation design based on the optimal geometric shaping (OGS) for the MIMO-VLC system under the peak-power and total average-power (PTA) constraints. The main contributions of our work can be summarized as follows: We mathematically derive the expression for the optimal geometric shaping region of the MIMO-VLC system under the PTA constraints, which is determined by a single parameter ν. Subsequently, we intersect the optimal geometric shaping region with the scaled integer lattice to obtain the equivalent transmitted signal constellation, which is then mapped to the transmitted signal constellation using the minimum-energy mapping.

The remainder of this paper is organized as follows. The channel model presented in Section II. In Section III, we provide a comprehensive review of the theoretical basis to facilitate the introduction of the proposed constellation design. Subsequently, in Section IV, we propose a spatial constellation design based on optimal geometric shaping. Furthermore, simulation results demonstrating the superior performance of our scheme compared with conventional MIMO–VLC schemes are presented in Section V. Finally, Section VI concludes this paper.

Notations: The operator $\mathsf{E}[\cdot]$ represents the expectation, while $\mathtt{Pr}\{\cdot\}$ denotes the probability of an event. The sets $\mathbb{R}$, $\mathbb{N}$, and $\mathbb{Z}$ refer to the real numbers, natural numbers, and integers respectively. The operator $\|\cdot\|_p$ signifies the l_p-norm of a vector, and $|\cdot|$ indicates the cardinality of a given set. The operator $\mathrm{vol}(\cdot)$ is the volume of an Euclidean space.

II. Channel Model

The channel considered in this paper, as illustrated in Fig. 1, is an $n_\mathrm{R} \times n_\mathrm{T}$ MIMO-VLC channel whose output can be modeled by

$$\mathbf{y} = \mathsf{H}\mathbf{x} + \mathbf{z}, \tag{1}$$

where the n_T-dimensional real-valued vector $\mathbf{x} = (x_1, \ldots, x_{n_\mathrm{T}})^\top$ is the transmitted optical intensity signal, which is selected with equal probability from a constellation set denoted as $\mathcal{X}$, and $\mathbf{y} = (y_1, \ldots, y_{n_\mathrm{R}})^\top$ denotes the n_R-dimensional received signal; where the n_R-dimensional vector $\mathbf{z} = (z_1, \ldots, z_{n_\mathrm{R}})^\top$ represents the channel noise, which is modeled as additive white Gaussian noise (AWGN) with a zero mean and variance of σ^2; where the channel gain matrix $\mathsf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{n_\mathrm{T}}]$ is an $n_\mathrm{R} \times n_\mathrm{T}$ deterministic real-valued matrix and $\mathbf{h}_i$ is an n_R-dimensional column vector that represents the channel gains from the i-th transmitter to the n_R receivers. The element h_{ij} of the channel gain matrix H can be calculated by the Lambertian model [4]:

$$h_{ij} = \begin{cases} \frac{g T_S A(m+1)}{2\pi D_{ij}^2} \cos^m(\phi_{ij}) \cos\psi_{ij}, & 0 \le \psi_{ij} \le \Psi_C \\ 0, & \psi_{ij} > \Psi_C \end{cases} \tag{2}$$

where A represents the physical area of the PD, D_{ij} denotes the distance between the j-th LED and the i-th PD, m signifies the order of Lambertian radiation determined by the half-power angle of the LED $\Phi_{1/2}$, which can be calculated as $m = -\ln 2/\ln(\cos\Phi_{1/2})$. ϕ_{ij}, ψ_{ij}, T_S, and g refer to the incidence angle, irradiance angle, optical filter gain, and concentrator gain respectively. Ψ_C represents the field of view (FOV) of the PD.

In scenarios where the MIMO channels have more receivers than transmitters (i.e., $n_\mathrm{R} \ge n_\mathrm{T}$), the equivalent transmitted signal (defined in (6)) can be uniquely mapped onto the transmitted signal. The constellation design procedures are the same with the scenario where $n_\mathrm{T} > n_\mathrm{R}$ except for the zonotope partition and the minimum-energy mapping. Therefore, considering a more complex and general case, in this paper, we mainly focus on the MIMO channels with more transmitters than receivers. Without loss of generality, we make the following assumptions:

$$n_\mathrm{T} > n_\mathrm{R} > 1, \mathrm{rank}\,(\mathsf{H}) = n_\mathrm{R}, \tag{3}$$

Such a system arises when the transmitter is based on an existing illumination system with many LEDs, while the receiver PDs are procured at additional cost [15].

Two main constraints in the MIMO-VLC system is the peak-power constraint and the total average-power constraint which are attributed to the limited dynamic range of LED devices and the need for dimming control or energy consumption. In this paper, we normalize the peak-power constraint on the transmitted signals as

$$\mathbf{x} \in [0, 1]^{n_\mathrm{T}}, \tag{4}$$

and the total average-power constraint is

$$\mathsf{E}\left[\|\mathbf{x}\|_1\right] \le \alpha, \tag{5}$$

where the parameter α denotes the allowed total average power of all LEDs together. Equation (4) and (5) constitute the PTA constraints.

We denote the transmitted signal constellation by $\mathscr{X} \subseteq \mathbb{R}_{+}^{n_{\mathrm{T}}}$ due to the nonnegativity of optical signals, and the cardinality of it can be represented by $M \in \mathbb{N}_{+}$. Therefore, the normalized bit rate of the constellation $\mathscr{X}$ is $\beta = \log_2^M$. Then we define the equivalent transmitted signal as

$$\mathbf{s} = \mathsf{H}\mathbf{x}, \tag{6}$$

and the equivalent transmitted signal constellation is $\mathscr{S}$.

Indoor VLC systems generally operate at a high signal-to-noise ratio (SNR) regime [4]. With the perfect CSI at the receiver in the high-SNR, the error performance is primarily determined by the MED of the equivalent transmitted signal constellation $\mathscr{S}$, and the figure of merit can be expressed as

$$d_{\min} = \min_{\substack{\mathbf{s}_1,\mathbf{s}_2 \in \mathscr{S} \\ \mathbf{s}_1 \neq \mathbf{s}_2}} \|\mathbf{s}_1 - \mathbf{s}_2\|_2 = \min_{\substack{\mathbf{x}_1,\mathbf{x}_2 \in \mathscr{X} \\ \mathbf{x}_1 \neq \mathbf{x}_2}} \|\mathsf{H}\mathbf{x}_1 - \mathsf{H}\mathbf{x}_2\|_2 . \tag{7}$$

Therefore, the target of this paper is to design a size-M transmitted signal constellation satisfies the PTA constraints while maximizing $d_{\min}$.

For the VLC system, the channel is regarded as time-invariant, but it may change with the locations of transmitters and receivers according to the Lambertian model in (2). In this paper, the LEDs are assumed to be installed on the ceiling, so the CSI changes with the locations of the PDs. When the user is moving, the CSI can be estimated and fed back to the transmitters by the receivers through the feedback channel in negligible time without considerably affecting performance [13], [16]. The constellation can be designed by leveraging the CSI, thereby enabling adaptability to user mobility.

III. Theoretical Basis

To facilitate a clear presentation of our proposed constellation design, we illustrate some theoretical basis in this section.

A. Minimum-Energy Mapping and Zonotope Decomposition

From (3) and (6), we know the support set of the equivalent transmitted signal is the graph mapped from the support set of the transmitted signal by a rank deficient matrix. It means that an equivalent transmitted signal may correspond to multiple transmitted signals. Therefore, it is necessary to find a unique mapping with minimum energy between the equivalent transmitted signal and the transmitted signal.

The minimum-energy mapping is essentially a solution to the following linear programming (LP) problem:

$$\begin{aligned} \min \quad & \mathbf{1}_{\mathbf{n}_{\mathrm{T}}}^{\top}\mathbf{x} \\ \text{s.t.} \quad & \mathsf{H}\mathbf{x} = \mathbf{s} \\ & \mathsf{L}\mathbf{x} \leq \begin{bmatrix} \mathbf{1}_{\mathbf{n}_{\mathrm{T}}} \\ \mathbf{0}_{\mathbf{n}_{\mathrm{T}}} \end{bmatrix} \end{aligned} \tag{8}$$

where $\mathbf{1}_{\mathbf{n}_{\mathrm{T}}}$ denotes the n_{T}-dimensional all ones column vector, $\mathbf{0}_{\mathbf{n}_{\mathrm{T}}}$ represents the n_{T}-dimensional all zeros column vector, the matrix

$$\mathsf{L} = \begin{bmatrix} \mathbb{I}_{n_{\mathrm{T}}} \\ -\mathbb{I}_{n_{\mathrm{T}}} \end{bmatrix},$$

and $\mathbb{I}_{n_{\mathrm{T}}}$ is the n_{T}-dimensional identity matrix.

The LP problem has been solved in [15] and the minimum-energy mapping can be summarized as follows.

The equivalent transmitted signal $\mathbf{s}$ takes value in the zonotope $\mathcal{Z}(\mathsf{H})$, which is defined as

$$\mathcal{Z}(\mathsf{H}) \triangleq \left\{ \sum_{i=1}^{n_{\mathrm{T}}} \lambda_i \mathbf{h}_i : \lambda_1, \ldots, \lambda_{n_{\mathrm{T}}} \in [0,1] \right\}.$$

The MEM strategy partitions the zonotope into at most N_p parallelepipeds, where $N_p = \binom{n_{\mathrm{T}}}{n_{\mathrm{R}}}$. Each parallelepiped $\mathcal{P}_i$ is spanned by a n_{R}-dimensional full-rank matrix $\mathsf{H}_{\mathcal{U}_i}$, where

$$\mathcal{P}_i \triangleq \left\{ \sum_{j=1}^{n_{\mathrm{R}}} \lambda_j \mathbf{h}_j : \lambda_1, \ldots, \lambda_{n_{\mathrm{R}}} \in [0,1], j \in \mathcal{U}_i \right\},$$

and

$$\mathsf{H}_{\mathcal{U}_i} \triangleq [\mathbf{h}_j : j \in \mathcal{U}_i]$$

is a subset of n_{R} linearly independent columns of the channel matrix, and $\mathcal{U}_i$ is an index set.

Then we define $\mathscr{U}$ as the set of all choices of $\mathcal{U}_i$:

$$\begin{aligned} \mathscr{U} &\triangleq \{\mathcal{U}_i : i = 1, \ldots, N_p\} \\ &= \{\mathcal{U} = \{j_1, \ldots, j_{n_{\mathrm{R}}}\} \subseteq \{1, \ldots, n_{\mathrm{T}}\} : \\ &\qquad \mathbf{h}_{\mathbf{j}_1}, \ldots, \mathbf{h}_{\mathbf{j}_{n_{\mathrm{R}}}} \text{ are linearly independent} \}. \end{aligned}$$

While the complementary set of $\mathcal{U}_i$ is represented by

$$\mathcal{U}_i^c \triangleq \{1, \ldots, n_{\mathrm{T}}\} \setminus \mathcal{U}_i.$$

The n_{R}-dimensional coefficient vector is defined as:

$$\gamma_{\mathcal{U}_i,j} \triangleq \mathsf{H}_{\mathcal{U}_i}^{-1}\mathbf{h}_j, \mathcal{U}_i \in \mathscr{U}, j \in \mathcal{U}_i^c. \tag{9}$$

The sum of $\gamma_{\mathcal{U}_i,j}$ is

$$a_{\mathcal{U}_i,j} \triangleq \mathbf{1}_{n_{\mathrm{R}}}^{\top}\gamma_{\mathcal{U}_i,j}, \mathcal{U}_i \in \mathscr{U}, j \in \mathcal{U}_i^c. \tag{10}$$

Then we let

$$g_{\mathcal{U}_i,j} \triangleq \begin{cases} 1, & a_{\mathcal{U}_i,j} > 1 \\ 0, & \text{otherwise} \end{cases} \quad \mathcal{U}_i \in \mathscr{U}, j \in \mathcal{U}_i^c, \tag{11}$$

and the basic cost vector is given by

$$\mathbf{g}_{\mathcal{U}_i} = (g_{\mathcal{U}_i,j} : j \in \mathcal{U}_i^c)^{\top}. \tag{12}$$

We define $\mathbf{v}_{\mathcal{U}_i}$ as the translation vector:

$$\mathbf{v}_{\mathcal{U}_i} \triangleq \sum_{j \in \mathcal{U}_i^c} g_{\mathcal{U}_i,j}\mathbf{h}_j, \mathcal{U}_i \in \mathscr{U}. \tag{13}$$

For a equivalent transmitted signal $\mathbf{s}$ within the parallelepiped $\mathcal{P}_i$ spanned by $\mathsf{H}_{\mathcal{U}_i}$, the corresponding transmitted signal $\mathbf{x} = (x_1, \ldots, x_{n_{\mathrm{T}}})^{\top}$ is given by

$$x_j = \begin{cases} g_{\mathcal{U}_i,j}, & j \in \mathcal{U}_i^c, \\ q_{\mathcal{U}_i,j}, & j \in \mathcal{U}_i, \end{cases} \tag{14}$$

where the vector $\mathbf{q}_{\mathcal{U}_i} = (q_{\mathcal{U}_i,j} : j \in \mathcal{U}_i)^\top$ is calculated by

$$\mathbf{q}_{\mathcal{U}_i} \triangleq \mathsf{H}_{\mathcal{U}_i}^{-1} \left(\mathbf{s} - \mathbf{v}_{\mathcal{U}_i}\right). \tag{15}$$

Therefore, the minimum-energy mapping procedures from an n_{R}-dimensional equivalent transmitted signal $\mathbf{s}$ to an n_{T}-dimensional transmitted signal $\mathbf{x}$ can be summarized the two steps

- *Step 1:* Identify the specific parallelepiped within the zonotope $\mathcal{Z}(\mathsf{H})$ to which the equivalent transmitted signal $\mathbf{s}$ belongs. Refer to this parallelepiped as $\mathcal{P}_i$, and denote its corresponding full-rank matrix as $\mathsf{H}_{\mathcal{U}_i}$.
- *Step 2:* Calculate the $(n_{\mathrm{T}} - n_{\mathrm{R}})$-dimensional vector $\mathbf{g}_{\mathcal{U}_\mathrm{i}}$ and the n_{R}-dimensional vector $\mathbf{q}_{\mathcal{U}_\mathrm{i}}$, and map $\mathbf{s}$ to $\mathbf{x}$ by (14).

B. Optimal Geometric Shaping for SISO-VLC

Constellation shaping technology is an important approach to improve the performance of communication system [17], [18]. The constellation shaping can be divided into probabilistic shaping [19], [20], [21] and geometric shaping [14], [22], [23]. In this paper, we focus on the geometric shaping, which obtains the shaping gain by uniformly distributing the constellation points in a specific subset of Euclidean space. Therefore, the crucial aspect of the spatial constellation design is to determine the optimal geometric shaping region of the MIMO-VLC system under the PTA constraints.

Before determining the optimal geometric shaping region for MIMO-VLC, we first introduce the optimal shaping region for SISO-VLC as a theoretical basis. In [14], the optimal geometric shaping region is provided for the SISO-VLC system under the peak- and average-power constraints. According to [14], the multidimensional optimal geometric shaping region refers to the cost-optimal region that maximizes the volume while ensuring uniformly distributed constellation bounded by the region satisfying the power constraints. The cost of an n-dimensional region $\mathcal{X}$, which can be approximated by a uniformly distributed constellation with a sufficiently large cardinality within $\mathcal{X}$ [18], [24], is defined as

$$f_{\mathrm{cost}}(\mathcal{X}) \triangleq \frac{1}{n \times \mathrm{vol}\left(\mathcal{X}\right)} \int_{\mathcal{X}} \sum_{i=1}^{n} x_i \, d\Omega, \tag{16}$$

where $\mathbf{x} = (x_1, \ldots, x_n)^\top \in \mathscr{X}$, and $\mathscr{X}$ is the uniformly distributed constellation with sufficiently large cardinality bounded by the region $\mathcal{X}$.

The optimal geometric shaping region for the multidimensional constellation of the SISO-VLC system, subject to the peak- and average-power constraints, has been proposed as a truncated cube determined by a single parameter $t \in (0, n_{\mathrm{R}}]$. The region ensures that the sum of all elements in the constellation points $\mathbf{x}$ bounded by it satisfies $\sum_{i=1}^{n} x_i \leq t$. The volume of an n-dimensional truncated cube is

$$V_n(t) = \frac{1}{n!} \sum_{j=0}^{n} \binom{n}{j} (-1)^j (t-j)^n \mathbf{1}_+(t-j), \tag{17}$$

where the value of $\mathbf{1}_+(x)$ equals 1 if $x \geqslant 0$, and 0 otherwise. The cost of the n-dimensional truncated cube can be calculated by

$$P_n(t) = \frac{1}{n}\Big(t - \frac{1}{V_n(t)(n+1)!} \sum_{j=0}^{n} \binom{n}{j} (-1)^j (t-j)^{n+1} \mathbf{1}_+(t-j)\Big). \tag{18}$$

IV. Constellation Design Based on Optimal Geometric Shaping

In this section, we initially derive the optimal geometric shaping region for the MIMO-VLC system, followed by proposing a constellation design based on the obtained result combined with the scaled integer lattice.

A. Optimal Geometric Shaping

For the MIMO-VLC with $\mathrm{rank}(\mathsf{H}) = \mathrm{n_R}$, we focus on the n_{R}-dimensional equivalent transmitted signal. The cost-optimal region of the equivalent transmitted signal is the maximum n_{R}-dimensional volume over all closed regions $\mathcal{S}$ satisfying the following constraints:

$$\mathcal{S} \subseteq \mathcal{Z}\left(\mathsf{H}\right) \subseteq \mathbb{R}^{n_{\mathrm{R}}}, \tag{19a}$$

$$f_{\mathrm{cost}}(\mathcal{S}) \leq \alpha. \tag{19b}$$

Analog to (16), the cost metric $f_{\mathrm{cost}}(\mathcal{S})$ is defined as

$$f_{\mathrm{cost}}(\mathcal{S}) \triangleq \frac{1}{\mathrm{vol}\left(\mathcal{S}\right)} \int_{\mathcal{S}} \|f_{\min}(\mathbf{s})\|_1 \, d\Omega, \tag{20}$$

where $f_{\min}(\mathbf{s})$ can be obtained by the minimum-energy mapping procedures in Section III-A, and $\mathbf{s}$ is uniformly distributed over $\mathcal{S}$.

From [15], we know that the zonotope can be partitioned into at most N_p parallelepipeds. Therefore the closed region $\mathcal{S}$ satisfies (19) can also be partitioned into at most N_p parts.

$$\mathcal{S} = \bigcup_{i \in \{1,\ldots,N_p\}} \mathcal{S}_i, \tag{21}$$

where $\mathcal{S}_i = \mathcal{S} \bigcap \mathcal{P}_i$. Theorem 1 provides the cost-optimal region for the MIMO-VLC system.

Theorem 1: The image of the cost-optimal region for the MIMO-VLC under the PTA constraints is

$$\mathcal{S}_{opt} = \bigcup_{i \in \{1,\ldots,N_p\}} \left\{ \mathcal{S}_i \subseteq \mathcal{P}_i : \forall \mathbf{s} \in \mathcal{S}_\mathbf{i}, \sum_{\mathbf{j} \in \mathcal{U}_\mathbf{i}} \mathbf{x_j} \leq \mathbf{t_i} \right\}, \tag{22}$$

where $\mathbf{x} = f_{\min}(\mathbf{s})$ is obtained by the minimum-energy mapping and t_i is a parameter which is given by:

$$t_i = \begin{cases} 0, & \nu - \mathsf{N}_i \leqslant 0 \\ \nu - \mathsf{N}_i, & 0 < \nu - \mathsf{N}_i < n_{\mathrm{R}} \\ n_{\mathrm{R}}, & \nu - \mathsf{N}_i \geqslant n_{\mathrm{R}} \end{cases} \tag{23}$$

where $\mathsf{N}_i = \mathbf{1}^\top \mathbf{g}_{\mathcal{U}_i}$ represents the basic cost of $\mathcal{P}_i$.

Proof: The volume of $\mathcal{S}$ is represented by

$$\mathrm{vol}\,(\mathcal{S}) = \sum_{i\in\{1,\ldots,N_p\}} \mathrm{vol}\,(\mathcal{S}_i) \tag{24}$$

$$= \sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i \times \mathrm{vol}\,(\mathcal{X}_i)\,, \tag{25}$$

$$= \sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i \times V_{n_\mathrm{R}}\,(t_i)\,, \tag{26}$$

where (26) follows from the zonotope decomposition and the volume of the n_R-dimensional cost-optimal region for SISO-VLC,[1] and D_i can be calculated by

$$\mathsf{D}_i = \mid \det\,(\mathsf{H}_{\mathcal{U}_i})\mid . \tag{27}$$

It is known that the cost-optimal region maximizes the volume while satisfying the constraints in (19). However, this problem is challenging and can be transformed into its dual form, which aims to minimize the cost of a given volume V_s to obtain parameters of the cost-optimal region.

The cost of $\mathcal{S}$ is given by (20). Substituting (26) into (20), we can conclude that

$$f_\mathrm{cost}(\mathcal{S}) = \frac{1}{\mathrm{vol}\,(\mathcal{S})}\int_\mathcal{S} \|f_\mathrm{min}(\mathbf{s})\|_1\, d\Omega, \tag{28}$$

$$= \frac{1}{\mathrm{vol}\,(\mathcal{S})}\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i \int_{\mathcal{X}_i}\left(\sum_{j\in\mathcal{U}_i} x_j + \mathsf{N}_i\right) d\Omega', \tag{29}$$

$$= \frac{1}{\mathrm{vol}\,(\mathcal{S})}\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i V_{n_\mathrm{R}}(t_i)\left(n_\mathrm{R} P_{n_\mathrm{R}}(t_i) + \mathsf{N}_i\right), \tag{30}$$

$$= \frac{1}{\mathsf{V}_s}\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i V_{n_\mathrm{R}}(t_i)\left(n_\mathrm{R} P_{n_\mathrm{R}}(t_i) + \mathsf{N}_i\right), \tag{31}$$

where (29) follows from the minimum-energy mapping, and (30) is obtained by the cost of SISO-VLC in (16).

Then we let

$$\Phi_n(t) = nV_n(t)P_n(t) \tag{32}$$

$$= tV_n(t) - \frac{1}{(n+1)!}\sum_{j=0}^{n}\binom{n}{j}(-1)^j(t-j)^{n+1}\mathbf{1}_+(t-j), \tag{33}$$

whose derivative satisfies

$$\Phi'_n(t) = tV'_n(t) + V_n(t) - \frac{1}{n!}\sum_{j=0}^{n}\binom{n}{j}(-1)^j(t-j)^{n}\mathbf{1}_+(t-j) \tag{34}$$

$$= tV'_n(t). \tag{35}$$

[1]It should be noted that the dimension of the spatial constellation is the rank of the channel matrix rather than the number of channel uses.

where (33) and (35) follow from (18) and (17), respectively. Therefore, the optimal problem is given by

$$\begin{aligned} \min \quad & \sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i\left[V_{n_\mathrm{R}}(t_i)\,\mathsf{N}_i + \Phi_{n_\mathrm{R}}(t_i)\right] \\ \text{s.t.} \quad & \sum_{\mathrm{i}\in\{1,\ldots,\mathrm{N_p}\}} \mathsf{D}_\mathrm{i}\mathsf{V}_{\mathrm{n_R}}(\mathrm{t_i}) = \mathsf{V}_\mathrm{s}, \end{aligned} \tag{36}$$

By the Lagrange condition, we can obtain

$$L(\mathbf{t},\nu) = \left(\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i\left[\mathsf{N}_i V_{n_\mathrm{R}}(t_i) + \Phi_{n_\mathrm{R}}(t_i)\right]\right) - \nu\left(\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i V_{n_\mathrm{R}}(t_i) - \mathsf{V}_s\right), \tag{37}$$

where ν is the Lagrange multiplier, and $\mathbf{t} = (\mathbf{t_1},\ldots,\mathbf{t_{N_p}})$ represents the optimization variable. The necessary condition for extreme values is given by

$$\mathsf{D}_i\left[\mathsf{N}_i V'_{n_\mathrm{R}}(t_i) + \Phi'_{n_\mathrm{R}}(t_i)\right] = \nu \mathsf{D}_i V'_{n_\mathrm{R}}(t_i) \tag{38}$$

Then we replace $\Phi'_{n_\mathrm{R}}(t_i)$ by (35), the parameter t_i should satisfy

$$t_i = \nu - \mathsf{N}_i. \tag{39}$$

Due to $t_i \in (0, n_\mathrm{R}]$, the final condition t_i should satisfy is summarized as (23). ■

As demonstrated in Theorem 1, the cost-optimal region of the MIMO-VLC system under the PTA constraints is determined by a single parameter ν. The parameter t_i increases as the parameter ν increases within the range of $\nu \in [0, n_\mathrm{T}]$, in accordance with (23). Additionally, due to the monotonicity of $V_n(t)$ in (17), the volume of the optimal geometric shaping region also increases. Consequently, an increase in the parameter ν leads to a corresponding increase in the volume of the optimal geometric shaping region.

Moreover, the parameter ν can be accurately determined through the bisection method using α due to the cost metric $f_\mathrm{cost}(\mathcal{S})$ exhibits a monotonic increase with respect to the parameter ν, within the range of $\nu \in [0, n_\mathrm{T}]$. The proof of the monotonicity property is presented in Appendix A. The optimization process can be mainly divided into the bisection method and the calculation of $f_\mathrm{cost}(S)$. The computational complexity of the bisection method is $O(\log(n_\mathrm{R}))$, and the computational complexity of the calculation of $f_\mathrm{cost}(S)$ by (30) is $O(N_p \cdot n_\mathrm{R})$. Therefore, the computational complexity of the optimization process is $O(N_p \cdot n_\mathrm{R}\log(n_\mathrm{R}))$.

When the parameter $\nu \to \infty$, $t_i = n_\mathrm{R}$ for all index $i \in \{1,\ldots,N_p\}$. Therefore, the volume of $\mathcal{X}_i$ is

$$\mathrm{vol}(\mathcal{X}_i) = V_{n_\mathrm{R}}(n_\mathrm{R}) = 1,$$

and the volume of $\mathcal{S}$ is given by

$$\mathrm{vol}(\mathcal{S}) = \sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i\,\mathrm{vol}(\mathcal{X}_i) = \sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i, \tag{40}$$

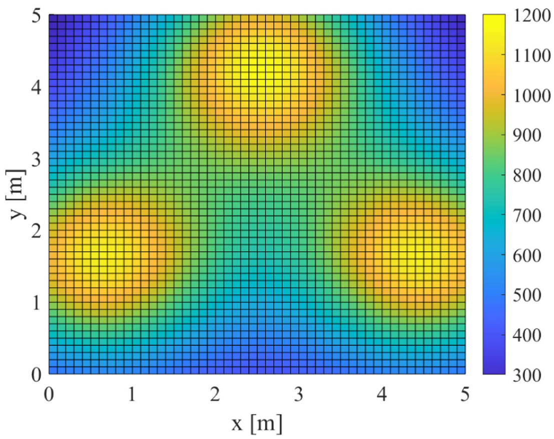


Fig. 2. The distribution of illuminance in the 5 m×5 m×3 m room with three LED arrays on the ceiling.

TABLE I
LED PARAMETERS

Number of LEDs	6400(80 × 80)
Center luminous intensity	0.73 [cd]
LED interval	0.01 [m]
Size of LED light	0.59 × 0.59
Height of user terminals	0.9 [m]

TABLE II
CONFIGURATION PARAMETERS

LED1 coordinate	[2.5m, 4.32m, 3m]
LED2 coordinate	[4.5m, 1.5m, 3m]
LED3 coordinate	[0.5m, 1.5m, 3m]
FOV (Ψ_C)	60°
Semi-angle at half power $(\Phi_{1/2})$	60°
Physical area of PD (A)	1 [cm^2]
Gain of optical filter (T_s)	1
Gain of concentrator (g)	1.5

TABLE III
PD POSITIONS

	PD1	PD2
H_1	[2.7m, 3m, 0.8m]	[2.2m, 2.3m, 1m]
H_2	[3m, 1.8m, 0.8m]	[2.8m, 1.2m, 1m]

where D_i is provided in (27). Then we substitute $\mathrm{vol}(\mathcal{S})$ and $\mathrm{vol}(\mathcal{X}_i)$ into (30), we can conclude that

$$\alpha_{\mathrm{th}} = \sum_{i\in\{1,\ldots,N_p\}} \frac{\mathsf{D}_i}{\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i} \left(n_{\mathrm{R}} P_{n_{\mathrm{R}}}(t_i) + \mathsf{N}_i\right) \quad (41)$$

$$= \frac{n_{\mathrm{R}}}{2} + \sum_{i\in\{1,\ldots,N_p\}} \frac{\mathsf{D}_i \mathsf{N}_i}{\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i}, \quad (42)$$

TABLE IV
SNR GAINS OF THE PROPOSED SCHEME COMPARED WITH THE TWO CONTRAST SCHEMES

(a) H_1

Gains / Parameter α / Bit rate	0.8		1.2	
5 bpcu	RC	SMP	RC	SMP
	4.1 dB	3 dB	3.8 dB	2.7 dB
6 bpcu	RC	SMP	RC	SMP
	5.5 dB	2.9 dB	5.1 dB	2.5 dB

(b) H_2

Gains / Parameter α / Bit rate	0.8		1.2	
5 bpcu	RC	SMP	RC	SMP
	2.5 dB	2.7 dB	2.1 dB	2.3 dB
6 bpcu	RC	SMP	RC	SMP
	4 dB	3.5 dB	3.5 dB	3 dB

where α_{th} is the threshold of the average-power constraint parameter α, as proposed in [15]. When $\alpha \geq \alpha_{\mathrm{th}}$, the average-power constraint becomes inactive. In this paper, we establish its validity by considering the optimal shaping parameter approaching positive infinity.

B. Constellation Design

As discussed in Section IV-A, the cost-optimal region for the MIMO-VLC system is characterized by an n_{R}-dimensional zonotope comprising a maximum of N_p pieces. Hence, the primary challenge lies in arranging M uniformly distributed equivalent transmitted signal constellation points within the zonotope.

According to [25], a lattice refers to a regular array in Euclidean space that exhibits geometric uniformity. In this paper, we employ the scaled n_{R}-dimensional integer lattice $\mathbb{Z}_{n_{\mathrm{R}}}$ to obtain uniformly distributed equivalent transmitted signal constellation points within the cost-optimal region, where

$$\mathbb{Z}_{n_{\mathrm{R}}} = \{(x_1, x_2, \ldots, x_{n_{\mathrm{R}}}) : x_i \in \mathbb{Z}\}, 1 \leqslant i \leqslant n_{\mathrm{R}},$$

and $\mathbb{Z}$ is the set of all integers.

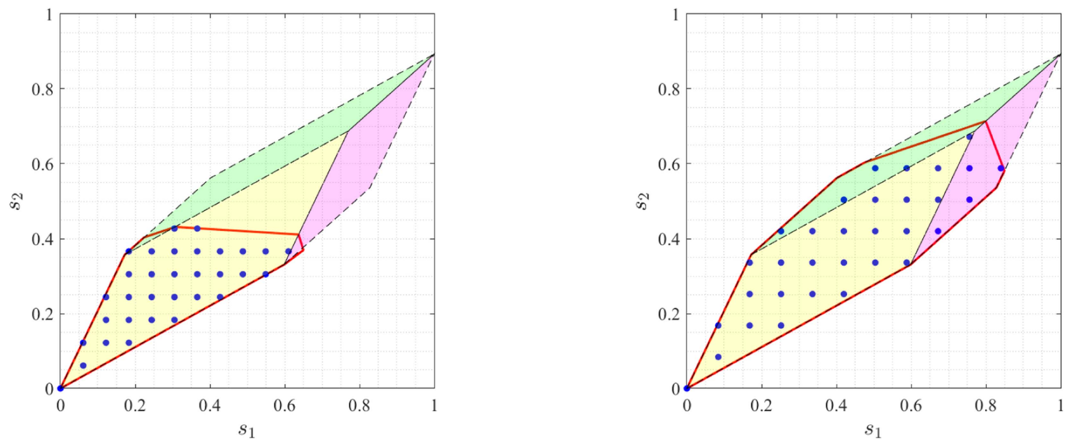
The initial scaling factor κ_{in} is chosen to ensure that the intersection of $\kappa_{in}\mathbb{Z}_{n_{\mathrm{R}}}$ and $\mathcal{S}_{opt}$ contains more than M elements. Subsequently, we incrementally increase the scaling factor κ with a fixed step size until we determine the maximum scaling factor $\kappa_{\max}$ that satisfies the given conditions. The constellation of equivalent transmitted signals refers to the size-M average-power-minimized subset of the intersection $\kappa_{\max}\mathbb{Z}_{n_{\mathrm{R}}} \cap \mathcal{S}_{opt}$.

The constellation design is provided in Algorithm 1.

V. SIMULATION RESULT

In this section, we conduct simulations on an indoor MIMO-VLC system to evaluate the performance of the proposed constellation design and compare it with other conventional MIMO–VLC schemes under different average-power constraints.

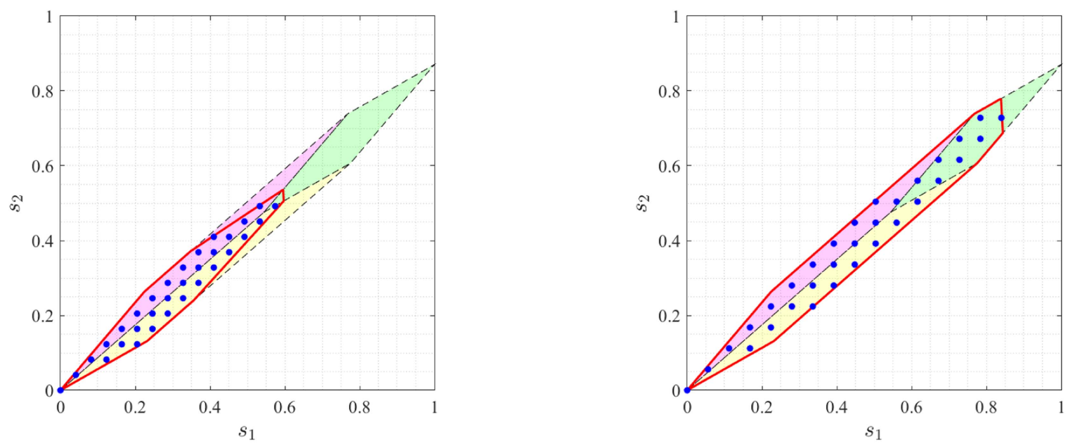
For the multi-LED VLC system, it is more general to assume a square LED layout with four LED arrays on the ceiling. However, for the sake of simplicity, in this paper, we consider a



(a) Cost-optimal region and constellation points with $\alpha = 0.8$ under the channel condition of H_1.

(b) Cost-optimal region and constellation points with $\alpha = 1.2$ under the channel condition of H_1.

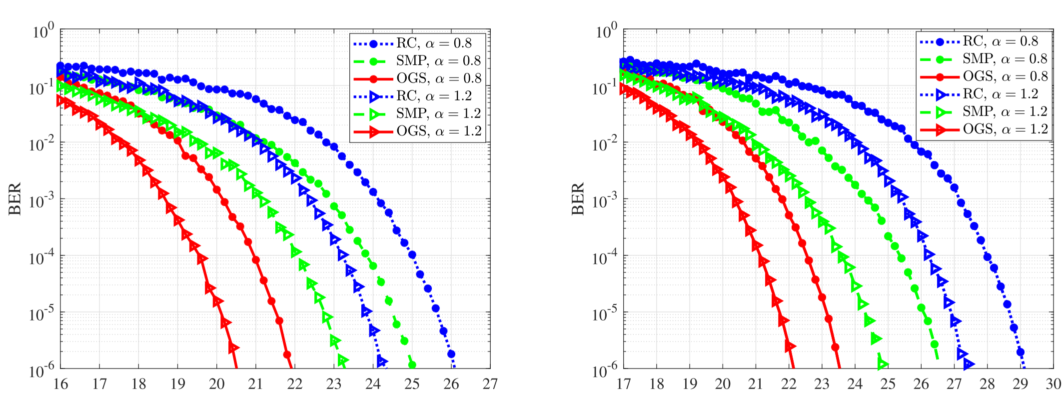
Fig. 3. Cost-optimal region and constellation points under the channel condition of H_1.



(a) Cost-optimal region and constellation points with $\alpha = 0.8$ under the channel condition of H_2.

(b) Cost-optimal region and constellation points with $\alpha = 1.2$ under the channel condition of H_2.

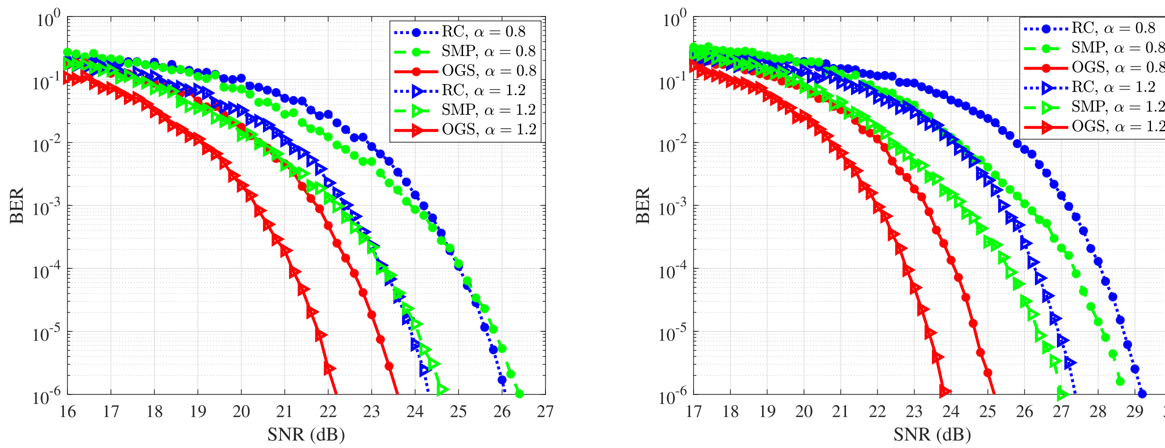
Fig. 4. Cost-optimal region and constellation points under the channel condition of H_2.



(a) BER curves of the three schemes with $\beta = 5$ bpcu under the channel condition of H_1.

(b) BER curves of the three schemes with $\beta = 6$ bpcu under the channel condition of H_1.

Fig. 5. BER curves of the three schemes under the channel condition of H_1.



(a) BER curves of the three schemes with $\beta = 5$ bpcu under the channel condition of H_2.

(b) BER curves of the three schemes with $\beta = 6$ bpcu under the channel condition of H_2.

Fig. 6. BER curves of the three schemes under the channel condition of H_2.

Algorithm 1: Constellation Design Based on the Optimal Geometric Shaping.

Input: The average-power constraint parameter α, the channel gain matrix H, the bit rate β.
Output: The transmitted signal constellation $\mathscr{X}$.
$M = 2^{\beta}$; $N_p = \binom{n_{\mathrm{T}}}{n_{\mathrm{R}}}$;
Partition the zonotope $\mathcal{Z}(H)$ into N_p parallelepipes;
Employ Theorem 1 to ascertain the shaping parameter ν based on the parameter α, and determine the cost-optimal region $\mathcal{S}_{opt}$;
Determine an initial scaling factor κ_{in} which satisfies

$$| \kappa_{in}\mathbb{Z}_{n_{\mathrm{R}}} \cap \mathcal{S}_{opt} | > M;$$

Select a proper step size δ;
$\kappa = \kappa_{in}$;
Loop
  $\kappa = \kappa + \delta$;
Until $| \kappa\mathbb{Z}_{n_{\mathrm{R}}} \cap \mathcal{S}_{opt} | < M$
  $\kappa_{\max} = \kappa - \delta$;
  End
Utilize minimum-energy mapping to map the points in the set $\kappa_{\max}\mathbb{Z}_{n_{\mathrm{R}}} \cap \mathcal{S}_{opt}$ onto the corresponding transmitted signals;
The constellation $\mathscr{X}$ is obtained by selecting M points that minimize the total average power.
End

2×3 MIMO-VLC system in a room measuring 5 m $\times$5 m$\times$3 m, and a triangular LED layout with three LED arrays on the ceiling. We should first prove the system we considered is practical in terms of illumination.

According to International Organization for Standardization (ISO), the illuminance of 300 to 1500 lx in required for offices work [4]. From [4], we also know that the horizontal illuminance at a point (x, y) is given by

$$E_{hor} = I(0)\cos^{m}(\phi_{ij})/D_{ij}^{2} \cdot \cos(\psi_{ij}), \tag{43}$$

where $I(0)$ denotes the center luminous intensity of an LED, ϕ_{ij}, ψ_{ij}, and D_{ij} are the parameters of Lambertian model in (2). Then we provide the simulated illuminance of the 5 m $\times$5 m $\times$3 m room with a triangular LED layout consisting of three LED arrays on the ceiling in Fig. 2. The other simulated parameters of LEDs are summarized in Table I. It is should be noted that the height of user terminals in the simulation is 0.9 m, which is the average height of PD1 and PD2 in Table III.

From Fig. 2, we can see that the illuminance of the room with three LED arrays on the ceiling is 300 to 1200 lx, which satisfies the standard of ISO. Therefore, this result shows that the assumption of three LED arrays on the ceiling is practical in terms of illumination.

The configuration parameters are listed in Table II. In our simulations, the three LEDs are installed at fixed positions on the ceiling, while the two PDs are randomly located within the FOV, which reflects practical conditions. To provide different zonotope decompositions, we select two representative channels for simulation. The PD positions for these two channels are listed in Table III.

The channel gain matrix is computed using (2) with the parameters specified in Table II and Table III. For the convenience of subsequent presentation, the deterministic real-valued channel gain matrix can be normalized without loss of generality by dividing it with the maximum sum value of the rows in the channel matrix. Consequently, the two resulting normalized channel gain matrices are given by

$$\mathsf{H}_1 = \begin{bmatrix} 0.5984 & 0.2295 & 0.1721 \\ 0.3308 & 0.2053 & 0.3570 \end{bmatrix},$$

$$\mathsf{H}_2 = \begin{bmatrix} 0.2316 & 0.5440 & 0.2244 \\ 0.1314 & 0.4757 & 0.2634 \end{bmatrix}.$$

A. Cost-Optimal Region and Constellation Analysis

According to the previous findings, it is evident that the cost-optimal region primarily relies on two key factors: the channel gain matrix H and the average-power constraint parameter α. Hence, we present the cost-optimal region along with corresponding constellation points for different channel conditions and various average-power constraint parameters when considering a bit rate of $\beta = 5$ bits per channel use (bpcu) (resulting in a constellation cardinality of $M = 2^{\beta} = 32$).

In Figs. 3 and 4, the solid red line surrounds the optimal geometric shaping region $\mathcal{S}_{opt}$, while the dashed line encloses the zonotope $\mathcal{Z}(\mathsf{H})$, and the regions with different colors depict the parallelograms $\mathcal{P}_i$ spanned by $\mathsf{H}_{\mathcal{U}_i}$. Additionally, blue points within $\mathcal{S}_{opt}$ are used to represent the equivalent transmitted signal constellation points.

As observed from Figs. 3 and 4, the cost-optimal region, which is associated with the parameter α can be decomposed into at most N_p regions, and each region corresponds to a subset of the parallelepiped $\mathcal{P}_i$. Furthermore, it is evident that the proposed scheme achieves uniform distribution of constellation points within the cost-optimal region.

B. Error Performance

In this subsection, we compare the error performance of the proposed OGS scheme with two traditional MIMO-VLC schemes, the RC scheme and the SMP scheme.

The RC scheme involves the simultaneous transmission of identical signals from all transmitters, as expressed by

$$\mathbf{x}_{\mathrm{RC}}^{\top} = \left(\frac{2\alpha i}{(M-1)\, n_{\mathrm{T}}}\right)^{n_{\mathrm{T}}}, i \in \{0, 1, \ldots, M-1\}. \quad (44)$$

Optical signals transmitted from different LEDs in the RC scheme are combined at the PD receiver through distinct channels, resulting in significant diversity gain.

The SMP scheme enables each LED to independently transmit signals, thereby providing multiplexing gain. The signal is given by

$$\mathbf{x}_{\mathrm{SMP}}^{\top} = \left(\frac{2\alpha i_1}{(M_1-1)\, n_{\mathrm{T}}}, \frac{2\alpha i_2}{(M_2-1)\, n_{\mathrm{T}}}, \ldots, \frac{2\alpha i_{n_{\mathrm{T}}}}{((M_{n_{\mathrm{T}}}-1))\, n_{\mathrm{T}}}\right), \quad (45)$$

where for all $k \in \{1, \ldots, n_{\mathrm{T}}\}$, i_k is selected from the set $\{0, 1, \ldots, M_k - 1\}$, and M_k represents the cardinality of the constellation set for the k-th LED, subject to the constraint $\sum_{k=1}^{n_{\mathrm{T}}} M_k = M$.

To assess the error performance, we present the bit error rate (BER) curves for the three schemes across diverse channel conditions, average-power constraints, and bit rates. For scaling invariance, the SNR is defined as the maximum received signal power divided by the standard deviation σ of the noise [26], [27]. Mathematically, it can be expressed as follows:

$$\mathrm{SNR} \triangleq \frac{1}{\sigma}. \quad (46)$$

Figs. 5 and 6 provide the BER curves of the three schemes when the channel gain matrix is H_1 and H_2 respecitvely. As depicted in Figs. 5 and 6, the proposed scheme exhibits superior error performance compared to the two contrast schemes while also demonstrating adaptability across various optical channels. Given the targeted BER of 10^{-5}, the gain of the proposed scheme compared with RC and SMP under different conditions is shown in Table IV.

As depicted in Table IV, the gains of the proposed scheme compared with other schemes under channel condition H_1, are observed to be larger than those under channel condition H_2 when all other conditions remain constant. This discrepancy can be attributed to the fact that the cost-optimal region is greater under H_1 than under H_2.

VI. Conclusion

In this paper, we propose a geometrically-shaped constellation design for MIMO-VLC systems under the peak-power and total average-power constraints. The proposed scheme combines the cost-optimal region with a scaled integer lattice to achieve a spatial constellation design. Simulation results demonstrate that the proposed constellation design outperforms conventional methods in indoor MIMO-VLC scenarios. In conclusion, our proposed scheme represents a promising solution for the MIMO-VLC system under the aforementioned constraints.

Appendix A
Proof of the Monotonicity Between ν and $f_{\mathrm{cost}}(\mathcal{S})$

In [14], another align for $P_n(t)$ is given by

$$P_n(t) = \frac{1}{n}\mathrm{E}\left[Z | Z \leq t\right], \quad (47)$$

where $Z = \sum_{i=1}^{n} X_i$, and $X_1, \ldots, X_n$ are independent and uniformly distributed random variables on the interval [0,1]. The expression $\mathrm{E}[Z|Z \leq t]$ represents the conditional expectation of Z given that $Z \leq t$.

Combine (24), (30), and (32), we can obtain that

$$f_{\mathrm{cost}}(\mathcal{S}) = \frac{\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i \left[\mathsf{N}_i V_{n_{\mathrm{R}}}(t_i) + \Phi_{n_{\mathrm{R}}}(t_i)\right]}{\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i V_{n_{\mathrm{R}}}(t_i)}, \quad (48)$$

whose derivative with respect to $\mathbf{t}$ is given by

$$f'_{\mathrm{cost}}(\mathcal{S}, \mathbf{t}) = \frac{\left(\sum_{i\in\{1,\ldots,N_p\}} (t_i - nP_{n_{\mathrm{R}}}(t_i))\right) \cdot \left(\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i V'_{n_{\mathrm{R}}}(t_i)\right)}{\sum_{i\in\{1,\ldots,N_p\}} \mathsf{D}_i V_{n_{\mathrm{R}}}(t_i)}, \quad (49)$$

which follows from (32) and (35).

The denominator of the derivative is positive due to the definition of D_i and $V_n(t)$. From (47), we know that $nP_n(t) = \mathrm{E}[Z|Z \leq t]$, thus $nP_n(t) < t$ and the first term in the numerator is positive. As stated in [14], within the range of $t \in [0, n]$, it is known that $V_n(t)$ strictly increases with respect to t. Therefore, we have $V'_n(t) > 0$ and the second term in the numerator is positive for values of t_i within $[0, n_{\mathrm{R}}]$. Consequently, it can be concluded that the derivative of $f_{\mathrm{cost}}(\mathcal{S})$ with respect to $\mathbf{t}$ is positive within the range of $t_i \in [0, n_{\mathrm{R}}]$.

Furthermore, based on the aforementioned derivation, we know that the range of t_i lies within $t_i \in [0, n_{\mathrm{R}}]$. From (23), we know that the parameter t_i is monotonically increasing with the paremeter ν within the range of $t_i \in [0, n_{\mathrm{R}}]$, for all $i \in \{1, \ldots, N_p\}$. Considering the definition of N_i and (12), it is evident that the maximum value of N_i is $(n_{\mathrm{T}} - n_{\mathrm{R}})$. Thus, the monotone interval of ν is given by $\nu \in [0, n_{\mathrm{T}}]$. Combining this conclusion with the above derivation, we can conclude that $f_{\mathrm{cost}}(\mathcal{S})$ exhibits a monotonic increase with respect to the parameter ν, within the range of $\nu \in [0, n_{\mathrm{T}}]$.

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