Manipulation Modeling and Control under Pure Rolling Contact Constraints

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***Abstract*—A novel control method is proposed for a dexterous robotic hand in pure rolling contact with an object. For dynamics modeling, the extended Udwadia-Kalaba equation is applied to formulate the motion equation of the robotic hand (which inertia matrix is singular due to the pure rolling constraints) without any auxiliary variables. A control strategy based on the formulated contact forces model is introduced to tackle with the trajectory tracking issues of the robotic hand from the constraint-following perspective. Both the theoretical proof and the simulations in the three-dimensional workspace could verify the effectiveness of the proposed control method.**

Keywords: dexterous robotic hand; pure rolling contact; dynamic manipulation control; Udwadia-Kalaba equation

I. INTRODUCTION

As a kind of flexible end-effector mechanism with a high degree of freedom, a dexterous robotic hand could accomplish various complex and precise tasks. Those tasks can be divided into two groups: "grasping" and "manipulation". The former one requires fingers (might include palm) to meet the form or force closure with the target object to ensure stable grasping. The latter one manipulates the object by moving the finger joints, which is based on the grasp of the object. In comparison to the conventional grasping tasks, the robotic hand manipulation covers a broader range of contact types, and each of these distinct contact types exerts different effects on the object manipulation task (the variation in contact forces exerted by the hand on the object, intuitively). Furthermore, the differing contact forces increase the difficulty of the manipulation control design. Hence, how to realize the high-precision dynamic manipulation control of the dexterous robotic hand under differing contact types becomes the focus.

The study we did previously in [1] discusses the dynamic control design when the multi-fingered hand robot is in fixed contact with the target object. However, the non-fixed contacts also can be found in those manipulation tasks that aim to adjust the position of the target object [2]. The most common non-fixed contacts involve pure rolling contact, twist rolling contact, and slide rolling contact. When the robotic hand maintains slide rolling contact while manipulating the target, there will be the relative velocity at the contact point, which may impact the accuracy of the object manipulation. Hence, slide rolling contact is always needing to be avoided during the manipulation. When there is a twist rolling contact between the robotic hand and the object, the kinematic analysis of this contact type can be simplified based on the pure rolling contact. Therefore, we consider discussing the dynamic control method of the dexterous robotic hand manipulation system which incorporates pure rolling contact constraints.

The hand-object manipulation system's inertial matrix is singular caused by the pure rolling constraints. Therefore, to formulate the system dynamic model, the extended Udwadia-Kalaba equation mentioned in [3] is used in this paper. The fingertip contact forces model in the joint space is further derived from the dynamic model. Then, the contact forces model-based manipulation control strategy is formulated form the constraint-following perspective. This control methodology discussed in [4][5] treats the control target as servo constraints and proposes the control strategy using the servo constrained forces. The theoretical analysis guaranteed the proposed control strategy's effectiveness, and further simulation results demonstrate that the dexterous robotic hand can perform precise manipulation with object under the proposed control.

II. UDWADIA-KALABA THEORY

In a mechanical system consists of particles and rigid bodies, the system's configuration can be described by a set of generalized coordinates $x \in \boldsymbol{R}^n$. The corresponding generalized velocities and accelerations are denoted as $\dot{x} \in \boldsymbol{R}^n$ and $\ddot{x} \in \boldsymbol{R}^n$, respectively. By employing Lagrange equation, the unconstrained motion equation of the system can be represented as

$$\mathcal{M}(x,t)\ddot{x} = F(x,\dot{x},t) \tag{1.}$$

where $\mathcal{M}(x,t) = \mathcal{M}^T(x,t) \in \boldsymbol{R}^{n\times n}$ is the inertia matrix and $F(x,\dot{x},t) \in \boldsymbol{R}^n$ encompasses various effects include contributions from gravitational forces, externally applied forces, as well as Coriolis and centrifugal forces due to the system's motion.

Assume that the system is subjected to l Pfaffian constraints, where $l < n$, these constraints can be either holonomic or non-holonomic in nature

$$\sum_{s=1}^{n} A_{rs}(x,t)\dot{x}_s + c_r(x,t) = 0 \quad (r = 1,2,\cdots,l) \tag{2.}$$

where $A_{rs}(\cdot) \coloneqq \boldsymbol{R}^n \times \boldsymbol{R} \to \boldsymbol{R}$ and $c_r(\cdot) \coloneqq \boldsymbol{R}^n \times \boldsymbol{R} \to \boldsymbol{R}$ are both C^l. The l constraints can be combined and represented in a compact matrix form as

$$A(x,t)\dot{x} + c(x,t) = 0 \tag{3.}$$

where $A(x,t) \in \boldsymbol{R}^{l\times n}$ and $c(x,t) \in \boldsymbol{R}^{l}$. Differentiating (3) with respect to t yields

$$A(x,t)\ddot{x} = -\dot{c}(x,t) - \dot{A}(x,t)\dot{x} =: b(x,\dot{x},t) \quad (4.)$$

According to [3], there will be

$$\begin{bmatrix}\left(I - A^{+}(x,t)A(x,t)\right)\mathcal{M}(x,t) \\ A(x,t)\end{bmatrix}\ddot{x} = \begin{bmatrix}\left(I - A^{+}(x,t)A(x,t)\right)\left(F(x,\dot{x},t) + C(x,\dot{x},t)\right) \\ b(x,\dot{x},t)\end{bmatrix} \quad (5.)$$

where $C(x,\dot{x},t)$ is an n-vector explaining the properties of the non-ideal constraints, which can be determined by the mechanician and could be obtained by experimentation and/or observation according to [6].

$$F^{c}(x,\dot{x},t) = \mathcal{M}(x,t)\begin{bmatrix}\left(I - A^{+}(x,t)A(x,t)\right)\mathcal{M}(x,t) \\ A(x,t)\end{bmatrix}^{+}\begin{bmatrix}\left(I - A^{+}(x,t)A(x,t)\right)\left(F(x,\dot{x},t) + C(x,\dot{x},t)\right) \\ b(x,\dot{x},t)\end{bmatrix} - \left(F(x,\dot{x},t) + C(x,\dot{x},t)\right) \quad (6.)$$

The extended Udwadia-Kalaba equation remains applicable to the constrained mechanical systems, irrespective of the rank of the inertia matrix. Therefore, it can handle situations where the inertia matrices are singular, without imposing any restrictions. Based on the motion equation (5), the constrained force $F^{c}(x,\dot{x},t)$ can be expressed as (6).

The constrained force, obtained by the extended Udwadia-Kalaba equation, is expressed in an analytic form, and notably, it does not involve any auxiliary variables. When the inertia matrix $\mathcal{M}(x,t)$ in (6) to be non-singular, the constrained force $F^{c}(x,\dot{x},t)$ can be simplified as the one obtained by the Udwadia-Kalaba equation in [7]. (The proof can be seen in [6]).

III. THE DYNAMIC MODEL OF HAND-OBJECT SYSTEM SUBJECT TO PURE-ROLLING-CONTACT CONSTRAINTS

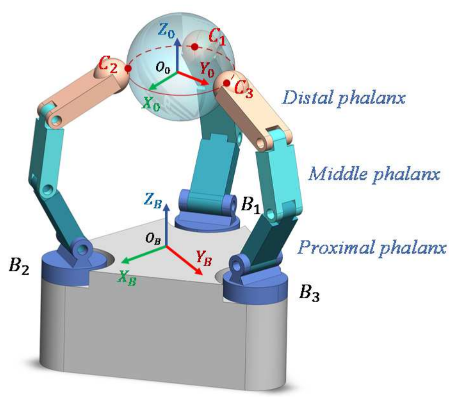

Figure 1. Dexterous robotic hand in contact with an object

Consider the scenario where a dexterous robotic hand in contact with an object (Fig. 1). The robotic hand consists of three identical fingers, all sharing the same structure with a distal phalanx, middle phalanx, and proximal phalanx (Fig. 2). Each finger of the robotic hand is equipped with three joints, all actuated by servo motors. Moreover, each finger possesses four degrees of freedom: the first joint, connected to the palm base, has two degrees of freedom, while the second and third joint each have one degree of freedom.

Let $\{O_B\}$ and $\{O_{Bi}\}$ be the primary coordinate frame of the robot and the i^{th} finger primary coordinate frame, without a relative rotation relationship between those two. Set $\{O_{i(j-1)}\}$ to be the coordinate frame fixed in the k^{th} knuckle and $\{O_{i0}\}$ is overlapped with $\{O_{Bi}\}$. Set $\{O_{fi}\}$ to be the fixed coordinate frame in the mass center of the i^{th} fingertip. Lastly, let $\{O_o\}$ denote the coordinate frame located at the mass center of the object. Define the parameters of the i^{th} finger as following:

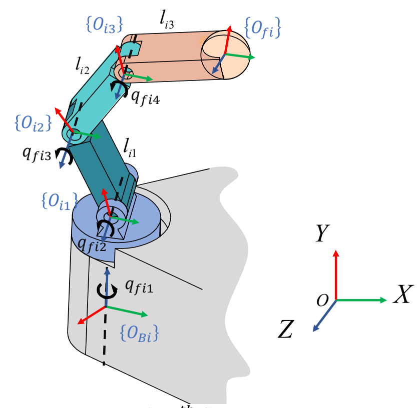

Figure 2. The i^{th} finger $(i = 1,2,3)$

q_{fij}: the j^{th} rotation angle ($j = 1,2,3,4$);
l_{ik}: the length of the k^{th} phalanx;
m_{ik}: the mass of the k^{th} phalanx;
I_{ik}: the inertia moment of the k^{th} phalanx.

A. The Pure Rolling Contact Constraints

To represent the positions of the points on the fingertips and the object surface, we establish the mappings: ${}^{fi}C_{fi}(\cdot): \boldsymbol{R}^2 \rightarrow \boldsymbol{R}^3$ and ${}^{o}C_{oi}(\cdot): \boldsymbol{R}^2 \rightarrow \boldsymbol{R}^3$. Suppose α_{fi}, $\alpha_{oi} \in \boldsymbol{R}^2$ are local parameters of the i^{th} fingertip surface and the object surface. Then, ${}^{fi}C_{fi}(\alpha_{fi})$ is a point on the surface of the i^{th} fingertip in $\{O_{fi}\}$ coordinate frame, and ${}^{o}C_{oi}(\alpha_{oi})$ is a point on the surface of the object in $\{O_o\}$ coordinate frame.

Choose $q_{fi} = [q_{fi1}, q_{fi2}, q_{fi3}, q_{fi4}, \alpha_{fi}]^T$ and $q_{oi} = [x_o, y_o, z_o, \phi_o, \theta_o, \psi_o, \alpha_o]^T$ as the generalized coordinates of the i^{th} finger and the target object. Where, $\{x_o, y_o, z_o\}$ are the positions of the object's mass center and $\{\phi_o, \theta_o, \psi_o\}$ are the object's Euler angles. Then, the positions of the i^{th} fingertip surface point $P_{f_{ci}}$ and the object surface point $P_{o_{ci}}$ in the primary coordinate frame $\{O_B\}$ can be presented as

$$P_{f_{ci}}(q_{fi}) = P_{fi}(q_{fi}) + R_{fi}(q_{fi})\,{}^{fi}C_{fi}(\alpha_{fi}) \quad (7.)$$

$$P_{o_{ci}}(q_{oi}) = P_o(q_{oi}) + R_o(q_{oi})\,{}^{o}C_{oi}(\alpha_{oi}) \quad (8.)$$

where $P_{fi}(q_{fi}) \in \boldsymbol{R}^3$ is the position vector of mass center of the i^{th} fingertip, and $P_o(q_{oi}) \in \boldsymbol{R}^3$ is the position vector of the object mass center in the primary coordinate frame $\{O_B\}$. $R_{fi}(q_{fi}) \in \boldsymbol{R}^{3\times3}$ is the rotation matrix of the coordinate frame $\{O_{fi}\}$ relative to the primary coordinate frame $\{O_B\}$, and $R_o(q_{oi}) \in \boldsymbol{R}^{3\times3}$ is the rotation matrice of the coordinate frame $\{O_o\}$ relative to the primary coordinate frame $\{O_B\}$.

To perform object manipulation, the robotic hand utilizes multiple fingers and is in point contact with the object (we solely consider the point contact situation). Suppose the i^{th} fingertip and object in contact at time t and set the contact points are $P_{f_{ci}}(q_{fi})$ and $P_{o_{ci}}(q_{oi})$, the positions of those two contact points will always equal to each other as

$$P_{f_{ci}}(q_{fi}) = P_{o_{ci}}(q_{oi}) \quad (9.)$$

When the fingertip is in pure rolling contact with the target object, there will be relative moving between the contact point and the surface it is located. However, there is no relative velocity between those two contact points, which means

$$R_{fi}(q_{fi})^{fi}\dot{C}_{fi}(\alpha_{fi}, \dot{\alpha}_{fi}) = R_o(q_{oi})^{o}\dot{C}_{oi}(\alpha_{oi}, \dot{\alpha}_{oi}) \quad (10.)$$

Besides, suppose the fingertips have continuous contact with the target object, which means the fingertips and the object will not separate or penetrate at the contact points. The contact considered in this paper is a rigid contact between the hemispherical fingertips and the spherical object. Hence, the relationship between those two outward unit normal vectors at the contact points should be

$$R_{fi}(q_{fi})^{fi}e_{fi}(\alpha_{fi}) = -R_o(q_{oi})^{o}e_{oi}(\alpha_{oi}) \quad (11.)$$

where ${}^{fi}e_{fi}(\alpha_{fi})$ is the outward unit mormal vector of the contact point $P_{f_{ci}}(q_{fi}, \alpha_{fi})$ in coordinate frame $\{O_{fi}\}$, ${}^{o}e_{oi}(\alpha_{oi})$ is the outward unit mormal vector of the contact point $P_{o_{ci}}(q_{oi}, \alpha_{oi})$ in coordinate frame $\{O_o\}$.

Take the derivative of (10) and yields

$$\dot{P}_{f_{ci}}(q_{fi}, \dot{q}_{fi}) + \dot{R}_{fi}(q_{fi}, \dot{q}_{fi})^{fi}C_{fi}(\alpha_{fi}) + R_{fi}(q_{fi})^{fi}\dot{C}_{fi}(\alpha_{fi}, \dot{\alpha}_{fi})$$
$$= \dot{P}_o(q_{oi}, \dot{q}_{oi}) + \dot{R}_o(q_{oi}, \dot{q}_{oi})^{o}C_{oi}(\alpha_{oi}) + R_o(q_{oi})^{o}\dot{C}_{oi}(\alpha_{oi}, \dot{\alpha}_{oi}) \quad (12.)$$

where

$$\dot{R}_{fi}(q_{fi}, \dot{q}_{fi}) = (\omega_{fi}\times)R_{fi}(q_{fi})$$
$$= \begin{bmatrix} 0 & -\omega_{fi3} & \omega_{fi2} \\ \omega_{fi3} & 0 & -\omega_{fi1} \\ -\omega_{fi2} & \omega_{fi1} & 0 \end{bmatrix} R_{fi}(q_{fi}, t) \quad (13.)$$

$$\dot{R}_o(q_{oi}, \dot{q}_{oi}) = (\omega_{oi}\times)R_o(q_{oi})$$
$$= \begin{bmatrix} 0 & -\omega_{o3} & \omega_{o2} \\ \omega_{o3} & 0 & -\omega_{o1} \\ -\omega_{o2} & \omega_{o1} & 0 \end{bmatrix} R_o(q_{oi}, t) \quad (14.)$$

Here, $\omega_{fi} = [\omega_{f1}, \omega_{f2}, \omega_{f3}]^T$ is the angular velocity of the $\{O_{fi}\}$ coordinate frame, $\omega_{oi} = [\omega_{o1}, \omega_{o2}, \omega_{o3}]^T$ is the angular velocity of the $\{O_o\}$ coordinate frame.

Differentiating (11) to get

$$(\omega_{fi}\times)R_{fi}(q_{fi})^{fi}e_{fi}(\alpha_{fi}) + R_{fi}(q_{fi})^{fi}\dot{e}_{fi}(\alpha_{fi}, \dot{\alpha}_{fi}) +$$
$$(\omega_{oi}\times)R_o(q_{oi})^{o}e_{oi}(\alpha_{oi}) + R_o(q_{oi})^{o}\dot{e}_{oi}(\alpha_{oi}, \dot{\alpha}_{oi}) = 0 \quad (15.)$$

Combining (12), (10), and (15), the pure rolling contact constraints can be presented as

$$\begin{bmatrix} I_{3\times3} & -(R_{fi}{}^{fi}C_{fi}\times) & R_{fi}(\partial^{fi}C_{fi}/\partial\alpha_{fi}) \\ 0_{3\times3} & 0_{3\times3} & R_{fi}(\partial^{fi}C_{fi}/\partial\alpha_{fi}) \\ 0_{3\times3} & -(R_{fi}{}^{fi}e_{fi}\times) & R_{fi}(\partial^{fi}e_{fi}/\partial\alpha_{fi}) \end{bmatrix} \begin{bmatrix} \dot{P}_{fi} \\ \omega_{fi} \\ \alpha_{fi} \end{bmatrix} +$$

$$\begin{bmatrix} -I_{3\times3} & R_o{}^{o}C_{oi}\times & -R_o(\partial^{o}C_{oi}/\partial\alpha_{oi}) \\ 0_{3\times3} & 0_{3\times3} & -R_o(\partial^{o}C_{oi}/\partial\alpha_{oi}) \\ 0_{3\times3} & -(R_o{}^{o}C_{oi}\times) & R_o(\partial^{o}e_{oi}/\partial\alpha_{oi}) \end{bmatrix} \begin{bmatrix} \dot{P}_o \\ \omega_o \\ \alpha_o \end{bmatrix} = 0 \quad (16.)$$

Using the coordinates q_{fi} and q_{oi}, (16) can be represented in a matrix form as

$$[A_{fi} \quad A_{0i}] \begin{bmatrix} \dot{q}_{fi} \\ \dot{q}_{0i} \end{bmatrix} = 0 \quad (17.)$$

where

$$A_{fi} = \begin{bmatrix} D_{fi}J_{fi} & 0_{3\times2} \\ 0_{3\times4} & R_{fi}(\partial^{fi}C_{fi}/\partial\alpha_{fi}) \\ E_{fi}J_{fi} & R_{fi}(\partial^{fi}e_{fi}/\partial\alpha_{fi}) \end{bmatrix}$$
$$A_{0i} = \begin{bmatrix} D_{oi} & 0_{3\times2} \\ 0_{3\times4} & -R_o(\partial^{o}C_{oi}/\partial\alpha_{oi}) \\ E_{oi} & R_o(\partial^{o}e_{oi}/\partial\alpha_{oi}) \end{bmatrix}$$

Here $D_{fi} := [I_{3\times3} \quad -(R_{fi}{}^{fi}C_{fi}\times)]$, $E_{fi} := [0_{3\times3} \quad -(R_{fi}{}^{fi}e_{fi}\times)]$, $D_{oi} := [-I_{3\times3} \quad R_o{}^{o}C_{oi}\times]$, $E_{oi} := [0_{3\times3} \quad -(R_o{}^{o}e_{oi}\times)]$. J_{fi} is the Jacobian matrix mapping of the i^{th} finger motion from the joint to the fingertip.

When the i^{th} finger in pure rolling contact with the object, there will be two holonomic constraints in (9) and (11) and one non-holonomic constraint in (10). Due to the surface mapping function ${}^{fi}C_{fi}(\cdot)$ and ${}^{o}C_{oi}(\cdot)$, the contact positions in $\boldsymbol{R}^3$ can be expressed by α_{fi} and α_{oi} in $\boldsymbol{R}^2$, which means (9) will limit two degrees of freedom with three equations. Similarly, the unit normal vector in $\boldsymbol{R}^3$ can be calculated with two parameters, and (12) will only reduce two degrees of freedom of the kinematics. Hence, there will be two degrees of freedom for the relative motion when the i^{th} finger is in pure rolling contact with the object.

B. The Contact Forces Model of the dexterous robotic hand in the Work Space

Considering the robotic hand and the target object as a unified system, the dynamic model of the system obtained by the approach to multi-body system modeling introduced [8]. According to this hierarchical and decentralized approach, the manipulation system should be partitioned into four subsystems, comprising three finger subsystems and one object subsystem. Combining these four "unconstrained" subsystems together yields

$$\mathcal{M}(q, t)\ddot{q} = B\tau_f(t) + F(q, \dot{q}, t) \quad (18.)$$

where $\mathcal{M} = \begin{bmatrix} \mathcal{M}_{f1} & & & \\ & \mathcal{M}_{f2} & & \\ & & \mathcal{M}_{f3} & \\ & & & \mathcal{M}_o \end{bmatrix}$ ($\mathcal{M}_{fi} \in \boldsymbol{R}^{6\times6}$ $(i = 1, 2, 3)$ is the singular inertial matrix of the i^{th} finger subsystem, and $\mathcal{M}_o \in \boldsymbol{R}^{12\times12}$ is the singular mass matrix of the target object), $B = \begin{bmatrix} I_{18\times18} \\ 0_{12\times18} \end{bmatrix}$, $\tau_f = \begin{bmatrix} \tau_{f1}^T \\ \tau_{f2}^T \\ \tau_{f3}^T \end{bmatrix}$ ($\tau_{fi} = [\tau_{i1}, \tau_{i2}, \tau_{i3}, \tau_{i4}, 0, 0]^T \in \boldsymbol{R}^6$ $(i = 1, 2, 3)$ is the control torque applied by the joint motors), $\ddot{q} = \begin{bmatrix} \ddot{q}_{f1}^T \\ \ddot{q}_{f2}^T \\ \ddot{q}_{f3}^T \\ \ddot{q}_o^T \end{bmatrix}$, and $F = \begin{bmatrix} F_{f1}^T \\ F_{f2}^T \\ F_{f3}^T \\ F_0^T \end{bmatrix}$ ($F_{fi}^T \in \boldsymbol{R}^6$ $(i = 1, 2, 3)$ and $F_o \in \boldsymbol{R}^{12}$ contains the centrifugal/Coriolis force and the gravity force of the i^{th} finger subsystem and the target object, respectively). Details of the derivation can be found in our previous work [1].

Based on the pure rolling contact constraints (17) between the i^{th} fingertip and the object, the second derivation of these constraints can be given by

$$A_{fi}\ddot{q}_{fi} + A_{oi}\ddot{q}_{oi} = -\dot{A}_{fi}\dot{q}_{fi} + \dot{A}_{oi}\dot{q}_{oi} =: b_{fi} \quad (19.)$$

Hence, the pure rolling contact constraints that the dexterous robotic hand manipulation system subject can be presented as

$$A_k\ddot{q} = b_k \quad (20.)$$

where $A_k = \begin{bmatrix} A_{f1} & \cdots & 0_{9\times6} & A_{bo1} \\ \vdots & A_{f2} & \vdots & A_{bo2} \\ 0_{9\times6} & \cdots & A_{f3} & A_{bo3} \end{bmatrix}$ and $b_k = \begin{bmatrix} b_{f1} \\ b_{f2} \\ b_{f3} \end{bmatrix}$.

Here, $A_{bo1} = \begin{bmatrix} D_{o1} & 0_{3\times2} & 0_{3\times4} \\ 0_{3\times6} & -R_o(\partial^o C_{o1}/\partial\alpha_{o1}) & 0_{3\times4} \\ E_{o1} & R_o(\partial^o e_{o1}/\partial\alpha_{o1}) & 0_{3\times4} \end{bmatrix}$

$$A_{bo2} = \begin{bmatrix} D_{o2} & 0_{3\times2} & 0_{3\times2} & 0_{3\times2} \\ 0_{3\times6} & 0_{3\times2} & -R_o(\partial^o C_{o2}/\partial\alpha_{o2}) & 0_{3\times2} \\ E_{o2} & 0_{3\times2} & R_o(\partial^o e_{o2}/\partial\alpha_{o2}) & 0_{3\times2} \end{bmatrix}$$

$$A_{bo3} = \begin{bmatrix} D_{o3} & 0_{3\times4} & 0_{3\times2} \\ 0_{3\times6} & 0_{3\times4} & -R_o(\partial^o C_{o3}/\partial\alpha_{o3}) \\ E_{o3} & 0_{3\times4} & R_o(\partial^o e_{o3}/\partial\alpha_{o3}) \end{bmatrix}$$

Since the mass matrix $\mathcal{M}$ of the system is singular, the motion equation (21) and the constrained forces (22) due to the contact constraints can be established by the extended Udwadia-Kalaba equation.

The motion equation of the multi-fingered hand robot manipulation system can be written as

$$\ddot{q} = \begin{bmatrix} (I - A_k^+A_k)\mathcal{M} \\ A_k \end{bmatrix} \begin{bmatrix} (I - A_k^+A_k)(F + B\tau_f) \\ b_k \end{bmatrix} =: \widehat{\mathcal{M}} \begin{bmatrix} (I - A_k^+A_k)(F + B\tau_f) \\ b_k \end{bmatrix} \quad (21.)$$

The constrained force of the multi-fingered hand robot manipulation system

$$F^c(q, \dot{q}, t) = \mathcal{M}[\mathcal{M}(I - A_k^+A_k)\mathcal{M} + A_k^TA_k]^{-1}[\mathcal{M}(I - A_k^+A_k)(F + B\tau_f) + A_k^Tb_k] - (F + B\tau_f) \quad (22.)$$

This constraint forces introduced in (22) is the pure rolling contact forces in the joint space, which is used to control the target object in the work space.

IV. THE DYNAMIC MANIPULATION CONTROL DESIGN OF THE DEXTEROUS ROBOTIC HAND

To manipulate the target object tracking the desired trajectory, the dynamic control designed based on the contact forces needs to provide the appropriate finger joint driving torques. Before the control design, the motion equation of the dexterous robotic hand manipulation system (21) can be further decoupled as

$$\ddot{q} = [\mathcal{M}(I - A_k^+A_k)\mathcal{M} + A_k^TA_k]^{-1}[\mathcal{M}(I - A_k^+A_k)F + A_k^Tb_k] + [\mathcal{M}(I - A_k^+A_k)\mathcal{M} + A_k^TA_k]^{-1}[\mathcal{M}(I - A_k^+A_k)B\tau_f] \quad (23.)$$

Suppose the desired trajectory of the object to be $q_o^d = [x_o^d, y_o^d, z_o^d] \in \boldsymbol{R}^3$, the desired velocity and acculation to be $\dot{q}_o^d$ and $\ddot{q}_o^d$. Then, the error between the actual position of the object and the desired trajectory q_o^d can be presented as

$$e(q_o, \dot{q}_o, t) = q_o - q_o^d \quad (24.)$$

To achieve the desired velocity while tracking the required trajectory, the servo constraints can be formalized as

$$\dot{e}(q_o, \dot{q}_o, t) + Ke(q_o, t) = 0 \quad (25.)$$

where the constant matrix $K = diag[k_i]_{3\times3}$, $k_i > 0$, $k_i = \lambda_{min}(K)(i = 1,2,3)$. Here, $\lambda_{min}(\cdot)$ denotes the minimum eigenvalue of the designated matrix.

Differentiating the servo constraints (25) with respect to t yields

$$\ddot{e}(q_o, \dot{q}_o, t) + K\dot{e}(q_o, \dot{q}_o, t) = 0 \quad (26.)$$

Equation (26) can be represented in the form of matrix as

$$A_s\ddot{q} = b_s(q_o, \dot{q}_o, t) \quad (27.)$$

Where $A_s = [0_{3\times18}, I_{3\times3}, 0_{3\times9}]$ and $b_s = \ddot{q}_o^d - K(\dot{q}_o - \dot{q}_o^d)$.

Assumption 1. Equation (27) is consistent: for any $A_s(q, t)$ and $b_s(q, \dot{q}, t)$, there exists at least one solution $\ddot{q}$ to (27).

According to Assumption 1, there will be

$$A_sA_s^+b_s = b_s \quad (28.)$$

Considering the motion equation of the dexterous robotic hand manipulation system (23) and the servo constraints (27), there can be

$$A_s\Phi_{\tau_f} = \bar{b} \quad (29.)$$

Where $\Phi = [\mathcal{M}(I - A_k^+A_k)\mathcal{M} + A_k^TA_k]^{-1}\mathcal{M}(I - A_k^+A_k)B$ and $\bar{b} = b_s - A_s[\mathcal{M}(I - A_k^+A_k)\mathcal{M} + A_k^TA_k]^{-1}[\mathcal{M}(I - A_k^+A_k)F + A_k^Tb_k]$. (29) can be consider as a constraint for the control torques τ_f.

Assumption 2. Equation (29) is constant: for any $A_s(q, t)$, $\Phi(q, t)$ and $\bar{b}(q, \dot{q}, t)$, there exists at least one solution τ_f to (29).

According to assumption 2, there will be

$$A_s\Phi(A_s\Phi)^+\bar{b} = \bar{b} \quad (30.)$$

Theorem 1. Subject to Assumptions 1-2, the motion equation of the mechanical system (1) is servo constraints controllable for the constraints (27) if and only if

$$Rank[A_s(q, t)\Phi(q, \dot{q}, t)] \geq 1 \quad (31.)$$

for any $(q, \dot{q}, t) \in \boldsymbol{R}^{30} \times \boldsymbol{R}^{30} \times \boldsymbol{R}$. Furthermore, for all $(q, \dot{q}, t) \in \boldsymbol{R}^{30} \times \boldsymbol{R}^{30} \times \boldsymbol{R}$, the constraints-following control τ_f can be presented as

$$\tau_f = (A_s\Phi)^+\bar{b} + [I - (A_s\Phi)^+A_s\Phi]S \quad (32.)$$

where $S \in \boldsymbol{R}^{30}$ is an arbitrary vector.

Proof of Theorem 1:

(Sufficiency) Suppose $Rank[A_s(q, t)\Phi(q, \dot{q}, t)] \geq 1$, which means $[A_s(q, t)\Phi(q, \dot{q}, t)]^+$ exists according to [9] and control (32) is meaningful. Using the constraint-following control τ_f in system (23) and remultiplying both sides of (23) by A_s result in

$$A_s\ddot{q} = A_s[\mathcal{M}(I - A_k^+A_k)\mathcal{M} + A_k^TA_k]^{-1}[\mathcal{M}(I - A_k^+A_k)F + A_s\Phi\tau_f$$

$$= b_s - \bar{b} + A_s\Phi\left[(A_s\Phi)^+\bar{b} + [I - (A_s\Phi)^+A_s\Phi]S\right]$$
$$= b_s - \underbrace{\bar{b} + A_s\Phi(A_s\Phi)^+\bar{b}}_{=0} + \underbrace{A_s\Phi[I - (A_s\Phi)^+A_s\Phi]S}_{=0}$$
$$= b_s \tag{33.}$$

(Necessity) Suppose that $Rank[A_s(q,t)\Phi(q,\dot{q},t)] = 0$ for some $(q,\dot{q},t)$, which means $A_s(q,t)\Phi(q,\dot{q},t) = 0$ for some $(q,\dot{q},t)$, remultiplying both sides of (23) by A_s, there will be

$$A_s\ddot{q} = A_s[\mathcal{M}(I - A_k^+A_k)\mathcal{M} + A_k^TA_k]^{-1}[\mathcal{M}(I - A_k^+A_k)F + A_k^Tb_k] + A_s\Phi\tau_f \tag{34.}$$

since $A_s\Phi = 0$ for some $(q,\dot{q},t)$, (34) can be simplified as

$$A_s\ddot{q} = A_s[\mathcal{M}(I - A_k^+A_k)\mathcal{M} + A_k^TA_k]^{-1}[\mathcal{M}(I - A_k^+A_k)F + A_k^Tb_k]$$
$$= b_s - \bar{b} \tag{35.}$$

it can be seen that (35) is not equal to the servo constraints (27).

Hence, $Rank[A_s(q,t)\Phi(q,\dot{q},t)] \geq 1$ should satisfied for all$(q,\dot{q},t) \in \boldsymbol{R}^{30} \times \boldsymbol{R}^{30} \times \boldsymbol{R}$.

Since the number of the finger joint driving torques of the dexterous robotic hand manipulation system is less than the number of the chosen coordinates, which means the system is a typical underdrive system. Based on the constraint-following control method, paper [10] introduced a control designing method for such an underdrive system with a non-singular inertia matrix. However, the inertia matrix M of the hand manipulation system under pure rolling contact constraints is singular, which means the inverse of M cannot be solved. Compared with the controller proposed in paper [10], the constraint-following control (10) can accomplish the manipulation tasks even when the inertia matrix of the system does not have a full rank.

Assumption 3. For constant matrix $P \in \boldsymbol{R}^{3\times3}, P > 0$, there exists a constant $\lambda > 0$ such that

$$\lambda_{min}\left(PA_s(q,t)\Phi(q,\dot{q},t)\left(A_s(q,t)\Phi(q,\dot{q},t)\right)^TP^T\right) \geq \lambda \tag{36.}$$

According to assumption 3, the minimum eigenvalue of the matrix $PA_s(q,t)\Phi(q,\dot{q},t)\left(A_s(q,t)\Phi(q,\dot{q},t)\right)^TP^T$ needs to be larger than 0, which implies that the matrix $PA_s(q,t)\Phi(q,\dot{q},t)\left(A_s(q,t)\Phi(q,\dot{q},t)\right)^TP^T$ will be symmetrically positively definite for any $(q,\dot{q},t) \in \boldsymbol{R}^{30} \times \boldsymbol{R}^{30} \times \boldsymbol{R}$.

Set the servo constraints error of the dexterous robotic hand manipulation system as $v(t) \coloneqq \dot{e}(q_o,\dot{q}_o,t) + Ke(q_o,t)$. The dynamics manipulation control of the system which is suffered by the pure rolling contact constraints is proposed as

$$\tau_f(t) = u_1(q,\dot{q},t) + u_2(q,\dot{q},t) \tag{37.}$$

with

$$u_1(q,\dot{q},t) = [A_s(q,t)\Phi(q,\dot{q},t)]^+\bar{b}(q,\dot{q},t) \tag{38.}$$

$$u_2(q,\dot{q},t) = -\gamma\left(A_s(q,t)\Phi(q,\dot{q},t)\right)^TP^Tv(t) \tag{39.}$$

where the constant $\gamma > 0$.

Theorem 2. Consider the dexterous robotic hand manipulation system (23), which is under pure rolling contact constraints, subjects to Assumptions 1-3. Under the constraint-following control (37), the following performance of the target object trajectory tracking error $v(t) \coloneqq \dot{e}(q,\dot{q},t) + Ke(q,\dot{q},t)$ is guaranteed: (t_o is the initial time)

(1) Uniformly stability: for any $\hat{k} > 0$ ($\hat{k}$ is a constant) and $\|v(t_0)\| < \hat{k}$, there will be $\|v(t)\| < \hat{k}$ as $t > t_0$.

(2) Converge to zero: for any $t > t_0$, $\lim_{t\to\infty} v(t) = 0$.

Proof of Theorem 2:

Choose the Lyapunov function as

$$V(v) = v^TPv \tag{40.}$$

where the constant matrix $P > 0$. Then, there will be

$$V \geq \|v\|^2 \tag{41.}$$

Since $V(0,t) = 0$ and $V(v,t) > 0$ ($v \neq 0$), the function V in (40) is said to be decrescent.

Taking the derivative of (37) yields

$$\dot{V} = 2v^TP\dot{v} = 2v^TP(A_s\ddot{q} - b_s) \tag{42.}$$

With the proposed control design (37), $(A_s\ddot{q} - b_s)$ in (42) can be rewritten according to assumption 2 as

$$A_s\ddot{q} - b_s = -\gamma A_s\Phi(A_s\Phi)^TP^Tv \tag{43.}$$

substitute (43) in (32) to get

$$\dot{V} = -2\gamma v^TPA_s\Phi(A_s\Phi)^TP^Tv \tag{44.}$$

According to assumption 3, (41) can be rewritten as

$$\dot{V} \leq -2\gamma\lambda v^Tv = -2\gamma\lambda\|v\|^2 \tag{45.}$$

since $\gamma > 0$, $\lambda > 0$, $\dot{V}(v,t) = 0$ only when $v = 0$, and $\dot{V}(v,t) \leq 0$ with $\forall v \neq 0$, the constraint-following error $v(t)$ is uniformly stable.

Suppose there exist a constant $\delta > 0$ for $\|v(t_1) - v(t_2)\| < \delta$, the following equation based on (45) will be satisfied

$$\begin{aligned}\dot{V}_1 - \dot{V}_2 &\leq 2\gamma\lambda(\|v(t_2)\|^2 - \|v(t_1)\|^2)\\ &= 2\gamma\lambda(\|v(t_2) - v(t_1)\|\|v(t_2) + v(t_1)\|)\\ &< 2\gamma\lambda\delta\|v(t_2) + v(t_1)\|\end{aligned} \tag{46.}$$

According to the definition of uniformly stable: $\|v(t)\| < \hat{k}$ for any $t > t_0$, (46) can be rewritten as

$$\dot{V}_1 - \dot{V}_2 < 2\gamma\lambda\delta\|v(t_2) + v(t_1)\| < 4\gamma\lambda\delta\hat{k} \tag{47.}$$

Hence, the function $\dot{V}(v,t)$ will be uniformly continuous at time t. Besides, since $V(v,t) \geq 0$ is lower unbounded and $\dot{V}(v,t) \leq 0$ is negative semi-definite, there will be $\dot{V}(v,t) = -2\gamma\lambda\|v\|^2 \to 0$ as $t \to \infty$ according to the Lyapunov-Like lemma discussed in [11], which means $\lim_{t\to\infty} v(t) = 0$.

The design dynamic control (37) of the dexterous robotic hand manipulation system includes two portions: The feedforward portion (38), which is formulated by the contact forces model, and the feedback portion (39) that can eliminate the initial constraint-following error. With the aforementioned control strategy, the hand robot can achieve high-precision tracking of the trajectory with the target object without requiring the contact force feedback information measured by force sensors.

V. Simulation and Discussion

TABLE I. The Properties of the Dexterous Robotic Hand Manipulation System

Symbol	The properties of the dexterous robotic hand manipulation system		
	Means	*Data*	*Unit*
m_1	the mass of the proximal phalanx	0.06	kg
m_2	the mass of the middle phalanx	0.033	kg
m_3	the mass of the distal phalanx	0.018	kg
m_o	the mass of the target object	0.053	kg

Symbol	The properties of the dexterous robotic hand manipulation system		
	Means	*Data*	*Unit*
r_1	the radius of the fingertip	10	mm
r_2	the radius of the target object	30	mm
l	The length of the phalanx	40	mm
g	gravitatianal acceleration	9800	mm/s^2

TABLE II. THE SIMULATION PARAMETERS OF THE DEXTEROUS ROBOTIC HAND MANIPULATION SYSTEM

Symbol	The simulation parameters of the dexterous robotic hand manipulation system	
	Data	*Unit*
q_{f1}	[−1.513689; 1.439038; 0.346388; −2.039030; 0.066831; 1.792217]	rad
q_{f2}	[2.5408120; 1.621548; −0.158905; −1.695082; −0.0347450; 1.705898]	rad
q_{f3}	[0.601601; 1.919066; −0.806517; −1.204788; 0.082720; 1.647304]	rad
q_o	[1.632884; 4.672300; 69.422381; −0.041982; 1.535519; −0.032036; −0.020233; 1.645094; 1.034046; −1.659032; −1.018308; −1.618006]	mm or rad
$\dot{q}_{f1}$	[0.089534; −1.089919; 2.439512; −1.386070; 0.052346; 0.059651]	rad/s
$\dot{q}_{f2}$	[−0.226253; −0.567053; 0.950041; −0.908325; −0.185272; 0.368132]	rad/s
$\dot{q}_{f3}$	[0.106493; 0.247942; −0.737890; 0.691595; 0.017312; 0.104417]	rad/s
$\dot{q}_o$	[4.082141; 10.429187; −0.777873; −0.043177; −0.011827; 0.019302; −0.016664; 0.020479; −0.071678; −0.229078; 0.017312; 0.104417]	mm/s or rad/s

To validate the dynamic modeling and the control design of the dexterous robotic hand manipulation system, a series of experiments are simulated by the MATLAB software. The system's properties and simulation parameters are presented in Tables I and II, respectively. Set the desired trajectory as $q_o^d = [-4, 8,\ 40\sqrt{3}]^T$, choose the control parameters as $\gamma = 10$ and $K = diag(8, 8, 8)$, the simulation results of the dexterous robotic hand manipulation system are demonstrated in Fig.3-Fig.7.

With Assumptions 1-2, (27) and (29) needed to be consistent to ensure the solutions of those equations exist, which means there will be, $A_s{A_s}^+b_s = b_s$ and $A_s\Phi(A_s\Phi)^+\bar{b} = \bar{b}$. According to the calculation result, the magnitude of $\|A_s{A_s}^+b_s - b_s\|$ and $\|A_s\Phi(A_s\Phi)^+\bar{b} - \bar{b}\|$ are to be 10^{-5}, which means Assumptions 1-2 met. Besides, Assumption 3 needs the minimum eigenvalue of the matrix $PA_s\Phi A_s\Phi^T P^T$ is always large than 0, which is satisfied in the simulation according to Fig.3. Hence, with the design control (34) the multi-fingered hand robot manipulation system under pure rolling contact constraints is always subjects to those Assumptions that needed for the dynamics modeling and control design.

Set $e \coloneqq [e_x, e_y, e_z]^T$, Fig.4 depicts the trajectory tracking error, which refers to the error between the actual and desired positions of the object's mass center. It can be seen that the tracking error e decreases closely to 0 after 3.5s. The control torques τ_{f1}, τ_{f2}, and τ_{f3} acting on the finger joints are shown in Fig.5 - Fig.7, where $\tau_{fi} = [\tau_{fi1}, \tau_{fi2}, \tau_{fi3}, \tau_{fi4}] (i = 1, 2, 3)$. The control torques of each finger are bounded and less than $8\ N \cdot m$, which suggests its compatibility with the current actuator.

Considering the simulation results mentioned above, with the proposed control design (34), the dexterous robotic hand manipulation system in pure rolling contact constraints, can achieve high-precision manipulation of the target object by accurately tracking the desired trajectory.

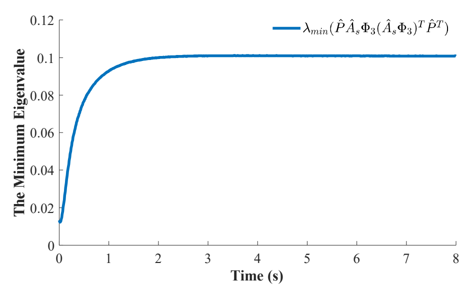

Figure 3. The Minimum Eigenvalue of the Matrix $PA_s\Phi A_s\Phi^T P^T$

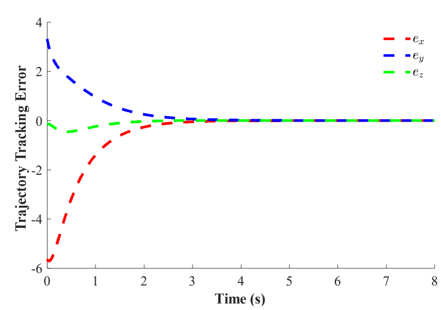

Figure 4. The Trajectory Tracking Error

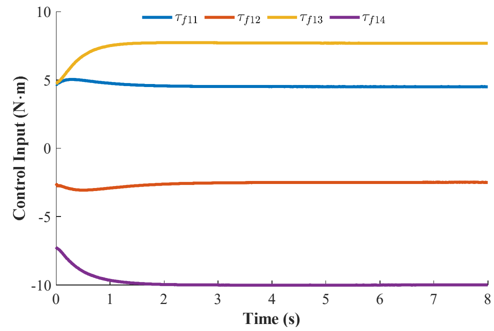

Figure 5. The Control Input of Finger 1

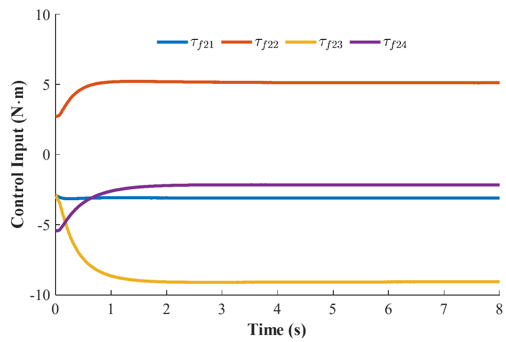

Figure 6. The Control Input of Finger 2

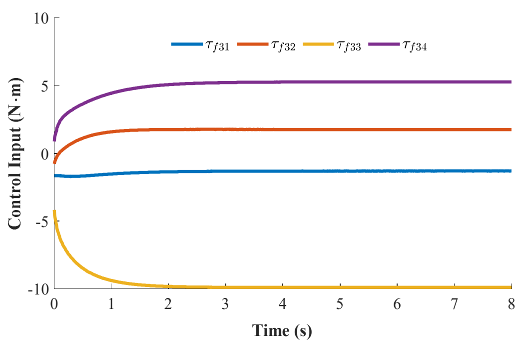

Figure 7. The Control Input of Finger 3

VI. Conclusion

A novel control method is proposed for a dexterous robotic hand in pure rolling contact with an object. The entire system, including the dexterous robotic hand and the object, is considered for analysis.

The kinematic constraint model under pure rolling contact is derived at first. The kinematic analysis suggests that two degrees of freedom for the relative motion when the i^{th} finger is in pure rolling contact with the object. Using the kinematic constraint model, the dynamic model of the hand-object manipulation system is constructed by the extended Udwadia-Kalaba equation without any auxiliary variables. This modeling approach effectively addressed the problem of singularity in the system's inertia matrix arising from pure rolling contact constraints. Based on the dynamic model, the paper derives the fingertip contact forces model in the joint space using only the generalized coordinates of the system. By treating the manipulation task as a constraint-following problem, the contact forces model-based manipulation control strategy is formulated, which includes two portions. The feedforward portion is formulated by the contact forces model, and the feedback portion that can eliminate the initial constraint-following error.

The proposed control strategy 's effectiveness is substantiated through theoretical analysis. Moreover, the simulation results demonstrate that the robotic hand can precisely manipulate the target object by accurately tracking the intended trajectory under the proposed control.

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