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RBF Neural Networks Model-Free Feedforward Control of Piezoelectric Actuator

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***Abstract*—Piezoelectric actuator is widely utilized in ultra-precision position fields. However, it is challenged by difficulty modeling dynamic properties, such as hysteresis, creep, vibration, etc. Therefore, this paper presents a model-free feedforward control strategy of Radial Basis Functions Neural Network (RBFNN) for piezoelectric actuator. RBFNN is used to estimate the inverse dynamic model of the piezoelectric actuator, and avoiding the difficult modeling and impossible to invert problems. This feedforward controller has model-free advantage that it does not depend on the specific physics-based model and is only data-driven by references and control signals. The parameters of RBFNN are training by offline methods and online learning strategy especially. The offline learning method can compensate the trained trajectory with high precision, but for the untrained trajectory, the compensation accuracy is insufficient. For unknown target trajectories, the online learning strategy is proposed, which utilizes current data for parameter iteration and updating to improve control accuracy. Finally, simulation verify that the proposed model-free feedforward control strategy is effective. Especially the online learning strategy has the adaptive ability and generalization ability for unknown trajectory targets.**

***Index Terms*—difficulty modeling, radial basis function neural network, feedforward compensation, online learning**

I. Introduction

The requirement for precise positioning of ultra-precision manufacturing equipment has led to extensive research and development in precision engineering. Piezoelectric actuator has been extensively used over the past few years as a source of precision positioning in a wide range of commercial applications. The main application areas include precision manufacturing, medical technology, robotics, aerospace, and consumer electronics. In particular, they are most widely used in components such as micropumps/microreactors/micromixers [1] [2], micromanipulators [3] [4], microvalves [5], microjet dispensers [6] [7], atomic force microscopes [8] [9], tool feeding mechanisms [10] [11], vibration isolation systems [12], etc. In addition, piezoelectric actuator has the advantages of fast response speed, high stiffness/load capacity, low energy consumption and cleanroom compatibility, which is very suitable for application in the field of chip manufacturing [13]-[15]. Although there are many advantages of piezoelectric actuator, its difficult modeling problem leads to model inaccuracy and low control accuracy, which limits the application and development of piezoelectric actuator. For the model-based control method, a model is built based on the mechanism of the piezoelectric actuator and then its inverse is solved to design the controller.

In recent years, many physics-based and phenomenological models have been built to solve difficult modeling problems such as hysteresis, creep, and vibration, these models mainly include Jiles-Atherton, Domain Wall, Duhem, Dahl, Bouc - Wen, Backlash, Maxwell - Slip, Preisach, Prandtl - Ishlinskii, Viscoelastic creep model, etc. Common physics-based models include the Jiles-Atherton model and the Domain Wall model. These models are usually very complex and limited to a single object. The physical characteristics of the piezoelectric actuator are highly complex, which presents a significant challenge in modelling [16]. Phenomenological models are also the most widely used model at present, including Duhem model, Dahl model, Bouc-Wen model [17]- [19], etc. Lin used a generalized Duhem hysteresis model to model the hysteresis characteristics of the biaxial piezoelectric positioning platform, and designed a controller to compensate the hysteresis for platform tracking control [20]. Ahmad employed the Dahl hysteresis model to characterise the hysteretic nonlinear behaviour of the piezoelectric actuator and designed a Dahl feed-forward compensator that did not involve inverse model calculation in order to circumvent the potential complexity of such a calculation. [21]. The hysteresis nonlinear model based on the model is capable of describing the hysteresis dynamic characteristics of a piezoelectric actuator. However, the structure is complex,

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the parameters are challenging to determine, and the model's accuracy is not optimal. Although the integral hysteresis model improves the accuracy of the model, it has no theoretical inverse model and can only approximate inverse model, which is not conducive to controller design. For phenomenological models, model errors due to the unmodeled portion, and errors in the identification process can combine to affect the control accuracy. In addtiton, the problem of non-theoretical inverse modules and difficult inverse modules is also very tricky. If only these models are used for control, there will be a large comprehensive error [22].

Thus, model-free control approach is specially appropriate for this case, which does not require accurate modeling of the piezoelectric actuator. Only the input and output data of the piezoelectric actuator are used for training and the resulting controller is able to compensate accurately. This approach is a good solution to the problem of difficult modeling and no theoretical inverse modeling. For example, Al-Mahasneh proposed a reinforcement learning algorithm based on network supervised control, which compensates nonlinear errors through reinforcement learning feedforward control structure, and solves the problem of difficult modeling of typical nonlinear time-varying systems [23]. Li presents a finite-time sliding-mode controller (SMC) based on the disturbance observer (DOB) and a radial basis function neural network (RBFNN). This control method avoids the establishment of complex models and can meet the requirements of high frequency control [24]. Yang proposed a combined control of feedforward and feedback In the feedforward loop, a multilayer feedforward neural network is used to directly obtain the inverse of the model and improve the control bandwidth [25]. Takemura et al. proposed an iterative learning feedforward control that does not require a mathematical model. Compared with the traditional based model, this method can more easily fit the nonlinear characteristics, avoid the problems of inaccurate and difficult modeling [26].

However, the above method only trains and compensates the hysteresis nonlinearity of piezoelectric actuator, and do not realize the static and dynamic compensation of the whole piezoelectric actuator. When the target trajectory changes, the compensation ability of the model-free controller decreases, and the whole control system is poor in generalization and adaptability. Therefore, this paper proposes the RBFNN model-free feedforward controller for piezoelectric actuator, which takes four trajectory-related parameters as training inputs and control quantities as training outputs, and the RBFNN parameters include offline training and online training methods. The compensation effect of the proposed model-free control is simulated under trigonometric function trajectory tracking targets with different frequencies. For the unknown target trajectory, an online learning strategy is proposed to utilize the current data for parameter iteration and updating to improve the control accuracy. Finally, the effectiveness of the proposed model-free feedforward control method is verified by simulation, especially the online learning strategy has the adaptive ability and generalization ability for unknown trajectory targets.

The main contribution of this paper is that the RBFNN is model-free control. It overcomes the problem of difficult modeling/no theoretical inverse modeling due to hysteresis nonlinearity, improves the adaptability and generalization ability of the control system to unknown trajectories. It is suitable for complex, nonlinear, time-varying, or unknown systems, and improves the overall descriptive ability of the system.

Part A of section II illustrates the main structure of the RBFNN model-free feedforward controller. Part B of section II derives the iterative equation for offline parameter optimization. Part C of section II is based on the RBFNN model-free feedforward controller proposed in part A, and the controller parameters are updated by acquiring the current piezoelectric actuator data in real time in an online learning manner. Section III simulates of the proposed RBFNN controller's is carried out by targeting the trigonometric function trajectories with different frequencies and the adaptability in the case of unknown trajectories is verified.

II. RBFNN Model-free feedforward controller

A. RBFNN model-free feedforward controller structure

The RBFNN is used to design a model-free feedforward controller for the piezoelectric actuator neural network. It has three layers, including input layer, hidden layer and output layer. The input layer is the data related to the reference trajectory, and the expression of the input layer is (1).

$$X = \left[\begin{array}{c} r(k), v(k), a(k), jerk(k) \end{array} \right]^T \tag{1}$$

where *k* is the Current time step, *r*, *v*, *a*, *jerk* are the reference trajectory, velocity, acceleration, and jerk. Too many hidden layer nodes can lead to long learning time, resulting in redundant resources, while too few hidden nodes can lead to poor generalization. Therefore, it is necessary to select the best number of neuron nodes in the hidden layer. Equation (2) is an empirical equation for the range of nodes in the hidden layer of the neural network.

$$j = \sqrt{n_{in} + n_{out}} + a \quad j = (1, 2, 3, ..., 15) \tag{2}$$

where *j* is the number of selected hidden layer neuron nodes, n_{in} is the number of neurons in the input layer of the network, n_{out} is the number of neurons in the output layer of the network, and a is a constant between 1-15. Therefore, the selection range of the number of hidden layer nodes of the neural network should be 7-17, and the specific number should be determined by comparison with the minimum sum of squares due to error(SSE) as the selection criterion during neural network training. By summarizing relevant studies, *j* is about 15, the mean square error is the smallest, so 15 nodes are first given priority to train the network to analyze the training effect. The kernel function of the hidden layer is the Gaussian radial basis function as (3).

$$h_j = \exp(-\frac{\|X - C_j\|^2}{2b_j^2}) \tag{3}$$

where C_j is the center vector of node j and b_j is the base width parameter of node j, as shown in (3), (4).

$$C_j = \left[\ c_{1j}, c_{2j}, c_{3j}, c_{4j}\ \right]^{\mathrm{T}} \tag{4}$$

$$b_j = [b_1, b_2, b_3, ..., b_{15}]^{\mathrm{T}} \tag{5}$$

The network weights is (6).

$$w_j = [w_1, w_2, w_3, ..., w_{15}]^{\mathrm{T}} \tag{6}$$

The control signal is a single output voltage signal, so the output layer node is 1, the function h_j and weights ω of each node in the kernel hidden layer are linearly weighted, the neural network is required to give the control signal U_n of the piezoelectric actuator , n is the current sample. The feedforward control signal is calculated as (7).

$$U_n\,(n) = h_1\omega_1 + h_2\omega_2 + h_3\omega_3 + \cdots + h_{15}\omega_{15} \tag{7}$$

Therefore, the RBFNN model-free feedforward controller structure is shown in Fig. 1. The boundary condition of the basis width parameter b_j can be obtained as [0,2], and the boundary condition of the voltage control signal u_n of the piezoelectric actuator is [0,10] volts.

B. Offline Parameter optimization

$U(n)$ is the sum of control signals, including feedforward and feedback. The effectiveness of the training of a neural network is judged by the SSE of one sample.The internal parameters C_j, b_j, ω are all optimized in a gradient descent manner, and the loss/performance function is (8).

$$E_o(n) = \frac{1}{2}\sum_{k=1}^{n}(U(k) - U_n(k))^2 \tag{8}$$

Therefore, the gradient expressions and iterative expressions for the center vector, base width parameter, and weights are obtained as in (9) to (14).

$$\Delta\omega_{jo}(n) = \sum_{k=1}^{n}\frac{\partial E_o(n)}{\partial \omega_{jo}(n)} = -\sum_{k=1}^{n}[U\,(k) - U_n\,(k)]h_{jo} \tag{9}$$

$$\Delta c_{jio}(n) = \sum_{k=1}^{n}[U\,(k) - U_n\,(k)]w_{jo}\frac{x_{io} - c_{jio}}{b_{jo}^2} \tag{10}$$

$$\Delta b_j o(n) = \sum_{k=1}^{n}[U\,(k) - U_n\,(k)]w_{jo}h_{jo}\frac{\|X - C_{jo}\|^2}{b_{jo}^3} \tag{11}$$

$$c_{jio}(n+1) = c_{jio}(n) - \eta_1 \times \Delta c_{jio}(n) \tag{12}$$

$$b_{jo}(n+1) = b_{jo}(n) - \eta_2 \times \Delta b_{jo}(n) \tag{13}$$

$$\omega_{jo}(n+1) = \omega_{jo}(n) - \eta_3 \times \Delta\omega_{jo}(n) \tag{14}$$

The offline optimization algorithm of RBFNN model-free feedforward controller is Table. 1.

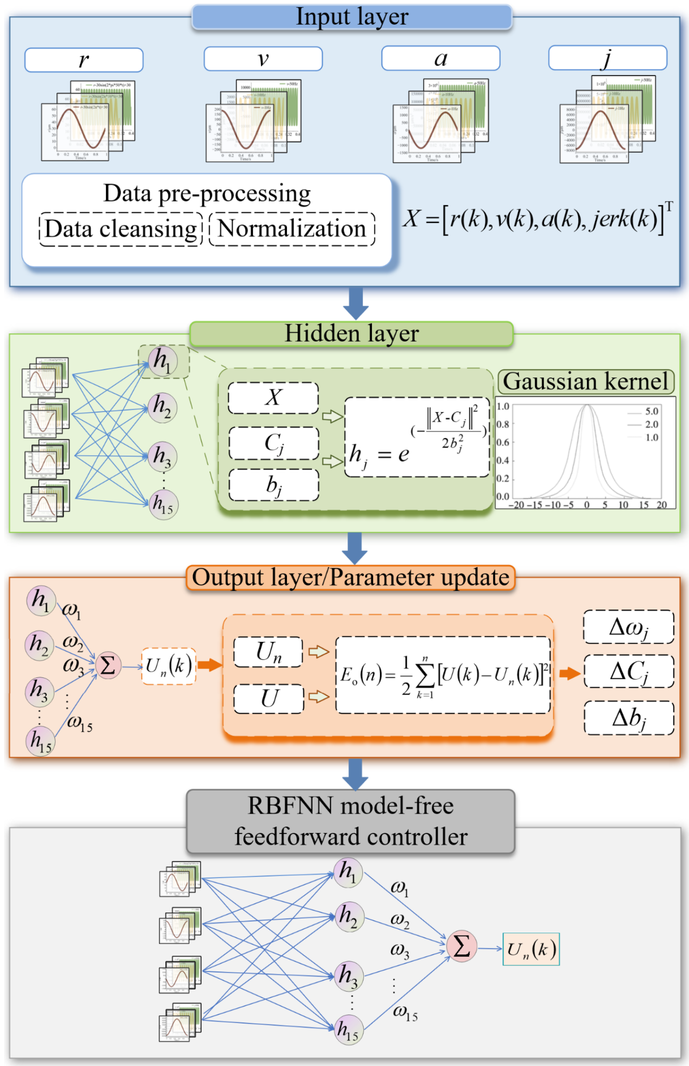


Fig. 1. The framework of RBFNN model-free feedforward controller

TABLE I
THE OFFLINE OPTIMIZATION ALGORITHM OF RBFNN MODEL-FREE FEEDFORWARD CONTROLLER

Algorithm 1: Offline RBFNN model-free feedforward controller Designing

1. *Conduct different frequencies trigonometric trajectory tracking closed-loop tracking experiments, clean and normalize data*;
2.*Initialize C, b, ω by random assignment*;
2. *Get X, U_n, U, b_j, C_j, ω_j for the current sample n*;
3. *Calculate the Gaussian kernel function h by (3) and the feedforward control signal $U_n(n)$ by (7)*;
4. *Calculate the feedforward controller cost function $E_o(n)$ by (8)*;
5. *If $E_o(n)$ does not converge to the target value, calculate the $\Delta c_{jio}, \Delta b_j o, \Delta\omega_j o$ by (9)-(11), and update the parameters by (12)-(14)*;
6. *Repeat step 2-5 until the parameters iterate to the threshold.*

C. Online learning Parameter optimization

The internal parameters of the RBFNN model-free feedforward controller established in part A are obtained by offline learning and are not updated and corrected in real-time operation. Although it solves the problem of difficult modeling or no theoretical inverse model, the RBFNN model-free feedforward controller will regard the trajectory not involved in the training as an unknown target, and it cannot guarantee that it still has good control effect and stability.

To solve the above problems, on the basis of the RBFNN model-free feedforward control, the parameter update mode is improved to the online learning mode, and the RBFNN model-free feedforward control strategy is formed. When the new trajectory was used as the tracking target and the tracking error increased abnormally, RBFNN responded to the data changes in real time, learned and corrected the control quantity. Therefore, updating the parameters online can dynamically adapt to the trajectory changes and maintain good control accuracy. The online learning flow of the RBFNN model-free feedforward controller is Fig. 2.

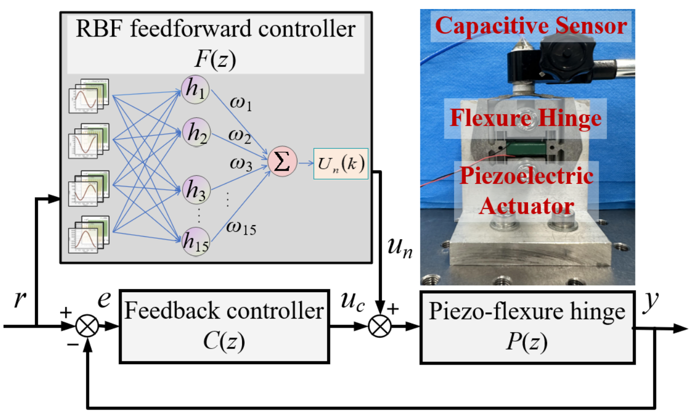


Fig. 2. The online learning flow of the RBFNN model-free controller

The main content of online learning RBFNN model-free feedforward control strategy is as follows: when the piezoelectric actuator runs for the first time, the RBFNN model-free feedforward controller learns the internal relationship between the reference trajectory related parameters and the feedback control quantity through the closed-loop control system. Starting from the second run of the piezoelectric actuator, the total control amount of feedforward + feedback is taken as the learning objective. The RBFNN model-free controller learns the latest total control quantity through multiple times and updates the parameters online. By updating and modifying the parameters, the RBFNN model-free feedforward controller gradually plays the main control role, while the feedback controller is mainly used to improve the stability of the system and further compensate the accuracy.

Offline parameter optimization can prevent RBFNN from overfitting by cross-validation(CV), but online parameter optimization does not have samples consistent with training to prevent overfitting. Here, the regularization is used to optimize the parameters online to avoid overfitting. A regularization term is added to the online parameter optimization objective function $E_l(n)$ as (16).

$$E_l(n) = E_o(n) + \frac{\lambda}{2}\sum_{i,j}[w_l(n)^2 + c_l(n)^2 + b_l(n)^2] \tag{15}$$

Therefore, the gradient expression for online parameter optimization is (16)-(18).

$$\Delta c_{jil}(n) = \sum_{k=1}^{n}[U(k) - U_n(k)]w_{jl}\frac{x_{il} - c_{jil}}{b_{jl}^2} + \lambda c_{jil} \tag{16}$$

$$\Delta b_j l(n) = \sum_{k=1}^{n}[U(k) - U_n(k)]w_{jl}h_{jl}\frac{\|X - C_{jl}\|^2}{b_{jl}^3} + \lambda b_{jl} \tag{17}$$

$$\Delta\omega_{jl}(n) = -\sum_{k=1}^{n}[U(k) - U_n(k)]h_{jl} + \lambda\omega_{jl} \tag{18}$$

When there are unknown changes in the trajectory tracking target and the tracking error increases, it means that the RBFNN model-free feedforward cannot compensate the target well, and the online parameter optimization recursive (19)-(21) is used to reduce the tracking error.

$$c_{jil}(n+1) = c_{jil}(n) + \eta_1 \times \Delta c_{jil}(n) \tag{19}$$

$$b_{jl}(n+1) = b_{jl}(n) + \eta_2 \times \Delta b_{jl}(n) \tag{20}$$

$$\omega_{jl}(n+1) = \omega_{jl}(n) + \eta_3 \times \Delta\omega_{jl}(n) \tag{21}$$

The online optimization algorithm of RBFNN model-free feedforward controller is Table. 2.

TABLE II
THE ONLINE OPTIMIZATION ALGORITHM OF RBFNN MODEL-FREE FEEDFORWARD CONTROLLER

Algorithm 2: Online RBFNN model-free feedforward controller Designing
1. *Set the RBFNN feedforward control to the initial value (0),and initialize C, b, ω by random assignment*; 2. *Conduct trajectory tracking experiment to obtain reference trajectories and control signal as training samples*; 3. *Get X, U_n, U, b_j, C_j, ω_j for the current sample*; 4. *Calculate the Gaussian kernel function h by (3) and the feedforward control signal $U_n(n)$ by (7)*; 5. *Calculate the feedforward controller cost function $E_l(n)$ by (15)*; 6. *If $E_l(n)$ does not converge to the target value, calculate the $\Delta c_{jil}, \Delta b_{jl}, \Delta\omega_{jl}$ by (16)-(18), and update the parameters by (19)-(21)*; 7. *Repeat step 2-6 until the parameters iterate to the threshold.*

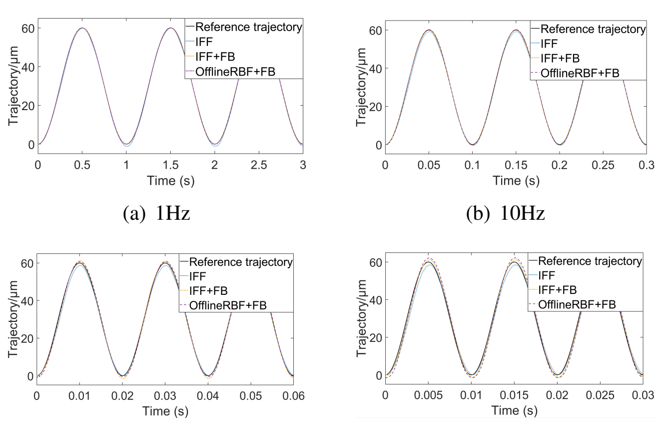
III. SIMULATION

A. simulation setup

Other researchers in this subject have established the mathematical model of the piezoelectric table with a flexure hinge and completed the model identification experiment, which lays the foundation for this simulation [27]. The simulation model is a piezoelectric table with a flexure hinge, witch includes a piezoelectric actuator, a flexhinge amplification mechanism(The displacement amplification ratio of the flexure hinge mechanism is 6.1), a drive amplifier with a magnification of 15 times, a capacitive displacement sensor with a sensitivity of 20 $\mu\mathrm{m/V}$ and a resolution of 2.5 nm. In order to verify the effectiveness of the proposed RBFNN model-free feedforward controller, trigonometric signals with different frequencies are used as the tracking target. Two groups of comparative experiments were designed:

Simulation 1: Different controllers are used to track the trigonometric reference trajectories at different frequencies. The controller includes open-loop feedforward control based on inverse Hammerstein model(IFF), feedforward feedback composite control based on inverse MPI lag model(IFF+FB) [27], and RBFNN model-free feedforward feedback composite controller (OfflineRBFNN+FB). The curves and error curves for tracking trigonometric reference trajectories at 1Hz, 10Hz, and 100Hz using different controllers are shown in Fig. 3 to Fig. 7.

Simlation 2: The dynamic learning effect of the online learning RBFNN model-free feedforward controller is tested with a 100 Hz high-frequency trajectory tracking signal as an example. The iterative trend of the tracking error with the number of runs using the online parameter optimization method is shown in Fig. 8. When the tracking target changes from a 100Hz trigonometric function curve to the fourth-order function curve (not train, which is an unknown trajectory for RBFNN), the iterative process through the online parameter optimization method is shown in Fig. 9.



(a) 1Hz (b) 10Hz

(c) 50Hz (d) 100Hz

Fig. 3. Simulation results of trajectory tracking curve

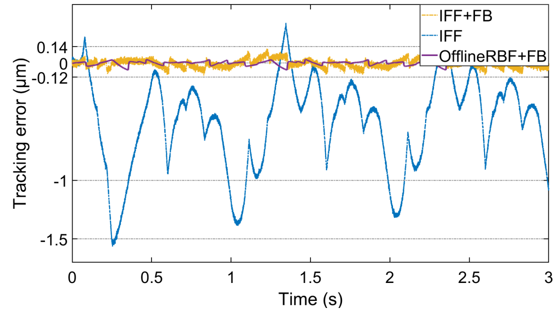


Fig. 4. Simulation results of 1Hz trajectory tracking error

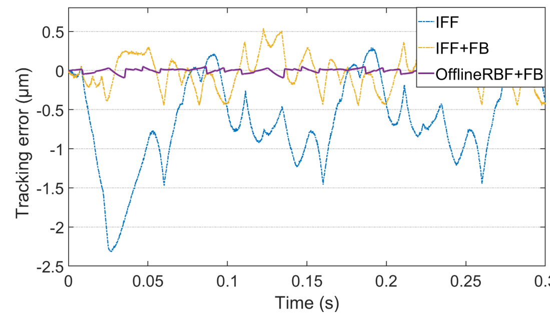


Fig. 5. Simulation results of 10Hz trajectory tracking error

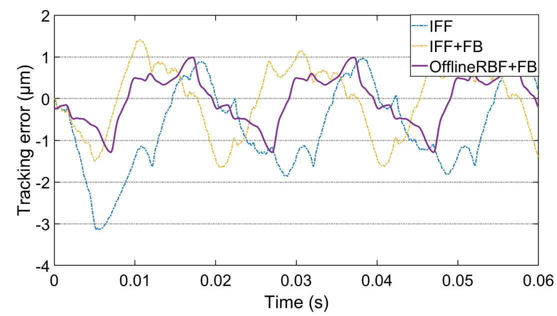


Fig. 6. Simulation results of 50Hz trajectory tracking error

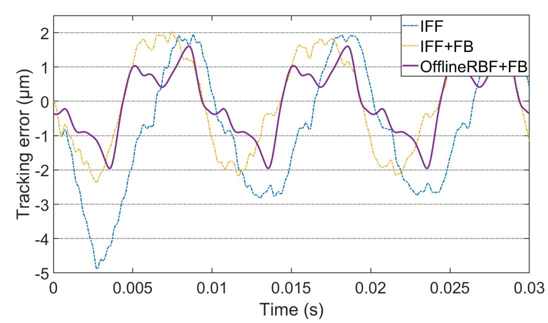


Fig. 7. Simulation results of 100Hz trajectory tracking error

B. simulation results and discussion

Discussion 1: The maximum error and root mean square error of the three control modes are Table. 3. It can be verified that the RBFNN model-free feedforward controller has higher accuracy than the model-based controller at different tracking

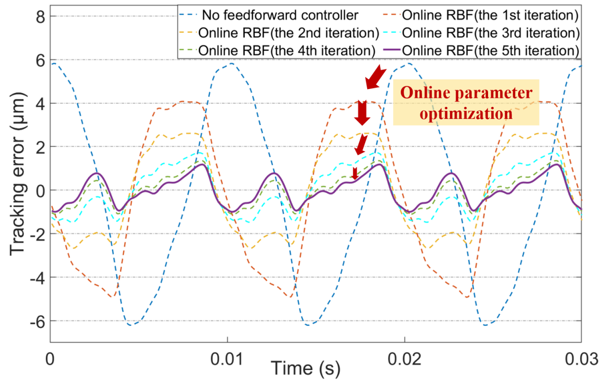


Fig. 8. Error iteration procedure using Online RBF+FB at 100Hz

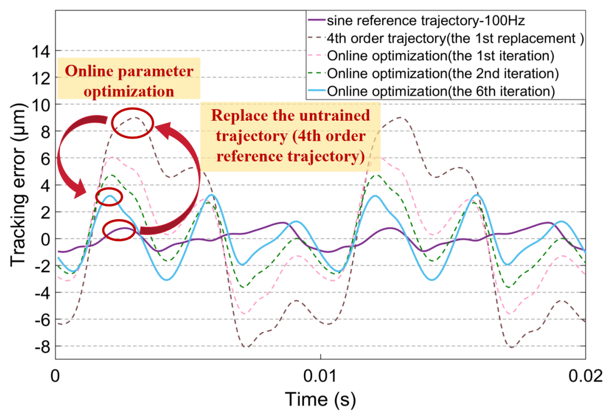


Fig. 9. Iterative procedure for Online parameter optimization under unknown training trajectories(the fourth-order function curve)

frequencies, especially in terms of modeling difficulties and model uncertainties caused by hysteresis nonlinearity, the model-free feedforward controller has a great advantage.

TABLE III
COMPARISON OF THE MAXIMUM ERROR AND RMS OF THE ERROR OF THE THREE CONTROL MODES

Trajectory	Controller	e_{max} (μm)	e_{rms} (μm)
	IFF	1.5648	0.5973
1Hz	IFF+FB	0.1396	0.0361
	OfflineRBFNN+FB	0.0605	0.0163
	IFF	2.3229	0.7118
10Hz	IFF+FB	0.5410	0.2377
	OfflineRBFNN+FB	0.0931	0.0268
	IFF	3.1484	1.0548
50Hz	IFF+FB	1.6544	0.8004
	OfflineRBFNN+FB	1.2890	0.5899
	IFF	4.8841	1.7496
100Hz	IFF+FB	2.3757	1.3476
	OfflineRBFNN+FB	1.9490	0.9610

Discussion 2: From Fig. 8 and Fig. 9, it can be verified that compared with the offline parameter optimization method, the online parameter optimization can effectively reduce the tracking error caused by the unknown trajectory, and has the ability of online adjustment and learning.

IV. Conclusion

Aiming at the difficult modeling problem of Piezoelectric actuator due to hysteresis nonlinearity, creep and vibration, this paper proposes the RBFNN model-free feed-forward controller with data-driven, neural network, and model-free as the characteristics. The whole paper derives and details the controller in terms of its structure, parameter optimization, online learning, and then simulates the effect of the RBFNN model-free feedforward controller. The main contribution of this paper is that RBFNN is model-free control, which overcomes the difficult modeling/no theory inverse modeling problem. It is suitable for complex, nonlinear, time-varying, or unknown systems, and improves the overall descriptive capability of the system.

However, there are still some limitations in this study, such as the inconvenient stability analysis of the neural network controller and the lack of physical interpretation, which hinder the application and development in engineering.

Future work will be guided by this strategy and further verify the effect of the strategy on the piezoelectric actuator stage using the simulation results as a reference; in addition, a disturbance observer can be introduced for further compensation and correction.

References

[1] N. Naik and P. Suresh, "A review on composite materials for energy harvesting in electric vehicles," *Energies*, vol. 16, no. 8, pp. 334801-334819, 2023.

[2] S. Mohith, P. Karanth and S. Kulkarni, "Experimental investigation on performance of disposable micropump with retrofit piezo stack actuator for biomedical application," *Microsystem Technologies*, vol. 25, pp. 4741–4752, 2019.

[3] A. El-Sayed, A. Ahmed, T. Moumen, N. Hamzaid and A. Azuan, "Development of a micro-gripper using piezoelectric bimorphs," *Sensors*, vol. 13, no. 5, pp. 5826-5840, 2013.

[4] T. Abondance, K. Jayaram, N. Jafferis, J. Shum and R. Wood, "Piezoelectric grippers for mobile micromanipulation," *IEEE Robotics and Automation Letters*, vol. 5, no. 3, pp. 4407-4414, 2020.

[5] J. Jeon, C. CHan, Y. Han and S. Choi, "A new type of a direct-drive valve system driven by a piezostack actuator and sliding spool." *Smart Materials and Structures*, vol. 23, pp. 07500201-07500213, 2014.

[6] S. Woo and S. Choi, "Identification of operating parameters most strongly influencing the jetting performance in a piezoelectric actuator-driven dispenser," *Applied Sciences*, vol. 8, no. 2, pp. 24301-24315, 2018.

[7] G. Deng, W. Cui, C. Zhou and J. Li, A piezoelectric jetting dispenser with a pin joint, Optik, vol. 175, pp. 163-171, 2018.

[8] K. Cai, Y. Tian, F. Wang, D. Zhang and B. Shirinzadeh, "Development of a piezo-driven 3-DOF stage with T-shape flexible hinge mechanism," *Robotics and Computer-Integrated Manufacturing*, vol. 37, pp. 125–138, 2016.

[9] B. Eslami and S. Solares, "Experimental approach for selecting the excitation frequency for maximum compositional contrast in viscous environments for piezo-driven bimodal atomic force microscopy," *Journal of Applied Physics*, vol. 119, pp. 0849011-0849017, 2016.

[10] Muralidhara, J. Nilesh and M. Singaperumal, "Investigations on a directly coupled piezoactuated tool feed system for micro-electro-discharge machine," *International Journal of Machine Tools and Manufacture*, Vol. 49, pp. 1197-1203, 2009.

[11] X. Tian, B. Zhang, Y. Liu, S. Chen and H. Yu, “A novel U-shaped stepping linear piezoelectric actuator with two driving feet and low motion coupling: Design, modeling and experiments,” *Mechanical Systems and Signal Processing*, vol. 124, pp. 679-695, 2019.

[12] Z. Lu, D. Shao, Z. Fang, D. He and L. Chen, “Integrated vibration isolation and energy harvesting via a bistable piezo-composite plate,” *Journal of Vibration and Control*, vol. 26, no. 9-10, pp. 779-789, 2020.

[13] Muralidhara and R. Rao, “Displacement characteristics of a piezo actuator-based prototype microactuator with a hydraulic displacement amplification system,” *Journal of Mechanical Science and Technology*, vol. 29, pp. 4817–4822, 2015.

[14] S. Mohith, P. Karanth and S. Kulkarni, “Performance analysis of valveless micropump with disposable chamber actuated through Amplified Piezo Actuator (APA) for biomedical application,” *Mechatronics*, vol. 67, pp. 10234701-10234715, 2020.

[15] M. Ling, J. Cao, M. Zeng, J. Lin and D. Inman, “Enhanced mathematical modeling of the displacement amplification ratio for piezoelectric compliant mechanisms,” *Smart Materials and Structures*, vol. 25, pp. 75022–75032, 2016.

[16] M. Kanchan, M. Santhya, R. Bhat and N. Naik, “Application of modeling and control approaches of piezoelectric actuators: a review,” *Technologies*, vol. 11, no. 6, pp. 15501-15552, 2023.

[17] Y. Liu, R. Zhong, D. Li and Z. Li, “A fractional-order Duhem model of rate-dependent hysteresis for piezoelectric actuators,” *Measurement and Control*, vol. 55, no. 9-10, pp. 974-982, 2022.

[18] N. Sidra, M. Raja, A. Mehmood and A. Jaafery, “Intelligent predictive solution dynamics for Dahl hysteresis model of piezoelectric actuator,” *micromachines*, vol. 13, no. 12, pp. 220501-220522, 2022.

[19] J. Zhao, Y. Li, Y. Cao, F. Zhang, M. Cui and R. Xu, “High-precision position tracking control with a hysteresis observer based on the Bouc–Wen model for smart material-actuated systems,” *Actuators*, vol. 13, no. 3, pp. 105, 2024.

[20] C. Lin and P. Lin, “Tracking control of a biaxial piezo-actuated positioning stage using generalized Duhem model,” *Computers & Mathematics with Applications*, vol. 64, no. 5, pp. 766-787, 2012.

[21] I. Ahmad, M li and W. Ko, “Robust μ-synthesis with Dahl model based feedforward compensator design for piezo-actuated micropositioning stage,” *IEEE Access*, vol. 8, pp. 141799-141813, 2020.

[22] B. Hashwan, M. Khir and Nawi, “A review of piezoelectric MEMS sensors and actuators for gas detection application,” *Nanoscale Research Letters*, vol. 18, no. 1, 2023.

[23] A.Mahasneh, A. Jobran, Anavatti, S. Garratt and A. Matthew, “Online model-free reinforcement learning for output feedback tracking control of a class of discrete-time systems With input saturation,” *IEEE ACCESS*, vol. 10, pp. 104966-104979, 2022.

[24] Q. Li, W. Chen, L. Tian, and Y. Xie, “A finite-time sliding-mode controller based on the disturbance observer and neural network for hysteretic systems with application in piezoelectric actuators,” *Sensors*, vol. 23, no. 14, pp. 624601-624613, 2023.

[25] Q. Yan, Y. Zhang, H. Duan, and J. Han, “High-bandwidth hysteresis compensation of piezoelectric actuators via multilayer feedforward neural network based inverse hysteresis modeling,” *Micromachines*, vol. 12, no. 11 , pp. 132501-132513, 2021.

[26] T. Takemura and H. Fujimoto, “Proposal of novel rolling friction compensation with data-based friction model for ball screw driven stage,” *IEEE Industrial Electronics Society*, pp. 1932-1937, 2010.

[27] T. Huang, Y. Wang, Z. Luo, H. Cao, G.Tao and M. Ling, “Feedback linearization and equivalent-disturbance compensation control strategy for piezoelectric stage,” *Nanotechnology and Precision Engineering*, vol. 2024-7, no. 2, pp. 0230061-02300611, 2024.