

# Motion Analysis of Micro-swimmers in Liquid with a Low Reynolds Number

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**Abstract**—The main purpose of this research is to provide a basic idea for controlling micro-swimming robots. This report is focused on exploring the way micro-swimmers move in a liquid environment with a low Reynolds number and how their physical parameters affect their speed of movement and finally, simulate the movement trajectory of cells. In conclusion, the way of motion of a cell is divided into "Run Phase" and "Tumble Phase", and cells move at different speeds in different states. Additionally, the relationship between the speed of cells and the pitch angle of the tail flagella is roughly a quadratic function with the opening facing downwards, which helps to control the speed of micro-swimmers. Lastly, the trajectory of a cell is simulated based on the analysis above and the results showed that they matched perfectly.

**Keywords**—micro-swimmer, Reynolds number, Pitch angle control, simulate, flagella

## I. INTRODUCTION

Medical robots can extend human medical capabilities, better plan and execute medical instructions, and also save costs. Microvascular medical robots are one of the new and promising research directions. Micro-vascular medical robots offer a novel and innovative approach to studying and potentially treating cardiovascular disease. These tiny, technologically advanced devices can navigate the complex vascular network of the human body, further complicating the study of the brain's micro-vascular structure and potentially paving the way for targeted treatment strategies.

This research paper focuses on understanding the motion mechanism and trajectory of these microscopic robots influenced by the behavior of microorganisms in various types of liquid environments. Firstly, based on the Scallop theory [1], in low Reynolds number environments, objects cannot move forward through reciprocating motion. Thus the spiral flagella of bacteria are a good propulsion device. Due to the asymmetric movement of spiral flagella, bacteria can freely move in low Reynolds number liquids. That's also why this article uses monoflagellar bacteria as a model to construct a swimming robot. Park, Yunyoung, and others state that bacteria with flagella move forward or backward through the unidirectional rotation of flagella; In addition, they can turn counterclockwise by changing the direction of flagella rotation [2]. Secondly, the various physical parameters of bacteria and the surrounding environmental parameters can affect their movement speed. Mathematical formulas can be used to derive the functional relationship between any parameter and its velocity [3]. From Berg, Howard C. and Douglas A. Brown

"Three-dimension tracking" [4] is a good way to display the movement images of bacteria and help verify simulation results. The movement pattern of bacteria can be simply simulated as two steps: straight forward and turning, and the trajectory can be drawn by three-dimensional tracking. This study will build a biomimetic model based on Monoflagellar Escherichia coli and analyze how seven opposite numbers affect movement speed. Subsequently, based on these analyses, the approximate movement trajectory of the bacteria was simulated.

## II. CELL MOVEMENT MECHANISM

### A. The movement of micro-swimmer in liquid with low Reynolds number

To better design microrobots that can move smoothly in designated liquids, it is necessary to refer to the motion mechanism of microcells. According to Lyman Laboratory and others, an object undergoing reciprocal motion cannot generate a net translational motion in a time-irreversible fluid flow at low Reynolds numbers based on the Scallop theory. [1]. In other words, at low Reynolds numbers, a swimmer that engaged in a reciprocating circular motion cannot maintain forward motion. It is the dominant effect of viscous forces on inertial forces that cause this limitation [5]. Therefore, cells within an organism should have a special mechanism of movement that enables them to continue advancing and turning in low Reynolds number liquids. Lyman Laboratory and others argue that "run and tumble" is a general strategy micro-swimmers use to navigate through the surrounding environment [1]. For instance, single flagellar bacteria move forward or backward through the unidirectional rotation of flagella and turn counterclockwise by changing the direction of flagella rotation [2]. This movement pattern allows them to effectively explore their surroundings and locate favorable regions for nutrient uptake or avoid harmful conditions. The run-and-tumble behavior is a key mechanism for bacterial chemotaxis, which is the directed movement of cells in response to chemical stimuli.

Run and tumble behavior can be divided into two basic principles. The first part is called the "Run Phase". During the "Run Phas", the bacterial cell swims in a relatively straight line with a consistent velocity and direction. According to Kundu, Pijush K and others, the cell's flagella, which are whip-like appendages, rotates in a coordinated manner, propelling the cell forward [6]. The run phase persists for a short period, during which the bacterium can cover some distance in a specific direction. The second part is "Tumble Phase". After the run phase, the bacterium enters the "Tumble Phase". In this

phase, the flagella briefly switches to a chaotic, disorganized rotation. As a result, the cell undergoes a random reorientation, causing it to change its direction of motion. The tumble phase is short-lived and serves to disrupt the cell's previous trajectory.

### B. Mathematical explanation of swimmer's speed related to the viscosity

In order to better control the motion trajectory of micro swimming robots, mathematical expressions for their motion are needed. Yukio Magariyama and Seishi Kudo combine the physical parameters of the cell itself, such as the radius of the cell, the length of the flagella, pitch, pitch angle, and the physical parameters of the liquid, such as viscosity, for force analysis and motion analysis. By using free body diagram and differential methods, they obtain the relationship between the velocity of cell movement and flagella angular velocity based on the above physical quantities [3]. The equations can be concluded as follows:

$$\alpha_c = -6\pi\mu R \quad (1)$$

$$\beta_c = -8\pi\mu R^3 \quad (2)$$

$$P = \pi r \tan(\theta) \quad (3)$$

$$\alpha_f = \frac{2\pi\mu L}{(\log[d/2p]+1/2)(4\pi^2 r^2 + p^2)} (8\pi^2 r^2 + p^2) \quad (4)$$

$$\beta_f = \frac{2\pi\mu L}{(\log[d/2p]+1/2)(4\pi^2 r^2 + p^2)} (4\pi^2 r^2 + 2p^2)r^2 \quad (5)$$

$$\gamma_f = \frac{2\pi\mu L}{(\log[d/2p]+1/2)(4\pi^2 r^2 + p^2)} (-2\pi r^2 p) \quad (6)$$

$$v = \frac{-\omega\gamma_f}{(\alpha_c + \beta_c)} \quad (7)$$

Based on the above formula, let's conduct a quantitative analysis of cell velocity and attempt to change the values of some of the parameters and plot a function plot of them with cell velocity. The meanings and values of the symbols above can be found in TABLE I.

TABLE I. PARAMETERS NECESSARY FOR CALCULATING CELL MOTION

Symbol	Parameters	Value
$\alpha_c, \beta_c$	Drag coefficients of cell body	
$\alpha_f, \beta_f, \gamma_f$	Drag coefficients of flagellar filament	
R	Radius of the cell	$1 \times 10^{-6}$ m
d	Diameter of the tail	$20 \times 10^{-9}$ m
r	Radius of flagellar helix	$1 \times 10^{-6}$ m
L	Length of the tail	$10 \times 10^{-6}$ m
$\omega$	Frequency of the tail in Hz	$200 \times 2\pi$ Hz
$\mu$	Viscosity of the water at 25 °C	$8.9 \times 10^{-4}$ N·s·m <sup>-2</sup>

Symbol	Parameters	Value
p	Pitch of the tail	
v	Velocity of the cell	
$\theta$	Pitch angle of the flagellar helix	

Assuming that the body of the swimmer is an ideal sphere with a diameter of 2  $\mu$ m. The tail is a regular spiral-shaped flagella, with a diameter of 20 nm, a length of 10  $\mu$ m, a diameter of 2  $\mu$ m, and a pitch angle of 45°. This swimmer is placed in water and the tail is rotating at 200 Hz. Firstly, the plan is to find its speed based on the physical parameters above using the equations from Yukio Magariyama and Seishi Kudo [3].

A model of the micro-swimmer by SolidWorks (Fig. 1) was built to get a more intuitive understanding of its appearance characteristics. By bringing these parameter values into the program, it is easy to come up with the answer.

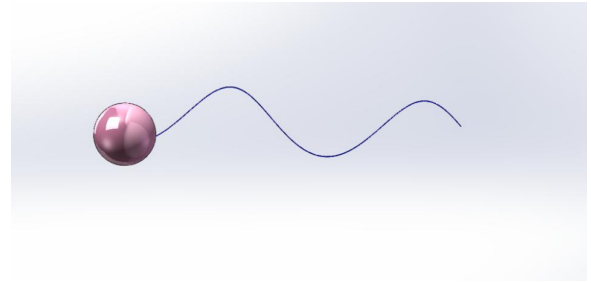


Fig. 1 The model of micro-swimmer

Then the issue is upgraded and a function plot of the micro-swimmer speed changing with the pitch angle is created while other parameters remain unchanged. Also, a function plot of the frequency of its flagellum vs speed while other parameters remain unchanged. This way, when other conditions are known and determined, the speed can be controlled by adjusting the pitch angle or the frequency of its flagellum. As shown in Fig. 2, the pitch angle here refers to the angle between the tangent of the endpoint position of the peak and valley of a continuous wave function and the horizontal direction, which can reflect the shape of the projected view of a spiral object [7]. Fig. 3 is the plot created by MATLAB.

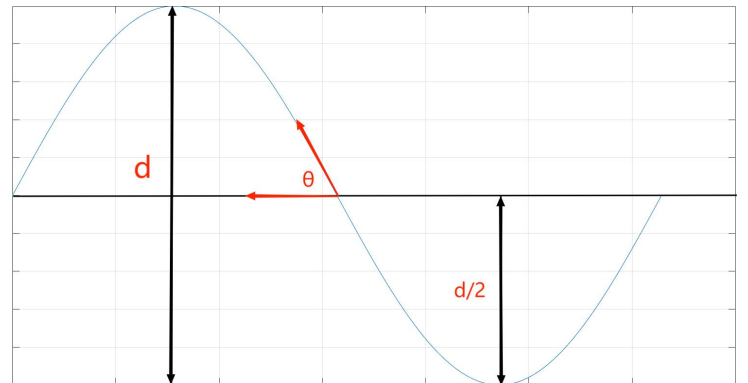


FIGURE 2 The pitch angle of a spiral on its projection

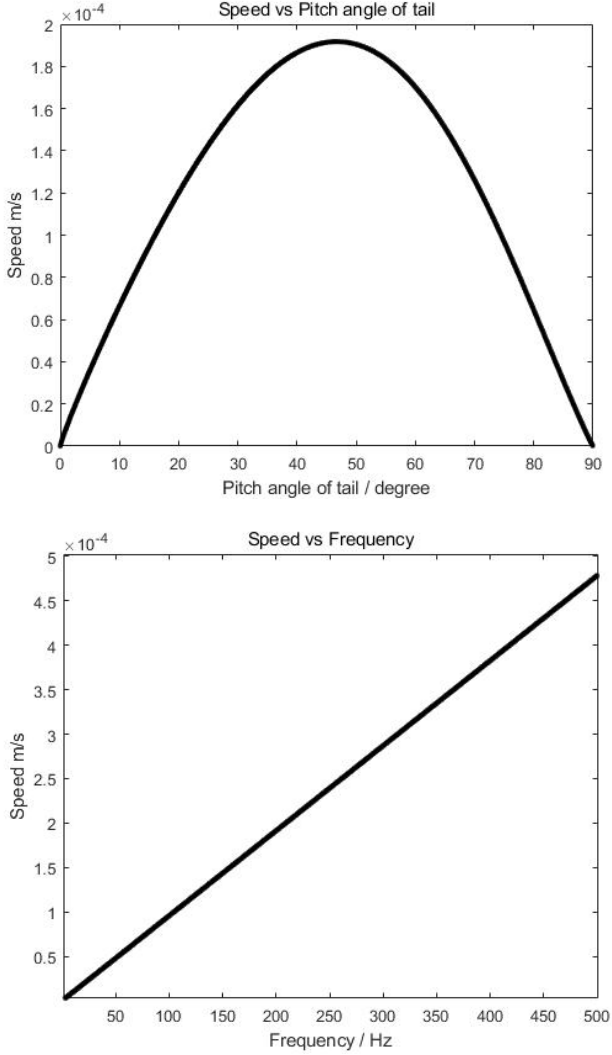


Fig. 3 Plot of velocity of the micro-swimmer vs pitch angle of the spiral tail (up) and the plot of velocity of the micro-swimmer vs frequency of flagellum (down), created by MATLAB.

The plot above in Fig. 3 shows the relationship between velocity and pitch angle. The plot is similar to a quadratic function with an opening downwards. When the pitch angle ranges from 0 to  $2\pi$ , the velocity first increases with the increase of the pitch angle and reaches a peak, then decreases with the increase of the pitch angle. With this graph, it is easier to precisely control the speed of a miniature swimmer by adjusting its pitch angle at the tail, given that all physical parameters remain unchanged. The plot below in Fig. 3 expresses the relationship between velocity and the frequency of flagellum. There is a linear positive correlation between cell movement speed and flagella frequency. The speed increases proportionally with the increase in frequency.

Based on the above analysis, it is possible to take control of the speed of the micro-swimmer. For example, if other parameters are known and the motor operates at a constant power, then its the velocity corresponding to each pitch angle can be obtained, and also an pitch angle that maximizes its

velocity. In addition, another way to control its speed is to adjust the frequency of its motor since there is a linear relation -ship between the speed and the frequency of motor.

### III. EXPLORATION AND SIMULATION OF CELL MOTION TRAJECTORIES

#### A. The speed of cells in different states of motion

As mentioned above, the motion of a cell can be divided into "Run Phase" and "Tumble Phase". Therefore, to better understand the motion of the swimmer, it is necessary to observe how a real bacteria moves by these two methods, which would help to understand the random motion of the swimmer better. Department of Molecular and others used an automatic tracking microscope to track the movement trajectory of mutants of E. coli K12 and do a run-twiddle analysis of mutants swimming in a homogeneous, isotropic medium. The measurements include all necessary physical quantities with their mean values separately, such as mean speed, mean twiddle length, and mean change in direction. In their opinion, cells spend most of their time in approximately linear motion and will slow down to change direction randomly at specific concentrations [4], then accelerate to normal speed and maintain a straightforward motion at that speed. During the process of turning again, the cell itself will have an angular velocity, which is controlled by the rotation of the tail flagella and the body of the cell.

To make the simulation more concise and clear, the cell can be treated as a particle with parameters: initial speed  $v_0$ , turning speed  $v_T$ , acceleration  $a_c$ , and the values of each symbol can be found in TABLE II. Suggest the duration of linear motion is  $t_d$  and the turning happens immediately, which means that the cell advances along a straight line with initial speed  $v_0$  for time  $t_d$ , then decelerate to  $v_T$  with acceleration  $a_c$  and change a random angle. After the turn is completed, accelerate to the initial speed with acceleration  $a$  and move in a straight line in the new direction. The process above is called a cycle of the motion.

TABLE II. PARAMETERS OF THE CELL

Symbol	Parameters	Value
$v_0$	Initial speed of the cell	20 $\mu\text{m/s}$
$v_T$	Turning speed of the cell	5 $\mu\text{m/s}$
$a_c$	Acceleration of the cell	20 $\mu\text{m/s}^2$
$t_d$	Duration of linear motion	1s

Based on the hypothesis and parameters above, the plot of the cell speed vs time can be generated as shown in Fig. 4. From Fig. 4, the plot of speed remains horizontal for 1 second, which shows the linear motion. The downward peak corresponds to the process of deceleration and turning process. The speed of cells will not remain constant, and here we are only based on theoretical analysis and taking the average value. So the actual image should be similar in shape to Figure 4, but with some noise.

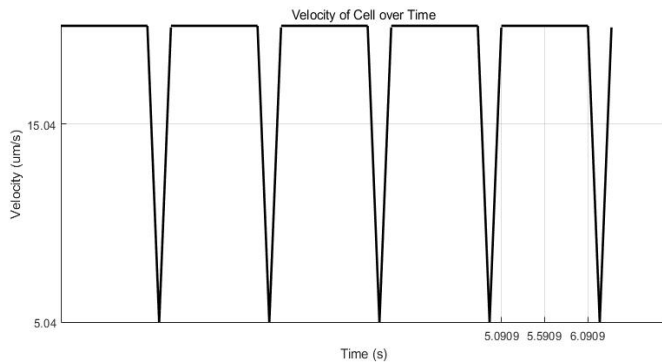


Fig. 4 The variation of cell speed over time during the movement cycle in an ideal situation

Fig. 4 represents the speed variation of *E. coli* K12 when moving in close proximity to reality, therefore it also has valuable reference value for speed control of micro swimmers. Due to the fact that micro swimmers are bionic robots modeled after *Escherichia coli*, their speed changes during movement should be similar to Figure 4. In order to control the variation of its speed, the control method discussed in part II can be applied. For instance, If the specific frequency corresponding to the highest efficiency of the motor is known, then it is possible to directly use this frequency and find the exact pitch angle of the motor through the function relationship between pitch angle and speed, so that the swimmer can maintain the desired constant speed and the motor can operate at the highest efficiency. During the turning phase, the speed of the motor can be reduced by briefly reducing its frequency, and after completing the turn, it can return to its original frequency and move forward in a straight line at the highest efficiency and uniform speed again

### B. Simulation of cell trajectories

Based on the speed-time plot, we can make further analysis of its motion. To make the entire random motion more intuitive, that is to simulate its trajectory. Here a way called "Three-dimensional Tracking" is used [4], which is mentioned by the Department of Molecular and others in their article. In a nutshell, it is to project a three-dimensional image onto three planes, XY, YZ, and XZ respectively, to two-dimensional the three-dimensional image. It's similar to the principle of three views in engineering drawings. Assume that the cell moves along a straight line at speed  $v_0$  for 1 second and then rotates with a mean angle of 68 degrees in a random direction. Then, it moves in a straight line toward the new direction and repeats the above process continuously. The total time of the motion is 10 seconds. Then record the cell position every 0.1 seconds and label it on the plot. Fig. 5 illustrates the 3D view of the cell trajectory and projected view on XY, YZ, and XZ planes.

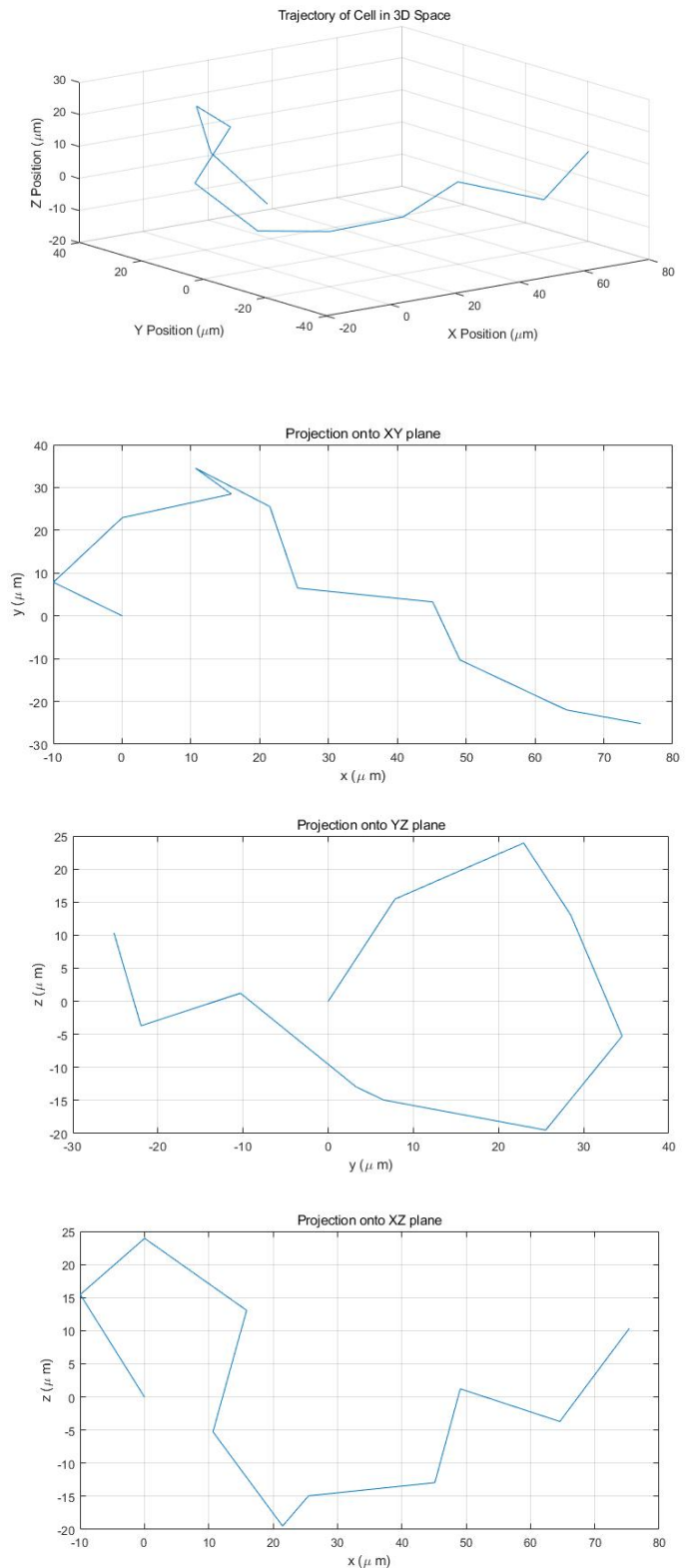


Fig. 5 Digital plot of the displacement of a cell at the rate of 10 words (data points) per second. The view in the top left corner is a 3D view of the cell trajectory, while the views in the top right corner, bottom left corner, and bottom right corner represent their projections on the XY, XZ, and YZ planes, respectively.

Fig. 5 shows clearly shows the ideal movement trajectory of bacteria from 3D to 2D. Due to the random rotation direction of real bacterial enzymatic turns, a random function was used in the program to represent the direction of each turn, and the average value ( $68^\circ$  here) was used as the angle of each turn. This can simulate the approximate movement of a bacterium. Due to its very short turning time, almost occurring in an instant, the turning process here is treated as a turning point (actually a very small arc).

This program that developed would help to understand the random motion of the swimmer better. The platform will be helpful when developing new artificial swimmers. For instance, if there is a need to design a nano robot for drug delivery with certain swimming pattern, researchers can directly borrow this existing program for the simulation.

The turning process has been simplified in the trajectory simulation mentioned above. In order to fully reproduce the real situation, considering the small time and displacement during turning, a random path was created using SOLIDWORKS, and the swimmer model is required to move along the path. Suggest that the test is in a water environment at  $25^\circ\text{C}$ . a rotating motor and a linear motor are added to mimic the flagella protein motor of *Escherichia coli* (driven by the rotation of the internal motor to rotate the flagella) [8]. Assume that when it moves along straight line, the frequency of tail flagella is 200 Hz and the speed is  $1.9 \times 10^{-4}$  m/s, and when it rotates, the frequency of motor is set at 50Hz where the speed is equal to  $0.48 \times 10^{-4}$  m/s. Suggest pitch angle of the tail is  $45^\circ$ (the values of speed above are calculated based on the physical parameters of the model and equations proposed by Yukio Magariyama and Seishi Kudo. [3]).

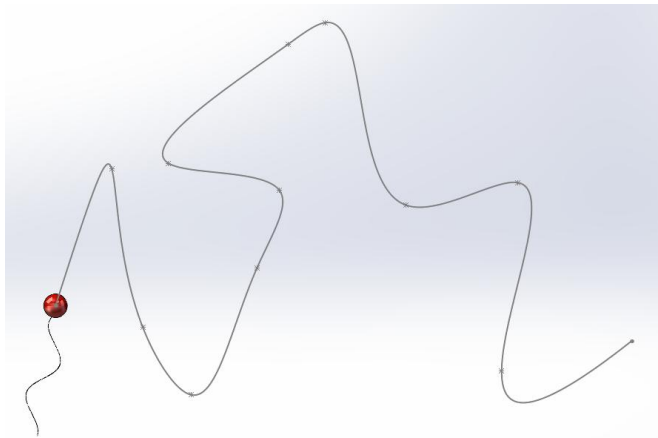


Fig. 6 The Motion Trajectory of Micro Swimmers.

The trajectory that built for the swimmer is shown in Fig. 6. The simulation is more like an explanatory way of presenting results. Behind the trajectory is a simulation of *E. coli* swimming. Because we know how *E. coli* swims, SOLIDWORKS is used here to demonstrate it. Especially for those who are not familiar with how bacteria swim, this will be helpful. In the future, if people want to simulate the swimming behavior of other things, such as micro robots, they

can first design the swimming mode of the robot and then combine it with the SOLIDWORKS model to make the results explanatory. However, in this case, the turning point is no longer simplified into a single point, but rather a small arc, which is more complicated. Therefore the total time that tested in SOLIDWORKS will be a little larger than that calculated from the simplified simulation above if the swimmer pass the same distance. So, it is worth noting that conducting experiments in reality requires considering the turning process within the measurement range, even though that is a very small value. How to choose the appropriate radius as the turning path for swimmers is a question worth considering.

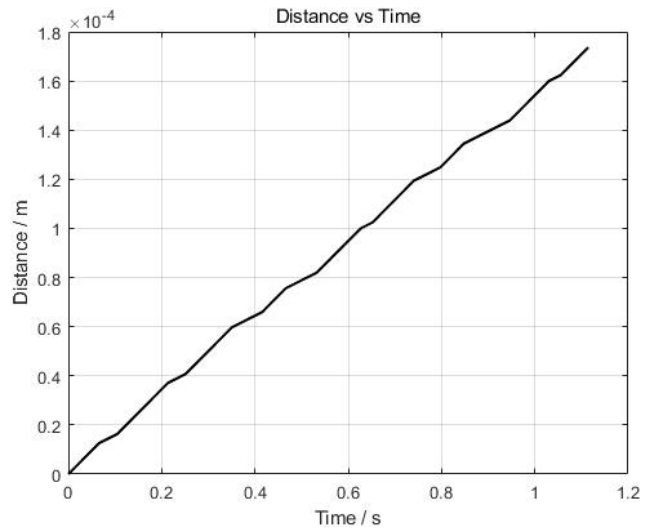


Fig. 7 The distance micro-swimmer passes vs time

In Fig.7, the curve segments with high slopes on the graph represent micro swimmers passing through the straight section at high speeds, while the line segments with low slopes represent them passing through the curve section at low speeds. This is a more realistic motion image, accompanied by some ups and downs, because the speed during turning is not the same as the straight line speed. It is a feasible method to control the speed of swimmers based on the speed frequency image and the speed and pitch angle image in the second part. By changing the frequency of the motor to achieve the appropriate speed and finding a suitable pitch angle so that the motor can operate at maximum efficiency for a longer period of time.

#### IV. DISCUSSION

Applying cellular methods to swimming robots is a highly innovative approach, which is an application of bionics. Its advantages lie in flexibility and simplicity. Since "Run" and "Tumble" is a general strategy for the swimmer's movement, only the rotation speed and direction of the swimmer's flagella need to be controlled to make it move in a two-dimensional plane. By changing physical parameters such as the size of the swimmer's body, the length of their flagella, and pitch angle,

their movement speed can be altered. Thus, for a fixed voltage, the required speed can be obtained by changing various physical parameters or by adjust the frequency of the motor if other parameters are already known. By using the above method, it is possible to achieve targeted adjustment of the speed of swimmers under different states of "Run" and "Tumble". In addition, In the subsequent simulation measurement process, "Three-dimensional Tracking" is an important method to test the quality of the fitting results, because it can not only display the 3-D view of the motion trajectory, but also display the projection view on three planes, which is more convenient to compare with the real cell trajectory, and observe whether the fitting results meet the expectations. Additionally, the program that can simulate the trajectory of a bacterial really help understanding random motion of the swimmer better because it can simulate the trajectory of the swimmer if running speed, steering angle and direction are provided.

However, the future is still full of challenges. Firstly, in the experiment of swimming, the actual operation is often more complex because the turning process cannot be ignored, even though it is an extremely small period of time. For example, in practical tasks, it is necessary to accurately calculate how to determine the turning radius, at what speed to turn, and the acceleration during the process of reducing from straight-line speed to turning speed and restoring to the original speed based on the target requirements. These are all worthy of deeper research and reflection. Secondly, how to control the three-dimensional motion of swimmers will be a more in-depth issue. At present, through the rotation of flagella, swimmers can move in two dimensions in a retrograde manner, but there is still a lack of dimension, which is the movement in height. The current feasible solution is to add a controllable valve that can control the inflow and outflow of water and change the weight of the swimmers themselves to achieve diving and floating. The principle is similar to that of a submarine. Another solution is to add another flagellum to control its movement in

another dimension. But these solutions will introduce new difficulties and challenges, such as how to accurately control the inflow and outflow of water and maintain body balance during this process? And where should the second flagella be installed? Will the eddy currents caused by its rotation interfere with the original plane motion? Therefore, further research is still needed.

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