2024 9th International Conference on Automation, Control and Robotics Engineering (CACRE) | 979-8-3503-5030-2/24/$31.00 ©2024 IEEE | DOI: 10.1109/CACRE62362.2024.10635080

Position stabilization control with specified performance of Lagurange systems under input saturations

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***Abstract*—Consider Lagrange systems with uncertain dynamics and unknown disturbances under input saturations in this paper. A disturbance observer is constructed to provide the estimate of the total disturbance lumped by uncertain dynamics and unknown disturbances of Lagrange systems. A new specified performance function with saturation characteristics (SPFSC) is developed to handle input saturations. Further, incorporating the developed SPFSC into a barrier function, based on the disturbance observer and the barrier function, a proportional-derivative controller with specified performance is proposed for position stabilization of Lagrange systems such that the position stabilization error settles within a specified tolerance error band in a specified settling time. Therein, the problem that the bandwidth of the controller approaches infinity when the time approaches the specified settling time is solved due to the SPFSC-incorporated barrier function being injected into the position stabilization controller as its bandwidth and properly choosing the parameters of the SPFSC. Simulation results on a three-degrees-of-freedom parallel robot show the effectiveness of the developed controller.**

Keywords*—nonlinear systems, uncertain dynamics, unknown disturbances, input saturations, position stabilization control, specified performance

I. INTRODUCTION

The settling time and the steady-state error for the position stabilization control of nonlinear systems are important control performance index. For the settling time, finite-time control method can guarantee that the position stabilization error of Lagrange system converges in a finite time which depends on the initial conditions of Lagrange system and the design parameters of the controllers [1-2]. Further, to solve the above problem, the fixed-time control method is proposed where the finite time is independent of the initial conditions of Lagrange system, but the design parameters of the controllers [3-4]. However the finite-time and the fixed-time control methods cannot preset the settling time of nonlinear systems. Fortunately, the specified-time control method is recently developed, which can set the settling time of Lagrange system in advance, while being independent of initial conditions of Lagrange system and the design parameters of the controller [5-8].

For the trajectory tracking control of multiagent systems, based on a time scaling function whose value approaches infinity when the time reaches a specified time, [5] developed a proportional controller whose gain is the time scaling function such that the trajectory tracking error converges to zero in the specified time where, however, the high gain of controller may make the system unstable. For the trajectory tracking control of a class of dynamical systems, [6-7] designed a sliding mode controller whose sliding surface is an exponential function of system states such that the specified-time trajectory tracking is achieved, which can be only sued for the sliding mode controller design. For a trajectory tracking control of a class of non-strict feedback nonlinear systems, based on a fractional power function of time, [8] used a specified performance control method and backstepping technique to design a finite-time adaptive tracking controller to achieve the predefined-time trajectory tracking. However, the predefined time relies on the parameters of the fractional power function.

On the other hand, for the trajectory tracking control of Lagrange system with unknown nonlinearities, Bechlioulis et al. [9] proposed a specified performance control method to design a robust adaptive controller using neural network, which firstly guarantees the trajectory tracking error converges to a specified tolerance error band in a finite time. In practice, input saturations can degrade the position stabilization control performance of Lagrange system and its stability. For the trajectory tracking control problem for a 6-degrees-of-freedom spacecraft rendezvous and docking operations under input saturations, [10] developed an adaptive anti-saturation appointed-time specified performance function (APPF) to design coordinated controller, which achieved the appointed-time specified performance trajectory tracking. For trajectory tracking control problem for an unmanned underwater vehicle, [11] designed a new APPF and use backstepping technique to design a H_∞ robust control strategy, which achieved the fixed-time prescribed performance trajectory tracking.

Inspired by the above discussions, this paper proposes a specified-performance position stabilization controller for Lagrange system with uncertain dynamics and unknown disturbances under input saturations. A disturbance observer is constructed to handle the total disturbance lumped by uncertain dynamics and unknown disturbances of Lagrange systems. A specified performance function with saturation characteristics (SPFSC) is developed to handle input saturations. A barrier function is introduced. Based on the disturbance observer and the barrier function on this SPFSC, A position stabilization controller with specified performance for Lagrange system is proposed, which can guarantee that the position stabilization error of Lagrange system settles within the specified tolerance error band in a specified settling time. Compared with [7], the bandwidth of the controller is bounded due to the introduced barrier function being injected into the position stabilization controller as its bandwidth and properly choosing the parameters of the SPFSC, according to the control performance requirements and the actuating capabilities of Lagrange system.

II. Problem Formulation And Preliminaries

A. Problem Formulation

Consider a three degrees-of-freedom Lagrange system

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q})=\boldsymbol{\tau}+\boldsymbol{\tau}_w \tag{1}$$

where $\boldsymbol{q}=[q_1,q_2,q_3]^{\mathrm{T}}$ is a state vector; $\boldsymbol{\tau}=[\tau_1,\tau_2,\tau_3]^{\mathrm{T}}$ is the control input vector; $\boldsymbol{\tau}_w=[\tau_{w1},\tau_{w2},\tau_{w3}]^{\mathrm{T}}$ is the disturbance vector; $\boldsymbol{M}(\boldsymbol{q})\in R^{3\times 3}$, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\in R^{3\times 3}$ and $\boldsymbol{G}(\boldsymbol{q})\in R^3$ are the parameter matrix or vector.

Introducing the state vectors $\boldsymbol{q}_1=\boldsymbol{q}$ and $\boldsymbol{q}_2=\dot{\boldsymbol{q}}$, and the output vector $\boldsymbol{y}=\boldsymbol{q}_1$, we can rewrite (1) as follows

$$\begin{cases}\dot{\boldsymbol{q}}_1=\boldsymbol{q}_2\\ \dot{\boldsymbol{q}}_2=\boldsymbol{d}+\boldsymbol{M}^{-1}(\boldsymbol{q}_1)\boldsymbol{\tau}\end{cases} \tag{2}$$

where $\boldsymbol{d}=[d_1,d_2,d_3]^{\mathrm{T}}=\boldsymbol{M}_0^{-1}(\boldsymbol{q}_1)[-\boldsymbol{C}(\boldsymbol{q}_1,\boldsymbol{q}_2)\boldsymbol{q}_2-\boldsymbol{G}(\boldsymbol{q}_1)+\boldsymbol{\tau}_w]$ denotes the total disturbances lumped by uncertain dynamics of the gangway and disturbance force and moments vector $\boldsymbol{\tau}_w$.

Assumption 1: $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ and $\boldsymbol{G}(\boldsymbol{q})$ are uncertain. τ_{wi}, $i=1,2,3$ and its first derivative are bounded.

The control objective of this paper is to propose a position stabilization control law with specified performance of Lagrange system with uncertain dynamics and unknown disturbances under Assumption 1 such that the position stabilization error can settle within a specified tolerance error band in a specified settling time.

B. Prelimilaries

Lemma 1 [12]: Suppose $\boldsymbol{A}=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\0&0&0&\cdots&0\end{bmatrix}_{n\times n}$, $\boldsymbol{C}=[1,0,\cdots 0]\in R^{1\times n}$ and $\boldsymbol{H}=\mathrm{diag}(n,...2,1)$ being a diagonal matrix. For any constant α_1, there exist a vector $\boldsymbol{L}\in R^n$ and a positive definite matrix $\boldsymbol{P}\in R^{n\times n}$ such that

$$\begin{cases}(\boldsymbol{A}-\boldsymbol{LC})\boldsymbol{P}+\boldsymbol{P}(\boldsymbol{A}-\boldsymbol{LC})\le \boldsymbol{A}^{\mathrm{T}}+\boldsymbol{A}-\alpha_1\boldsymbol{I}_{n\times n}\\ \alpha_2\boldsymbol{I}_{n\times n}\le \boldsymbol{PH}+\boldsymbol{HP}\le\alpha_3\boldsymbol{I}_{n\times n}\end{cases} \tag{3}$$

where $\boldsymbol{A}-\boldsymbol{LC}$ is Hurwitz; α_2 and α_3 are positive constants.

Specified performance function: A specified performance function is developed as follows

$$k(t)=\begin{cases}(a_0-a_\infty)(\frac{t_d-t}{t_d})^{\frac{r}{1-r}}+a_\infty & t<t_d\\ a_\infty & t\ge t_d\end{cases} \tag{4}$$

$$r=\vartheta_1-\frac{\vartheta_2(F-F_c)^2}{F^2+1} \tag{5}$$

$$F_c=\begin{cases}F_{\max}, & F>F_{\max}\\ F, & F_{\min}\le F\le F_{\max}\\ F_{\min}, & F<F_{\min}\end{cases} \tag{6}$$

where a_0, ϑ_1, ϑ_2 and a_∞ are positive constants; t_d is the specified time for error signals to settle within a specified tolerance error band $(-a_\infty,a_\infty)$; F is the control force or moment, and $F_{\max}$ and $F_{\min}$ are the maximum and the minimum control forces or moments, respectively.

III. Creation of Position stabilization Control Law with Specified Performance

A. Constrcution of disturbance observer

we construct the following disturbance observer [13]

$$\begin{cases}\hat{\boldsymbol{d}}=\bar{\boldsymbol{q}}+\boldsymbol{KMq}_2\\ \dot{\bar{\boldsymbol{q}}}=-\boldsymbol{K}\bar{\boldsymbol{q}}-\boldsymbol{K}(\boldsymbol{\tau}+\boldsymbol{KMq}_2)\end{cases} \tag{7}$$

where $\hat{\boldsymbol{d}}\in R^3$ is the estimate of $\boldsymbol{d}$, $\bar{\boldsymbol{q}}\in R^3$ is the auxiliary state vector of the disturbance observer, and $\boldsymbol{K}\in R^3$ is a positive definite design matrix.

Define the disturbance estimation error vector $\tilde{\boldsymbol{d}}\in R^3$ and the stability of the disturbance observer will be discussed later

$$\tilde{\boldsymbol{d}}=\hat{\boldsymbol{d}}-\boldsymbol{d} \tag{8}$$

B. Position stabilization control law design

Defining the position stabilization error $\varepsilon_{1i}=q_{di}-q_{1i}$, $i=1,2,3$ and the velocity error $\varepsilon_{2i}=\dot{q}_{di}-q_{2i}$ and letting $\boldsymbol{\varepsilon}_i=[\varepsilon_{1i},\varepsilon_{2i}]^{\mathrm{T}}$, we introduce a barrier function as follows

$$\rho_i(\varepsilon_{1i})=\frac{k_i^2}{k_i^2-\varepsilon_{1i}^2} \tag{9}$$

where k_i is the specified performance function (4).

Based on the disturbance observer and the introduced barrier function, we propose a position stabilization controller with specified performance for Lagrange systems as follows

$$F = M(q_1)\left[\beta_1(q_d - q_1) + \beta_2(\dot{q}_d - q_2) + \ddot{q}_d\right] - \hat{d} \tag{10}$$

where β_1 and β_2 are the controller gain matrices.

Substituting (10) into (2), we obtain the following position stabilization error dynamic equation

$$\begin{aligned}\dot{\varepsilon}_i &= \varphi_i'\varepsilon_i - B_i L_i'\varepsilon_i + \Xi_i \\ &= \bar{\varphi}_i'\varepsilon_i + \Xi_i \end{aligned} \tag{11}$$

where $\bar{\varphi}_i' = \varphi_i' - B_i L_i'$, $A_i' = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_i = [0,1]^{\mathrm{T}}$, $L_i' = [\beta_1, \beta_2]$ and $\Xi_i = \left(\Phi_i \tilde{d}\right)^{\mathrm{T}}$ with $\Phi_i = \begin{bmatrix} 0 & 0 & 0 \\ -2 & -1 & -1 \end{bmatrix}$.

Using the pole placement method, we place all the characteristic roots of $\bar{\varphi}_i'$ in (11) at the desired positions $-\Delta_i\omega_i$ with $\omega_i = \rho_i(\varepsilon_{1i})$ being the bandwidth of the controller and Δ_i being a positive design constant, and then $\bar{\varphi}_i'$ is Hurwitz. Let

$$\det(sI_{2\times 2} - \bar{\varphi}_i') = (s + \Delta_i\omega_i)^2 \tag{12}$$

According to (12), we have $L_i' = [\beta_{1i}, \beta_{2i}]$ with $\beta_{1i} = \gamma_i^2\omega_i^2$ and $\beta_{2i} = 2\gamma_i\omega_i$.

Let $E_i = \Psi_i\varepsilon_i \in R^2$ with $\Psi_i = \mathrm{diag}(\Delta_i\omega_i^2, \omega_i)$. Taking the time derivative of E_i, we get

$$\dot{E}_i = \dot{\Psi}_i\varepsilon_i + \Psi_i\dot{\varepsilon}_i \tag{13}$$

where $\dot{\Psi}_i = \dot{\omega}_i D_i H_i$ with $D_i = \mathrm{diag}(\Delta_i\omega_i, 1)$ and $H_i = \mathrm{diag}(2,1)$.

Substituting (13) and $E_i = \Psi_i\varepsilon_i$ into (11), we get

$$\begin{aligned}\dot{E}_i &= \dot{\Psi}_i\Psi_i^{-1}E_i + \Psi_i(\varphi_i' - B_i L_i')\Psi_i^{-1}E_i + \Psi_i\Xi_i \\ &= \dot{\omega}_i D_i H_i \Psi^{-1}E_i + \Psi_i\varphi_i'\Psi_i^{-1}E_i - \Psi_i B_i L_i'\Psi_i^{-1}E_i + \Psi_i\Xi_i \\ &= \frac{\dot{\omega}_i}{\omega_i}H_i E_i + \omega_i\Delta_i\varphi_i' E_i - \omega_i\Delta_i B_i L_i E_i + \Psi_i\Xi_i \\ &= \omega_i\chi_i' H_i E_i + \omega_i\Delta_i\bar{\varphi}_i E_i + \Psi_i\Xi_i \end{aligned} \tag{14}$$

where $\chi_i' = \dfrac{\dot{\omega}_i}{\omega_i^2}$, $L_i = [1,2]$, and $\bar{\varphi}_i = \varphi_i' - B_i L_i = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$. we know that $\bar{\varphi}_i$ is Hurwitz. Then $\chi_i' = \dfrac{-2\dot{k}_i\varepsilon_{1i}^2 + 2k_i\varepsilon_{1i}\dot{\varepsilon}_{1i}}{k_i^3}$ due to $\omega_i = \rho_i(\varepsilon_{1i})$.

According to Lemma 1 and $\bar{\varphi}_i$ being Hurwitz, there is a positive definite matrix $P_i = \begin{bmatrix} p_{11i} & p_{12i} \\ p_{21i} & p_{22i} \end{bmatrix}$ satisfying the following equation

$$\begin{cases} \bar{\varphi}_i^{\mathrm{T}} P_i + P_i\bar{\varphi}_i \le \bar{\varphi}_i'^{\mathrm{T}} + \bar{\varphi}_i' - \alpha_{1i}I_{2\times 2} \\ \alpha_{2i}I_{2\times 2} \le P_i H_i + H_i P_i \le \alpha_{3i}I_{2\times 2} \end{cases} \tag{15}$$

where α_{1i} can be any constant and α_{2i} and α_{3i} are positive constants.

Select the Lyapunov function candidate for (8) and (14) as

$$V_i = E_i^{\mathrm{T}} P_i E_i + \frac{1}{2}\tilde{d}^{\mathrm{T}}\tilde{d} \tag{16}$$

Taking the time derivative of (16), from (7), (14) and (15), we have

$$\begin{aligned}\dot{V}_i &= \dot{E}_i^{\mathrm{T}} P_i E_i + E_i^{\mathrm{T}} P_i \dot{E}_i + \tilde{d}^{\mathrm{T}}\dot{\tilde{d}} \\ &= (\omega_i\chi_i' H_i E_i + \Delta_i\omega_i\bar{\varphi}_i E_i + \Psi_i\Xi_i)^{\mathrm{T}} P_i E_i \\ &\quad + E_i^{\mathrm{T}} P_i(\omega_i\chi_i' H_i E_i + \Delta_i\omega_i\bar{\varphi}_i E_i + \Psi_i\Xi_i) + \tilde{d}^{\mathrm{T}}\dot{\tilde{d}} \\ &= \omega_i\chi_i' E_i^{\mathrm{T}}(P_i H_i + H_i P_i)E_i + E_i^{\mathrm{T}} P_i\Psi_i\Xi_i \\ &\quad + \Delta_i\omega_i E_i^{\mathrm{T}}(\bar{\varphi}_i P_i + P_i\bar{\varphi}_i)E_i + \Xi_i^{\mathrm{T}}\Psi_i^{\mathrm{T}} P_i E_i + \tilde{d}^{\mathrm{T}}\dot{\tilde{d}} \\ &\le \omega_i\chi_i E_i^{\mathrm{T}}(P_i H_i + H_i P_i)E_i - \Delta_i\omega_i(\alpha_{1i} - 2\|\bar{\varphi}_i'\|)\|E_i\|^2 \\ &\quad + \Xi_i^{\mathrm{T}}\Psi_i^{\mathrm{T}} P_i E_i + E_i^{\mathrm{T}} P_i\Psi_i\Xi_i + \tilde{d}^{\mathrm{T}}\dot{\tilde{d}} \\ &\le -\left[\Delta_i\omega_i(\alpha_{1i} - 2\|\bar{\varphi}_i'\|) - \omega_i\chi_i\alpha_{2i}\right]\|E_i\|^2 \\ &\quad + \Xi_i^{\mathrm{T}}\Psi_i^{\mathrm{T}} P_i E_i + E_i^{\mathrm{T}} P_i\Psi_i\Xi_i + \tilde{d}^{\mathrm{T}}\dot{\tilde{d}} \\ &\le -\left[\Delta_i(\alpha_{1i} - 2\|\bar{\varphi}_i'\|) - \chi_i\alpha_{2i}\right]\omega_i\|\tilde{d}\|^2 + \tilde{d}^{\mathrm{T}}\dot{\tilde{d}} \\ &\quad + \omega_i(2|p_{21i}|\|\tilde{d}\| + 2|p_{22i}|\|\tilde{d}\|)\|\Phi_i\|\|\varepsilon_i\| \\ &\le -\left[\Delta_i(\alpha_{1i} - 2\|\bar{\varphi}_i'\|) - \chi_i\alpha_{2i}\right]\omega_i\|E_i\|^2 + 6\omega_i\bar{p}_i\|\tilde{d}\|\|\varepsilon_i\| + \tilde{d}^{\mathrm{T}}\dot{\tilde{d}} \end{aligned}$$

where $\chi_i = \sup_{t\in[0,+\infty)}|\chi_i'|$ and $\bar{p}_i = \max\{|p_{21i}|, |p_{22i}|\}$.

According to Young's inequality and Assumption 1, we have

$$\begin{aligned}\dot{V}_i &\le -\left[\Delta_i(\alpha_{1i} - 2\|\bar{\varphi}_i'\|) - \chi_i\alpha_{2i}\right]\omega_i\|E_i\|^2 \\ &\quad + 3\omega_i\bar{p}_i^2\|\tilde{d}\|^2 + 3\omega_i\|\tilde{d}\|^2 + \tilde{d}^{\mathrm{T}}(-K\tilde{d} - \dot{d}) \\ &\le -\nabla_i\omega_i\|E_i\|^2 + 3\omega_i\bar{p}_i^2\|\tilde{d}\|^2 - \left[\lambda_{\min}(K) - \frac{1}{2}\right]\tilde{d}^{\mathrm{T}}\tilde{d} + \frac{1}{2}\dot{d}^2 \\ &\le -\mu_i V_i + C_i \end{aligned} \tag{17}$$

where $\mu_i = \min\{\phi_i\omega_i/\|P_i\|, 2\lambda_{\min}(K) - 6\omega_i\bar{p}_i^2 - 1\}$ with $\nabla_i = \Delta_i(\alpha_{1i} - 2\|\bar{\varphi}_i'\|) - \chi_i\alpha_{3i} - 3$ and $C_i = \frac{1}{2}\dot{d}^2$.

By properly choosing $\alpha_{1i} > 2\|\bar{\boldsymbol{\varphi}}_i'\|+1$ and $\Delta_i \ge 3+\psi_i\alpha_{3i}$, we have $\mu_i > 0$. Therefore, integrating the two sides of (17) yields

$$
\begin{aligned}
0 \le V_i &\le \varpi_i + \int_0^t \exp\left(\int_\tau^t \mu_i(s)ds\right) C_i d\tau \\
&\le \varpi_i + 3\bar{p}_i^2\zeta_i^2 \int_0^t \exp\left(-\nabla_i \int_0^t \omega_i(s)ds + \nabla_i \int_0^\tau \omega_i(s)ds\right)\omega_i d\tau \\
&\le \varpi_i + 3\bar{p}_i^2\zeta_i^2 \exp\left(-\int_0^t \mu_i(s)ds\right) \\
&\int_0^t \exp\left(\nabla_i \int_0^\tau \omega_i(s)ds\right) d\left(\int_0^\tau \omega_i(s)ds\right) \\
&\le \varpi_i + \frac{3\bar{p}_i^2\zeta_i^2}{\phi_i} \exp\left(-\int_0^t \mu_i(s)ds\right)\left[\exp\left(\nabla_i \int_0^t \omega_i(s)ds\right)-1\right] \\
&\le \varpi_i + \frac{3\bar{p}_i^2\zeta_i^2}{\nabla_i}\left[1-\exp\left(-\int_0^t \mu_i(s)ds\right)\right] \\
&\le \varpi_i + \frac{3\bar{p}_i^2\zeta_i^2}{\nabla_i}
\end{aligned} \tag{18}
$$

where $\varpi_i = \exp\left(-\int_0^t \mu_i(\tau)d\tau\right)V_i(0)$.

Furthermore, from $\boldsymbol{E}_i = \boldsymbol{\Psi}_i\boldsymbol{\varepsilon}_i$ and (18), we know

$$
\begin{aligned}
\|\boldsymbol{\varepsilon}_i\|^2 &= \|\boldsymbol{\Psi}_i^{-1}\boldsymbol{E}_i\|^2 \\
&\le \|\boldsymbol{\Psi}_i^{-1}\|^2 \|\boldsymbol{E}_i\|^2 \\
&\le \frac{\|\boldsymbol{\Psi}_i^{-1}\|^2}{\lambda_{\min}(\boldsymbol{P}_i)}\left[\varpi_i + \frac{3\bar{p}_i^2\zeta_i^2}{\nabla_i}\right]
\end{aligned} \tag{19}
$$

If

$$
\frac{\|\boldsymbol{\Psi}_i^{-1}\|^2}{\lambda_{\min}(\boldsymbol{P}_i)}\left[\varpi_i + \frac{3\bar{p}_i^2\zeta_i^2}{\nabla_i}\right] < k_i^2 \tag{20}
$$

then $|\varepsilon_{1i}| < k_i$ and further $|\varepsilon_{1i}| < a_{\infty i},\ \forall t \ge t_{di}$ from (4). Further, according to $\omega_i = \rho_i(\varepsilon_{1i})$, there is $\omega_i \ge \sigma_{1i}$, whereby we get from (19)

$$
\Delta_i > \sqrt{\frac{V_i(0)+3\bar{p}_i^2\zeta_i^2/\nabla_i}{\lambda_{\min}(\boldsymbol{P}_i)a_{\infty i}^2}} \tag{21}
$$

Therefore, if the parameters Δ_i satisfying (21), the position stabilization error ε_{1i} settles within $(-k_i, k_i)$, and then enters and no longer exceeds the specified tolerance error band $(-a_{\infty i}, a_{\infty i})$ in the specified settling time t_{di}.

The previous facts prove the following theorem.

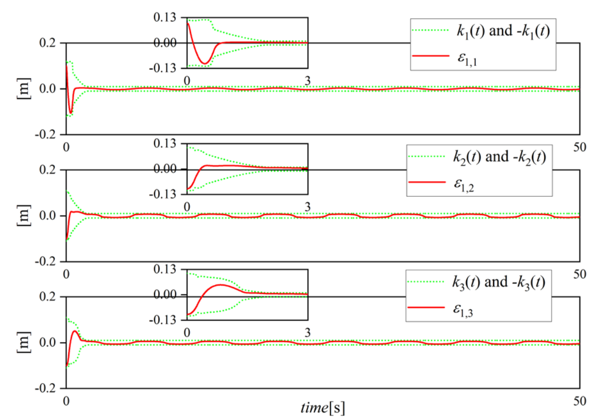
Theorem 1: Consider a closed-loop system consisting of a three degrees-of-freedom Lagrange system (1) with uncertain dynamics and unknown disturbances under input saturations and Assumption 1, and the position stabilization controller (10). By appropriately selecting the design constants Δ_i satisfying $\Delta_i \ge 3+\psi_i\alpha_{3i}$ and $\Delta_i > \sqrt{\frac{V_i(0)+3\bar{p}_i^2\zeta_i^2/\nabla_i}{\lambda_{\min}(\boldsymbol{P}_i)a_{\infty i}^2}}$, the position stabilization error ε_{1i} settles within $(-k_i, k_i)$, and then enters and no longer exceeds the specified tolerance error band $(-a_{\infty i}, a_{\infty i})$ in the specified settling time t_{di}.

IV. Simulations

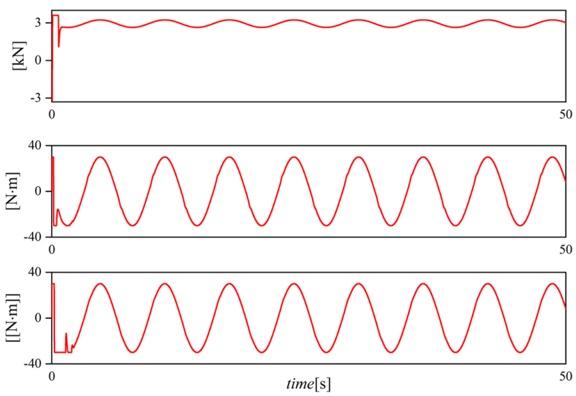
In this section, to illustrate the proposed position stabilization control law, we consider the stabilization of a three degrees-of-freedom parallel robot [14] whose mathematical model parameters are given in [14] in details.

A. Simulations under our proposed position stabilization control

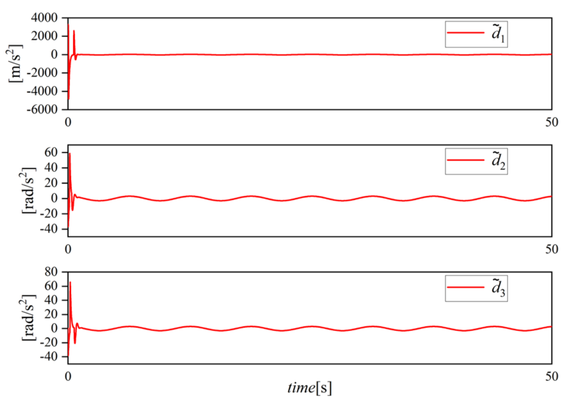
The parameters of the SPFSC are chosen as follows: $a_{0i} = 1.2$, $a_{\infty i} = 0.01$, $\vartheta_{1i} = 0.6$, $\vartheta_{2i} = 0.5$, $F_{\max 1} = 3500\text{N}$, $F_{\min 1} = -3500\text{N}$, $F_{\max 2} = 30\text{N}\cdot\text{m}$, $F_{\min 2} = -30\text{N}\cdot\text{m}$, $F_{\max 3} = 30\text{N}\cdot\text{m}$, $F_{\min 3} = 30\text{N}\cdot\text{m}$ and $t_{di} = 2$ s. The parameters of the disturbance observer are chosen as follows: $\boldsymbol{K} = \text{diag}(10,10,10)$. The parameters of controller are chosen as follows: $\Delta_i = 15$. The initial state of position and orientation vector of Lagrange system $\boldsymbol{q}$ is set as $\boldsymbol{q}(0) = [0.1\text{ m},\ -0.1\text{ rad},\ -0.1\text{ rad}]^{\text{T}}$, the desired state of position and orientation vector of Lagrange system $\boldsymbol{q}$ is $\boldsymbol{q}_d = [0\text{ m},\ 0\text{ rad},\ 0\text{ rad}]^{\text{T}}$ and the unknown disturbance is set as $\boldsymbol{\tau}_w = [100\sin(t)\text{ N},\ 30\sin(t)\text{ N}\cdot\text{m},\ 30\sin(t)\text{ N}\cdot\text{m}]$.



(a)



(b)



(c)

Fig. 1 The simulation results. (a) Position stabilization errors under F. (b) Control inputs. (c) The total disturbance estimation errors.

The simulation results are depicted in Fig. 1(a)-(c) using solid lines. It can be seen from Fig. 1(a) that the position stabilization error settles within a specified tolerance error band $(-0.01, 0.01)$ in a specified settling time $t_{di} = 2$ s, as proved in Theorem 1, which means the position stabilization control of Lagrange system satisfies the specified performance. Fig. 1(b) shows that the control inputs are reasonable. Fig. 1(c) shows that the disturbance observer can precisely estimate the total disturbance.

V. Conclusions

In this paper, a disturbance observer has been constructed to provide the total disturbance lumped by uncertain dynamics and unknown disturbances of Lagrange systems. a new SPFSC has been developed to handle input saturations and a barrier function on this SPFSC has been developed, based on which the position stabilization controller with specified performance for Lagrange system with uncertain dynamics and unknown disturbances under input saturations has been proposed. The problem that the bandwidth of the controller approaches infinity when the time approaches the specified settling time has been solved, and the bandwidth of the proposed controller can be guaranteed to be bounded due to the SPFSC-incorporated barrier function being injected into the position stabilization controller as its bandwidth and properly choosing the parameters of the SPFSC. The proposed controller can achieve that the position stabilization error settles within a specified tolerance error in a specified settling time. Theoretical analyses and simulation results have proved the effectiveness of our proposed position stabilization controller.

Acknowledgment

This work is supported by the Nature Natural Science Foundation of China (52371362), and partly by the Fundamental Research Funds for the Central Universities (3132023510).

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